HFAC Risk: Assignment 1.

October 2, 2015

In this assignment, we will be exploring basic R functions, as well as the capital asset pricing model and the Fama-French three factor model.

Set-Up

The environment we will be doing most of our work in is R. Please install R (`https://www.r-project.org`) and RStudio (`https://www.rstudio.com`). RStudio is a great IDE for R (and probably the best IDE I've ever seen for any language).

There are many great intros to R (a particularly good one is `https://cran.r-project.org/doc/contrib/Torfs+Brauer-Short-R-Intro.pdf`). Please read through this document carefully if you are not already familiar with the language, as I will assume working familiarity with R functionality throughout this document. As always, if you are not sure how to perform a particular task in R, please use Google and StackExchange (it is almost certain that someone else has already had the question you have).

Summary of Equity Returns

We will begin by downloading data for the "prices" of the S&P 500 since January 1, 1982. You can find the end-of-day prices of the S&P 500 from Yahoo Finance - it will probably be easiest to download this data as a .csv file and import the data into R. Google around a little bit to find out how to do this.

Having loaded the data into R, we will denote the closing price of day t as p_t. Please compute the following summary statistics, and think about what each of these numbers mean. Please use the Internet generously to compute these statistics - R often has built in functions that can save you a lot of time (you can imagine that computing the sample mean of a bunch of numbers is something people do quite regularly, and so a function probably exists to do that).

1. The sample mean of the returns,

$$\bar{R} = \frac{1}{n-1}\sum_{t=2}^{n} R_t, \text{ where } R_t = \frac{p_t - p_{t-1}}{p_{t-1}}$$

2. The sample mean of the log-returns,

$$\bar{r} = \frac{1}{n-1}\sum_{t=2}^{n} r_t, \text{ where } r_t = \log\left(\frac{p_t}{p_{t-1}}\right)$$

3. The sample variance of the returns,

$$\hat{\sigma}^2 = \frac{1}{n-1}\sum_{t=1}^{n} \left(r_t - \bar{r}\right)^2$$

   (a) Please also compute the square root of this quantity, the sample standard deviation.

4. The sample "realized variance" of the returns,

$$RV^2 = \frac{1}{n-1}\sum_{t=1}^{n} r_t^2$$

   (a) Please also compute the square root of this quantity, the sample realized volatility.

Something we often do in finance is compute "annualized" summary statistics. That is, the sample mean of the daily returns, which you computed in 1., represents the mean return each day. This is probably quite a small number, and it often is easier to talk about returns in yearly terms. To annualize a return, we think about the return we would have, if we had our daily return every day for an entire year. There are 252 trading days in a year, so usually Annualized Return = $252 \times$ Daily Return. Please also answer the following question.

1. How might we convert a sample variance of daily returns to annualized terms?
   (a) What about annualizing our sample standard deviation/volatility numbers?

Finally, please produce a plot of:

1. The price of the S&P 500 over time.
2. (Bonus) The trailing 1-year sample realized volatility of the S&P 500 over time.
   (a) That is, on January 1, 1983, plot the sample realized volatility computed over the period from January 1, 1982 - January 1, 1983, and on January 2, 1983, plot the sample RV over the period from January 2, 1982 - January 2, 1983, and so on.

Capital Asset Pricing Model

Please now download data for AAPL, starting from January 1, 1982. Import the data into R as before.

In this exercise, we will be exploring the capital asset pricing model. Recall this model says that for any stock i, we can model its log-returns as

$$r_i = \alpha + \beta r_{S\&P500} + \varepsilon_t, \varepsilon_t \sim \mathcal{N}\left(0, \sigma^2\right)$$

We notice that this looks like a regression. Please carry out the regresion this represents (AAPL log-returns as the dependent variable and S&P500 log-returns as the independent variable).

1. What is the intercept (alpha)?
2. What is the slope (beta)?
   (a) What is the interpretation of beta?
3. Plot the time series of $r_{AAPL} - \beta r_{S\&P500}$ for each day. What is this showing us?

Fama-French Three Factor Model

We end this assignment by exploring the Fama-French three factor model. Please refer to Ken French's website (`http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html#Research`), and download the data for the Fama/French 3 Factors [Daily] series. Recall that the Fama/French model says that

$$r_i = \alpha + \beta_m R_{S\&P500} + \beta_{SMB} r_{SMB} + \beta_{HML} r_{HML} + \varepsilon_t, \varepsilon_t \sim \mathcal{N}\left(0, \sigma^2\right)$$

The data in the Fama/French series is longer than the data we have for either the S&P or AAPL. You will need to trim the data so that the time series match up.

First,

1. Please compute the sample mean and sample realized volatility of r_{SMB} and r_{HML}. How do these numbers compare to the mean and realized volatility of the S&P 500?

Now, please carry out the multivariate regression that the Fama/French model represents.

1. Please report the intercept in this regression. Would you expect this intercept to be greater or less than the intercept in the CAPM?
2. Please report the values of the various βs. What do these numbers represent?
3. Please carry out the regression only on data from January 1, 1982 - January 1, 1995, and then again only on data from January 1, 1995 - present. How do you expect the βs to change? How did they change? What is the interpretation of these changes?