STAT 302 - Chapter 2 : Inference for Simple Linear Regression - Part 1

Harsha Perera

Inference for Regression Slope

- t-Test for Slope
- Confidence Interval for Slope

t-Test for the Slope Parameter - β_1

Question : We want to assess whether the slope for sample data provides significant evidence that the slope for the population differs from zero ? ($H_a : \beta_1 \neq 0$)

- We need an estimate of β_1 which is $\hat{\beta}_1$ (from R)
- We need an estimate of $\sigma_{\hat{\beta}_1}$ which is $SE_{\hat{\beta}_1}$ (from R)
- t-test statistic is $t = \frac{\hat{\beta}_1}{SE(\hat{\beta}_1)}$

- Think about the null hypothesis of ($H_0 : \beta_1 = 0$) of no linear relationship. Under the null hypothesis, the test statistic follows a **t-distribution** with $n - 2$ degrees of freedom.
- R will provide the p-value we need, based on this distribution.
- The p-value measures the probability of obtaining extreme or more extreme values than what we observed as the test statistic in the direction of H_a .
- If the p-value is below our significance level α, we reject the null hypothesis and conclude that the slope differs from zero.

t-Test for the Slope Parameter - β_1

t-TEST FOR THE SLOPE OF A SIMPLE LINEAR MODEL

To test whether the population slope is different from zero, the hypotheses are

$H_0 : \beta_1 = 0$
$H_a : \beta_1 \neq 0$

and the test statistic is

$$t = \frac{\hat{\beta}_1}{SE_{\hat{\beta}_1}}$$

If the conditions for the simple linear model, including normality, hold, we may compute a *P*-value for the test statistic using a *t*-distribution with $n-2$ degrees of freedom.

*Be careful to avoid confusing the **standard error of the slope**, $SE_{\hat{\beta}_1}$, with the **standard error of regression**, $\hat{\sigma}_\epsilon$.

t-Test for the Slope Parameter - β_1

- In some cases, we might be interested in testing for a relationship in a particular direction (positive slope or negative slope).
- Here we would use a one-sided alternative (such as $H_a : \beta_1 > 0$) in the t-test and compute the one-sided p-value.
- In general R provides the two-sided p-value. Therefore we need to divide by 2 for a one-sided p-value.
- Then we compare the one-sided p-value with our significance level α. If the p-value is below our significance level α, we reject the null hypothesis and conclude that the alternative is true.

Confidence Interval for Slope

- A confidence interval for the slope may be more useful than a test because it tells us more than just whether or not the slope is zero.
- Confidence Interval for the slope β_1 has the form

$$\hat{\beta}_1 \pm \mathrm{t}^* SE(\hat{\beta}_1)$$

  Where t^* is the critical value for the t_{n-2} distribution corresponding to the desired confidence level.
- Read example 2.1 on page 64 and refer section 2.1 R code and output.