Expanding Brackets

Learning Objectives

- Simplify algebraic expressions
- Expand brackets
- Solve equations in which the unknown appears on both sides

Basic Principles

Algebra uses letters, often x, to stand for numbers. Algebraic expressions can be treated in the same way as number expressions.

- $x + 3$ means "add three to the unknown number."
- $3x$ means "3 times the unknown number."
- x^2 means "square the unknown number."

Example: Combining Like Terms

$$a + 3ab - 4ba = a - ab$$

Note

Note: $ab = ba$, so $3ab$ and $-4ba$ are like terms.

$$3p^3 + 2p^2 - 2p^3 + 5p^2 = (3p^3 - 2p^3) + (2p^2 + 5p^2) = p^3 + 7p^2$$

Example: Multiplying Algebraic Terms

$$4r \times 5t = 20rt$$

$$(3b)^2 \times 3b = 9b^2 \times 3b = 27b^3$$

Tip

- The multiplication sign is often not included between letters, e.g. $3ab$ means $3 \times a \times b$
- When multiplying, add like powers: $3a^2b \times 2a^5b^4 = 6a^8b^5$. (Think of a as a^1).

Example: Expanding Brackets

$$2(3+x) = 2 \times 3 + 2 \times x = 6 + 2x$$

Tip

- Multiply each term inside by the outside term.
- The multiplication sign is usually left out: $3(x+y)$ means $3 \times (x+y) = 3 \times x + 3 \times y = 3x + 3y$
- When multiplying, the number 1 is usually left out: $-(2x+3)$ means $-1 \times (2x+3) = (-1) \times (2x) + (-1) \times 3 = -2x + 3$

Warning

Be very careful with negative signs outside a bracket.

$$-2 \times (a - 3) = (-2) \times a + (-2) \times (-3) = -2a + 6$$

Example: Expanding with Indices

Expand $5x^2(12x^3 + 4x^5)$

$$5x^2 \times 12x^3 + 5x^2 \times 4x^5 = 60x^5 + 20x^7$$

Test Your Understanding

Expand

1) $4y(2y - 8)$

2) $3x^2(x^3 + y^4)$

3) $c^{\frac{1}{2}}(c^{\frac{3}{2}} - c^{\frac{1}{2}} + c^{-\frac{1}{2}})$

4) $5x^2(6x - x^2 + 7y - x^{-2})$

Answers

1) $4y(2y-8) = 8y^2 - 32y$

2) $3x^2(x^3+y^4) = 3x^5 + 3x^2y^4$

3) $c^{\frac{1}{2}}(c^{\frac{3}{2}} - c^{\frac{1}{2}} + c^{-\frac{1}{2}}) = c^2 - c + 1$

4) $5x^2(6x - x^2 + 7y - x^{-2}) = 30x^3 - 5x^4 + 35x^2y - 5$

Exercise 1

Please complete the worksheet

Example: Expanding Double Brackets

To expand $(x+3)(x+2)$, you can use the grid method.

	x	$+2$
x	x^2	$2x$
$+3$	$3x$	6

Summing the terms on the inside and simplifying gives:

$$x^2 + 3x + 2x + 6 = x^2 + 5x + 6$$

Example: Squared Bracket

A squared bracket is simply the same bracket twice. For example to expand

$$(2x + 3)^2$$

the grid method is:

	$2x$	$+3$
$2x$	$4x^2$	$6x$
$+3$	$6x$	9

Adding the terms on the inside and simplifying gives

$$4x^2 + 12x + 9$$

Test Your Understanding

Expand and simplify:

1) $(2y + 3)^2$

2) $(4 + x)(3x - 1)$

3) $(4a + 3)(4a - 3)$

4) $(5 - 2a)(b - 6)$

Answers

1) $(2y+3)^2 = 4y^2 + 12y + 9$

2) $(4+x)(3x-1) = 3x^2 + 11x - 4$

3) $(4a+3)(4a-3) = 16a^2 - 9$

4) $(5-2a)(b-6) = 5b + 12a - 2ab - 30$

Exercise 2

Please complete the worksheet

Example: 2×3 Grids

Expand and simplify $(x^2 + 6x - 3)(x - 4)$

	x	-4
x^2	x^3	$-4x^2$
$6x$	$6x^2$	$-24x$
-3	$-3x$	12

$$\begin{aligned} & x^3 + 6x^2 - 3x - 4x^2 - 24x + 12 \\ = \; & x^3 + 2x^2 - 27x + 12 \end{aligned}$$

Test Your Understanding

Expand and simplify:

1) $(a - 3b + 2c)(a + b)$

2) $(cd + 4c - 5d)(3e + 2f)$

3) $(x^2 - x + 7)(2x + 3)$

4) $(1 + 3x - 2x^2)(7x - 2)$

Answers

1) $(a-3b+2c)(a+b) = a^2 - 2ab + 2ac + 2bc - 3b^2$

2) $(cd+4c-5d)(3e+2f) = 3cde+2cdf+12ce+8cf-15de-10df$

3) $(x^2 - x + 7)(2x + 3) = 2x^3 + x^2 + 11x + 21$

4) $(1 + 3x - 2x^2)(7x - 2) = -14x^3 + 25x^2 + x - 2$

Exercise 3

Please complete the worksheet.

Example: Triple Brackets

A three-bracket expansion such as $(x+1)(x-3)(x+2)$ can be done in two grid-method steps. First, expand $(x-3)(x+2)$:

	x	$+2$
x	x^2	$2x$
-3	$-3x$	-6

This gives a result of

$$x^2 - x - 6$$

Then expand $(x+1)(x^2-x-6)$ with a 2×3 grid:

	x^2	$-x$	-6
x	x^3	$-x^2$	$-6x$
$+1$	x^2	$-x$	-6

So altogether $(x+1)(x-3)(x+2) = x^3 - 7x - 6$

Tip

The order you multiply the brackets does not matter.

Test Your Understanding

Expand and simplify:

1) $(x+2)^3$

2) $(2x+1)(x-1)^2$

3) $(3+y)(3-y)^2$

4) $(a-bx)^3$

Answers

1) $(x+2)^3 = x^3 + 6x^2 + 12x + 8$

2) $(2x+1)(x-1)^2 = 2x^3 - x^2 + 1$

3) $(3+y)(3-y)^2 = y^3 - 3y^2 - 9y + 27$

4) $(a-bx)^3 = a^3 - 3a^2bx + 3ab^2x^2 - b^3x^3$

Exercise 4

Please complete the worksheet.