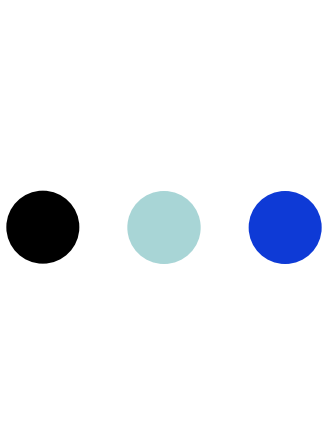


Parsing 2

LR parser construction.

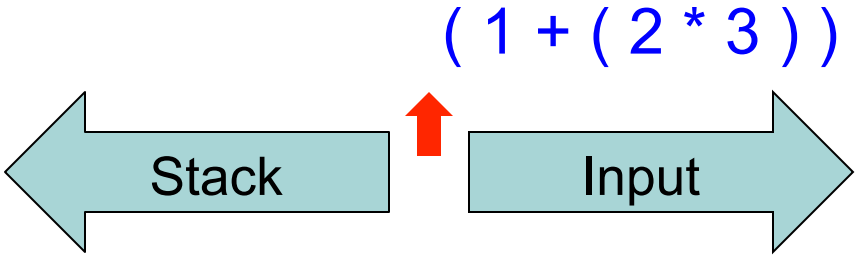
Slides by Rohit Chadha, Bill Harrison, Chris Hathhorn,
& some stolen wholesale from David Walker at Princeton.

So far…

- Previously: top-down, recursive decent parsing.
- Today: bottom-up, shift-reduce parsing.
  - LR(0).
  - Also mention: SLR, LR(1), LALR.

Shift-reduce parsing

Exp → *Num*
Exp → (*Exp* + *Exp*)
Exp → (*Exp* - *Exp*)
Exp → (*Exp* * *Exp*)
Exp → (*Exp* \ *Exp*)



Actions:

- **Shift**: shift the next input symbol onto the stack.
- **Reduce**: replace a string of symbols on top of the stack with a corresponding non-terminal from the grammar.
- **Accept**: announce successful completion of parsing.
- **Error**: indicates a syntax error.

Shift-reduce parsing

Exp → *Num*
Exp → (*Exp* + *Exp*)
Exp → (*Exp* - *Exp*)
Exp → (*Exp* * *Exp*)
Exp → (*Exp* \ *Exp*)

(1 + (2 * 3))

Shift-reduce parsing

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(1 + (2 * 3))

Shift!

Shift-reduce parsing

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Exp → (*Exp* \ *Exp*)

(1 + (2 * 3))

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Shift-reduce parsing

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Exp → (*Exp* * *Exp*)
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(*Exp* + (2 * 3))

Reduce!

Shift-reduce parsing

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(*Exp* + (2 * 3))

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Shift-reduce parsing

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Exp

Reduce!

Shift-reduce parsing

Exp → *Num*
Exp → (*Exp* + *Exp*)
Exp → (*Exp* - *Exp*)
Exp → (*Exp* * *Exp*)
Exp → (*Exp* \ *Exp*)

Exp

Accept!

Shift-reduce parsing

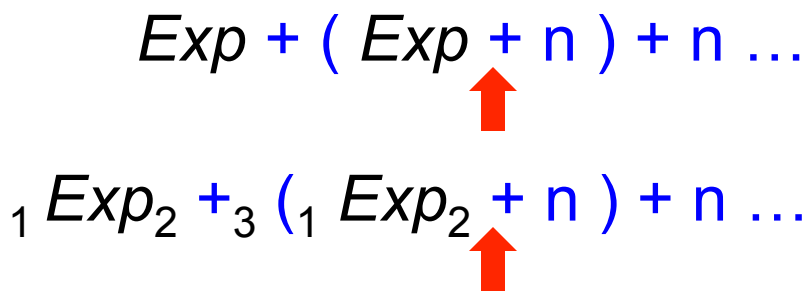
- Shift-reduce parsing is general technique for bottom-up parsing.
- Many practical parsing algorithms are based on this technique:
  - LR, SLR, GLR, LALR, …
- The basic problem: deciding when to shift and when to reduce.
  - Solution: use DFAs!

The magic: constructing an LR parser table

Exp + (*Exp* + n) + n …

- At every point in the parse, the LR parser table tells us what to do next.
  - shift, reduce, error or accept
- To do so, the LR parser keeps track of the parse "state" on the stack (i.e., the state of the finite automaton).

The magic: constructing an LR parser table



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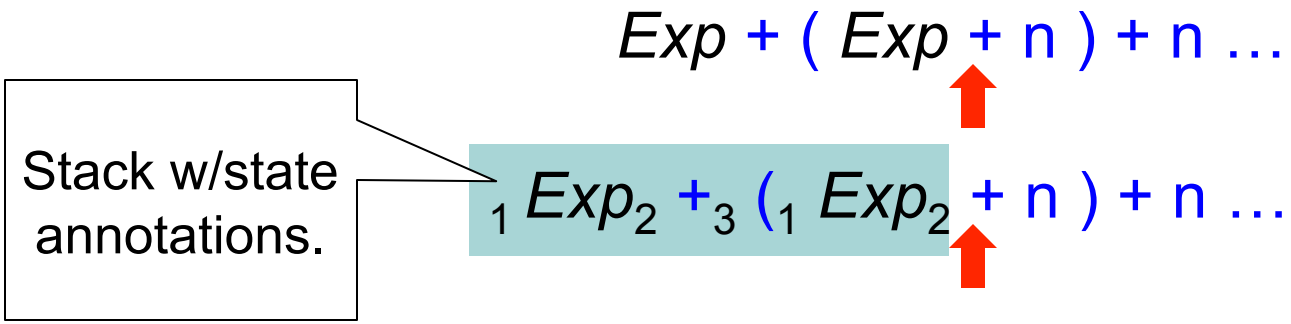

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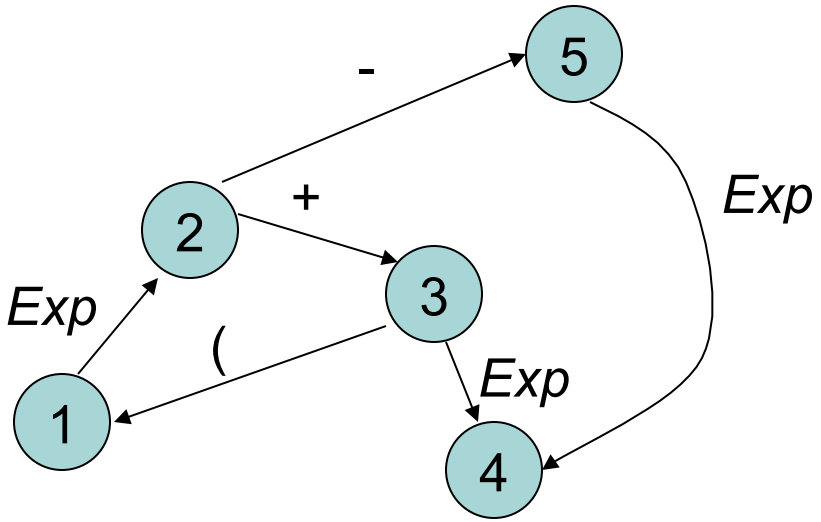
  - shift, reduce, error or accept
- To do so, the LR parser keeps track of the parse "state" on the stack (i.e., the state of the finite automaton).

The magic: constructing an LR parser table

This state & input tell us what to do next.

$_1 Exp_2 +_3 (_1 Exp_2 + n) + n \ldots$

finite automaton; terminals and non terminals label edges



The magic: constructing an LR parser table

$_{1}\,Exp_{2}\ +_{3}\ (_{1}\ Exp_{2}\ +\ n\ )\ +\ n\ \ldots$

finite automaton;
terminals and
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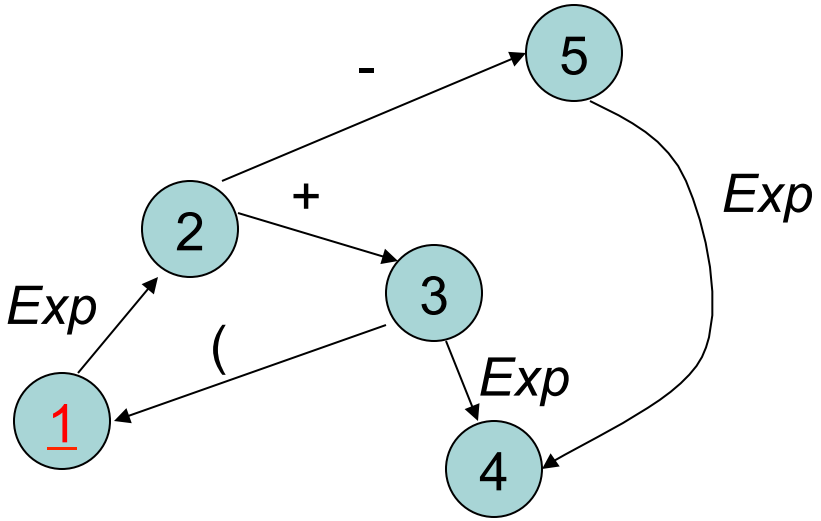

The magic: constructing an LR parser table

$_1 Exp_2 +_3 (_1 Exp_2 + n) + n \ldots$

finite automaton;
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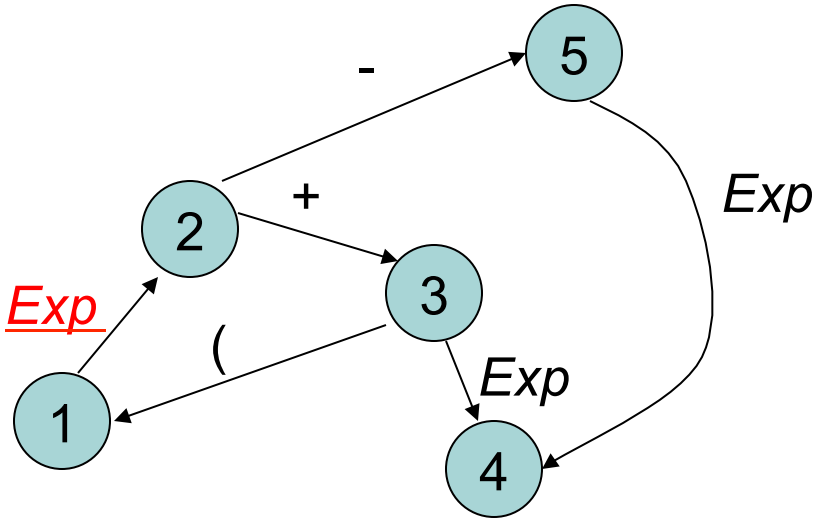

The magic: constructing an LR parser table

$_1 Exp_{\underline{2}} +_3 (_1 Exp_2 + n) + n \ldots$

finite automaton;
terminals and
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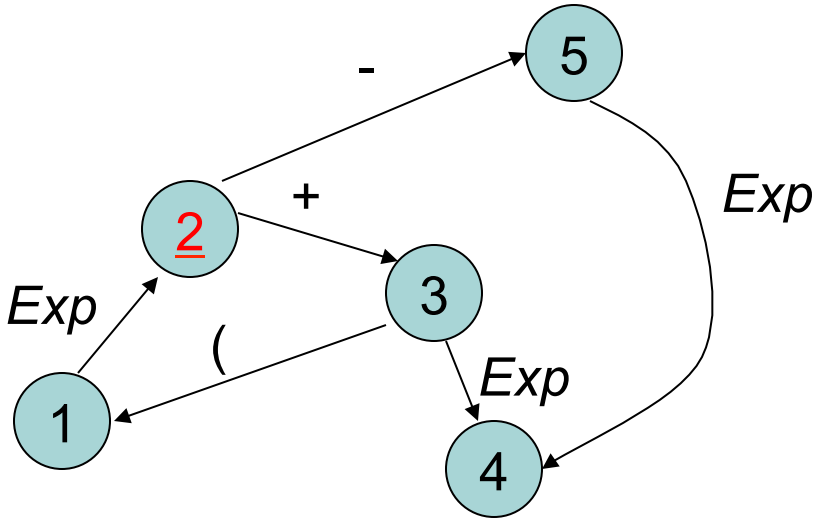

The magic: constructing an LR parser table

$_1\,Exp_2\ \pm_3\ (_1\,Exp_2\ +\ n\ )\ +\ n\ \ldots$

finite automaton;
terminals and
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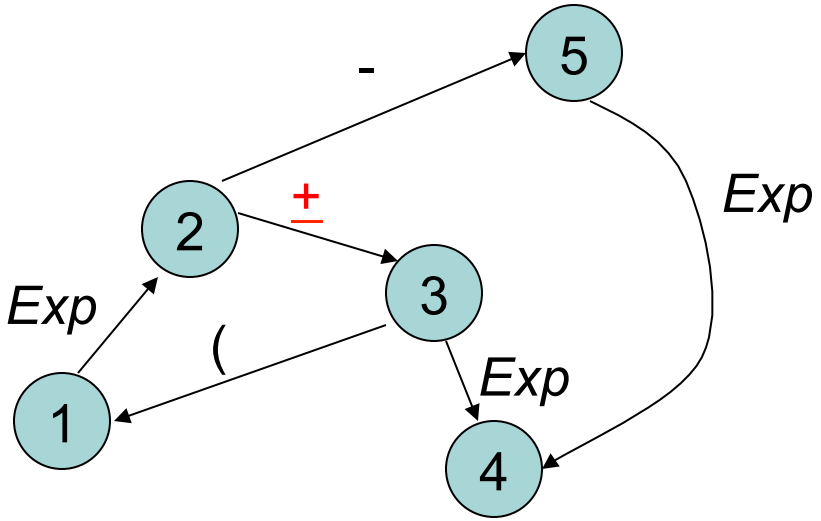

The magic: constructing an LR parser table

$_1 Exp_2 +_{\underline{3}} (_1 Exp_2 + n) + n \ldots$

finite automaton;
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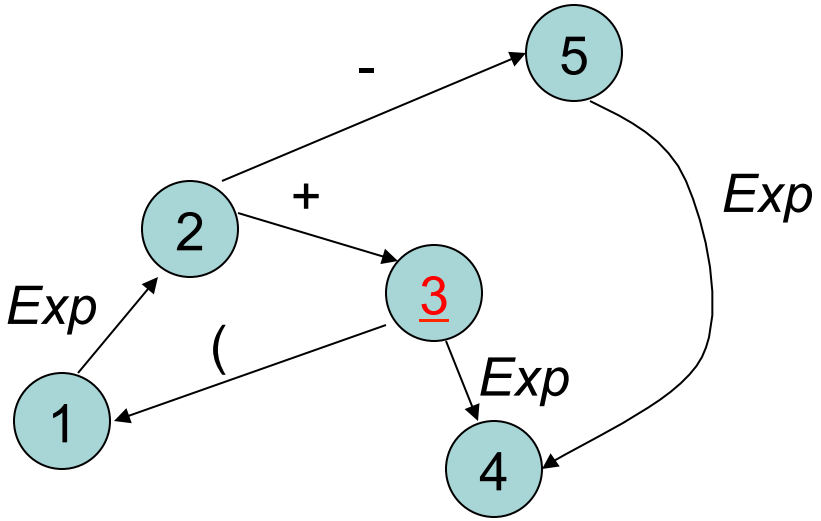

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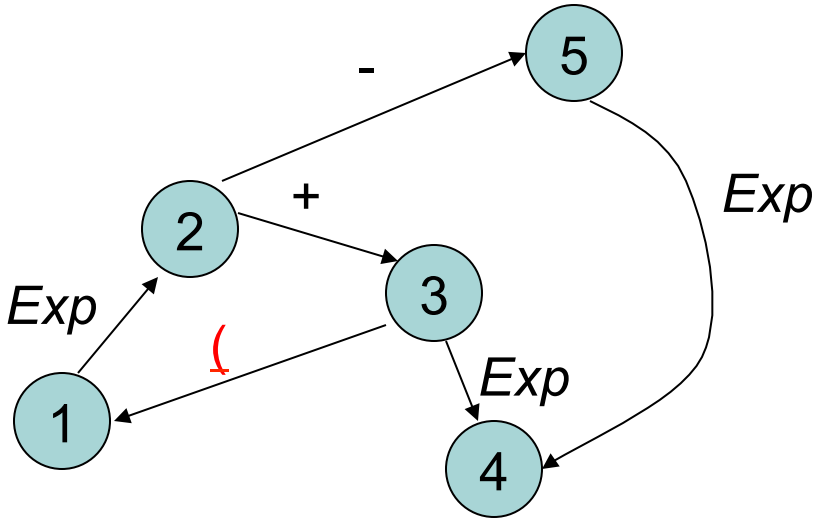

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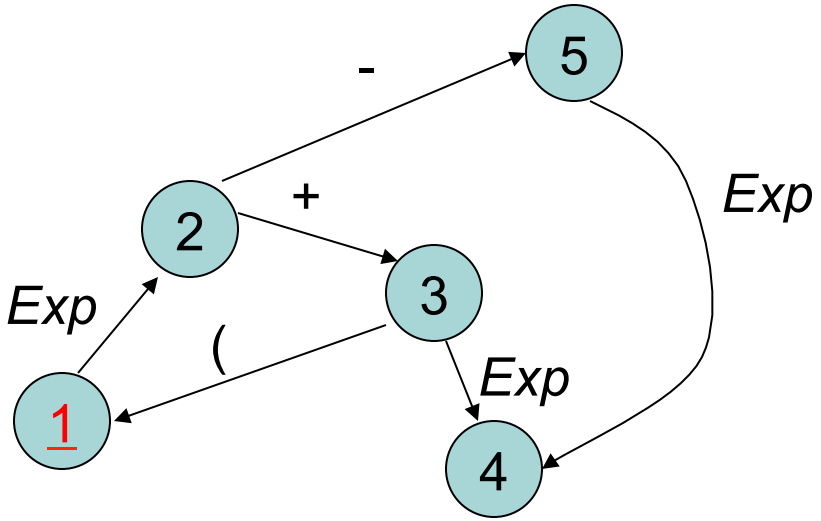

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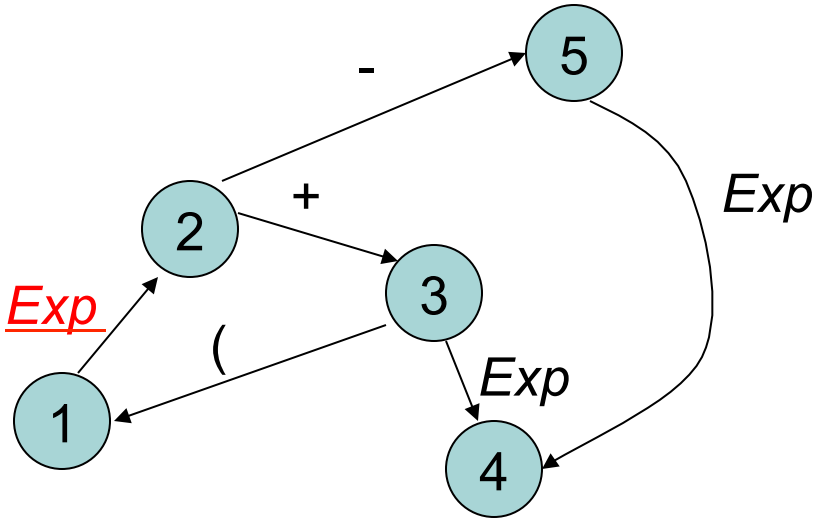

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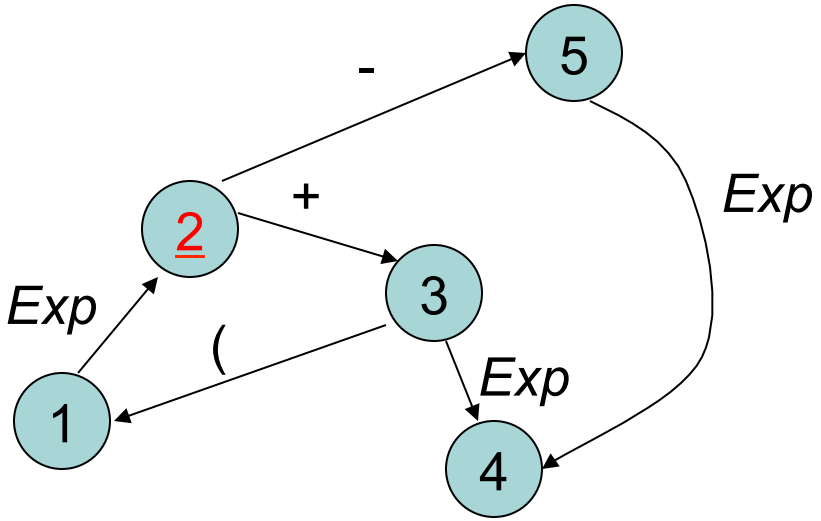


The parse table

At every point in the parse, the LR parser table tells us what to do next according to the automaton state at the top of the stack:

- shift, reduce, error or accept.

states	Terminal seen next ID, NUM, := ...
1	
2	s*n* = shift & goto state *n*
3	r*k* = reduce by rule *k*
...	a = accept
n	= error

The parse table

Reducing by rule k is broken into two steps:

- Current stack:

  $A_8\ B_3\ C_2\ \ .....{}_7\ RHS_{12}$

- Rewrite the stack according to X → RHS:

  $A_8\ B_3\ C_2\ \ .....{}_7\ X$

- Figure out new state based on state on top (i.e.: goto 13 from 7):

  $A_8\ B_3\ C_2\ \ .....{}_7\ X_{13}$

states	Terminal seen next ID, NUM, := ...	Non-terminals X,Y,Z ...
1		
2	s*n* = shift & goto state *n*	g*n* = goto state *n*
3	r*k* = reduce by rule *k*	
...	a = accept	
n	= error	

The parse table

0. S' → S $
1. S → (L)
2. S → x
3. L → S
4. L → L , S

Terminals

Non-terminals

	(	)	x	,	$	S	L
1	s3		s2			g4	
2	r2	r2	r2	r2	r2		
3	s3		s2			g7	g5
4					a		
5		s6		s8			
6	r1	r1	r1	r1	r1		
7	r3	r3	r3	r3	r3		
8	s3		s2			g9	
9	r4	r4	r4	r4	r4		

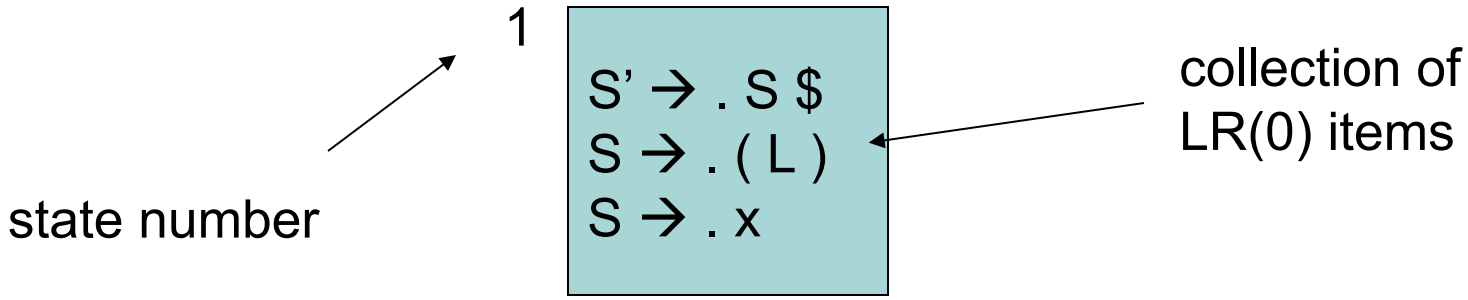
States go here

action

goto

LR(0) parsing

- Each state in the automaton represents a collection of LR(0) items:
  - an item is a rule from the grammar combined with "." to indicate where the parser currently is in the input
    - e.g.: S' → . S $ indicates that the parser is just beginning to parse this rule and it expects to be able to parse S then $ next
- A whole automaton state looks like this:



LR(0) parsing

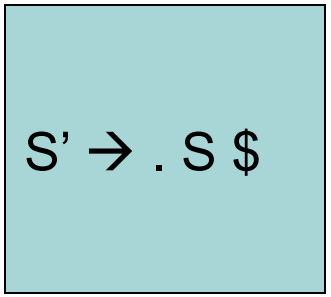
To construct states, we begin with a particular LR(0) item and construct its closure:

- The closure adds more items to a set when the "." appears to the left of a non-terminal.
- If the state includes X → s . Y s' and Y → t is a rule then the state also includes Y → . t.

Grammar:

S' → S $
S → (L)
S → x
L → S
L → L , S

1



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Grammar:

S' → S $
S → (L)
S → x
L → S
L → L , S

1

S' → . S $
S → . (L)

LR(0) parsing

To construct states, we begin with a particular LR(0) item and construct its closure:

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Grammar:

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S → (L)
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L → S
L → L , S

1

S' → . S $
S → . (L)
S → . x

Full Closure

LR(0) parsing

To construct an LR(0) automaton:

- Start with start rule & compute initial state with closure.
- Pick one of the items from the state and move “.” to the right one symbol (as if you have just parsed the symbol)
  - this creates a new item ...
  - ... and a new state when you compute the closure of the new item
  - mark the edge between the two states with:
    - a terminal T, if you moved “.” over T
    - a non-terminal X, if you moved “.” over X
- Continue until there are no further ways to move “.” across items and generate new states or new edges in the automaton.

Grammar:

S' → S $
S → (L)
S → x
L → S
L → L , S

S' → . S $
S → . (L)
S → . x

Grammar:

S' → S $
S → (L)
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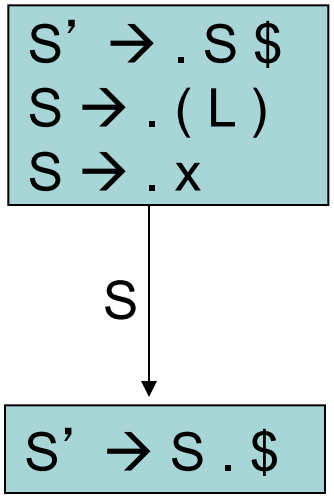
L → L , S



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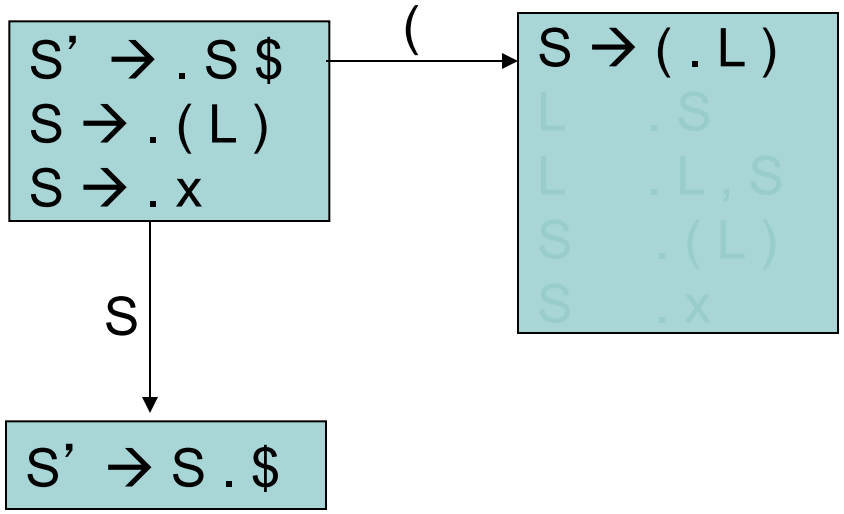

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S → x
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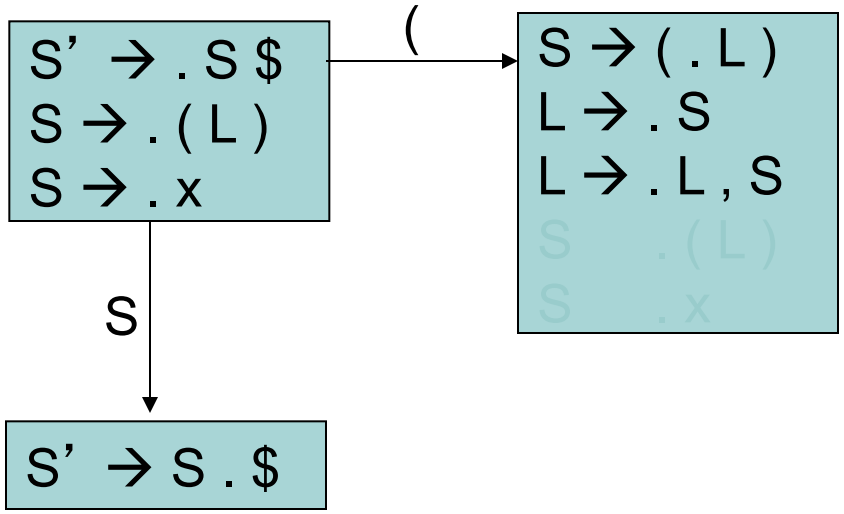

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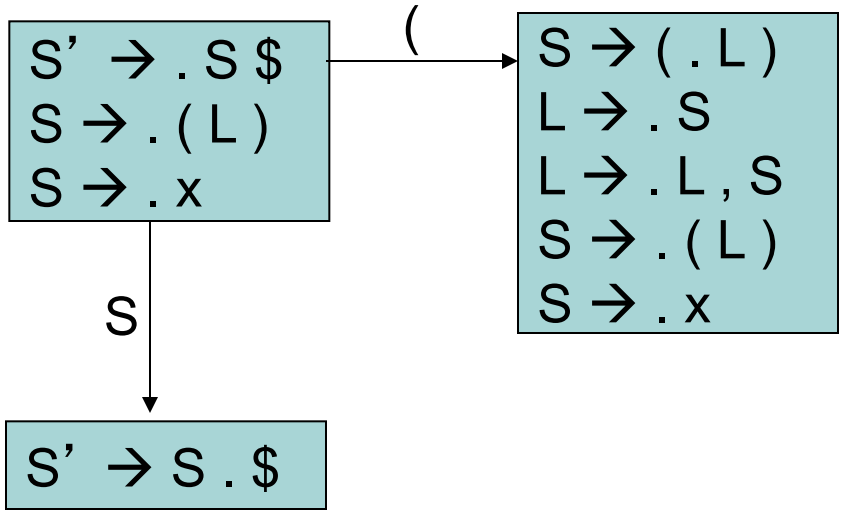

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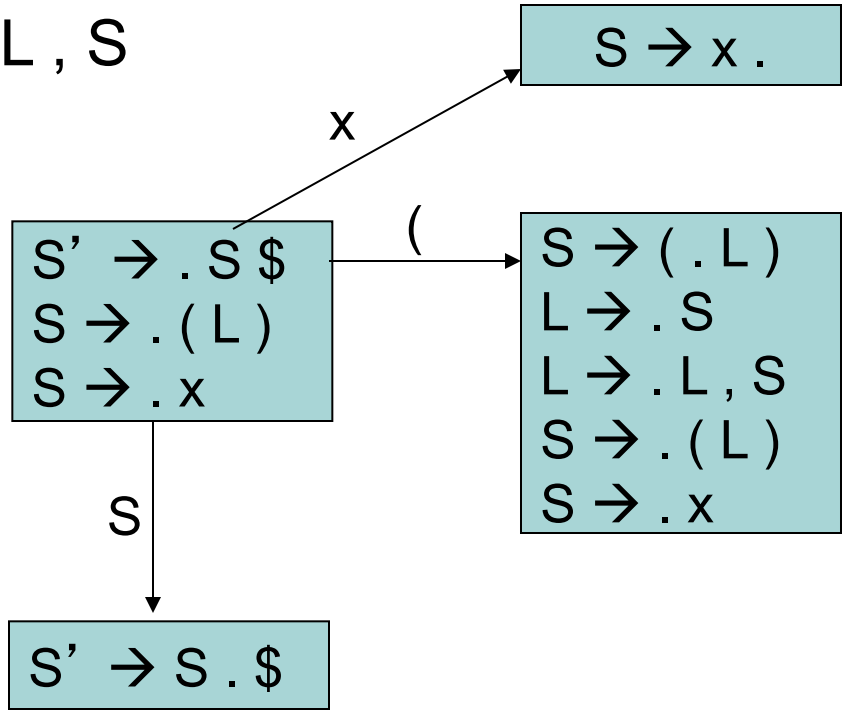

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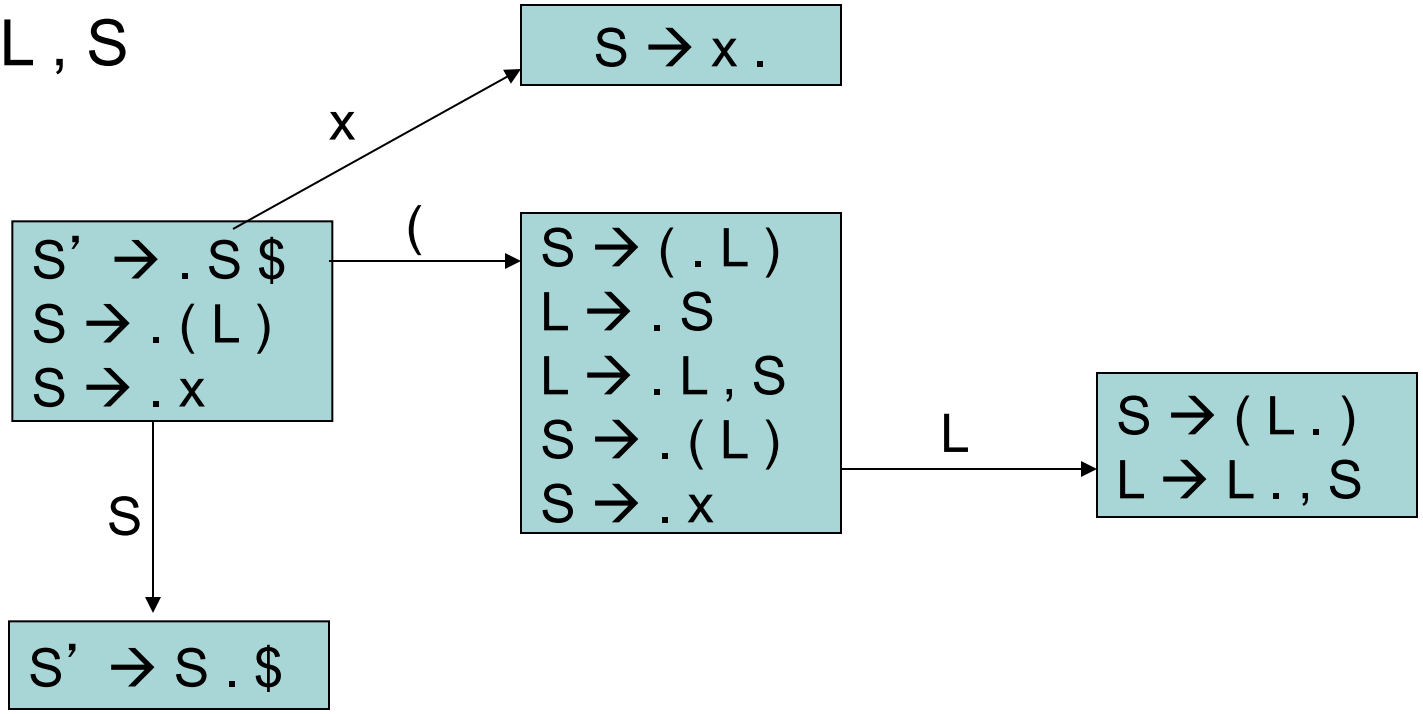

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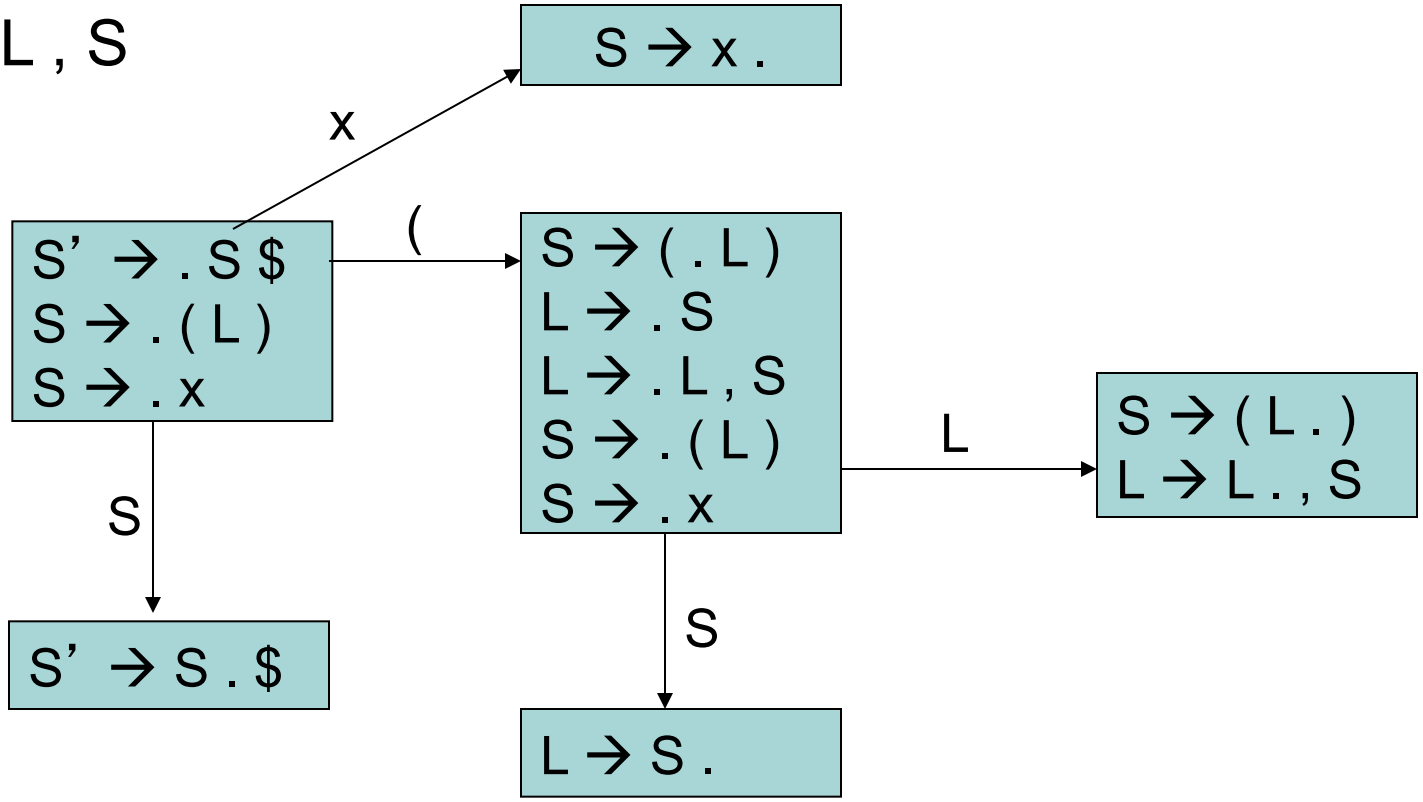

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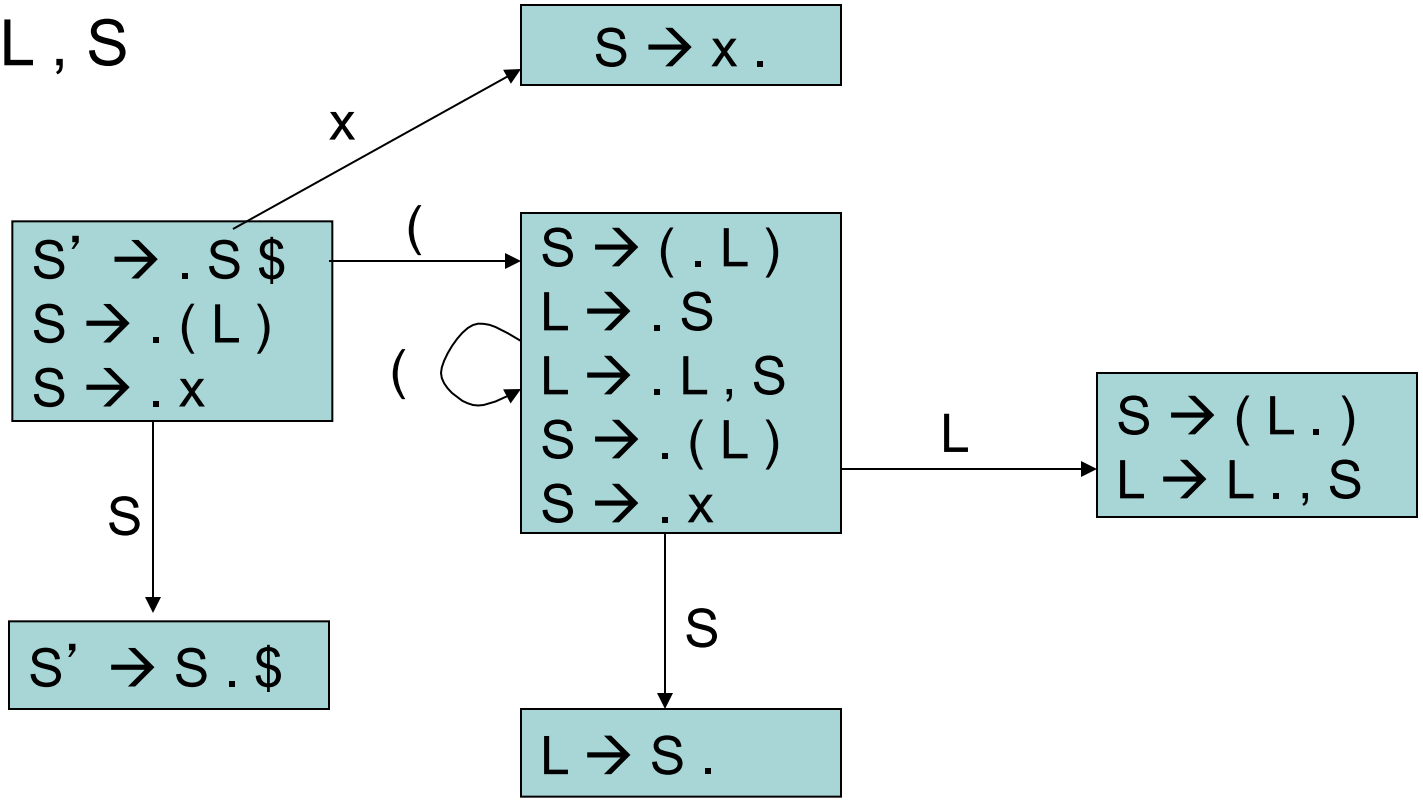

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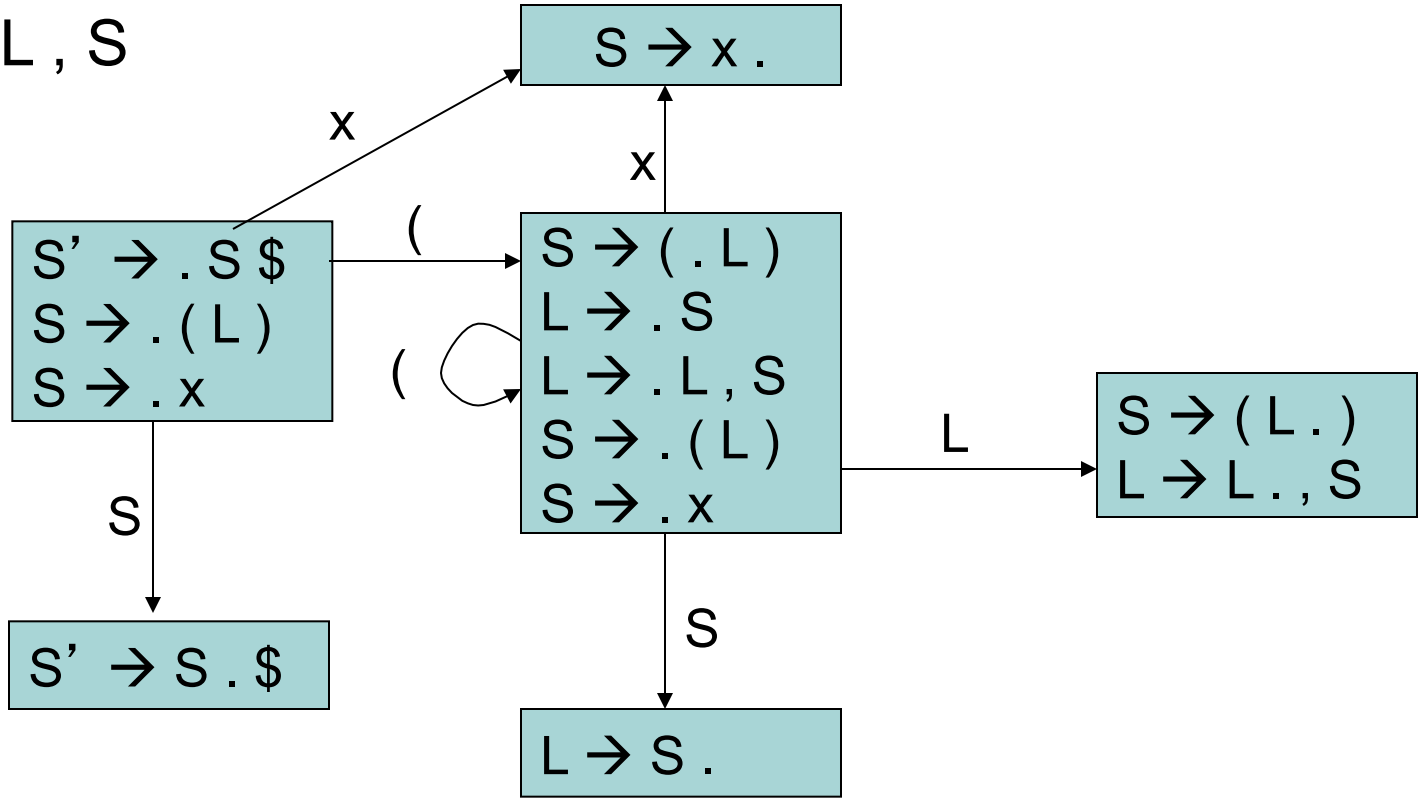

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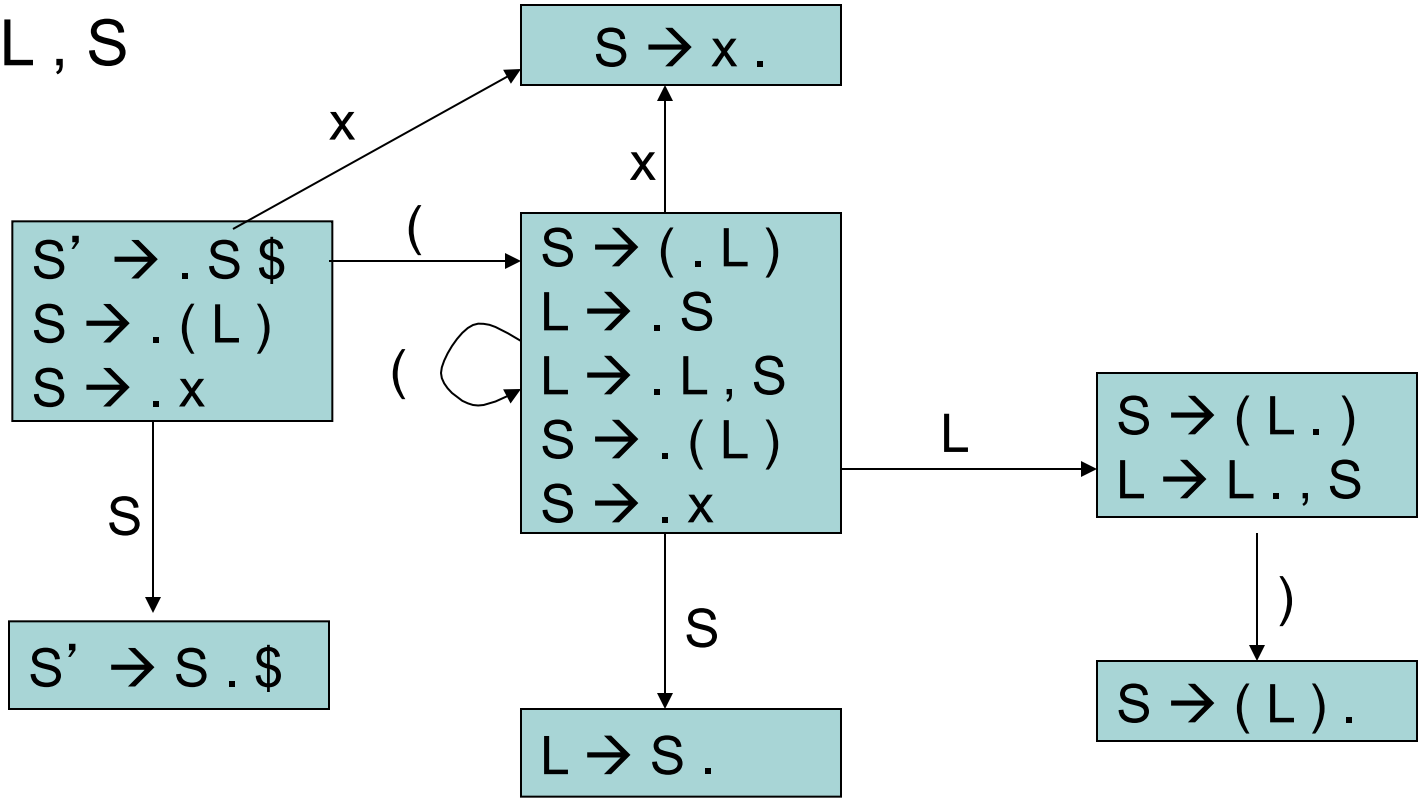

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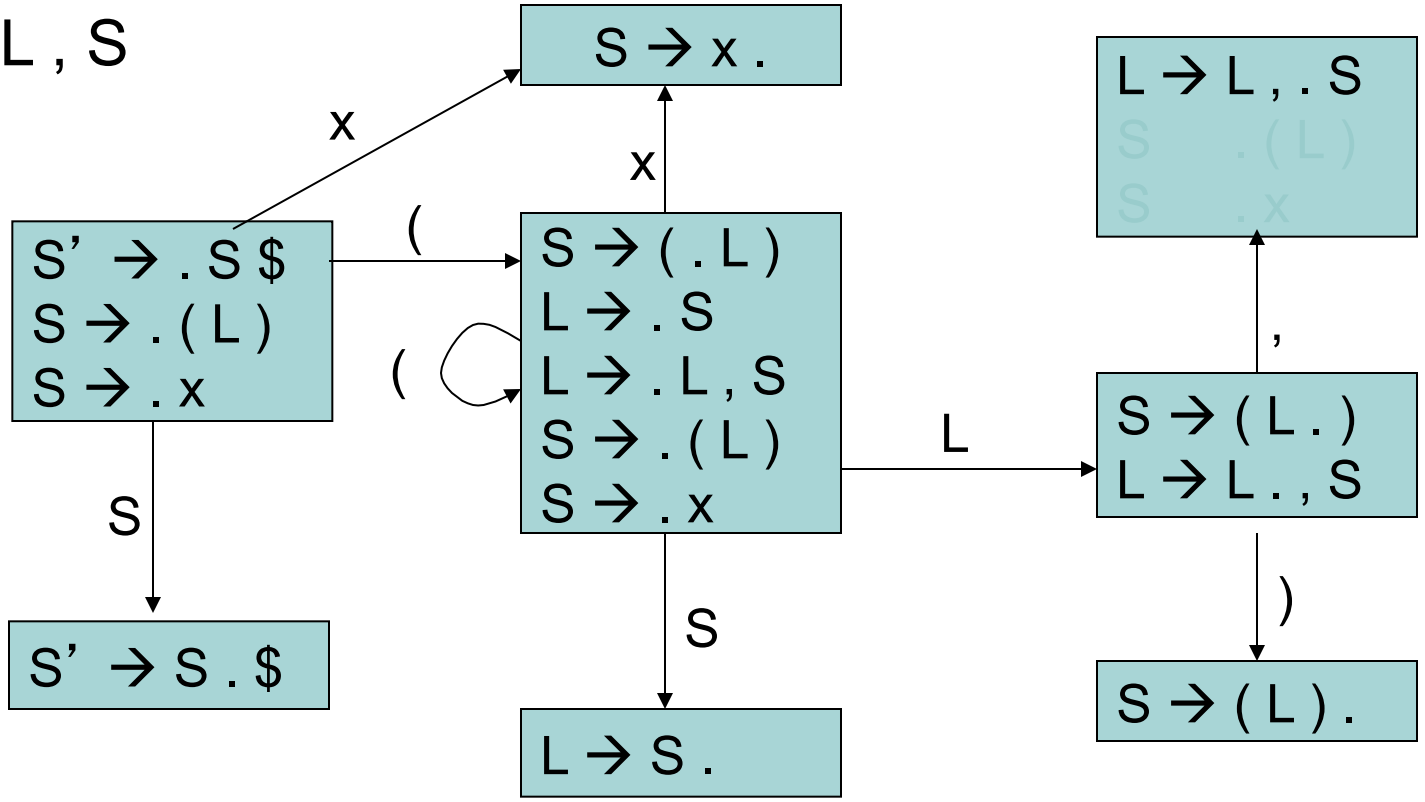

Grammar:

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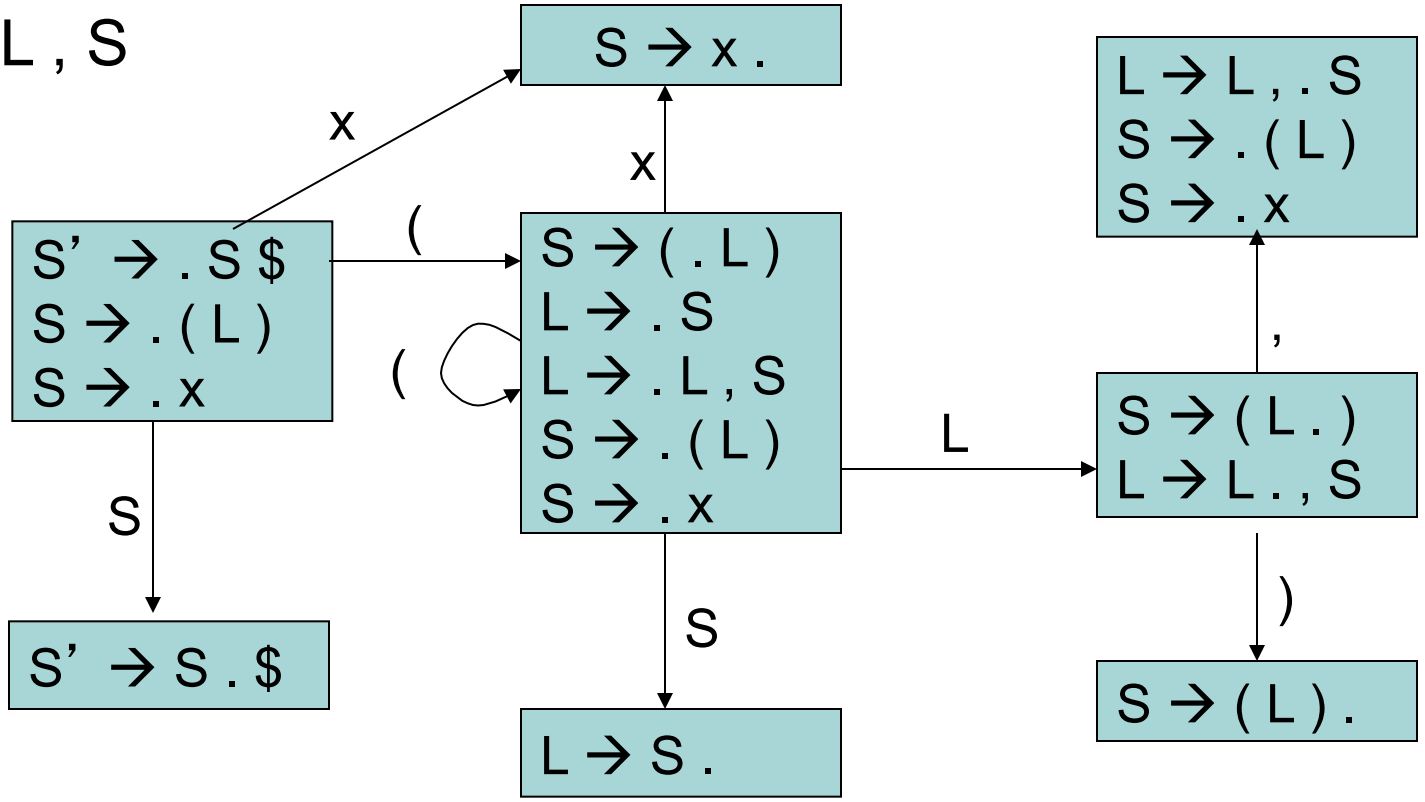

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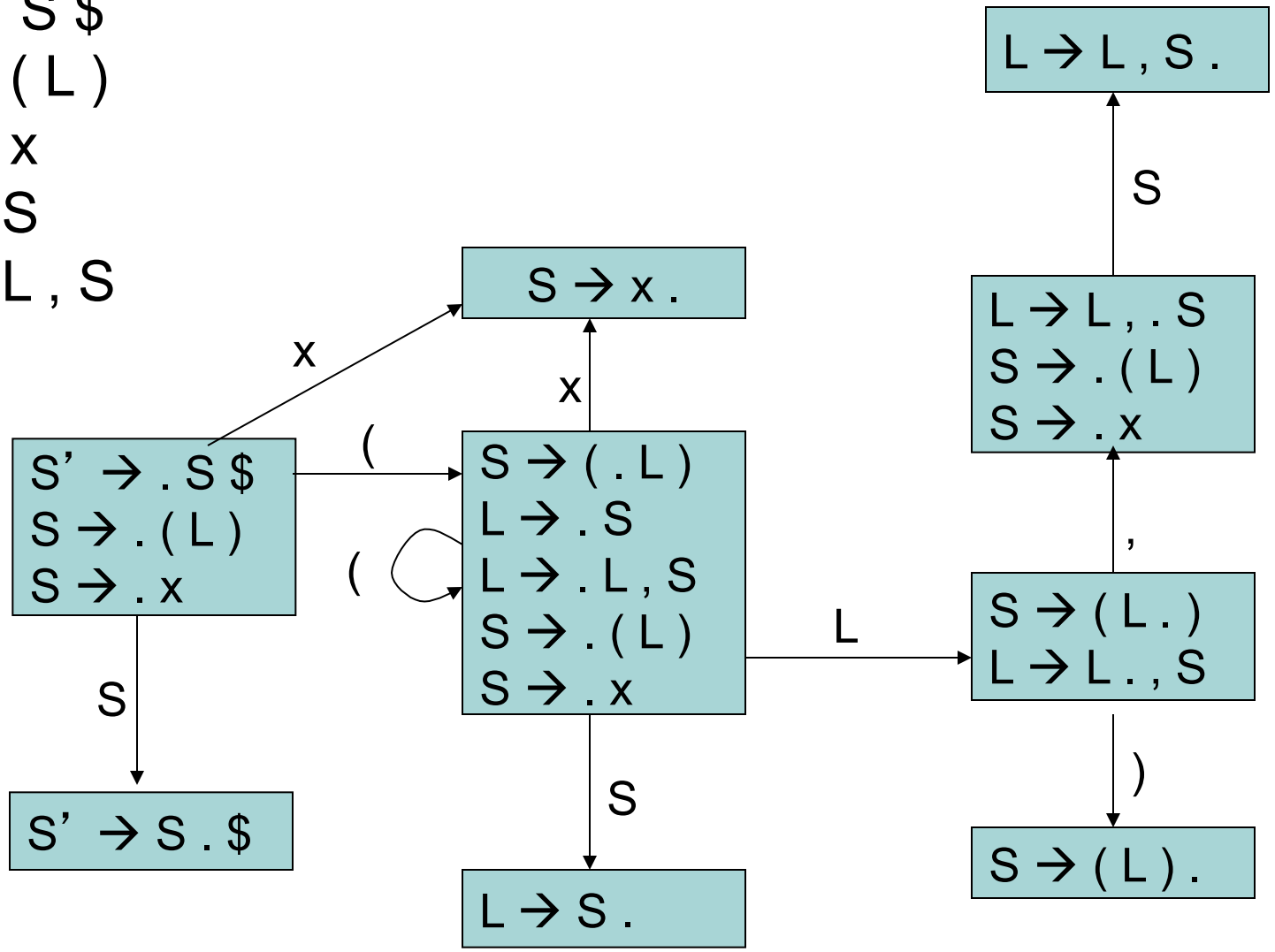

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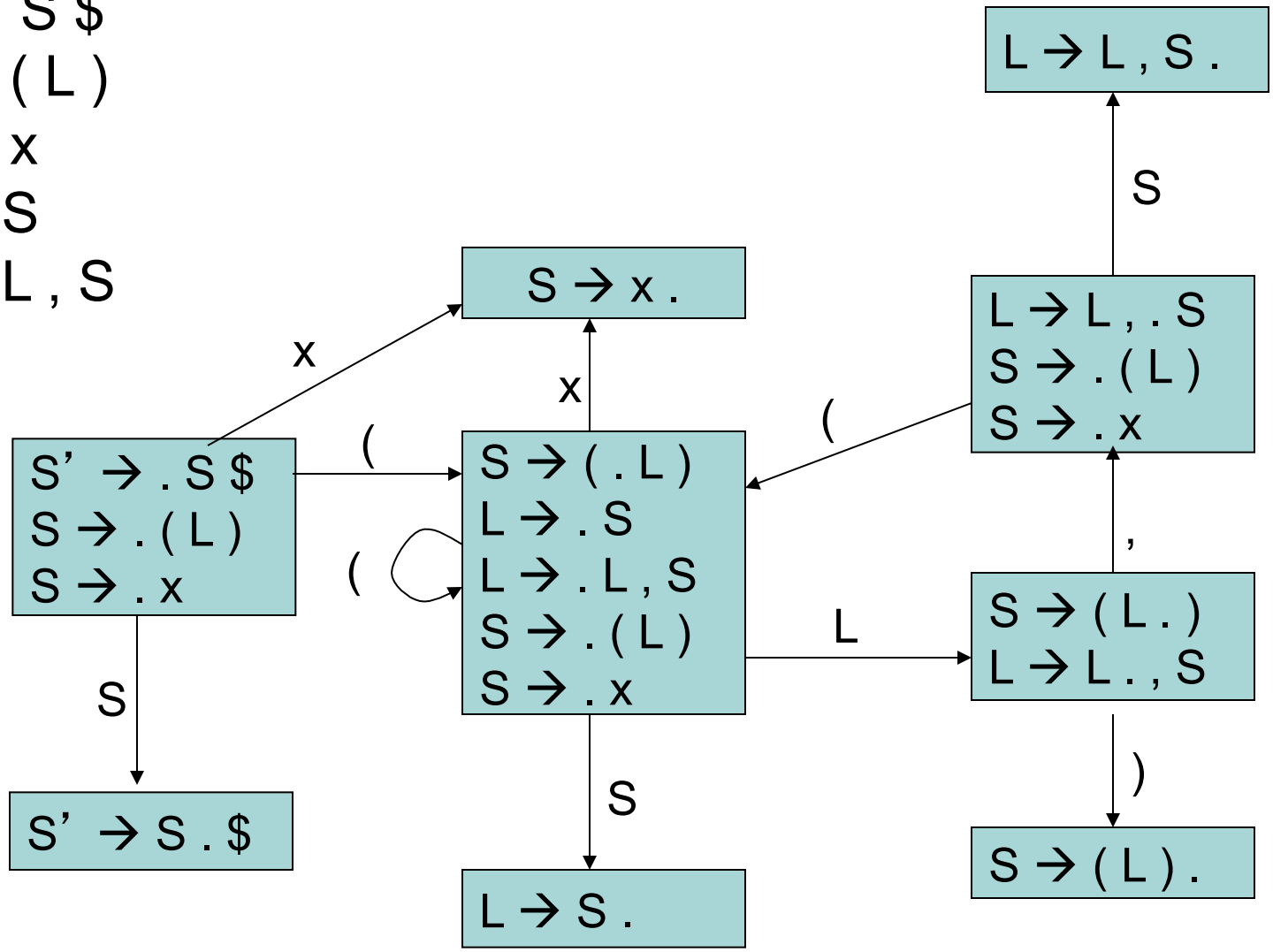
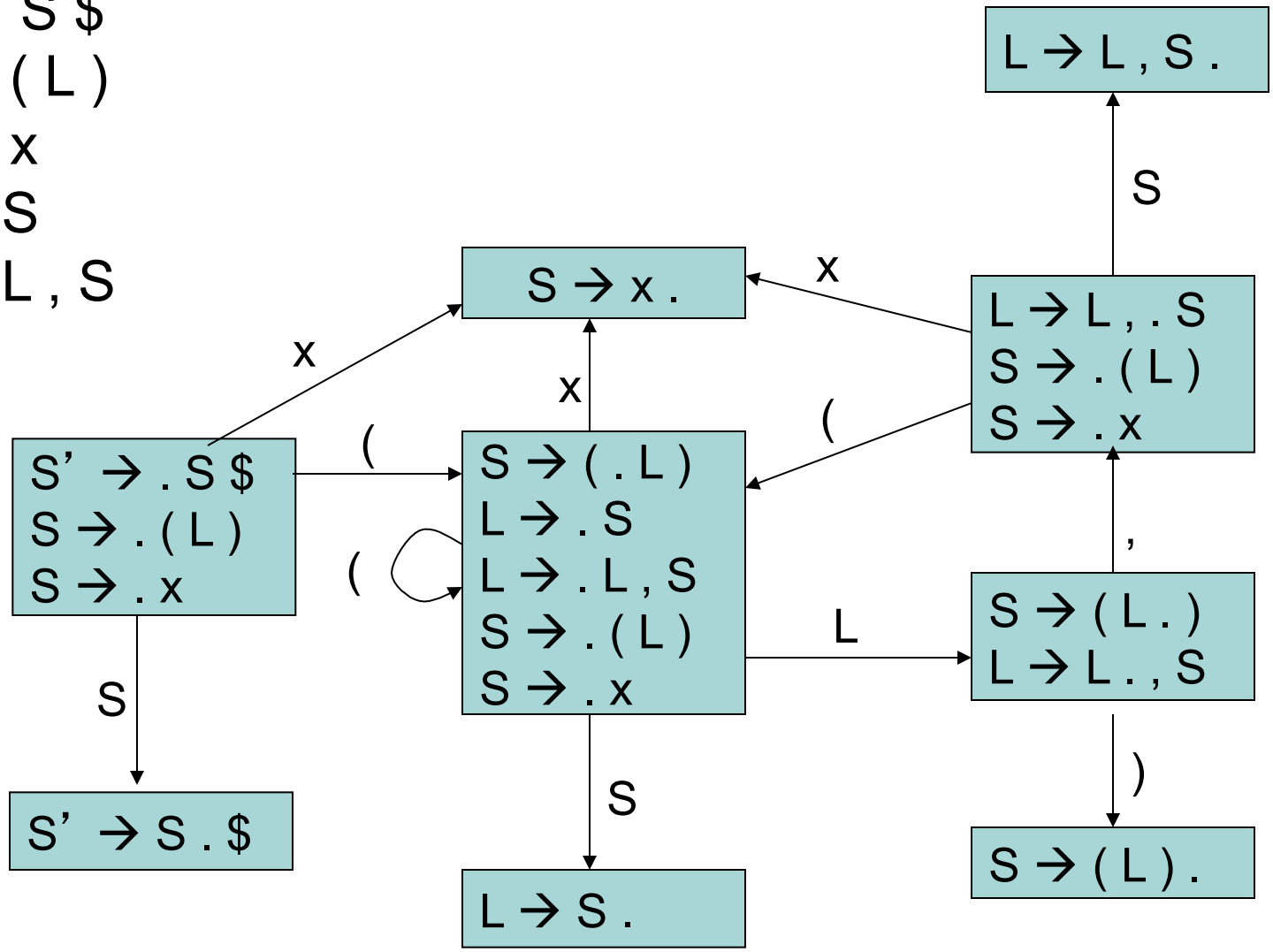
L → S
L → L , S



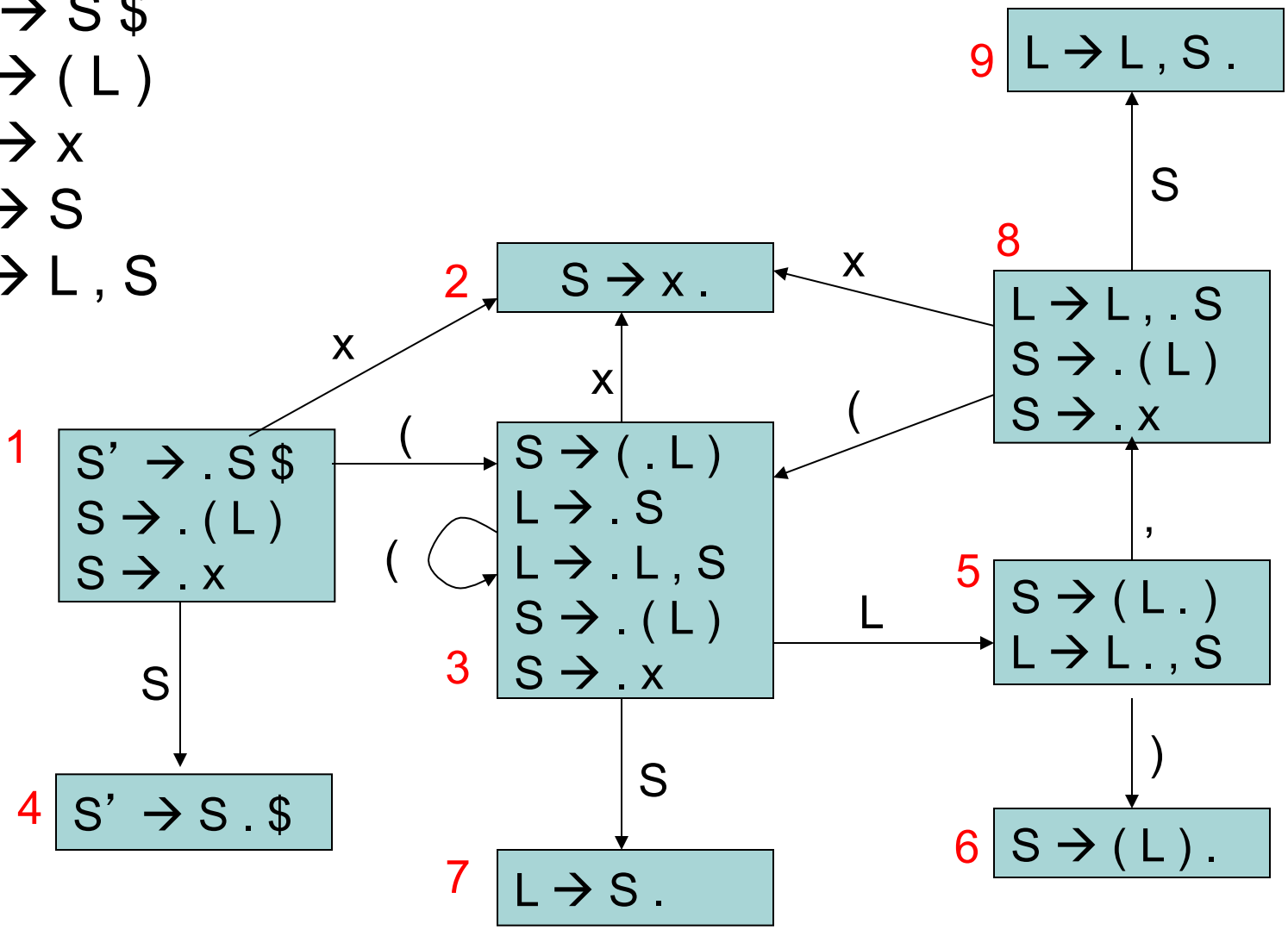
Grammar:

S' → S $
S → (L)
S → x
L → S
L → L , S



Grammar:

S' → S $
S → (L)
S → x
L → S
L → L , S

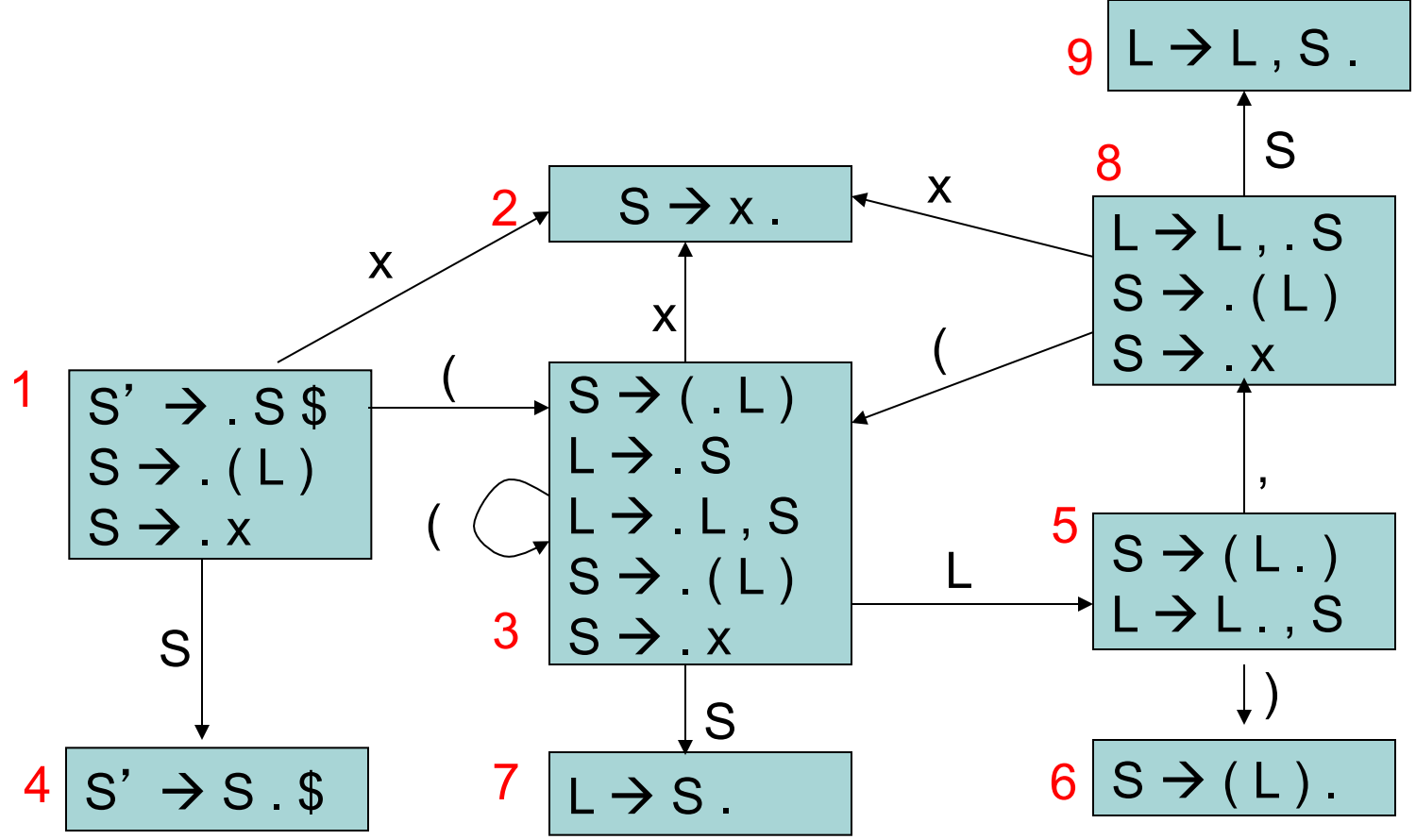


Finally, assign numbers to states.

Computing the parse table

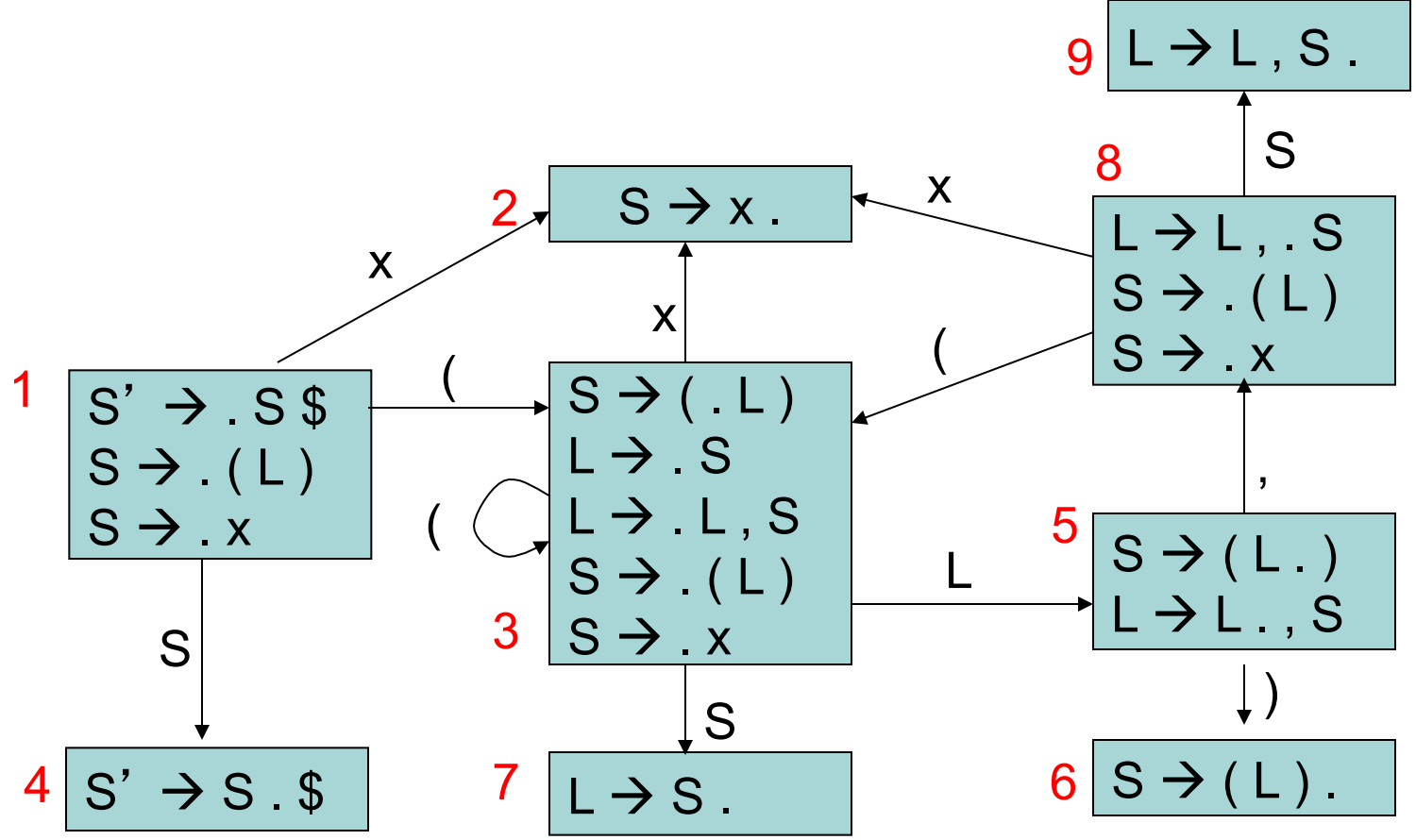
- If state *i* contains *X* → s . $, then action[*i*,$] = a.
- If state *i* contains rule *k*: *X* → s ., then table[*i*,*t*] = r*k*, for all terminals *t*.
- If there's a transition from *i* to *j* marked with *t*, then action[*i*,*t*] = s*j*.
- If there's a transition from *i* to *j* marked with *X*, then goto[*i*,*X*] = g*j*.

states	Terminal seen next ID, NUM, := ...	Non-terminals X,Y,Z ...
1		
2	s*n* = shift & goto state *n*	g*n* = goto state *n*
3	r*k* = reduce by rule *k*	
...	a = accept	
n	= error	



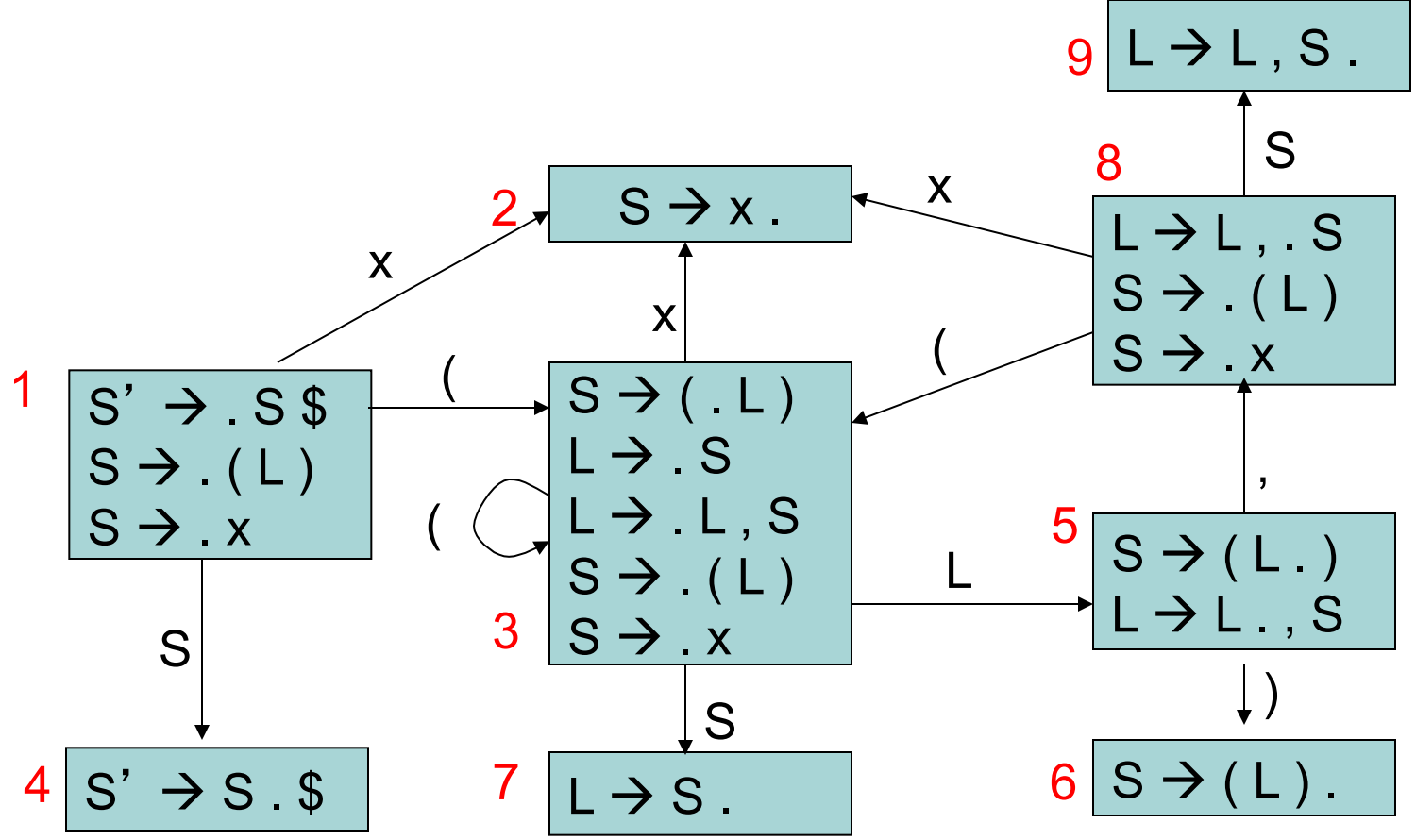
0. S' → S $
1. S → (L)
2. S → x
3. L → S
4. L → L , S

states	(	)	x	,	$	S	L
1							
2							
3							
4							
...							



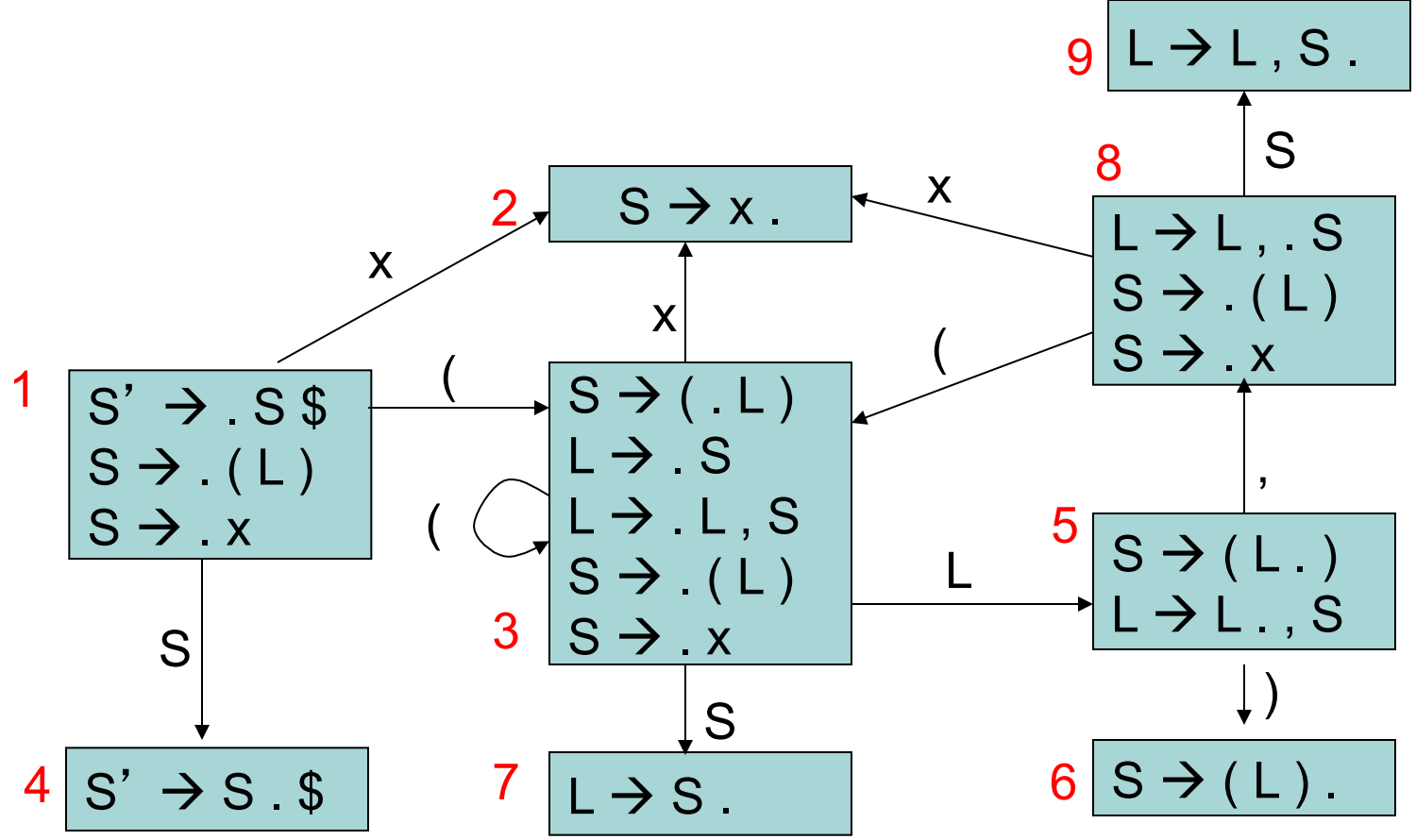
0. S' → S $
1. S → (L)
2. S → x
3. L → S
4. L → L , S

states	(	)	x	,	$	S	L
1	s3						
2							
3							
4							
...							



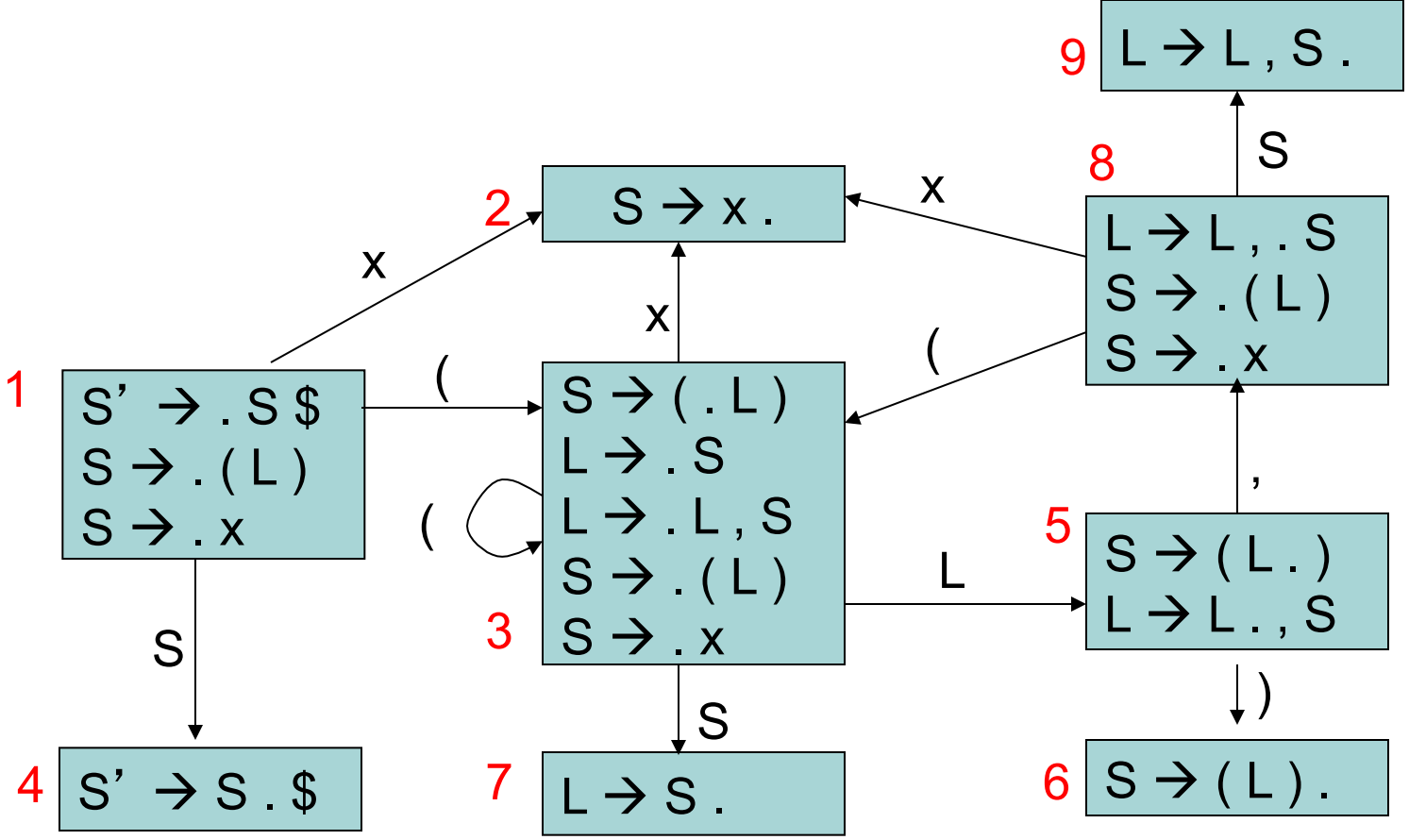
0. S' → S $
1. S → (L)
2. S → x
3. L → S
4. L → L , S

states	(	)	x	,	$	S	L
1	s3		s2				
2							
3							
4							
...							



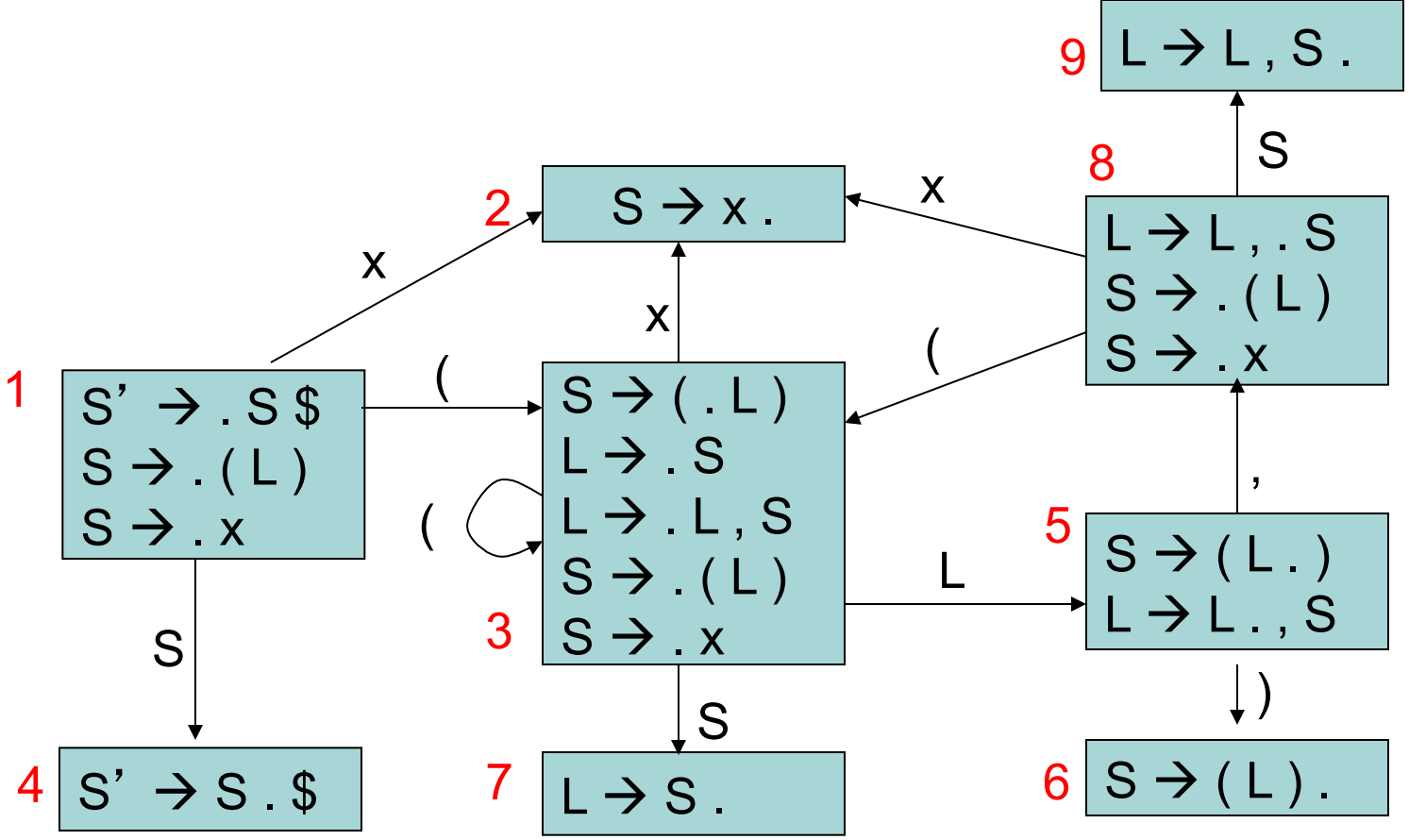
0. S' → S $
1. S → (L)
2. S → x
3. L → S
4. L → L , S

states	(	)	x	,	$	S	L
1	s3		s2			g4	
2							
3							
4							
...							



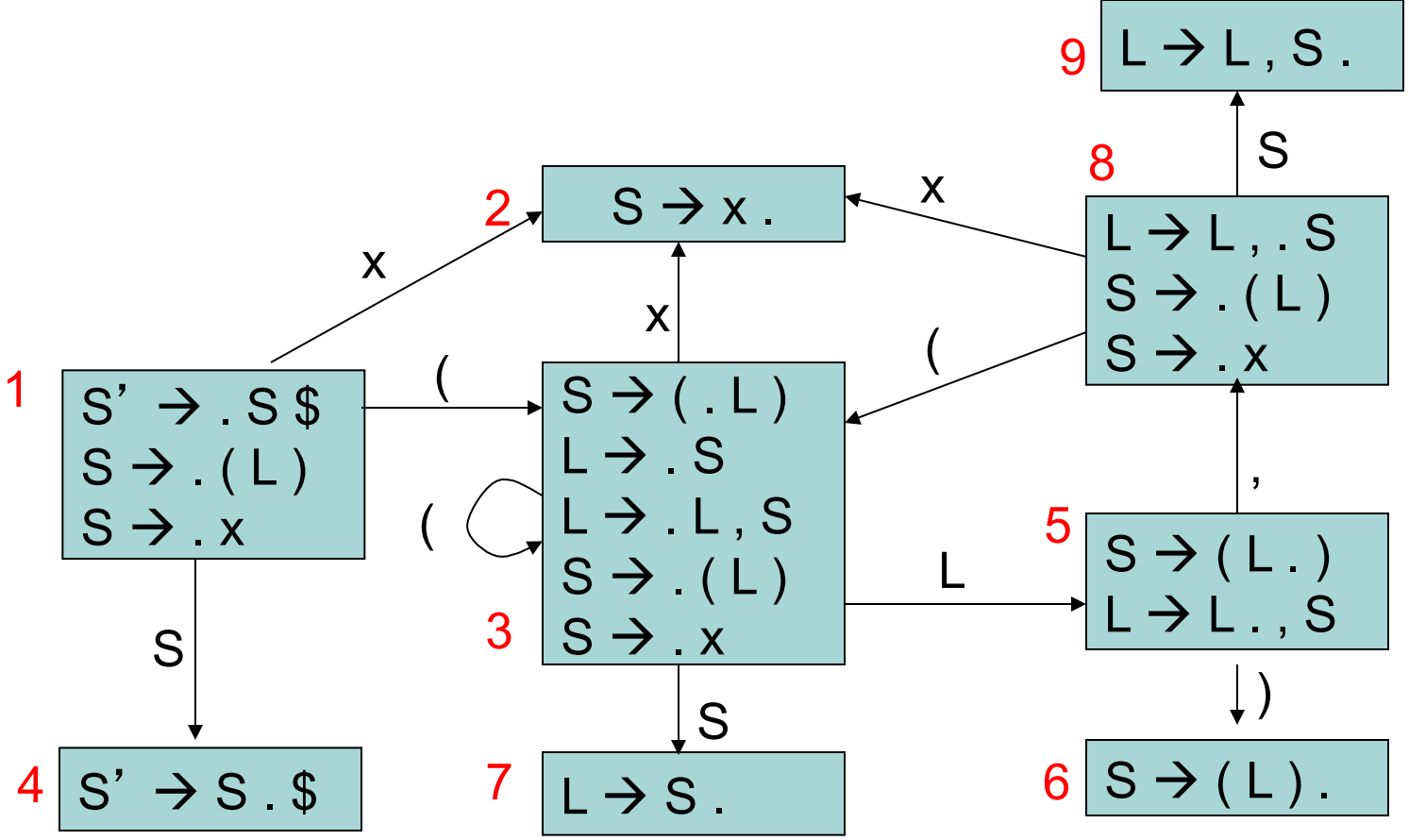
0. S' → S $
1. S → (L)
2. S → x
3. L → S
4. L → L , S

states	(	)	x	,	$	S	L
1	s3		s2			g4	
2	r2	r2	r2	r2	r2		
3							
4							
...							



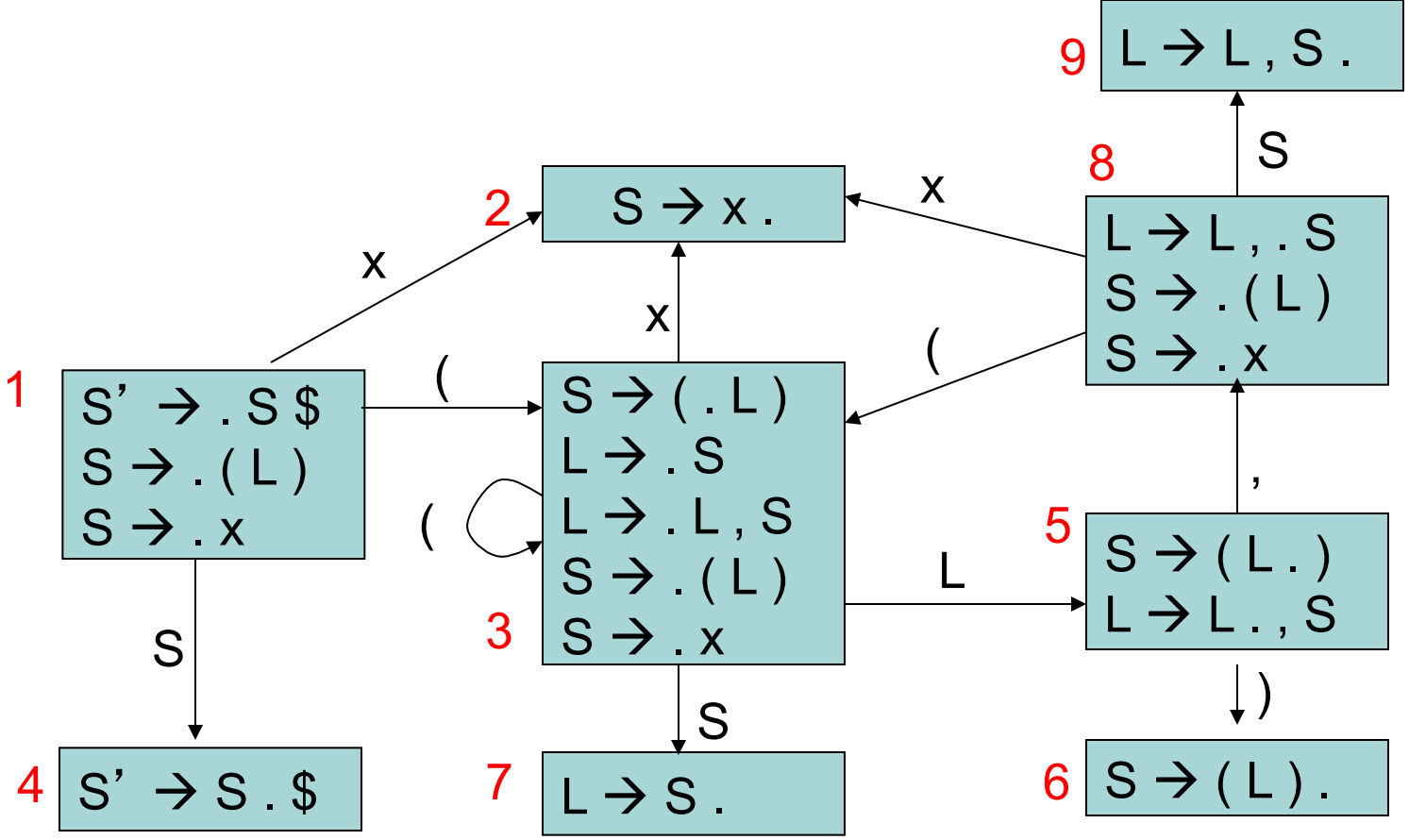
0. S' → S $
1. S → (L)
2. S → x
3. L → S
4. L → L , S

states	(	)	x	,	$	S	L
1	s3		s2			g4	
2	r2	r2	r2	r2	r2		
3	s3		s2				
4							
...							



0. S' → S $
1. S → (L)
2. S → x
3. L → S
4. L → L , S

states	(	)	x	,	$	S	L
1	s3		s2			g4	
2	r2	r2	r2	r2	r2		
3	s3		s2			g7	g5
4							
...							



0. S' → S $
1. S → (L)
2. S → x
3. L → S
4. L → L , S

states	(	)	x	,	$	S	L
1	s3		s2			g4	
2	r2	r2	r2	r2	r2		
3	s3		s2			g7	g5
4					a		
...							

The parse table

0. S' → S $
1. S → (L)
2. S → x
3. L → S
4. L → L , S

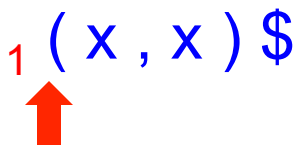
	(	)	x	,	$	S	L
1	s3		s2			g4	
2	r2	r2	r2	r2	r2		
3	s3		s2			g7	g5
4					a		
5		s6		s8			
6	r1	r1	r1	r1	r1		
7	r3	r3	r3	r3	r3		
8	s3		s2			g9	
9	r4	r4	r4	r4	r4		

action

goto

states	(	)	x	,	$	S	L
1	s3		s2			g4	
2	r2	r2	r2	r2	r2		
3	s3		s2			g7	g5
4					a		
5		s6		s8			
6	r1	r1	r1	r1	r1		
7	r3	r3	r3	r3	r3		
8	s3		s2			g9	
9	r4	r4	r4	r4	r4		

0. S' → S $
1. S → (L)
2. S → x
3. L → S
4. L → L , S



states	(	)	x	,	$	S	L
1	s3		s2			g4	
2	r2	r2	r2	r2	r2		
3	s3		s2			g7	g5
4					a		
5		s6		s8			
6	r1	r1	r1	r1	r1		
7	r3	r3	r3	r3	r3		
8	s3		s2			g9	
9	r4	r4	r4	r4	r4		

0. S' → S $
1. S → (L)
2. S → x
3. L → S
4. L → L , S

$_1(_3$ x , x) $

states	(	)	x	,	$	S	L
1	s3		s2			g4	
2	r2	r2	r2	r2	r2		
3	s3		s2			g7	g5
4					a		
5		s6		s8			
6	r1	r1	r1	r1	r1		
7	r3	r3	r3	r3	r3		
8	s3		s2			g9	
9	r4	r4	r4	r4	r4		

0. S' → S $
1. S → (L)
2. S → x
3. L → S
4. L → L , S

$_1 (_3 x_2 , x) \$$

states	(	)	x	,	$	S	L
1	s3		s2			g4	
2	r2	r2	r2	r2	r2		
3	s3		s2			g7	g5
4					a		
5		s6		s8			
6	r1	r1	r1	r1	r1		
7	r3	r3	r3	r3	r3		
8	s3		s2			g9	
9	r4	r4	r4	r4	r4		

0. S' → S $
1. S → (L)
2. S → x
3. L → S
4. L → L , S

$_1 (_3$ S , x) $

states	(	)	x	,	$	S	L
1	s3		s2			g4	
2	r2	r2	r2	r2	r2		
3	s3		s2			g7	g5
4					a		
5		s6		s8			
6	r1	r1	r1	r1	r1		
7	r3	r3	r3	r3	r3		
8	s3		s2			g9	
9	r4	r4	r4	r4	r4		

0. S' → S $
1. S → (L)
2. S → x
3. L → S
4. L → L , S

$_1 (_3 S_7 , x) \$$

states	(	)	x	,	$	S	L
1	s3		s2			g4	
2	r2	r2	r2	r2	r2		
3	s3		s2			g7	g5
4					a		
5		s6		s8			
6	r1	r1	r1	r1	r1		
7	r3	r3	r3	r3	r3		
8	s3		s2			g9	
9	r4	r4	r4	r4	r4		

0. S' → S $
1. S → (L)
2. S → x
3. L → S
4. L → L , S

$_1 (_3 L , x) \$$

states	(	)	x	,	$	S	L
1	s3		s2			g4	
2	r2	r2	r2	r2	r2		
3	s3		s2			g7	g5
4					a		
5		s6		s8			
6	r1	r1	r1	r1	r1		
7	r3	r3	r3	r3	r3		
8	s3		s2			g9	
9	r4	r4	r4	r4	r4		

0. S' → S $
1. S → (L)
2. S → x
3. L → S
4. L → L , S

$_1$ ($_3$ L_5 , x) $

states	(	)	x	,	$	S	L
1	s3		s2			g4	
2	r2	r2	r2	r2	r2		
3	s3		s2			g7	g5
4					a		
5		s6		s8			
6	r1	r1	r1	r1	r1		
7	r3	r3	r3	r3	r3		
8	s3		s2			g9	
9	r4	r4	r4	r4	r4		

0. S' → S $
1. S → (L)
2. S → x
3. L → S
4. L → L , S

$_1$ ($_3$ L_5 ,$_8$ x) $

states	(	)	x	,	$	S	L
1	s3		s2			g4	
2	r2	r2	r2	r2	r2		
3	s3		s2			g7	g5
4					a		
5		s6		s8			
6	r1	r1	r1	r1	r1		
7	r3	r3	r3	r3	r3		
8	s3		s2			g9	
9	r4	r4	r4	r4	r4		

0. S' → S $
1. S → (L)
2. S → x
3. L → S
4. L → L , S

$_1 (_3 \; L_5 \; ,_8 \; x_2 \;) \; \$$

states	(	)	x	,	$	S	L
1	s3		s2			g4	
2	r2	r2	r2	r2	r2		
3	s3		s2			g7	g5
4					a		
5		s6		s8			
6	r1	r1	r1	r1	r1		
7	r3	r3	r3	r3	r3		
8	s3		s2			g9	
9	r4	r4	r4	r4	r4		

0. S' → S $
1. S → (L)
2. S → x
3. L → S
4. L → L , S

$_1 (_3 \; L_5 \; ,_8 \; S \;) \; \$$

states	(	)	x	,	$	S	L
1	s3		s2			g4	
2	r2	r2	r2	r2	r2		
3	s3		s2			g7	g5
4					a		
5		s6		s8			
6	r1	r1	r1	r1	r1		
7	r3	r3	r3	r3	r3		
8	s3		s2			g9	
9	r4	r4	r4	r4	r4		

0. S' → S $
1. S → (L)
2. S → x
3. L → S
4. L → L , S

$_1 (_3 \; L_5 \; ,_8 \; S_9 \;) \; \$$

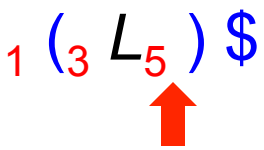
states	(	)	x	,	$	S	L
1	s3		s2			g4	
2	r2	r2	r2	r2	r2		
3	s3		s2			g7	g5
4					a		
5		s6		s8			
6	r1	r1	r1	r1	r1		
7	r3	r3	r3	r3	r3		
8	s3		s2			g9	
9	r4	r4	r4	r4	r4		

0. S' → S $
1. S → (L)
2. S → x
3. L → S
4. L → L , S

$_1 (_3 \; L \;) \; \$$

states	(	)	x	,	$	S	L
1	s3		s2			g4	
2	r2	r2	r2	r2	r2		
3	s3		s2			g7	g5
4					a		
5		s6		s8			
6	r1	r1	r1	r1	r1		
7	r3	r3	r3	r3	r3		
8	s3		s2			g9	
9	r4	r4	r4	r4	r4		

0. S' → S $
1. S → (L)
2. S → x
3. L → S
4. L → L , S



states	(	)	x	,	$	S	L
1	s3		s2			g4	
2	r2	r2	r2	r2	r2		
3	s3		s2			g7	g5
4					a		
5		s6		s8			
6	r1	r1	r1	r1	r1		
7	r3	r3	r3	r3	r3		
8	s3		s2			g9	
9	r4	r4	r4	r4	r4		

0. S' → S $
1. S → (L)
2. S → x
3. L → S
4. L → L , S

$_1 (_3 \; L_5 \;)_6 \; \$$

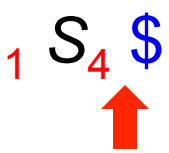
states	(	)	x	,	$	S	L
1	s3		s2			g4	
2	r2	r2	r2	r2	r2		
3	s3		s2			g7	g5
4					a		
5		s6		s8			
6	r1	r1	r1	r1	r1		
7	r3	r3	r3	r3	r3		
8	s3		s2			g9	
9	r4	r4	r4	r4	r4		

0. S' → S $
1. S → (L)
2. S → x
3. L → S
4. L → L , S

1 S $

states	(	)	x	,	$	S	L
1	s3		s2			g4	
2	r2	r2	r2	r2	r2		
3	s3		s2			g7	g5
4					a		
5		s6		s8			
6	r1	r1	r1	r1	r1		
7	r3	r3	r3	r3	r3		
8	s3		s2			g9	
9	r4	r4	r4	r4	r4		

0. S' → S $
1. S → (L)
2. S → x
3. L → S
4. L → L , S



LR(0)

- Even though we are doing LR(0) parsing we are using some look ahead (there is a column for each non-terminal).
- However, we only use the terminal to figure out which state to go to next, not to decide whether to shift or reduce.

states	(	)	x	,	$	S	L
1	s3		s2			g4	
2	r2	r2	r2	r2	r2		
3	s3		s2			g7	g5

LR(0)

- Even though we are doing LR(0) parsing we are using some look ahead (there is a column for each non-terminal).
- However, we only use the terminal to figure out which state to go to next, not to decide whether to shift or reduce.

states	(	)	x	,	$	S	L
1	s3		s2			g4	
2	r2	r2	r2	r2	r2		
3	s3		s2			g7	g5

states	no look-ahead	S	L
1	shift	g4	
2	reduce 2		
3	shift	g7	g5

LR(0)

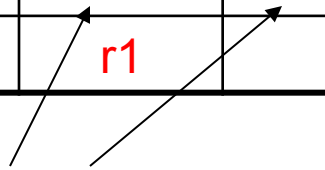
- Even though we are doing LR(0) parsing we are using some look ahead (there is a column for each non-terminal).
- However, we only use the terminal to figure out which state to go to next, not to decide whether to shift or reduce.
- If the same row contains both shift and reduce, we will have a conflict – i.e., the grammar is not LR(0).
- Likewise, if the same row contains reduce by two different rules.

states	no look-ahead	S	L
1	shift, reduce 5	g4	
2	reduce 2, reduce 7		
3	shift	g7	g5

SLR

- SLR (simple LR) is a variant of LR(0) that reduces the number of conflicts in LR(0) tables by using a tiny bit of look ahead.
- To determine when to reduce, 1 symbol of lookahead is used.
- Only put reduce by rule (X → RHS) in column T if T is in FOLLOW(X).

states	(	)	x	,	$	S	L
1	s3		s2			g4	
2	r2	s5	r2				
3	r1		r1	r5	r5	g7	g5



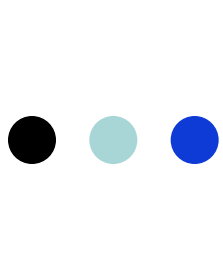
cuts down the number of r*k* slots & therefore cuts down conflicts

LR(1) & LALR

- LR(1) automata are identical to LR(0) except for the "items" that make up the states:
  - LR(0) items: X → s1 . s2
  - LR(1) items: X → s1 . s2, T

Lookahead symbol added.

- Idea: sequence s1 is on stack; input stream is s2 T.
- Find closure with respect to X → s1 . Y s2, T by adding all items Y → s3, U when Y → s3 is a rule and U is in FIRST(s2 T).
- Two states are different if they contain the same rules but the rules have different look-ahead symbols
  - Leads to many states.
  - LALR(1) = LR(1) where states that are identical aside from look-ahead symbols have been merged.
  - YACC/Bison & most parser generators use LALR by default.



Summary

- LR parsing is more powerful than LL parsing, given the same look ahead.
- To construct an LR parser, it is necessary to compute an LR parser table.
- The LR parser table represents a finite automaton that walks over the parser stack.
- YACC and Bison use LALR, a compact variant of LR(1).