0.1 Electrostatics

0.1.1 Electric Field $\vec{E}$

The electric field is a vector field that describes the electrical force experienced by a unit positive charge at any point in space.

- **Definition**: The electric field $\vec{E}$ at a point is defined as the force $\vec{F}$ per unit positive charge q_0 placed at that point.

$$\vec{E} = \frac{\vec{F}}{q_0}$$

0.1.2 Electric Field Due to a Point Charge

For a point charge q, the electric field at a distance r is given by:

$$\vec{E} = k\frac{q}{r^2}\hat{r}$$

where:

- k is Coulomb's constant ($k = \frac{1}{4\pi\epsilon_0}$)
- ϵ_0 is the permittivity of free space
- $\hat{r}$ is the unit vector pointing radially outward from the charge.

0.1.3 Electric Field Due to a System of Point Charges

For a system of N point charges $q_1, q_2, ..., q_N$ located at positions $\vec{r}_1, \vec{r}_2, ..., \vec{r}_N$, the electric field at a point $\vec{r}$ is the vector sum of the electric fields due to each individual charge (superposition principle).

$$\vec{E} = \sum_{i=1}^{N} k\frac{q_i}{|\ \vec{r} - \vec{r}_i|^2}\frac{\vec{r} - \vec{r}_i}{|\ \vec{r} - \vec{r}_i|}$$

0.1.4 Electric Field Lines

Electric field lines are imaginary lines that represent the direction of the electric field.

- They originate from positive charges and terminate on negative charges.
- The density of the lines indicates the strength of the electric field.
- Field lines never cross each other.

[DIAGRAM: Sketch of electric field lines originating from a positive point charge and terminating on a negative point charge, showing outward radial lines from the positive charge and inward radial lines to the negative charge.]

0.1.5 Electric Dipole

An electric dipole consists of two equal and opposite charges separated by a small distance.

- The dipole moment $\vec{p}$ is defined as:

$$\vec{p} = q\vec{d}$$

  where q is the magnitude of the charge and $\vec{d}$ is the vector pointing from the negative to the positive charge.
- The electric field of a dipole falls off as $1/r^3$ at large distances.

0.1.6 Gauss's Law

Gauss's law relates the electric flux through a closed surface to the enclosed charge.

- **Statement**: The total electric flux Φ_E through any closed surface is proportional to the total electric charge Q_{enc} enclosed within that surface.

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$

Gauss's law is particularly useful for calculating the electric field of charge distributions with high symmetry (spherical, cylindrical, planar).

0.1.6.1 Application: Electric Field of a Uniformly Charged Sphere

For a uniformly charged sphere of radius R and total charge Q:

- **Outside the sphere ($r > R$)**: The electric field is the same as that of a point charge Q located at the center.

$$\vec{E} = k\frac{Q}{r^2}\hat{r}$$

- **Inside the sphere ($r < R$)**: The electric field depends on the charge enclosed within a radius r.

$$\vec{E} = k\frac{Qr}{R^3}\hat{r}$$

[DIAGRAM: Cross-section of a uniformly charged sphere, showing a Gaussian surface inside and outside.]