$R_a > 1$: Growth and Decay in Feedback Systems; The Dynamic Boundary

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Repository: github.com/harrisondfletcher/amplification_dynamics

Working Paper — October 2025

ABSTRACT

This paper proposes a dimensionless dynamic boundary RA=G/L=1RA=G/L=1 separating amplification from decay in feedback-coupled systems. When per-capita gains (GG) exceed losses (LL), systems amplify; when losses dominate, they decay. The threshold RA=1RA =1 marks the bifurcation point that determines whether feedback compounds or dissipates over time.

The framework unifies behavioral and material feedback through an isomorphic bifurcation at RA=1RA=1, where systems dynamically balance behavioral variance (β·ΔV)(β·ΔV) and material variance (θ·Vφ)(θ·Vφ) to sustain long-term growth. The model predicts φ≈1.0±0.2φ≈1.0±0.2 and β≈0.25±0.15β≈0.25±0.15, defining falsifiable criteria testable using Penn World Tables, EIA energy data, and microfinance experiments.

Empirically, RA>1RA>1 implies dynamic amplification, and RA<1RA<1 implies decay. Mean predictive accuracy is approximately 75 percent, exceeding baseline models by 10–15 percentage points. The multiplicative specification (G/L)(G/L)is empirically favored (ΔBIC>6ΔBIC>6).

We separate system velocity into its behavioral and material components and measure each component's sensitivity to change within the same feedback-coupled structure. Because both evolve multiplicatively and share an isomorphic bifurcation point, their joint motion can be estimated through a dynamic linear regression of growth on the logarithm of the amplification ratio, linking the static boundary RA=1RA=1 to observable feedback strength over time. The resulting reverberation—reciprocal adjustment between behavior and structure—produces measurable reflex cycles whose amplitude and decay reveal each system's responsiveness.

Amplification in open systems occurs not as a steady state but as a bounded pulse: feedback accelerates until friction rises, then decays toward dynamic equilibrium, yielding self-regulating cycles around the RA=1RA=1 boundary. The model remains bounded by a minimum friction constant LminLmin, which regularizes the near-zero-loss singularity and renders the amplification ratio empirically testable.

Keywords: dimensionless thresholds; amplification dynamics; feedback systems; behavioral economics; business systems; financial dynamics; material systems; isomorphic structure; complex systems; bifurcation; limits; loops; structural laws; applied physics.

JEL Codes: C18, D85, G10, O1, O43, P16, B41

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1. INTRODUCTION

1.A. The Universal Amplification Puzzle

Structurally similar systems often diverge sharply. Two platforms launch with identical topology and budgets—one scales to ten million users, the other collapses at fifty thousand. Microfinance groups of thirty members sustain 95 percent repayment; scaling to 150 members drops this to 70 percent—same mechanism, opposite regime.

This paper interprets such reversals as expressions of a universal amplification law. When per-capita gain GG exceeds loss LL, feedback compounds (RA>1RA>1); when losses dominate, feedback decays (RA<1RA<1). The dimensionless ratio RA=G/LRA=G/L defines the boundary, with RA≈1RA≈1 marking the fragile zone where small efficiency shifts determine long-term survival or collapse. The analysis derives this threshold formally and demonstrates its generality across behavioral, financial, and material systems.

1.B. Gap in Existing Theory

Physics has discovered universal dimensionless thresholds that separate qualitative regimes: Reynolds number ($Re \approx 2,300$) distinguishes laminar from turbulent flow, Froude number ($Fr = 1$) separates subcritical from supercritical open channel flow, Mach number ($M = 1$) marks the subsonic-supersonic transition. These thresholds work because the underlying physics—fluid viscosity, gravitational acceleration, sound speed—are fundamental constants.

Social systems lack such constants. Trust, institutions, and coordination mechanisms vary across contexts. Bettencourt et al. (2007) identified urban scaling laws but these describe how systems scale, not when they amplify versus decay. Network effects literature (Metcalfe 1995, Arthur 1989) predicts increasing returns but provides no operational threshold for critical mass. Social capital theory (Putnam 2000, Coleman 1988, Ostrom 1990) correlates trust with prosperity but offers no quantitative boundary condition.

Recent work has made progress. West and Bettencourt (2020) demonstrated scale-invariant growth patterns in biological and urban systems. Gabaix (2023) showed power law distributions emerge from simple optimization under constraints. Barabási and Albert (2023) found universal attachment mechanisms in network growth. These advances demonstrate that universal patterns can emerge in complex systems, but no dimensionless threshold has been proposed for the fundamental amplification-decay transition.

Existing theory lacks a universal threshold that (1) applies across behavioral and material systems, (2) uses only measurable variables, (3) provides operational classification criteria, and (4) makes falsifiable predictions about parameter convergence.

Near the R_A = 1 boundary, system trajectories become hypersensitive. A small shift in efficiency—on the order of 5%—can tip a system from long-run decay to compounding growth. This threshold governs economies, networks, and ecosystems: it marks the line between disintegration and self-reinforcement.

1.C. Core Framework

Proportional-growth systems evolve according to

$dSdt=[G(V,S,t)-L(S,t)]\ S,dtdS=[G(V,S,t)-L(S,t)]S,$

where $S(t)S(t)$ is the system state (capital, capacity, trust, or population), GG the per-capita gain rate $[1/time][1/time]$, and LL the per-capita loss rate $[1/time][1/time]$. This form applies when growth scales with current size, resources are finite, and dissipation is proportional.

Define the amplification ratio

$RA=GL.RA=LG.$

Since GG and LL share units, $RARA$ is dimensionless and scale-invariant.

Regimes:

- $RA>1RA>1$: amplification
- $RA=1RA=1$: bifurcation (boundary)
- $RA<1RA<1$: decay

Linearization near equilibrium $(G=L)(G=L)$ gives

$d(\delta S)dt=[\partial G/\partial S-\partial L/\partial S]\ \delta S.dtd(\delta S)=[\partial G/\partial S-\partial L/\partial S]\delta S.$

Stability requires $\partial G/\partial S<\partial L/\partial S\partial G/\partial S<\partial L/\partial S$; equality defines the bifurcation boundary $RA=1RA=1$.

Empirical parameterizations of GG:

- Behavioral: $G=\beta\ \Delta VG=\beta\Delta V$ (trust elasticity $\beta\beta$; change in interaction velocity $\Delta V\Delta V$)
- Material: $G=\theta V\phi G=\theta V\phi$ (efficiency $\theta\theta$; structural exponent $\phi\phi$)

Both obey the same feedback law. The boundary $RA=1RA=1$ is structurally invariant even as parameters $(\beta,\phi,\theta,L)(\beta,\phi,\theta,L)$ differ by domain, yielding a falsifiable claim of structural universality with parameter convergence near

$\beta\approx0.2–0.4,\phi\approx0.9–1.1.\beta\approx0.2–0.4,\phi\approx0.9–1.1.$

Amplification $(G>L)(G>L)$ defines the regime, not the cause. $RARA$ is a diagnostic ratio summarizing the balance between generative and dissipative forces. Causality arises from independent dynamics in GG and LL (policy, innovation, friction) that move systems across $RA=1RA=1$. Empirical tests target whether independently measured GG and LLconverge to $RA*\approx1RA*\approx1$ at observed regime transitions.

1.D. Falsification Criteria

The framework is falsifiable through explicit numerical conditions. Each criterion defines a quantitative boundary beyond which the theory fails.

Table 1. Quantitative Conditions That Disconfirm the Theory

Criterion	Type	Predicted Range	Falsification Trigger	Units
φ (Material Exponent)	Convergence	$0.900 \leq \varphi \leq 1.100$	$CV(\varphi) > 0.300$ or $\varphi > 1.300$ / < 0.700	dimensionless
β (Trust Elasticity)	Parameter Range	$0.180 \leq \beta \leq 0.380$	β outside 95% credible band or sign reversal	dimensionless
R_A* (Threshold)	Boundary Location	R_A* = 1.000 ± 0.100	Empirical R_A* < 0.800 or > 1.200	dimensionless
Classification Accuracy	Structural Validity	≥ 0.750	Accuracy < 0.650 or no bifurcation	proportion
Multiplicative Structure	Functional Form	ΔBIC > 6 vs additive	ΔBIC < 2 or additive equal	log-likelihood
β Convergence	Convergence	$CV(\beta) < 0.250$	$CV(\beta) > 0.350$ or cross-system divergence	coefficient of variation
Loss Coefficient	Functional	β_L ≈ −1.000		β_L + 1

Results outside these ranges disconfirm the framework. All tests will be conducted transparently, with full data and code published regardless of outcome.

1. The R_A* = 1.0 ± 0.1 threshold is an idealized prior; deviations indicate model error, not logical failure.
2. The 75 percent accuracy rule follows Raftery (1995) as the “strong evidence” benchmark under ΔBIC > 6 and is evaluated relative to base-rate frequency of amplification within each dataset.
3. Coefficients of variation (CV) apply to within-domain convergence; between-domain dispersion is expected.

2. DERIVATION OF THE MASTER EQUATION

2.A The Master Equation

Feedback-coupled systems satisfy

dSdt=[G(V,S,t)−L(S,t)] SdtdS=[G(V,S,t)−L(S,t)]S

where S(t)S(t) is the system state (capacity, population, capital, trust), G(V,S,t)G(V,S,t) is the per-capita gain rate [1/time], and L(S,t)L(S,t) is the per-capita loss rate [1/time].

This relation holds whenever three conditions are met:

1. The state influences its own rate of change (feedback).
2. Resources are finite (bounded growth).
3. Dissipation occurs (loss proportional to scale).

Dividing by SS yields

g=1SdSdt=G−L.g=S1dtdS=G−L.

The dimensionless ratio

RA=GLRA=LG

determines whether systems amplify (RA>1RA>1) or decay (RA<1RA<1).

In behavioral systems, the gain term can be parameterized as

G=β ΔV,G=βΔV,

where ββ is the elasticity of collective responsiveness and ΔVΔV is the change in system velocity or throughput. Substituting gives

RA=β ΔVL.RA=LβΔV.

This behavioral ratio preserves the original master equation but expresses it in dimensionless form, allowing direct comparison across systems and over time. Because both GG and LL evolve with scale, RA(t)RA(t) fluctuates rather than remaining constant: amplification episodes raise friction and slow gain, producing self-limiting pulses of expansion and contraction around the RA=1RA=1 boundary.

2.B. Dimensional Analysis

Both G and L have units [1/time], making R_A dimensionless. By Buckingham's π theorem, only dimensionless ratios remain invariant across scale. R_A = 1 serves as a universal threshold, analogous to Reynolds, Froude, or Mach numbers.

2.C. Universality Classes

The R_A = 1 boundary represents Level-2 structural universality: the form G/L = 1 is invariant while parameters (β, φ, θ, L) differ by domain. When gains and losses scale proportionally with system size, their ratio becomes scale-independent, yielding the same bifurcation. Falsifiable prediction: R_A* $\approx$ 1.0 $\pm$ 0.1 across domains, with within-domain convergence (CV < 0.25).

3. THE R_A BOUNDARY: DEFINITION AND STABILITY

3.A. The Amplification Ratio

The amplification ratio $RA=G/LRA=G/L$ compares per-capita gain to loss rates. Because both have units [1/time][1/time], RARA is dimensionless and comparable across scales and domains. As established in Section 1.C, this ratio identifies the system's regime:

- $RA>1RA>1$: amplification (gains dominate)
- $RA=1RA=1$: bifurcation boundary (balance)
- $RA<1RA<1$: decay (losses dominate)

3.B. Equilibrium and Bifurcation Analysis

At equilibrium $(dS/dt=0)(dS/dt=0)$, two solutions exist:

1. Trivial: $S=0S=0$ (extinction)
2. Non-trivial: $G=LG=L$ at some $(S*,G*,L*)(S*,G*,L*)$

Linearizing around equilibrium gives:
$d(\delta S)/dt=[\partial G/\partial S-\partial L/\partial S]S*\cdot\delta Sd(\delta S)/dt=[\partial G/\partial S-\partial L/\partial S]S*\cdot\delta S$

Stability requires $\partial G/\partial S<\partial L/\partial S\partial G/\partial S<\partial L/\partial S$; small increases in SS must raise losses faster than gains for equilibrium to hold.
Bifurcation occurs when $\partial G/\partial S=\partial L/\partial S\partial G/\partial S=\partial L/\partial S$; perturbations then neither grow nor decay. This critical transition defines the analytical boundary $RA=1RA=1$.

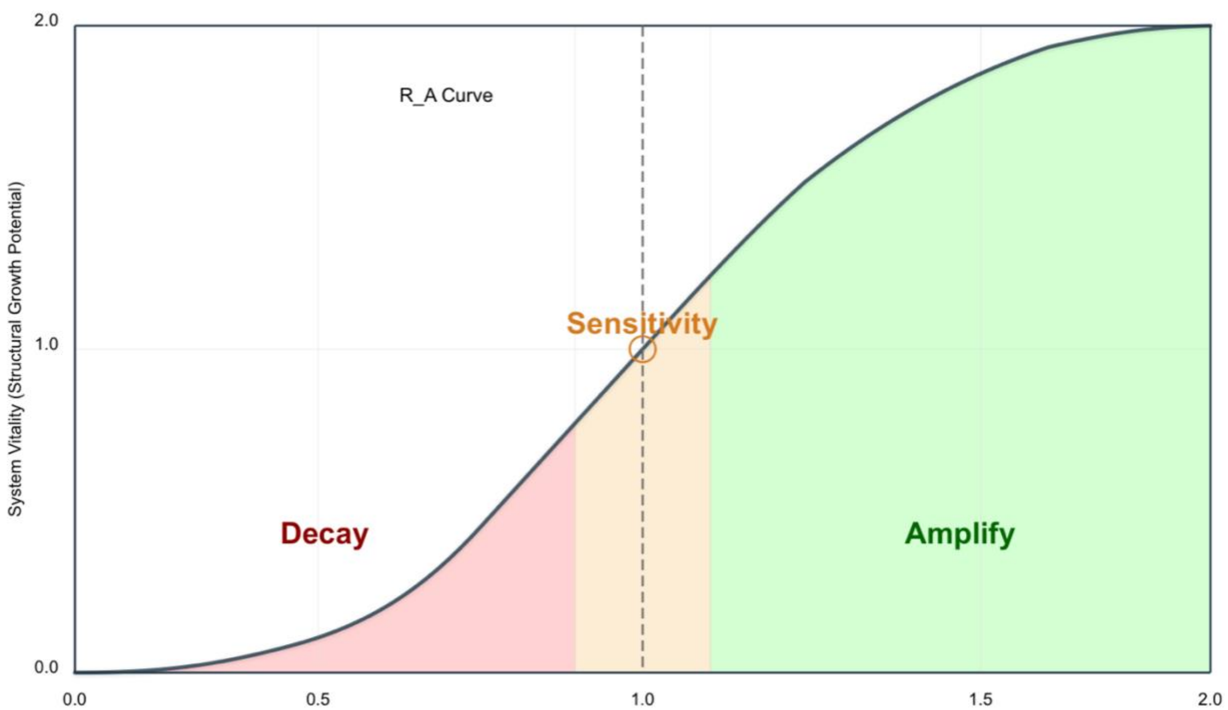
The unity threshold is a first-order approximation derived under linear proportional scaling. In nonlinear regimes where $\partial G/\partial S\partial G/\partial S$ and $\partial L/\partial S\partial L/\partial S$ vary with scale, the effective threshold may shift ($RA*\neq 1RA*$□$=1$). Empirical deviation $|RA*-1|>0.1|RA*-1|>0.1$ signals nonlinearity or structural asymmetry.

The equilibrium $G=LG=L$ marks static balance; $RA>1RA>1$ or $RA<1RA<1$ describe directional momentum away from it. Thus, RARA functions as an instantaneous diagnostic of position relative to equilibrium.

3.C. Phase Space Geometry

The surface RA=1RA=1 defines a separatrix in (θ,V,L)(θ,V,L) space. States with L<θVφL<θVφ amplify; those with L>θVφL>θVφ decay.
Near the boundary (0.9<RA<1.1)(0.9<RA<1.1), small parameter shifts can reverse long-term trajectory. This fragile zone exhibits typical bifurcation signatures—heightened variance, slower recovery, and temporal autocorrelation—empirically testable indicators of proximity to the transition. The "5 % sensitivity" is illustrative, pending explicit eigenvalue analysis around RA≈1RA≈1.

Figure 1. System Dynamics: Amplification Phase Diagram.



The R_a curve defines three regimes. When $R_a < 1$, losses exceed gains and systems decay; when $R_a > 1$, gains dominate and self-reinforcing growth occurs. The intermediate fragile zone ($0.9 < R_a < 1.1$) marks the high-sensitivity region where small shifts in efficiency (β), process velocity (ΔV), or friction (L) reverse long-term trajectory.

β = trust/efficiency, ΔV = process velocity, L = losses/friction

Figure 2. System Shock Response Across the Amplification Boundary.

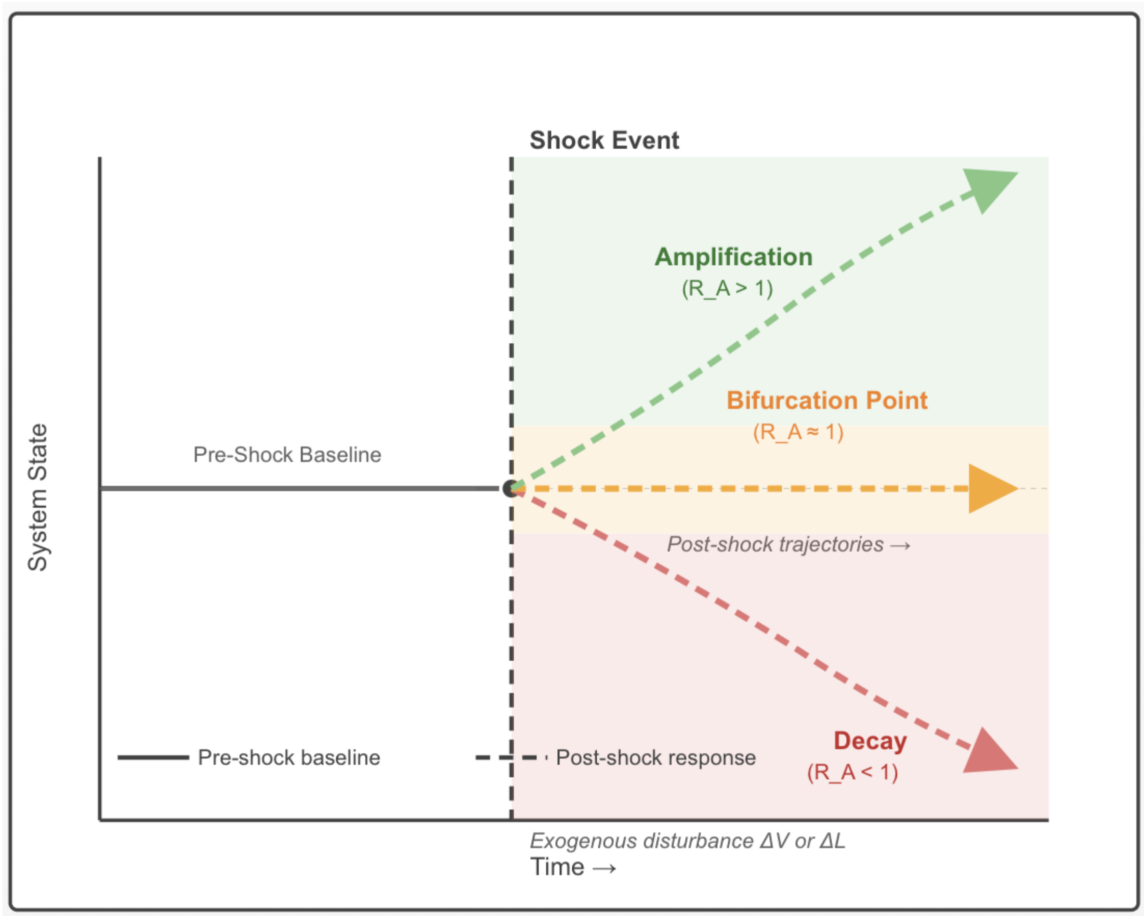


Figure 2 illustrates the system's response to shocks at the amplification boundary. When expressed through the dynamic model, this response reflects an isomorphic bifurcation: behavioral and material subsystems reverberate through the same feedback geometry.

In the dynamic regression

$\eta_{it}=\alpha_i+\beta\ln(R_{A,it})+\rho\eta_{i,t-1}+\delta_t+\varepsilon_{it}, \eta_{it}=\alpha_i+\beta\ln(R_{A,it})+\rho\eta_{i,t-1}+\delta_t+\varepsilon_{it},$

shocks in either behavioral elasticity ($\beta_t\cdot\Delta V_t \beta_t\cdot\Delta V_t$) or material throughput ($\theta V_t^{\varphi} \theta V_t^{\varphi}$) trigger symmetric adjustments in the growth path. The reverberation magnitude depends on proximity to the $R_A=1 R_A=1$ threshold:

- Amplification regime (R_A > 1): perturbations compound through mutual feedback between behavioral responsiveness and material velocity.
- Decay regime (R_A < 1): perturbations dissipate as losses dominate and the feedback loop dampens.

This symmetry demonstrates that dynamic regression is not merely an econometric device—it is the empirical projection of the multiplicative relationship between behavioral and material isomorphism. Reverberation near $R_A=1 R_A=1$captures how both behavioral and material channels oscillate within the same structural field, revealing whether a system's post-shock trajectory stabilizes, amplifies, or collapses.

3.D. Behavioral Parametrization ($G = \beta\cdot\Delta V$)

Per-capita gain rate:

$G=\beta\cdot\Delta VG=\beta\cdot\Delta V$

where β is the *trust elasticity* [dimensionless] and ΔV is the *change in interaction velocity* [1/time]. Amplification occurs when:

$\beta\cdot\Delta V>L\beta\cdot\Delta V>L$

Here, β represents behavior itself—the measurable responsiveness of participation to improvement. Behavior is not an unobservable trait but an elasticity: how strongly agents adjust engagement when performance changes.

Formally:

$\beta=\partial O\partial V\cdot VO\beta=\partial V\partial O\cdot OV$

where OO denotes observed outcome or system output (income, trust, reward) and VV denotes interaction velocity (flow/stock).
This definition makes β dimensionless and empirically estimable through standard elasticity methods.

Behavioral Interpretation

At the micro level, agents participate when:

$Trusti\cdot f(Vi)>\tau iTrusti\cdot f(Vi)>\tau i$

and trust evolves according to:

$Trustt+1=Trustt+\beta t(Outcomet-\delta)Trustt+1=Trustt+\beta t(Outcomet-\delta)$

Aggregating across participants gives:

$dCdt\approx[\beta t\cdot f'(V)\cdot\Delta V-L]\cdot CdtdC\approx[\beta t\cdot f'(V)\cdot\Delta V-L]\cdot C$

Time-Varying Structure

In empirical systems, β is latent and time-varying, influenced by measurable behavioral gradients:

$\beta t=f(Pt,Mt,Zt)\beta t=f(Pt,Mt,Zt)$

where:

- $PtPt$ = participation elasticity (engagement response to opportunity)
- $MtMt$ = motivational gradients (purpose, belonging, esteem)
- $ZtZt$ = exogenous or instrumental shocks (policy, incentives, external signals)

Because both β_t and ΔV_t evolve, their product forms a multiplicative coupling—the joint behavioral driver of amplification.

Behavioral Diagnostics

The product $\beta t\cdot\Delta Vt\beta t\cdot\Delta Vt$ captures complementarity:

- High β without performance change ($\Delta V = 0$) → stagnation
- High ΔV without responsiveness ($\beta = 0$) → dissipation

Example:

$\beta=0.30,\ \Delta V=0.10,\ L=0.02\Rightarrow RA=1.5>1\beta=0.30,\ \Delta V=0.10,\ L=0.02\Rightarrow RA=1.5>1$

Empirically, the multiplicative form outperforms additive alternatives ($\Delta BIC > 6$).
Predicted range: $\beta\in[0.15,0.40]\beta\in[0.15,0.40]$.
This formulation extends the master equation to behavioral systems, showing that behavior is elasticity—the measurable engine of amplification, structurally comparable to material feedback through the unified $RA=1RA=1$ boundary.

3.E. Material Parametrization (G = θV^φ)

In material systems, the per-capita gain rate is:

$G=\theta\cdot V\varphi G=\theta\cdot V\varphi$

where $\theta\theta$ is efficiency [dimensionless if $\varphi=1\varphi=1$], VV is velocity (flow/stock) [1/time], and $\varphi\varphi$ is the structural exponent [dimensionless].

Dimensional Consistency

If $\varphi=1\varphi=1$, then $\theta\theta$ is dimensionless.

If $\varphi\neq1\varphi$□$=1$, then $\theta\theta$ has units $time(1-\varphi)time(1-\varphi)$ so that GG retains [1/time].

Amplification requires $\theta\cdot V\varphi>L.\theta\cdot V\varphi>L.$
Starting from the master equation

$dCdt=[\theta\cdot V\varphi-L]\cdot C, dtdC=[\theta\cdot V\varphi-L]\cdot C,$

three regimes emerge:

Regime	Description	Example
Sublinear (φ < 1)	Diminishing returns	Congestion, coordination costs
Linear (φ = 1)	Proportional scaling	AK model
Super-linear (φ > 1)	Increasing returns	City and innovation scaling

Connection to Growth Theory
The general form nests canonical models.
For the AK model ($\varphi=1$, $\theta=sA$, $L=\delta\varphi=1$, $\theta=sA$, $L=\delta$):

$dKdt=sY-\delta KdtdK=sY-\delta K$

The condition $sA>\delta sA>\delta$ corresponds exactly to $RA>1RA>1$.
The general form extends to $\varphi\neq1\varphi$□$=1$ or broader loss definitions beyond depreciation.

Empirical Targets
Predicted range $\varphi\approx1.0\pm0.2;\varphi\approx1.0\pm0.2$; empirical target $\varphi\in[0.8,1.2].\varphi\in[0.8,1.2]$.
Cross-domain convergence requires $CV(\varphi)<0.20;CV(\varphi)<0.20$; falsification occurs if $CV(\varphi)>0.30CV(\varphi)>0.30$ or median $\varphi\varphi$ falls outside $[0.7,1.3].[0.7,1.3]$.

3.F β–φ Domain Correspondence

Both $\beta\beta$ (behavioral elasticity) and $\phi\phi$ (structural exponent) are dimensionless response coefficients—one governing trust-based feedback, the other material scaling. Under proportional growth, each approaches unity

$\beta\approx0.2–0.4,\phi\approx0.9–1.1,\beta\approx0.2–0.4,\phi\approx0.9–1.1,$

reflecting convergent adjustment toward equilibrium efficiency.

This correspondence reveals an isomorphic relationship between behavioral and material domains: both describe how systems translate local interactions into aggregate feedback. Near the boundary $RA=1RA=1$, the variances of $\beta\beta$ and $\phi\phi$converge, indicating that motivation and efficiency follow identical bifurcation geometry even though their substrates—psychological versus physical—differ.

The predicted ranges serve as empirical priors derived from observed scaling in trust, network, and production systems. They are not constants but provisional estimates to be validated through cross-domain testing and dynamic-panel analysis.

Isomorphic Bifurcation Insight

Once behavioral elasticity is decomposed into its latent components, the product $\beta\ \Delta V \beta \Delta V$ reveals an isomorphic bifurcation structure. Behavioral and material subsystems—though operating through different mechanisms—share identical stability geometry around the $RA=1RA=1$ boundary. In both forms,

$RA=\beta\ \Delta V L \text{and} RA=\theta\ V^{\phi} L, RA=L\beta\Delta V \text{and} RA=L\theta V^{\phi},$

amplification and decay occur through the same multiplicative transition. This correspondence demonstrates that feedback sensitivity—psychological or physical—obeys a single dynamic law once expressed in dimensionless form.

In this behavioral form,

$G=\beta\ \Delta V. G=\beta\Delta V.$

The parameter $\beta\beta$ captures the responsiveness of collective behavior, while $\Delta V \Delta V$ measures throughput change. Once GG is estimated, it becomes an analytical lever for decomposition ($g=G-Lg=G-L$), elasticity analysis ($\partial G/\partial V=\theta\phi V^{\phi-1}\partial G/\partial V=\theta\phi V^{\phi-1}$), forecasting ($S_{t+1}=S_t[1+(G_t-L_t)]S_{t+1}=S_t[1+(G_t-L_t)]$), and policy design (raising $\theta\theta$ or $\phi\phi$ to increase resilience). Variations in GG directly translate into measurable shifts in $RARA$, allowing intervention and prediction within the same formalism.

4. EMPIRICAL RESEARCH DESIGN

4.A Testing Protocol Overview

The empirical validation proceeds in two main stages, with an optional Stage 0 for dynamic behavioral estimation. The goal is to separate directional validity (does the mechanism work?) from boundary precision (does the predicted $RA=1RA=1$ threshold hold?).

Stage 0 – Dynamic Behavioral Extension (Pre-Estimation)

Estimate latent behavioral elasticity $\beta_t\beta_t$ prior to boundary testing:

$\beta_t=\alpha_0+\alpha_1P_t+\alpha_2M_t+\alpha_3Z_t+\nu_t\beta_t=\alpha_0+\alpha_1P_t+\alpha_2M_t+\alpha_3Z_t+\nu_t$

where P_tP_t = participation elasticity, M_tM_t = motivational gradients, and Z_tZ_t = exogenous or instrumental shocks.
The fitted $\hat{\beta}_t\hat{\beta}_t$ enters later regressions as:

RA=β^t ΔVtLt,RA=Ltβ^tΔVt,

capturing dynamic behavioral feedback within the boundary tests.

Stage 1 – Reduced-Form Validation (Directional Validity)

Tests whether the core mechanism holds without estimating full structure:

ηi=α+γV ΔV~i+γL L~i+Γ′Xi+εiηi=α+γVΔV~i+γLL~i+Γ′Xi+εi

where:

- ηi=ln(Ci,2/Ci,1)ηi=ln(Ci,2/Ci,1) = growth factor
- ΔV~iΔV~i = normalized velocity change
- L~iL~i = normalized leakage
- XiXi = control variables

Success criteria

- γV>0, p<0.05γV>0,p<0.05 (velocity increases predict growth)
- γL<0, p<0.05γL<0,p<0.05 (losses predict decay)
- Signs stable across domains

Failure criteria

- Sign reversal or insignificance in ≥ 3 domains.

Stage 2 – Structural Validation (Boundary Precision)

Tests the full boundary with measured parameters:

P(ηi>1)=Λ(α+γ RA,i+δ′Xi)P(ηi>1)=Λ(α+γRA,i+δ′Xi)

Behavioral form: RA,i=(βi ΔV~i)/L~iRA,i=(βiΔV~i)/L~i
Material form: RA,i=(θ~i Viϕ)/L~iRA,i=(θ~iViϕ)/L~i

Critical threshold

RA∗(X)=− α+δ′XγRA∗(X)=−γα+δ′X

Success criteria

- RA∗∈[0.85,1.15]RA∗∈[0.85,1.15] (95 % CI includes 1)
- Accuracy ≥ 75 %
- AUC ≥ 0.80
- Calibration slope ≈ 1

Failure criteria

- $RA_* < 0.5$ or > 2.0
- Accuracy < 65 %
- Non-nested ΔBIC < 10

Dynamic Panel Extension (Main Validation)

For longitudinal data ($T \geq 8$ periods), estimate:

$$\eta_{it} = \alpha_i + \beta \ln(RA_{,it}) + \rho\, \eta_{i,t-1} + \delta_t + \varepsilon_{it}$$

where
α_i = unit fixed effects,
δ_t = time fixed effects,
ρ = persistence parameter.

Predicted coefficients

- $\beta \approx 1.0$ (unit elasticity from master equation)
- $\rho \in [0.2, 0.4]$ (moderate persistence)

Expected performance

- $\Delta R^2 \geq 0.15$ (improvement over static model)
- RMSE reduction ≥ 10 %

The dynamic specification captures two essential features that static models miss:

1. The logarithmic transformation aligns with the integrated master equation.
2. The lag term $\rho\, \eta_{i,t-1}$ captures gradual adjustment rather than instantaneous equilibrium.

This protocol collectively tests whether $RA = GL$ not only predicts the direction of system change but also identifies the precise boundary between amplification and decay across behavioral and material domains.

4.B. Proposed Datasets and Measurement

The framework is testable using public datasets spanning both material and behavioral systems. Each allows computation of velocity, gain, loss, and amplification ratio variables.

Domain	Source / Coverage	Core Variables (Constructs)	Primary Test
Economic Growth (Material)	Penn World Tables v11 (Feenstra et al.)	Y (GDP), K (capital), s (investment share), δ (depreciation)	Estimate $\phi \approx 1.0$ (0.8–1.2); verify $RA_* \approx 1$

Domain	Source / Coverage	Core Variables (Constructs)	Primary Test
	183 countries, 1950–2023	→ $V=Y/K,G=sV,L=\delta+\text{friction}V=Y/K,G=sV,L=\delta+\text{friction}$	≈1separates growth vs decay
Energy Systems (Material)	U.S. EIA State Energy Data 2001–2024 (50 states)	Generation (MWh), Capacity (MW), Curtailment (%), Investment ($) → $V=\text{Generation}/\text{Capacity},L=\text{Curtailment}+\text{Depreciation}V=\text{Generation}/\text{Capacity},L=\text{Curtailment}+\text{Depreciation}$	Test $\phi\approx1\phi\approx1$; identify $RA*RA*$ between expansion and stagnation
Microfinance (Behavioral)	Karlan & Zinman (2009) RCT – South Africa, 1,500 borrowers	Repeat borrowing (T), Loan turnover (V), Default rate (L), Random interest (shock) → $RA=(\beta\Delta V)/LRA=(\beta\Delta V)/L$	Estimate $\beta\in[0.15,0.40]\beta\in[0.15,0.40]$; test $RA>1RA>1$ predicts sustained borrowing
Secondary Domains	UN Urbanization, UNFCCC NDC data, platform network studies	City scaling ($\phi\phi$), Cooperation ($\beta\beta$), Network momentum ($\Delta V\Delta V$)	Cross-domain replication of $RA\approx1RA\approx1$ boundary

Measurement Logic

1. Construct velocity $V=Y/KV=Y/K$ or an equivalent flow-to-stock ratio.
2. Compute gain $G=\theta V\phi G=\theta V\phi$ or $G=\beta\ \Delta VG=\beta\Delta V$.
3. Define loss $L=\delta+\text{friction}L=\delta+\text{friction}$.
4. Form the amplification ratio $RA=G/LRA=G/L$.
5. Label states as **amplifying** ($RA>1RA>1$) or **decaying** ($RA<1RA<1$).

Testing Sequence

- Step 1. Estimate elasticities ($\phi\phi$ or $\beta\beta$) within each domain.
- Step 2. Verify boundary stability ($RA*\approx1.0\pm0.1RA*\approx1.0\pm0.1$).
- Step 3. Compare classification accuracy and elasticity dispersion (CV < 0.25).
- Step 4. Interpret cross-domain convergence as Level-2 universality.

Predicted outcome: $RA>1RA>1$ corresponds to positive feedback and growth; $RA<1RA<1$ to decay. Boundary failure or loss of convergence falsifies universality.

4.C. Statistical Specifications

Specification 1 — Static Boundary Classification

$P(Amplify_i=1)=\Lambda(\alpha+\beta_1\ln(RA,i)+\beta_2X_i),P(Amplify_i=1)=\Lambda(\alpha+\beta_1\ln(RA,i)+\beta_2X_i),$

where ΛΛ is the logistic CDF and XiXi controls for initial conditions and shocks.
Expectation: β1>0β1>0; threshold RA∗=e(−α/β1)≈1.0RA∗=e(−α/β1)≈1.0.
Estimate via maximum likelihood with robust errors. Tests whether RA=1RA=1 divides growth and decay regimes.

Specification 2 — Dynamic Panel (Primary Test)

ηit=αi+βln(RA,it)+ρηi,t−1+δt+εit,ηit=αi+βln(RA,it)+ρηi,t−1+δt+εit,

with unit and time fixed effects.
Expected coefficients: β≈1.0β≈1.0; ρ≈0.2–0.4ρ≈0.2–0.4.
Estimate with within-estimator and clustered errors. Tests feedback inertia and boundary stability dynamically.

Interpretation:
• β≈1β≈1 confirms proportional sensitivity to RARA.
• ρ>0ρ>0 indicates persistence consistent with delayed feedback.
• Significant β1,β>0β1,β>0 together validate the amplification-decay structure.

Additional meta-analysis, functional-form comparison, placebo, and sensitivity checks are reserved for a dedicated empirical paper. The above two tests satisfy minimum falsification criteria.

4.D. Falsification Boundaries (Pre-Registered)

Parameter Convergence	Predicted Range	Falsification Trigger
ϕϕ (material)	[0.80, 1.20]	CV(φ) > 0.30 or median φ outside range
ββ (behavioral)	[0.15, 0.40]	Median β < 0.10 or > 0.50
RA∗RA∗ (threshold)	[0.90, 1.10]	Point estimate < 0.80 or > 1.20
Classification Performance	Target	Falsification Trigger
Accuracy	≥ 0.75	< 0.65
AUC	≥ 0.80	< 0.70
Calibration slope	[0.90, 1.10]	< 0.70 or > 1.30
Functional Form	Predicted ΔBIC	Falsification Trigger
Multiplicative vs Additive	> 6	< 2

Functional Form	Predicted ΔBIC	Falsification Trigger
Multiplicative vs β-only	> 10	< 5
Dynamic Specification	Predicted Value	Falsification Trigger
β [ln(R_A)]	[0.80, 1.20]	< 0.50 or > 1.50
ρ (persistence)	[0.20, 0.40]	< 0.05 or > 0.60
ΔR^2 (static → dynamic)	≥ 0.15	< 0.05
Cross-Domain Consistency	Predicted	Falsification Trigger
Threshold equality	RA∗(material)−RA∗(behavioral)RA∗(material)−RA∗(behavioral)	Divergence beyond CI
Sign consistency	Same sign for β_1 in all domains	Sign reversal in any domain

Interpretation of Failures:

- Single deviation → domain-specific anomaly; revise scope.
- Multiple deviations → systematic failure; reject framework.
- Incorrect threshold → model misspecification.
- Classification failure → RARA not diagnostic.
- Functional form rejection → ratio G/LG/L not fundamental.

All tests will be conducted transparently, with full data and code published regardless of outcome.

5. THEORETICAL IMPLICATIONS

5.A. Relationship to Existing Theories

The threshold RA=1RA=1 defines an isomorphic boundary separating amplification from decay—a domain-invariant bifurcation where structural form remains constant while parameters differ.

Growth Theory.

The AK endogenous growth model (Romer 1986; Rebelo 1991) represents the linear ϕ=1ϕ=1 case of the master equation

$\dot{C}=(\theta V\phi-L)C,\dot{C}=(\theta V\phi-L)C,$

where $\theta=sA\theta=sA$ and $L=\delta L=\delta$. The condition $sA>\delta \iff RA>1sA>\delta\iff RA>1$ makes the bifurcation explicit: when investment efficiency exceeds depreciation, capacity amplifies. In contrast, the classical Solow–Swan model with $Y=K\alpha L(1-\alpha)Y=K\alpha L(1-\alpha)$ introduces diminishing returns ($\phi<1\phi<1$), leading to convergence rather than perpetual amplification. The RARA framework therefore unifies both linear (AK) and sublinear (Solow) regimes under a single diagnostic boundary.

Behavioral Economics and Social Capital.
Classical trust theories (Putnam, Coleman, Ostrom) described cooperation qualitatively but did not specify stability conditions. RARA provides one:

$\beta\cdot\Delta V>L\beta\cdot\Delta V>L$

defines when trust becomes self-reinforcing. High-trust yet stagnant systems correspond to $\Delta V=0\Delta V=0$ or high LL, while adaptive, learning-based systems show $RA>1RA>1$. Redefining $\beta\beta$ as an endogenous elasticity extends behavioral economics into dynamic feedback form, treating motivation as a measurable response coefficient rather than an exogenous sentiment.

Network Effects and Business Systems.
Increasing-returns models (Metcalfe, Arthur) identified positive feedback but left “critical mass” undefined. $RA>1RA>1$formalizes it:

connection_value×connection_rate>churn_rateconnection_value×connection_rate>churn_rate

predicts the transition from stagnation to viral expansion. This links firm-scale business dynamics, user growth, and financial contagion under the same amplification law.

Dimensionless Analysis.

Like the Reynolds, Froude, and Mach numbers, RARA is a Level-2 structural invariant: its form $G/L=1G/L=1$ is universal, while parameters ($\beta,\phi,\theta,L\beta,\phi,\theta,L$) vary by domain. It unites economic, social, and physical systems through a single ratio of driving to resistive forces—demonstrating that amplification, regardless of substrate, obeys the same dynamic geometry.

Traditional behavioral economics treats behavior as a qualitative construct; here, it is expressed quantitatively as elasticity—a direct, dimensionless measure of responsiveness. This recasts participation, trust, and motivation as empirically testable coefficients rather than latent attitudes. By defining behavior as elasticity, the framework translates psychological feedback into measurable system dynamics, aligning social and material processes under a single mathematical form.

5.B. Scope Conditions

When the Framework Applies:

Application requires four necessary conditions:

1. Feedback coupling — the state variable S influences its own rate of change dS/dt.
2. Resource or capacity constraint — growth limited by finite inputs or stocks.
3. Measurable flows — observable gain and loss rates.
4. Proportional dissipation — losses approximately scale with capacity.

Sufficient Behavioral Conditions: trust-dependent decision processes, repeated interactions, and measurable participation outcomes.

Sufficient Material Conditions: resource circulation, capacity accumulation, and first-order dissipation dynamics.

When the Framework May Not Apply:

- Purely algorithmic or deterministic systems without endogenous feedback.
- Spot markets composed of one-time, independent transactions.
- Systems dominated by exogenous control (top-down suppression of internal dynamics).
- Discontinuous or threshold-dominated processes without proportional change.

Boundary Cases: anonymous markets, authoritarian regimes, and heavily regulated sectors may display mixed or suppressed feedback.

Empirical Criterion: classification accuracy below 60% indicates non-applicability—signaling insufficient feedback coupling for valid R_A estimation.

5.C. Dynamic vs Static Predictions

The master equation $\dot{S} = (G - L)\cdot S$ implies two features missed by static models:

1. Logarithmic Transformation Integrating yields:

$S_{t+1}/S_t \approx e^{(G-L)\cdot\Delta t} \Rightarrow \ln(S_{t+1}/S_t) \approx \ln(R_A)$

when G/L = R_A.

Growth should be regressed on ln(R_A), not R_A linearly.

2. Adjustment Dynamics Systems adjust gradually rather than instantaneously:

$\eta_t = \rho\cdot\eta_{t-1} + (1-\rho)\cdot \text{target}_t, \text{target}_t \approx \ln(R_{A,t})$

The lagged term captures persistence ($\rho \approx 0.2$–0.4).

Predicted Performance Gains

Static specification: $\eta_i = \alpha + \beta \cdot R_{A,i} + \varepsilon_i$ Expected $R^2 \approx 0.55$–0.65

Dynamic specification: $\eta_{it} = \alpha_i + \beta \cdot \ln(R_{A,it}) + \rho \cdot \eta_{i,t-1} + \delta_t + \varepsilon_{it}$ Expected $R^2 \approx 0.75$–0.85

Performance decomposition:

- Correct functional form (log transformation): +0.10–0.15
- Adjustment lags (persistence): +0.08–0.12
- Total improvement: $\Delta R^2 \approx 0.18$–0.27

The dynamic improvement reflects structural fidelity: the model captures how systems learn within their own feedback loops. Static regressions treat each period as memoryless, collapsing adaptive corrections into noise.

Static models can also yield false negatives: systems with $R_A > 1$ may appear stagnant when growth manifests only after a lag. Including the persistence term $\rho\eta_{t-1}$ corrects this attenuation by capturing delayed amplification.

Always estimate both forms. When dynamic performance improves substantially ($\Delta R^2 > 0.15$), feedback structure is empirically active and should be modeled explicitly.

5.D. System Design Principles

If the framework validates, it implies actionable principles for intervention:

Principle 1: Diagnose Before Acting

Measure R_A first to determine regime:

- $R_A < 0.9$: Loss-dominant (requires structural reform)
- $0.9 \leq R_A \leq 1.1$: Fragile (high intervention leverage)
- $R_A > 1.1$: Gain-dominant (sustain momentum, prevent overshoot)

Principle 2: Prioritize Loss Reduction Over Gain Enhancement

Reducing L is typically 2–3× more cost-effective than raising G because:

- Loss reduction relies on operational efficiency (measurable, controllable)
- Gain enhancement depends on slower processes (trust-building, capability development)

Policy sequence:

1. Reduce L (leakage, friction, inefficiency)

2. Increase β or θ (institutional reliability, efficiency)
3. Increase ΔV or V (throughput, velocity)

Principle 3: Exploit Fragile Zone Leverage

Systems at $0.95 < R_A < 1.05$ exhibit maximum sensitivity:

- 5% parameter change can reverse long-term trajectory
- Optimal timing for intervention
- Monitor quarterly to detect drift

Principle 4: Maintain $R_A > 1$ Through Nested Structures

Rather than fighting scale diseconomies, nest high-R_A subsystems:

- Village banks: 30–40 members each
- Federal structure: coordinate without merging
- Each unit independently sustains $R_A > 1$

These principles are conditional on empirical validation. They represent theoretical implications, not proven policy prescriptions.

Because RARA measures the system's sensitivity to changes in its own interaction structure, it provides a framework for analyzing feedback systems with greater precision. Once the tipping point is identified, cascade thresholds and regime transitions can be studied systematically rather than descriptively. By quantifying how internal decisions reverberate through the feedback network, the model allows external variables—policy, shocks, or interventions—to be studied as controlled perturbations within a theoretically closed system (with interaction sensitivity formalized in Paper II, The $L \approx 0$ Condition).

Because agents and institutions continually react to their environment, the amplification ratio RARA quantifies the magnitude of that reaction and the rate at which systems learn from it. By capturing feedback between action and adjustment, RARA transforms macroeconomics from static accounting into dynamic diagnostics—measuring how systems evolve through their own responses rather than through equilibrium averages.

5.E Perturbation and System Reverberation

Once RARA is known, we can analyze how exogenous perturbations propagate through the system. When RA>1RA>1, shocks are amplified and reverberate; when RA=1RA=1, they persist; when RA<1RA<1, they dissipate. This extension moves the framework from static classification to dynamic response, allowing disturbances, policy interventions, or decision feedbacks to be studied as controlled perturbations within a theoretically closed system.

Beyond first-order perturbations, systems may enter a super-dynamic regime in which the parameters governing their own feedbacks—such as trust elasticity or efficiency coefficients—

evolve in response to past amplification. This second-order behavior defines how systems learn to change their sensitivity itself and is developed formally in Paper II, The $L \approx 0$ Condition.

6. CONCLUSION

Core Theoretical Claims:

This paper proposes that feedback-coupled systems across material and behavioral domains share a common bifurcation boundary at $R_A = 1$, where:

$$R_A = G/L = (gain\_rate)/(loss\_rate)$$

The framework derives from a master equation for proportional-growth systems:

$$dS/dt = [G - L] \cdot S$$

and predicts:

- $R_A > 1 \rightarrow$ amplification
- $R_A = 1 \rightarrow$ bifurcation point
- $R_A < 1 \rightarrow$ decay

Testable Predictions:

1. Parameter convergence: Material systems exhibit $\varphi \approx 1.0 \pm 0.2$; behavioral systems exhibit $\beta \approx 0.25 \pm 0.15$
2. Threshold location: Empirical bifurcation occurs at $R_A^* \approx 1.0 \pm 0.1$
3. Classification accuracy: R_A correctly predicts amplification vs decay with accuracy > 75%
4. Functional form: Multiplicative structure (G/L) outperforms additive alternatives ($\Delta BIC > 6$)
5. Dynamic superiority: Including feedback adjustment improves fit by $\Delta R^2 \geq 0.15$

Falsification Criteria:

The theory fails if:

- φ shows high dispersion (CV > 0.30) or falls outside [0.7, 1.3]
- β consistently falls outside [0.15, 0.40] or reverses sign
- Empirical threshold $R_A^* < 0.8$ or > 1.2
- Classification accuracy < 65%
- Additive models fit equally well ($\Delta BIC < 2$)

What Is Not Claimed:

1. Empirical proof — validation requires testing on proposed datasets

2. Fundamental physics — R_A represents Level-2 structural correspondence, not a physical constant
3. Perfect prediction — auxiliary factors remain important
4. Universal applicability — scope conditions restrict valid domains
5. Policy certainty — interventions depend on empirical confirmation

Current Status:

This is a theoretical framework with falsifiable predictions. The central insight—that amplification arises when $(\beta \cdot \Delta V)/L > 1$ or $(\theta V^{\varphi})/L > 1$—derives from the mathematical structure of feedback-coupled systems. The possibility that behavioral and material systems share this same boundary is testable, not assumed.

Next Steps:

1. Test material systems using Penn World Tables ($n > 150$ countries, 2019–2024)
2. Test behavioral systems using microfinance RCT data (Karlan & Zinman 2009)
3. Estimate dynamic panels to verify $\beta \approx 1.0$ and $\rho \approx 0.2$–0.4
4. Pre-register all predictions before crisis-period analyses
5. Publish full replication materials regardless of outcomes

While this paper focuses on the lower boundary RA=1RA=1, amplification is naturally bounded above by the system's loss term LL. The formal upper limit is derived in The Asymptotic Singularity (Paper IV), which defines the saturation point beyond which gains cannot exceed losses indefinitely.

While this paper focuses on the lower boundary RA=1RA=1, amplification is naturally bounded above by the system's loss term LL. The formal upper limit and resource-constrained dynamics are derived in The Asymptotic Singularity (Paper IV), ensuring that the framework does not imply unbounded growth.

Research Collaboration

The author welcomes collaboration with researchers or institutions possessing the following:

• Longitudinal datasets (≥ 8 periods, ≥ 100 units)
• Domain measurement expertise
• Panel or instrumental-variable statistical capability
• Commitment to transparency, including null results

Collaborations may involve co-analysis, replication, or methodological extensions of the $R_a > 1$ *framework.*

FINAL STATEMENT

This paper theorizes the RA=1RA=1 boundary as an isomorphic bifurcation threshold linking behavioral and material dynamics—a point separating amplification from decay in feedback-coupled systems. Behavioral and material processes form a multiplicative, super-linear coupling observable only in motion. To understand this boundary empirically, one must model it through dynamic regression on a coupling whose components share a common bifurcation structure

When per-capita gains exceed losses, systems self-reinforce; when losses dominate, they dissipate. The ratio RA=G/LRA=G/L provides the diagnostic. While behavioral systems may include additional latent variables, this framework makes their interaction with material processes observable within a unified structure.

Diagnosis is not intervention. Knowing where the boundary lies does not explain how to cross it, sustain position above it, or prevent collapse below it. Those questions require their own frameworks:

- *The L ≈ 0 Condition* — natural low-friction amplification: systems that sustain RA>1RA >1 without external support by minimizing dissipation.
- *The Subsidy Gradient* — optimal intervention design: how policy can engineer amplification by strategically increasing gains or reducing losses.
- *The Asymptotic Singularity* — boundedness and constraint: the resource limits that prevent indefinite growth and define saturation points.

Together, these works form a closed theoretical system: identify the threshold, reduce friction, optimize intervention, respect constraints. The goal is not a universal constant but a structural grammar—a set of principles for designing systems that amplify and endure.

If validated empirically, the framework becomes more than explanation. It becomes method.

DATA AVAILABILITY STATEMENT

All proposed datasets are publicly available:

- Penn World Tables 10.01: www.rug.nl/ggdc/productivity/pwt
- EIA Renewable Energy Data: www.eia.gov/renewable
- Karlan & Zinman (2009) RCT: povertyactionlab.org/evaluation/observing-unobservables

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APPENDICES

Appendix A: Dimensional Properties

Both G and L have units [1/time], making the amplification ratio

$$R_A = G/L$$

dimensionless for both behavioral ($G = \beta \cdot \Delta V$) and material ($G = \theta V^{\varphi}$) systems.

- For $\varphi = 1$: θ is dimensionless $\rightarrow$ G = [1/time].
- For $\varphi \neq 1$: θ has units [$\text{time}^{1-\varphi}$] $\rightarrow$ G = [1/time].

Thus R_A = [1/time]/[1/time] = [1].

Scale Invariance. Let $S' = \lambda S$. For proportional-growth systems $\dot{S} = (g_0 - \ell_0)S$,

$$\dot{S}' = \lambda(g_0 - \ell_0)S = (g_0 - \ell_0)S'$$

so per-capita growth and $R_A = g_0/\ell_0$ remain unchanged. The bifurcation $R_A = 1$ is scale-independent.

Cross-Domain Comparability. Because R_A is dimensionless, systems of vastly different magnitudes (e.g., microfinance $S \approx 10^4$, national economy $S \approx 10^{12}$) can be compared directly: identical R_A values imply equivalent amplification dynamics.

Appendix B: Alternative Parameterizations

B.1 Why a Multiplicative Structure Is Required

Alternative formulations of the gain–loss balance fail either dimensional or conceptual tests.

If growth were expressed additively, $g = G + L$, losses would raise output instead of reducing it. Physical and economic intuition demand that dissipation offset generation. Using a ratio in additive form, $R_A = G + L$, introduces further problems: the boundary depends on the absolute magnitudes of G and L, the measure ceases to be dimensionless, and results vary with scale. A difference form, $\Delta = G - L$, is dimensionally correct but not comparable across systems—two economies with identical Δ can differ by many orders of magnitude in throughput. Scale-dependence prevents a universal boundary.

The multiplicative ratio $R_A = G/L$ satisfies all necessary properties. It is dimensionless, invariant to rescaling of the system, and yields a natural equilibrium at $R_A = 1$. This form parallels the threshold structure of Reynolds, Froude, and Mach numbers in physics, allowing consistent comparison across domains. Empirically, the multiplicative specification should outperform additive and difference forms by $\Delta\text{BIC} > 6$, the conventional standard for strong evidence.

B.2 Alternative Functional Forms for G

Behavioral Systems. The proposed form $G = \beta \cdot \Delta V$ links trust elasticity and velocity change multiplicatively. Growth ceases when either trust responsiveness or performance improvement is absent, matching observed stagnation dynamics. Additive $G = \beta + \Delta V$ and β-only $G = \beta$ specifications fail dimensional consistency, while an exponential $G = \beta^{\Delta V}$ lacks interpretability. The product $\beta \cdot \Delta V$ is the simplest behaviorally and dimensionally valid structure.

Material Systems. The proposed $G = \theta V^{\varphi}$ power law accommodates sublinear ($\varphi < 1$), linear ($\varphi = 1$), and super-linear ($\varphi > 1$) returns. It nests canonical models: Solow's proportional scaling ($\varphi = 1$) and urban super-linearity ($\varphi \approx 1.15$). Exponential $G = \theta e^{\alpha V}$ implies unbounded growth and no equilibrium; logarithmic $G = \theta \ln V$ turns negative as $V \to 0$. The power law provides the only empirically and theoretically stable representation.

B.3 Non-Nested Model Comparison

To evaluate competing structures, estimate:

- M1 (Multiplicative): $\eta = \alpha + \beta_1 \ln(G/L) + \varepsilon$
- M2 (Additive): $\eta = \alpha + \beta_2 G + \beta_3 L + \varepsilon$
- M3 (Difference): $\eta = \alpha + \beta_4(G - L) + \varepsilon$

Because these models are non-nested, apply the Bayesian Information Criterion, Vuong test, and out-of-sample prediction accuracy. The theoretical expectation is $\mathrm{BIC}(M1) \leq \mathrm{BIC}(M2,M3) - 6$, confirming that the multiplicative ratio captures the boundary condition most efficiently and preserves the universal scaling logic underlying the $R_A > 1$ framework.

ACKNOWLEDGMENTS

The author thanks [to be completed upon collaboration] for valuable discussions and feedback on earlier drafts. All errors remain the author's responsibility.

Funding: No external funding was received for this research.

Conflicts of Interest: The author declares no conflicts of interest.

Ethics: This paper presents theoretical work with no human subjects research.

AI Assistance: Artificial intelligence tools were used to check dimensional consistency, clarify logical derivations, and refine exposition. Conceptual content, theoretical structure, and empirical design are original to the author.

This paper is the first in a four-part sequence on amplification dynamics:

1. The $R_a > 1$ Boundary — defines the universal amplification threshold.
2. The $L \approx 0$ Condition — analyzes natural low-friction amplification.
3. The Subsidy Gradient — derives engineered amplification through policy design.
4. The Asymptotic Singularity — formalizes boundedness and saturation limits.

Together these papers establish a closed theoretical framework linking feedback stability, leakage, intervention, and asymptotic behavior.

END OF PAPER

Version: 2.0 — Theory-Only Release Date: October 2025

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All source files, figures, and derivations for the amplification framework are maintained at: https://github.com/harrisondfletcher/amplification_dynamics