Data Flow Analysis 2

Iterative Data Flow Analysis

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CS 4430 Compilers I

Iterative Data Flow Analysis

- Generally, an analysis of data dependency within a program
  - Like liveness
  - Liveness for programs with loops is solved with IDFA
- First, **attributes** are associated with each node/basic block and given initial values
- Second, relationships between these attributes are specified as **data flow equations**
- Third, a solution to these equations is solved by **iteration**

Example: Reaching Definitions

- A **definition** is an assignment of some value to a variable
- A particular definition is said to **reach** a given point within a procedure if
  - There is an execution path from the definition to that point s.t. the variable may have the value given in the definition
- **Reaching analysis** asks, for each variable, which definitions of it may apply
  - Just like use-def

Reaching Definitions Analysis

```
int g(int m, int i);
int f(n)
{   int i=0, j;
    if (n==1) i=2;
    while (n>0) {
        j = i+1;
        n = g(n,i);
    }
    return j;
}
```

…asks the question: at which points further in a program do particular definitions reach?

Reaching Definitions Analysis

```
int g(int m, int i);

int f(n)
{   int i=0, j;
    if (n==1) i=2;
    while (n>0) {
        j = i+1;
        n = g(n,i);
    }
    return j;
}
```

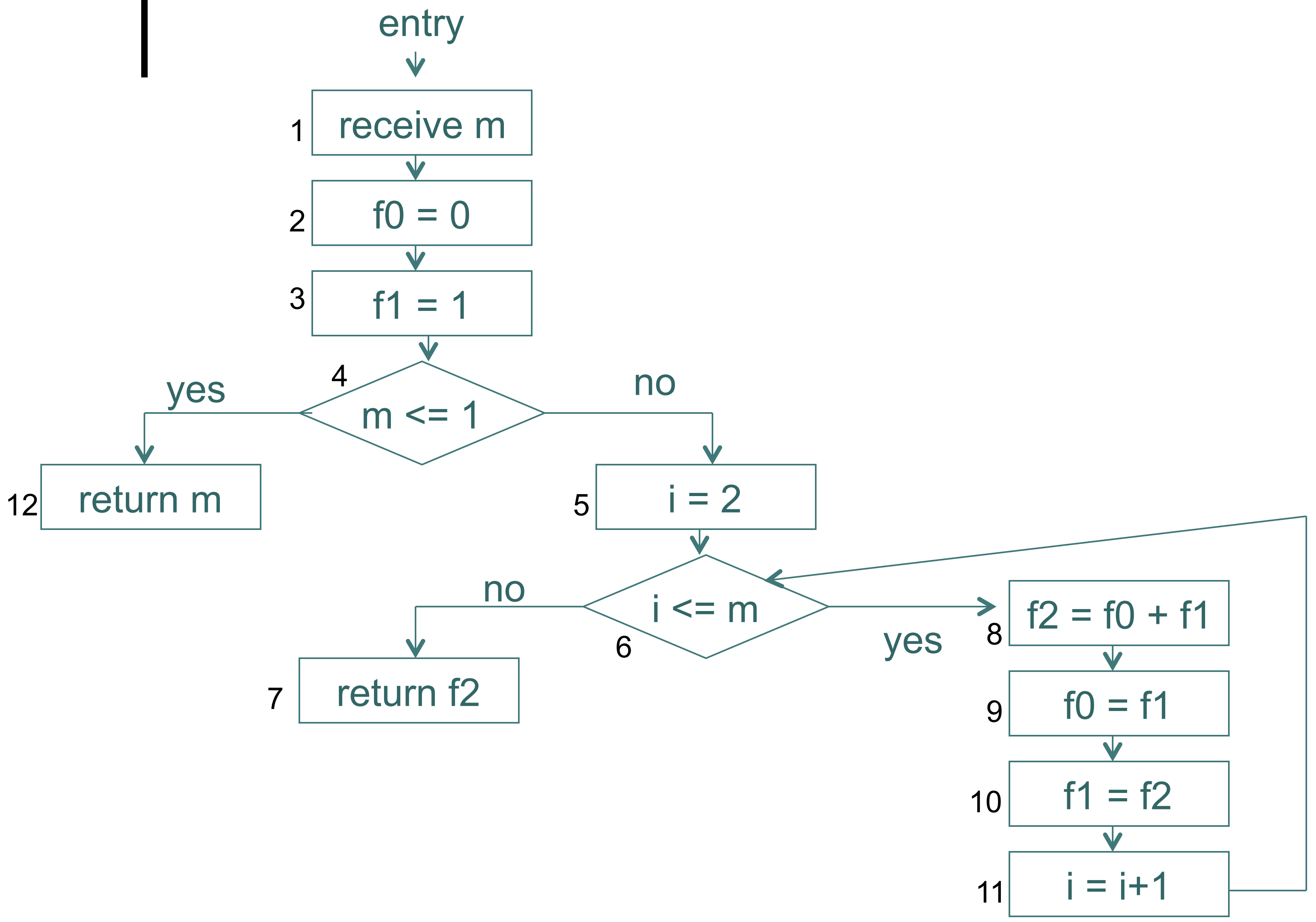
Does this definition of variable `i` apply at this use?

Reaching, like liveness, can generally only be **estimated**

An Example

```
int fib(int m)
{   int f0=0, f1=1, f2, i;
    if (m<=1) {
        return m;
    } else {
        for (i=2; i<=m; i++) {
            f2 = f0 + f1;
            f0 = f1;
            f1 = f2;
        }
        return f2;
    }
}
```

Control Flow Graph for Fibonacci



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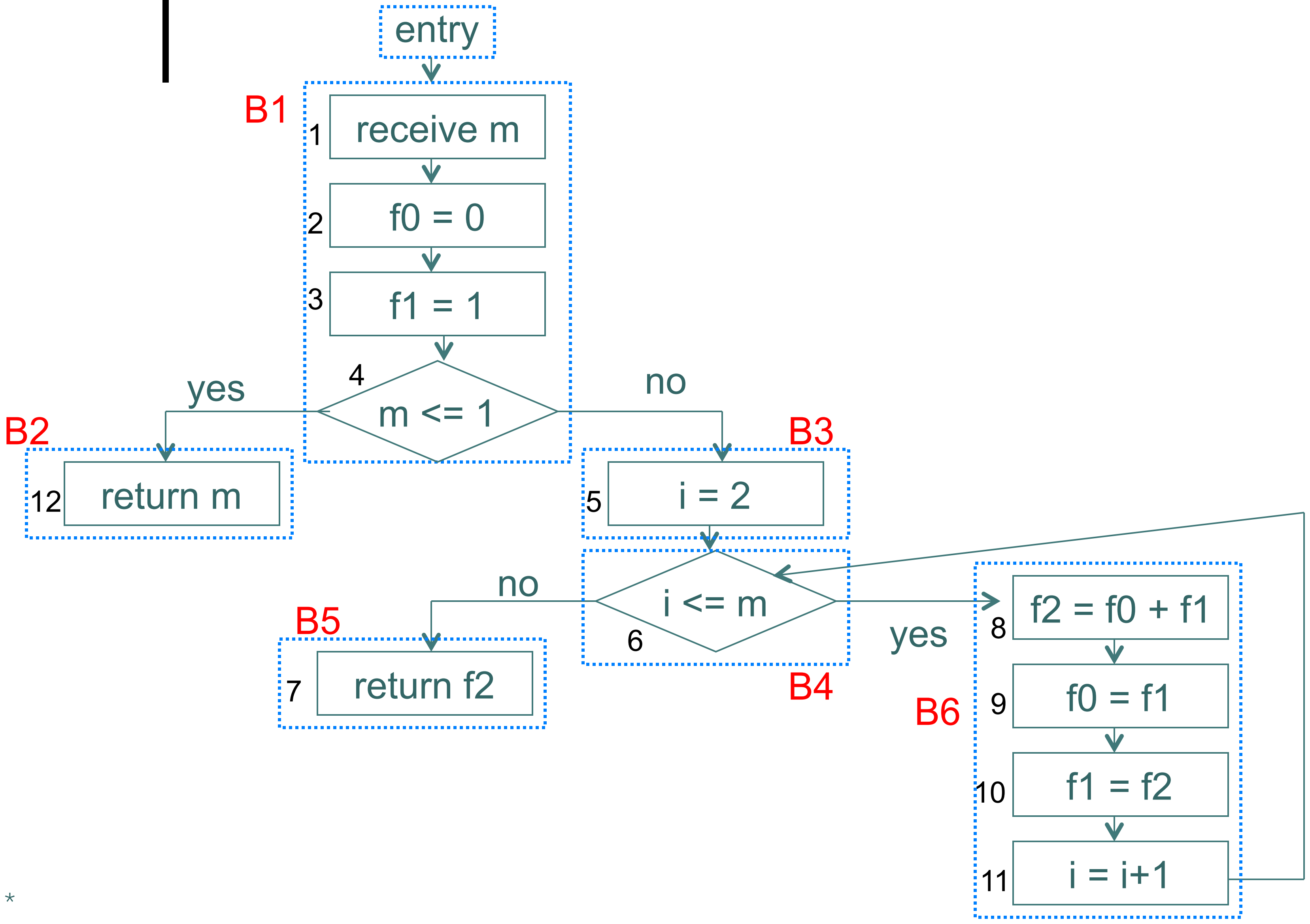
Control Flow Graph for Fibonacci with Basic Blocks



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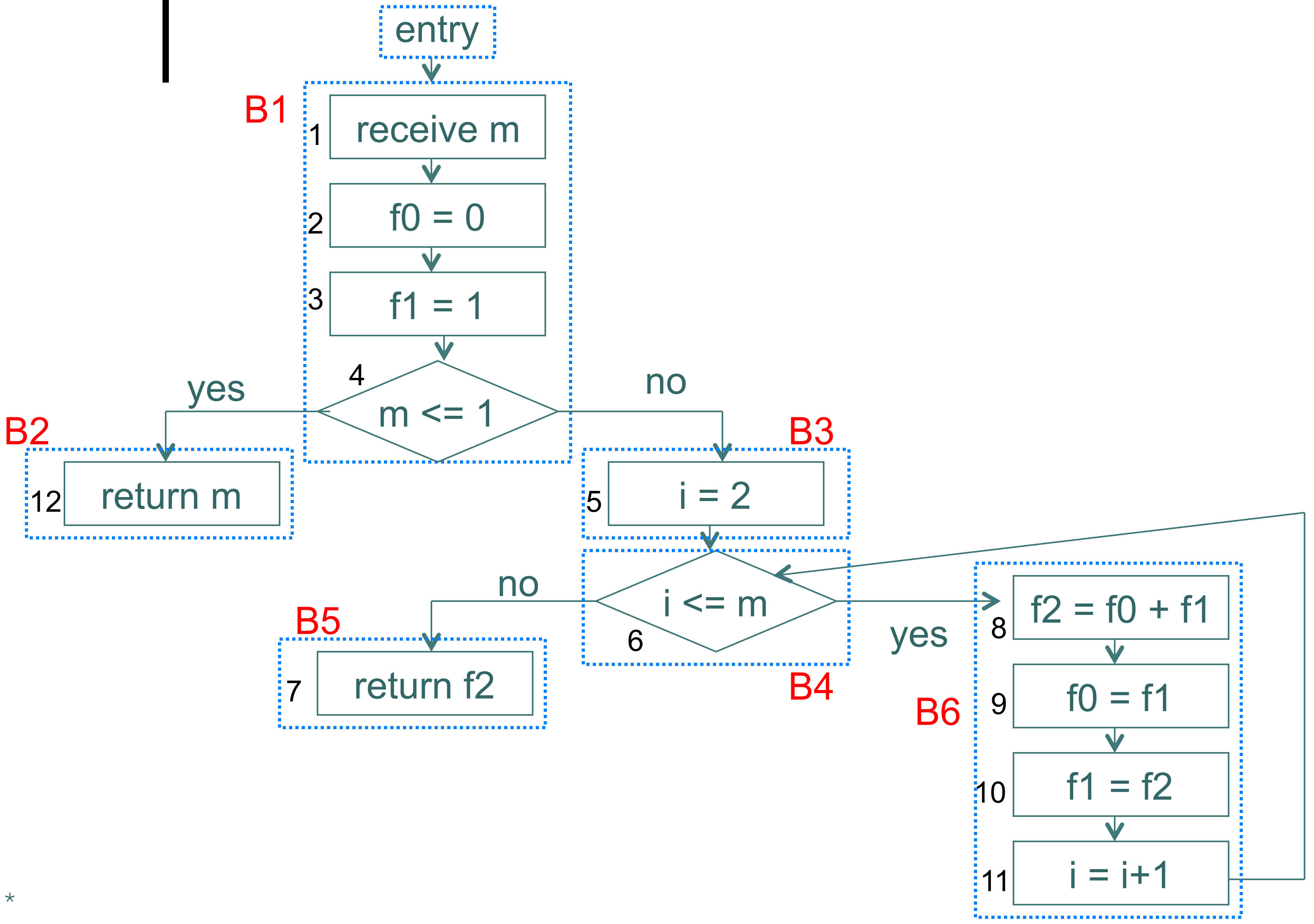
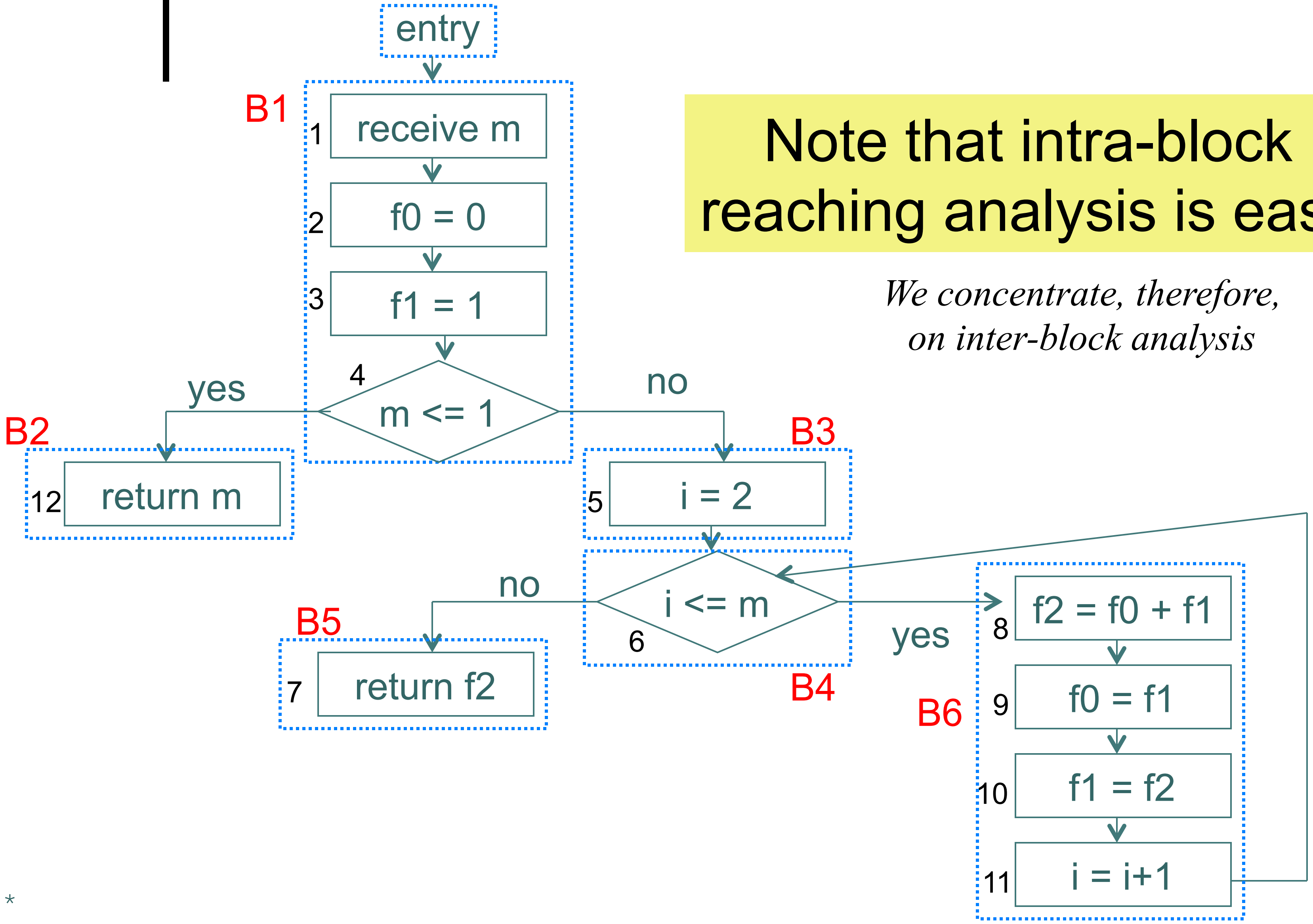
Nodes 1, 2, 3, 5, 8, 9, 10, 11 contain “Definitions”



Intra- vs. Inter-block Reaching Analysis



Note that intra-block reaching analysis is easy

We concentrate, therefore, on inter-block analysis

Attributes

- Reaching Definitions

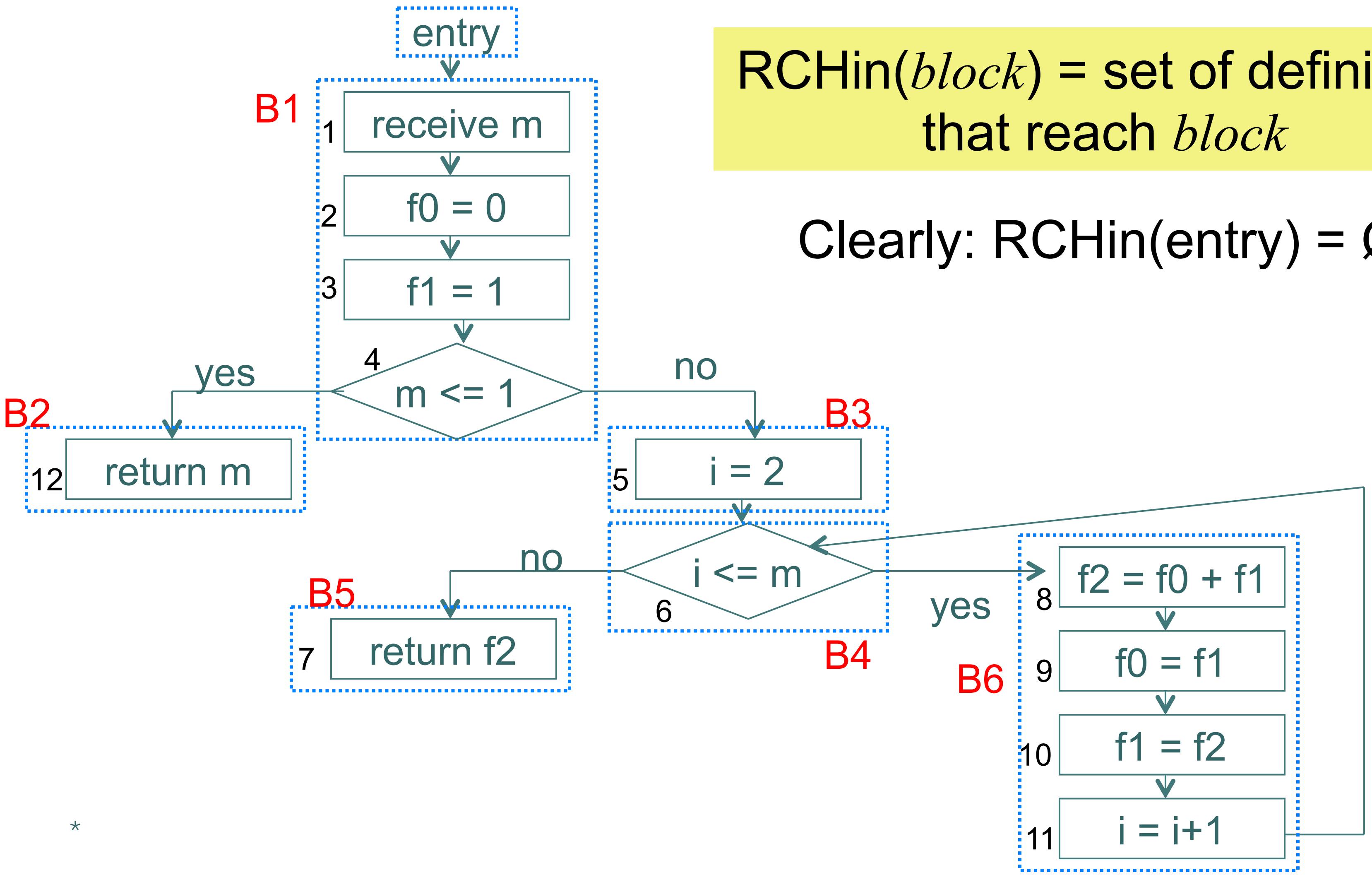
  RCHin(*block*) = set of definitions that reach *block*
- Preserved Definitions

  PRSV(*block*) = set of definitions preserved by *block*
- Generated Definitions

  GEN(*block*) = set of definitions in *block* not
  subsequently killed in *block*
- Out-reaching Definitions

  RCHout(*block*) = set of definitions reaching end of *block*

Reaching Definitions: RCHin(*node*)



RCHin(*block*) = set of definitions that reach *block*

Clearly: RCHin(entry) = Ø

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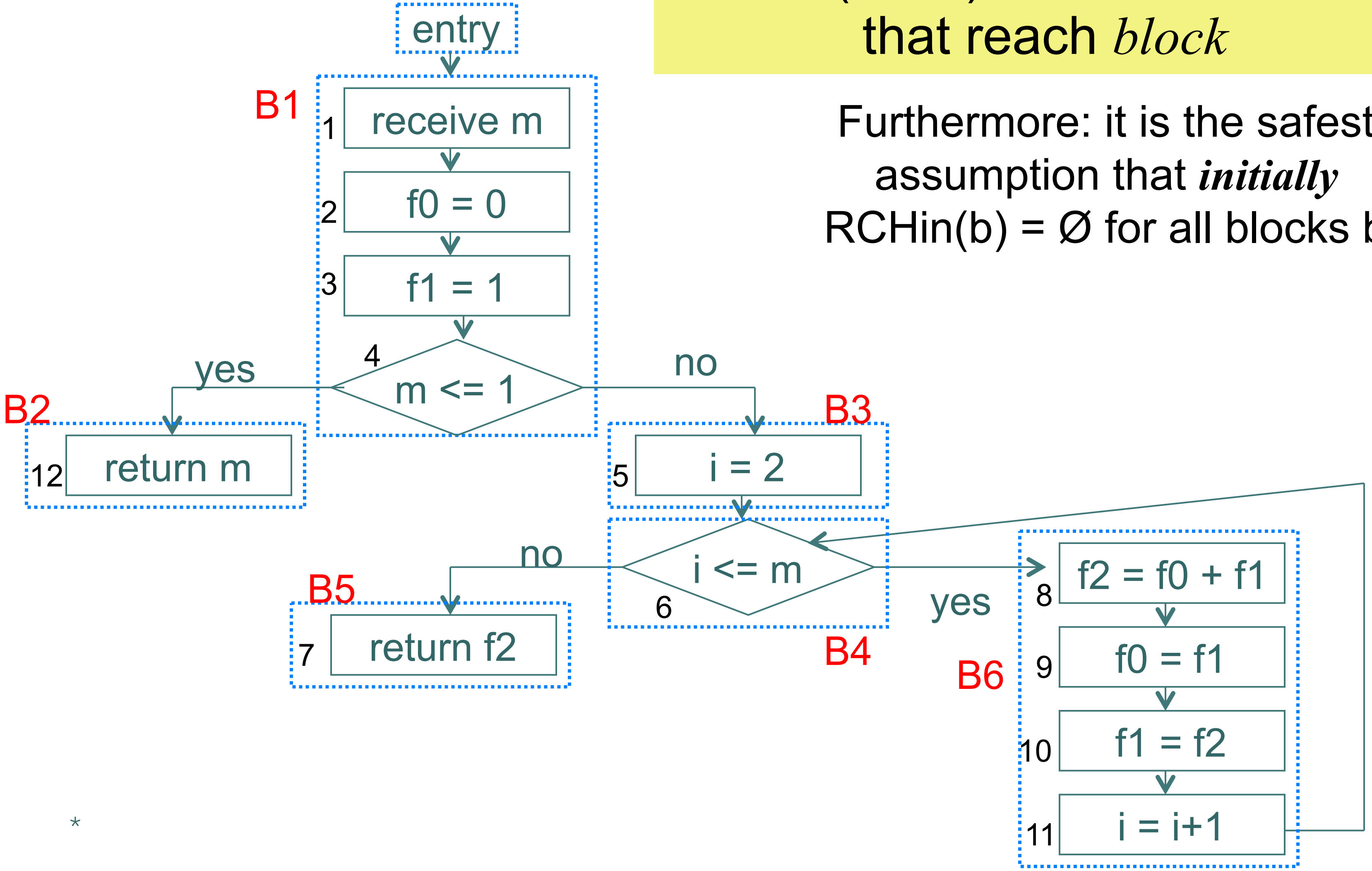
Reaching Definitions: RCHin(*node*)

RCHin(*block*) = set of definitions that reach *block*

Furthermore: it is the safest assumption that ***initially*** RCHin(b) = Ø for all blocks b



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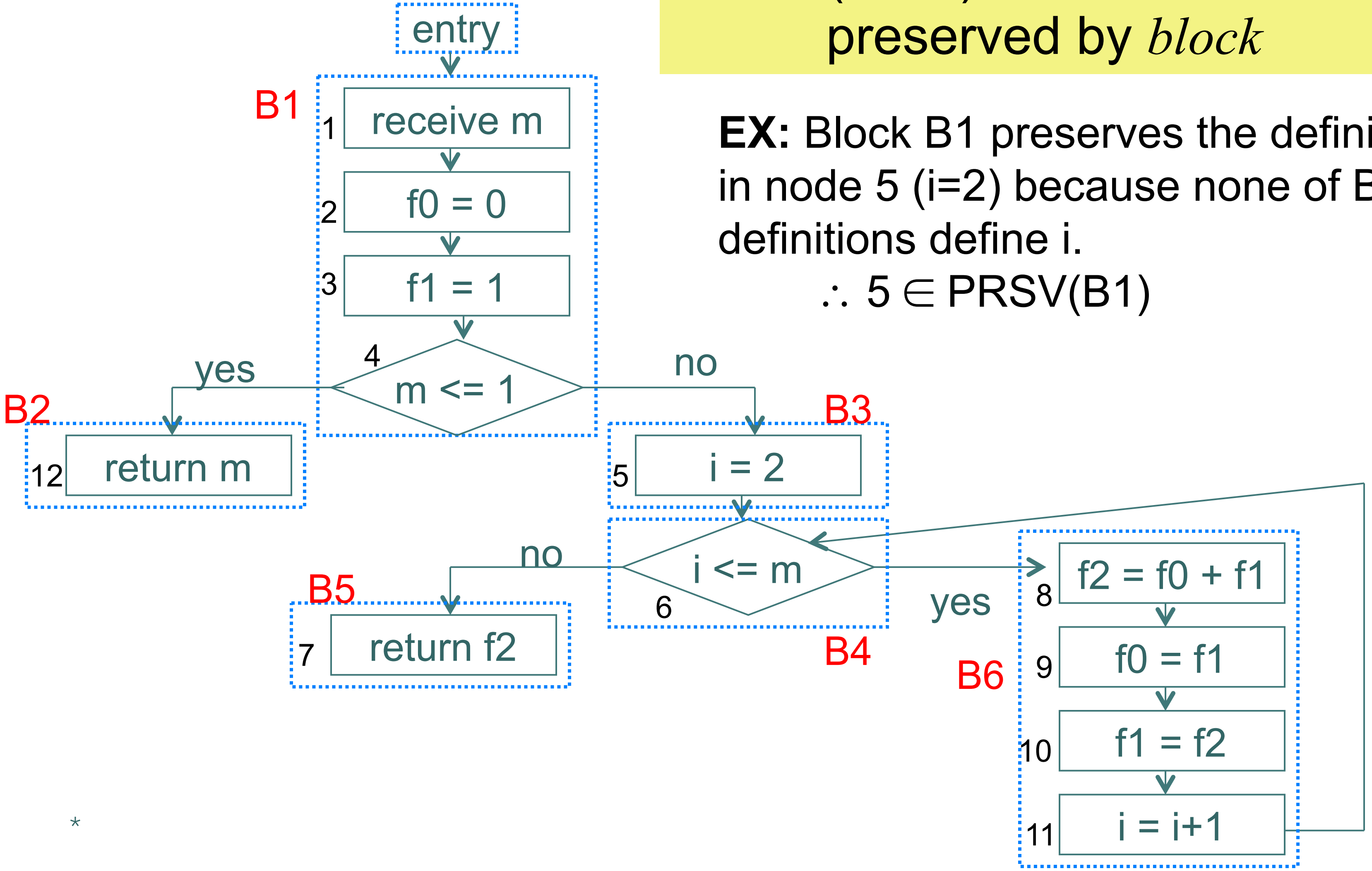
Preserved Definitions: PRSV(*block*)

PRSV(*block*) = set of definitions preserved by *block*



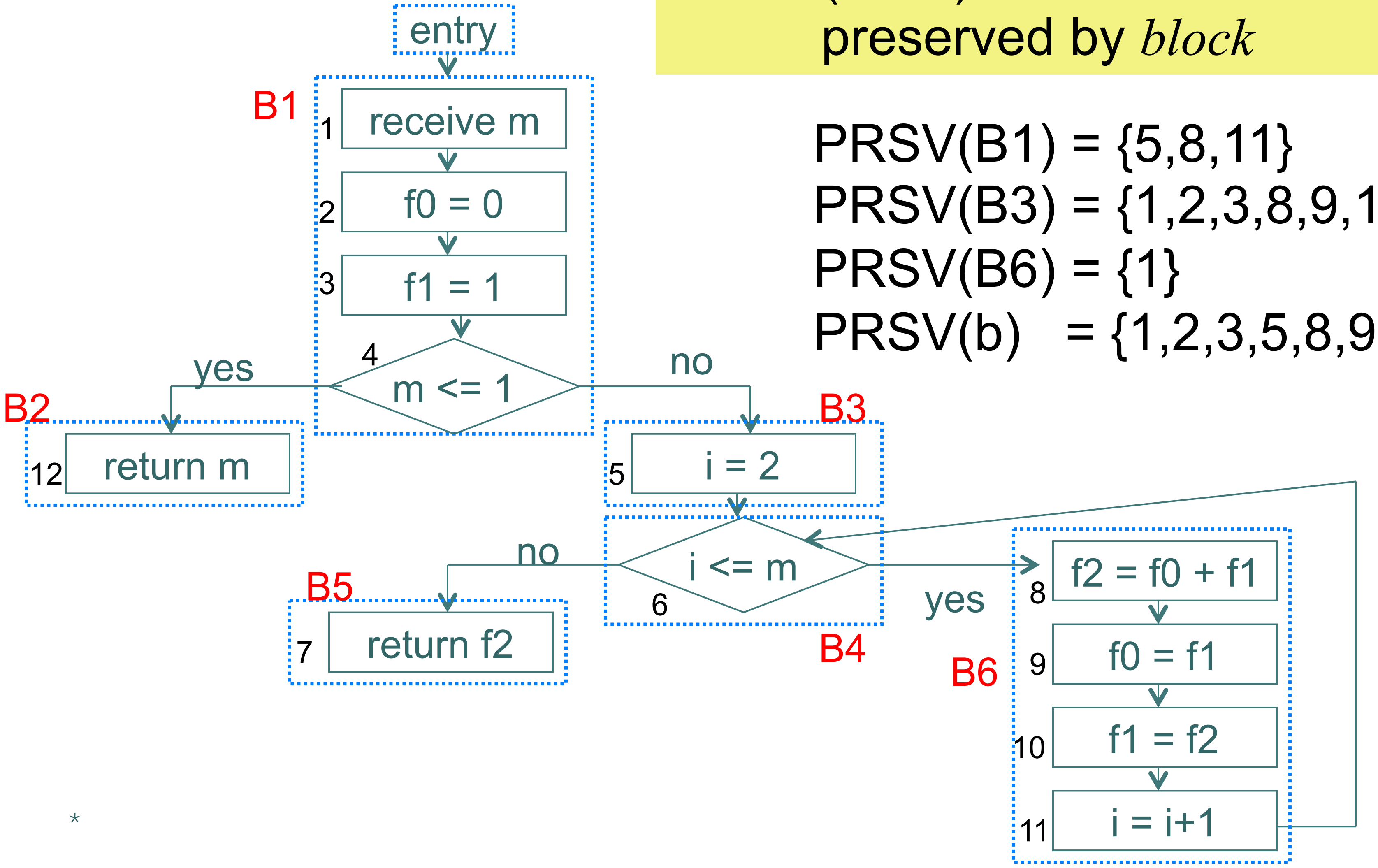
EX: Block B1 preserves the definition in node 5 (i=2) because none of B1's definitions define i.

$\therefore$ 5 $\in$ PRSV(B1)

Preserved Definitions: PRSV(*block*)

PRSV(*block*) = set of definitions preserved by *block*

PRSV(B1) = {5,8,11}
PRSV(B3) = {1,2,3,8,9,10}
PRSV(B6) = {1}
PRSV(b) = {1,2,3,5,8,9,10,11}



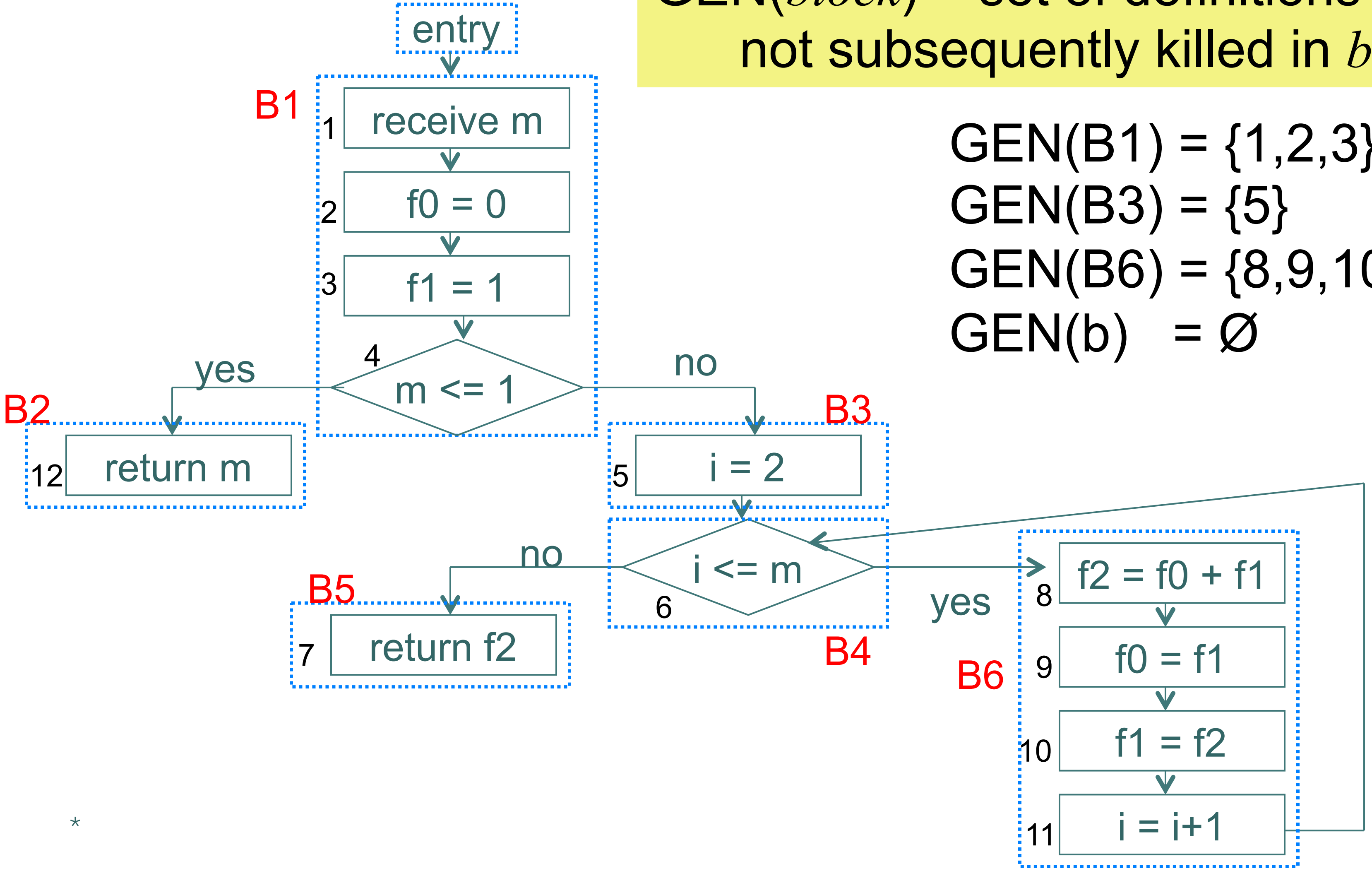
“Killed” Definitions

A definition is said to “kill” another definition if they write to the same location

EX: “x = y * z” kills “x = 2 + u”

Generated Definitions: GEN(*block*)

GEN(*block*) = set of definitions in *block* not subsequently killed in *block*



GEN(B1) = {1,2,3}
GEN(B3) = {5}
GEN(B6) = {8,9,10,11}
GEN(b) = Ø

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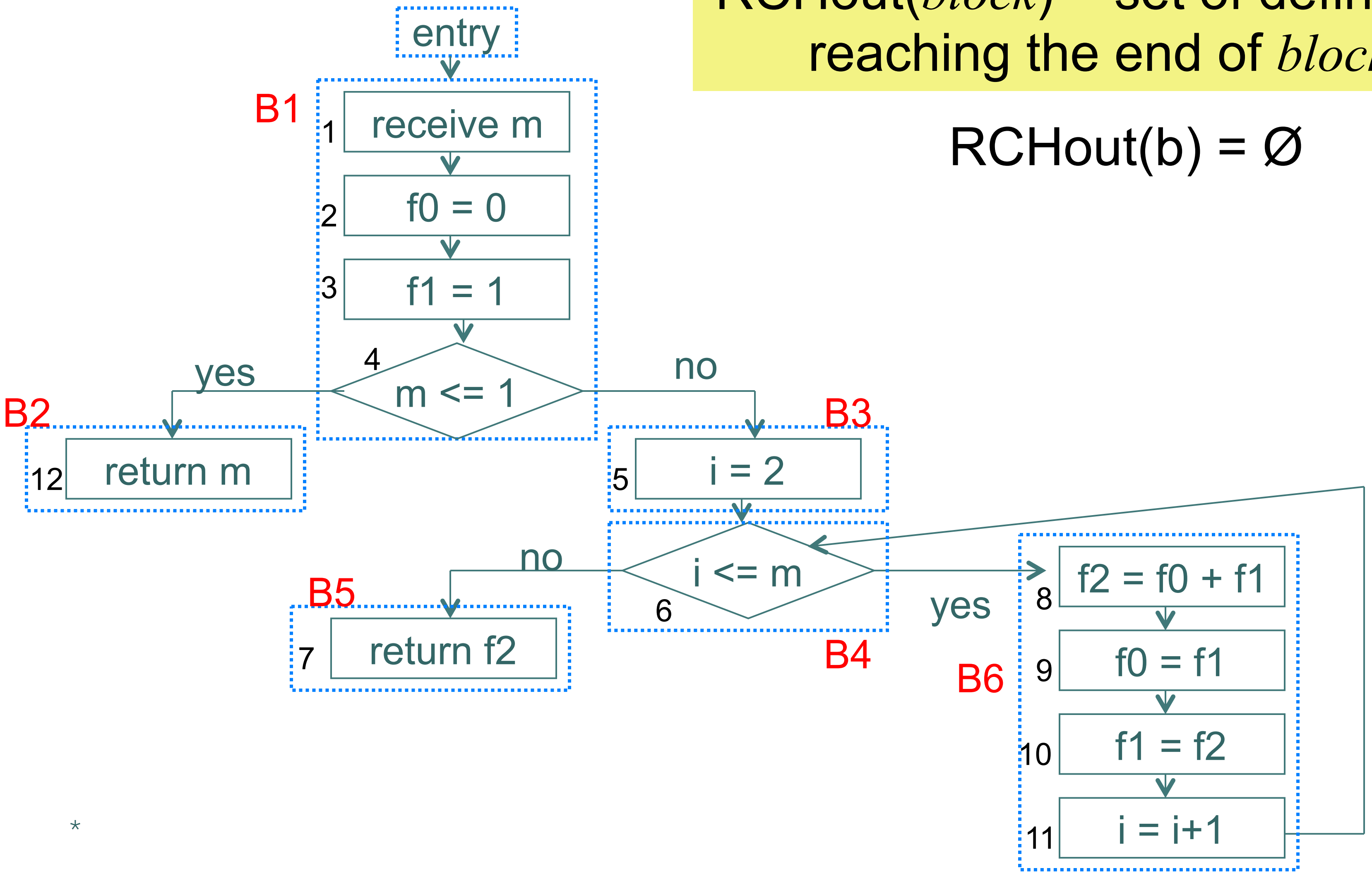
Out Reaching Definitions: RCHout(*block*)

RCHout(*block*) = set of definitions reaching the end of *block*

RCHout(b) = Ø



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Data Flow Equations

The definitions out of a block are:
- those generated by it and
- those reaching it that are preserved

$$\text{RCHout}(b) = \text{GEN}(b) \cup (\text{RCHin}(b) \cap \text{PRSV}(b)) \textit{ for all } b$$

The definitions reaching a block are those out-reaching from its predecessors

$$\text{RCHin}(b) = \bigcup_{p \in \text{Pred}(b)} \text{RCHout}(p) \textit{ for all } b$$

Solving Data Flow Equations Iteratively

1. Initialize the attributes
2. Treat the data flow equations as assignments
3. If there has been a change to the computed attributes, go to 2 otherwise halt

Solving Data Flow Equations

Here is a code outline

```
*Initialization code*
repeat
    $RCHout(b) := GEN(b) \cup (RCHin(b) \cap PRSV(b))$ *for all* b
    $RCHin(b) := \bigcup_{p \in Pred(b)} RCHout(p)$ *for all* b
until
    *no change to* RCHin/RCHout
```

Next time

- Justifying Iterative Solution
    - I.e., why does this give us a solution?
- Liveness as iterative data flow analysis

Iterative Data Flow Analysis

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  - Like liveness
  - Liveness for programs with loops is solved with IDFA
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Liveness Analysis

- …determines when the value within a virtual register may still be used
    - a.k.a. its value is “live”
- …and when it won't
    - a.k.a. its value is “dead”
- This property, “liveness”, may be **approximated** statically

More Precise Definition of Liveness

Definition
– **assignment** of a value to a variable
– def[v] = set of nodes that define variable v
– def[n] = set of variables defined at node n

a🡐0

Use
– **reading** the value to a variable
– use[v/n] = analogous to def[v/n]

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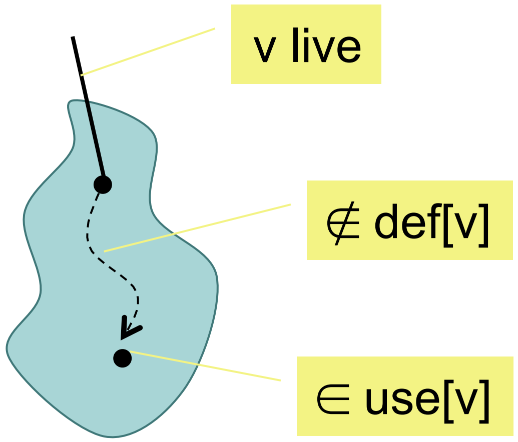
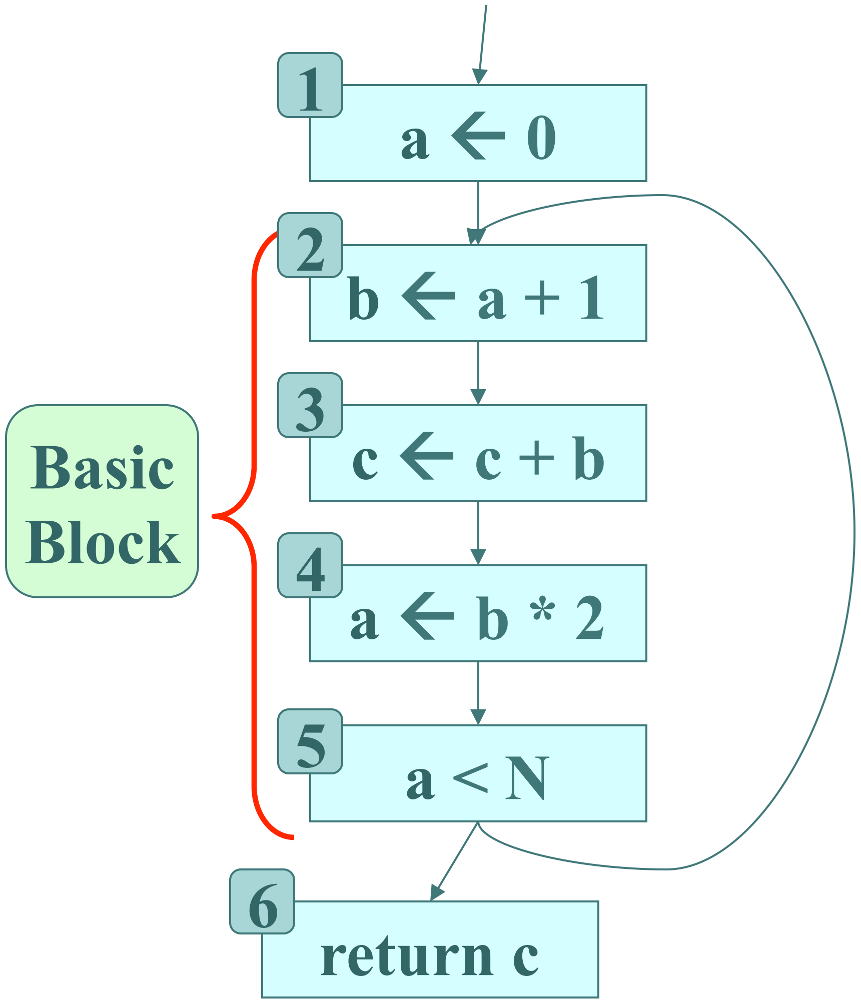
Liveness
v is **live on a CFG edge** if

1. ∃ a directed path from that edge to a use of v
2. The path does not go through any defn of v



Small Control Flow Graph

```
   a ← 0
L: b ←a + 1
   c ← c + b
   a ←b * 2
   if (a < N) goto L
   return c
```



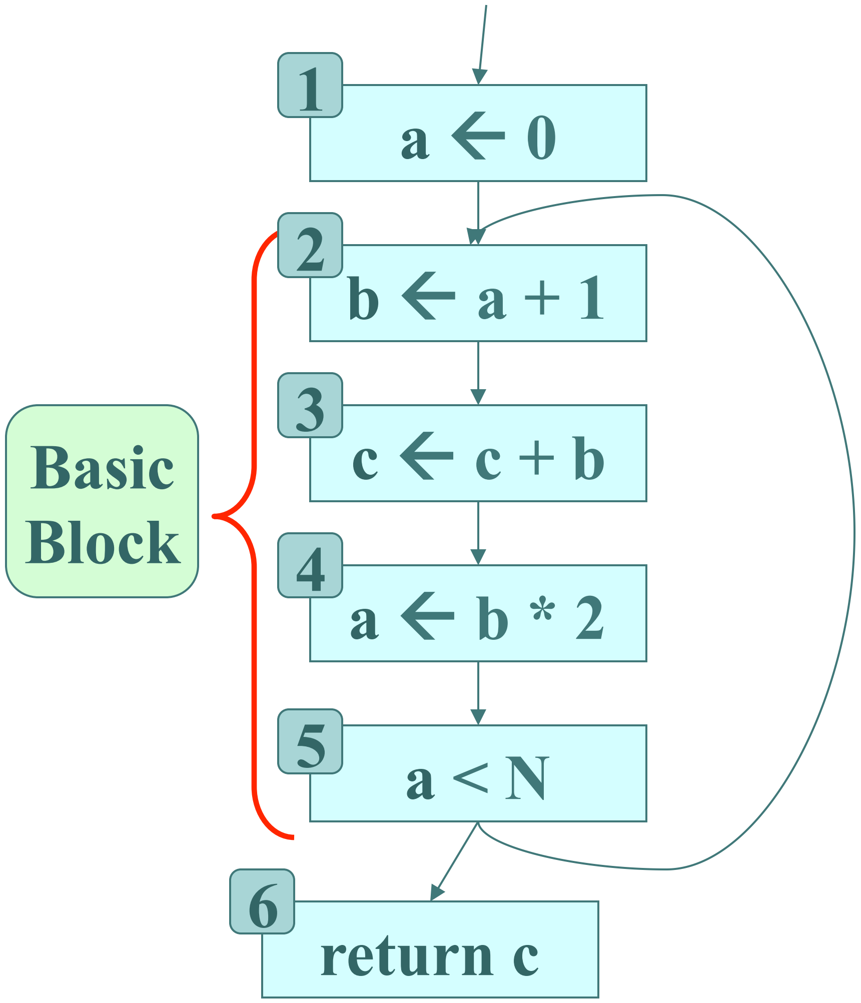
The Flow of Liveness

Data-flow

- Liveness flows through the edges of the CFG

Direction of Flow

- Liveness flows backwards because
    - Behavior at future nodes determines liveness at given node
- Consider a or b
- “Forward” properties exist (e.g., reaching)



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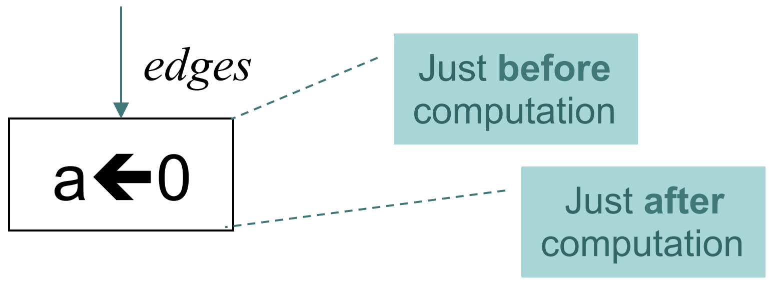
Liveness at Nodes

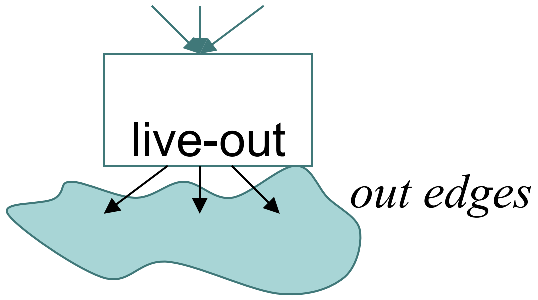
We have liveness at edges

- How do we talk about liveness at nodes?

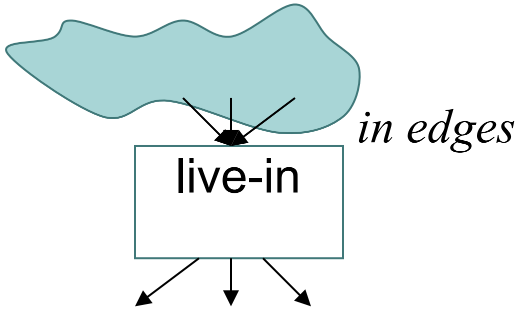


Two more definitions

- A variable is **live-out** at a node if it is live on **any** of that node's out-edges

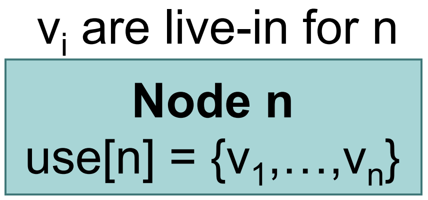


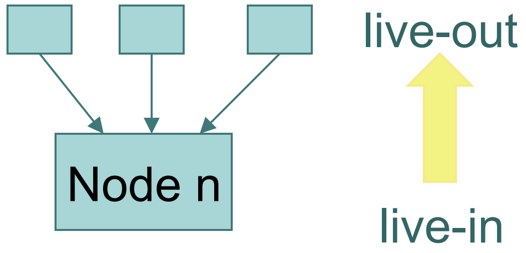
- A variable is **live-in** at a node if it is live on **any** of the node's in-edges

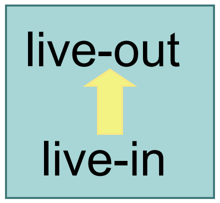


Computing Liveness

1. **Generate liveness:** if a variable is in use[n], then it is in live-in[n]
2. **Push liveness across edges:** if a variable is in live-in[n], then it is in live-out for all nodes in pred[n]
3. **Push liveness across nodes:** if a variable is in live-out[n] and not in def[n] then the variable is in live-in[n]







Step 1. The Attributes

1. **Generate liveness:** if a variable is in use[n], then it is in live-in[n]
2. **Push liveness across edges:** if a variable is in live-in[n], then it is in live-out for all nodes in pred[n]
3. **Push liveness across nodes:** if a variable is in live-out[n] and not in def[n] then the variable is in live-in[n]

$$\left.\begin{array}{ll} \text{live-in}[n] & = \\ \text{live-out}[n] & = \\ \text{use}[n] & = \\ \text{def}[n] & = \end{array}\right\} \text{Initial values?}$$

Step 1. The Attributes

1. **Generate liveness:** if a variable is in use[n], then it is in live-in[n]
2. **Push liveness across edges:** if a variable is in live-in[n], then it is in live-out for all nodes in pred[n]
3. **Push liveness across nodes:** if a variable is in live-out[n] and not in def[n] then the variable is in live-in[n]

$$\begin{array}{ll} \text{live-in}[n] & = \varnothing \\ \text{live-out}[n] & = \varnothing \\ \text{use}[n] & = \\ \text{def}[n] & = \end{array}$$

$\left.\begin{array}{l} \\ \\ \end{array}\right\}$ constant, determined by node n

Step 2. Data Flow Equations

1. **Generate liveness:** if a variable is in use[n], then it is in live-in[n]
2. **Push liveness across edges:** if a variable is in live-in[n], then it is in live-out for all nodes in pred[n]
3. **Push liveness across nodes:** if a variable is in live-out[n] and not in def[n] then the variable is in live-in[n]

live-in[n] = ?

live-out[n] = ?

Step 2. Data Flow Equations

1. **Generate liveness:** if a variable is in use[n], then it is in live-in[n]
2. **Push liveness across edges:** if a variable is in live-in[n], then it is in live-out for all nodes in pred[n]
3. **Push liveness across nodes:** if a variable is in live-out[n] and not in def[n] then the variable is in live-in[n]

$$\text{live-in}[n] = \text{use}[n] \cup (\text{live-out}[n] - \text{def}[n])$$

$$\text{live-out}[n] = \bigcup_{s \in \text{succ}[n]} \text{live-in}[s]$$

Step 3. Solving DFE Iteratively

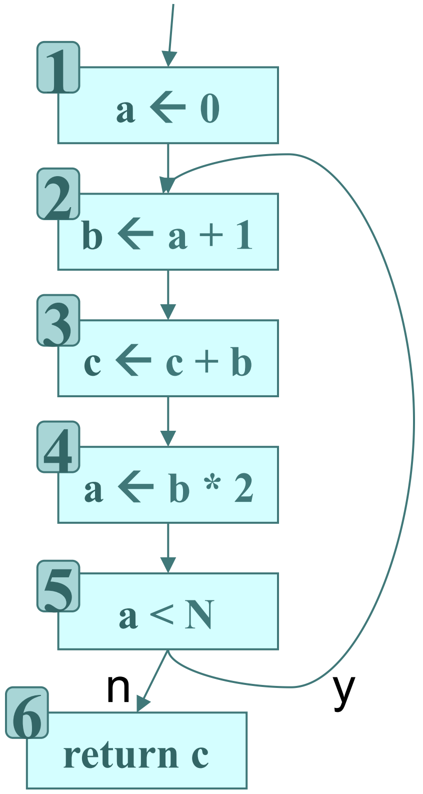
```
foreach node n in CFG
        in[n] = ∅ ; out[n] = ∅                          } initialization

repeat
    foreach node n in CFG
        in’[n] = in[n]                                   } save current results
        out’[n] = out[n]
        in[n]   = use[n] ∪ (out[n] - def[n])             } iterative solution
        out[n] = $\bigcup_{s \in succ[n]} in[s]$

until  in’[n] == in[n] & out’[n] == out[n] for all nodes n   } convergence test
```

Example

$$in[n] = use[n] \cup (out[n] - def[n])$$

$$out[n] = \bigcup_{s \in succ[n]} in[s]$$

node	use	def	iteration step 1		2		3		4		5		6		7	
			in	out	in	out	in	out	in	out	in	out	in	out	in	out
1		a				a		a		ac	c	ac	c	ac	c	ac
2	a	b	a		a	bc	ac	bc	ac	bc	ac	bc	ac	bc	ac	bc
3	bc	c	bc		bc	b	bc	b	bc	b	bc	b	bc	bc	bc	bc
4	b	a	b		b	a	b	a	b	ac	bc	ac	bc	ac	bc	ac
5	a		a	a	a	ac	ac	ac	ac	ac	ac	ac	ac	ac	ac	ac
6	c		c		c		c		c		c		c		c	



in = red
out = blue

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Example (cont’d)

$$in[n] = use[n] \cup (out[n] - def[n])$$

$$out[n] = \bigcup_{s \in succ[n]} in[s]$$

Improving Performance

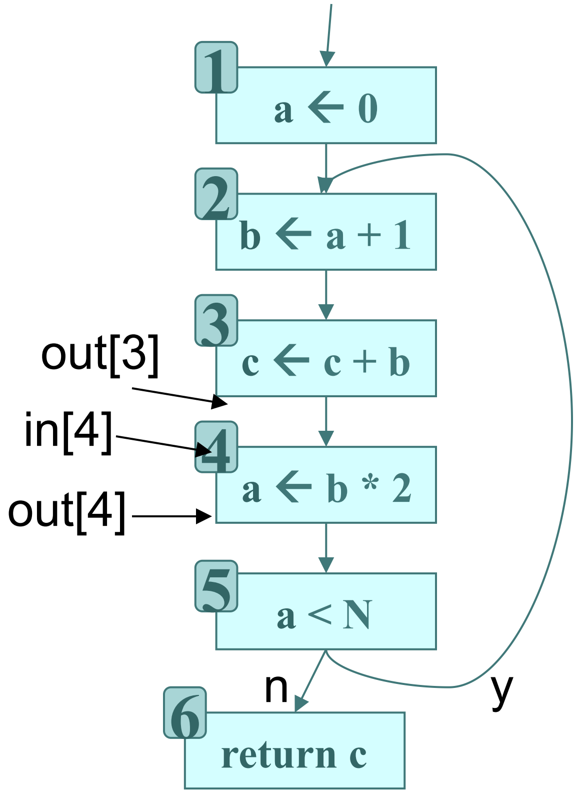
consider the (3➔4) edge in the graph:

out[4] is used to compute in[4],

in[4] is used to compute out[3],…

So, we should compute in the

order: out[4], in[4], out[3], in[3],…



Order of computation should follow flow direction

Step 3. Solving DFE Iteratively revisited

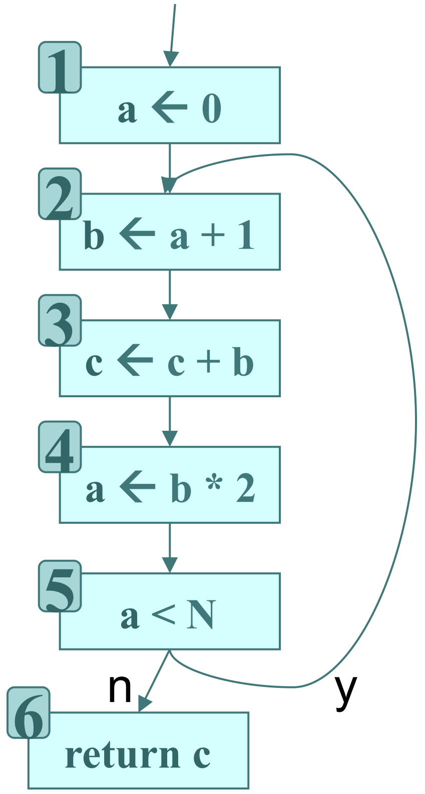
```
foreach node n in CFG
        in[n] = ∅ ; out[n] = ∅                      } initialization

repeat
    foreach node n in CFG in reverse topological sort order
        in'[n] = in[n]                              } save current results
        out'[n] = out[n]
        out[n] = $\bigcup_{s \in succ[n]}$ in[s]    } Note the change in order
        in[n]  = use[n] ∪ (out[n] - def[n])

until in'[n] == in[n] & out'[n] == out[n] for all nodes n   } convergence test
```

Example

$$out[n] = \bigcup_{s \in succ[n]} in[s]$$

$$in[n] \;\; = use[n] \cup (out[n] - def[n])$$

node	use	def	1		2		3		4		5		6		7	
6	c			c		c		c								
5	a		c	ac	ac	ac	ac	ac								
4	b	a	ac	bc	ac	bc	ac	bc								
3	bc	c	bc	bc	bc	bc	bc	bc								
2	a	b	bc	ac	bc	ac	bc	ac								
1		a	ac	c	ac	c	ac	c								



out = red
in = blue

Note the change in order!

Time Complexity (rough)

- Consider a program of size N
  - N = max(nodes in CFG, number of vars)
    - ∴ each live-in, live-out set has at most N elements
  - Each set union takes O(N) time

Step 3. Solving DFE Iteratively revisited

```
foreach node n in CFG
        $in[n] = \emptyset$ ; $out[n] = \emptyset$                    } O(N)

repeat
    foreach node n in CFG in reverse topological sort order       }
        $in'[n] = in[n]$                                           }
        $out'[n] = out[n]$                                         } O(N^2)
        $out[n] = \bigcup_{s \in succ[n]} in[s]$                   }          } O(N^2)
        $in[n] = use[n] \cup (out[n] - def[n])$                    }
until $in'[n] == in[n]$ & $out'[n] == out[n]$ for all nodes n     } O(N)
```

$\therefore$ Worst case is $O(N^2 \times N^2) = O(N^4)$

More Performance Considerations

- Use basic blocks instead of nodes
  - Merge nodes into basic blocks to decrease size of CFG
- Representation of sets
  - For dense sets, use a bit vector representation
    - This can reduce the cost of set operations to O(1)
  - For sparse sets, use sorted (linked) lists
- Typical Case: 2 to 3 iterations with good ordering and sparse sets
  - In the O(N) to O(N^2) range for average

Termination Guarantees: possible values of live-in (for example)

$$\begin{tikzcd}
 & & \{a,b,c,d\} \arrow[dll] \arrow[dl] \arrow[dr] \arrow[drrr] & & & \\
\{a,b,c\} \arrow[d] & \{a,b,d\} & & \{a,c,d\} & & \{b,c,d\} \\
\{a,b\} & \{a,c\} & \{a,d\} & \{b,c\} & \{b,d\} & \{c,d\} \\
 & \{a\} & \{b\} & \{c\} & \{d\} & \\
 & & & \varnothing & &
\end{tikzcd}$$

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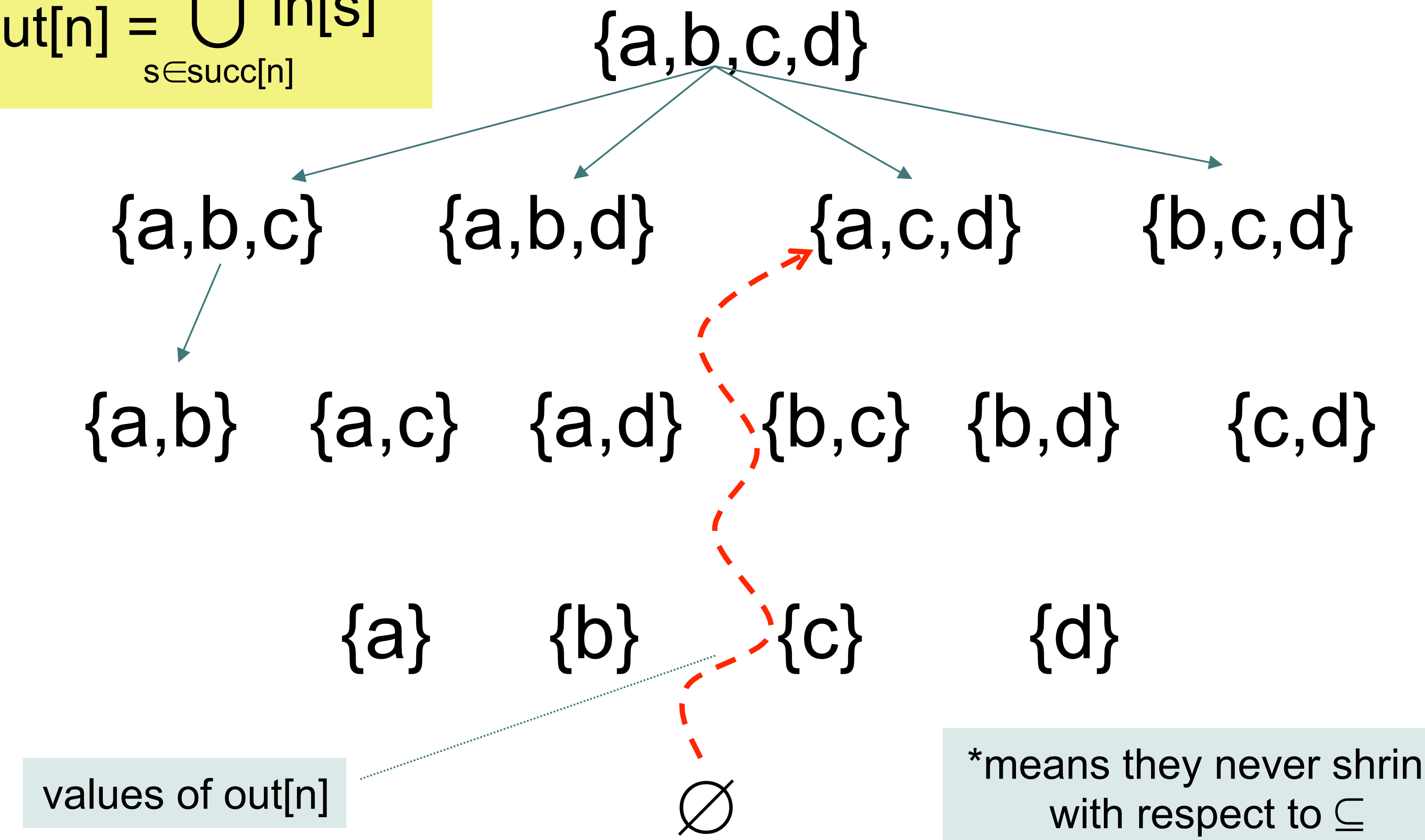
*Not all links shown

$\longrightarrow$ means is a “subset of”

Attribute values are **monotonically increasing***

$$out[n] = \bigcup_{s \in succ[n]} in[s]$$



*means they never shrink with respect to ⊆