Percentages

Methods for Percentages

Previously, when working out percentages of amounts, we would use a method called "chunking". For example, to find 35% of 120, we might do something like:

$$\begin{aligned}
100\% &= 120 \\
10\% &= 12 \\
5\% &= 6 \\
20\% &= 24 \\
35\% &= 20\% + 10\% + 5\% \\
&= 24 + 12 + 6 \\
&= 42
\end{aligned}$$

This is a good **non-calculator** method, but with a calculator there is a far better way.

Decimal Multipliers

The key concept of this topic is the idea of **decimal multipliers**. In short, when we represent percentages (or percentage changes) as decimals, we can multiply to work things out.

It turns out that in many cases this makes questions **much easier**, particularly when it comes to compound percentage changes.

To see how well they work and how much they can simplify percentage problems, let's look at an example.

Example: Pocket Money

You get £10 a week pocket money. Unfortunately, you get a little lazy about revising for your maths exam, and so you don't score as well as you could have if you'd worked harder. Your parents are concerned, and so they decide to reduce your pocket money by 10% until your results improve.

You pull up your socks and study hard for the next maths exam, and your score is much better. As a reward your parents increase your pocket money by 10%.

$$100\% - 10\% + 10\% = 100\%$$

All good then! Are you happy with the result?

In fact this logic is incorrect, because the two 10%s are not 10% of the same amount.

$$£10 \xrightarrow{-10\%} £9 \xrightarrow{+10\%} £9.90$$

Percentage changes do not add up.

However, they **do multiply**. Let's see.

$$10\% \text{ increase} = 100\% + 10\% = 110\% = \frac{110}{100} = 1.1$$

$$10\% \text{ decrease} = 100\% - 10\% = 90\% = \frac{90}{100} = 0.9$$

$$\pounds 10 \times 1.1 \times 0.9 = \pounds 9.90$$

The decimal multipliers give the correct result.

This is the key to this whole topic. Percentages do not add up, but they **do multiply**.

This way of calculating percentages is important in several areas of applied mathematics, particularly financial mathematics.

Example: Finding a Percentage of an Amount

To find a percentage **of** an amount, we simply convert the percentage to a decimal and multiply it by the amount.

Calculate 13% of 300:

$$300 \times 0.13 = 39$$

Calculate 87% of 400:

$$400 \times 0.87 = 348$$

Test Your Understanding

Calculate the following. You **must** use a decimal multiplier.Please show your working by writing down the calculation. You may use a calculator.

1) 84% of 15

2) 9% of 78

3) 70% of 33.2

4) 150% of 13

5) 317% of 2.8

6) 84.2% of 68

Answers

1) 84% of $15 = 12.6$

2) 9% of $78 = 7.02$

3) 70% of $33.2 = 23.24$

4) 150% of $13 = 19.5$

5) 317% of $2.8 = 8.876$

6) 84.2% of $68 = 57.256$

Example: Percentage Increase

When you increase something by a percentage, you keep the original 100% and add on an additional percentage. Therefore to calculate the decimal multiplier we add on to 100%.

Increase 170 by 10%: $170 \xrightarrow[\times 1.1]{100\%+10\%=110\%} 187$

Increase 40 by 12%: $40 \xrightarrow[\times 1.12]{100\%+12\%=112\%} 44.8$

Note

This way of representing percentage changes on an arrow is useful, particularly for reverse percentage changes (coming up soon).

Example: Percentage Decrease

When you decrease something by a percentage, you take the original 100% and remove a percentage. Therefore to calculate the decimal multiplier we subtract from 100%.

Decrease 86 by 61%:

$$86 \xrightarrow[\times 0.39]{100\% - 61\% = 39\%} 33.54$$

Test Your Understanding

Calculate the following. You **must** use decimal multipliers and write down your calculations.

1) Find 62% of 97

2) Find 3% of 52.8

3) Find 14.7% of 44

4) Find 178% of 52

5) Increase 24 by 9%

6) Increase 5.2 by 64%

7) Decrease 97 by 38%

8) Decrease 540 by 8.7%

Answers

1) 60.14

2) 1.584

3) 6.468

4) 92.56

5) 26.16

6) 8.528

7) 60.14

8) 493.02

Example: Reverse Percentage Changes

The price of an item was increased by 20% to £66. What was the original price before the increase? We can use the same arrow method as before:

$$? \xrightarrow[\times 1.2]{100\%+20\%=120\%} 66$$

The answer is then found by **dividing** by the multiplier:
$66 \div 1.2 = 79.2$

What Not To Do

- Don't find 20% of £66. The 20% is a percentage of the original unknown amount.
- Don't use a decimal multiplier of 0.8. Reversing an increase is not the same as a decrease.

Example: Reverse Percentage Decrease

The price of an item in a sale was reduced by 18% to a new price of £60.68. What was the price before the sale?

$$? \xrightarrow[\times 0.82]{100\% - 18\% = 82\%} £60.68$$

$$£60.68 \div 0.82 = £74$$

Test Your Understanding

Find the original value. You must divide by a decimal multiplier.

1) A number was increased by 64% to give a new value of 60.68. Find the original value.

2) A number was decreased by 23% to give a new value of 37.73. Find the original value.

3) A number was decreased by 6.3% to give a new value of 76,834. Find the original value.

4) A number was increased by 104% to give a new value of 142.8. Find the original value.

Answers

1) 37

2) 49

3) 82,000

4) 70

Exercise 1

Please complete the worksheet.

Example: Compound Percentage Change

Matthias invests £100 in a savings account at 2% interest. How much does he have at the end of year 1, year 2, and year 3?

$$\begin{aligned}\text{Year 1: } & £100 \times 1.02 = £102 \\ \text{Year 2: } & £102 \times 1.02 = £104.04 \\ \text{Year 3: } & £104.04 \times 1.02 = £106.12\end{aligned}$$

Notice here that we are simply repeatedly multiplying by the same decimal multiplier.

Matthew invests £3000 in a savings account with an interest rate of 3.2%. How much will he have after six years?

$$100\% + 3.2\% = 103.2\% = 1.032$$

$$£3000 \xrightarrow{\times 1.032} \xrightarrow{\times 1.032} \xrightarrow{\times 1.032} \xrightarrow{\times 1.032} \xrightarrow{\times 1.032} \xrightarrow{\times 1.032} \; ?$$

Once we notice that we are multiplying repeatedly, we can simplify this method using powers:

$$£3000 \xrightarrow{\times 1.032^6} £3624.09$$

Note

Amounts of money should always be rounded to the nearest penny or cent, i.e. to two decimal places.

Example: Compound Percentage Decrease

Thomas buys a new car for £15,690. The car depreciates at a rate of 4% each year. How much will it be worth in 11 years?

$$\pounds 15690 \xrightarrow{\times 0.96^{11}} \pounds 10013.98$$

Code: Compound Percentage Formula

```
initial_amount = 100
interest_rate = 5
number_of_years = 6

multiplier = 1 + interest_rate/100

final_amount = initial_amount * (multiplier ** number_of_ye

print(round(final_amount, 2))
```

Test Your Understanding

1) An investment of £500 grows by 4% each year. How much will it be worth after 5 years?

2) A rare book increases in value by 7% each year. If it is currently worth £1,200, what will it be worth in 8 years?

3) A car is bought for £18,000 and decreases in value by 12% each year. What will it be worth after 6 years?

4) A population of 250,000 increases by 2.5% each year. What will the population be after 10 years?

5) A laptop costs £900 and depreciates by 15% each year. What will it be worth after 4 years?

6) A forest area is shrinking by 1.8% each year. If its current size is 5,000 acres, what will its size be after 12 years?

Answers

1) 608.33

2) 2060.37

3) 8042.05

4) 320634.62

5) 522.68

6) 3997.19

Example: Multiple Compound Changes

Andrew buys a watch for £225. It decreased in value by 3.5% for 1 year, then decreased in value by 7.5% per year for 2 years. Find the new value of the watch.

Again this is a case where decimal multipliers make this **much easier**.

$$\begin{aligned} 3.5\% \text{ decrease} &= 0.965 \\ 7.5\% \text{ decrease for 2 years} &= 0.925^2 \\ \text{new value} &= £225 \times 0.965 \times 0.925^2 \\ &= £185.78 \end{aligned}$$

Example: Reverse Compound Change

Philip invests some savings into an account that pays 4% compound interest per year. After 4 years the savings are worth £17,075. How much did Philip have in the account at the beginning?

An arrow representation will be useful here:

$$? \xrightarrow{\times 1.04^4} £17075$$

This shows that we should divide:

$$17075 \div 1.04^4 = 14595.78$$

Example: Missing Interest Rate

Consider the same question but with a missing interest rate.

Philip invests £14,595.78 into an savings account. After 4 years the savings are worth £17,075. What was the interest rate?

In this case, we can form and solve an equation:

$$\begin{aligned} 14595.78 \times x^4 &= 17075 \\ x^4 &= 17075 \div 14595.78 \\ x &= \sqrt[4]{17075 \div 14595.78} \\ x &= 1.04 \end{aligned}$$

which represents a 4% increase. This is the interest rate.

Example: Unknown Length of Time

Consider the same question but with a missing length of time.

Philip invests £14,595.78 into an savings account at an interest rate of 4%. After how many years are they worth £17,075?

There is an A-Level method called a *logarithm* that can help with this, but for now we will use trial and error. You can do this on your calculator to speed things up.

$$
\begin{aligned}
14595.78 \times 1.04 &= 15,179.61 \\
14595.78 \times 1.04^2 &= 15,786.80 \\
14595.78 \times 1.04^3 &= 16,418.27 \\
14595.78 \times 1.04^4 &= 17,075.00
\end{aligned}
$$

Test Your Understanding

1) The value of the house increases each year by 2.1%. How much is her house worth after 6 years?

2) The value of the house increases each year by 3%. How many years will it take for her house to be worth £163,909.05?

3) The value of the house increases each year by x% and after 3 years is worth £168,729.60. Find x.

4) The value of the house increases each year by 14%. How many years will it take for her house price to double?

Answers

1) £169,920.47

2) 3 years

3) 4

4) 6 years

Exercise 2

Please complete the worksheet.