Linear Equations

Inverse Pairs

For each mathematical operation, there is an inverse operation.

$$\times \longleftrightarrow \div$$

$$+ \longleftrightarrow -$$

To solve equations, we use inverse pairs to isolate the variable x for which we are trying to solve.

Example: One-Step Equations

To solve one-step equations, we simply use the relevant inverse pair:

1)

$$x + 3 = 2 \tag{1}$$
$$x + 3 - 3 = 2 - 3 \tag{2}$$
$$x = -1 \tag{3}$$

2)

$$y - 5 = 7 \tag{4}$$
$$y - 5 + 5 = 7 + 5 \tag{5}$$
$$y = 12 \tag{6}$$

3)

$$3u = 12 \tag{7}$$
$$u \times 3 \div 3 = 12 \div 3 \tag{8}$$
$$u = 4 \tag{9}$$

4)

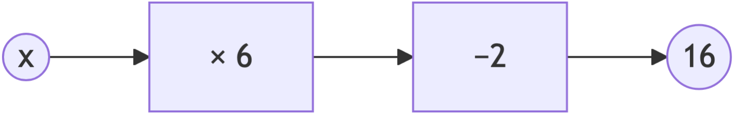
$$v \div 2 = 8 \tag{10}$$
$$v \div 2 \times 2 = 8 \times 2 \tag{11}$$
$$v = 16 \tag{12}$$

Example: Two-Step Equations

When there is more than one step, it's important to think about the **order of operations**. Drawing a diagram can help. For the equation

$$6x - 2 = 16$$

we can draw the following diagram:

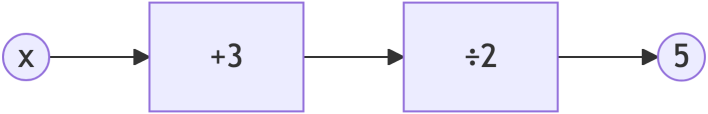


We need to work backwards using inverse pairs, so we first add 2 then divide by 6:

$$6x - 2 + 2 = 16 + 2 \tag{13}$$
$$6x = 18 \tag{14}$$
$$x \times 6 \div 6 = 18 \div 6 \tag{15}$$

$$\frac{x+3}{2} = 5$$

The diagram for this equation is



We need to work backwards using inverse pairs,

$$\frac{x+3}{2} = 5 \tag{17}$$

$$(x+3) \div 2 \times 2 = 5 \times 2 \tag{18}$$

$$x + 3 = 10 \tag{19}$$

$$x + 3 - 3 = 10 - 3 \tag{20}$$

$$x = 7 \tag{21}$$

Test Your Understanding

Solve these equations:

1) $5x - 8 = 27\$$
2) $\frac{x}{3} + 4 = 9$
3) $\frac{x-3}{4} = 1$
4) $3 - 2x = 7$

Answers

1) $5x - 8 = 27 \Rightarrow x = 7$
2) $\frac{x}{3} + 4 = 9 \Rightarrow x = 15$
3) $\frac{x-3}{4} = 1 \Rightarrow x = 7$
4) $3 - 2x = 7 \Rightarrow x = -2$

Exercise 1

First question: $3a - 2 = 19$

Example: Equations with Brackets

There are often two ways to solve these kinds of questions:

$$3(x+7) = 15 \tag{22}$$
$$3x + 21 = 15 \tag{23}$$
$$3x = -6 \tag{24}$$
$$x = -2 \tag{25}$$

$$3(x+7) = 15 \tag{26}$$
$$x + 7 = 5 \tag{27}$$
$$x = -2 \tag{28}$$

As you can see, expanding the brackets is not always the quickest method.

But sometimes if there is more than one bracket you need to expand the brackets and simplify first:

$$
\begin{aligned}
5(x+2)-3(x-4) &= 28 \qquad (29)\\
5x+10-3x+12 &= 28 \qquad (30)\\
2x+22 &= 28 \qquad (31)\\
2x &= 6 \qquad (32)\\
x &= 3 \qquad (33)
\end{aligned}
$$

Test Your Understanding

Solve these equations:

1) $6(x-3)+5=29$
2) $8(x+2)-3(2x-1)=39$

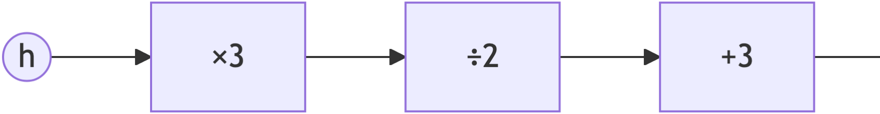
Answers

1) $6(x-3)+5=29 \Rightarrow x=7$
2) $8(x+2)-3(2x-1)=39 \Rightarrow x=10$

Exercise 2

First question: $3(a+5) = 36$

Example: Equations with Fractions



We can use the same method for questions like this

$$\frac{3h}{2} + 3 = 12 \tag{34}$$

$$\frac{3h}{2} = 9 \tag{35}$$

$$h = 6 \tag{36}$$

There are two methods for questions like this:

Subtract 4 first:

$$\frac{x}{3} + 4 = 9 \tag{37}$$

$$\frac{x}{3} = 5 \tag{38}$$

$$x = 15 \tag{39}$$

Multiply by 3 first:

$$\frac{x}{3} + 4 = 9 \tag{40}$$

$$x + 12 = 27 \tag{41}$$

$$x = 15 \tag{42}$$

> **Warning**
>
> When you multiply both sides by 3 you must multiply **every term** by 3.

Test Your Understanding

1) $\dfrac{x}{2} - 8 = 16$

2) $\dfrac{2x}{5} + 6 = 19$

Answers

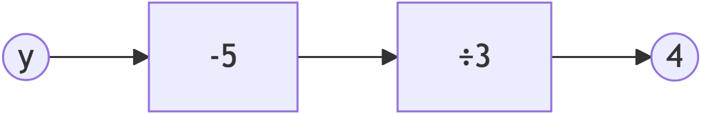
1) $\dfrac{x}{2} - 8 = 16 \Rightarrow x = 48$

2) $\dfrac{2x}{5} + 6 = 19 \Rightarrow x = 32.5$

Example: Multiplying First

$$\frac{y-5}{3} = 4$$

In some questions, you should multiply first because of the order of operations:



$$\frac{y-5}{3} = 4 \tag{43}$$

$$y - 5 = 12 \tag{44}$$

$$y = 17 \tag{45}$$

Example: Fractions on Both Sides

If both sides of the equation are fractions, you can mutiply by the lowest common multiple of the denominators.

$$\frac{3y+2}{3} = \frac{2}{5} \tag{46}$$

$$(3y+2) \div 3 \times 15 = 2 \div 5 \times 15 \tag{47}$$

$$(3y+2) \times 5 = 2 \times 3 \tag{48}$$

$$15y + 10 = 6 \tag{49}$$

$$15y = -4 \tag{50}$$

$$y = -\frac{4}{15} \tag{51}$$

Test Your Understanding

1) $\dfrac{a+2}{3} = 4$

2) $\dfrac{g-4}{10} = 2$

3) $6 = \dfrac{2w+4}{3}$

4) $\dfrac{4r-3}{10} = \dfrac{1}{5}$

Answers

1) $\dfrac{a+2}{3} = 4 \Rightarrow a = 10$

2) $\dfrac{g-4}{10} = 2 \Rightarrow g = 24$

3) $6 = \dfrac{2w+4}{3} \Rightarrow w = 7$

4) $\dfrac{4r-3}{10} = \dfrac{1}{5} \Rightarrow r = \dfrac{5}{4}$

Example: Variable on the Denominator

If the variable is on the denominator, the first step is always to multiply **both sides** of the equation by whatever the denominator is.

$$\frac{7}{z} = 42 \tag{52}$$

$$7 \div z \times z = 42 \times z \tag{53}$$

$$7 = 42z \tag{54}$$

$$7 \div 42 = z \times 42 \div 42 \tag{55}$$

$$\frac{1}{6} = z \tag{56}$$

This same method applies even when the denominator is more complex:

$$\begin{aligned}
\frac{7}{2v+3} &= 2 && (57)\\
7 \div (2v+3) \times (2v+3) &= 2 \times (2v+3) && (58)\\
7 &= 4v+6 && (59)\\
1 &= 4v && (60)\\
\frac{1}{4} &= v && (61)
\end{aligned}$$

Example: Comparing Methods

With equations like

$$\frac{2}{v} + 2 = 9$$

there are two possible methods, but one has a high rate of error.

$$\frac{2}{v} + 2 = 9 \tag{62}$$
$$\frac{2}{v} = 7 \tag{63}$$
$$2 = 7v \tag{64}$$
$$\frac{2}{7} = v \tag{65}$$

$$\frac{2}{v} + 2 = 9 \tag{66}$$
$$2 + 2v = 9v \tag{67}$$
$$2 = 7v \tag{68}$$
$$\frac{2}{7} = v \tag{69}$$

Warning

A very common error in the second method is to forget to multiply the 2 by v. Remember, when you multiply both sides of an equation, you must multiply **every term**. Terms are expressions separated by a plus or minus

Test Your Understanding

1) $3 = \dfrac{15}{y}$

2) $\dfrac{8}{3p} = 12$

3) $1 = \dfrac{5}{y+2}$

4) $\dfrac{3}{3a-4} = 5$

Answers

1) $3 = \dfrac{15}{y} \Rightarrow y = 5$

2) $\dfrac{8}{3p} = 12 \Rightarrow p = \dfrac{2}{9}$

3) $1 = \dfrac{5}{y+2} \Rightarrow y = 3$

4) $\dfrac{3}{3a-4} = 5 \Rightarrow a = \dfrac{23}{15}$

Exercise 3

First question: $\dfrac{c}{2} = 3$

Example: Unknown on Both Sides

When the unknown appears on both sides, the goal is to combine those terms together so that the unknown appears only once. Then we can solve it in the same way as the previous examples. To do this, we can add or subtract one of the terms to both sides.

$$11 - 2x = 6x - 13 \tag{70}$$

$$11 - 2x + 2x = 6x - 13 + 2x \tag{71}$$

$$11 = 8x - 13 \tag{72}$$

$$24 = 8x \tag{73}$$

$$3 = x \tag{74}$$

If there are brackets, we can expand those first:

$$
\begin{aligned}
2(x-4) &= 6x-28 && (75)\\
2x-8 &= 6x-28 && (76)\\
-8 &= 4x-28 && (77)\\
20 &= 4x && (78)\\
5 &= x && (79)
\end{aligned}
$$

Test Your Understanding

Solve these equations:

1) $8 - 3x = 4x + 22$
2) $3(x+5) = 10x + 1$
3) $4(x+5) = 3(3x-5)$

Answers

1) $8 - 3x = 4x + 22 \Rightarrow x = -2$
2) $3(x + 5) = 10x + 1 \Rightarrow x = 2$
3) $4(x + 5) = 3(3x - 5) \Rightarrow x = 7$

Exercise 4

Start with question 3: $7x - 2 = 4x + 13$