$$\frac{1}{2}m_1{v_1}^2 + \frac{1}{2}m_2{v_2}^2 = \text{sum of initial KE} = s_{\text{KE}}$$

$$m_1v_1 + m_2v_2 = \text{sum of initial momentum} = s_p$$

Solution for v_1:

$$m_1v_1 + m_2v_2 = s_p$$

$$v_1 = \frac{s_p - m_2v_2}{m_1}$$

Solution for v_2:

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2{v_2}^2 = s_{\text{KE}}$$

$$m_1{v_1}^2 + m_2{v_2}^2 = 2s_{\text{KE}}$$

$${v_2}^2 = \frac{2s_{\text{KE}} - m_1{v_1}^2}{m_2}$$

$$v_2 = \sqrt{\frac{2s_{\text{KE}} - m_1{v_1}^2}{m_2}}$$

Substitute v_1:

$$v_2 = \sqrt{\frac{2s_{\text{KE}} - m_1\left(\frac{s_p - m_2v_2}{m_1}\right)^2}{m_2}}$$

$$v_2 = \sqrt{\frac{2s_{\text{KE}} - \frac{(s_p - m_2v_2)^2}{m_1}}{m_2}}$$

$${v_2}^2 = \frac{2s_{\text{KE}} - \frac{(s_p - m_2v_2)^2}{m_1}}{m_2}$$

$$m_2{v_2}^2 = 2s_{\text{KE}} - \frac{(s_p - m_2v_2)^2}{m_1}$$

$$2s_{\mathrm{KE}} - m_2{v_2}^2 = \frac{(s_p - m_2 v_2)^2}{m_1}$$

$$m_1(2s_{\mathrm{KE}} - m_2{v_2}^2) = (s_p - m_2 v_2)^2$$

$$m_1(2s_{\mathrm{KE}} - m_2{v_2}^2) = {s_p}^2 - 2m_2 v_2 s_p + {m_2}^2{v_2}^2$$

$$2s_{\mathrm{KE}}m_1 - m_2{v_2}^2 m_1 = {s_p}^2 - 2m_2 v_2 s_p + {m_2}^2{v_2}^2$$

$$0 = {s_p}^2 - 2m_2 s_p v_2 + {m_2}^2{v_2}^2 - 2s_{\mathrm{KE}}m_1 + m_2 m_1 {v_2}^2$$

$$0 = (m_2 m_1 + {m_2}^2){v_2}^2 - (2m_2 s_p)v_2 + ({s_p}^2 - 2s_{\mathrm{KE}}m_1)$$

$$v_2 = \frac{2m_2 s_p + \sqrt{(-2m_2 s_p)^2 - 4(m_2 m_1 + {m_2}^2)({s_p}^2 - 2s_{\mathrm{KE}}m_1)}}{2(m_2 m_1 + {m_2}^2)}$$

Simplify v_2:

$$v_2 = \frac{2m_2 s_p + \sqrt{4{m_2}^2{s_p}^2 - 4(m_2 m_1 + {m_2}^2)({s_p}^2 - 2s_{\mathrm{KE}}m_1)}}{2(m_2 m_1 + {m_2}^2)}$$

$$\begin{aligned}&(m_2 m_1 + {m_2}^2)({s_p}^2 - 2s_{\mathrm{KE}}m_1)\\ &= m_2 m_1 {s_p}^2 - 2s_{\mathrm{KE}}{m_1}^2 m_2 + {m_2}^2{s_p}^2 - 2s_{\mathrm{KE}}m_1{m_2}^2\end{aligned}$$

$$v_2 = \frac{2m_2 s_p + 2\sqrt{{m_2}^2{s_p}^2 - (m_2 m_1 + {m_2}^2)({s_p}^2 - 2s_{\mathrm{KE}}m_1)}}{2(m_2 m_1 + {m_2}^2)}$$

$$v_2 = \frac{m_2 s_p + \sqrt{{m_2}^2{s_p}^2 - (m_2 m_1 + {m_2}^2)({s_p}^2 - 2s_{\mathrm{KE}}m_1)}}{m_2 m_1 + {m_2}^2}$$

$$v_2 = \frac{m_2 s_p + \sqrt{2{m_2}^2{s_p}^2 - m_1 m_2 {s_p}^2 - 2s_{\mathrm{KE}}{m_1}^2 m_2 - 2s_{\mathrm{KE}}m_1{m_2}^2}}{m_2 m_1 + {m_2}^2}$$

$$v_2 = \frac{m_2 s_p + \sqrt{{s_p}^2 m_2(2m_2 - m_1) - 2s_{\mathrm{KE}}m_1 m_2(m_1 - m_2)}}{m_2(m_1 + m_2)}$$

$$\mathbf{v_2} = \frac{\mathbf{m_2 s_p} + \sqrt{\mathbf{m_2}(\mathbf{s_p}^2(\mathbf{2m_2} - \mathbf{m_1}) - \mathbf{2}s_{\mathrm{KE}}\mathbf{m_1}(\mathbf{m_1} - \mathbf{m_2}))}}{\mathbf{m_2}(\mathbf{m_1} + \mathbf{m_2})}$$