Ques :- A hydraulic press exerts a load of 3.5 MN. The load is supported by two steel columns is the shaper stress is 85MPa. modulus of elasticity is 20 $\frac{kN}{mm^2}$. find the diameter of column and change in length, if the ~~column~~ column ~~and~~ length is 2.5m.

solution :-

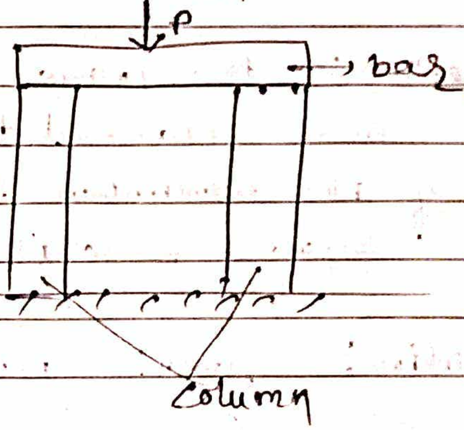
given : load (P) = $3.5 MN = 3.5 \times 10^6 N$

stress (σ) = 85 MPa

$$= 85 \times 10^6 \, N/m^2 = 85 \times 10^6 \frac{N}{10^6 mm^2} = 85 \frac{N}{mm^2}$$

E = modulus of elasticity $= 20 \frac{kN}{mm^2} = 20 \times 10^3 \frac{N}{mm^2}$

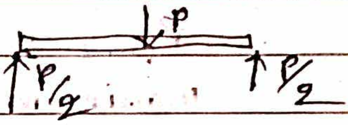
L = 2.5 m



Now, $\text{Com. Stress} = \frac{\text{Load}}{\text{Area}}$

$$\text{Com. stress on column} = \frac{P/2}{\frac{\pi}{4} D^2}$$

$$85 \frac{N}{mm^2} = \frac{3.5 \times 10^6 \, N}{2 \times \frac{\pi}{4/2} D^2}$$



$$D^2 = \frac{3.5 \times 10^6 \times 2}{85} \, mm^2$$

$$D = \sqrt{\frac{7.0}{85} \times 10^6} = 0.28697 \times 10^3 \, mm$$

$$= 286.97 \, mm$$

Let δl be change in length

Also,

$$\delta l = \frac{PL}{AE} \quad \text{or} \quad \sigma = E\varepsilon$$

$$\sigma = E \times \frac{\delta L}{l}$$

$$\delta l = \frac{\sigma l}{E}$$

$$\delta l = \frac{\sigma L}{E} = \frac{85 \frac{N}{mm^2} \times 10^6 \frac{N}{m^2} \times 2.5\,m}{20 \times 10^3 \frac{N}{mm^2}}$$

$$= \frac{85 \times 10^6 \times 2.5 \;\; \frac{N}{m^2} \times m}{20 \times 10^3 \times 10^6 \frac{N}{m^2}}$$

$$= \frac{85 \times 2.5 \times 10^{-3}}{20}$$

$$= 10.625 \times 10^{-3}\, m = 10.625\, mm$$

$$= 0.010625\, m$$

Question :- A square bar of cross-section $20 \times 20\, mm^2$. The square base attached to 6 bolts. Calculate the diameter of bolt if the maximum stress in the bolt is $75 \frac{N}{mm^2}$, and that in square base is $150 \frac{N}{mm^2}$.

Solution :- given:

Area of cross-section of square $= 20\,mm \times 20\,mm = 400\, mm^2$

P_{load} = Load on square = Load on 6 bolts

Maximum stress on bolts $= 75 N/mm^2$

Maximum stress on square base $= 150 \frac{N}{mm^2}$

Now,

$$\text{Stress on square} = \frac{P_{load}}{\text{Area of square}}$$

$$150 \frac{N}{mm^2} = \frac{P_{load}}{400\, mm^2}$$

$$P_{Load} = 150 \times 400\, N$$

Again,

$$\text{Stress on Bolts} = \frac{P_{load}}{\text{Area of bolts}}$$

$$7.5\frac{N}{mm^2} = \frac{150 \times 400\ N}{\frac{1}{6}\times\frac{\pi}{4}D^2}$$

Area of bolts $= \frac{\pi}{4}D^2$

Area of 6 bolts $= \frac{\pi}{4}D^2$

1 bolts $= \frac{1}{6}\times\frac{\pi}{4}D^2$

$$D^2 = \frac{\overset{2}{\cancel{150}} \times 400 \times 4 \times 6}{3.14 \times \cancel{75}}$$

$$D^2 = \frac{2\times4\times6\times400}{3.14} = \frac{19200}{3.14} = 6114.649$$

$$D = \sqrt{\frac{19200}{3.14}} = 78.196\ mm$$

Ques:- The diameter of a piston in steam engine is 300 mm and maximum pressure acting is $0.7\frac{N}{mm^2}$, maximum compression strength is $40\frac{N}{mm^2}$. find the diameter of connecting rod?

Solution:- Given: D = piston diameter = 300 mm

stress in piston $\sigma_P = 0.7\ N/mm^2$

stress on connecting rod $\sigma_R = 40\frac{N}{mm^2}$

d = diameter of connecting rod = ?

$$\sigma_P = \frac{P_{load}}{\frac{\pi}{4}D^2}$$

$$P_{Load} = \sigma_P \times \frac{\pi}{4}D^2$$

$$= 0.7 \times \frac{3.14}{4} \times (300)^2 = 49455\ N$$

$$\sigma_R = \frac{P_{load}}{\frac{\pi}{4}d^2}$$

$$d^2 = \frac{P_{Load}}{\frac{\pi}{4}\times\sigma_R} = \frac{0.7 \times \cancel{\frac{3.14}{4}} \times (300)^2}{\cancel{\frac{3.14}{4}} \times 40} = \frac{0.7}{40}\times(300)^2$$

$$d = \sqrt{\frac{0.7\times(300)^2}{40}} = 0.1322875 \times 300$$

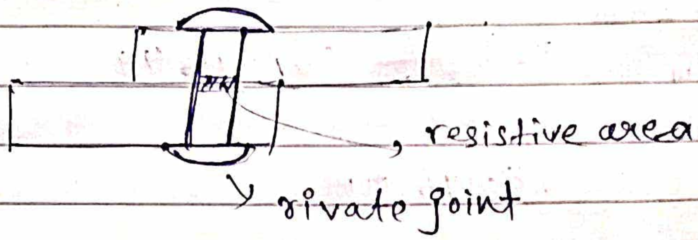
39.6862 mm (Ans)

Date: 16/08/24

* Shear stress :- It is the ratio of tangential force to the resistive area. It denoted by τ (tau).

$$\tau = \frac{\text{tangential force}}{\text{Resisting area}}$$

*



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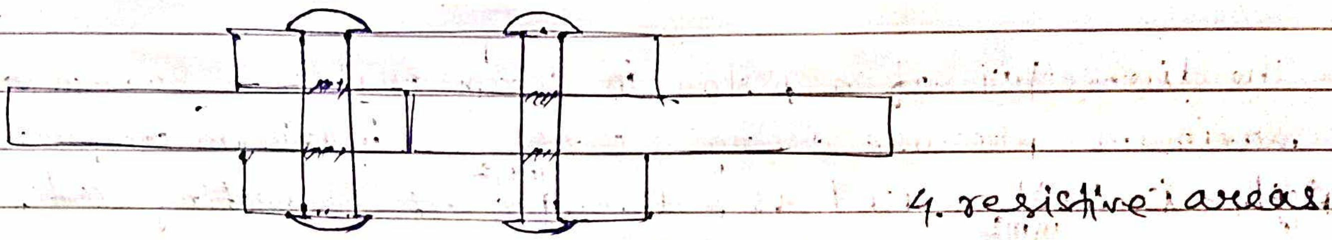

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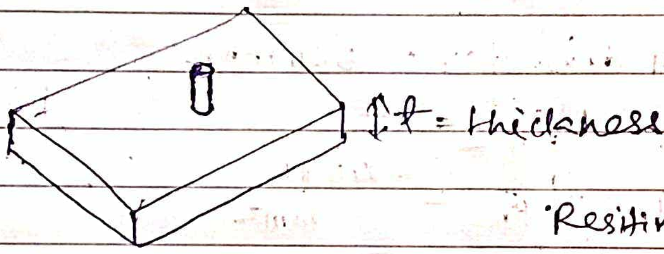
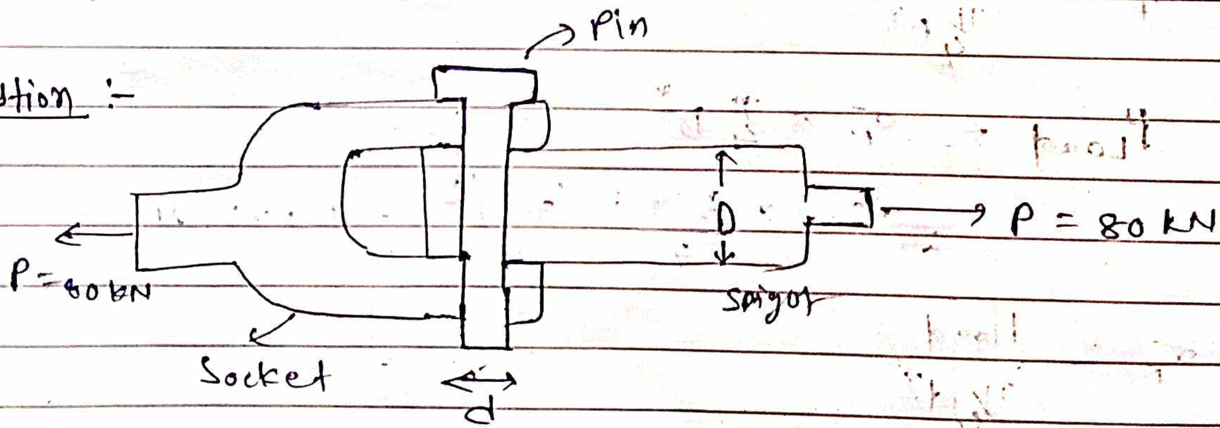


Resistive area $= 2\pi r t$

$= \pi D t$

Question :-



For a given socket and cotter joint find out diameter of the free end of the rod and diameter of the cotter pin when it carrying 80 kN load. If the allowable tensile stress of the rod is 100 N/mm^2, allowable stress of the pin is 80 N/mm^2.

Solution :- Given: tensile stress $(\sigma_t) = 100 \frac{N}{mm^2}$

Shear stress $(\tau) = 80 N/mm^2$

Load applied $(P) = 80\,kN = 80 \times 10^3 N$

No. of Resistive area of pin = 2

Area of pin $= \frac{\pi}{4} d^2 \times 2$, d is diameter of pin

Area of cotter/spigot $= \frac{\pi}{4} D^2$, D = diameter of spigot/cotter

$$\text{Tensile stress } (\sigma_t) = \frac{P}{\text{Area of Cotter}}$$

$$\frac{100 N}{mm^2} = \frac{80 \times 10^3 N}{\frac{\pi}{4} D^2}$$

$$D^2 = \frac{80 \times 4 \times 10^3}{\pi \times 100} mm^2$$

$$D^2 = \frac{3200}{3.14} mm^2$$

$$D^2 = 1019.10828\, mm^2$$

$$\boxed{D = \sqrt{1019.10828\, mm^2} = 31.9234\, mm}$$

$$\tau = \text{shear stress} = \frac{P}{\text{Resistive Area of Pin}} = \frac{80 \times 10^3 N}{2 \times \frac{\pi}{4} d^2}$$

$$\frac{80 N}{mm^2} = \frac{80 \times 10^3 N}{2 \times \frac{\pi}{4} d^2}$$

$$d^2 = \frac{2 \times 10^3}{3.14} mm^2 \qquad d = \sqrt{\frac{2 \times 10^3}{3.14}} mm$$

$$\boxed{d = 25.23\, mm}$$

Ques :- find the size of hole that can be produced if thickness 20 mm, having ultimate tensile stress is 300 N/mm^2, maximum permissiable compressive stress on the punch material is 1200 N/mm^2.

Solution :- thickness $t = 20$ mm

Ultimate tensile stress $\sigma_t = 300\ N/mm^2$

maximum compressive stress $\sigma_c = 1200\ N/mm^2$

Let the diameter of punch be D.

Now,

$$\text{tensile stress} = \frac{P_{allowable}}{\pi D t}$$

$$P_{allowable} = \text{tensile stress} \times \pi D t$$

$$= 300 \times 3.14 \times D \times 20 \quad \text{--- (I)}$$

$$\text{compressive stress} = \frac{P_{allowable}}{\frac{\pi}{4} D^2}$$

$$P_{allowable} = \text{Compressive stress} \times \frac{\pi}{4} D^2$$

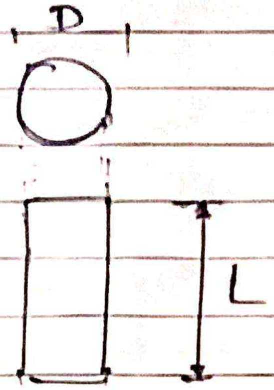
$$= 1200 \times \frac{3.14}{4} \times D^2 \quad \text{--- (II)}$$

Equating eq^n (I) and eq^n (II), we get

$$\cancel{300} \times \cancel{3.14} \times \cancel{D} \times 20 = \cancel{1200}^{4} \times \frac{\cancel{3.14}}{\cancel{4}} \times D^{\cancel{2}}$$

$$\boxed{20\ MM = D}$$

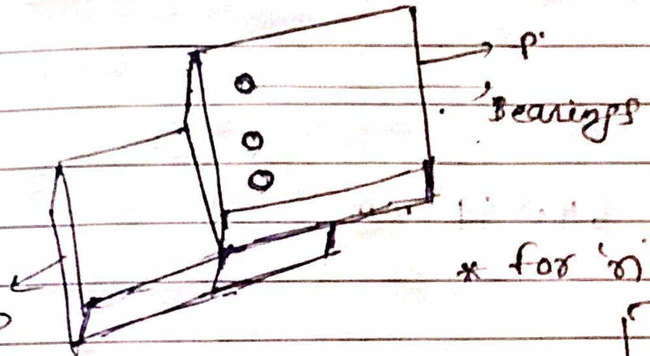
* Bearing Stress (Localized compressive stress) $= \dfrac{\text{Load}}{\text{projected Area}}$



$$= \frac{\text{Load}}{L \times D}$$

where, D = diameter of bearing
L = length of Bearing

ex:-



* for Single Bearing

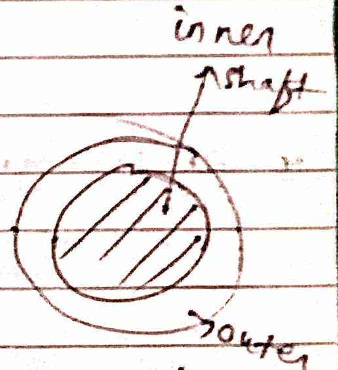
$$\sigma = \frac{P}{L \times D}$$

* for 'n' no. of bearing

$$\boxed{\sigma = \frac{P}{(L \times D)\, n}}$$

$\sigma \downarrow$ when $n \uparrow$

* Journal Bearing - for high load Carring Capacity



Question:-

Crank pin of an IC engine sustains maximum load of 35 kN. If the allowable bearing pressure is $7 \frac{N}{mm^2}$. Find the dimension of pin whoose $\frac{L}{D}$ is 1.2 ?

Solution:- given; $P = 35\ kN = 35 \times 10^3 N$

$$\sigma = 7\ N/mm^2 = 7 \times 10^6 \frac{N}{m^2}$$

$$L/D = 1.2$$

$$L = 1.2 D$$

$$\sigma_{Bearing} = \frac{load}{Projected\ Area}$$

$$7 \times \frac{N}{mm^2} = \frac{35 \times 10^3\ N}{L \times D}$$

$$7/mm^2 = \frac{35 \times 10^3}{1.2 D \times D}$$

$$7/mm^2 = \frac{35 \times 10^3}{1.2\ D^2}$$

$$D^2 = \frac{\overset{5}{\cancel{35}} \times 10^3}{1.2 \times \cancel{7}}\ mm^2$$

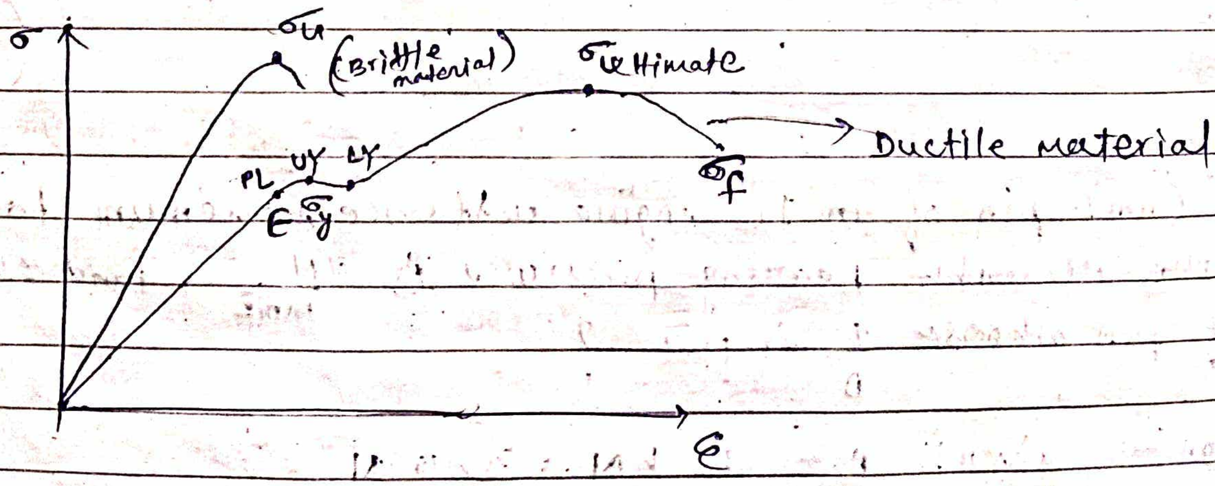
$$D^2 = \frac{5}{12} \times 10^4\ mm^2$$

$$D = \sqrt{\frac{5}{12} \times 10^4\ mm^2} = 64.549\ mm$$

$$\therefore \boxed{D = 64.55\ mm}$$

and $L = 1.2 D = 1.2 \times 64.55\ mm = 77.46\ mm$

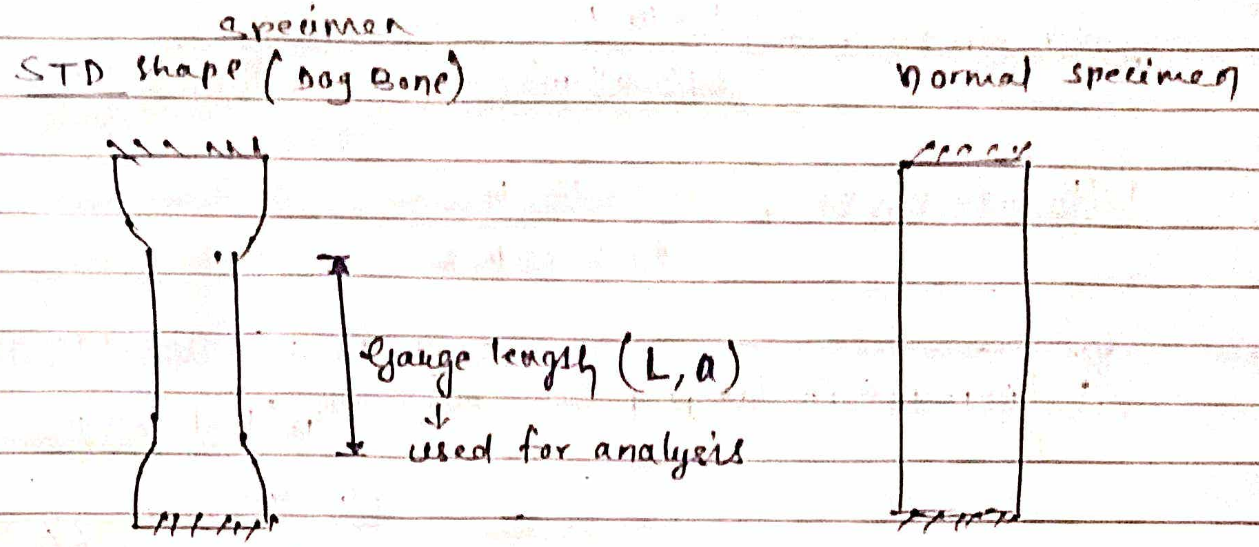
* Stress-Strain Curve :-



⇒ Limiting Load = σ_y (Ductile)
σ_u (Brittle)

Date

Tensile test :-



$$\sigma_Y = \frac{\text{Yield load}}{\text{Original Area}} \qquad \sigma_u = \frac{\text{Ultimate load}}{\text{Original Area}} \qquad \sigma_f = \frac{\text{fracture load}}{\text{Original Area}}$$

$$\%\ \text{increase in length} = \frac{\text{change in length}}{\text{original length}} \times 100 = \frac{\text{finale length} - \text{Initial length}}{\text{Initial length}} \times 100$$

$$\%\ \text{Reduction in Area} = \frac{\text{Change in area}}{\text{Original Area}} = \frac{\text{Original Area} - \text{Finale area}}{\text{Original Area}} \times 100$$

ues:- A mild steel of diameter 12mm, gauge length 60mm is subjected to tensile load, finale length 80mm, finale diameter is 7 mm, yield load 3.5kN, ultimate load 6.1kN, find the yield stress, ultimate stress, % increase in length and % reduction in length.

solution:- given: $d_i = 12\,mm$ $\quad P_Y = 3.5\,kN$

$L_i = 60\,mm$ $\quad P_u = 6.1\,kN$

$L_f = 80\,mm$

$d_f = 7\,mm$

$$a_i = \frac{\pi}{4} d_i^2 = \frac{\pi}{4} \times (12)^2 = 113.09\ mm^2 \qquad a_f = \frac{\pi}{4}(d_f)^2 = \frac{\pi}{4}(7)^2 = 38.48\ mm^2$$

$$\text{yield stress} = \frac{3.5 \times 10^3 N}{113.09 \, m^2}$$

$$\text{Ultimate stress} = \frac{6.1 \times 10^3 N}{113.09 \, m^2}$$

$$\%\ \text{increase in length} = \frac{\text{finale length} - \text{initial length}}{\text{initial length}}$$

$$= \frac{80 - 60}{60} = \frac{20}{60} = \frac{1}{3} \times 100$$

$$= 33.33\%$$

$$\%\ \text{Reduction of area} = \frac{\text{Initial area} - \text{finale area}}{\text{Initial area}} \times 100$$

$$= \frac{113.09 - 38.48}{113.09} \times 100$$

$$= \frac{74.61}{113.09} \times 100$$

$$= 0.6597 \times 100$$

$$= 65.97\%$$

* factor of safety :- $FOS = \frac{\text{Maximum stress}}{\text{Allowable stress}}$

$$= \frac{\sigma_u}{\sigma_a}$$

FOS > 1

		Steady	ive	shock
Brittle	CI	5	8	15
Ductile	MS	3	4	8
Elastic	Rubber	10	12	15

Ques: A bar is 3 m long is made of two material copper and steel. Young's modulus of copper is $105 \frac{GN}{m^2}$ and steel is 210. Area of cross section is 25 mm × 12.5 mm. The composite is acted a load of 50 kN; find the stress induced in steel and copper.

Solution:- Given: $L = 3m$

$$E_{cu} = 105 \frac{GN}{m^2} = 105 \times 10^9 \frac{N}{m^2} = \frac{105 \times 10^9}{10^6} \frac{N}{mm^2} = 105 \times$$

$$E_{st} = 210 \frac{GN}{m^2} = 210 \times 10^9 \frac{N}{m^2} = 210 \times 10^3 \frac{N}{mm^2}$$

$$P = 50 kN = 50 \times 10^3 N$$

$$C/s = 312.5\ mm^2 \qquad A_{cu} = 156.25\ mm^2$$

$$A_{st} = 156.25\ mm^2$$

$$\sigma_{cu} = \left(\frac{\sigma l}{E}\right)_{st} \left(\frac{E}{l}\right)_{cu}$$

$$P = P_{cu} + P_{st} \qquad A_{cu} = A_{st}$$

$$P = \left(\frac{\sigma L}{E}\right)_{st} \left(\frac{E}{l}\right)_{cu} A_{cu} + \sigma_{st} A_{st}$$

$$50 \times 10^3 = \frac{\sigma_{cu} \cancel{l_{st}}}{E_{st}} \times \frac{E_{cu}}{\cancel{l_{cu}}} \times A_{cu} + \sigma_{st} A_{st}$$

$$50 \times 10^3 = \frac{105 \times 10^3\, \sigma_{st} A_{cu}}{210 \times 10^3} + \sigma_{st} A_{st}$$

$$50 \times 10^3 = 0.5\, \sigma_{st} A_{cu} + \sigma_{st} A_{st}$$

$$50000 = \sigma_{st} \left(0.5 A_{cu} + A_{st}\right)$$

$$\sigma_{steel} \quad \frac{50\times10^3 - 0.5\sigma_{st} A_{cu}}{A_{steel}}$$

$$= 50\times10^3 - 0.5x$$

$A_{st} + A_{cu} = 312.5$

$A_{st} = 312.5$

$$50000 = (0.5 A_{cu} + A_{st})\,\sigma_{st}$$

$$\sigma_{st} = \frac{50000}{0.5 A_{cu} + A_{st}} = \frac{50000}{0.5\times156.25+156.25}$$

$$A_{st} + A_{cu} = 312. \qquad = \frac{50000}{78.125+156.25}$$

$$= \frac{50000}{234.375}$$

$$\sigma_{st} = 213.33\ \frac{N}{mm^2}$$

$$106.6\ \frac{N}{mm^2}$$

$$50000 = (0.5 A_{cu} + A_{st})\,\sigma_{st}$$

$$= (0.5 A_{cu} + 156.25)\,\sigma_{st}$$

$$= (0.5\times156.25 + 156.25)\,\sigma_{st}$$

$$50000 = (78.125 + 156.25)\,\sigma_{st}$$

$$50000 = (234.375)\,\sigma_{st}$$

$$\sigma_{st} = \frac{50000}{234.375} = 213.33\ \frac{N}{mm^2}$$

$$\sigma_{cu} = 0.5\times213.33 = 106.665\ \frac{N}{mm^2}$$

$A_{cu} + A_{st} = 312.5$

$2A_{st} = 312.5$

$A_{st} = 156.25$

$A_{cu} = 156.25$

$P_{st} = \sigma_{st} \cdot A_{st}$

$= 213.33 \times 156.25$

$= 33332.8125$ N

$= 33.332$ kN

$P_{cu} = \sigma_{cu} A_{cu}$

$= 106.665 \times 156.25$

$= 16666.40$ N

$= 16.66$ kN

(μ)

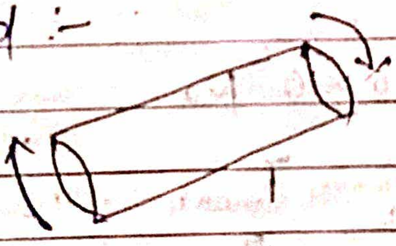
Poisson's Ratio :- It is Ratio of lateral strain to linear strain

$\boxed{\mu << 1}$

$\mu = \dfrac{\text{lateral strain / transverse strain}}{\text{linear strain / axial strain}}$

$= \dfrac{\delta b / b}{\delta l / l}$

Torsional Load :-



$\dfrac{T}{J} = \dfrac{\tau}{R} = \dfrac{G\theta}{L}$

T = torsion N.mm

J = polar moment of inertia (mm⁴)

τ = shear stress N/mm²

R = Extreme fibre distance

G = modulus of rigidity

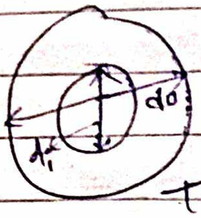
θ = angle of twist

L = length of shaft / bar

GJ = torsional rigidity

* $\frac{\tau}{R} = \frac{T}{J}$

$T = \frac{\tau}{R} \times J = \frac{\tau}{\frac{d}{2}} \times \frac{\pi}{32} d^4 = \tau \times \frac{\pi}{16} d^3$



* $T = \frac{\tau}{R} \frac{\pi}{32} (d_o^4 - d_i^4) = \frac{\tau}{16 d_o} (d_o^4 - d_i^4)$

$R = d/2$

Q. A shaft transmitting 100 kW at 160 rpm, the maximum torque induce is 25% excess of mean. find the diameter of shaft allowable shear is 70 MPa.

Solution:- Power = 100 kW $= 100 \times 10^3$ W

N = 160 rpm

τ = 70 MPa $= 70 \times 10^6$ Pa

T_{max} = 25% excess Maximum T_{mean} $= 0.25 T_{mean}$

$= 1.25 T_{mean}$

$P = \frac{2\pi N T}{60}$

$10^5 = \frac{2 \times 3.14 \times 160\ T_{mean}}{60}$

$T_{mean} = \frac{60 \times 10^5}{2 \times 3.14 \times 160}$

$$T_{mean} = \frac{\ldots}{2\times3.14\times60} = 5968.3\times10^3 \text{ Nmm}$$

$$T = 1.25\, T_{mean} = 1.25\times5968.3$$

$$= 7460.375 \text{ Nm}$$

$$7460.375\times10^3 \text{ N mm}$$

$$T = \tau\times\frac{\pi}{16}d^3$$

$$d^3 = \frac{16T}{\tau\times\pi} = \left(\frac{16\times7460.375\times10^3}{70\times10^6\times3.14}\right)\text{mm}$$

$$\boxed{d = 81.5 \text{ mm}}$$

<u>Question</u>:- Design a circular shaft transmitted 90 kW at 180 rpm the maximum torque is exceding the mean by 40%. Take shear stress is 70 MPa, length of shaft is 2 m and modulus of rigidity 90 GPa. Also find the angle of twist.

<u>Solution</u>: Power = 90 kW = 90×10^3 W

N = 180 rpm

$$\omega = \frac{2\pi N}{60}$$

$$P = \frac{2\pi N T_{mean}}{60} \Rightarrow T_{mean} = \frac{60P}{2\pi N}$$

$$T_{mean} = \frac{\cancel{2\times 3.14} \; 60 \times 90 \times 10^3}{2 \times 3.14 \times 180} \; Nm$$

$$= 4777.07 \; Nm$$

$$T_{max} = \left(1 + \frac{40}{100}\right) \times T_{mean}$$

$$= 1.40 \, T_{mean}$$

$$= 1.40 \times 4777.07$$

$$= 6687.898 \; Nm$$

$$T_{max} = \frac{\tau}{16} \pi d^3$$

$$d^3 = \frac{6687.898 \times 16}{\tau \times \pi} = \left(\frac{6687.898 \times 16}{70 \times 10^6 \times 3.14}\right)$$

$$d = \left(4.868 \times 10^{-4}\right)^{1/3}$$

$$= 0.0786 \; m$$

$$\boxed{d = 78.6 \; mm}$$

$$\left(\frac{T}{J} = \frac{\tau}{R}\right) = \frac{G\theta}{L}$$

$$\tau = \frac{TR}{J}$$

$$\frac{\tau}{R} = \frac{G\theta}{L}$$

$$\theta = \frac{\tau L}{GR} = \frac{2\tau L}{Gd} = \frac{2 \times 70 \times 10^6 \times 2}{90 \times 10^9 \times 0.0786}$$

2.2689 rad

$$= 0.03958 \;$$ Ans

Date 23|04|24

$$\cancel{\frac{0.124344}{180}}$$

$$= \cancel{(6.908 \times 10^{-4})}^{\circ}$$

$$180^{\circ} = \pi$$

$$\frac{180}{\pi} = 1$$

$$\theta = \left(\frac{0.03958}{\pi} \times 180\right)^{\circ}$$

$$= \frac{7.1244}{3.14} = 2.268^{\circ} \text{ (Ans)}$$

Ques:- A Hollow shaft required to transmit 11.2 kW ~~at~~ of power at speed of 300 rpm. the maximum allowable shear stress 80 MPa and the ratio of inner diameter to outer diameter is 3/4.

Solution:- Given: $P = 11.2\text{ kW} = 11.2 \times 10^3\text{ W}$

$N = 300\text{ rpm}$

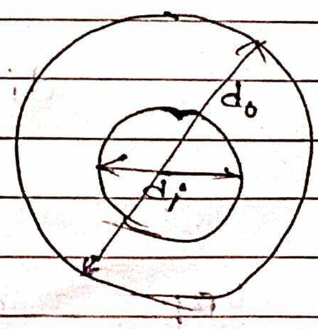
$\tau = 80 \times 10^6\text{ Pa}$

Let d_i and d_o be inner and outer diameter respectively

Then $\frac{d_i}{d_o} = 3/4$

$$d_i = \frac{3}{4} d_o$$

$$4 d_i = 3 d_o$$



$$\frac{\tau}{d_o/2} = \frac{T}{J}$$

$$\frac{\tau}{d_o/2} = \frac{T}{\frac{\pi}{32}(d_o^4 - d_i^4)}$$

16

$$\frac{T}{\frac{\pi}{16}(d_o^4 - d_i^4)} = \frac{\tau}{d_o}$$

$T = \frac{\tau \times \frac{\pi}{16}(d_o^4 - d_i^4)}{d_o}$

$T = \frac{80\times10^6}{} \times \frac{3.14}{16}\left(\frac{d_o^4 - d_i^4}{d_o}\right)$

$= 5 \times 3.14 \times 10^6 \left(1 - \left(\frac{d_i}{d_o}\right)^4\right) \times \frac{d_o^4}{d_o}$ $\frac{d_i}{d_o} = \frac{3}{4}$

$\frac{60\times11.2\times10^3}{2\times3.14\times300} = 5\times3.14\times10^6\left(1 - \left(\frac{3}{4}\right)^4\right) \times d_o^3$

$\frac{672}{1884} = 15.7\times10^3\left(\frac{1}{256}\right) d_o^3$

$\frac{172032}{2157000} = d_o^3$ $T = \frac{2\pi N T}{60}$

$(5.81605\times10^{-3}) = d_o^3$ $T = \frac{60P}{2\pi N}$

$d_o = (5.81605\times10^{-3})^{1/3}$

$= 0.179$

$d_i = \frac{3}{4}\times0.179$

$T = \tau \times \frac{\pi}{16}\left(1 - \left(\frac{d_i}{d_o}\right)^4\right) d_o^3$

$\frac{60P}{2\pi N} = \tau \times \frac{\pi}{16}\left(1 - \left(\frac{3}{4}\right)^4\right) d_o^3$

$\frac{60\times11.2\times10^3}{2\times3.14\times300} = \frac{80\times10^6\times3.14}{16}\left(1 - \left(\frac{3}{4}\right)^4\right) d_o^3$

$\frac{672000}{1884} = \frac{251200000}{16}\left(1 - \frac{81}{256}\right) d_o^3$

$1884 \quad 16 \quad 256$

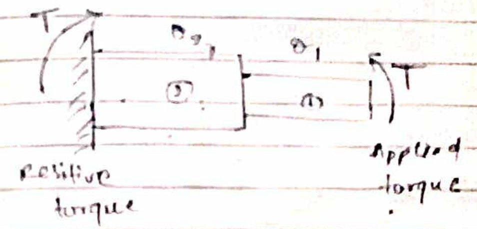
$d_o = \left(\frac{672\times16\times256}{1884\times251200\times175}\right)^{1/3} = \left(\frac{2752512}{82820640\times1000}\right)^{1/3} = \left(\frac{27648}{828206400}\right)^{1/3}$

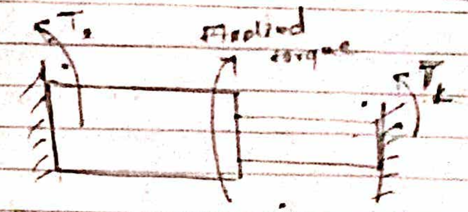
$= (3.338\times10^{-5})^{1/3}$

$\boxed{d_o = 0.0319\,m = 32.19\,mm}$

$d_i = \frac{3}{4}\times32.19 = 24.14\,mm$

* shaft in series & parallel





$\theta = \theta_1 + \theta_2$

shaft in series

$\frac{T}{J} = \frac{G\theta}{L}$

$\theta = \frac{Tl_1}{G_1J_1} + \frac{Tl_2}{G_2J_2}$

$\boxed{\theta = \frac{T}{G}\left(\frac{l_1}{J_1} + \frac{l_2}{J_2}\right)}$

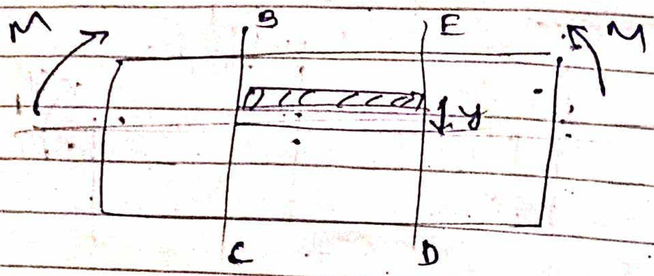
$T = T_1 + T_2$

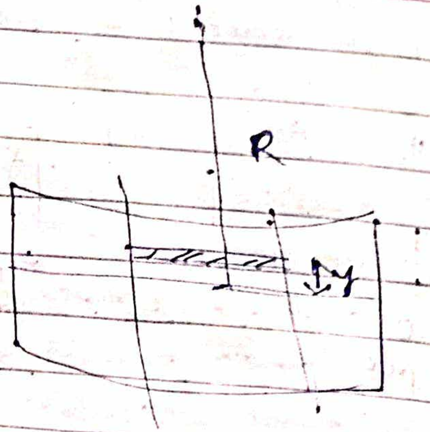
$\theta_1 = \theta_2$

$\frac{T_1 l_1}{G_1 J_1} = \frac{T_2 l_2}{G_2 J_2}$

$\boxed{\frac{T_1}{T_2} = \frac{l_2\times J_1}{l_1\times J_2}}$

* Bending stress in strength





$$\boxed{\frac{\sigma}{Y} = \frac{E}{R} = \frac{M}{I}}$$

σ = tensile stress

Y = distance of fibre (extreme fibre)

E = modulus of elasticity

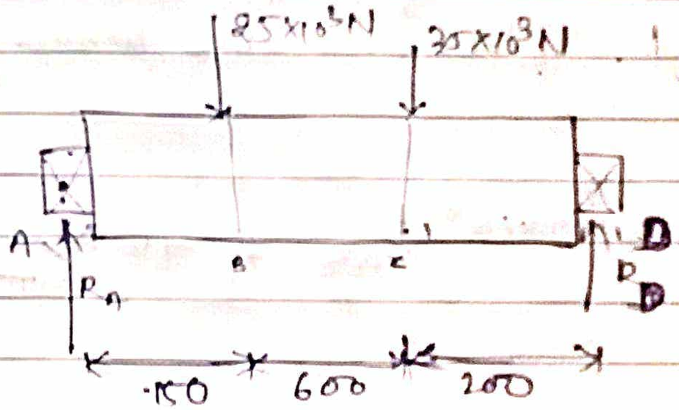
M = Moment (Bending)

R = Radius of curvature

I = Moment of Inertia

Q. A pump lever of shaft exerts a force of 25 kN and 35 kN that act a distance of 150 mm and 200 mm from the left and right hand bearing respectively. Find the diameter of centre portion of lever. If the maximum allowable stress is 100 MPa. Take the length of lever as 950 mm.

solution:-



$$M_{max} = \frac{\pi}{36}\sigma d^3$$

Reactions at A and D

$R_A + R_D = 25+35$

$R_A + R_D = 60$

Moments about R_A

$25\times150 + 35\times750 = 950 R_D$

$25\times15 + 35\times75 = 95 R_D$

$R_D = 31.57\times10^3$ N

$R_A = 28.43\times10^3$ N

(B M) at A $\quad M_A = 0$

BM at B $\quad M_B = 25\times150\times10^3 = 3750\times10^3$ Nm

BM at C $\quad M_C = 35\times10^3\times750 = 26250\times10^3$ Nm

BM at D $\quad M_D = -28.42\times10^3\times950 = -27008.5\times10^3$ Nm

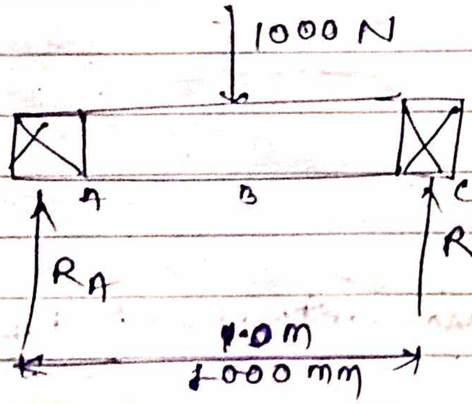
28/08/24

Q A shaft supported by pair of bearing with centre t[illegible] which carries pulley at centre of weight 1kN, find the diameter of the shaft. If the allowable bending stress 40 MPa ?

solution :-

$\sigma_{max} = 40 MPa = 40 \times 10^6 N/m^2 = \boxed{40 \frac{N}{mm^2}}$ $10^3 mm$

$P = 1000 N$



$M_{max} = \frac{\pi}{32} \sigma d^3$

$R_A + R_B = 1000$

$\cancel{0.5 R_B}$

$R_B = 1000 \times 0.5$

$= 100 \times 5$

$\boxed{R_B = 500 N}$

$\boxed{R_A = 500 N}$

BM at A.

$M_A = 0$

BM at B

$M_B = \cancel{500} \times 0.5$

$= 250 Nm$

$= 250 \times 10^3 N$

BM at C $M_C = \cancel{500 \times 1}$

$= 500 \times 10^3$

$= 500 \times 10^3$ N.mm

Now, $M_{max} = \frac{\pi}{32} \sigma d^3$

$$d^3 = \frac{250 \times 32}{3.14 \times 40 \times 10^6}$$

$$d = \left(\frac{250 \times 32}{3.14 \times 40 \times 10^6}\right)^{1/3}$$

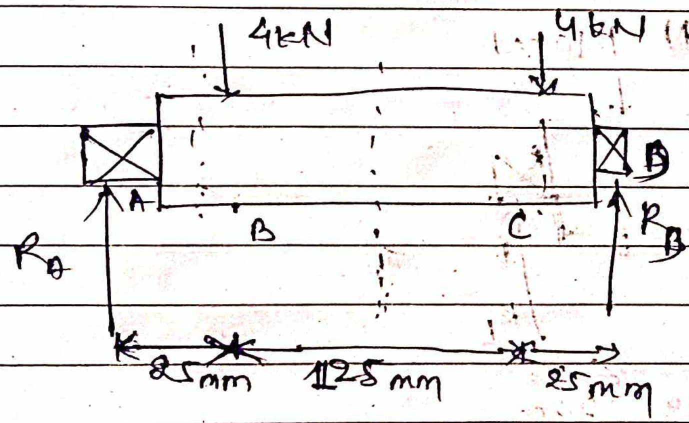
$$d = \left(1.2738 \times 10^{-4}\right)^{1/3}$$

$$d = 0.050316 \text{ m}$$

$$d = 50.3 \text{ mm}$$

(Ans)

Ques :-



$$R_A + R_D = 4 + 4$$

$$R_A + R_D = 8$$

Moment about 'A'

$$4 \times 25 + 4 \times 150 = R_D \times 175$$

$$100 + 600 = 175 R_D$$

$$\boxed{R_D = 4 \text{ kN}}$$

$$\boxed{R_A = 4 \text{ kN}}$$

BM at A

$M_A = 0$

BM at B

$M_B = 4000 \times 25 \times 10^{3}$

$= 100$ N-mm $= 100$ kN-mm

BM at C

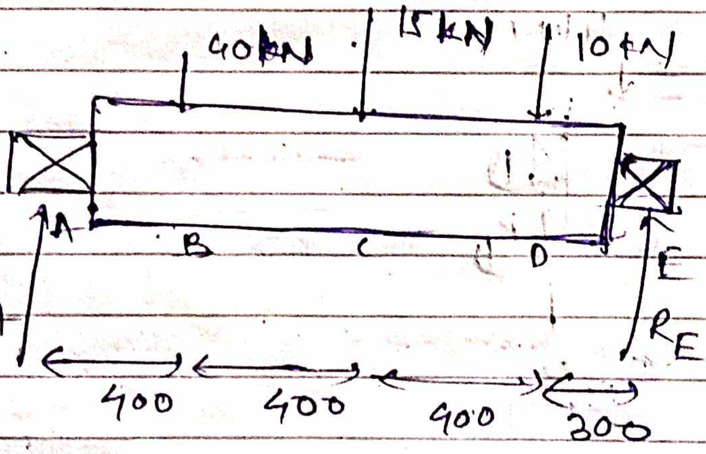
$M_C = 100N = 100$ kN.mm

BM at D

$M_D = 0$

So, No ~~maximum~~ bending moment

Q



$R_A + R_B = 40 + 15 + 10$

$R_A + R_B = 65$ kN

$\Sigma M_A = 0$

$40 \times 400 + 15 \times 800 + 10 \times 1200 = R_E \times 1500$

$16000 + 12000 + 12000 = R_E \times 1500$

$\frac{40000}{1500} = R_E$

$\boxed{R_E = 26.66 \text{ kN}}$

$R_A = 65 - 26.66$

$\boxed{R_A = 38.34 \text{ kN}}$

Now,

BM at A

$M_A = 0$

BM at B

$\cancel{M_B = 40 \times 400}$
$\cancel{= 16000 \text{ kN-mm}}$

$M_B = 38.34 \times 400 = 15336 \text{ kN-mm}$
$= 15.336 \text{ MN-mm}$

BM at C

$M_C = 38.34 \times 800 - 40 \times 400 = 14672 \text{ kN-mm} = 1.4672 \times 10^6 \text{ N-mm}$

~~15336 − 15 × 800 = 3336 kN-mm~~ = 3.336 MN-mm

BM at D

$M_D = \cancel{3336 +}\ 26.666 \times 300 = 7999.8 \text{ kN-mm}$
$= 7.9998 \times 10^6 \text{ N-mm}$

~~15336 − 15 × 800 − 10 × 1900~~

~~= 15336 − 24000~~

~~= −8864 kN-mm~~

Ques:- A cast iron pulley transmitts 10 kW at 400 rpm. The diameter of the pulley is 1.2 m and it has 4 arms of elliptical cross-section in which the major axis is 2 times the minor axis. Find the diamensions of the elliptical cross, if allowable bending stress of the material is 15 MPa.

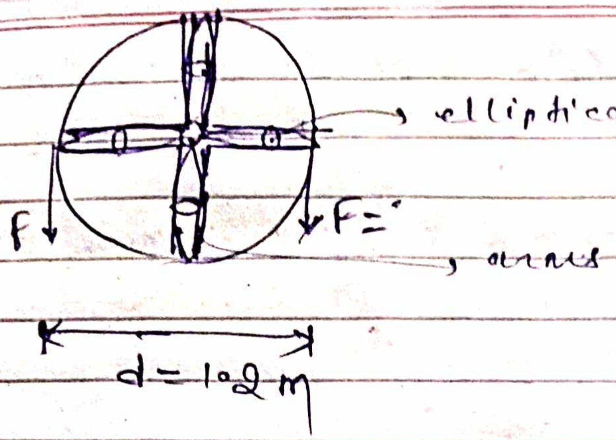
Solution:- $P = 10 \text{ kW}$ $d = 1.2 \text{ m}$

$N = 400 \text{ rpm}$

$\frac{0}{a}$ ⇕ b

a = major axis

b = minor axis



$f/_4$ = force on each arm

σ_b = 15 MPa

$$\frac{M}{I} = \frac{\sigma}{Y}$$

$$* \quad M = \sigma\left(\frac{I}{Y}\right)_{ellipse}$$

$\frac{I}{Y}$ = Z = section modulus

$$= \frac{\pi}{4} a^2 b$$

$$* \quad P = \frac{2\pi N T}{60}$$

$$10 \times 10^3 = \frac{2 \times 3.14 \times 40\cancel{0}}{6\cancel{0}} T$$

$$T = \frac{10^4 \times 6}{2 \times 3.14 \times 40} = 238.8535\text{0 N-m}$$

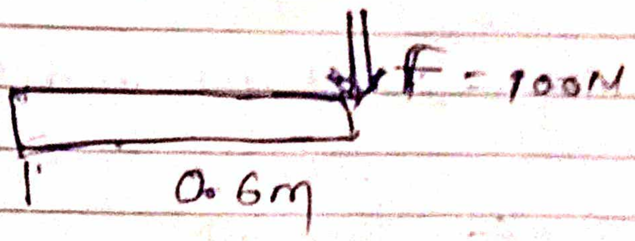
$$= 238.85 \text{ Nm}$$

$$T = F \times \text{distance}$$

$$F = T/_{dis} = \frac{238.85}{0.6}$$

$$= 398.08$$

Force acting in each arm $= \frac{F}{4} = \frac{398.08}{4} = 99.52$



$BM = 0.6 \times 100$

$M = 60\ N.m$

$M = \sigma \frac{I}{y}$

$60 = 15 \times 10^6 \times \frac{\pi}{4} a^2 b$

$60 = 15 \times 10^6 \times \frac{\pi}{4} \times (2b)^2\, b$

but $\boxed{a = 2b}$

$60 = \frac{15 \times 10^6}{\cancel{4}} \times 3.14 \times \cancel{4} \times b^3$

$\overset{4}{\cancel{60}} = \cancel{15} \times 3.14 \times 10^6\, b^3$

$b = \left(\frac{4}{3.14 \times 10^6} \right)^{\frac{1}{3}}$

$b = (1.273 \times 10^{-6})^{1/3}$

$= 0.01083\ m$

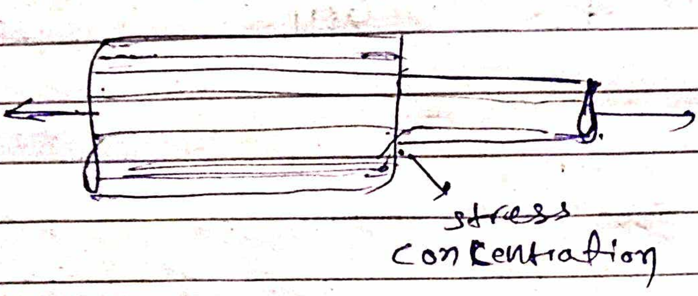
$= 10.83\ mm$

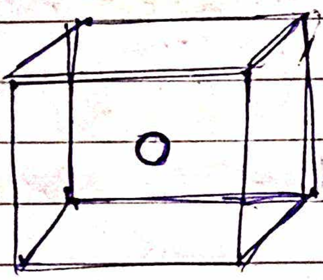
So, $a = 2b = 2 \times 10.83 = 21.675\ mm$

$\frac{2\pi NT}{60}$ $\omega = \frac{2\pi N}{60}$

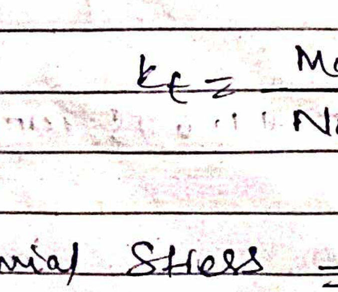
* Stress Concentration factor (K_t) = $\dfrac{\text{maximum stress}}{\text{nominal stress at thenet section}}$

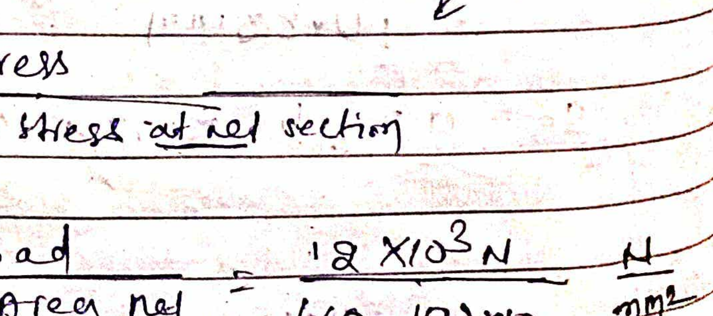
⇒ Stress concentrated where there is variation in geometry.





Ques: find the maximum stress induced in the following cases :-

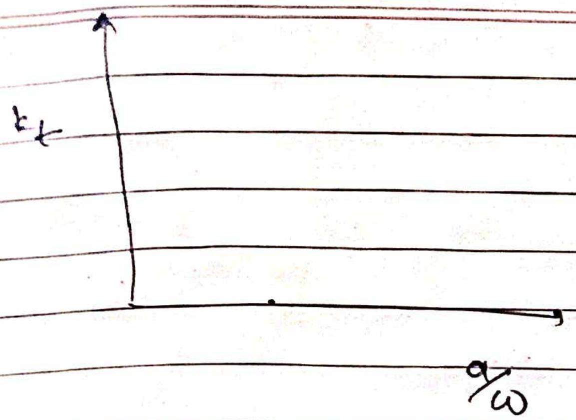




$$k_t = \frac{\text{Max}^m \text{ Stress}}{\text{Nominal stress at net section}}$$

& Nomial Stress $= \dfrac{\text{load}}{\text{Area net}} = \dfrac{12 \times 10^3 N}{(60-12) \times 10} \ \dfrac{N}{mm^2}$

$= 25 \times 10^6 \ Pa$



$$\frac{a}{w} = \frac{12}{60} = 0.2$$

$$k_t = 2.5$$

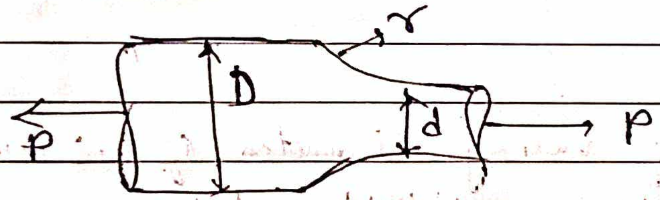
But $k_t = \frac{\text{Maximum Stress}}{\text{Nominal Stress at net section}}$

$$2.5 = \frac{\text{Maximum stress}}{25 \times 10^6}$$

$$\text{Maximum stress} = 2.5 \times 25 \times 10^6 \frac{N}{m^2}$$

$$= 62.5 \frac{MN}{m^2}$$

Question :-



$P = 12\ kN$

$D = 50\ mm$

$d = 25\ mm$

$$k_t = \frac{\text{Max}^m \text{ Stress}}{\text{Nomial stress at net section}}$$

$$\text{Nominal stress at net section} = \frac{\text{load}}{\text{Nominal area}}$$

$$= \frac{12 \times 10^3\ N}{\pi/4\ d^2}$$

$$= \frac{12 \times 10^3}{\frac{3.14}{4}(25 \times 10^{-3})^2}$$

$$24.45\ MPa = \frac{4 \times 12 \times 10^3}{3.14 \times 625 \times 10^{-6}}$$

$$\frac{r}{d} = \frac{5\,mm}{25\,mm} = \frac{1}{5} = 0.2$$

$$\frac{D}{d} = \frac{50}{25} = 2$$

$$k_t = 1.5$$

Maximum Stress = $k_t \times$ Nominal Stress

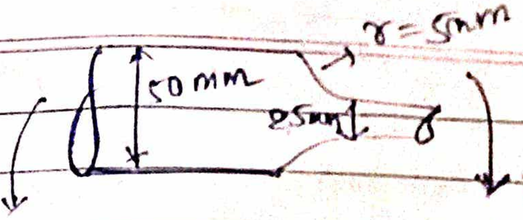
$= 1.5 \times 24.45$ MPa

$= \frac{1467}{40} = 36.675$ MPa

HW Questions

Question ①: ~~A Shaft~~ a
width = 15 mm; thickness 10 mm is having an eccentric hole distance 30 mm of diameter = 10 mm is subjected 10 kN of tensile load. Find the maximum stress?

Ques ② A step shaft of varing diameter of 45 mm and 30 mm diameter with a fillet radius 6 mm, find the maximum stress when it subjected to a tensile load of 10 kN.



* find maximum stress induced in shaft when it subjected to twist moment 50 N m

$$\boxed{M = \frac{\pi}{16} \tau d^3}$$

$\tau =$

$$50 \times 10^3 = \frac{3.14}{16} \times \tau \times (25)^3$$

$$\tau_{nominal} = \frac{50 \times 10^3 \times 16}{3.14 \times (25)^3} = 16.305 \frac{N}{mm^2}$$

$$\frac{r}{d} = \frac{5}{25} = 0.2$$

$$D/d = \frac{50}{25} = 2$$

$$k_t = 1.30$$

Maximum shear = k_t × Nominal stress

$$= 1.3 \times 16.305 \frac{N}{mm^2}$$

HW

(A)

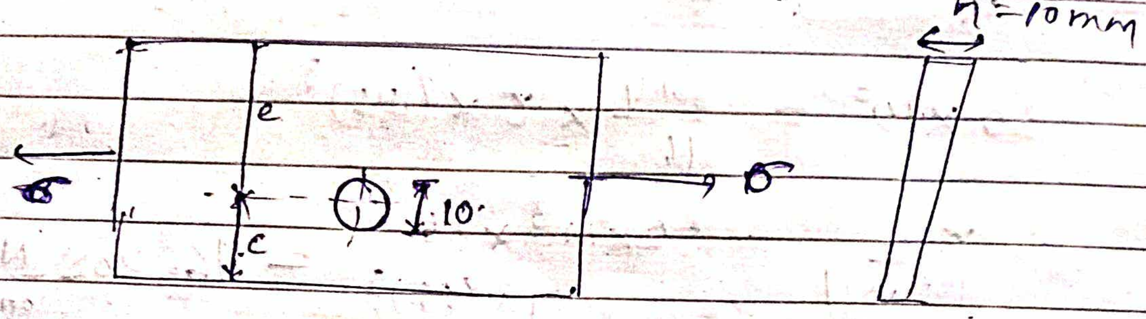
Solution (1) Width = 15 mm

thickness (t) = 10 mm

d = diameter of eccentric hole = 10 mm

P = tensile load = 10 kN

e = distance of eccentric hole = 30 mm



$\sigma = P/A$

$= \frac{10 \times 10^3}{(45) \times}$

$r/c = \frac{5}{15} = \frac{1}{3} = 0.333$

$e/c = \frac{30}{15} = 2 =$

Nominal stress $\sigma = \frac{\text{Load}}{\text{Nomial section}} = \frac{10 \times 10^3 \text{ N}}{}$

$k_t = \frac{\text{Stress maximum}}{\text{nominal stress at net section}} = \frac{\sigma_{max}}{\sigma}$

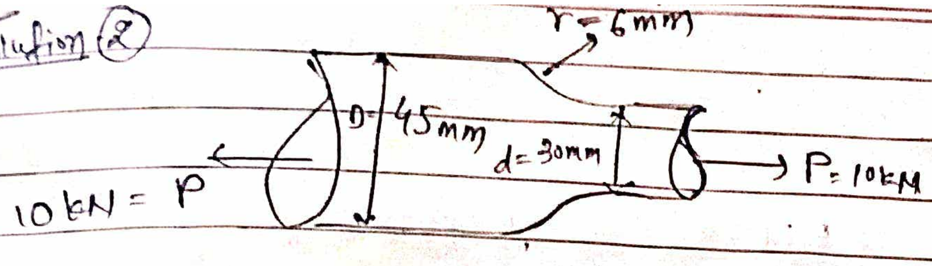
But $k_t = 2.4$

Stress maximum

$= k_t \times$ nominal stress

$= 2.4 \times$

Solution (2)



$P = 10\,kN = 10^4\,N$

$D = 45\,mm$

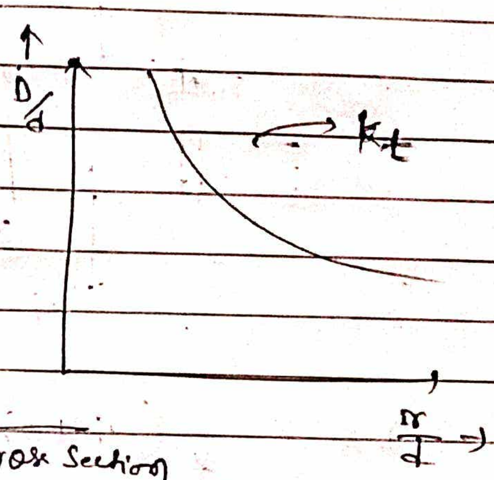
$d = 30\,mm$

$r = 6\,mm$

$\sigma_{max} = ?$

$\frac{r}{d} = \frac{6}{30} = \frac{1}{5} = 0.2$

$\frac{D}{d} = \frac{45}{30} = \frac{15}{10} = \frac{3}{2} = 1.5$



So, $k_t = 1.45$

Nominal Stress at net section = $\frac{\text{Load}}{\text{nominal Cross Section}}$

$= \frac{P}{\frac{\pi}{4} d^2} = \frac{10^4\,N}{\frac{3.14}{4} \times (30)^2} = \frac{4 \times 10^4\,N}{3.14 \times (30)^2\,mm^2}$

$= 14.154\,\frac{N}{mm^2}$

or

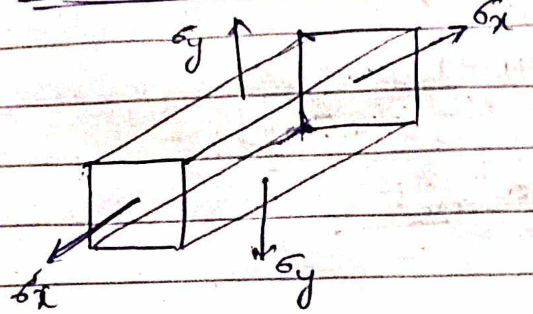
$14.154\,\frac{MN}{m^2}$

Maximum Stress = $k_t \times$ Nominal stress at net section

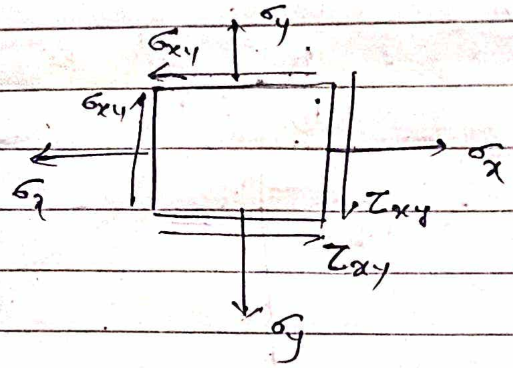
$= 1.45 \times 14.154 = 20.5233\,\frac{N}{mm^2}$

$= 20.5233\,\frac{MN}{m^2}$

* Principle stress



Bidirection stresses.



$$\sigma_{1,2} = \frac{1}{2}(\sigma_x + \sigma_y) \pm \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

$$\tau = \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

for unidirectional stress

$$\sigma = \frac{\sigma_1}{2} \pm \frac{1}{2}\sqrt{\sigma_1^2 + 4\tau^2}$$

$$\sigma_{max} = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} = \frac{\sigma}{2} + \frac{1}{2}\sqrt{\sigma^2 + 4\tau^2}$$

$$\sigma_{min} = \frac{1}{2}(\sigma_x + \sigma_y) - \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} = \frac{\sigma}{2} - \frac{1}{2}\sqrt{\sigma^2 + 4\tau^2}$$

$$\cancel{\tau_{max} = \sqrt{\sigma^2 + 4\tau^2}}$$

$$\tau_{max} = \frac{\sigma_{max} - \sigma_{min}}{2}$$

Date

Ques: A hollow shaft of 40 mm OD and 25 mm ID is subjected to twisting moment of 120 N m and an axial thrust of 10 kN and bending moment 80 N/m. Calculate the maximum shear stress induced in the hollow shaft and maximum compressive stress.

Solution :- OD = 40 mm

ID = 20 mm

Twisting moment = 120 Nm = 120×10^3 N mm

~~axial thrust = 10 kN = 10^4 N~~

Bending Moment = 80 Nm = 80×10^3 N mm

$$\sigma_{max}(\text{compress}) = \frac{\sigma}{2} + \frac{1}{2}\sqrt{\sigma^2 + 4\tau^2}$$

σ = ~~$\sigma_{bending} + \sigma_{twist}$~~

Bending moment (M) $= \frac{\pi}{32} d_o^3 \sigma (1 - k^4)$

$$80 \times 10^3 = \frac{3.14}{32} (40)^3 \times \sigma \left(1 - \left(\frac{25}{40}\right)^4\right)$$

$$8 \times 10^4 = \frac{3.14}{32} \times (40)^3 \times 0.8474 \times \sigma$$

$$\sigma_{bending} = \frac{8 \times 10^4 \times 32}{3.14 \times (40)^3 \times 0.8474} = 15.032 \frac{N}{mm^2}$$

$= \cancel{24052.59099 \frac{N}{mm^2}} = \cancel{24.052 \frac{kN}{mm^2}}$

$\sigma_{axial} = \frac{Load}{Area}$

$$= \frac{10^4}{\frac{3.14}{4}\left((40)^2-(25)^2\right)} = 13.065 \ \frac{N}{mm^2}$$

τ_{twist},

$$T = \frac{\pi}{16}\tau d_o^3\left(1-k^4\right) \qquad k = \frac{d_i}{d_o}$$

$$120\times10^3 = \frac{3.14}{16}\times\tau\times(40)^3\left(1-\left(\frac{25}{40}\right)^4\right)$$

$$\tau = \frac{120\times10^3\times16}{3.14\times(40)^3\left(1-\left(\frac{25}{40}\right)^4\right)}$$

$$= 11.274 \ \frac{N}{mm^2}$$

$\sigma = \sigma_{bending} + \sigma_{axial} = 15.03 + 13.06 = 28.09 \ \frac{N}{mm^2}$

$$\tau_{max} = \frac{1}{2}\sqrt{\sigma^2+4\tau^2}$$

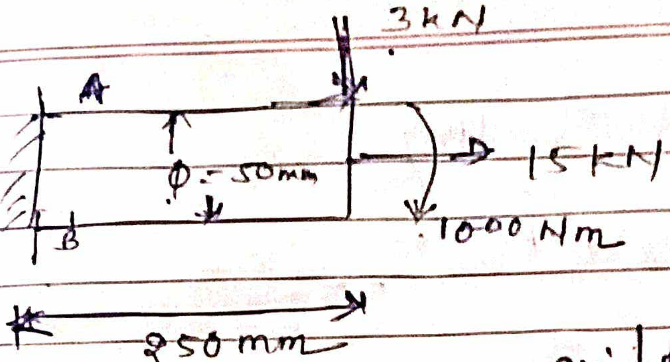
$$= \frac{1}{2}\sqrt{(28.09)^2+4(11.27)^2}$$

$$= 18.00 \ N/mm^2$$

$$\sigma_{max} = \frac{1}{2}\sigma + \frac{1}{2}\sqrt{\sigma^2+4\tau^2} = \frac{28.09}{2} + 18.00$$

$$= 32.045 \ N/mm^2$$

Ans :-



axial load 15 kN = 15×10^3 N

Bending load = 3 kN
= 3×10^3 N

Twist = 1000 Nm
= $10^3 \times 10^3$ N mm
= 10^6 N mm

Length = 250 mm
d = 50 mm

Solution:-

σ_{axial}

$(\sigma_{max})_A = ?$

$(\sigma_{max})_B = ?$

$\tau_{max} = ?$

$(\sigma_{max})_A = \sigma_{axial} + \sigma_{bending}$

$(\sigma_{max})_B = \sigma_{axial} - \sigma_{bending}$

$$\sigma_{axial} = \frac{\text{Load}}{\text{Area}} = \frac{15 \times 10^3}{\frac{\pi}{4} \times (50)^2} = 7.6433 \ \frac{N}{mm^2}$$

$\sigma_{bending}$

$$M = 3 \times 10^3 \times 250$$

$$\cancel{M = \frac{\pi}{32} d_0^3 \sigma}$$

$$M = \frac{\pi}{32} \sigma d_0^3 \Rightarrow 3 \times 10^3 \times 250 = \frac{3.14}{32} \times \sigma_{bending} (50)^3$$

$\sigma_{bending} = 61.14$ N/mm²

$$(\sigma)_A = \sigma_{axial} + \sigma_{bending}$$

$$= 7.64 + 61.14 \ N/mm^2$$

$$= 68.78 \ N/mm^2$$

$$(\sigma)_B = 61.14 - 7.64$$

$$= 53.5 \ N/mm^2$$

Twisting stress

$$T = \frac{\pi}{16} \tau d^3$$

$$10^6 = \frac{3.14}{16} \times \tau \times (50)^3$$

$$\tau = \frac{16 \times 10^6}{3.14 \times (50)^3} = 40.764 \ N/mm^2$$

$$(\sigma_{max})_A = \frac{\sigma}{2} + \frac{1}{2}\sqrt{\sigma^2 + 4\tau^2}$$

$$= \frac{68.78}{2} + \frac{1}{2}\sqrt{(68.78)^2 + 4(40.76)^2}$$

$$= 34.39 + \cancel{\tfrac{1}{2}}\ 53.32 = 87.71 \ N/mm^2$$

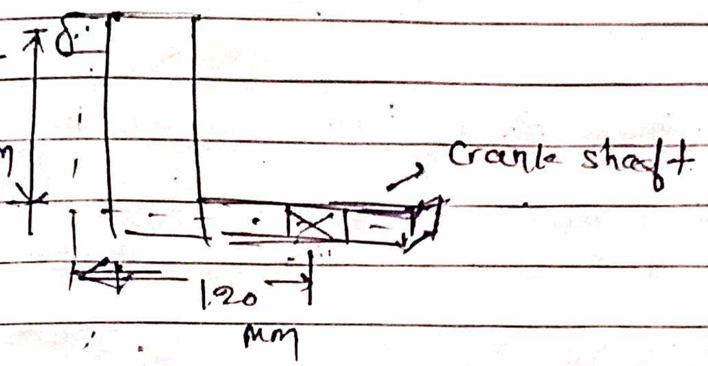
$$(\sigma_{max})_B = \frac{\sigma}{2} + \frac{1}{2}\sqrt{\sigma^2 + 4\tau^2}$$

$$= \frac{53.5}{2} + \frac{1}{2}\sqrt{(53.5)^2 + 4(40.76)^2} = ?$$

* Comparision of failure Theories:

Ques :- An overhanged crank shaft as shown in fig. is carrying a tangential load of 15 kN which is active on the crank pin. Determine the maximum principle stress & max shear stress induced at the centre of crankshaft

Solution :-



Twisting moment $= 15 \times 10^3 \times 140$ mm.

Bending moment $= 15 \times 10^3 \times 120$mm

$$\tau_{max} = \frac{1}{2}\sqrt{\sigma^2 + 4\tau^2}$$

$$= 27.51 \ N/mm^2$$

Principle stress.

$$\sigma_1 = \frac{\sigma}{2} + \frac{1}{2}\sqrt{\sigma^2 + 4\tau^2}$$

$$M = \frac{\pi}{32}\sigma d^3$$

$$15 \times 10^3 \times 120 = \frac{\pi}{32} \times \sigma \times 80^3$$

$$\boxed{\sigma = 35.82 \ N/mm^2}$$

$$T = \frac{\pi}{16}\tau d^3 \qquad \tau = \qquad \tau = 20.89 \ N/mm^2$$

$$15 \times 10^3 \times 140 = \frac{\pi}{16}\tau (80)^3$$

N/mm2

$$\sigma_1 = \frac{\sigma}{2} + \frac{1}{2}\sqrt{\sigma^2 + 4\tau^2}$$

$$\sigma_1 = \frac{35.82}{2} + \frac{1}{2}\sqrt{(2.0829)^2 + 4(20.87)^2}$$

* Theories Of failure :

1) Rankine Theory (Max^m stress theory / Max^m normal stress)

$$\text{Max}^m \text{ stress} \left(\begin{array}{c}\text{whenever is maximum} \\ \sigma_1 \text{ or } \sigma_2 \text{ or } \sigma_3\end{array}\right) = \sigma_y \text{ yeild}$$

Avoid failure, $\underset{(\text{max}^m)}{\sigma_1} = \frac{\sigma_y}{F.O.S}$

failure in cylinder occurs at a ~~point~~ when maximum principle stress / normal stress in a biaxial stress system reaches the maximum limiting value of material strength in a simple test.

(2) Guest's Theory or coulomb's theory or Maximum shear theory

$$\boxed{\sigma_1 - \sigma_2 \text{ or } \sigma_2 - \sigma_3 \text{ or } \sigma_3 - \sigma_1 \left(\begin{array}{c}\text{which ever is} \\ \text{maximum}\end{array}\right) = \sigma_y}$$

failure or yeilding occurs at a point in member when max shear in a biaxial stress system reaches the limiting value of shear in a simple tension test.

** Above theorem is applicable to Ductile material.

(3) St. venant's theory or Maximum strain theory :

$$\left.\begin{aligned} &\sigma_1 - \nu(\sigma_2 + \sigma_3) \\ &\text{or} \\ &\sigma_2 - \nu(\sigma_3 + \sigma_1) \\ &\text{or} \\ &\sigma_3 - \nu(\sigma_1 + \sigma_2) \end{aligned}\right\} = \frac{\sigma_y}{FOS}$$

σ_y = yeild strength
ν = poission ratio
FOS = factor of safety

(4) Maximum strain energy theorems :

$$\boxed{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1) = \sigma_y^2}$$

to avoid failure $= \left(\frac{\sigma_y}{FOS}\right)^2$

(5) Octahedral or distortion energy theory (von-mess theory)

$$\boxed{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1\sigma_2 - \sigma_2\sigma_3 - \sigma_3\sigma_1 = \sigma_y^2}$$

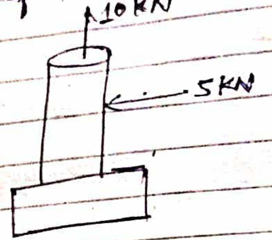
$$\boxed{\text{Prevent failer} = \left(\frac{\sigma_y}{FOS}\right)^2}$$

Question :- A bolt of a machine is subjected to axial pull of 10 kN or a transverse shear load of 5 kN. find the diameter of bolt using all theories of failure ?

solution

· Tension load = 10 kN

Transfer shear Load = 5 kN $\qquad \nu = 0.3$ poission ratio



σ_t = permissible tensile = allowable tensile

$= \dfrac{\sigma_y}{FOS} = 100 \dfrac{N}{mm^2}$

(i) Max Principle stress theory

$\sigma_1 = \dfrac{\sigma_y}{FOS}$

$= 100 \dfrac{N}{mm^2}$

$= 100 \times 10^6 \dfrac{N}{m^2}$

$mm^2 = 1m$

$\sigma_1 = \dfrac{\sigma}{2} + \dfrac{1}{2}\sqrt{\sigma^2 + 4\tau^2}$

$\sigma = \dfrac{\text{Tensile Load}}{\frac{\pi}{4}d^2} = \dfrac{10\times10^3}{\pi/4\, d^2} = \dfrac{12.73285}{d^2}$

$\tau = \dfrac{\text{Load}}{\text{Area}} = \dfrac{5\times10^3}{\pi/4\, d^2} = \dfrac{6369.42}{d^2}$

$\sigma_1 = \dfrac{\sigma}{2} + \dfrac{1}{2}\sqrt{\sigma^2 + 4\tau^2}$

$100 = \dfrac{12732.4}{2d^2} + \dfrac{1}{2}\sqrt{\left(\dfrac{12732.4}{d^2}\right)^2 + 4\left(\dfrac{6369.42}{d^2}\right)^2}$

$100 = \dfrac{6366.2}{d^2} + \dfrac{1}{2d^2}\sqrt{162114009.8 + 162278044.5}$

$100 = \dfrac{6366.2}{d^2} + \dfrac{9005.44}{d^2}$

$100 = \dfrac{15371.64}{d^2}$

$d^2 = \dfrac{15371.64}{100}$

$d = \sqrt{\dfrac{15371.64}{100}}$

$\boxed{d = 12.398\ mm}$

(ii) $\sigma_1 - \sigma_2 = \left(\dfrac{\sigma_y}{FOS}\right)$ Max^m shear theory

$\sigma_2 = \dfrac{\sigma}{2} - \dfrac{1}{2}\sqrt{\sigma^2 + 4\tau^2}$

$= \dfrac{6366.2}{d^2} - \dfrac{9005.44}{d^2}$

$\sigma_2 = -\dfrac{2639.24}{d^2}$

Now,

$\sigma_1 - \sigma_2 = \dfrac{15371.64}{d^2} + \dfrac{2639.24}{d^2}$

$100 = \dfrac{18010.88}{d^2}$

$d = \sqrt{\dfrac{18010.88}{100}}$

$\boxed{d = 13.420\ mm}$

(III) Max^n Strain theory :—

$$\sigma_1 - \nu(\sigma_2 + \sigma_3)$$

$$\sigma_1 - \nu\sigma_2 = \frac{\sigma_y}{FOS}$$

$$100 = \frac{15371.64}{d^2} + 0.3 \times \frac{2639.34}{d^2}$$

$$d^2 = \frac{15371.64 + 0.3 \times 2639.34}{100}$$

$$d^2 = \frac{16163.442}{100}$$

$$d = \sqrt{161.63442}$$

$$d = 12.71 \text{ mm}$$

(IV) Maximum strain energy theory

$$\cancel{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu}$$

$$\sigma_1^2 - 2\nu\sigma_1\sigma_2 = \left(\frac{\sigma_y}{FOS}\right)^2$$

$$\frac{15371.64 + 0.3 \times 2639.34}{d^2} = (100)^2$$

$$d^2 = \frac{16163.442}{(100)^2}$$

$$d = \sqrt{\frac{16163.442}{(100)^2}}$$

$= 12.7$ ~~mm~~

(V) Octahedral or Distortion energy theory :—

$$\sigma_1^2 - \nu\sigma_1\sigma_2 = \left(\sigma_y/FOS\right)^2$$

$d = 12.97$

Ques A rotating shaft of 16 mm diameter is subjected to an axial load 5000 N. Steady torque 50 Nm and maximum bending moment of 75 Nm. Calculate the probable factor of safety using maximum normal stress theory and maximum shear stress theory by taking the limiting strength of shaft material as 400 MPa.

Solution:- Shaft diameter = 16 mm

Axial load = 5000 N

Torque = 50 Nm = 50×10^3 N mm

Bending Moment = 75×10^2 N.mm

$\sigma_y = 400 \text{ MPa} = 400 \times 10^6 \frac{N}{m^2} = 400 \frac{N}{mm^2}$

theories, I, II

$$\sigma_1 = \frac{\sigma}{2} + \frac{1}{2}\sqrt{\sigma^2 + 4\tau^2}$$

$$M = \frac{\pi}{32} \sigma_{bending} d^3$$

$$\sigma_{bending} = \frac{32 M}{\pi d^3}$$

$$= \frac{32 \times 75 \times 10^3}{\pi (16)^3}$$

$$= 186.50 \frac{N}{mm^2}$$

$$\sigma_{axial} = \frac{load}{Area} = \frac{5000}{\frac{\pi}{4} d^2}$$

$$= \frac{5000}{\frac{\pi}{4} (16)^2}$$

$$= 24.867 \frac{N}{mm^2}$$

$$T = \frac{\pi}{16}\tau d^3$$

$$\tau = \frac{16 \times T}{\pi d^3} = \frac{16 \times 50 \times 10^3}{\pi (16)^3} = 62.169 \ N/mm^2$$

$$\sigma_{total} = \sigma_{bending} + \sigma_{axial}$$

$$= 186.5 + 24.867$$

$$= 211.36 \ N/mm^2$$

Now,

$$\sigma_1 = \frac{\sigma}{2} + \frac{1}{2}\sqrt{\sigma^2 + 4\tau^2}$$

$$\sigma = \frac{211.36}{2} + \frac{1}{2}\sqrt{(211.36)^2 + 4(62.169)^2}$$

$$\sigma_1 = 228.39 \ N/mm^2$$

So,

$$\sigma_1 = \frac{\sigma_y}{FOS}$$

$$FOS = \sigma_y / \sigma_1 = \frac{400}{228.83}$$

$$= 1.748$$

$$\sigma_2 = \frac{\sigma}{2} - \frac{1}{2}\sqrt{\sigma^2 + 4\tau^2}$$

$$= \frac{211.36}{2} - \frac{1}{2}\sqrt{(211.36)^2 + 4(62.161)^2}$$

$$= 105.68 - 122.610$$

$$= -16.93 \ N/mm^2$$

II

$$\sigma_1 - \sigma_2 = \frac{\sigma_y}{FOS}$$

$$228.39 - (-16.93) = \frac{400}{FOS}$$

$$228.39 + 16.93 = \frac{400}{FOS}$$

$$FOS = \frac{400}{245.32} = 1.6305$$

Dt - 18/09/24

cylindrical steel yeild strength

Question: A shaft of the ~~distance~~ 700 MPa. is subject to static consisting of bending Moment 10 kNm and twisting moment 20 kNm. Find the shaft diameter using maximum principle and maxm shear theory, FOS = 2 and young's modulus 210 GPa, ν = 0.25

Question: A mild shaft of 50mm diameter is subject a BM of 2000 Nm and torque (T) table ~~e~~ yeild of material is permisiable 200 MPa . Find the maximum value of the torque that can be applied on the shaft by using maximum . 1) Maxm strain theory, assume poission ration is 0.3.

Solution:-

$d = 50\,mm$

$BM = 2000\,Nm = 2000 \times 10^3\,Nmm = 2 \times 10^6\,Nmm$

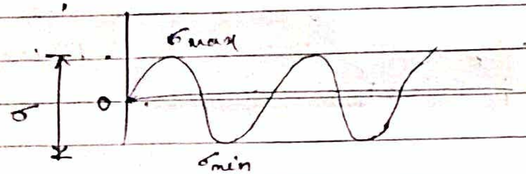
$\left(\sigma_y / FOS\right) = 200 \times 10^6 \frac{N}{m^2} = 200 \times 10^6 \frac{N}{10^6\,mm^2}$

$= 200 N/mm$

$\nu = 0.3$

* Variable stress in m/c parts :-

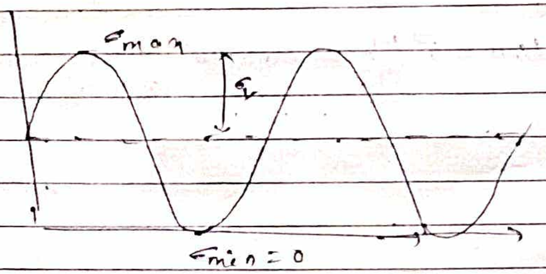
* Cyclic stress (completely reverse)



R = stress ratio

$= \frac{\sigma_{min}}{\sigma_{max}}$ $\boxed{R = -1}$

* Repeated stress



$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$

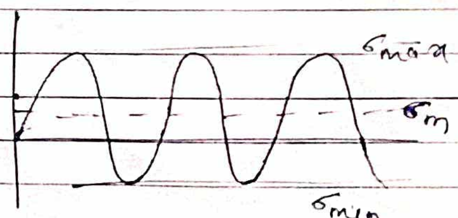
$= \frac{\sigma_{max} + 0}{2}$

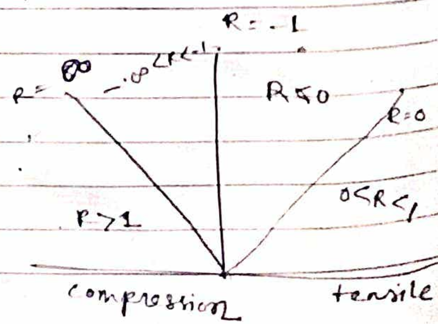
$= \frac{\sigma_{max}}{2}$

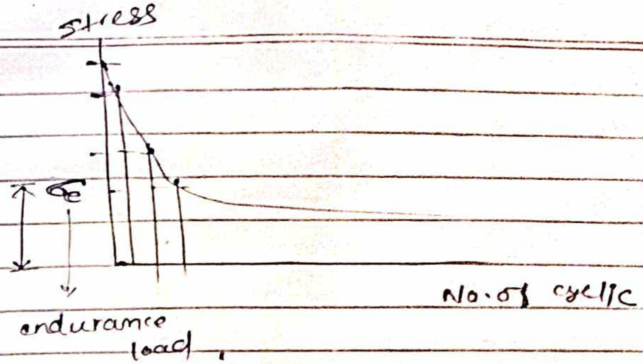
$\sigma_a = \frac{\sigma_{max}}{2}$

$\boxed{R = 0}$

* Fluctuating







* K_f = Fatigue stress concentration factor,
It is defined as the ratio of endurance stress without stress concentration to that of endurance stress with concentration

$K_f = \frac{\sigma_e \max . S_f}{\sigma_e \min S_f}$

Question

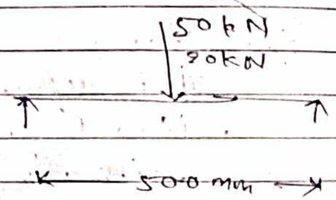
$n = \text{fos} = 1.5$

size factor $= 0.85$

Surface factor $= 0.9$

$\sigma_u = 650$ MPa

$\sigma_y = 500$ MPa



(B)max $M = \frac{PL}{4}$

maximum BM

$$M = \frac{\pi}{32} \sigma_{max} d^3$$

$$\frac{50 \times 10^3 \times 500}{4} = \frac{\pi}{32} \sigma_{max} d^3$$

$$\sigma_{max} = \frac{8 \times 50 \times 10^3 \times 500}{\pi d^3} = \frac{63661977.24}{d^3}$$

$$= \frac{63.661 \times 10^6}{d^3} \frac{N}{mm^2}$$

BM minimum

$$BM = \frac{20 \times 10^3 \times 500}{4}$$

$$M = \frac{\pi}{32} \sigma_{min} d^3$$

$$\frac{20 \times 10^3 \times 500}{4} = \frac{\pi}{32} \sigma_{min} d^3$$

$$\sigma_{min} = \frac{\cancel{101859163.6}}{\cancel{d^3}} \quad \frac{25464.79 \times 10^3}{d^3}$$

$$= \frac{25.464 \times 10^6}{d^3} \frac{N}{mm^2}$$

$$\sigma_{mean} = \frac{\sigma_{max} + \sigma_{min}}{2}$$

$$= \frac{89.121 \times 10^6}{2 d^3}$$

$$= \frac{44.5605 \times 10^6}{d^3} \frac{N}{mm^2}$$

$$\sigma_a = \sigma_{avg} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{63.66 - 25.46}{2 d^3} \times 10^6$$

$$= \frac{19.1 \times 10^6}{d^3} \frac{N}{mm^2}$$

* Soderberg Method

σ_y taken as limiting

$$\frac{1}{n} = \frac{\sigma_m}{\sigma_y} + \frac{k_f \sigma_a}{\sigma_{-1} \times K_{su} \times K_{size}}$$

$\boxed{k_f = 1}$ if no given

$$\frac{1}{1.5} = \frac{44.56 \times 10^6}{d^3 \times 500} + \frac{1 \times 19.1 \times 10^6}{d^3 \times 350 \times 0.9 \times 0.85}$$

$$d^3 = \frac{160455.2007}{\cancel{d^3}} \times 1.5$$

$$= 240682.8011$$

$$\boxed{d = 62.20 mm}$$

goodman Relation

$$\frac{1}{\eta} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_a}{\sigma_{-1} \times k_{size} \times k_{surf}}$$

$$\frac{1}{1.5} = \frac{44.56 \times 10^6}{650\, d^3} + \frac{19.1 \times 10^6}{350 \times d^3 \times 0.9 \times 0.85}$$

$$d^3 = 139889.04 \times 1.5$$

$$d^3 = 209833.57$$

$$d = (209833.57)^{1/3} = 59.4235 \text{ mm}$$

Ques A still rod is subjected reverse axial loading of 180 kN. find the diameter taking factor of safety as 2, yield stress 910 MPa, ultimate stress as 1070 MPa, $\sigma_{-1} = \frac{1}{2}\sigma_u$, take endurance correction factor;

Load factor = 0.7, surface finish factor 0.8, size factor 0.85, fatigue stress concⁿ factor is 1. Reversed axial load $= 180 \times 10^3$ N

Solution - given:

load factor = 0.7

$K_{su} = 0.8$

$K_{size} = 0.85$

Reverse axial load $= 180 \times 10^3$ N

FOS = 2

$\sigma_u = 1070 \frac{N}{mm^2}$ $\therefore \sigma_{-1} = \frac{1}{2} \times 1070$

$= 535 \frac{N}{mm^2}$

$\sigma_y = 910 \frac{N}{mm^2}$

$$\sigma_{max} = \frac{\text{max}^m \text{ load}}{\frac{\pi}{4} d^2} = \frac{180 \times 10^3}{\frac{\pi}{4} d^2} = \frac{229.18 \times 10^3}{d^2}$$

$$\sigma_{min} = \frac{-180 \times 10^3}{\frac{\pi}{4} d^2} = -\frac{229.18}{d^2} \times 10^3$$

$\sigma_{mean} = 0$ $\quad \sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{2.29 \times 10^5}{d^2}$

Soderberg Relation

$$\frac{1}{\eta} = \frac{\sigma_{mean}}{\sigma_y} + \frac{k_f\, \sigma_a}{\sigma_{-1} \times \text{factor's}}$$

$$\frac{1}{2} = \frac{1 \times \frac{2.29 \times 10^5}{d^2}}{535 \times 0.7 \times 0.8 \times 0.85}$$

$$d^2 = 1798.47$$

$$\boxed{d = 42.408 \text{ mm}}$$

Question: A bar of circular cross-section is subjected to varying tensile load 200 kN to 500 kN. Determine the diameter by taking FOS related to ultimate stress is 3.5 and 4 with endurance. Take $k_f = 1.65$, $\sigma_u = 900$ MPa, $\sigma_{-1} = 700$ MPa.

Solution :- given:

Modified Soderberg.

$$\frac{1}{n} = \frac{\sigma_m}{\sigma_u} + \frac{k_f \sigma_a}{\sigma_{-1}}$$

$$1 = n\left(\frac{\sigma_m}{\sigma_u} + \frac{k_f \sigma_a}{\sigma_{-1}}\right)$$

$$1 = \frac{\sigma_m}{(\sigma_u/n)} + \frac{k_f \sigma_a}{\left(\frac{\sigma_{-1}}{n}\right)}$$

$$\sigma_{max} = \frac{\text{Maximum load}}{\frac{\pi}{4}d^2} \qquad \sigma_{min} = \frac{\text{min load}}{\frac{\pi}{4}d^2}$$

$$= \frac{500 \times 10^3}{\frac{\pi}{4}d^2} \qquad = \frac{200 \times 10^3}{\frac{\pi}{4}d^2}$$

$$= \frac{6.36 \times 10^5}{d^2} \qquad = \frac{2.54 \times 10^5}{d^2}$$

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} \qquad \sigma_a = \frac{\sigma_{max} \mp \sigma_{min}}{2}$$

$$= \frac{4.45 \times 10^5}{d^2} \qquad = \frac{1.91 \times 10^5}{d^2}$$

Now

$$1 = \frac{\sigma_m}{(\sigma_u/n)} + \frac{\sigma_a}{(\sigma_{-1}/n)}$$

$$1 = \frac{4.45 \times 10^5}{d^2\left(\frac{900}{3.5}\right)} + \frac{1.91 \times 10^5}{d^2\left(\frac{700}{4}\right)}$$

$$d^2 = 2821.984$$

$$\boxed{d = 53.12 \text{ mm}}$$