Chapter 5: One-way ANOVA and Randomized Experiments - Part 2

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One-Way ANOVA: F-Test

- Is there a difference in mean response among the groups?
- For K groups,
- Null hypothesis: all group means are the same

$$H_0 : \mu_1 = \mu_2 = ... = \mu_{K-1} = \mu_K$$

- Alternative hypothesis: at least two groups have different means

$$H_1 : \mu_i \neq \mu_j$$

at least for one combination of i and j

One-Way ANOVA: F-Test...

- An alternate way to express the hypotheses
- Null hypothesis: all group means are the same

$$H_0 : \alpha_1 = \alpha_2 = ... = \alpha_{K-1} = \alpha_K = 0$$

- Alternative hypothesis: at least two groups have different means

$$H_1 : \alpha_i \neq 0$$

Conditions for One-way ANOVA

- Model: $Y = \mu + \alpha_i + \in$
- Assumption on α_i
  - The effects are constant and additive
- Assumptions on $\in$
  - The residuals are random and independent
  - The residuals have the same variability in each group
  - The residuals are normally distributed

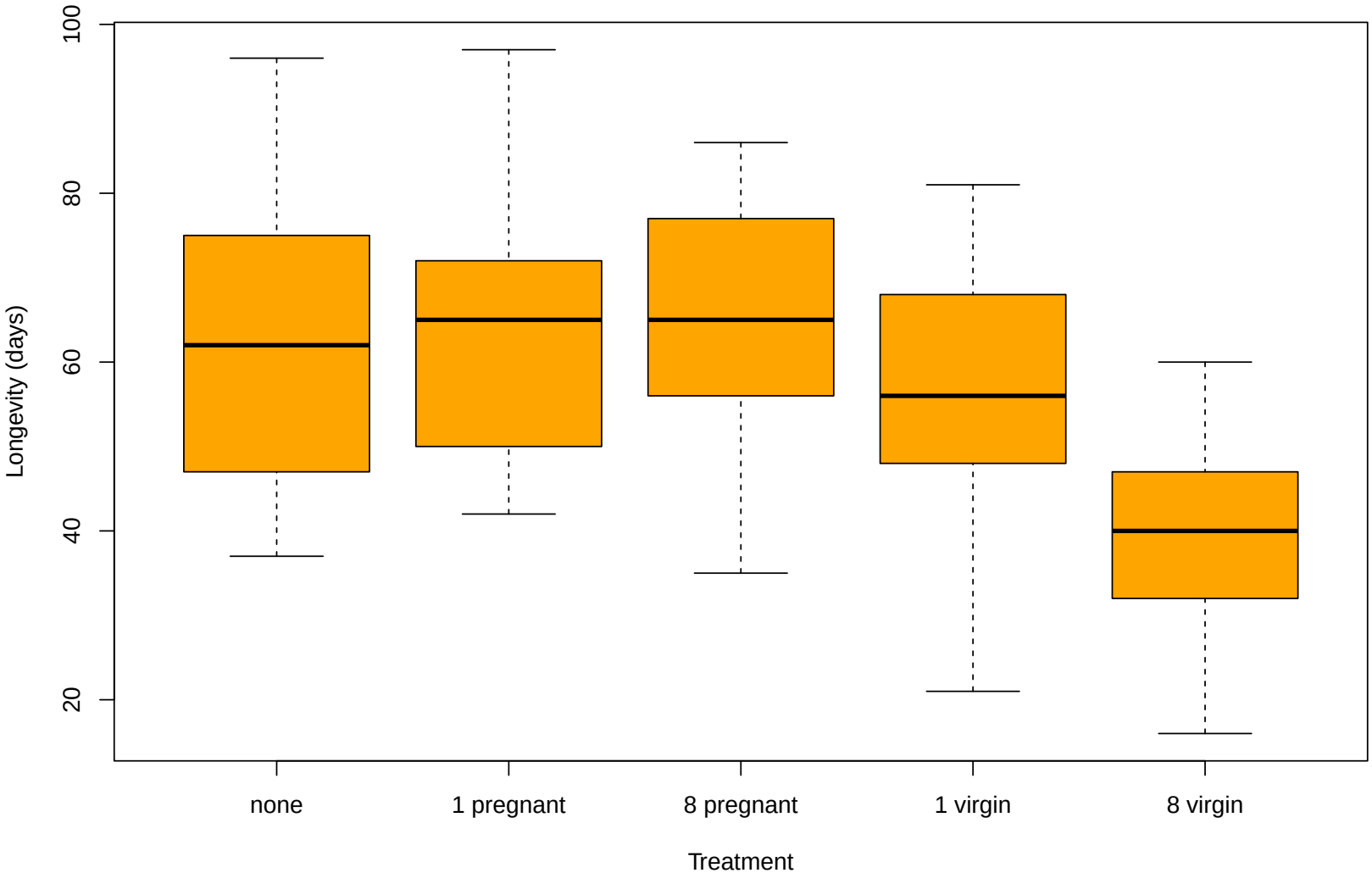
Example 5.11 - Fruit fly lifetimes

- An experiment:
  - 125 male fruit flies were randomly assigned into one of the 5 treatment groups ($K = 5$)
  - Treatment groups
    - None: no tube-mates
    - Pregnant1: one pregnant female
    - Pregnant8: eight pregnant females
    - Virgin1: one virgin female
    - Virgin8: eight virgin females
  - Response: *Longevity* : lifetime of each male fruit fly

- Research question: Does the sexual activity of male fruit flies shorten length of life?

- Note: no mating occurs in control/pregnant groups

Fruit fly lifetimes: Longevity vs. Treatment groups

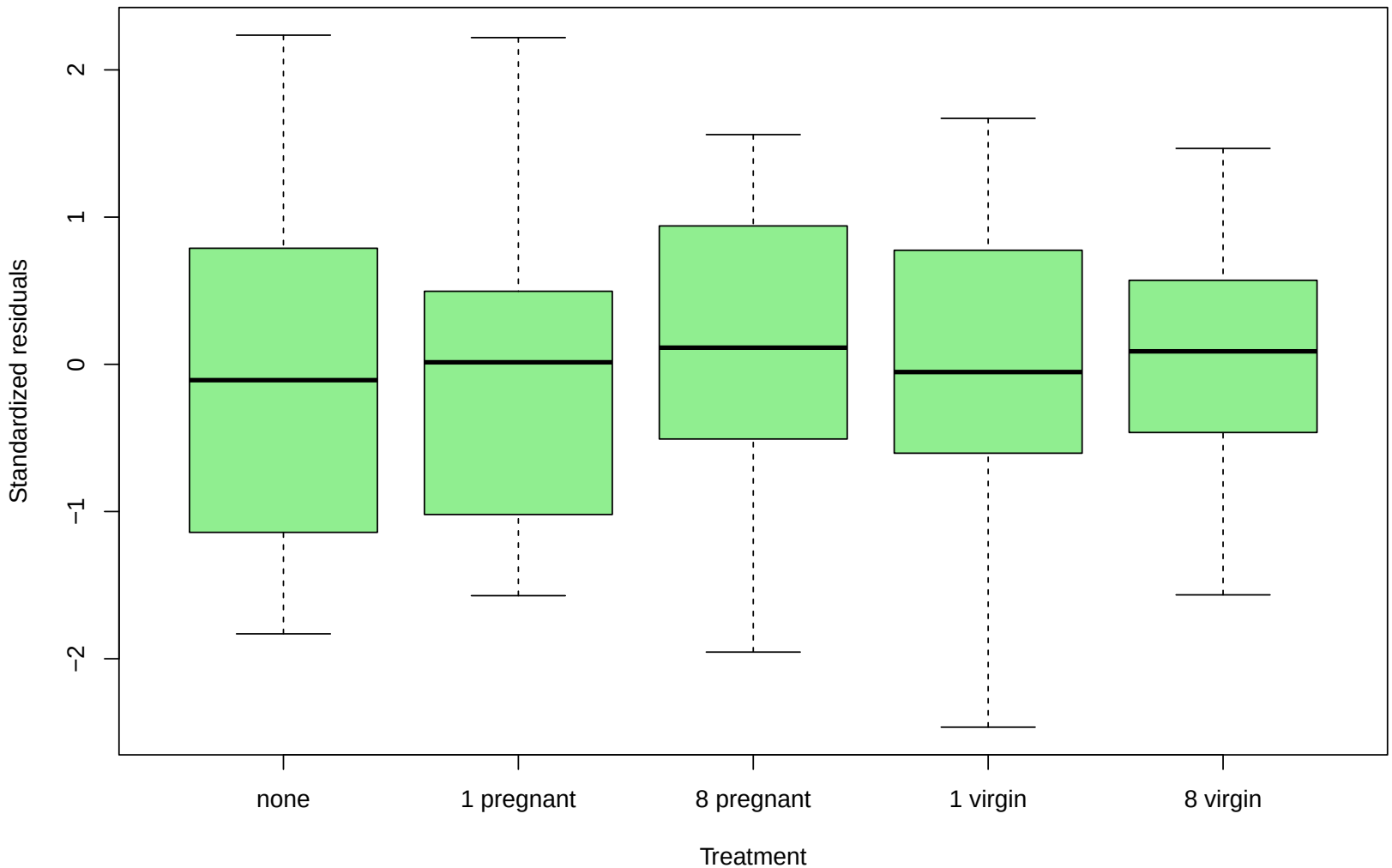


Fruit fly lifetimes: Summary statistics by Treatment groups

```
##              n Median Q1 Q3       SD  Mean
## none        25     62 47 75 16.45215 63.56
## 1 pregnant  25     65 50 72 15.65248 64.80
## 8 pregnant  25     65 56 77 14.53983 63.36
## 1 virgin    25     56 48 68 14.92838 56.76
## 8 virgin    25     40 32 47 12.10207 38.72
```

- There is no significant difference between means and corresponding medians
  - Distributions are symmetric
- The standard deviations are similar except in '8 virgin' group
  - Needs further investigation
- '8 virgin' group seems to be different from the rest: look at the inter-quartile range

Fruit fly lifetimes: Residuals have the same variability in each group?



Fruit fly lifetimes: Residuals have the same variability in each group?

- Ratio between largest group standard deviation to smallest group standard deviation
  - If ratio is less than 2: we can assume residuals have the same variability

- $\max(S_i) = 16.45$

- $\min(S_i) = 12.10$

- $\frac{max(S_i)}{min(S_i)} = \frac{16.45}{12.10} = 1.36 < 2$

Fruit fly lifetimes: Inference

- Hypotheses
  - $H_0 : \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = 0$
  - $H_1 : Some \alpha_i \neq 0$

```
##              Mean Grand_mean Treatment_effect
## none       63.56      57.44             6.12
## 1 pregnant 64.80      57.44             7.36
## 8 pregnant 63.36      57.44             5.92
## 1 virgin   56.76      57.44            -0.68
## 8 virgin   38.72      57.44           -18.72
```

- Test statistic: F $= 13.61$
- P-value < 0.05
- Decision / Conclusion:
  - We reject H_0 at 0.05 level of significance
  - At least one of the groups means are different from the rest. Type of tube-mates can affect the mean lifetime of male fruit flies.