# Iterative Data Flow Analysis

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CS 4430 Compilers I

# Iterative Data Flow Analysis

- Generally, an analysis of data dependency within a program
  - Like liveness
  - Liveness for programs with loops is solved with IDFA
- First, **attributes** are associated with each node/basic block and given initial values
- Second, relationships between these attributes are specified as **data flow equations**
- Third, a solution to these equations is solved by **iteration**

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# Example: Reaching Definitions

- A **definition** is an assignment of some value to a variable
- A particular definition is said to **reach** a given point within a procedure if
  - There is an execution path from the definition to that point s.t. the variable <u>may</u> have the value given in the definition
- **Reaching analysis** asks, for each variable, which definitions of it may apply
  - Just like use-def

# Reaching Definitions Analysis

```
int g(int m, int i);
int f(n)
{   int i=0, j;
    if (n==1) i=2;
    while (n>0) {
        j = i+1;
        n = g(n,i);
    }
    return j;
}
```

…asks the question: at which points further in a program do particular definitions reach?

# Reaching Definitions Analysis

```
int g(int m, int i);

int f(n)
{   int i=0, j;
    if (n==1) i=2;
    while (n>0) {
        j = i+1;
        n = g(n,i);
    }
    return j;
}
```

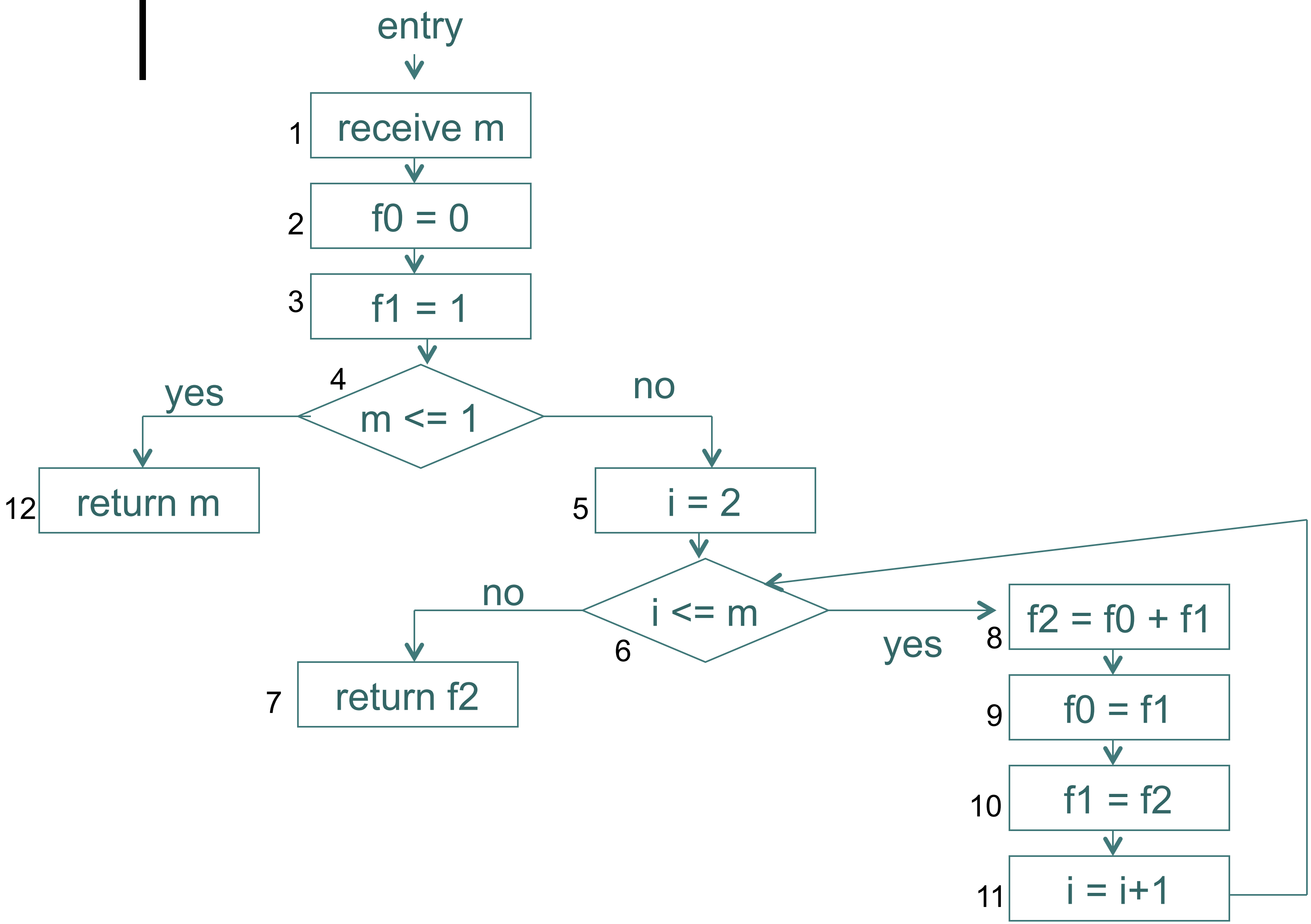
Does this definition of variable `i` apply at this use?

Reaching, like liveness, can generally only be **estimated**

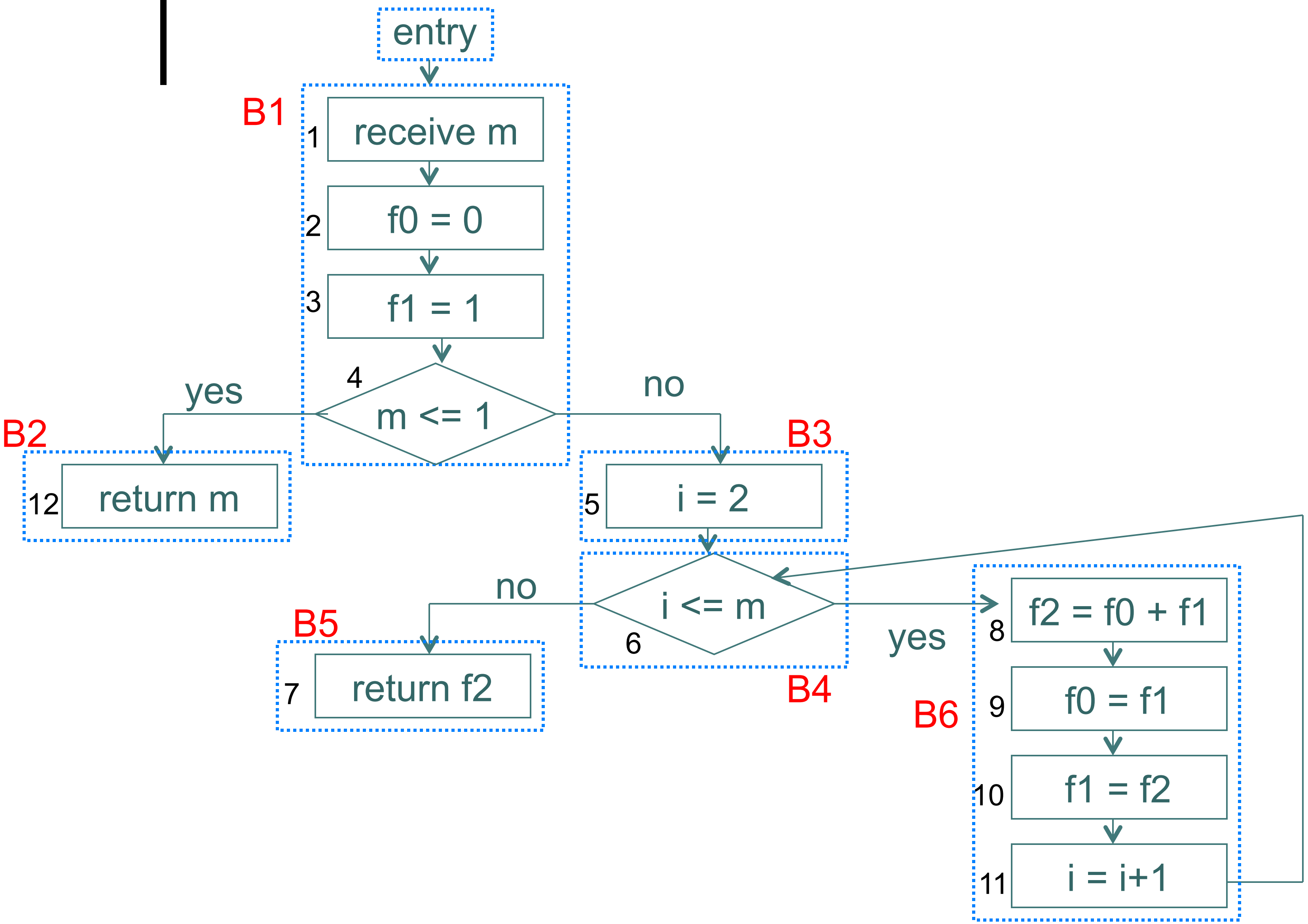
# An Example

```
int fib(int m)
{   int f0=0, f1=1, f2, i;
    if (m<=1) {
        return m;
    } else {
        for (i=2; i<=m; i++) {
            f2 = f0 + f1;
            f0 = f1;
            f1 = f2;
        }
        return f2;
    }
}
```

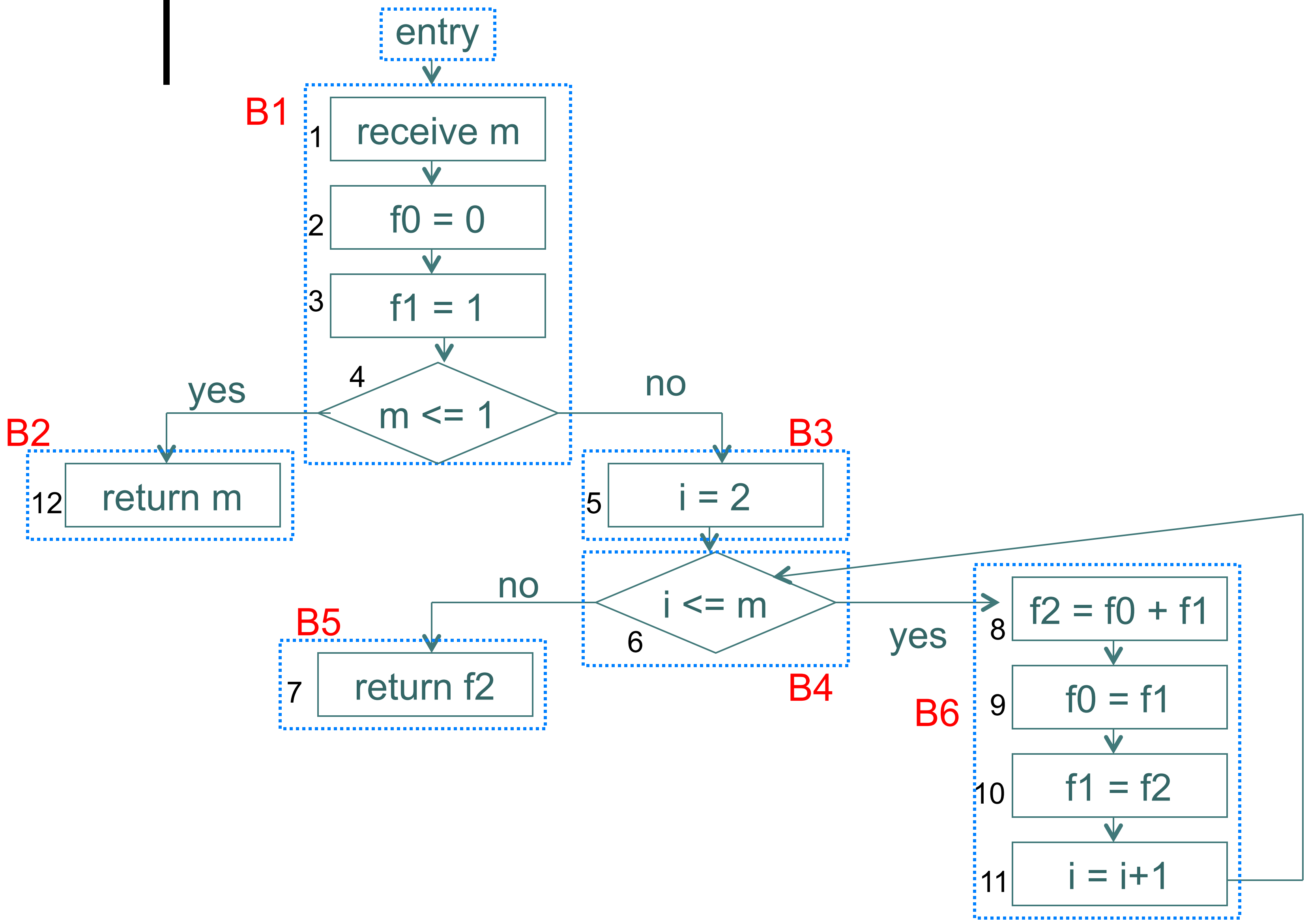
# Control Flow Graph for Fibonacci



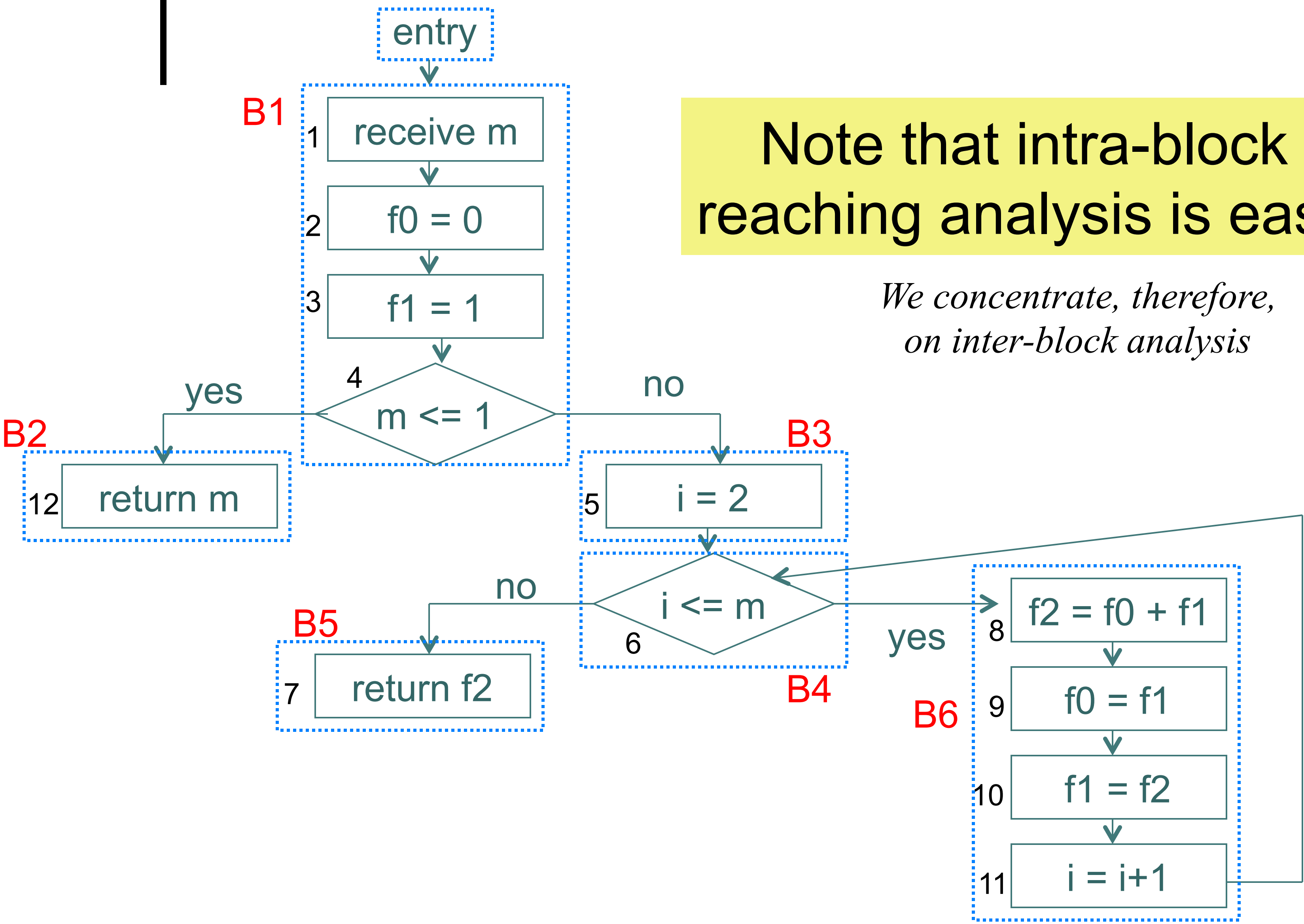
# Control Flow Graph for Fibonacci with Basic Blocks



## Nodes 1, 2, 3, 5, 8, 9, 10, 11 contain “Definitions”



# Intra- vs. Inter-block Reaching Analysis




Note that intra-block reaching analysis is easy

*We concentrate, therefore, on inter-block analysis*

# Attributes

- Reaching Definitions

  RCHin(*block*) = set of definitions that reach *block*

- Preserved Definitions

  PRSV(*block*) = set of definitions preserved by *block*

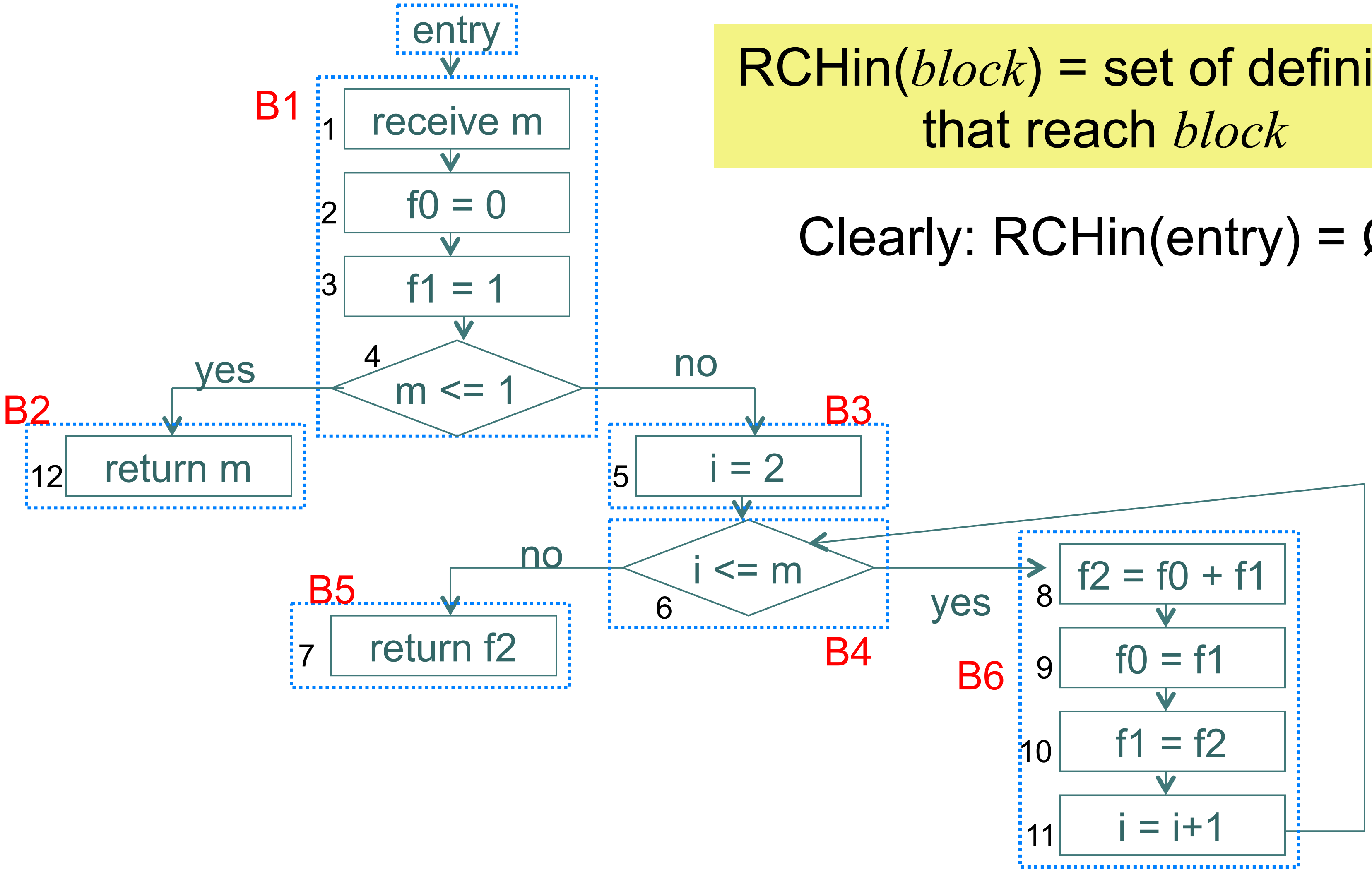
- Generated Definitions

  GEN(*block*) = set of definitions in *block* not
  subsequently killed in *block*

- Out-reaching Definitions

  RCHout(*block*) = set of definitions reaching end of *block*

# Reaching Definitions: RCHin(*node*)



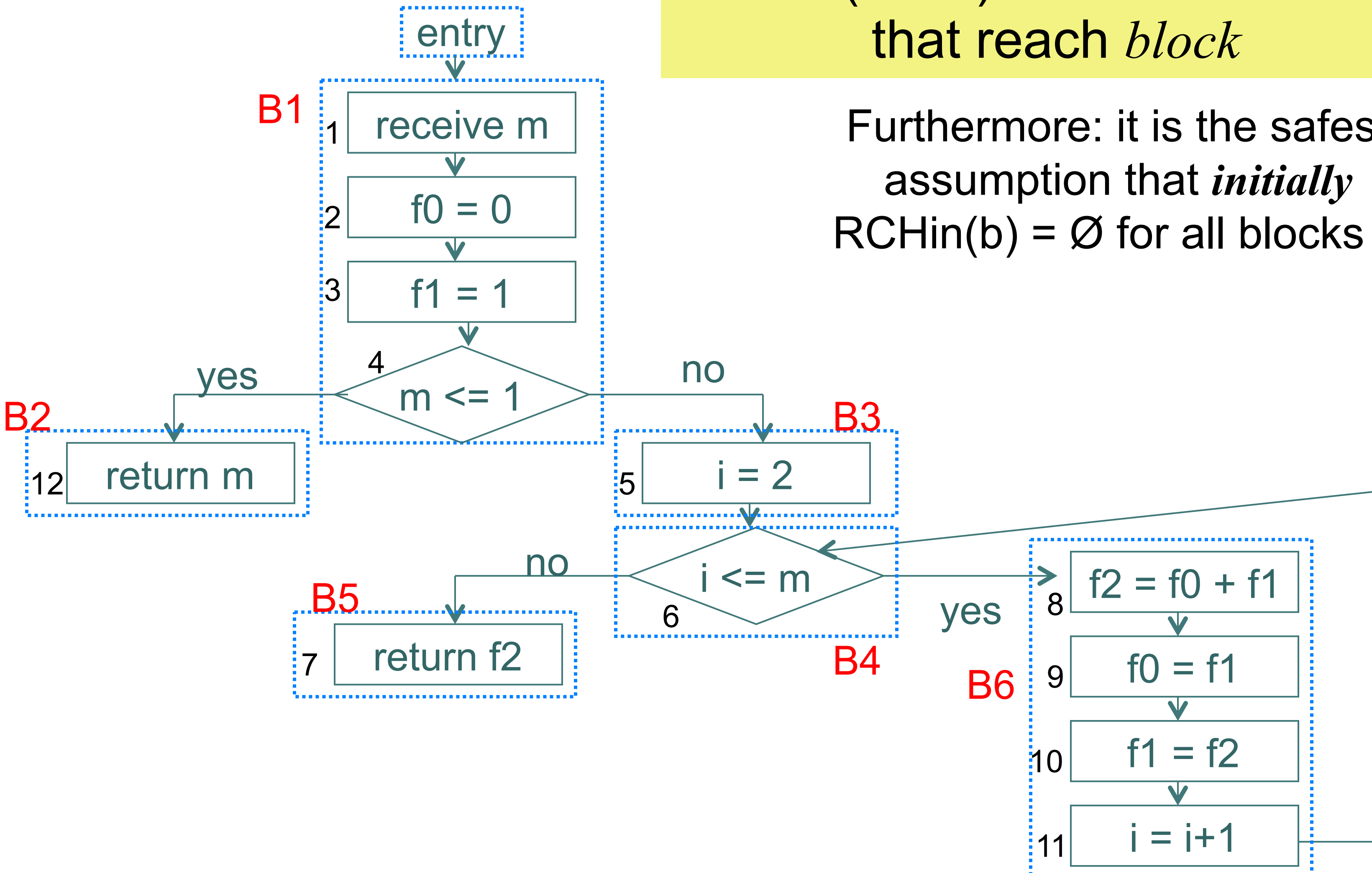

RCHin(*block*) = set of definitions that reach *block*

Clearly: RCHin(entry) = Ø

# Reaching Definitions: RCHin(*node*)

RCHin(*block*) = set of definitions that reach *block*

Furthermore: it is the safest assumption that ***initially*** RCHin(b) = Ø for all blocks b

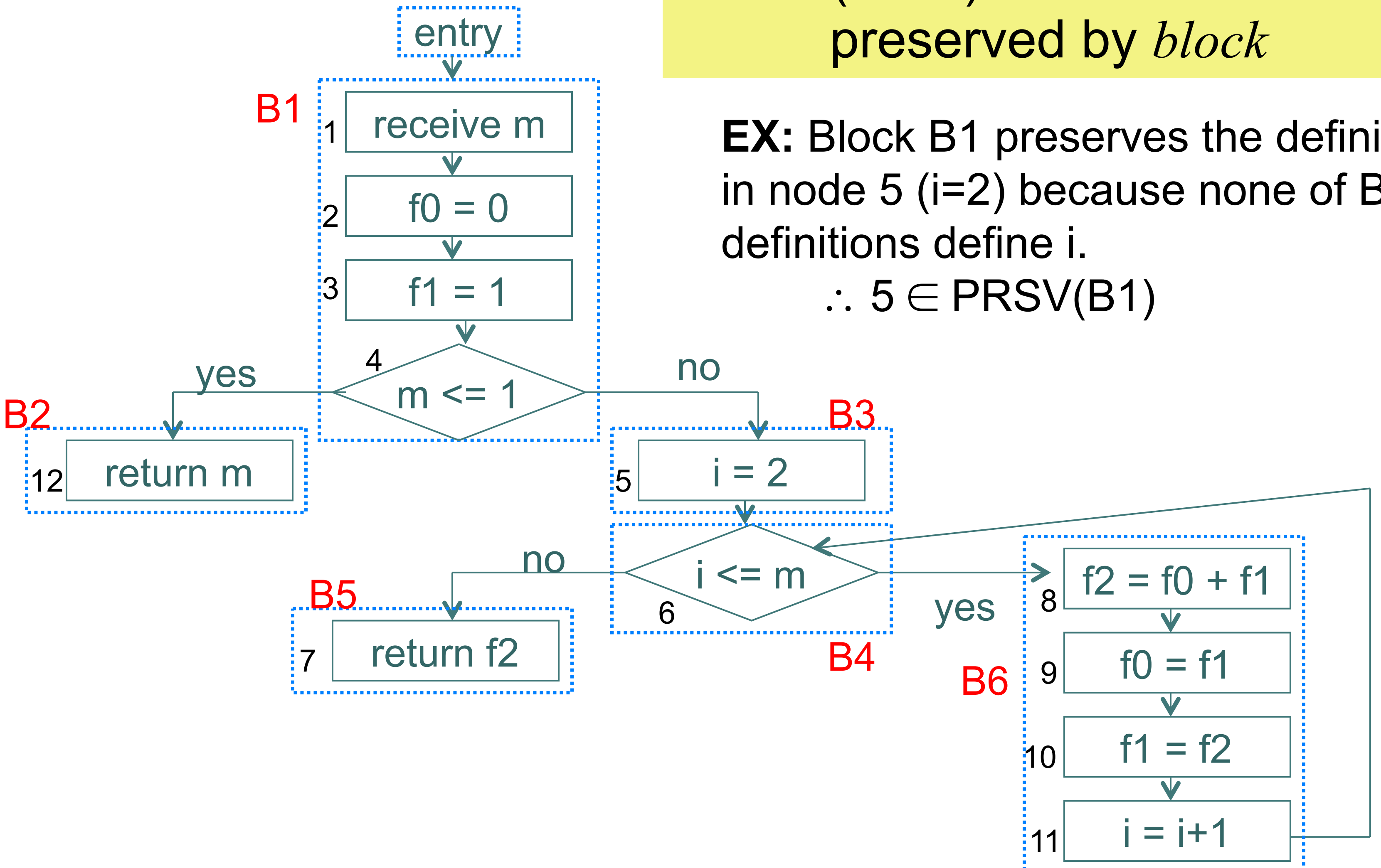


# Preserved Definitions: PRSV(*block*)

PRSV(*block*) = set of definitions preserved by *block*

**EX:** Block B1 preserves the definition in node 5 (i=2) because none of B1's definitions define i.

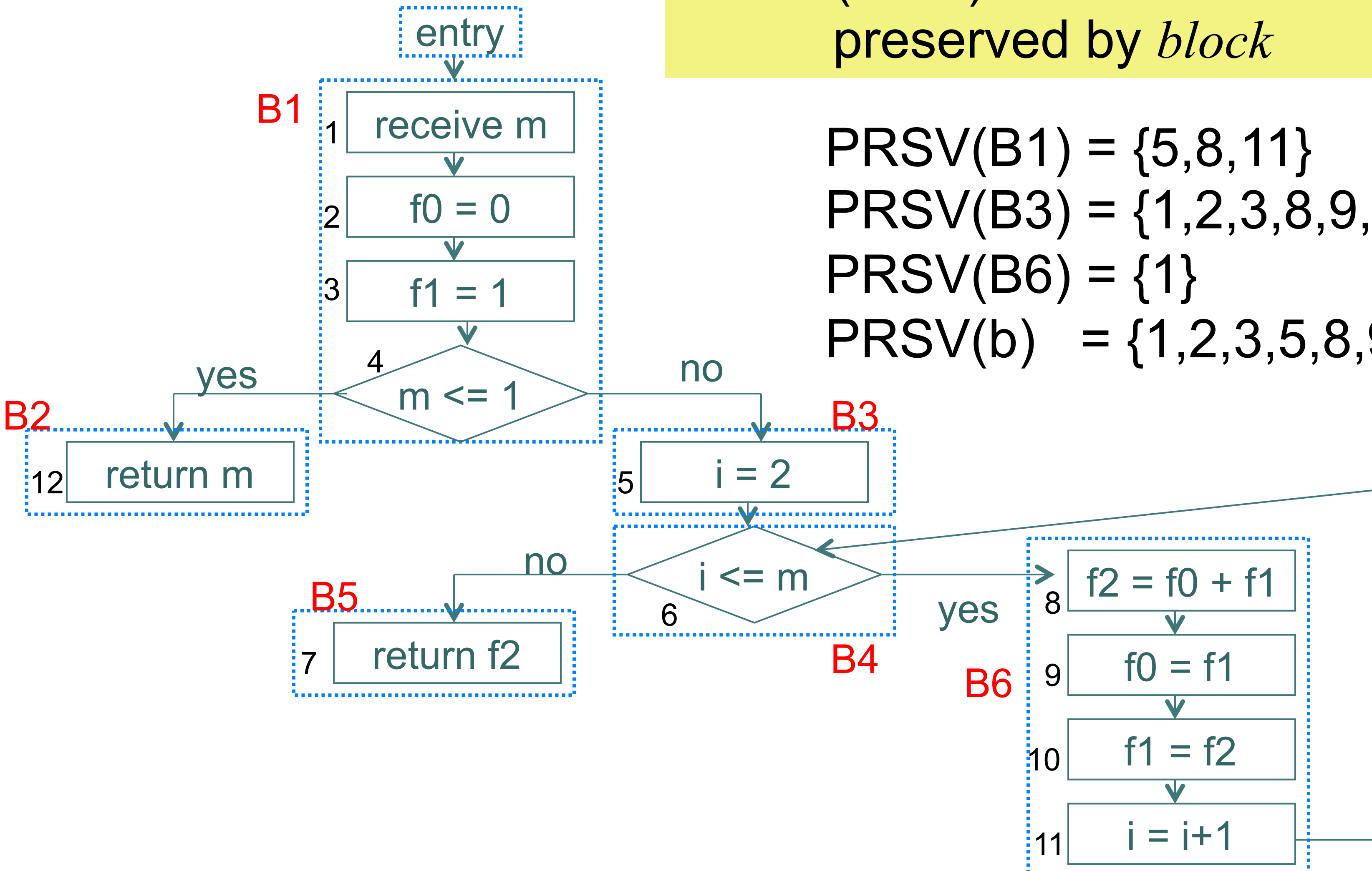
$\therefore\ 5 \in \text{PRSV(B1)}$



# Preserved Definitions: PRSV(*block*)

PRSV(*block*) = set of definitions preserved by *block*

PRSV(B1) = {5,8,11}
PRSV(B3) = {1,2,3,8,9,10}
PRSV(B6) = {1}
PRSV(b) = {1,2,3,5,8,9,10,11}



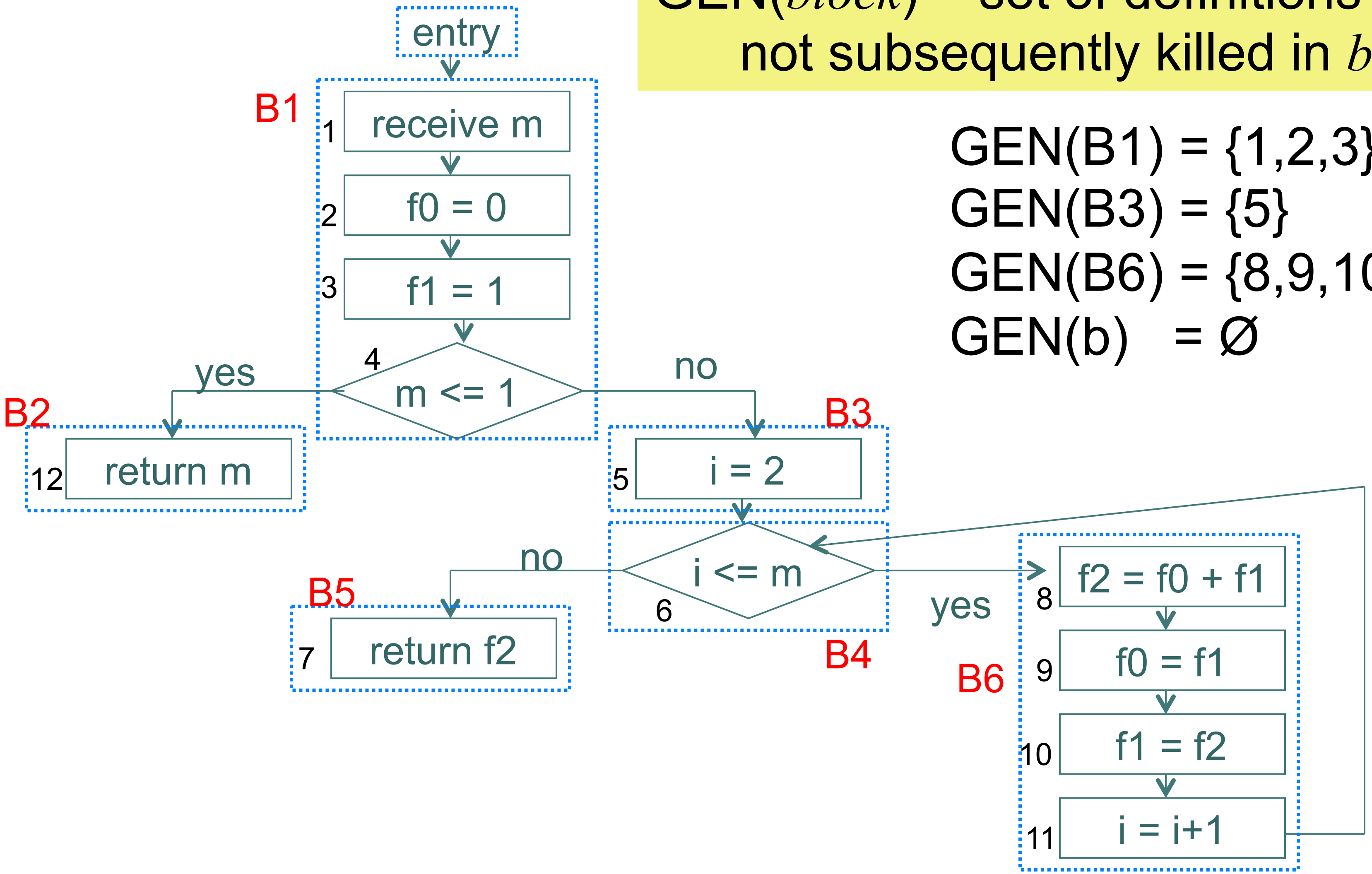
# “Killed” Definitions

A definition is said to “kill” another definition if they write to the same location

EX: “x = y * z” kills “x = 2 + u”

# Generated Definitions: GEN(*block*)

GEN(*block*) = set of definitions in *block* not subsequently killed in *block*

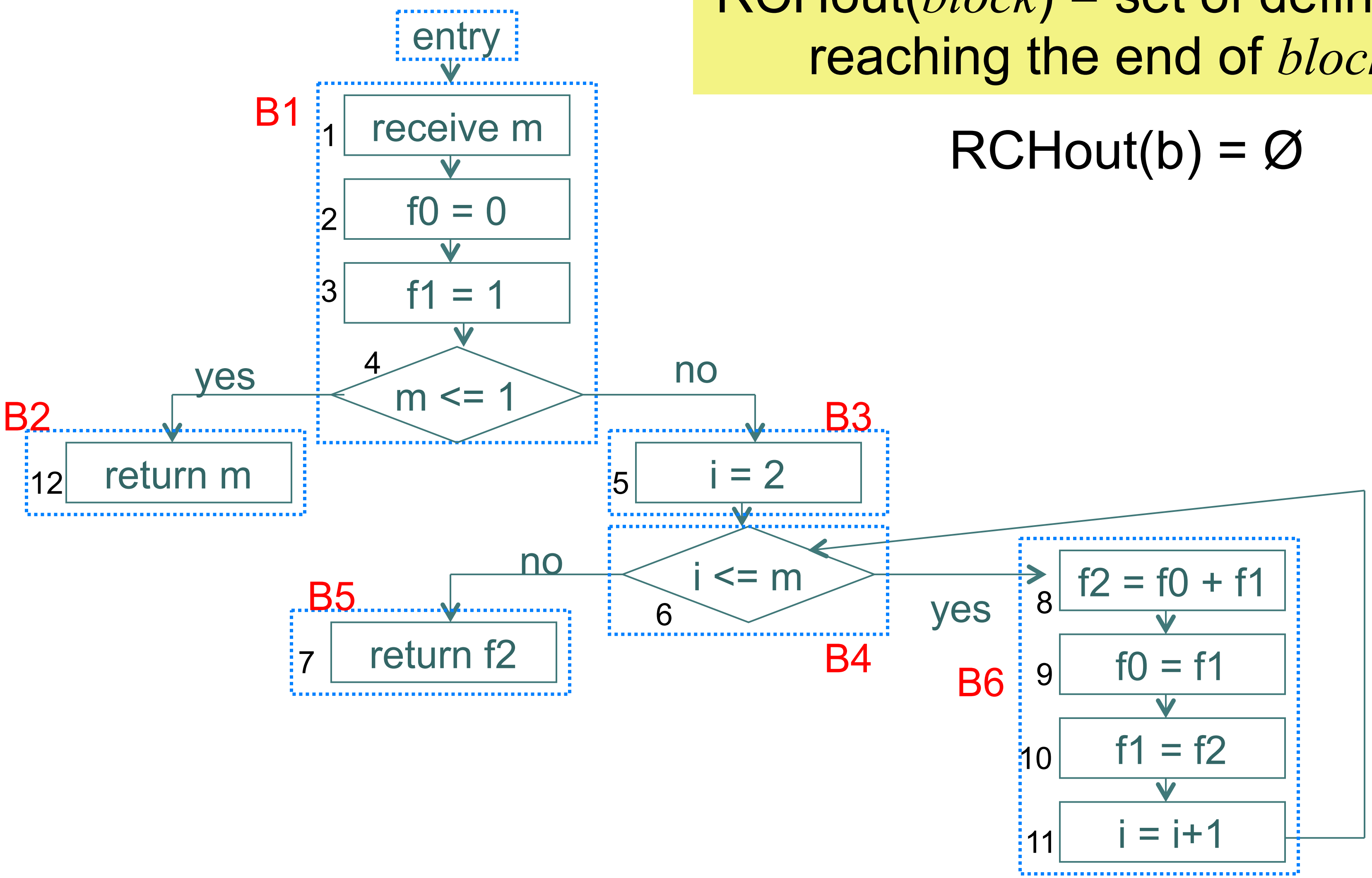



GEN(B1) = {1,2,3}
GEN(B3) = {5}
GEN(B6) = {8,9,10,11}
GEN(b) = Ø

# Out Reaching Definitions: RCHout(*block*)

RCHout(*block*) = set of definitions reaching the end of *block*

RCHout(b) = Ø



# Data Flow Equations

The definitions out of a block are:
- those generated by it and
- those reaching it that are preserved

$$\mathrm{RCHout}(b) = \mathrm{GEN}(b) \cup (\mathrm{RCHin}(b) \cap \mathrm{PRSV}(b)) \; \textit{for all } b$$

The definitions reaching a block are those out-reaching from its predecessors

$$\mathrm{RCHin}(b) = \bigcup_{p \in \mathrm{Pred}(b)} \mathrm{RCHout}(p) \; \textit{for all } b$$

# Solving Data Flow Equations Iteratively

1. Initialize the attributes
2. Treat the data flow equations as assignments
3. If there has been a change to the computed attributes, go to 2 otherwise halt

# Solving Data Flow Equations

Here is a code outline

*Initialization code*

**repeat**

$$\text{RCHout}(b) := \text{GEN}(b) \cup (\text{RCHin}(b) \cap \text{PRSV}(b)) \textit{ for all } b$$

$$\text{RCHin}(b) := \bigcup_{p \in \text{Pred}(b)} \text{RCHout}(p) \textit{ for all } b$$

**until**

*no change to* RCHin/RCHout

# Next time

- Justifying Iterative Solution
  - I.e., why does this give us a solution?
- Liveness as iterative data flow analysis

# Iterative Data Flow Analysis

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# Liveness Analysis

- …determines when the value within a virtual register <u>may</u> still be used
  - a.k.a. its value is “live”
- …and when it <u>definitely</u> won't
  - a.k.a. its value is “dead”
- This property, “liveness”, may be **approximated** statically

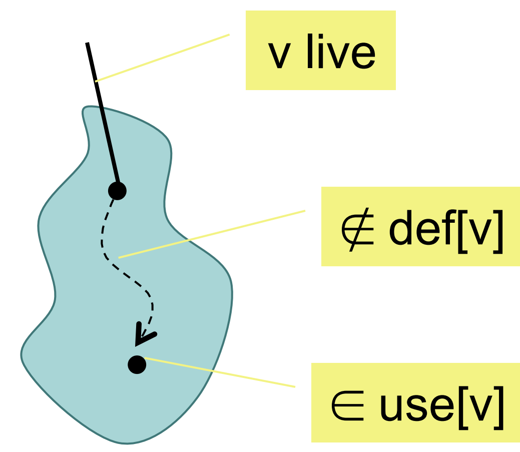
# More Precise Definition of Liveness

**Definition**
– **assignment** of a value to a variable
$a \leftarrow 0$
– def[v] = set of nodes that define variable v
– def[n] = set of variables defined at node n

**Use**
– **reading** the value to a variable
$a > 9$
– use[v/n] = analogous to def[v/n]

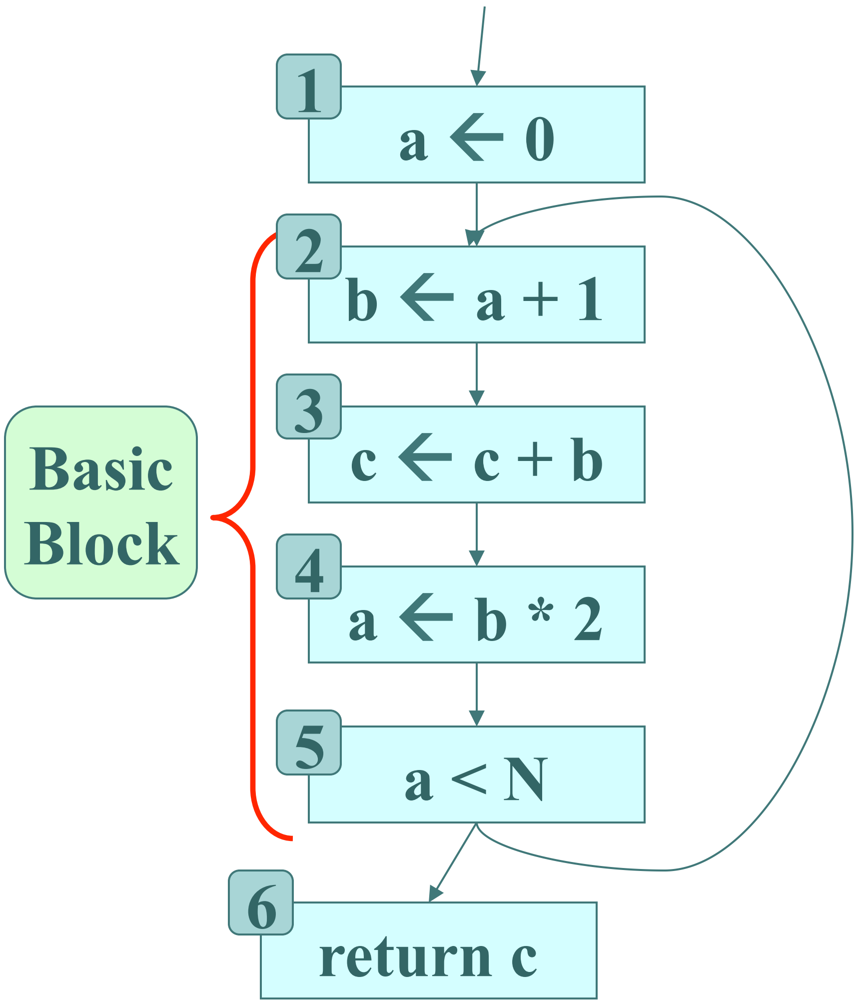
**Liveness**
v is **live on a CFG edge** if

1. ∃ a directed path from that edge to a use of v
2. The path does not go through any defn of v



# Small Control Flow Graph

a ← 0
L: b ← a + 1
c ← c + b
a ← b * 2
if (a < N) goto L
return c



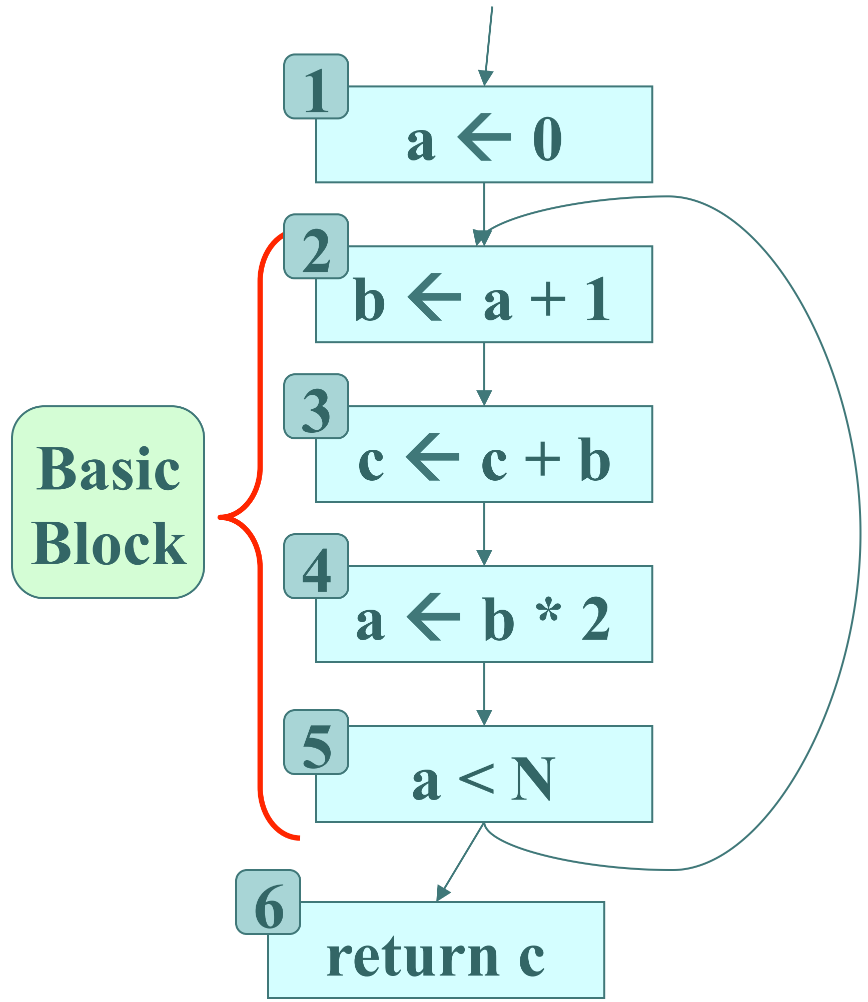
# The Flow of Liveness

**Data-flow**

- Liveness flows through the edges of the CFG

**Direction of Flow**

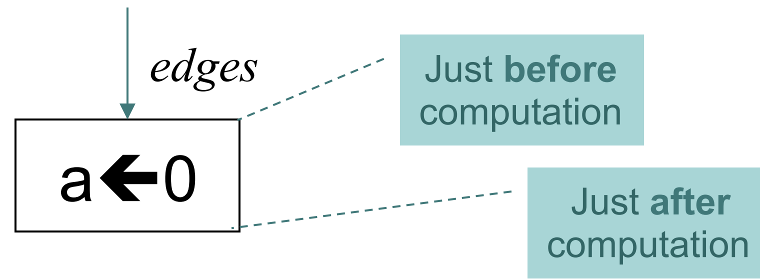
- Liveness flows backwards because
  - Behavior at future nodes determines liveness at given node
- Consider a or b
- “Forward” properties exist (e.g., reaching)



# Liveness at Nodes

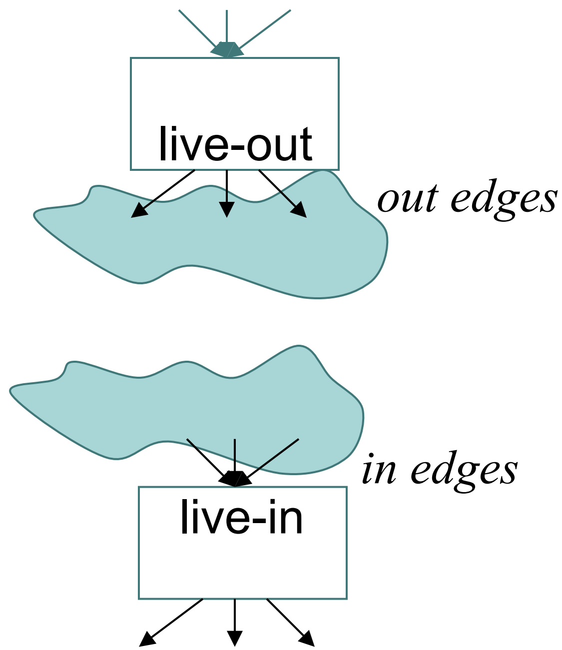
**We have liveness at edges**

- How do we talk about liveness at nodes?



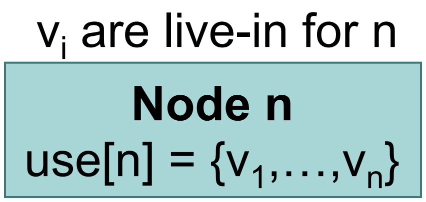

**Two more definitions**

- A variable is **live-out** at a node if it is live on **any** of that node's out-edges
- A variable is **live-in** at a node if it is live on **any** of the node's in-edges

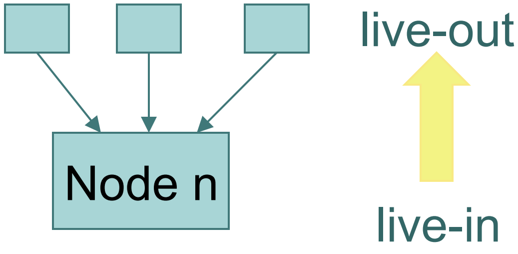


# Computing Liveness

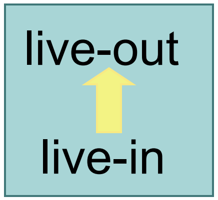
1. **Generate liveness:** if a variable is in use[n], then it is in live-in[n]




2. **Push liveness across edges:** if a variable is in live-in[n], then it is in live-out for all nodes in pred[n]




3. **Push liveness across nodes:** if a variable is in live-out[n] and not in def[n] then the variable is in live-in[n]



# Step 1. The Attributes

1. **Generate liveness:** if a variable is in use[n], then it is in live-in[n]
2. **Push liveness across edges:** if a variable is in live-in[n], then it is in live-out for all nodes in pred[n]
3. **Push liveness across nodes:** if a variable is in live-out[n] and not in def[n] then the variable is in live-in[n]

live-in[n] =
live-out[n] =
use[n] =
def[n] =

Initial values?

# Step 1. The Attributes

1. **Generate liveness:** if a variable is in use[n], then it is in live-in[n]
2. **Push liveness across edges:** if a variable is in live-in[n], then it is in live-out for all nodes in pred[n]
3. **Push liveness across nodes:** if a variable is in live-out[n] and not in def[n] then the variable is in live-in[n]

$$
\begin{aligned}
&\text{live-in[n]} = \emptyset \\
&\text{live-out[n]} = \emptyset \\
&\left.\begin{aligned}&\text{use[n]} = \\ &\text{def[n]} =\end{aligned}\right\} \text{constant, determined by node n}
\end{aligned}
$$

# Step 2. Data Flow Equations

1. **Generate liveness:** if a variable is in use[n], then it is in live-in[n]
2. **Push liveness across edges:** if a variable is in live-in[n], then it is in live-out for all nodes in pred[n]
3. **Push liveness across nodes:** if a variable is in live-out[n] and not in def[n] then the variable is in live-in[n]

live-in[n] = ?

live-out[n] = ?

# Step 2. Data Flow Equations

1. **Generate liveness:** if a variable is in use[n], then it is in live-in[n]
2. **Push liveness across edges:** if a variable is in live-in[n], then it is in live-out for all nodes in pred[n]
3. **Push liveness across nodes:** if a variable is in live-out[n] and not in def[n] then the variable is in live-in[n]

$$\text{live-in}[n] = \text{use}[n] \cup (\text{live-out}[n] - \text{def}[n])$$

$$\text{live-out}[n] = \bigcup_{s \in \text{succ}[n]} \text{live-in}[s]$$

# Step 3. Solving DFE Iteratively

**foreach** node n in CFG

$in[n] = \varnothing$ ; $out[n] = \varnothing$

initialization

**repeat**

**foreach** node n in CFG

$in'[n] = in[n]$

$out'[n] = out[n]$

save current results

$in[n] = use[n] \cup (out[n] - def[n])$

$$out[n] = \bigcup_{s \in succ[n]} in[s]$$

iterative solution

**until** in' [n] == in[n] & out' [n] == out[n] for all nodes n

convergence test

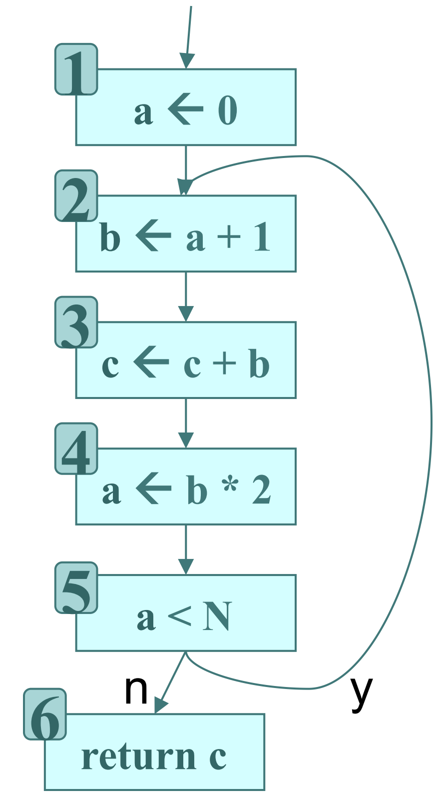
# Example

$$in[n] = use[n] \cup (out[n] - def[n])$$

$$out[n] = \bigcup_{s \in succ[n]} in[s]$$

iteration step

| node | use | def | 1 in | 1 out | 2 in | 2 out | 3 in | 3 out | 4 in | 4 out | 5 in | 5 out | 6 in | 6 out | 7 in | 7 out |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | a | | | | a | | a | | ac | c | ac | c | ac | c | ac |
| 2 | a | b | a | | a | bc | ac | bc | ac | bc | ac | bc | ac | bc | ac | bc |
| 3 | bc | c | bc | | bc | b | bc | b | bc | b | bc | b | bc | bc | bc | bc |
| 4 | b | a | b | | b | a | b | a | b | ac | bc | ac | bc | ac | bc | ac |
| 5 | a | | a | a | a | ac | ac | ac | ac | ac | ac | ac | ac | ac | ac | ac |
| 6 | c | | c | | c | | c | | c | | c | | c | | c | |




in = red
out = blue

## Example (cont’d)

$$in[n] = use[n] \cup (out[n] - def[n])$$

$$out[n] = \bigcup_{s \in succ[n]} in[s]$$

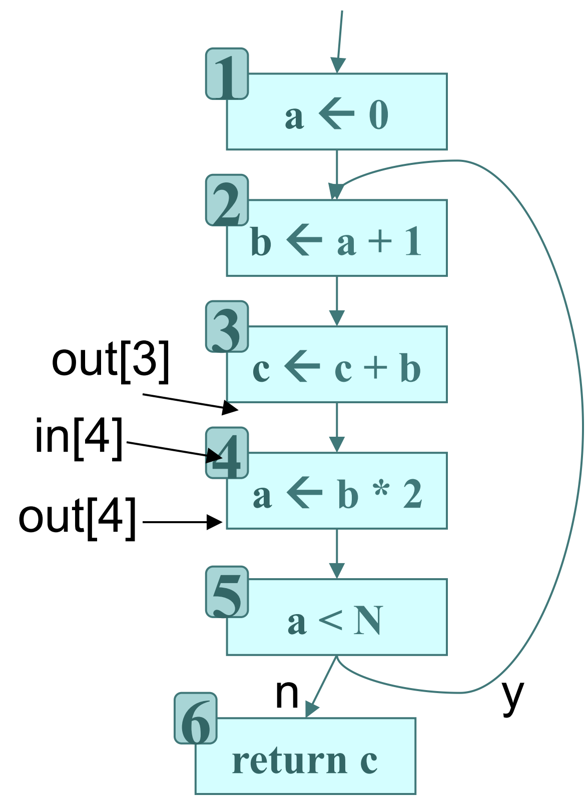
# Improving Performance

consider the (3➔4) edge in the graph:

out[4] is used to compute in[4],

in[4] is used to compute out[3],…

**So,** we should compute in the order: out[4], in[4], out[3], in[3],…




Order of computation should follow flow direction

# Step 3. Solving DFE Iteratively revisited

**foreach** node n in CFG

$in[n] = \emptyset$ ; $out[n] = \emptyset$ — initialization

**repeat**

**foreach** node n in CFG in reverse topological sort order

$in'[n] = in[n]$
$out'[n] = out[n]$ — save current results

$out[n] = \bigcup_{s \in succ[n]} in[s]$
$in[n] = use[n] \cup (out[n] - def[n])$ — Note the change in order

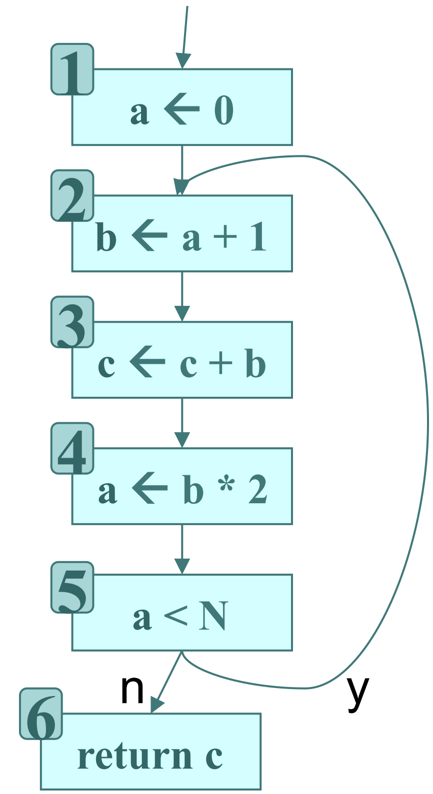
**until** $in'[n] == in[n]$ & $out'[n] == out[n]$ for all nodes n — convergence test

# Example

$$out[n] = \bigcup_{s \in succ[n]} in[s]$$

$$in[n] \;\; = use[n] \cup (out[n] - def[n])$$

| node | use | def | 1 | | 2 | | 3 | | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 6 | c | | | c | | c | | c | | | | |
| 5 | a | | c | ac | ac | ac | ac | ac | | | | |
| 4 | b | a | ac | bc | ac | bc | ac | bc | | | | |
| 3 | bc | c | bc | bc | bc | bc | bc | bc | | | | |
| 2 | a | b | bc | ac | bc | ac | bc | ac | | | | |
| 1 | | a | ac | c | ac | c | ac | c | | | | |




out = red
in = blue

Note the change in order!

# Time Complexity (very rough)

- Consider a program of size N
  - N = max(nodes in CFG, number of vars)
    - $\therefore$ each live-in, live-out set has at most N elements
  - Each set union takes O(N) time

# Step 3. Solving DFE Iteratively revisited

**foreach** node n in CFG
$$\left.\begin{array}{l} in[n] = \emptyset \ ; \ out[n] = \emptyset \end{array}\right\} O(N)$$

**repeat**

**foreach** node n in CFG in reverse topological sort order

$$\left.\begin{array}{l} in'[n] = in[n] \\ out'[n] = out[n] \\ out[n] = \bigcup_{s \in succ[n]} in[s] \\ in[n] \ = use[n] \cup (out[n] - def[n]) \end{array}\right\} O(N^2)$$

**until** $\left.in'[n] == in[n] \ \& \ out'[n] == out[n] \text{ for all nodes } n\right\} O(N)$

Whole repeat loop: $O(N^2)$

$\therefore$ Worst case is $O(N^2 \times N^2) = O(N^4)$

# More Performance Considerations

- Use basic blocks instead of nodes
  - Merge nodes into basic blocks to decrease size of CFG
- Representation of sets
  - For dense sets, use a bit vector representation
    - This can reduce the cost of set operations to O(1)
  - For sparse sets, use sorted (linked) lists
- Typical Case: 2 to 3 iterations with good ordering and sparse sets
  - In the O(N) to O($N^2$) range for average

# Termination Guarantees: possible values of live-in (for example)

{a,b,c,d}

{a,b,c} {a,b,d} {a,c,d} {b,c,d}

{a,b} {a,c} {a,d} {b,c} {b,d} {c,d}

{a} {b} {c} {d}

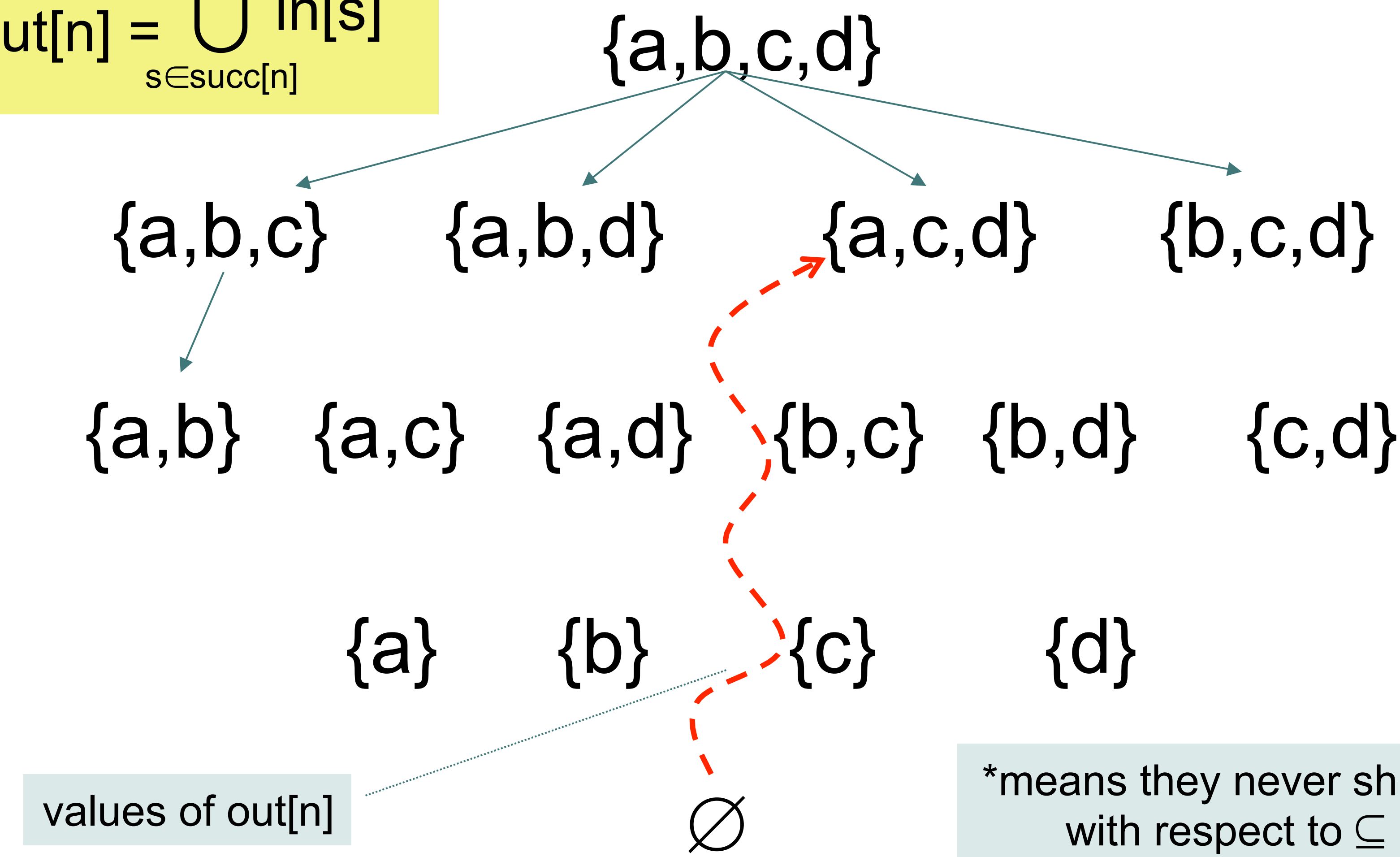
$\emptyset$

*Not all links shown

→ means is a “subset of”

# Attribute values are **monotonically increasing***

$$out[n] = \bigcup_{s \in succ[n]} in[s]$$




*means they never shrink with respect to ⊆