EECE 5643: Simulation and Performance Evaluation
Professor Ningfang Mi

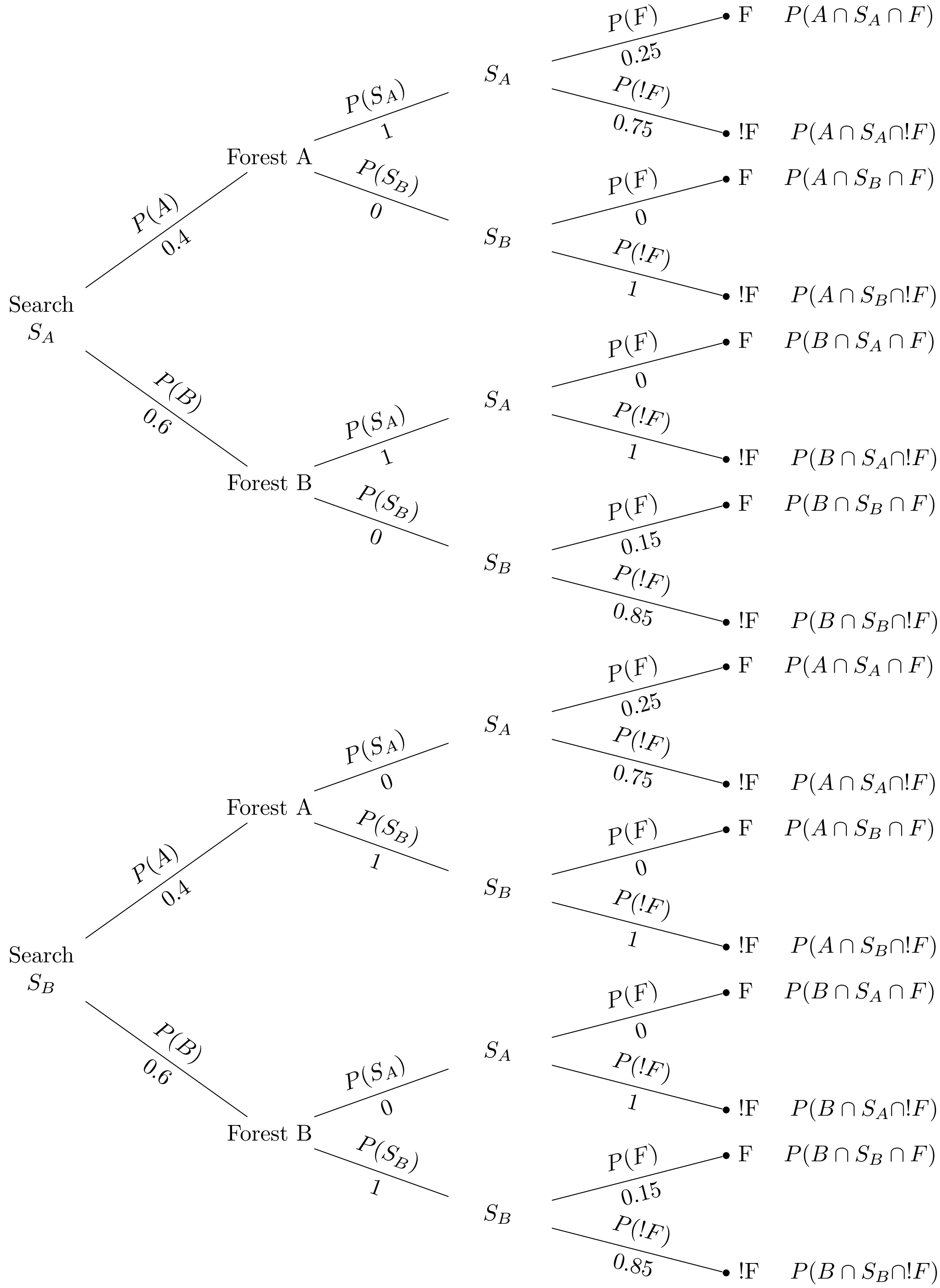
Homework 1

- Assignment Due: 01/19/2023 -

Harrison Sun
Monday, Thursday 11:45 am - 1:25 pm
Completed: January 18, 2023

1 Oscar # 1

1.1 Maximizing Day 1



Given:
$P(A) = 0.4 \quad P(B) = 0.6 \quad P(D_N) = n/n + 2 \quad P(F_N|S_A \cap A) = 0.25 \quad P(F_N|S_B \cap B) = 0.15$

Solution:
It is better to search Forest A.

Explanation:
Oscar is given the choice between searching Forest A or searching Forest B. Therefore, $P(S_A)$ or $P(S_B)$ can be set to 1, but the other will take the value of 0.

Equations used to solve problem:

$$\begin{gathered} S_A = S_A \cap A + S_A \cap B \\ S_B = S_B \cap A + S_B \cap B \end{gathered} \tag{1}$$

$$\begin{gathered} P(F \cap S_A \cap B) = 0 \\ P(F \cap S_B \cap A) = 0 \end{gathered} \tag{2}$$

$$\begin{gathered} P(S_A \cap A) + P(S_A \cap B) = P(S_A) \\ P(S_B \cap A) + P(S_B \cap B) = P(S_B) \end{gathered} \tag{3}$$

$$\begin{gathered} P(F|S_A \cap A) = P(F \cap S_A \cap A)/P(A) \\ P(F|S_B \cap B) = P(F \cap S_B \cap B)/P(B) \end{gathered} \tag{4}$$

$$\begin{gathered} P(F \cap S_A \cap A) = P(F_N|S_A \cap A) * P(A) \\ P(F \cap S_B \cap B) = P(F_N|S_B \cap B) * P(B) \end{gathered} \tag{5}$$

Assume that $P(S_A) = 1$.

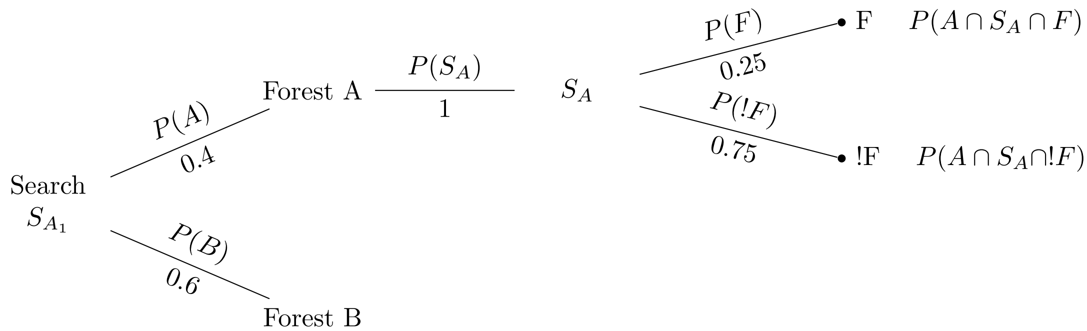
$$\begin{aligned}
&P(F|S_A) = P(F \cap S_A)/P(S_A) \\
&P(F|S_A) = \{P(F \cap S_A \cap A) + P(F \cap S_A \cap B)\}/\{P(S_A \cap A) + P(S_A \cap B)\} \\
&P(F|S_A) = P(F \cap S_A \cap A)/\{P(S_A \cap A) + P(S_A \cap B)\} \\
&P(F|S_A) = P(F \cap S_A \cap A) \\
&P(F|S_A) = P(F_N|S_A \cap A) * P(A) \\
&P(F|S_A) = 0.25 * 0.4 \\
&P(F|S_A) = 0.1
\end{aligned}$$

Now assume that $P(S_B) = 1$.

$$\begin{aligned}
&P(F|S_B) = P(F \cap S_B)/P(S_B) \\
&P(F|S_B) = \{P(F \cap S_B \cap B) + P(F \cap S_B \cap A)\}/\{P(S_B \cap B) + P(S_B \cap A)\} \\
&P(F|S_B) = P(F \cap S_B \cap B)/\{P(S_B \cap B) + P(S_B \cap A)\} \\
&P(F|S_B) = P(F \cap S_B \cap B) \\
&P(F|S_B) = P(F_N|S_B \cap B) * P(B) \\
&P(F|S_B) = 0.15 * 0.6 \\
&P(F|S_B) = 0.09
\end{aligned}$$

Since P($F|S_A$) > P($F|S_B$), Oscar should search Forest A.

1.2 Probability of Forest A Given No Success on Day 1



Given:
$P(A) = 0.4 \quad P(B) = 0.6 \quad P(D_N) = n/n+2 \quad P(F_N|S_A \cap A) = 0.25 \quad P(F_N|S_B \cap B) = 0.15$

Solution:
$\mathrm{P}(A|S_{A_1} \cap !F_1) = \dfrac{1}{3}$

Explanation:
Oscar looked in Forest A on day 1 but did not find his dog. The probability that the dog is in Forest A is described by $\mathrm{P}(A|S_{A_1} \cap !F_1)$. In this solution, the probability that the dog is in Forest A is independent of whether the dog is alive.

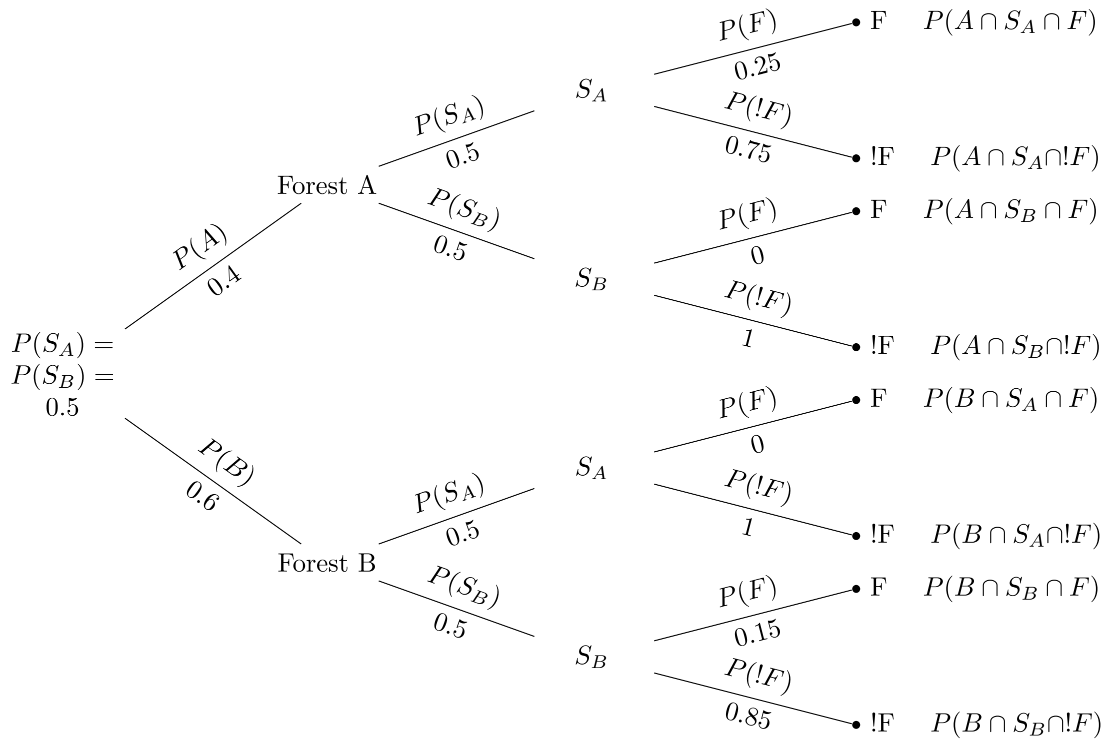
Equations used to solve problem:

$$\begin{aligned} P(S_{A_1} \cap !F_1) &= P(S_A \cap !F_1 \cap A) + P(S_A \cap !F_1 \cap B) \\ P(S_{A_1} \cap !F_1) &= 0.4 * 0.75 + 0.6 = 0.9 \end{aligned} \tag{6}$$

Using Eq. 6:

$$\begin{aligned} P(A|S_{A_1} \cap !F_1) &= \frac{P(A \cap S_{A_1} \cap !F_1)}{P(S_{A_1} \cap !F_1)} \\ P(A|S_{A_1} \cap !F_1) &= \frac{0.3}{0.9} \\ P(A|S_{A_1} \cap !F_1) &= \frac{1}{3} \end{aligned}$$

1.3 Probability of Forest A Given a Coin Flip and the Dog was Found



Given:
$P(A) = 0.4 \quad P(B) = 0.6 \quad P(D_N) = n/n+2 \quad P(F_N|S_A \cap A) = 0.25 \quad P(F_N|S_B \cap B) = 0.15$
$P(S_A) = P(S_B) = 0.5$

Solution:
P($S_A|F$) = 0.526

Explanation:
Oscar flipped a fair coin to determine which forest he looked in and found his dog. The probability that he looked in Forest A is given by P($S_A|F$).

$$P(S_A|F) = \frac{P(S_A \cap F)}{P(F)}$$

For the dog to be found while searching Forest A, the dog must be in Forest A. Therefore, $S_A|F$ implies $A \cap S_A \cap F$. Furthermore, since the forest searched is chosen by a fair coin flip, it is independent from which forest the dog is in.

$$P(A \cap S_A) = P(A) * P(S_A)$$
$$P(S_A|F) = \frac{P(S_A \cap F)}{P(F)}$$

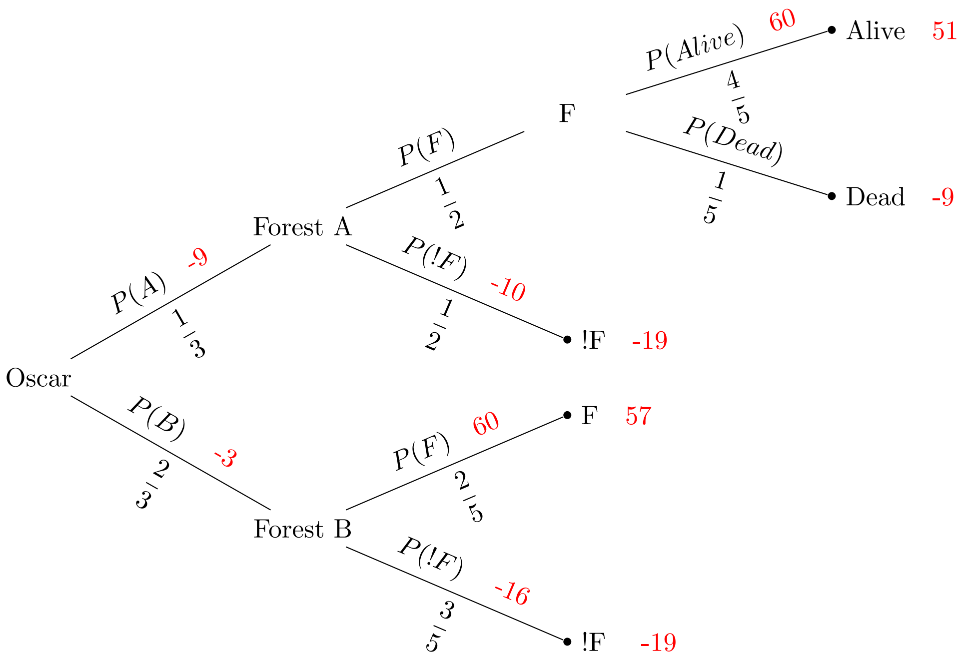
The probability of finding the dog P(F) is described by $P(A \cap S_A) * P(F|A \cap S_A) + P(B \cap S_B) * P(F|B \cap S_B)$.

$$P(F) = P(A \cap S_A) * P(F|A \cap S_A) + P(B \cap S_B) * P(F|B \cap S_B) \tag{7}$$

Thus,

$$
\begin{aligned}
P(S_A|F) &= \frac{P(S_A \cap F)}{P(A \cap S_A) * P(F|A \cap S_A) + P(B \cap S_B) * P(F|B \cap S_B)} \\
P(S_A|F) &= \frac{0.4 * 0.5 * 0.25}{0.4 * 0.5 * 0.25 + 0.6 * 0.5 * 0.15} \\
P(S_A|F) &= 0.526
\end{aligned}
$$

2 Oscar #2



Given:
$P(A) = \frac{1}{3}$ $P(B) = \frac{2}{3}$ $P(D_N|A) = \frac{4}{5}$ $P(D_N|B) = \frac{3}{5}$ $P(F_N|S_A \cap A) = \frac{1}{2}$ $P(F_N|S_B \cap B) = \frac{2}{5}$

Finding the dog alive	\$60
Each day of search	-\$3
Finding dog dead	\$0
Not finding dog	-\$10
Searching in both forests	-\$3

Solution: Expectation is \$10.93.

Explanation:
Oscar searches for his dog in Forest B and then in Forest A if he does not find his dog on the first day. If he finds his dog on the first day, it is guaranteed to be alive. Therefore, if the dog is in Forest B, the possible situations are Oscar finds his dog on the first day ($v = \$57$) or he does not find his dog and moves on to Forest A. In the latter case, Oscar has a value of ($v = -\$19$) as he will not find his dog when he looks in Forest A and will waste two days worth of time. Likewise, if the dog is in Forest A and Oscar does not find it, the value is ($v = -\$19$). If the dog is in Forest A and is found alive, the value is ($v = \$51$). If the dog is in Forest A and is found dead, the value is ($v = -\$9$).

Knowing this, the following table can be concluded. Red indicates value lost and blue indicates value gained.

$P(A \cap F \cap Alive)$	$\frac{1}{3} * \frac{1}{2} * \frac{4}{5}$	$\frac{2}{15}$	\$51
$P(A \cap F \cap Dead)$	$\frac{1}{3} * \frac{1}{2} * \frac{1}{5}$	$\frac{1}{30}$	\$9
$P(A \cap !F)$	$\frac{1}{3} * \frac{1}{2}$	$\frac{1}{6}$	\$19
$P(B \cap F)$	$\frac{2}{3} * \frac{2}{5}$	$\frac{4}{15}$	\$57
$P(B \cap !F)$	$\frac{2}{3} * \frac{3}{5}$	$\frac{2}{5}$	\$19

Expectation is calculated as the sum of the multiplication of probability and value divided by the total probability. In this case, the total value of probability adds up to 1.

$$\sum_{i=1}^{\infty} p_i * v_i$$

Using the values from the table above, the expected value of searching for Oscar's dog is \$10.93.