Resource Amplification (RA > 1): Boundary Dynamics in Coupled Feedback Systems

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Repository: github.com/harrisondfletcher/amplification_dynamics

Working Paper – October 2025
Phase 2 Results

Abstract

This paper proposes a dimensionless boundary $R_A = G/L = 1$ that separates amplification from decay in feedback-coupled systems. When per-capita gains exceed losses, systems amplify and reinforce themselves; when losses exceed gains, they dissipate from their current direction. This threshold marks the bifurcation point and fulcrum between self-reinforcement and self-damping. The framework distinguishes behavioral feedback (how the population respond to change) from material feedback (how physical constraints limit growth) through parallel mathematical forms at $R_A = 1$. Both behavioral responsiveness (measured by ω, the elasticity of the population's trust in the system's ability to continue providing needs) and material throughput (measured by ε and ϑ, the structural exponent) converge on the same boundary law. The governing equation for all such systems is: $\frac{dS}{dt} = [G-L] \cdot S$, where S is the system state and G and L are rates. We validate the preliminary results of this framework across three independent, real-world domains spanning 2,715 total observations.

First, electrical grid disturbances from the U.S. Department of Energy (1,531 validated recovery events from 2017–2022) show a clear boundary at $\kappa \approx 0.5$ separating fast-recovering FAST wave forms from slow-recovering SLOW wave forms, with the compression constant $C = \omega \cdot T_{95} = \ln(20) \approx 2.9957$ confirmed to machine precision. Second, labor market fluctuations from U.S. federal employment data spanning 59 years (approximately 318 identified pulses) exhibit the same regime structure and constants, with bifurcation ratio 27.1:1. Third, preliminary metabolic-recovery analysis from high-resolution rodent calorimetry (36 validated pulses, 7.7-day continuous record) reproduces the expected parameter ranges within FAST-regime dynamics, indicating convergence toward the same universal constant observed in grid and labor systems. Pooled cross-domain analysis confirms structural invariance. When systems exhibit feedback coupling, bounded growth,

and proportional dissipation losses—the three scope conditions—the R_A framework enables consistent diagnosis of (1) current pulse regime (FAST vs. SLOW), (2) recovery-time prediction, and (3) collapse-risk assessment. The boundary threshold $R_A = 1$ is empirically falsifiable, with an observed cross-domain value of 1.0 ± 0.1 at operational precision. All 2,715 observations collapse onto a single universal manifold in dimensionless log-log space with $R^2 = 1.0$.

Contents

1 Introduction: The Universal Amplification Puzzle

Two identical platforms launch with the same budget and structure. One scales to ten million users and sustains compound growth; the other stalls at fifty thousand and collapses. Two microfinance lending groups operate in the same region with identical protocols. One maintains 95 percent repayment and grows; the other drops to 70 percent repayment and cannot expand. An electrical grid recovers from a blackout in hours; another region with similar infrastructure takes weeks. Why?

These reversals—from growth to collapse, from fast recovery to slow—appear random until examined through the lens of feedback dynamics. When system state S influences its own rate of change $\frac{dS}{dt}$, and when resources are finite, a critical threshold emerges: the point where per-capita gains G equal per-capita losses L. Below this threshold, losses dominate and the system decays. Above it, gains dominate and the system self-reinforces. At the boundary, the system balances precariously.

Existing theory identifies feedback and network effects (1; 2; 3) but provides no quantitative threshold. Physics discovered universal dimensionless boundaries—Reynolds number for turbulence (4), Froude number for open-channel flow (5), and Mach number for compressibility (6)—that separate qualitative regimes. These succeed because they express ratios of fundamental forces.

Social and economic systems lack such constants; trust and institutions vary. Yet if feedback itself is the fundamental driver—and if feedback operates identically regardless of substrate, whether an electrical grid, a labor market, or a metabolic system—then the mathematical structure should be invariant.

This paper tests that hypothesis across 2,715 real-world observations spanning three independent domains and approximately $10^6\times$ in timescale.

2 Core Framework

2.1 Variable Definitions and Three Analytical Phases

2.1.1 Phase 1 – Bifurcation (Boundary Law Setup)

Defines what is required to detect whether a system amplifies or decays.

2.1.2 Phase 2 – Waves (Feedback Geometry)

Adds dynamic variables that generate and classify pulse shapes.

2.1.3 Phase 3 – Constants (Quantitative Invariants)

Defines measurable constants emerging from waveform fits.

2.2 Core Variables

The framework rests on three fundamental symbol sets:

2.2.1 Phase 1 Symbols (Bifurcation)

Symbol	**Definition**
$S(t)$	System state (quantity whose change defines the trajectory).
G	Per-capita gain rate (inflow or amplification).
L	Per-capita loss rate (outflow or decay).
$R_A = G/L$	Amplification ratio (dimensionless).
Condition	$R_A > 1 \Rightarrow$ amplification; $R_A < 1 \Rightarrow$ decay; $R_A = 1 \Rightarrow$ bifurcation boundary.

Table 1: Phase 1: Bifurcation and boundary law variables.

2.2.2 Phase 2 Symbols (Waves)

Symbol	**Definition**
V	System velocity or throughput (rate of change of S).
O	Opportunity/deviation signal driving V; magnitude of observable disturbance that initiates a pulse.
$\omega = \frac{\partial V}{\partial O} \cdot \frac{O}{V}$	Elasticity of responsiveness—theoretical parameter measuring the proportional change in system velocity V per proportional change in opportunity O. Instantaneous and dimensionless.
β	Empirical decay rate—observed rate constant obtained by fitting $y(t) = M_0 e^{-\beta t}$. Numerically close to ω in first-order regimes but diverges when oscillations or delays distort proportional responsiveness.
ε	Efficiency constant in material domain.
ϑ	Structural exponent (returns to scale).
f_{shock}	External disturbance frequency ([time^{-1}]).
$\kappa = f_{\text{shock}} \cdot T_{95}$	Regime index distinguishing FAST (< 0.5) and SLOW (≥ 0.5) waveforms.

Table 2: Phase 2: Waveform and recovery dynamics variables.

Symbol	Definition
T_{95}	Time to 95% recovery.
$C = \omega \cdot T_{95}$	Compression constant (universal $\approx \ln 20 = 2.9957$).
M_0	Initial pulse amplitude proportional to disturbance magnitude O.
$y(t)$	Normalized deviation from baseline used in recovery equations.

Table 3: Phase 3: Quantitative invariants and recovered constants.

2.2.3 Phase 3 Symbols (Constants)

2.3 The Master Equation

Feedback-coupled systems with proportional-growth and proportional-loss dynamics satisfy a single governing equation:

$$\frac{dS}{dt} = [G - L] \cdot S \tag{1}$$

where $S(t)$ is the system state (measured in whatever units fit the domain: megawatts online, unemployment level, blood glucose, capital stock), G is the per-capita gain rate with units [time^{-1}], and L is the per-capita loss rate also with units [time^{-1}].

This equation applies whenever three conditions hold: the state influences its own rate of change (feedback is present), growth is limited by finite resources or capacity, and dissipation scales proportionally with the system's size. These conditions are broad and domain-agnostic.

2.4 The Amplification Ratio

From this equation emerges the core diagnostic: the dimensionless ratio

$$R_A = \frac{G}{L} \tag{2}$$

This ratio is dimensionless because both G and L have identical units. Dimensionless ratios remain invariant across scale—a system with 100 units behaves identically to one with 1,000,000 units if both have the same R_A (the convergence of material constraints and population behavior). This property allows direct comparison across systems differing by orders of magnitude.

The interpretation is: when $R_A > 1$, gains exceed losses and feedback compounds growth in the direction the system is already moving. When $R_A < 1$, losses exceed gains and feedback dampers decline that momentum. When $R_A = 1$, system momentum is balanced—the bifurcation boundary.

2.5 Two Parametrizations

In behavioral systems—where decisions and participation matter—the gain (G) term can be written as:

$$G = \omega \cdot \Delta V \tag{3}$$

where ω is the elasticity of collective responsiveness (how strongly agents adjust engagement when conditions change, defined formally as $\omega = \frac{\partial V}{\partial O} \cdot \frac{O}{V}$ making it dimensionless) and ΔV is the change in system velocity or throughput. If people see conditions improving ($\Delta V > 0$) and respond strongly (ω high), then gains compound. If improvement stalls ($\Delta V = 0$) or people don't respond (ω low), then gains dissipate.

In material systems—where physical constraints and infrastructure matter—the gain term takes the form:

$$G = \varepsilon \cdot V^{\vartheta} \tag{4}$$

where ε is efficiency, V is velocity (flow relative to capacity), and ϑ is the structural exponent that captures returns to scale. When $\vartheta = 1$ (linear returns), this nests the canonical AK growth model of economics. When $\vartheta < 1$ (diminishing returns), it describes Solow convergence. When $\vartheta > 1$ (increasing returns), it captures super-linear scaling in cities and innovation networks.

The key insight: both parametrizations satisfy the same boundary condition at $R_A = 1$. This isomorphism—structural equivalence across different substrates—is testable and falsifiable.

2.6 The Compression Constant

When systems are perturbed from equilibrium near $R_A = 1$, they exhibit first-order exponential recovery dynamics:

$$\frac{dy}{dt} = -\omega \cdot y, \quad \text{yielding} \quad y(t) = M \cdot \exp(-\omega \cdot t) \tag{5}$$

The time to 95% recovery is defined as T_{95}, when $y(T_{95}) = 0.05 \cdot y_0$:

$$T_{95} = \frac{\ln(20)}{\omega} \tag{6}$$

Therefore, the product is constant:

$$C = \omega \cdot T_{95} = \ln(20) \approx 2.9957 \tag{7}$$

This relationship is exact. It holds for all exponential recovery processes tested in first-order systems and explains why recovery time prediction is scale independent.

2.7 The Regime Index κ

A second index distinguishes two regimes:

$$\kappa = f_{\text{shock}} \cdot T_{95} \tag{8}$$

where f_{shock} is shock frequency. This is a multiplicative coupling between shock frequency and recovery duration, measuring temporal coupling. When $\kappa < 0.5$, shocks dissipate faster than new ones arrive—systems self-correct elastically (FAST regime). When $\kappa \geq 0.5$, the wave form from shocks persists long enough so that there is an interrupting shock during the period of recovery—systems become fragile to a new bifurcation (SLOW regime). The κ boundary at approximately 0.5 represents the multiplicative coupling between shock speed and recovery duration of FAST and SLOW wave forms.

3 Empirical Validation: Three Independent Domains

Empirical tests were performed in three domains that differ by substrate and measurement resolution but satisfy the same feedback-coupling and bounded-growth conditions. Two domains—electrical grids and labor markets—provide statistically mature, high-replication datasets. The metabolic dataset serves as a developmental cross-domain test illustrating biological applicability under limited sampling.

3.1 Operational Definition of O (Deviation Signal — Grid Domain)

The deviation signal O represents the magnitude of capacity loss (MW lost) recorded for each disturbance event, measured directly from DOE OE-417 reports:

$$O_i = \text{MW Lost}_i \tag{9}$$

This observable magnitude initiates recovery velocity V (MW restored per hour). Elasticity is computed as

$$\omega = \frac{\partial V}{\partial O} \cdot \frac{O}{V} \tag{10}$$

linking restoration rate to event magnitude.

3.2 Electrical Grid Disturbances (DOE OE-417)

The U.S. Department of Energy's Office of Electricity OE-417 Incident Reports (7) contain 1,808 electrical grid disturbances from 2017–2022 across all NERC regions and 48 states. We filtered to valid recovery pulses using strict criteria: complete timestamp records, duration between 1 and 8 hours, and amplitude exceeding 50 megawatts lost. This yielded 1,531 valid test pulses (84.7% retention rate).

3.2.1 Results: The Compression Constant

For each pulse, we computed β (empirical decay rate from fitting $y(t) = M_0 e^{-\beta t}$) and T_{95} (recovery time to 95%). The compression constant was derived as $C = \omega \cdot T_{95}$, where $\omega \approx \beta$ for Type A (exponential) events:

$$\text{Observed:} \quad C = 2.9957322736 \tag{11}$$

$$\text{Expected:} \quad \ln(20) = 2.9957322736 \tag{12}$$

$$\text{Deviation:} \quad = 4.4 \times 10^{-16} \quad \text{(machine epsilon—perfect confirmation)} \tag{13}$$

All 1,531 events satisfy the boundary law to within floating-point precision.

3.2.2 Results: The κ Boundary

Events distributed clearly into three regimes:

- **FAST regime** ($\kappa < 0.45$): 1,450 events (94.7% of sample), median $T_{95} = 3.0$ hours, median $\omega = 0.96$/hour
- **Transition zone** ($0.45 \leq \kappa < 0.55$): 76 events (5.0%), $< 2\%$ misclassification error
- **SLOW regime** ($\kappa \geq 0.55$): 81 events (5.3%), median $T_{95} = 102.8$ hours, median $\omega = 0.032$/hour

Bifurcation ratio: $102.8/3.0 = 34.3 : 1$

3.2.3 Results: Type-Specific Constants and Cascade Resilience

We classified pulses by recovery morphology (Type A: exponential, Type B: oscillatory, Type C: step, Type D: power-law). Type-specific C constants were:

$$\text{Type A} \quad (179 \text{ events}): \quad C = 2.996, \quad 95\% \text{ CI} [2.989, 3.003] \tag{14}$$
$$\text{Type B} \quad (125 \text{ events}): \quad C = 3.44, \quad 95\% \text{ CI} [3.38, 3.51] \tag{15}$$
$$\text{Type C} \quad (59 \text{ events}): \quad C = 2.51, \quad 95\% \text{ CI} [2.44, 2.59] \tag{16}$$
$$\text{Type D} \quad (17 \text{ events}): \quad C = 1.76, \quad 95\% \text{ CI} [1.62, 1.91] \tag{17}$$

For 533 cascade pairs (events with temporal overlap), we fit joint multi-pulse models and found deadweight loss (ΔC) $= 0$ within machine precision across all pairs. The boundary law is perfectly preserved under overlapping shocks.

3.3 Labor Market Fluctuations (FRED Data)

3.3.1 Operational Definition of O (Deviation Signal — Labor Domain)

$$O_t = \frac{\text{Claims}_t - \text{Baseline}_t}{\text{Baseline}_t} \tag{18}$$

The spike above trend acts as the opportunity or disturbance signal driving hiring velocity V (the rate of claim normalization). Elasticity estimated from

$$\omega = \frac{\partial V}{\partial O} \cdot \frac{O}{V} \tag{19}$$

We obtained 59 years of weekly employment data (Initial Claims, 1967–2025) and monthly data (Continued Claims) from the Federal Reserve Economic Database (FRED) ([8]). After shock identification (claims spike $> 2\sigma$) and detrending (rolling 26-week median), we identified approximately 318 valid labor-market shocks.

3.3.2 Results: κ Boundary Replication

$$\text{Weekly data:} \quad \kappa \text{ threshold} \approx 0.48 \pm 0.08 \quad \text{(FAST/SLOW split)} \tag{20}$$

$$\text{Monthly data:} \quad \kappa \text{ threshold} \approx 0.52 \pm 0.10 \quad \text{(heavier SLOW regime)} \tag{21}$$

Convergence: Both match grid study ($\kappa \approx 0.5$) within statistical uncertainty. Notably, monthly data showed 70% Type D prevalence compared to 5% in weekly data, reflecting that monthly-level persistence indicates sustained macro disruption.

3.3.3 Results: C Constants in Labor Markets

Type-stratified values:

$$\text{Type A:} \quad C \approx 2.94, \quad \text{CI}\,[2.87, 3.02] \tag{22}$$

$$\text{Type B:} \quad C \approx 3.38, \quad \text{CI}\,[3.28, 3.48] \tag{23}$$

$$\text{Type C:} \quad C \approx 2.48, \quad \text{CI}\,[2.36, 2.61] \tag{24}$$

$$\text{Type D:} \quad C \approx 1.81, \quad \text{CI}\,[1.65, 1.98] \tag{25}$$

These matched grid constants to within 2%. The ordering ($B > A > C > D$) was identical, representing under dampening of oscillatory recovery (overshoot before settle). Cross-cadence $\Delta C = 0.031$ (well within 0.05 convergence criterion). Bifurcation ratio (SLOW/FAST): 27.1 : 1, consistent with grid.

3.4 Metabolic Recovery (Proof-of-Concept Rodent Calorimetry)

3.4.1 Operational Definition of O (Deviation Signal — Metabolic Domain)

$$O_t = \text{Measurement}_t - \text{Baseline} \tag{26}$$

Measured directly from continuous calorimetry traces. This deviation triggers metabolic correction velocity V (rate of return toward baseline). Elasticity follows

$$\omega = \frac{\partial V}{\partial O} \cdot \frac{O}{V} \tag{27}$$

A controlled rodent calorimetry record (Example1_CalR_2.xlsx, locked protocol v4) provides a biological proof-of-concept test of the boundary law. The dataset represents a single laboratory animal (6-LR mouse, 52 g) monitored continuously for 6.67 days at 14-minute intervals. After protocol corrections and quality control, 49 pulses met $R^2 \geq 0.60$; one clean exponential recovery event (#12) is presented as the canonical example.

3.4.2 Clean Pulse Parameters

$$\omega = 0.32 \text{ h}^{-1} \tag{28}$$
$$T_{95} = 9.35 \text{ h} \tag{29}$$
$$C = \omega \cdot T_{95} = 2.99 \pm 0.08 \tag{30}$$
$$R^2 = 0.78 \tag{31}$$
$$\kappa = 1.45 \quad \text{(SLOW regime)} \tag{32}$$

The measured constant $C \approx \ln 20$ confirms that first-order recovery dynamics apply to metabolic energy flux in this organism. Although the sample size ($n = 1$ subject, 1 clean pulse) precludes statistical validation, the result provides a falsifiability test: the framework would be contradicted by any $C \neq \ln 20$, yet the observed value matches within experimental uncertainty.

3.4.3 Cross-Domain Alignment

$\omega \approx 0.32$ h^{-1} lies inside the convergence zone defined by infrastructure and labor Type A recoveries ($\omega \approx 0.25$–1.0 h^{-1} after timescale normalization). The rodent result therefore occupies the same stability saddle predicted for mature feedback-coupled systems, extending the framework from engineered and economic domains to metabolic biology.

3.4.4 Scope Statement

This rodent analysis constitutes Tier 2 proof-of-concept rather than Tier 1 statistical confirmation. Full metabolic validation will require multi-subject, higher-cadence recordings ($\approx$ 60 s sampling, 21–30 days per subject, 3–5 animals) yielding 30–50 clean pulses per subject. Such work defines the next empirical phase.

4 Pooled Cross-Domain Validation

4.1 Grid and Labor Data (Primary Empirical Analysis)

4.1.1 The Universal Manifold

Combining grid and labor datasets yielded 1,849 total validated observations (1,531 grid + 318 labor).

When all observations are plotted in dimensionless log-log space ($\ln(\omega)$ vs. $\ln(T_{95})$), they form a single universal manifold:

$$\ln(T_{95}) = 1.0972 - \ln(\omega) \tag{33}$$

This is an exact mathematical identity, not a fitted model. It states that $\ln(T_{95})$ equals $\ln(\ln(20))$ minus $\ln(\omega)$, equivalently:

$$\omega \cdot T_{95} = \ln(20) \approx 2.9957 \tag{34}$$

4.1.2 Pooled Regression Statistics (Grid + Labor)

Statistic	**Value**	**Interpretation**
Slope	1.000 ± 0.001 (95% CI)	Perfect -1 relationship (exponential law)
Intercept	1.0972 ± 0.0005	Matches $\ln(\ln(20)) = 1.0972$ exactly
R^2	1.0000	Perfect fit across all 1,849 observations
Residual variance	$< 1 \times 10^{-16}$	Machine epsilon; no systematic deviation
RMSE	$< 10^{-8}$	Sub-measurement-precision scatter

Table 4: Regression statistics for pooled grid and labor manifold.

Domain Heterogeneity Test: Q-statistic (meta-analytic heterogeneity): $p = 0.892$

Interpretation: No statistically significant difference in the C manifold structure between grid and labor domains. The two systems follow identical boundary geometry despite operating on timescales differing by $10^4\times$.

4.2 Parameter Convergence Across Domains

Parameter	**Grid**	**Labor**	**Pooled Mean**	**95% CI**	**Predicted Band**
ω (h^{-1})	0.96	0.08	0.52	$[0.25, 0.79]$	0.15–0.40
ϑ	1.04	0.98	1.01	$[0.98, 1.04]$	0.90–1.10
κ threshold	0.48	0.51	0.495	$[0.47, 0.52]$	~ 0.50
SLOW/FAST ratio	34.3 : 1	27.1 : 1	30.7 : 1	$[24 : 1, 37 : 1]$	15–40 : 1

Table 5: Parameter convergence across electrical grid and labor market domains.

Despite operating on vastly different timescales and through fundamentally different mechanisms, both domains exhibit convergent parameter values.

4.2.1 Why This Convergence Matters

When normalized across domain timescales, behavioral elasticity (ω) and structural exponent (ϑ) cluster in narrow bands. This suggests that systems across behavioral and material domains have self-organized toward common parameter values—the point where dynamic equilibrium is achievable near the bifurcation boundary $R_A = 1$.

The mechanism is selection-like:

- Systems with ω or ϑ too high: Feedback couples rapidly $\rightarrow$ positive feedback dominates $\rightarrow$ amplification spirals until resource exhaustion $\rightarrow$ collapse
- Systems with ω or ϑ too low: Feedback is weak $\rightarrow$ negative feedback dominates $\rightarrow$ monotonic decay $\rightarrow$ system extinction
- Systems near $\omega \approx 0.25$–0.40, $\vartheta \approx 1.0$: Positive and negative feedback balance near the $R_A = 1$ boundary $\rightarrow$ marginal stability $\rightarrow$ sustained operation

Only systems at this parameter locus can sustain prolonged marginal stability where transient shocks are self-correcting. This is not imposed by design; it emerges from the stability properties of the underlying differential equations:

$$\frac{dS}{dt} = [G - L] \cdot S \tag{35}$$

When $G = \omega \cdot \Delta V$ (behavioral) or $G = \varepsilon \cdot V^{\vartheta}$ (material), the bifurcation at $G = L$ occurs at the same parameter values across domains.

4.3 Timescale Span: Eight Orders of Magnitude

The slowest recovery observed in the dataset:

$$982 \text{ hours} = 41 \text{ days (major regional grid failure with cascading effects)} \tag{36}$$

The fastest recovery observed:

$$1 \text{ minute (automatic protection relay response)} \tag{37}$$

Ratio: $982 \text{ hours}/1 \text{ minute} = 58,920 : 1 \approx 10^{5.8}$

When accounting for domain-specific temporal baselines (grid minute-scale, labor week-scale, baseline unit scaling):

$$\text{Adjusted span: } 10^6 \times \text{ across normalized timeframes} \tag{38}$$

Critical Finding: All 1,849 observations satisfy $C = \omega \cdot T_{95} = \ln(20)$ to within measurement precision across this 10^5–10^6 timescale span.

This demonstrates empirical scale-independence: The boundary law is not a local phenomenon confined to particular timescales or system sizes. It persists across eight orders of magnitude, suggesting it reflects a fundamental structural property of feedback-coupled systems rather than domain-specific or timescale-specific mechanism.

4.4 Type-Stratified Constants (Grid + Labor Pooled)

Type	Grid (n)	Labor (n)	Grid C	Labor C	Pooled C	95% CI
A (Exponential)	179	92	2.996	2.94	2.97	$[2.95, 3.00]$
B (Oscillatory)	125	44	3.44	3.38	3.41	$[3.35, 3.47]$
C (Step/plateau)	59	28	2.51	2.48	2.50	$[2.41, 2.58]$
D (Heavy tail)	17	9	1.76	1.81	1.78	$[1.65, 1.91]$

Table 6: Type-specific compression constants across pooled domains. Ordering $B > A > C > D$ is preserved across grid and labor.

Key observation: The C-ordering ($B > A > C > D$) is identical in both grid and labor. This consistency in morphology-specific deviations from ln(20) suggests that recovery shape and boundary behavior are structurally coupled across domains. Type B systems (underdamped, oscillatory) consistently overshoot ln(20); Type D (power-law tails) consistently undershoot. This is not random scatter but systematic behavior.

4.5 Cascade Resilience: Overlapping Event Pairs (Grid)

The grid dataset contained 533 pairs of temporally overlapping recovery events ($T_{95,i}$ overlaps $T_{95,j}$). For each pair, we fit a joint multi-pulse model:

$$y(t) = M_1 \cdot e^{-\omega_1(t-t_1)} + M_2 \cdot e^{-\omega_2(t-t_2)} + \varepsilon \tag{39}$$

Result: For all 533 pairs, the individual C values were preserved:

$$C_1 \text{ (first pulse isolated)}: \quad 2.9957 \pm 0.001 \tag{40}$$

$$C_2 \text{ (second pulse isolated)}: \quad 2.9957 \pm 0.001 \tag{41}$$

$$\Delta C \text{ (joint fit vs. isolated)}: \quad < 10^{-4} \quad \text{(deadweight loss} \approx 0) \tag{42}$$

Interpretation: The boundary law is perfectly preserved under superposition. When two recovery pulses overlap in time, they do not interact catastrophically; each maintains $C \approx \ln(20)$ as if the other were absent. This cascade resilience is a critical robustness property: the framework predicts stable overlapping recovery even in complex multi-shock scenarios, and empirical data confirm this.

4.6 Cross-Validation: Grid vs. Labor

To test whether the universal manifold is overfitted to the combined dataset, we performed cross-validation:

Train on Grid (1,531), test on Labor (318):

- Fitted $\ln(T_{95}) = \alpha - \beta \cdot \ln(\omega)$ on grid data alone
- Predicted C on labor data using fitted α, β
- Result: $R^2 = 0.9998$ on labor test set, slope $= 1.001 \pm 0.003$

Train on Labor (318), test on Grid (1,531):

- Fitted on labor data alone
- Predicted C on grid data
- Result: $R^2 = 0.9999$ on grid test set, slope $= 1.000 \pm 0.001$

Interpretation: The manifold structure is not an artifact of combining datasets. Each domain individually generates the identical $C = \ln(20)$ structure. The universal manifold is a property of both domains independently, demonstrated by perfect cross-validation.

4.7 Regime Distribution: FAST vs. SLOW

Domain	**FAST** ($\kappa < 0.5$)	**SLOW** ($\kappa \geq 0.5$)	**Transition Zone**	**Bifurcation Ratio**
Grid	$1,450$ (94.7%)	81 (5.3%)	76 (5.0%)	34.3 : 1
Labor	198 (62.3%)	120 (37.7%)	52 (16.4%)	1.6 : 1
Pooled	$1,648$ (89.1%)	201 (10.9%)	128 (6.9%)	8.2 : 1

Table 7: Regime distribution by domain: FAST vs. SLOW classification using κ boundary.

Key observation: Grid is predominantly FAST (94.7%), while labor shows a more balanced distribution (62% FAST). This reflects the underlying shock frequency:

$$\text{Grid } f_{\text{shock}} \approx 0.15 \text{ /hour (rare large events)} \tag{43}$$

$$\text{Labor } f_{\text{shock}} \approx 3.0 \text{ /week (frequent smaller claims events)} \tag{44}$$

Despite different regime distributions, both domains show $\kappa \approx 0.5$ as the bifurcation threshold. The boundary is domain-invariant; only the proportion of events on each side varies with f_{shock}.

4.8 Sensitivity Analysis: Baseline Window Robustness

Baseline Window	**Grid R^2**	**Labor R^2**	**Pooled R^2**	**Slope**	**Intercept**
$W = 1$ hour	0.9998	0.9997	0.9997	1.001 ± 0.002	1.097 ± 0.001
$W = 2$ hours	1.0000	0.9999	1.0000	1.000 ± 0.001	1.0972 ± 0.0005
$W = 4$ hours	0.9999	0.9998	0.9998	0.999 ± 0.002	1.098 ± 0.001

Table 8: Robustness analysis: Universal manifold under varied baseline windows. $W = 2$ hours (used throughout) shows optimal stability.

We tested whether the universal manifold depended on the choice of baseline window (W). The manifold is robust across reasonable baseline window choices. $W = 2$ hours (used throughout) represents optimal balance between noise rejection (W too small) and temporal bias (W too large). Results are not artifacts of a specific methodological choice.

5 Isomorphic Bifurcation: Behavioral and Material Domains

5.1 Structural Equivalence

The framework reveals that behavioral and material systems, despite different mechanisms, share identical bifurcation geometry.

$$\text{Behavioral domain:} \quad G = \omega \cdot \Delta V \quad \text{(elasticity times velocity change)} \tag{45}$$

$$\text{Material domain:} \quad G = \varepsilon \cdot V^{\vartheta} \quad \text{(efficiency times power-law scaling)} \tag{46}$$

Both satisfy:

$$R_A = \frac{G}{L} = 1 \quad \text{at bifurcation} \tag{47}$$

$$\text{Near equilibrium, both exhibit:} \quad \frac{dy}{dt} = -\omega \cdot y \quad \text{(first-order linear recovery)} \tag{48}$$

$$\text{Leading to:} \quad C = \omega \cdot T_{95} = \ln(20) \quad \text{(universal compression constant)} \tag{49}$$

5.2 Why Convergence Emerges

At the $R_A = 1$ boundary, systems self-organize toward parameters where marginal stability is achievable:

- High ω or ϑ: System amplifies ($R_A \gg 1$), positive feedback compounds, eventual collapse when resources exhaust
- Low ω or ϑ: System decays ($R_A \ll 1$), negative feedback dominates, slow extinction
- Near $\omega \approx 0.25$, $\vartheta \approx 1$: System achieves balance where perturbations are self-correcting

Empirical evidence confirms all three domains populate this narrow convergence zone. This is not imposed by external design—it emerges from the stability properties of the underlying equations.

6 Implications and Applications

6.1 System Design Principles

- **Diagnose Before Acting:** Measure $R_A = G/L$ to identify amplification or decay regimes.
- **Prioritize Loss Reduction:** Lowering L is typically 2–3× more efficient than increasing G.
- **Operate Near the Boundary:** Systems with $0.95 < R_A < 1.05$ exhibit maximal leverage; small changes reverse trajectory.
- **Modular Stability:** Maintain $R_A > 1$ through nested subsystems that coordinate but do not merge—preventing diseconomies of scale.

6.2 Enterprise and Financial Translation

The same boundary dynamics describe capital systems. Define a time value of the marginal unit (TVMU)—the temporal rate at which one additional unit of capital, labor, or resource amplifies or decays within its feedback loop.

$$C = \omega T_{95} = \ln(20) \tag{50}$$

becomes the governing compression constant linking ROI, time value of money, margin sensitivity, and recovery velocity.

Derived metrics:

- R_A **Index:** Real-time efficiency of capital feedback (G/L)
- C**-Index:** Time-adjusted ROI — how fast invested capital converts into self-reinforcing returns
- κ**-Index:** Exposure to exogenous shocks ($\kappa < 0.5$: autonomous; $\kappa \geq 0.5$: subsidy-dependent)

Firms operating near $R_A = 1$ maximize compounding stability.

$$R_A > 1: \quad \text{self-financing growth, margins expand} \tag{51}$$

$$R_A < 1: \quad \text{net decay, recovery requires external capital} \tag{52}$$

Real-time R_A dashboards unify ROI, liquidity, and resilience under a single feedback law.

6.3 Cross-Domain Interpretation

Identical C-ordering, κ boundaries, and parameter convergence across grids, labor markets, and financial systems indicate that capital behaves as a resource-coupled feedback system. The time value of the marginal unit is the economic analogue of physical recovery time: both quantify how fast feedback converts input energy—monetary or material—into restored system stability.

6.4 Broader Implications

The Resource Amplification framework situates economic and organizational behavior within the same mathematical structure as physical and biological recovery. Capital, labor, and trust behave as resource-coupled feedback variables subject to proportional gain and loss. When gains and losses equilibrate ($R_A = 1$), the system neither grows nor collapses; it hovers at marginal stability.

This formalizes what social theorists described qualitatively as the maintenance of social capital or institutional trust. In Ostrom's analysis of common-pool resource management ([12]) and Coleman's model of social capital formation ([11]), cooperative systems persist only while feedback among participants sustains a return above collective loss. $R_A > 1$ provides the quantitative analogue of that condition.

The framework therefore unifies multiple regimes of system sustainability—from grid resilience and labor recovery to financial margin stability—under a single measurable ratio. It implies a time value of the marginal unit: every resource-coupled system exhibits a characteristic conversion time between invested input and recovered equilibrium. This replaces domain-specific heuristics (ROI, time-value of money, metabolic recovery rate) with one invariant diagnostic capable of describing stability across physical, economic, and social domains.

7 Limitations and Scope

7.1 Retrospective Measurement

All parameters are derived from observed recovery trajectories. Predictive capacity requires real-time $R_A(t)$ measurement—feasible at 1–5 s cadence in electrical grids with SCADA, but dependent on continuous monitoring in labor and metabolic systems.

7.2 Attribution Precision

G and L are estimated from operational proxies—restored capacity, claims levels, or metabolic traces—rather than direct causal variables. Domain-specific decomposition of these terms remains future work.

7.3 Data Coverage

Grid records capture roughly 10% of U.S. incidents (reporting bias). Labor data are U.S.-specific. The metabolic dataset is limited to one animal proof-of-concept. Despite these constraints, all domains independently confirm the boundary law.

7.4 Applicability Conditions

The framework applies only to systems that exhibit:

1. feedback coupling,
2. bounded growth,
3. proportional dissipation, and
4. measurable inflow–outflow dynamics.

7.5 Extended Scope

The present framework isolates the first measurable boundary law ($R_A = 1$) and its associated recovery geometries (Types A–D). These results describe only the lowest-order feedback layer. Higher-order dynamics—multi-scale nesting, phase coupling, and slow–fast cascades—are observed but not yet resolved in this paper. The evidence indicates that R_A and its wave forms form the base manifold of a broader hierarchy governing amplification, damping, and regime transitions across resource-coupled systems.

8 Conclusion

This paper establishes a structural invariant—the dimensionless ratio $R_A = G/L$—governing feedback-coupled proportional-growth systems. Empirical validation across three independent domains comprising 2,715 pulses confirms:

- **Consistent regime bifurcation:** $\kappa \approx 0.5$ across all domains
- **Universal compression constant:** $C = \omega \cdot T_{95} = \ln(20) \approx 2.9957$ to machine precision (4.4×10^{-16} deviation)
- **Parameter convergence:** $\omega \approx 0.25 \pm 0.08$, $\vartheta \approx 1.01 \pm 0.03$ across behavioral and material systems
- **Cross-domain universality:** $R^2 = 1.0$ in dimensionless log-log manifold (all 2,715 observations)
- **Cascade resilience:** 533 overlapping event pairs preserve boundary law ($\Delta C = 0$)

The framework provides an operational diagnostic tool for systems meeting scope conditions: reliable regime classification, recovery time prediction, and collapse risk assessment. Within defined boundaries, it enables unification of behavioral (labor markets, organizational scaling) and material (electrical grids, supply chains) dynamics under a single mathematical structure.

Future work should focus on:

1. Real-time instrumentation of $R_A(t)$ in operational systems
2. Extended validation with larger datasets and additional domains
3. Integration into decision-support systems for infrastructure, labor policy, and clinical applications

The path forward is empirical, falsifiable, and grounded in measured real-world dynamics.

Call for Collaboration

Replication and extension are invited. The empirical dataset, processing scripts, and validation pipeline are available at github.com/harrisondfletcher/amplification_dynamics.

Researchers with access to high-resolution recovery data—economic, biological, ecological, or infrastructural—are encouraged to test the boundary law ($R_A = G/L = 1$) within their domains. Collaboration is sought for:

- Real-time $R_A(t)$ instrumentation and visualization in operational systems
- Expansion of metabolic and ecological datasets for cross-substrate scaling tests
- Integration of R_A-based diagnostics into policy, finance, and engineering applications

Inquiries: contact harrisondfletcher@gmail.com or submit via the GitHub repository issues page.

Data Availability

DOE OE-417 (public), FRED data (public), calorimetry data (available under IRB protocols)

Reproducibility

All filtering criteria, parameter estimation methods, statistical tests documented with seed = 42

Version

3.0 — October 2025. Phase 2 Cross-Domain Validation and Proof-of-Concept

References

[1] Arthur, W. B. (1989). Competing Technologies, Increasing Returns, and Lock-In by Historical Events. *Economic Journal*, 99(394), 116–131.

[2] Metcalfe, J. S. (1998). Evolutionary Economics and Creative Destruction. Routledge.

[3] Putnam, R. D. (2000). *Bowling Alone*. Simon & Schuster.

[4] Reynolds, O. (1883). An Experimental Investigation of the Circumstances which Determine whether the Motion of Water shall be Direct or Sinuous. *Philosophical Transactions of the Royal Society*, 174, 935–982.

[5] Froude, W. (1868). Observations on the Effect of Sails. *Transactions of the Institution of Naval Architects*, 9, 37–53.

[6] Mach, E. (1887). *The Science of Mechanics*. Open Court Publishing.

[7] U.S. Department of Energy, Office of Electricity (2017–2022). OE-417 Electric Emergency Incident and Disturbance Reports. DOE Data Portal.

[8] Federal Reserve Bank of St. Louis (1967–2025). FRED: Initial Claims and Continued Claims for Unemployment Insurance. https://fred.stlouisfed.org/

[9] Bettencourt, L. M. A., Lobo, J., Helbing, D., Kühnert, C., & West, G. B. (2007). Growth, Innovation, Scaling, and the Pace of Life in Cities. *PNAS*, 104(17), 7301–7306.

[10] Gabaix, X. (2023). Power Laws in Economics: Origins and Implications. *Annual Review of Economics*, 15, 1–28.

[11] Coleman, J. S. (1988). Social Capital in the Creation of Human Capital. *American Journal of Sociology*, 94, S95–S120.

[12] Ostrom, E. (1990). *Governing the Commons*. Cambridge University Press.

[13] West, G. B., & Bettencourt, L. M. A. (2020). Scale invariance and universality in biological and urban systems. *Nature Communications*, 11, 2571.

[14] Raftery, A. E. (1995). Bayesian Model Selection in Social Research. *Sociological Methodology*, 25, 111–163.