Compilers I

Loop Optimizations

Dr William Harrison

Loop Optimizations

- Empirical studies show that much of program execution occurs within loops
  - So, it makes sense to identify loops & concentrate optimization efforts on that code
  - But what is a loop?
    - Within source code, it's obvious
    - Within IR/CFG, however,…
      - Loops aren't precisely what your first guess would be

Simple loop optimization

within the source code, that is…

for (i = 1; i++ ; i = 10) { c } ➔ c ; … ; c

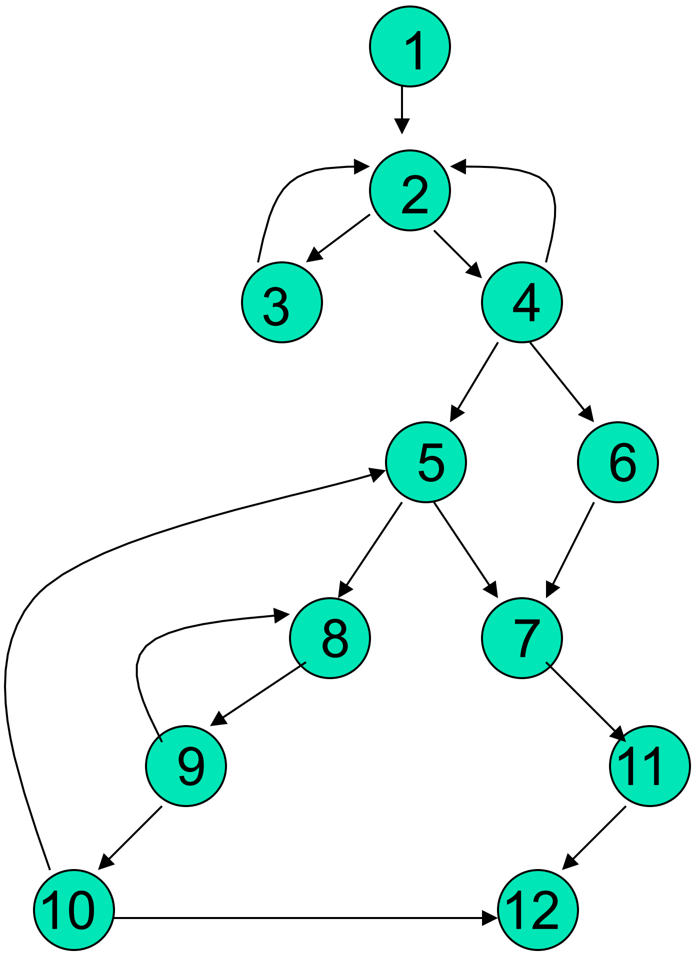
10 times

Caveat:
- seems reasonable (removes branches)
- not terribly general: would like to use on while loops but how do you identify the induction variable "i"?
- also, how does this apply at the IR/CFG level?

Ex: Loop invariant code hoisting

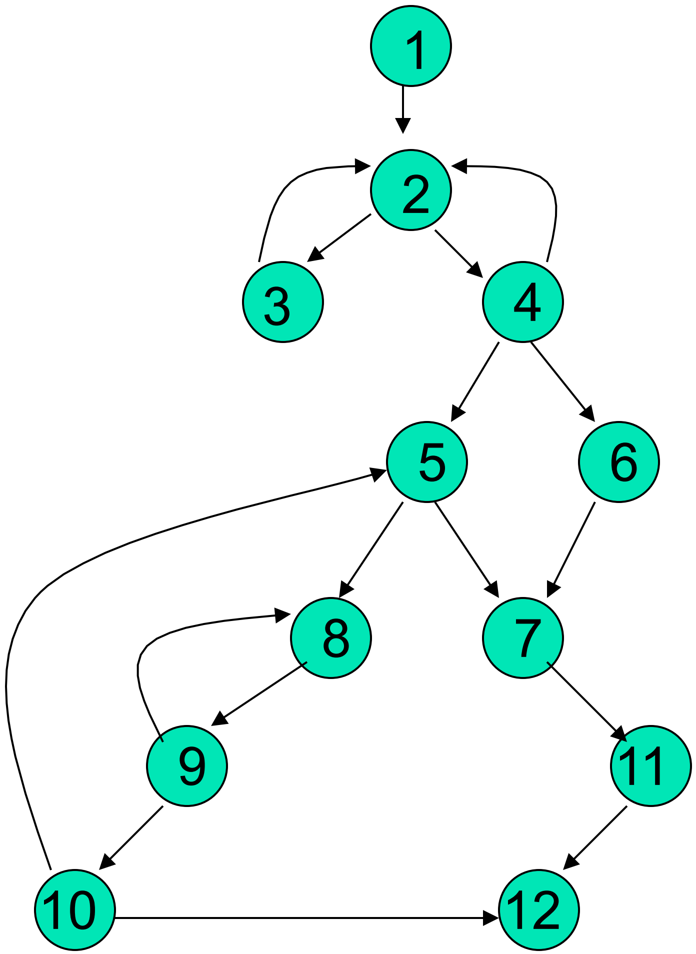
- Say the assignment "a←b+c" occurs within a loop
  - But "b" and "c" aren't assigned in the loop
  - Would like to move "a←b+c" somewhere "right in front of" the loop
    - Thereby avoiding redundant reassignments to "a"
- Problem IR/CFG is a directed graph of basic blocks
  - Where, for example, is the "front of" a loop?

A flow graph



Say "a←b+c" occurs in 9,
To where might we hoist it?

A flow graph



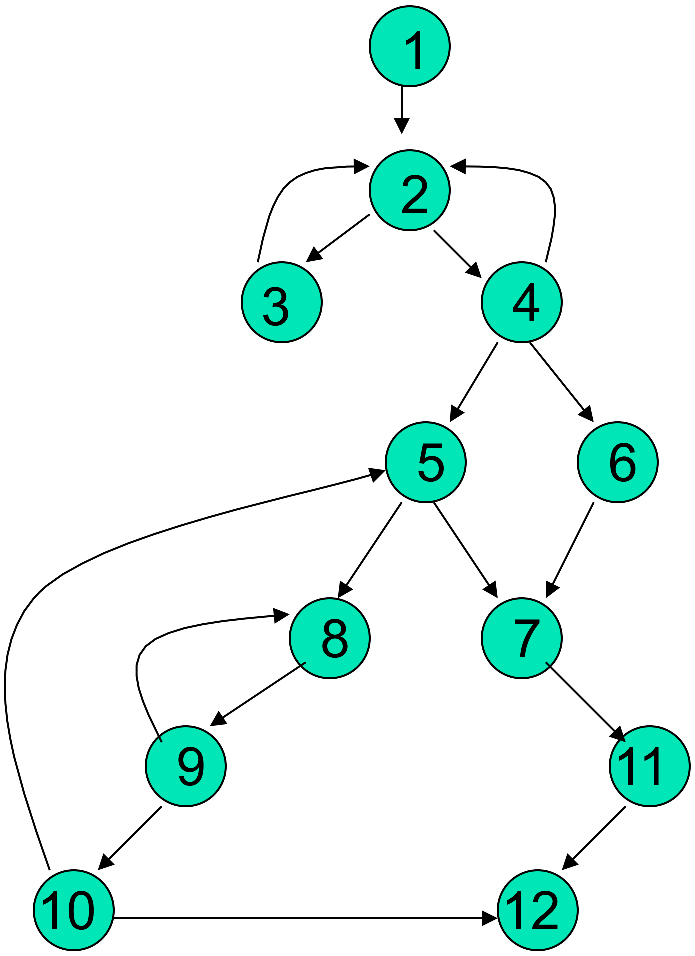
Say “a🡐b+c” occurs in 9,
To where might we hoist it?

- Nodes 5,4,2,1 seem likely
  - how would I determine that automatically?
  - Impact on other optimizations?
  - Identifying loops with “dominator tree"

Dominators

- Assume we have a flow graph with entry node s_0 with no predecessors
  - I.e., no edge into s_0
- Node d **dominates** n means that
  - d occurs in every path from s_0 to n
- Note that every node dominates itself

A flow graph

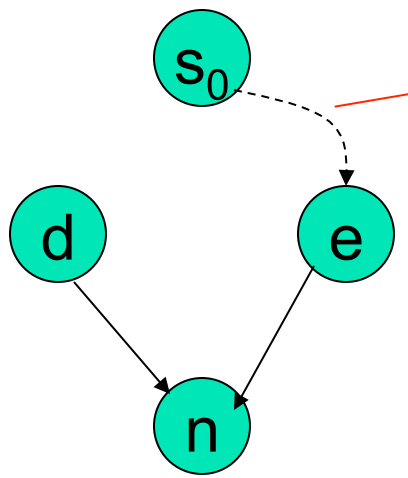


– for each pair of nodes d,n
  – does d dominate n?
  – can think of “d dominates n” as a new kind of directed arc in a new graph
– **Question**: is it possible that for d≠n that:
  – d dominates n, and
  – n dominates d?

Theorem

Let d ≠ e both dominate n. Then, either d dominates e or e dominates d (but not both)

Proof sketch. Assume neither d dominates e nor vice versa.



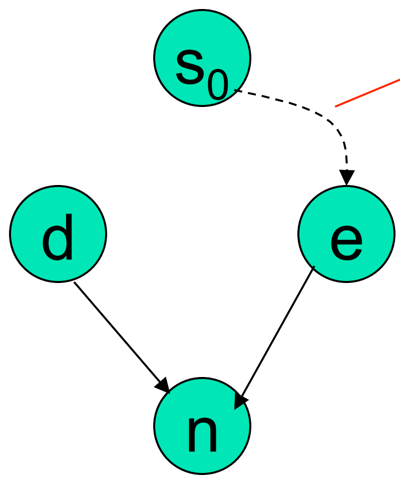
Path to e not containing d exists since d doesn't dominate e

Theorem

Let d ≠ e both dominate n. Then, either d dominates e or e dominates d (but not both)

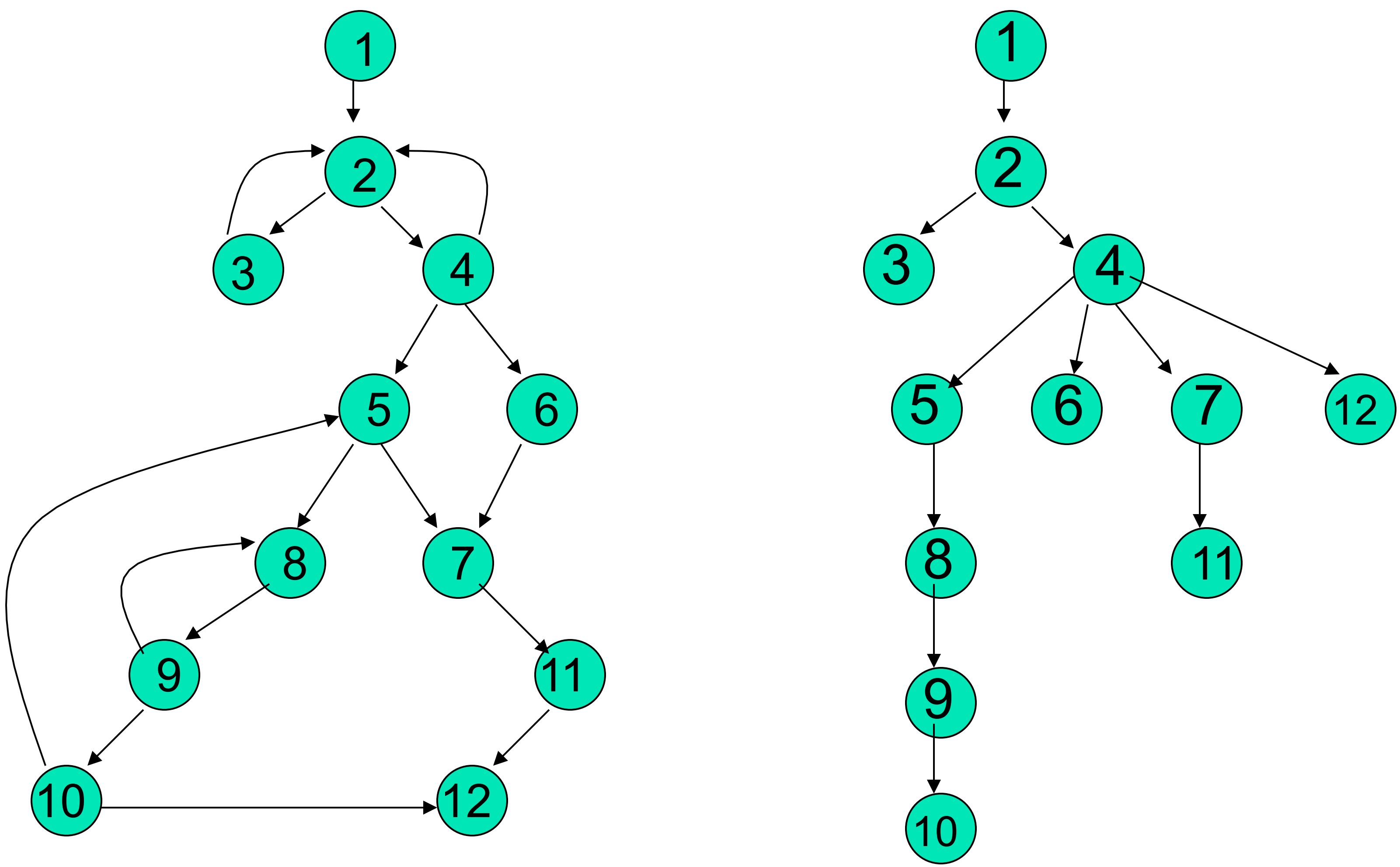
Proof sketch. Assume neither d dominates e nor vice versa.

Path to e not containing d exists since d doesn’t dominate e

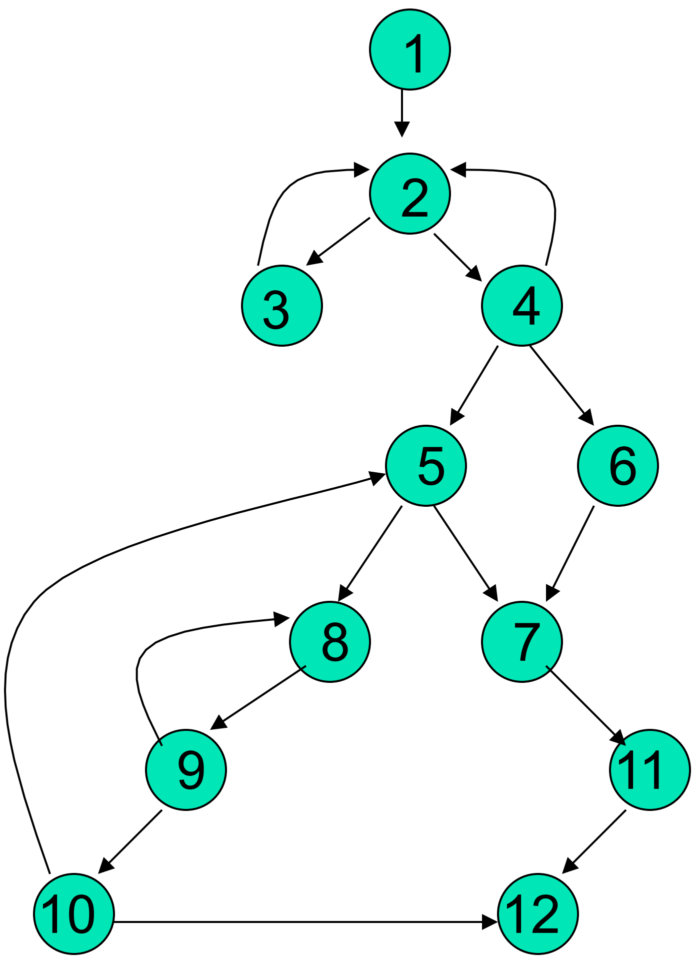


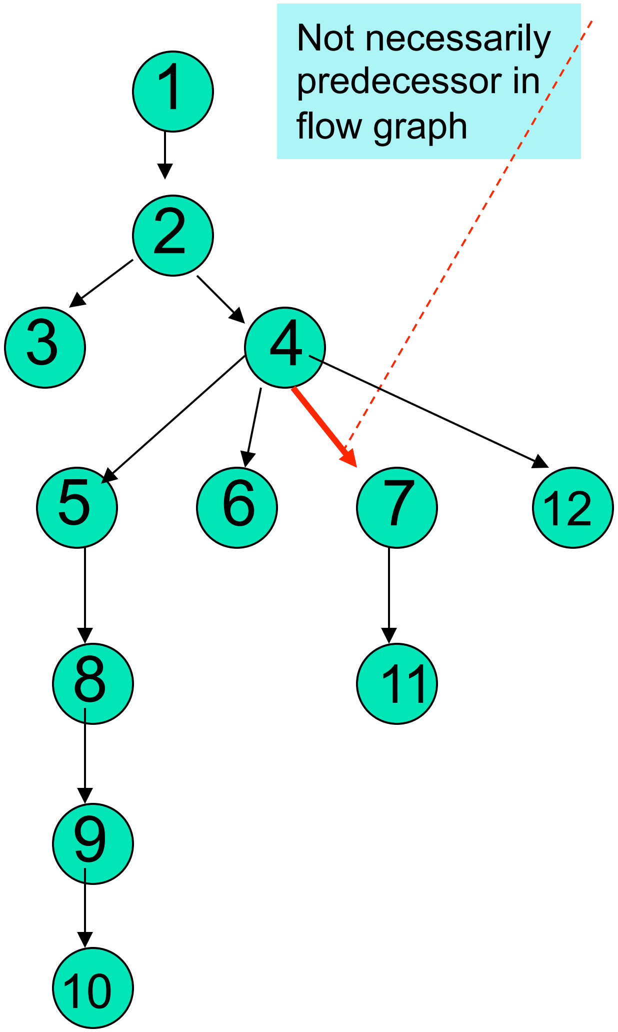
Path from s_0 to n not through d exists. ∴ d doesn’t dominate n ➔🡐

flow graph & its **dominator tree**



flow graph & its **dominator tree**





Immediate Dominators

- An **immediate dominator** of node n is a node idom(n) such that
  - idom(n) $\neq$n
  - idom(n) dominates n
  - idom(n) does not dominate any other dominator of n
- Every node has (at most) one immediate dominator
  - How do we know this?

Calculating Dominators

Let D[n] be the set of nodes that dominate n in a particular flow graph G, we get the following two simultaneous equations*

$$D[s_0] = \{s_0\}$$

$$D[n] = \{n\} \cup \left(\bigcap_{p \in pred(n)} D[p]\right) \text{ for } n \neq s_0$$

*I wonder how we'd solve equations like this?

Iterative solution

change := true

$D[s_0]$:= $\{s_0\}$

foreach $n \in (\text{Nodes}(G) \setminus \{s_0\})$ { D[n] := {n} }

repeat

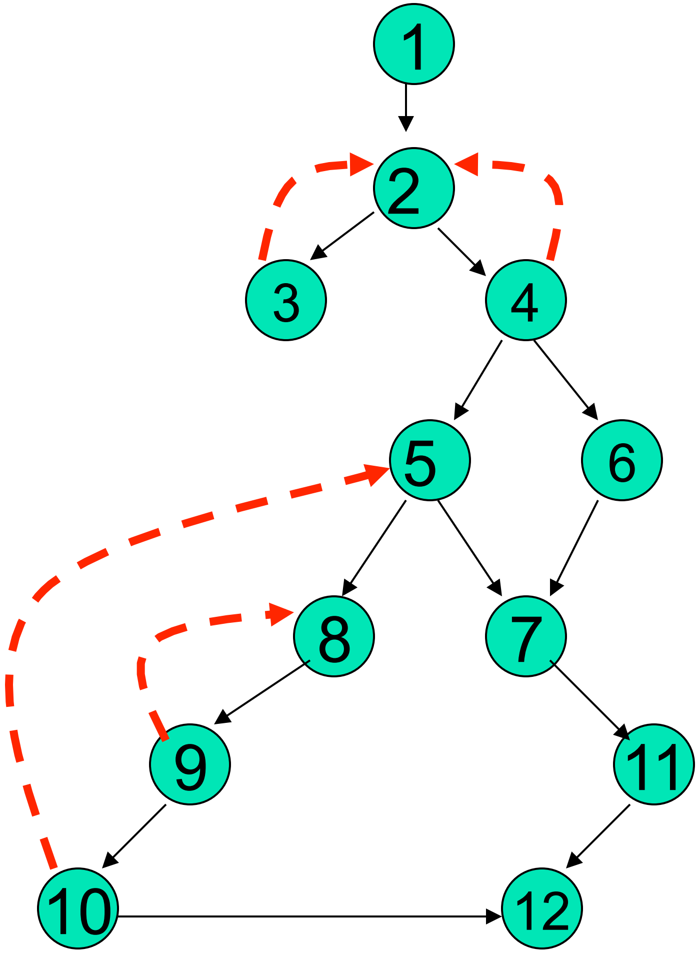
    change := false

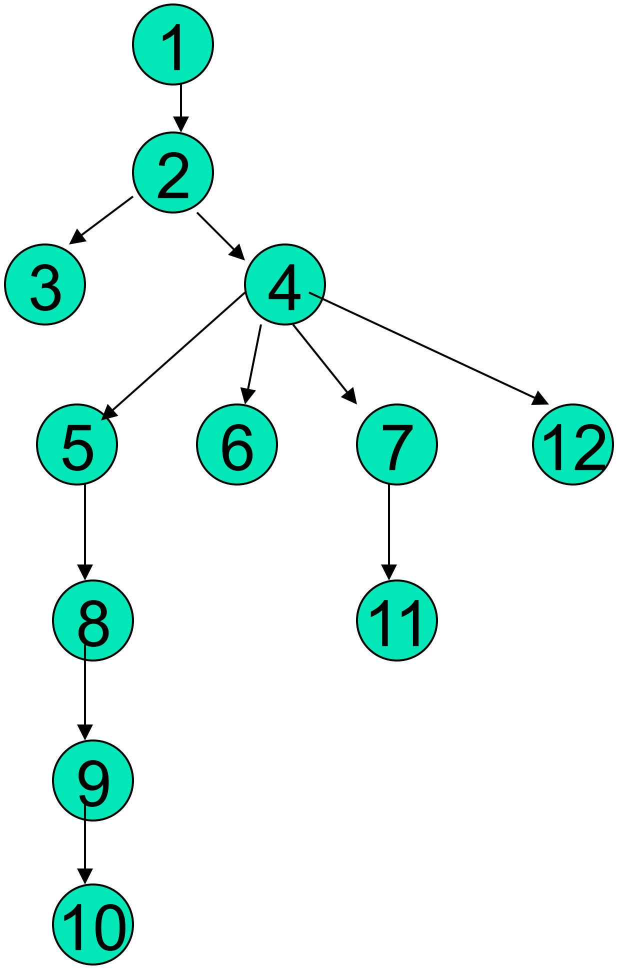
    foreach $n \in (\text{Nodes}(G) \setminus \{s_0\})$

$$\left(\begin{array}{l}
T := \text{Node} \\
\text{foreach } p \in (\text{pred}(n) \setminus \{s_0\}) \ \{ \ T := T \cap D[p] \ \} \\
X := \{n\} \cup T \\
\text{if } X \neq D[n] \text{ then} \\
\quad \text{change} := \text{true} \\
\quad D[n] = X
\end{array}\right)$$

until (not change)

Backedges are edges n→h where h dominates n

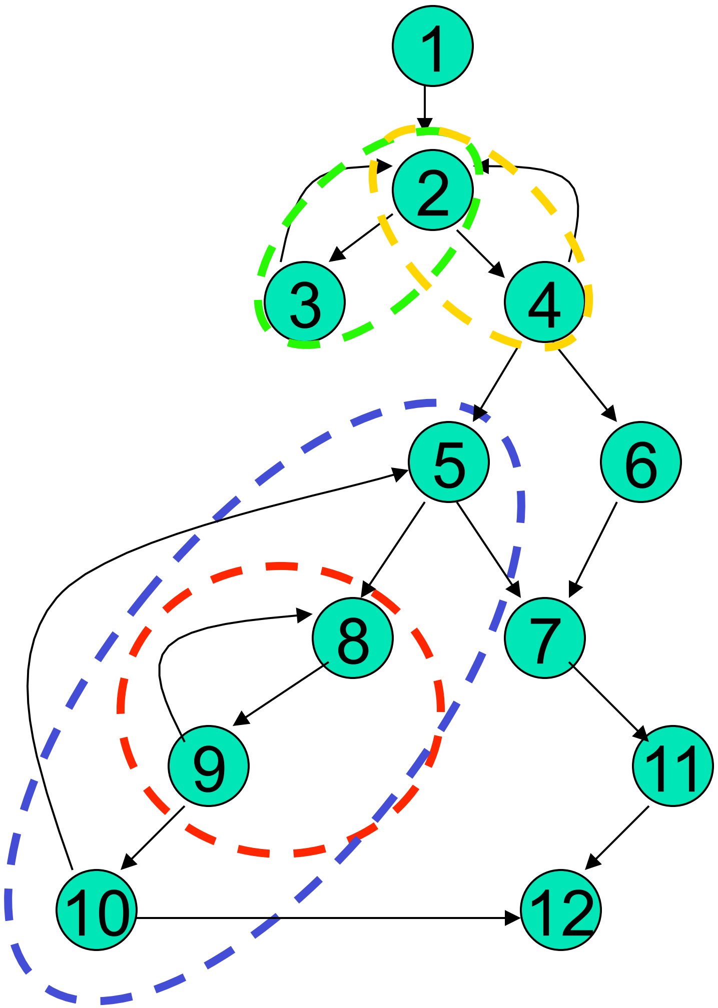




“Natural” Loops

- The natural loop of a backedge n➔h is
  - The set of x such that
    - h dominates x
    - there is a path from x to n not containing h
- h is called the **header** of this loop

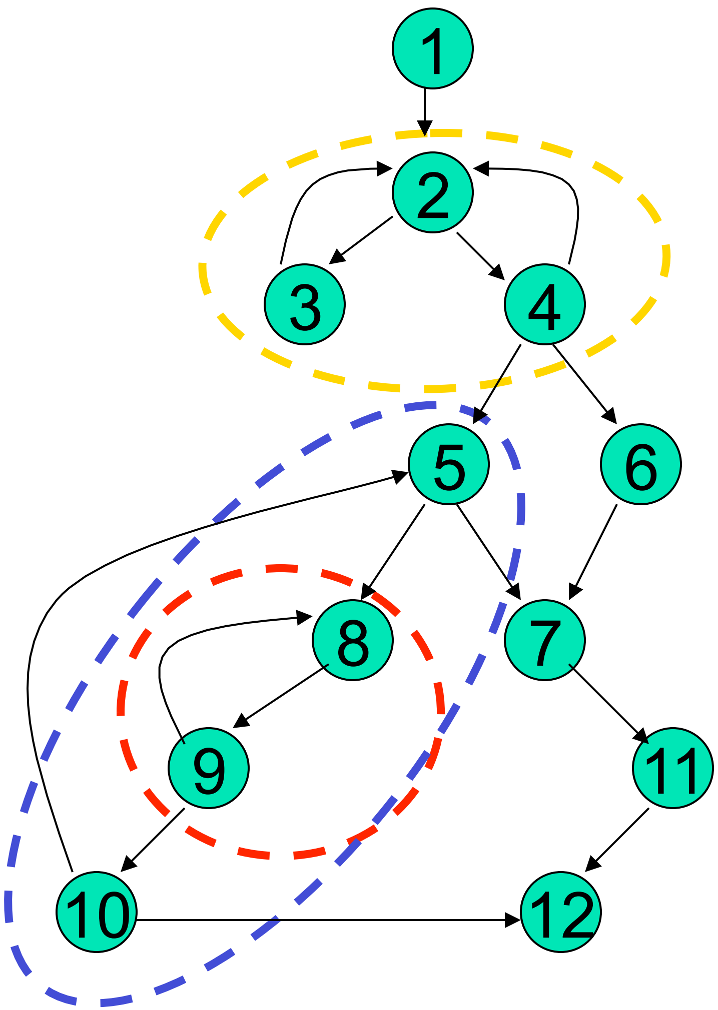
Backedges induce natural loops



The four natural loops are:

{2,3}
{2,4}
{5,8,9,10}
{8,9}

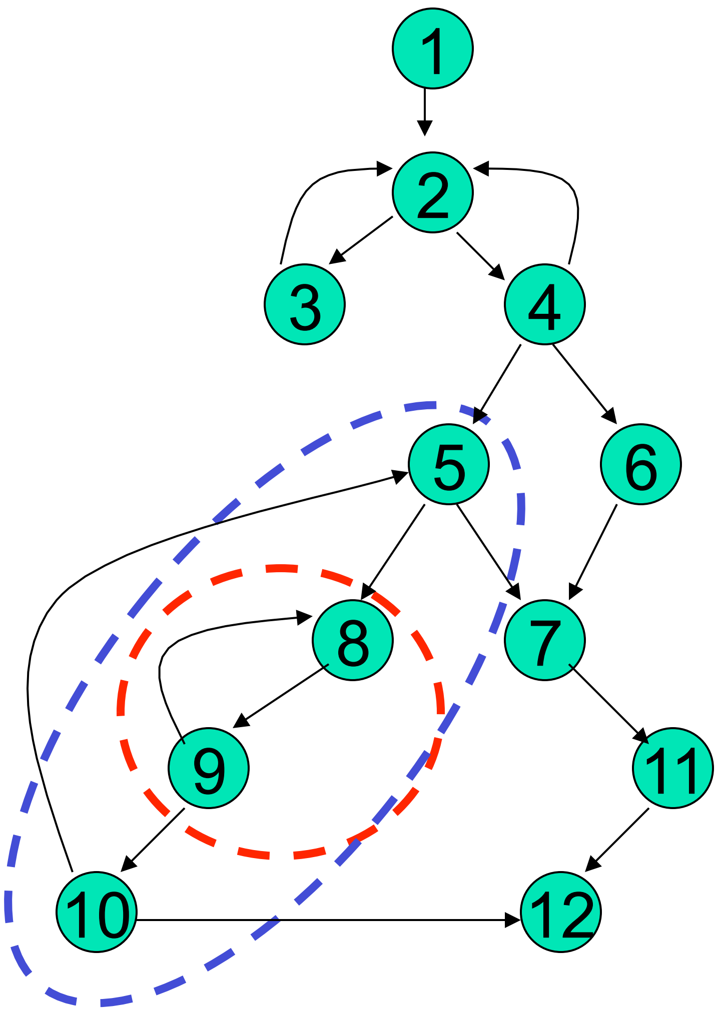
Identifying loops



By merging natural loops with identical headers, one identifies all **loops**

- note that a natural loop is what you think of as a loop
- while a loop inside a compiler is a slightly different animal

Identifying nested loops



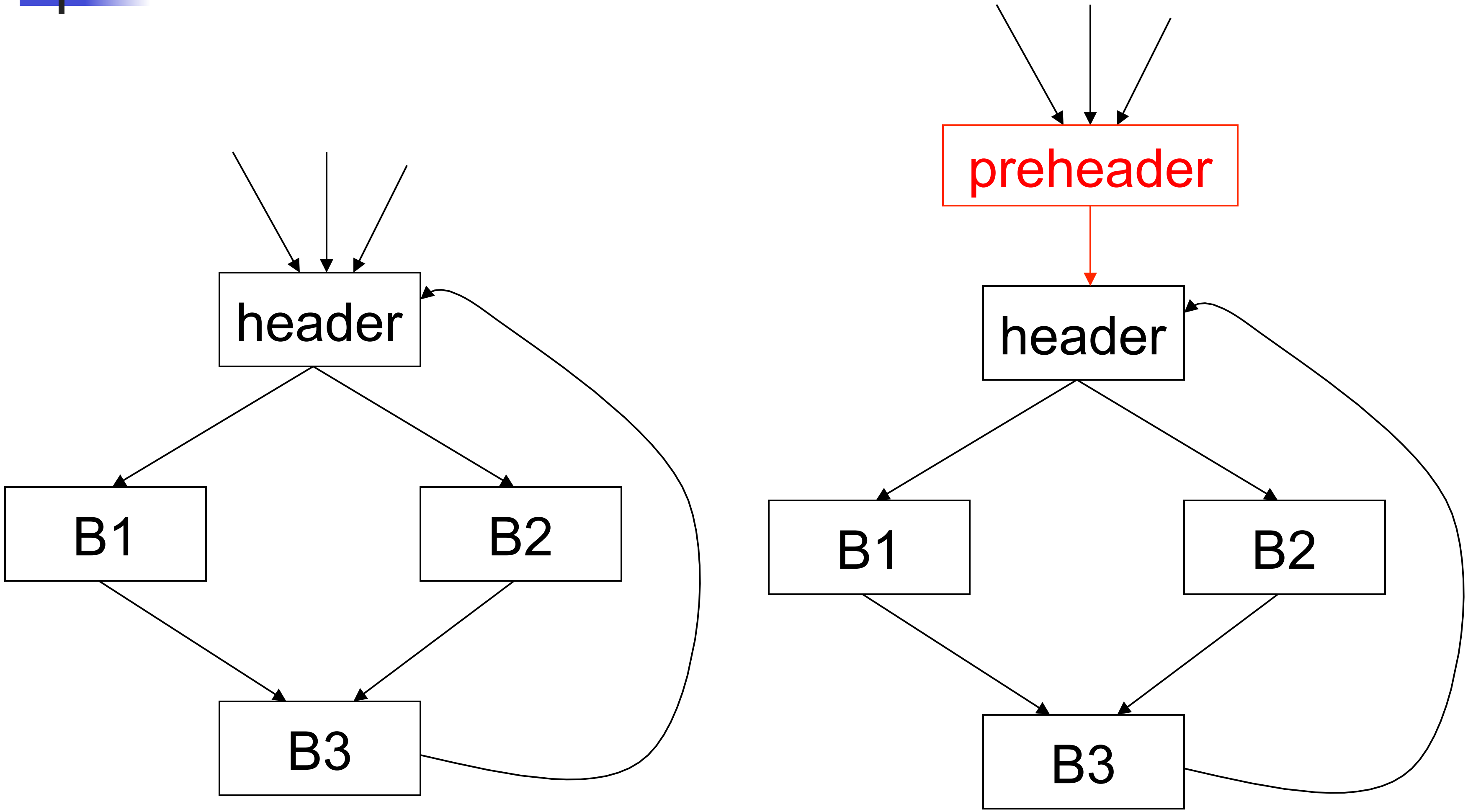
Defn: Let A,B be loops with headers a,b s.t. a≠b.
B is **nested** within A iff B⊂A

- generally start optimizing inside the innermost loop
- Ex: {8,9} nested within {5,8,9,10}
- Reminder: "⊂" means proper subset
  - i.e., A≠B

Loop pre-header

- Many loop optimizations require moving code from inside a loop to just before its header
- To guarantee that we uniformly have such a place available, we may insert a **loop pre-header**
  - initially empty basic block with a single edge into the header
  - potentially reduces code duplication, among other things
  - the pre-header block dominates the loop (that's important)

Inserting a loop pre-header



Loop invariant computations

- Let L be a loop and d be "t$\leftarrow$a$\otimes$b"
  - d is **loop invariant** for L iff
    - "a" and "b" are constant, or
    - definitions reaching "a" and "b" occur outside L, or
    - only one definition reaches "a" and one reaches " b" and they are loop invariant for L
- This is a conservative estimate of loop invariance
  - i.e., it may report d is not loop invariant when it is
    - but it will never say it is loop invariant when it is not

To hoist or not to hoist…

Q: when is it safe to hoist **t←a⊗b**?

```
L0
 t←0

L1
 i←i+1
 t←a⊗b
 M[i]←t
 if i<N
    goto L1

L2
 x←t
```

```
L0
 t←0

L1
 i←i+1
 t←a⊗b
 M[i]←t
 t←0
 M[j]←t
 if i<N
    goto L1

L2
 x←t
```

```
L0
 t←0

L1
 M[j]←t
 i←i+1
 t←a⊗b
 M[i]←t
 t←0
 if i<N
    goto L1

L2
 x←t
```

Ex 1: To hoist or not to hoist…

Before

```
L0
 t←0

L1
 i←i+1
 t←a⊗b
 M[i]←t
 if i<N
     goto L1

L2
 x←t
```

After

```
L0
 t←0
 t←a⊗b

L1
 i←i+1
 M[i]←t
 if i<N
     goto L1

L2
 x←t
```

* We must determine the criteria for hoisting

Determining the criteria for safe hoisting

Before

```
L0
 t←0

L1
 i←i+1
 t←a⊗b
 M[i]←t
 if i<N
    goto L1

L2
 x←t
```

irrelevant definition:
t not live-out

single definition of t

a⊗b is loop invariant

(t←a⊗b) dominates L2

After

```
L0
 t←0
 t←a⊗b

L1
 i←i+1
 M[i]←t
 if i<N
    goto L1

L2
 x←t
```

Ex 2: To hoist or not to hoist…

Before

```
L0
 t←0

L1
 branch (i≥N) L2
 i←i+1
 t←a⊗b
 M[i]←t
 goto L1

L2
 x←t
```

After

```
L0
 t←0
 t←a⊗b

L1
 branch (i≥N) L2
 i←i+1
 M[i]←t
 goto L1

L2
 x←t
```

To hoist or not to hoist…

Before

```
L0
 t←0

L1
 branch (i≥N) L2
 i←i+1
 t←a⊗b
 M[i]←t
 goto L1

L2
 x←t
```

only executes when branch fails

can be a⊗b or 0

After

```
L0
 t←0
 t←a⊗b

L1
 branch (i≥N) L2
 i←i+1
 M[i]←t
 goto L1

L2
 x←t
```

always executes

always a⊗b

Ex 3: To hoist or not to hoist…

Before

```
L0
 t←0

L1
 i←i+1
 t←a⊗b
 M[i]←t
 t←0
 if i<N
      goto L1

L2
```

After

```
L0
 t←0
 t←a⊗b

L1
 i←i+1
 M[i]←t
 t←0
 if i<N
      goto L1

L2
```

To hoist or not to hoist…

Before

```
L0
 t←0

L1
 i←i+1
 t←a⊗b
 M[i]←t
 t←0
 if i<N
     goto L1

L2
```

always
a⊗b

After

```
L0
 t←0
 t←a⊗b

L1
 i←i+1
 M[i]←t
 t←0
 if i<N
     goto L1

L2
```

sometimes
a⊗b

Observe: multiple definitions of `t` inside `L1` complicate matters

Ex 4: To hoist or not to hoist…

Before

```
L0
 t←0

L1
 M[j]←t
 i←i+1
 t←a⊗b
 M[i]←t
 if i<N
     goto L1

L2
 x←t
```

sometimes a⊗b

always a⊗b

After

```
L0
 t←0
 t←a⊗b

L1
 M[j]←t
 i←i+1
 M[i]←t
 if i<N
     goto L1

L2
 x←t
```

Safe hoisting of `d: t←a⊗b`

- `d` dominates all loop exits where `t` is live, and
- only one definition of `t` in the loop, and
- `t` is not live-out in the preheader
- assumes `d` is loop invariant

This is only a conservative approximation!
- will say some hoistings are unsafe when they are safe
- will <u>not</u> say a hoisting is safe when it isn't

To hoist or not to hoist…

Before

```
L0
 t←0

L1
 i←i+1
 t←a⊗b
 M[i]←t
 if i<N
    goto L1

L2
 x←t
```

t not live-out

one definition of t

a⊗b is loop invariant

(t←a⊗b) dominates L2

After

```
L0
 t←0
 t←a⊗b

L1
 i←i+1
 M[i]←t
 if i<N
    goto L1

L2
 x←t
```

*Therefore, hoist!

Induction Variable Analysis

- Some loops have an induction variable
  - a variable "i" that is incremented by a constant or loop invariant amount each iteration
    - for loop-inv. "c", only definitions of the form:
      - "i ← i + c" or "i ← i - c"
- Other variables may depend entirely on "i"
  - these are called **derived induction variables**
- Identifying induction variables within a loop enables a variety of loop optimizations
  - **strength reduction**: replacing an expensive operation by a less expensive one
  - **induction variable elimination**: removing the variable, thereby (perhaps) shortening the code and reducing register pressure

An Example

```
 s←0
 i←0
L1
 branch (i>n) L2
 j←i*4
 k←j+a
 x←M[k]
 s←s+x
 i←i+1
 goto L1
L2
```

Before

`i` is an induction variable

- `i*4` has values 0,4,8,12,…
- can perform **strength reduction**

```
 s←0
 i←0
 j←0
L1
 branch (i>n) L2
 j←j+4
 k←j+a
 x←M[k]
 s←s+x
 i←i+1
 goto L1
L2
```

After

Basic Induction Variables

- Given a loop L with header h
  - "i" is a basic induction variable within L iff
    - the only definitions of "i" have the form:
      - "i ← i + c" or "i ← i - c"
      - for loop-invariant "c"
- Detection of basic induction variables is done by inspecting their form

Derived Induction Variables in the family of "i"

- If "i" is an induction variable (basic or otherwise) for loop L, then
  - <u>"j" is a derived induction variable in the family of "i"</u> means
    - all definitions of "j" in L are of the form
      - j ← c*i + d
        - where c,d are loop invariant
        - may be more than one instruction
        - Lingo: j is **determined** by (i,c,d)

Derived induction variables in the family of "i"

- **Key Insight:** definitions of such a "j" may be rewritten w/o reference to "i"
  - That is, replace definition(s) with the effect of "j ← c*i + d" with "j ← j + c*a"
    - where "i" is incremented by "a"
    - c,d --- loop invariant for L
    - N.b., "c*a" is either
      - constant: in which case, calculate it
      - loop-inv, but not constant
        - in which case compute in the pre-header

Example: $i \leftarrow i+4$, $j \leftarrow 2*i+5$

i	2*i + 5
0	5
4	13
8	21
12	29
…	…

Initialize iteration	$j \leftarrow 5$ $j \leftarrow j+2*4$
0^{th}	5
1^{st}	13
2^{nd}	21
3^{rd}	29
…	…

after each iteration

a=4,c=2

Detecting derived induction variables

Let “j” be an induction variable for L in the family of “i”

- “k” is a derived induction variable for “j” in loop L when:
  - **(Basic)** there is only one definition of “k” in L
    - and that definition is of the form:
      - “k ← c * j”
      - “k ← j + d”
        - where c,d are loop invariant

“k” is a derived induction variable for “j” in loop L when:

- **(Basic)** only one definition of “k” in L
  - and that definition is of the form:
    - “k ← c * j”
    - “k ← j + d”
      - where c,d are loop invariant
- **(More Complex)**
  - the only definition of “j” that reaches “k” is in the loop
    - i.e., “k” depends entirely on a single definition of “j”
  - and, no definition of “i” occurs on any path between the definitions of “j” and “k”
    - i.e., the dependence of “k” on “i” via “j” maintained

Array bounds checking

"Safe" programming languages
insert dynamic checks to array references
to avoid "out of bounds" references

```
array m[1..100] of int;
…
m[i] := 77;
…
```

```
r1 ← fp + offset_i
branch (r1>100) L_error
branch (r1<1) L_error
<code for assignment>
…
```

Array bounds checking

Under certain circumstances, may be able to determine that the array reference is safe and eliminate the check

- relies on induction variable analysis

```
array m[1..100] of int;
…
m[i] := 77;
…
```

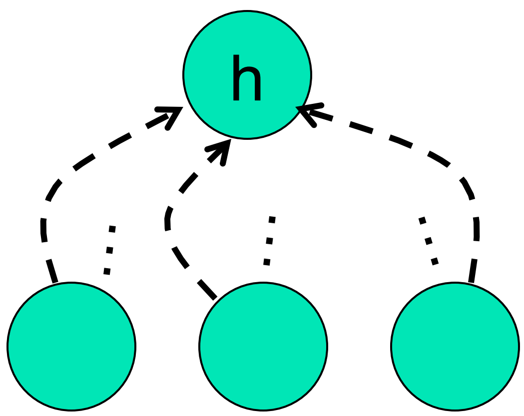
~~r1 ← fp + offset$_i$~~
~~branch (r1>100) L$_{error}$~~
~~branch (r1<1) L$_{error}$~~
<code for assignment>
…

Loop Unrolling Overview

- If the body of a loop L is small, it may be that it spends most of its execution "looping" rather than "computing"; i.e.,
  - incrementing induction variables
  - branching
- Simple example at source level; replace
  - "for (i=0; i++; i<2) { c }" with
  - "i=0; c; i=1; c
    - avoids branching, etc.
- The Problem: how do you do this at the machine code/IR level?
- AKA **Software Pipelining**

Unrolling a loop (brute force)

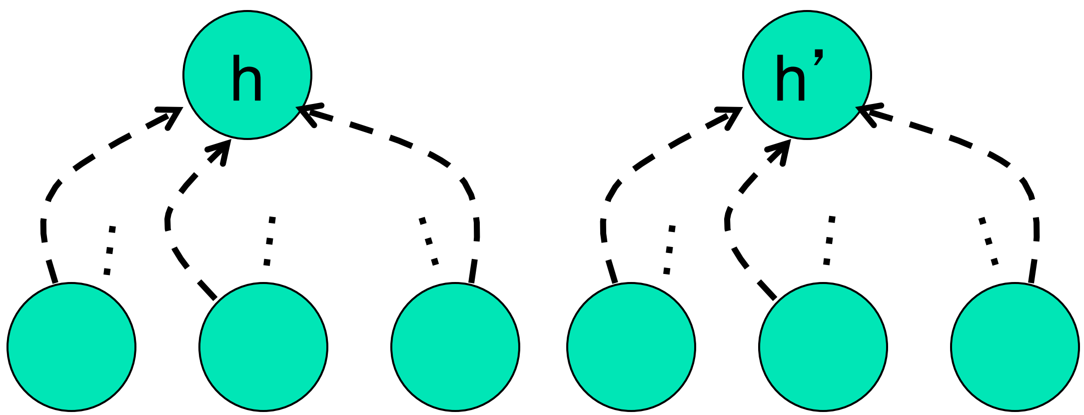
Let L be the loop:



where h is the header and ----→ are backedges

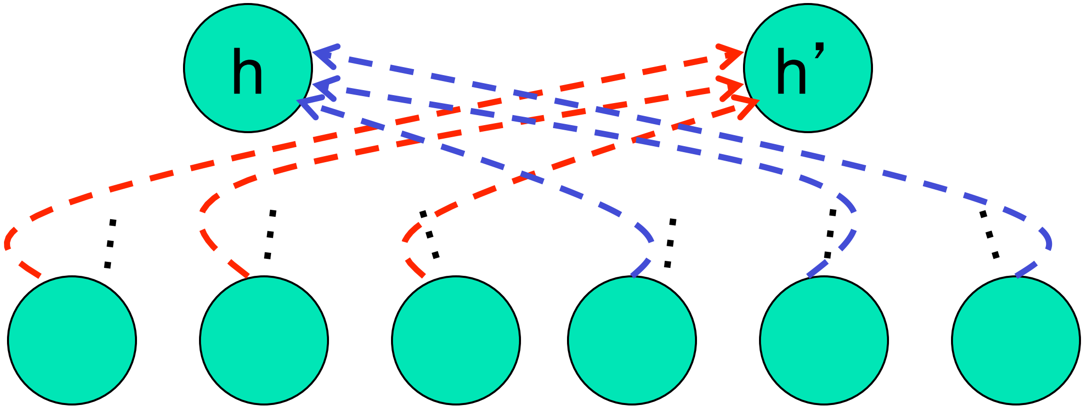
Unrolling a loop (brute force)

Make a copy of L



Unrolling a loop (brute force)

Reroute the backedges



Effects of "brute force" unrolling

Before	
L1:	x ← M[i]
	s ← s+x
	i ← i+4
	branch (i<n) L1
L2:	

After	
L1:	x ← M[i]
	s ← s+x
	i ← i+4
	branch (i≥n) L2
L1':	x ← M[i]
	s ← s+x
	i ← i+4
	branch (i<n) L1
L2:	

Q: Does this constitute an improvement?

Empirical Studies show: loops with the following are good candidates for unrolling

- **Single Basic Blocks**
  - i.e., straight line code
  - with a limited number of floating point & memory operations
    - why? Limits unrolling to loops that are most likely to benefit from instruction scheduling (i.e., ordering code w.r.t. architectural features such as data caches)
- **Small in Length**
  - otherwise unrolling may have negative impact on instruction cache performance
- **Simple Loop Control**
  - simplifies the unrolling transformation

Typical unrolling candidate

- Note that half of the work is in “looping”
- It’s a loop within a single basic block
- few instructions
- has a “repeat-until” structure

```
L1:    x←M[i]
       s←s+x
       i←i+4
       branch (i<n) L1
L2:
```

Fragile* unrolling (K=2)

Before

```
L1:     x←M[i]
        s←s+x
        i←i+4
        branch (i<n) L1
L2:
```

After

```
L1:     x←M[i]
        s←s+x
        i←i+4
        x←M[i]
        s←s+x
        i←i+4
        branch (i<n) L1
L2:
```

**Fragile*: must establish that middle branch may be removed and that loop iterates an even number of times

"Fragile" unrolling (K=2)

Before

```
L1:     x←M[i]
        s←s+x
        i←i+4
        branch (i<n) L1
L2:
```

After

```
L1:     x←M[i]
        s←s+x
        i←i+4
        x←M[i+4]
        s←s+x
        i←i+8
        branch (i<n) L1
L2:
```

…can do better with induction variable analysis

Robust* unrolling (K=2)

Before

```
L1:   x←M[i]
      s←s+x
      i←i+4
      if (i<n) L1 else L2
L2:
```

After

```
      if (i<n-8) L1 else L2
L1:   x←M[i]
      s←s+x
      x←M[i+4]
      s←s+x
      i←i+8
      if (i<n-8) L1 else L2
L2:   x←M[i]
      s←s+x
      i←i+4
      if (i<n) L2 else L3
L3:
```

**Robust*: i.e., works for any number of iterations.

Robust* unrolling (K=2)

Before

```
L1:     x←M[i]
        s←s+x
        i←i+4
        if (i<n) L1 else L2
L2:
```

After

```
        if (i<n-8) L1 else L2
L1:     x←M[i]
        s←s+x
        x←M[i+4]
        s←s+x
        i←i+8
        if (i<n-8) L1 else L2
L2:     x←M[i]
        s←s+x
        i←i+4
        if (i<n) L2 else L3
L3:
```

“twice” block

“once” block

**Robust*: i.e., works for any number of iterations.

That's all, folks!