STAT 302 - Chapter 2 : Inference for Simple Linear Regression - Part 3

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Intervals for Predictions

- Confidence interval for μ_Y - To estimate the **mean response**
- Prediction interval for y - To estimate the **individual response** y
- The standard error for predicting an individual response will always be **larger** than the standard error of predicting a mean response; therefore, prediction interval will always be **wider** than confidence interval.

Intervals for Predictions

CONFIDENCE AND PREDICTION INTERVALS FOR A SIMPLE LINEAR REGRESSION RESPONSE

A confidence interval for the mean response μ_Y when X takes the value x^* is

$$\hat{y} \pm t^* SE_{\hat{\mu}}$$

where the standard error is

$$SE_{\hat{\mu}} = \hat{\sigma}_\epsilon \sqrt{\frac{1}{n} + \frac{(x^* - \bar{x})^2}{\sum (x - \bar{x})^2}}$$

A prediction interval for a single observation of Y when X takes the value x^* is

$$\hat{y} \pm t^* SE_{\hat{y}}$$

where the standard error is

$$SE_{\hat{y}} = \hat{\sigma}_\epsilon \sqrt{1 + \frac{1}{n} + \frac{(x^* - \bar{x})^2}{\sum (x - \bar{x})^2}}$$

The value of t^* in both intervals is the critical value for the t_{n-2} density curve to obtain the desired confidence level.

Intervals for Predictions

- There are **two standard errors** : $SE_{\hat{\mu}}$ for estimating the mean response μ_Y and $SE_{\hat{y}}$ for predicting an individual response y.
- The only difference between the two standard errors is the extra 1 under the square root sign in the standard error for prediction.
- Therefore, prediction interval is always **wider** than confidence interval.
- Degrees of freedom for the t-distribution is **n-2**.
- Example 2.6 using R