Discussion of the Range and Endurance Performance Graphs in the Transport Canada Flight Training Manual (FTM)

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1 Drag and Thrust Required

The Flight Training Manual shows two diagrams depicting the resulting drag (or thrust required) and power required to maintain non-accelerated straight-and-level flight at various airspeeds. To maintain non-accelerated straight-and-level flight, the two opposing forces thrust and drag as well as lift and weight have to be in equilibrium, respectively.

Since, thrust and drag have to be equal in non-accelerated straight-and-level flight, the first diagram can be interpreted as both drag and thrust required depending on airspeed. Such a diagram is usually drawn using the True Air Speed (TAS) as the relevant airspeed and, hence, the resulting graph would change with changing air density. However, it could also be drawn using the Calibrated Air Speed (CAS) as the relevant airspeed giving a graph that is independent of the prevailing air density.

As can be seen from the graph, the dominant factor at lower airspeeds is lift-induced drag which decreases with increasing airspeed. The dominant factor at higher airspeeds is parasite drag which decreases with decreasing airspeed. The total drag is the sum of both forms of drag, that is, the sum of both drag curves, and is equal to the thrust required to maintain non-accelerated straight-and-level flight.

The lowest point of drag which is identified by the horizontal tangent to the total drag curve consequently corresponds to the lowest thrust required.

2 Thrust versus Power Rated Engines

For thrust-rated engines the thrust required is directly proportional to the fuel consumption of the engine. This applies to pure jet and rocket engines where the amount of air that passes through the engine is directly proportional to the amount of thrust created and fuel consumed by the engine.

Hence, for thrust producing jet engines, the lowest point of the drag or thrust required curve corresponds to the lowest fuel consumption and, therefore, maximum endurance of the jet aircraft.

Furthermore, by optimizing for both factors, fuel consumption and airspeed, the maximum range of a jet aircraft can be found using the drag or thrust required curve. By drawing a tangent from the origin of the diagram to the graph, the minimum *thrust to airspeed ratio* is found. This corresponds to the following equation (opposite leg divided by adjacent leg):

$$\boxed{\tan\phi = T_{req}/V} \tag{1}$$

where T_{req} is thrust required, and V is airspeed.

By minimizing the function $\tan\phi$ and, hence, applying the smallest angle ϕ, the tangent contacts the drag or thrust required curve at the point where the best trade-off to maximize airspeed while minimizing thrust required can be found.

Power-rated engines are those where the thrust (propulsion) created is not directly proportional to the fuel consumption of the engine. For a piston engine, the fuel consumption at a certain mixture ratio still depends on the mass of air that is drawn into the engine during a time interval, but the thrust developed by the propulsion system furthermore depends on the propeller and the mass of air that is drawn through it. Note that the mass of air in both cases relates to both its volume and density. Hence, the mass of air changes with changing density.

The propeller may be a fixed or constant speed propeller. If it is a fixed propeller, its efficiency might be optimized for acceleration at high RPM such as for training aircraft. It might also be optimized for cruise flight.

Consequently, the fuel flow of a power-rated engine depends on the engine RPM and its intake mass airflow (manifold pressure). The manifold pressure changes with mass of air that is moved by the propeller and relates to the airspeed of the aircraft taking into account propeller efficiency and slip. Hence, a power required curve has to be used to derive performance properties as opposed to a thrust required curve.

Note, that several engine types can be categorized as both thrust-rated and power-rated. Turbo-prop, turbo-fan (depending on their bypass ratio), and ramjet engines are usually grouped according to their dominating properties.

3 Power Required

The power required curve is created by multiplying the thrust required curve by the airspeed of the aircraft. The following derivation shows the relation between thrust required and power required:

$$\boxed{P = \frac{W}{t} = \frac{F * s}{t} = T * V} \tag{2}$$

where P is mechanical power, W is mechanical work, F is force with T being the thrust force, and V is airspeed.

The lowest point on the power required curve corresponds to the minimum fuel flow of the propeller-driven aircraft with a power-rated engine and, hence, gives the power required and airspeed for maximum endurance of the aircraft. It is found by drawing the horizontal tangent to the lowest point of the curve and minimizing power only.

Furthermore, by optimizing for both factors power and airspeed, the maximum range of a propreller-driven aircraft with a power-rated engine can be found. A tangent can be drawn from the origin of the diagram in order to minimize the *power to airspeed ratio*. This corresponds to the following equation (opposite leg divided by adjacent leg):

$$\boxed{\tan\psi = \frac{P_{req}}{V} = \frac{T_{req} * V}{V} = T_{req} = D} \tag{3}$$

where P_{req} is power required, T_{req} is thrust required, V is airspeed, and D is drag.

By minimizing the function $\tan\psi$ and, hence, applying the smallest angle ψ, the tangent contacts the power required curve at the point where the best trade-off to maximize airspeed while minimizing power required can be found. As can be seen from the equation, this point also corresponds to the the minimum drag. **Note that the minimum drag for a propeller driven aircraft with a power-rated engine is found at an airspeed which is higher than the airspeed to achieve best economy and, hence, maximum endurance.**

To determine the difference in *lift to drag ratio* between maximum endurance (minimum power) and maximum range (minimum drag) of an aircraft with a power-rated engine, the following derivation can be examined.

In non-accelerated straight-and-level flight thrust equals drag and lift equals weight:

$$\boxed{T = D, L = W \rightarrow W = L = D\frac{L}{D} = T\frac{L}{D} = T\frac{c_L}{c_D} \rightarrow T = L\frac{c_D}{c_L}} \tag{4}$$

Hence, for a propeller-driven aircraft with a power-rated engine in non-accelerated straight-and-level flight the power required is:

$$\boxed{P = T * V = L\frac{c_D}{c_L} * V} \tag{5}$$

Furthermore, the lift and airspeed required for non-accelerated straight-and-level flight are:

$$\boxed{L = c_L\frac{1}{2}\rho S v^2, v = \sqrt{\frac{L}{c_L\frac{1}{2}\rho S}}} \tag{6}$$

Substituting the airspeed in the power required equation gives:

$$\boxed{P = \sqrt{\frac{L^3}{\frac{1}{2}\rho S}\frac{c_D}{c_L^{\frac{3}{2}}}}} \tag{7}$$

Hence, the minimum power required and maximum endurance for propeller-driven aircraft with power-rated engines are achieved by maximizing the following *lift to drag ratio*:

$$\boxed{\frac{c_L^{\frac{3}{2}}}{c_D}} \tag{8}$$

Note that this ratio is different from the maximum lift to drag ratio required for aircraft with thrust-rated engines:

$$\boxed{\frac{c_L}{c_D}} \tag{9}$$

4 Slow Flight Regime

The slow flight regime or *regime of reverse control* applies to the airspeed range in which a higher thrust or power is required for thrust-rated or power-rated engines, respectively, in order to fly lower airspeeds in non-accelerated straight-and-level flight.

For power-rated engines the regime of reverse control applies to airspeeds slower than the airspeed for minimum power (maximum endurance). As discussed before, this airspeed is smaller than the airspeed for minimum drag or thrust required. For power-rated engines the *throttle* or *manifold pressure lever* has to be advanced to fly slower airspeeds in this regime.

For thrust-rated engines the regime of reverse control applies to airspeeds slower than the airspeed for minimum drag or thrust required (maximum endurance). For thrust-rated engines the *thrust lever* has to be advanced to fly slower airspeeds in this regime.