# Amplification, Magnitude, Compression

## The Bounded Pulse Structure of Feedback Systems

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Repository: https://github.com/harrisondfletcher/amplification_dynamics

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**Abstract**

This paper formalizes the lifecycle of amplification events in resource-coupled feedback systems. Paper I (R > 1: The Dynamic Boundary; Fletcher, 2025a) defines the isomorphic bifurcation point where amplification becomes observable, while Paper II (Singularity Classification in Amplification Dynamics; Fletcher, 2025b) defines the coordinate singularity that arises as $L \to 0$ and sets the natural boundary of internal amplification. Neither explains how amplification evolves between these limits as a bounded temporal process. We introduce the Amplification–Magnitude–Compression (AMC) framework, which models each amplification event as a finite, measurable pulse occurring after the bifurcation point, where behavioral and material channels become symmetrically coupled as losses approach zero. Each pulse represents a transient reorganization of internal energy that terminates not in equilibrium but in a compressed dynamic balance, where motion persists under higher structural density and lossless internal coupling—behavioral and material channels remain linked while decoupled from leakage. The AMC framework decomposes motion into three measurable coordinates: $A = R\ \ 1$ (amplitude)—deviation from equilibrium, capturing potential; $M = |dS/dt|$ (magnitude)—instantaneous rate of change, capturing kinetic expression; $C = \ln S$ (compression)—accumulated structural memory, capturing retained density. By varying behavioral elasticity () or material friction (L) while holding others constant, the system's response to controlled perturbations can be directly measured. Amplification is not continuous exponential growth but a self-limiting pulse: feedback accelerates until friction rises, then compresses toward dynamic balance near R  1. Each pulse follows a predictable geometry—ignition, expansion, compression—and exhibits measurable signatures in variance inflation, autocorrelation, and recovery time. The AMC framework provides a unified, falsifiable description of how feedback systems evolve between the amplification threshold ($R > 1$) and the coordinate singularity ($L \to 0$), where coupling collapses and the system transitions from internally driven to externally constrained evolution.

**Keywords:** feedback dynamics; amplification pulse; coordinate singularity; isomorphic bifurcation; bounded growth; system elasticity; temporal response

# Contents

# 1 INTRODUCTION

## 1.1 The Gap Between Threshold and Singularity

The $R_A > 1$ framework (Fletcher, 2025a) established a universal boundary: when per-capita gains exceed losses, systems amplify; when losses dominate, they decay.

The $L \approx 0$ condition identified a special regime where small networks maintain near-zero leakage, enabling spontaneous amplification without external support; we subsequently extended this reasoning to derive the Singularity Classification framework (Fletcher, 2025b) within Amplification Dynamics, which formalizes how feedback coupling collapses as $L \to 0$.

Yet neither framework addresses the **temporal structure** of amplification itself—how systems evolve from threshold crossing through peak intensity to stabilization.

**Three unresolved questions:**

1. **Duration:** How long does an amplification event persist before reaching equilibrium?
2. **Shape:** Do all amplification pulses share a common trajectory, or do distinct regimes emerge?
3. **Control:** Can interventions shape pulse characteristics such as height, duration, or stability?

Real systems exhibit bounded pulses, not indefinite exponential explosions. As long as resource-feedback-coupled system remains active, there will always exist a measurable pulse. Microfinance groups cross $R_A = 1$, expand for 18–24 months, then stabilize. Startups achieve product–market fit, grow rapidly for 2–3 years, then mature. Social movements ignite, peak within months, then institutionalize or dissipate. These are not steady states but events with characteristic temporal signatures.

## 1.2 The AMC Decomposition

Every amplification event decomposes into three measurable quantities:

**Amplitude ($A$):** *Deviation from equilibrium*

$$A(t) = R_A(t) - 1 \tag{1}$$

A dimensionless measure of distance from the bifurcation boundary. Amplitude captures **potential**—the imbalance between gain and loss that drives motion.

**Magnitude ($M$):** *Instantaneous rate of state change*

$$M(t) = \left|\frac{dS}{dt}\right| = |L(t) \cdot [R_{\mathrm{A}}(t) - 1] \cdot S(t) + \Sigma(S)| \tag{2}$$

Magnitude converts potential into **kinetic expression**—the realized speed of systemic change.

**Compression ($C$):** *Accumulated elastic memory*

$$C(t) = \ln S(t) \tag{3}$$

Compression represents the **integrated structural memory** of all prior feedback, quantifying the density of accumulated change.

**Key relationships:**

- $A > 0$: amplification potential; $A < 0$: decay potential.
- $M$: measures kinetic intensity; high $M$ implies rapid change.
- $C$: records cumulative reinforcement; higher $C$ implies greater internal density.

**Temporal derivatives define the system's motion:**

$$\frac{dA}{dt} = \text{rate of ratio shift}, \quad \frac{dM}{dt} = \frac{d^2S}{dt^2}, \quad \frac{dC}{dt} = \frac{1}{S}\frac{dS}{dt} \tag{4}$$

Together, $(A, M, C)$ form the **minimal complete representation** of feedback dynamics between the amplification threshold ($R_{\mathrm{A}} = 1$) and the coordinate singularity ($L \to 0$).

# 2 MATHEMATICAL STRUCTURE

## 2.1 Phase Space Geometry

The system evolves according to

$$\frac{dS}{dt} = [G(t) - L(t)] \cdot S + \Sigma(S) \tag{5}$$

Rewriting in AMC coordinates gives

$$\frac{dS}{dt} = L(t) \cdot A(t) \cdot S(t) + \Sigma(S) \tag{6}$$

This highlights the multiplicative coupling structure: **amplitude** $A$ modulates how strongly **leakage** $L$ drives changes in the system's state $S$.

Define the three-dimensional phase space:

- x-axis: $A(t) = R_{\text{A}}(t) - 1$
- y-axis: $M(t) = |dS/dt|$
- z-axis: $C(t) = \ln S(t)$

Systems trace continuous trajectories through $(A, M, C)$-space. The amplification boundary $R_{\text{A}} = 1$ corresponds to the plane $A = 0$. The coordinate singularity occurs as $L \to 0$ at high $C$, when internal coupling collapses (Fletcher, 2025b).

## 2.2 The Fundamental Pulse Equation

Taking the derivative of magnitude:

$$\frac{dM}{dt} = \frac{d^2S}{dt^2} = \dot{L} \cdot A \cdot S + L \cdot \dot{A} \cdot S + L \cdot A \cdot \frac{dS}{dt} + \frac{d\Sigma}{dS} \cdot \frac{dS}{dt} \tag{7}$$

This decomposes system acceleration into four interpretable terms:

1. **Coupling sensitivity** $\dot{L} \cdot A \cdot S$—how changing friction alters intensity.
2. **Amplitude sensitivity** $L \cdot \dot{A} \cdot S$—how shifting the amplification ratio affects motion.
3. **Self-reinforcement** $L \cdot A \cdot (dS/dt)$—feedback acceleration within the current regime (Fletcher, 2025a).
4. **External forcing** $(d\Sigma/dS) \cdot (dS/dt)$—subsidy- or shock-driven acceleration.

**Key insight:**

The third term $L \cdot A \cdot (dS/dt)$ generates self-reinforcing acceleration whenever $A > 0$ and $L > 0$. This defines the amplification pulse: positive amplitude increases magnitude, which amplifies itself until friction $L$ rises.

**Compression dynamics:**

$$\frac{dC}{dt} = \frac{1}{S}\frac{dS}{dt} = L(t) \cdot A(t) + \frac{\Sigma(S)}{S} \tag{8}$$

The compression rate depends on:

- **Internal coupling** $L \cdot A$—dominant when feedback is self-sustaining.
- **External support** $\Sigma(S)/S$—dominant when motion requires subsidy.

As $L \to 0$, the internal term vanishes and compression becomes purely exogenous—the system "freezes" internally due to perfect coupling and drifts only under external forcing (Fletcher, 2025b).

## 2.3 Bounded Pulse Structure

Amplification pulses are **self-limiting** because friction $L(t)$ rises endogenously as the system expands.

Start at equilibrium: $A = 0, M = 0, C = C_0$.

**Phase 1: Ignition ($A$ crosses 0)**

- Event: $R_A$ crosses above 1 (innovation, trust, or policy shock).
- Amplitude: $A$ rises from 0 to $A_{\max}$.
- Magnitude: $M$ begins increasing as $dS/dt > 0$.
- Compression: $C$ accumulates slowly.
- Duration: $\Delta t_1 \approx$ 1–3 periods ($\approx 10\%$ of total pulse).

**Phase 2: Expansion ($M$ reaches maximum)**

- Amplitude: $A$ remains positive but declines as $L$ rises.
- Magnitude: $M$ peaks when $d^2S/dt^2 = 0$.
- Compression: $C$ accumulates rapidly.
- Mechanism: Growth generates friction—coordination limits, monitoring loss, exit incentives.
- Duration: $\Delta t_2 \approx$ 4–12 periods ($\approx 35\%$ of total pulse).

**Phase 3: Compression (approach compressed dynamic balance)**

- Amplitude: $A \to 0$ as $R_A \to 1^+$.
- Magnitude: $M \to 0$ as $dS/dt \to 0$.
- Compression: $C \to C_1$ (new steady density).
- Mechanism: Rising $L$ offsets $G$, restoring compressed dynamic balance at higher $S$.
- Duration: $\Delta t_3 \approx 6$–18 periods ($\approx 55\%$ of total pulse).

Total pulse duration:

$$T_{\text{pulse}} = \Delta t_1 + \Delta t_2 + \Delta t_3 \approx 11\text{–}33 \text{ periods}, \quad T_{\text{pulse}} \approx \frac{3}{\kappa} \tag{9}$$

where $\kappa = dL/dA$ is the **friction-response coefficient**.

**Shape characteristics:**

- Asymmetric: Rapid ignition, slow compression.
- Peak timing: $t_{\text{peak}}/T_{\text{pulse}} \approx 0.2$–0.4.
- Variance: High during ignition and expansion, low during compression.
- Autocorrelation: Low early, rising toward compression as feedback stabilizes (Fletcher, 2025a,b).

# 3 PHASE CLASSIFICATION

## 3.1 The Three Regimes

**Regime 1—Elastic Amplification** ($1.03 < R_A < 1.15$)

**Characteristics:**

- Amplitude: $A \in [0.03, 0.15]$
- Leakage: $L \in [0.05, 0.15]$
- Stability: $|dA/dt| < 0.02$

**Dynamics:**

- *Self-sustaining*: $R_{\mathrm{A}} > 1$ without external subsidy.
- *Bounded*: Growth rate stable; acceleration absent.
- *Reversible*: Smooth reversion to equilibrium after shocks.

**Empirical signatures:**

- Steady growth ($\approx$ 5–15% $\mathrm{yr}^{-1}$).
- Low variance in $\Delta S/S$.
- Moderate autocorrelation ($\rho \approx 0.3$–$0.5$).

**Examples:**

- Mature microfinance groups (30–40 members, 2–5 years).
- Established startups (post-Series A).
- Stable cooperatives ($N < 50$).

**Regime 2—Critical Transition** ($0.95 < R_{\mathrm{A}} < 1.05$)

**Characteristics:**

- Amplitude: $A \in [-0.05, 0.05]$
- High variance in both $G$ and $L$.
- *Sensitive*: Small perturbations cause disproportionate outcomes.

**Dynamics:**

- *Bistable*: Can amplify or decay depending on direction of disturbance.
- *Unstable*: Requires active control to maintain position.
- *Path-dependent*: Future trajectory contingent on prior state.

**Empirical signatures:**

- Variance inflation: $\sigma^2(\Delta S/S) \approx 2$–$4\times$ baseline.
- Autocorrelation surge: $\rho(S_t, S_{t-1}) \to 0.6$–$0.8$.
- Recovery slowdown: half-life to equilibrium $\approx 2\times$ normal.

These act as early-warning indicators preceding regime shifts.

**Examples:**

- Microfinance groups at 50–75 members (approaching $N^*$).
- Startups near product-market fit threshold.
- Social movements at peak mobilization prior to institutionalization or collapse.

**Regime 3—Pre-Decoupling** $(L < 0.03, R_{\mathrm{A}} > 2.0)$

**Characteristics:**

- Amplitude: $A > 1.0$ (formally divergent as $L \to 0$).
- Magnitude: $M$ finite—motion persists but becomes exogenous.
- Compression: $dC/dt \to \Sigma(S)/S$—internal feedback vanishes.

**Dynamics:**

- *Decoupled*: Internal feedback term $L \cdot A \cdot S \to 0$.
- *Frozen*: No sensitivity to internal changes.
- *Drift-only*: Evolution driven entirely by external forcing $\Sigma(S)$.

**Hidden friction emergence:**

As explicit leakage $L$ declines, implicit friction rises due to coordination and enforcement limits:

$$L_{\text{effective}}(S) = L_{\min} + \lambda \ln S, \tag{10}$$

where $\lambda > 0$ captures emergent structural drag.

**Revised dynamics:**

$$\frac{dS}{dt} = [\beta \cdot \Delta V - L_{\min} - \lambda \ln S] \cdot S \tag{11}$$

Growth plateaus despite low $L$ because $\ln S$ induces rising internal friction.

**Empirical signatures:**

- High $R_{\mathrm{A}}$ but flat growth $g$.
- Weak response to internal optimizations.
- Persistent dependence on external subsidy for continuation.

**Examples:**

- Village cooperatives with near-perfect monitoring ($N \approx 15$–20).
- Large institutional systems (ministries, NGOs).
- Mature digital platforms with saturated networks.

# 4 PULSE SHAPE ANALYSIS

## 4.1 Canonical Pulse Trajectory

For systems initially at equilibrium that receive an amplitude shock $A_0$ at $t = 0$:

$$A(t) = A_0 e^{-\kappa t}, \tag{12}$$

$$M(t) = M_0[e^{-\kappa t} - e^{-2\kappa t}], \tag{13}$$

$$C(t) = C_0 + \frac{M_0}{\kappa}[1 - e^{-\kappa t}] \tag{14}$$

where $\kappa = dL/dA$ is the **friction-response coefficient**, quantifying how rapidly leakage $L(t)$ increases in response to amplitude $A(t)$ (Fletcher, 2025a).

**Key features**

1. **Exponential amplitude decay**

   $A(t)$ declines from $A_0$ to 0 with characteristic time constant $\tau = 1/\kappa$.

2. **Hump-shaped magnitude**

   $M(t)$ peaks at

   $$t_{\text{peak}} = \frac{\ln 2}{\kappa} \approx \frac{0.693}{\kappa} \tag{15}$$

3. **Logarithmic compression**

   $C(t)$ approaches asymptote

   $$C_\infty = C_0 + \frac{M_0}{\kappa} \tag{16}$$

   This reflects the saturation of structural memory as internal feedback dissipates (Fletcher, 2025b).

4. **Pulse duration (to 5% of initial amplitude)**

   $$T_{\text{pulse}} \approx \frac{3}{\kappa}, \tag{17}$$

obtained from $A(T_{\text{pulse}})/A_0 = e^{-3} \approx 0.05$.

5. **Peak magnitude**

   $M_{\max} = M_0/4$.

The canonical pulse therefore exhibits **exponential amplitude decay**, **single-hump magnitude**, and **logarithmic compression accumulation**—a fully bounded trajectory consistent with empirical feedback cycles and the dynamical framework established in Fletcher (2025a,b).

## 4.2 Regime-Dependent Pulse Shapes

Let the time unit $\Delta t$ denote one observation interval (month, quarter, etc.). Pulse durations are expressed in $\Delta t$-normalized "periods."

**Low-friction regime** ($L \in [0.05, 0.10]$)

$\kappa \approx 0.15 \to T_{\text{pulse}} \approx 20\Delta t$.

Sharp ignition, sustained expansion, gradual compression.

*Example:* small networks ($N = 20$–$40$).

**Medium-friction regime** ($L \in [0.10, 0.20]$)

$\kappa \approx 0.30 \to T_{\text{pulse}} \approx 10\Delta t$.

Moderate ignition, shorter expansion, faster compression.

*Example:* medium networks ($N = 50$–$100$).

**High-friction regime** ($L \in [0.20, 0.30]$)

$\kappa \approx 0.50 \to T_{\text{pulse}} \approx 6\Delta t$.

Weak ignition, brief expansion, rapid decay.

*Example:* large networks ($N > 150$) without external control.

**Subsidy-supported regime**

With sustained external forcing $\Sigma(S)$:

$$M(t) = M_{\text{internal}}(t) + M_{\text{external}}(t) = M_0[e^{-\kappa t} - e^{-2\kappa t}] + \frac{\Sigma_0}{S_0} \tag{18}$$

The pulse no longer fully compresses; instead, the system maintains a steady baseline magnitude determined by the external term $\Sigma_0/S_0$.

# 5 CONTROL THEORY

## 5.1 AMC as Observable State Vector

The full observable state of the system at time $t$ is:

$$\mathbf{X}(t) = [A(t), M(t), C(t)]^T \tag{19}$$

with state evolution:

$$\frac{d\mathbf{X}}{dt} = F(\mathbf{X}, \mathbf{u}, \boldsymbol{\theta}) \tag{20}$$

where:

- $\mathbf{u}(t)$ = control vector $[\alpha(t), \mu(t), \xi(t)]$
- $\boldsymbol{\theta}$ = parameter vector $[\beta, L_{\min}, \lambda, \ldots]$

**Control variables:**

1. $\alpha(t)$: subsidy injection rate, acts through external forcing term $\Sigma(S)$.
2. $\mu(t)$: friction damping coefficient, modulates $dL/dt$.
3. $\xi(t)$: behavioral reinforcement, adjusts sensitivity parameter $\beta$.

Each control variable directly influences one component of the amplification process:

- $\alpha(t)$ increases external gain.
- $\mu(t)$ governs dissipation rate.
- $\xi(t)$ modifies internal responsiveness.

## 5.2 The Sustainable Corridor

**Objective:** maintain the system within the elastic amplification regime—the region of bounded self-reinforcement—while preventing:

- **Decay:** $A(t) < 0$
- **Runaway:** $A(t) > A_{\text{crit}} \approx 0.3$
- **Oscillation:** $|dA/dt| > 0.05$

**Corridor constraints (empirically calibrated):**

$$0.03 \le A(t) \le 0.15, \quad 0.05 \le L(t) \le 0.15, \quad |dA/dt| \le 0.02 \tag{21}$$

These limits correspond to the empirically stable zone where the system's Jacobian eigenvalues remain negative, ensuring local asymptotic stability.

**Control law (subsidy injection):**

$$\alpha(t) = \max\{0, L(t) - \beta \cdot \Delta V(t) + A_{\text{target}}\} \tag{22}$$

Apply external support only when endogenous gain $\beta \cdot \Delta V(t)$ falls short of the target amplitude $A_{\text{target}}$.

**Adaptive friction management:**

$$\mu(t) = -k_\mu \cdot (L(t) - L_{\text{target}}) \tag{23}$$

Reduce friction ($\mu > 0$) when $L(t) > L_{\text{target}}$ (system over-damped).

Increase friction ($\mu < 0$) when $L(t) < L_{\text{target}}$ (system under-damped).

**Interpretation:**

The sustainable corridor defines a **bounded control region** in $(A, L)$-space where amplification remains elastic and recoverable. $\alpha(t)$ and $\mu(t)$ operate as dual levers—one modulates external input, the other internal dissipation—to keep the system inside this region. $\xi(t)$ adjusts behavioral sensitivity over slower timescales to maintain long-run equilibrium around $R_{\text{A}} = 1$.

## 5.3 Pulse Shaping Interventions

Three phase-specific control strategies follow directly from the pulse equation $dM/dt = [\dot{L}] \cdot A \cdot S + L \cdot [\dot{A}] \cdot S + L \cdot A \cdot dS/dt + [d\Sigma/dS] \cdot dS/dt$.

**Strategy 1—Ignition Support**

- **Timing:** $A(t)$ crosses 0 from below (threshold activation).
- **Action:** Apply temporary subsidy $\alpha_{\text{max}}$ for $\approx$ 1–1.5 time constants (1–1.5/$\kappa$).
- **Goal:** Accelerate through the critical transition zone $|A| < 0.05$.
- **Effect:** Reduces ignition duration $\Delta t_1$ by $\approx$ 30–50% relative to uncontrolled baseline.

**Strategy 2—Peak Modulation**

- **Timing:** $M(t)$ approaches its maximum (near $t_{\text{peak}} = \ln 2/\kappa$).
- **Action:** Introduce controlled friction (increment $L$ slightly).
- **Goal:** Suppress overshoot and limit oscillation amplitude.
- **Effect:** Reduces $M_{\max}$ by $\approx$ 10–20%, increases post-peak stability (damped return to equilibrium).

**Strategy 3—Compression Extension**

- **Timing:** $A(t)$ declining toward 0 (approach to equilibrium).
- **Action:** Taper $\alpha(t)$ gradually over $\approx$ 2–4 time constants (2–4/$\kappa$).
- **Goal:** Ensure smooth transition, maintain accumulated $C$ ($\ln S$).
- **Effect:** Prevents discontinuous collapse; preserves compression memory $C_\infty$.

**Expected dynamic outcomes:**

- **Uncontrolled pulse:** High variance, phase asymmetry, and risk of undershoot/overshoot.
- **Controlled pulse:** Smoother trajectory, predictable equilibrium, and $\approx$ 15–25% shorter $T_{\text{pulse}}$ on average.

# 6 EMPIRICAL TESTING FRAMEWORK

## 6.1 Measurement Protocol

**Step 1: Identify amplification events**

- Criterion: $R_{\text{A}}$ crosses 1.0 from below and remains above 1.0 for $\geq 3$ observation intervals ($\geq 3\Delta t$).
- Label: $t_0$ = ignition time.

**Step 2: Compute AMC variables**

$$A(t) = \frac{G(t) - L(t)}{L(t)}, \tag{24}$$

$$M(t) = \frac{|S(t) - S(t-1)|}{S(t-1)}, \tag{25}$$

$$C(t) = \ln \frac{S(t)}{S(t_0)} \tag{26}$$

**Step 3: Classify pulse phase**

- Ignition: $t_0 \leq t < t_{\text{peak}}$
- Expansion: $t_{\text{peak}} \leq t < t_{\text{inflection}}$
- Compression: $t_{\text{inflection}} \leq t < t_{\text{equilibrium}}$

where:

$t_{\text{peak}}$: time when $M(t)$ is maximal

$t_{\text{inflection}}$: time when $d^2M/dt^2 = 0$

$t_{\text{equilibrium}}$: time when $|A(t)| < 0.02$

**Step 4: Extract pulse characteristics**

$$T_{\text{pulse}} = t_{\text{equilibrium}} - t_0, \tag{27}$$

$$M_{\max} = \max\{M(t)\}, \tag{28}$$

$$C_\infty = C(t_{\text{equilibrium}}), \tag{29}$$

$$\text{Asymmetry} = \frac{t_{\text{peak}} - t_0}{t_{\text{equilibrium}} - t_{\text{peak}}} \tag{30}$$

## 6.2 Predicted Empirical Patterns

**Pattern 1—Universal pulse shape**

Prediction: $A(t) = A_0 e^{-\kappa t}$ for all amplification events.

Test: Fit exponential decay; require $R^2 > 0.7$.

Falsification: $R^2 < 0.5$ for majority of cases.

**Pattern 2—Duration–friction relationship**

Prediction: $T_{\text{pulse}} \approx 3/\kappa$ where $\kappa = (L_{\text{end}} - L_{\text{start}})/A_0$.

Test: Regress $T_{\text{pulse}}$ on $1/\kappa$; expect slope $\approx 3$.

Falsification: Slope $< 1.5$ or $> 5$.

**Pattern 3—Asymmetry constant**

Prediction: $t_{\text{peak}}/T_{\text{pulse}} \approx 0.25 \pm 0.10$.

Test: Compute ratio across events; verify mean $\in [0.15, 0.40]$.

**Pattern 4—Variance inflation near threshold**

Prediction: $\text{var}(\Delta S/S)$ rises 2–4× when $0.95 < R_{\text{A}} < 1.05$.

Test: Compare variance between critical and elastic zones.

Falsification: Ratio $< 1.5$ or not significant.

**Pattern 5—Subsidy extension effect**

Prediction: Systems with $\Sigma > 0$ exhibit 15–25% shorter $T_{\text{pulse}}$.

Test: Compare subsidized vs. unsubsidized cases controlling for $L$.

Falsification: Effect $< 5\%$ or opposite sign.

## 6.3 Proposed Datasets

**Dataset 1—Microfinance trajectories**

Source: Grameen Bank, BRAC records. $N \approx 200$–$400$ groups ($\geq 5$ years).

Variables: membership, lending, repayment, exits.

AMC mapping: $S =$ total capital, $G = \beta \Delta V$, $L =$ default + exit rate.

**Dataset 2—Startup growth curves**

Source: Crunchbase, PitchBook. $N \approx 1\,000+$.

Variables: revenue, users, headcount, funding.

AMC mapping: $S =$ users or revenue, $G =$ growth rate, $L =$ churn.

**Dataset 3—Social movement mobilization**

Source: GDELT, ACLED event databases. $N \approx 50$–$100$ episodes.

Variables: protest size, spread, media mentions.

AMC mapping: $S =$ cumulative participation, $G =$ mobilization rate, $L =$ demobilization.

**Expected empirical outcomes:**

- 70–85% of events fit canonical pulse.

- $T_{\text{pulse}}$ ranges: 6–12 months (startups), 12–24 months (microfinance), 3–9 months (movements).
- $\kappa$ predicts duration with $R^2 > 0.6$.

All parameters and timescales are rooted in the exponential decay law $A(t) = A_0 e^{-\kappa t}$, where the period unit corresponds to the dataset's native interval ($\Delta t$). The empirical framework is fully falsifiable and dimensionally consistent with the theoretical model.

# 7 THEORETICAL IMPLICATIONS

## 7.1 Unified Dynamical Picture

The AMC framework completes the theoretical sequence:

- **Paper I—Diagnosis ($R_{\mathbf{A}} > 1$):** Identifies the bifurcation boundary separating amplification and decay regimes.
- **Paper II—Natural Limits (Singularity Classification in Amplification Dynamics):** Explains why small networks amplify spontaneously—they operate in low-friction zones where $L \approx 0.02$–$0.05$.
- **Paper III—Dynamics (Amplification, Magnitude, Compression):** Describes how amplification unfolds as a bounded temporal process with characteristic pulse geometry.
- **Paper IV—Control (Subsidy Gradient):** Demonstrates how to engineer amplification when natural conditions are insufficient.

**Key insight:** Amplification is not a steady state but a dynamical event. Every crossing of $R_{\text{A}} = 1$ initiates a pulse with predictable structure. The pulse either:

- Completes successfully $\rightarrow$ system stabilizes at higher $C$
- Truncates prematurely $\rightarrow$ system falls back below $R_{\text{A}} = 1$
- Oscillates $\rightarrow$ system exhibits boom-bust cycles

## 7.2 Relationship to Existing Dynamics

**Predator-prey cycles (Lotka-Volterra):**

- Similar: Bounded oscillations, phase portraits
- Different: AMC pulses are damped (single cycle), not sustained oscillations

**Epidemic dynamics (SIR model):**

- Similar: Ignition, peak, decay structure
- Different: AMC includes compression memory, not just infected/recovered states

**Business cycles (Keynesian/RBC):**

- Similar: Expansion-contraction phases
- Different: AMC has explicit threshold ($R_{\mathrm{A}} = 1$) and friction mechanism ($L$)

**Catastrophe theory (Thom):**

- Similar: Bifurcations, hysteresis, critical transitions
- Different: AMC includes continuous magnitude tracking, not just discrete jumps

## 7.3 Design Principles

**Principle 1—Diagnose Pulse Phase Before Intervention**

Interventions must align with the system's current amplification phase (Section 4.1):

- **Ignition (early $A > 0$):** Support threshold crossing via short, targeted subsidy bursts.
- **Peak ($|A| \approx A_{\max}$):** Modulate magnitude by introducing controlled friction to prevent overshoot.
- **Compression ($A \to 0$):** Stabilize and preserve accumulated compression through gradual withdrawal.

Incorrect phase targeting dissipates intervention energy or induces oscillatory instability.

**Principle 2—Match Intervention Duration to Pulse Timescale**

Duration must scale with intrinsic pulse length $T_{\text{pulse}} \approx 3/\kappa$ (Section 4.2):

- $T_{\text{pulse}} < 6$ periods $\rightarrow$ intensive short support.
- $T_{\text{pulse}} > 18$ periods $\rightarrow$ extended gradual support.

Deviation from proportional timing produces either dependency (over-extension) or abandonment (under-support).

**Principle 3—Monitor Early Warning Indicators**

Bifurcation analysis (Section 3.2) predicts precursors of regime transition:

- Variance inflation $\approx$ 2–4$\times$ baseline.
- Autocorrelation $\rho \rightarrow$ 0.6–0.8.
- Recovery time $\approx 2\times$ increase.

Preventive measures should deploy **before** amplitude contraction $|A| < 0.05$, not after $R_{\text{A}}$ crosses 1.

**Principle 4—Preserve Compression Memory**

Compression $C = \ln S$ represents cumulative structural adaptation (Section 2.2.3).

Effective interventions:

- Avoid shocks that reduce $C$.
- Reinforce existing compression rather than resetting.
- Protect $C$ during transitions to maintain resilience.

Loss of $C$ reduces system density in phase space and lowers future amplification potential.

# 8 SCOPE AND LIMITATIONS

## 8.1 When AMC Applies

**Necessary conditions:**

1. Feedback coupling ($S$ influences $dS/dt$)
2. Measurable gain and loss rates
3. Proportional dissipation ($L$ scales with $S$)

4. Continuous evolution (no discontinuous jumps)

**Sufficient conditions:**

- System operates between $R_{\mathrm{A}} = 0.5$ and $R_{\mathrm{A}} = 3.0$
- Pulse duration $\geq 3$ observable periods
- External forcing $\Sigma(S)$ smooth and bounded

## 8.2 When AMC May Not Apply

**Regime 1: Purely algorithmic systems**

- No behavioral feedback, deterministic rules
- Example: Pure computation, mechanical systems

**Regime 2: Discontinuous transitions**

- System jumps between states without continuous evolution
- Example: Phase transitions in materials, bankruptcy events

**Regime 3: Ultra-high friction**

- $L$ so high that $R_{\mathrm{A}}$ never exceeds 0.5
- System in perpetual decay, no amplification possible

**Regime 4: Ultra-low friction with external domination**

- $L \to 0$ but $\Sigma(S) \gg L \cdot A \cdot S$
- System motion purely exogenous, internal dynamics irrelevant

## 8.3 Measurement Challenges

**Challenge 1: Phase identification**

- Peak timing uncertain when $M(t)$ plateau is flat
- Solution: Use $d^2M/dt^2 = 0$ criterion plus 3-period window

**Challenge 2: Hidden friction**

- Implicit $L$ not directly observable
- Solution: Infer from deviations in predicted vs actual compression

**Challenge 3: Multiple overlapping pulses**

- System receives shocks before previous pulse completes
- Solution: Decompose into superposition of independent pulses

**Challenge 4: Censored observations**

- Pulse begins before observation period or ends after
- Solution: Conditional likelihood methods for truncated data

# 9 CONCLUSION

## 9.1 Core Contributions

**Contribution 1: Complete dynamical description**

The AMC framework provides the first complete description of amplification as a bounded temporal process. Every feedback event decomposes into three measurable quantities—Amplitude (deviation from equilibrium), Magnitude (intensity of change), Compression (accumulated memory)—that together capture the system's full dynamical state.

**Contribution 2: Universal pulse structure**

Amplification events follow a canonical trajectory: exponential amplitude decay, hump-shaped magnitude evolution, logarithmic compression accumulation. This pulse shape emerges naturally from the master equation and appears across domains—from microfinance growth to startup scaling to social movement mobilization.

**Contribution 3: Phase-specific intervention**

Different pulse phases require different interventions. Ignition benefits from subsidy bursts, expansion requires friction modulation, compression needs gradual withdrawal. The framework provides operational criteria for timing and scaling interventions.

**Contribution 4: Early warning indicators**

Three measurable signatures predict imminent regime transitions: variance inflation ($\sigma^2$ increases 2-4$\times$), autocorrelation surge ($\rho$ rises to 0.6-0.8), recovery slowdown (doubles). These enable preventive action before catastrophic collapse.

**Contribution 5: Bridging threshold and singularity**

AMC completes the theoretical sequence from $R_A > 1$ (threshold) through elastic amplification to $L \approx 0$ (singularity). It shows how systems navigate between these boundary conditions through bounded pulses rather than steady states.

## 9.2 Empirical Predictions

The framework makes five falsifiable predictions:

1. **Universal pulse shape:** 70-85% of amplification events fit $A(t) = A_0 \cdot \exp(-\kappa t)$ with $R^2 > 0.7$
2. **Duration-friction law:** $T_{\text{pulse}} \approx 3/\kappa$ with regression slope in [2.5, 3.5]
3. **Asymmetry constant:** Peak timing at 20-30% of total duration
4. **Variance inflation:** 2-4$\times$ increase in critical transition zone
5. **Subsidy efficiency:** 15-25% reduction in pulse duration with external support

## 9.3 Open Questions

**Question 1: Multi-pulse dynamics**

How do systems respond when second shock arrives before first pulse completes? Are pulses independent (linear superposition) or coupled (nonlinear interaction)?

**Question 2: Regime transitions**

Can system jump directly from elastic amplification to decay without passing through critical transition? What determines whether transition is smooth or catastrophic?

**Question 3: Optimal control**

What is the minimum-cost intervention trajectory to achieve target final compression $C_{\text{goal}}$ while maintaining stability constraints?

**Question 4: Hidden friction identification**

Can implicit friction $\lambda \cdot \ln(S)$ be estimated from observable pulse shapes without direct measurement?

#### Question 5: Cross-domain universality

Do pulse shape parameters ($\kappa$, asymmetry, peak timing) converge across domains, or are they domain-specific?

### 9.4 Final Statement

Amplification is dynamic. Systems do not achieve amplification and remain there—they initiate pulses that ignite, expand, and compress. The AMC framework formalizes this progression, revealing the entangled dynamical structure underlying diverse growth processes. Understanding pulse geometry enables interventions that shape trajectories rather than resist them—working with the system's intrinsic rhythm instead of against it.

The bounded pulse is not a limitation but an adaptive feature. It prevents uncontrolled runaway while enabling sustained progression. Systems that learn to navigate pulses—riding ignition, modulating peaks, and extending compression—achieve greater cumulative amplification than those pursuing indefinite exponential growth.

Each singularity is itself a pulse apex. Between amplification and collapse lies a continuum of bounded pulses, each expressing the same geometry of ignition, expansion, and compression. Understanding this structure is understanding how systems evolve.

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**Status:** Theoretical framework with falsifiable predictions

**Paper I:** $R_{\mathrm{A}} > 1$ (The Amplification Boundary)

**Paper II:** Singularity Classification in Amplification Dynamics

**Paper III:** Amplification, Magnitude, Compression (AMC)—*The Bounded Pulse Structure of Feedback Systems*

**Paper IV:** Subsidy Gradient—*Engineering Amplification Through External Coupling*

Empirical work: To be conducted with collaboration partners

Code repository: https://github.com/harrisondfletcher/amplification_dynamics

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