Chapter 7: ANOVA with Interaction and Factorial Designs

Harsha Perera

Key Topics

- ANOVA with Interaction
- Design : The Two-Way Factorial Experiment

ANOVA with Interaction

- One-way ANOVA
  - Continuous response vs. one categorical predictor
- Two-way ANOVA
  - Continuous response vs. two categorical prediction (treatments)
- In two-way ANOVA we assumed,
  - response can be estimated by adding grand mean, treatment effects and Interaction effect
  - i.e. an additive model is adequate TO explore the source variation in response
- Most of the time the factors do interact
- When factors interact we need to introduce additional information to the two-way ANOVA model (Interaction Term)

ANOVA with Interaction

- Interaction occurs when,
  - Effect of one variable depends on the value of the other variable

- Simple example:
  - Two Factors:
    - Type of food (hot dog, ice cream)
    - Condiment (ketchup, chocolate sauce)
  - Response: satisfaction

ANOVA with Interaction...

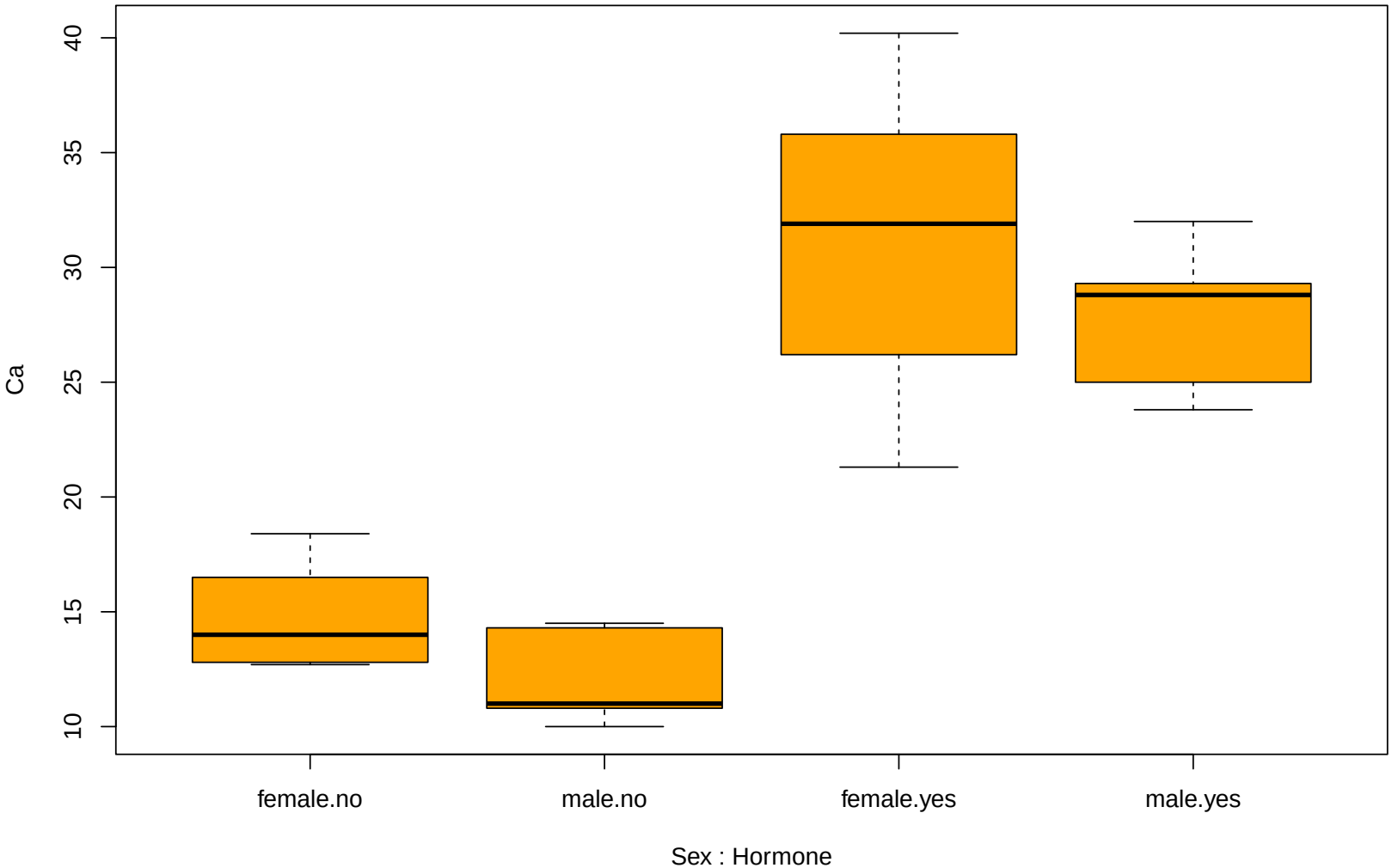
- Satisfaction of the type of food depends on the condiment
- Satisfaction will be higher
  - Hot dog with ketchup
  - Ice cream with chocolate sauce
- Satisfaction will be lower
  - Hot dog with chocolate sauce
  - Ice cream with ketchup
- Here, we can determine the need for an interaction term based on the context

ANOVA with Interaction: Numeric Differences

- Objective: investigates interaction numerically

- Example: Birds and hormones
  - Research question: do male and female birds respond in the same way to a hormone supplement added to their food?
  - Response: blood calcium levels
  - Factors: sex (male/female) vs. hormone supplement (with/without)

Example: Birds and hormones



Example: Birds and hormones - No Interaction

	Hormone: No	Hormone: Yes	Diff
Male	12.12	27.78	15.66
Female	14.88	31.08	16.20
Diff	2.76	3.30	

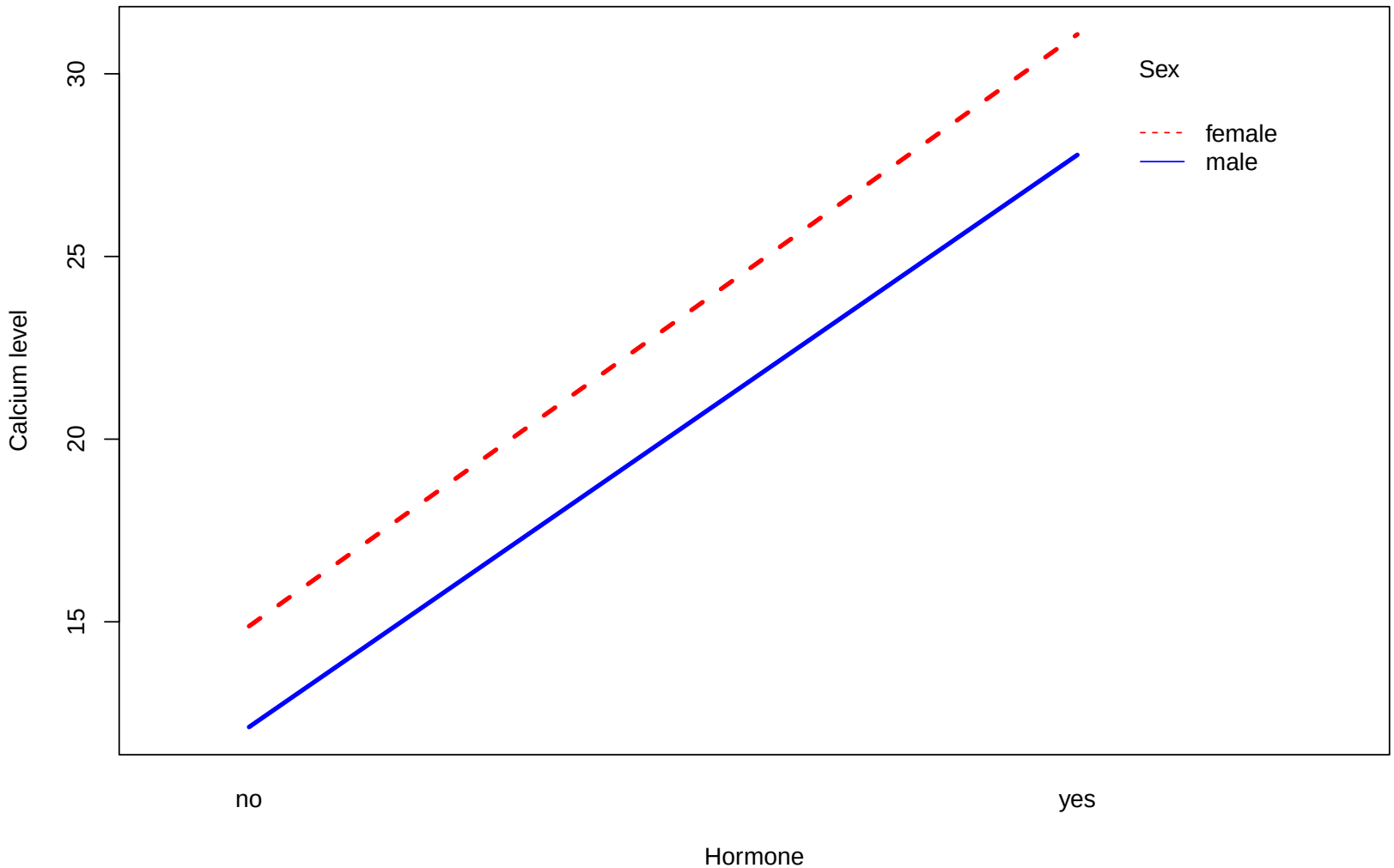
- Difference of row means are approximate same (≈ 16)
- Difference of column means are approximately same (≈ 3)
- Relationship between Calcium and presence and absence of hormone does not depend on sex of the bird
  - No interaction effect

Example: Pigs and Vitamins

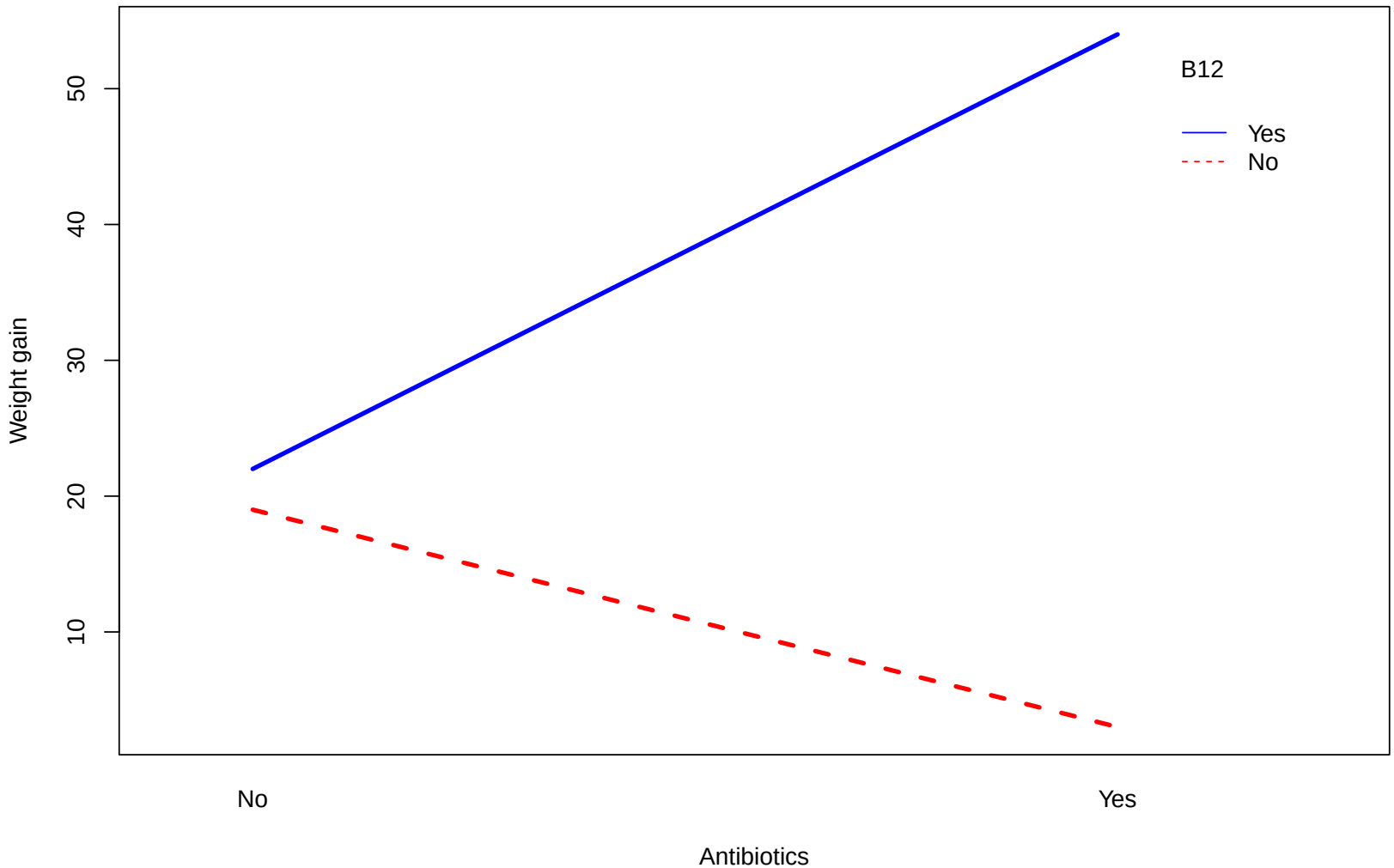
	No Antibiotics	with antibiotics	Diff
No B12	19	3	-16
With B12	22	54	32
Diff	3	51	

- Row means are different
- Column means are different
- Relationship between Weight and presence and absence of Vitamin B12 does depend on presence and absence of antibiotics in the diet
  - Need to include an interaction effect

Birds and hormones: interaction plot - parallel lines suggests no interaction



Pigs and vitamins: interaction plot - non-parallel lines suggests need for an interaction term



More on interaction effect

- In both examples, there are more than one measure for each combination of factors
- This is called replication
- Replication helps to measure unit-to-unit variation
- In order to measure interaction, we need to replicate
  - i.e to more than one measurement per cell
  - e.g we looked at the difference of means, not the difference of each observation
  - Interaction is a “difference of differences”

Two-Way Factorial Design

- A continuous response variable
- Two categorical predictors (2 factors)
- There are more than one measurements for each combination of factors
- If each cell has same number of measurements, we call the factorial design is balanced
  - Example: birds and hormones / pigs and vitamins