

Recurring Decimals

Example: Long Division

To turn fractions into decimals, we can use long division. For example, to calculate $\frac{17}{8}$:

$$\begin{array}{r} 2.125 \\ 8 \overline{) 17.000} \\ \underline{16} \\ 1.0 \\ \underline{8} \\ 20 \\ \underline{16} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

Notice that we stop when we get a remainder of zero.

Example: Fraction to Recurring Decimal

We can also do this to find a recurring decimal. For example, to find $\frac{3}{11}$, we can do:

$$\begin{array}{r} 0.\overline{27} \\ 11 \overline{) 3.00} \\ \underline{2.2} \\ 80 \\ \underline{77} \\ 3 \end{array}$$

As soon as we get a repeated digit, we know we will be stuck in a loop, so we mark the decimal as recurring.

Sometimes there is a non-recurring decimal part before the recurrence begins:

$$\begin{array}{r} 1.1\overline{3} \\ 15 \overline{) 17.00} \\ \underline{15} \\ 2.0 \\ \underline{1.5} \\ 50 \\ \underline{45} \\ 5 \end{array}$$

Notice that the vertical bar only extends over the repeating part.

And sometimes it takes a long time to repeat:

$$\begin{array}{r} 0.\overline{571428} \\ 7 \overline{)4.000000} \\ \underline{3.5} \\ 50 \\ \underline{49} \\ 10 \\ \underline{7} \\ 30 \\ \underline{28} \\ 20 \\ \underline{14} \\ 60 \\ \underline{56} \\ 4 \end{array}$$

Example: Recurring Decimal Notation

There are actually two ways to represent recurring decimals, the bar over the repeating part or a dot above the first and last digit of the repeating part, so for example, the recurring decimal

$$0.3456456456456456 \dots$$

Can be represented either as

$$0.3\overline{456}$$

or as

$$0.3\dot{4}5\dot{6}$$

Test Your Understanding

Write out what each of these recurring decimals looks like:

1) $0.\dot{3}$

2) $0.4\overline{3}$

3) $0.\dot{4}\dot{3}$

4) $0.1\overline{23}$

5) $0.\dot{1}2\dot{3}$

6) $0.12\overline{3}$

7) $0.4\dot{3}0\dot{3}$

Answers

1) $0.\dot{3} = 0.333 \dots$

2) $0.4\overline{3} = 0.4333 \dots$

3) $0.4\dot{3} = 0.434343 \dots$

4) $0.12\overline{3} = 0.1232323 \dots$

5) $0.\dot{1}2\dot{3} = 0.123123123 \dots$

6) $0.12\overline{3} = 0.12333 \dots$

7) $0.4\dot{3}0\dot{3} = 0.4303303303 \dots$

Exercise 1

Please complete the worksheet.

Example: Recurring Decimals to Fractions

To convert $0.\overline{54}$ to a fraction, we can use algebra. We let x equal the recurring decimal, then work out the value of x as fraction.

$$x = 0.545454 \dots$$

Write out the recurring decimal

$$100x = 54.545454 \dots$$

$\times 100$ because two recurring digits

$$99x = 54$$

Subtract the first line from the second

$$x = \frac{54}{99} = \frac{6}{11}$$

Divide and simplify

The trick here is that the recurring decimal parts cancel out when they are subtracted.

$$x = 0.133333 \dots$$

$$10x = 1.333333 \dots$$

$$9x = 1.2$$

$$x = \frac{1.2}{9} = \frac{12}{90} = \frac{2}{15}$$

Write out the recurring decimal

$\times 10$ because one recurring digit

Subtract the first line from the second

Divide and simplify

i Note

Note here that when we subtract the first digit after the decimal point does not match, so we are effectively doing $1.3 - 0.1$. The decimal digits after this *do* cancel because they are identical.

$$\begin{aligned}x &= 3.0868686 \dots \\100x &= 308.6868686 \dots \\99x &= 305.6 \\x &= \frac{305.6}{99} = \frac{3056}{990} = \frac{1528}{495}\end{aligned}$$

i Note

The calculation this time is $308.6 - 3$.

$$x = 0.5401401401 \dots$$

$$1000x = 540.1401401401 \dots$$

$$999x = 539.6$$

$$x = \frac{539.6}{999} = \frac{5396}{9990} = \frac{2698}{4995}$$



Warning

Be careful here. 5 is greater than 1 so the calculation we need to do is $540.1 - 0.5$.

Exercise 2

Please complete the worksheet.