目標：強度測定 → 波動関数を決定

Review

1. 状態

— 個数状態　$|0\rangle, |1\rangle, \cdots$

— コヒーレント〃　$|\alpha\rangle = \hat{D}(\alpha)|0\rangle$, $\hat{D}(\alpha) := e^{\alpha\hat{a}^\dagger - \alpha^*\hat{a}}$

— スクイーズド〃　$|\beta=0\rangle = \hat{S}(r)|0\rangle$, $\hat{S}(r) := e^{\frac{r}{2}(\hat{a}^2 - \hat{a}^{\dagger 2})}$

スクイーズド state

$_g\langle\beta=0|\hat{x}|\beta=0\rangle_g = 0$

$_g\langle\beta=0|\hat{p}|\beta=0\rangle_g = 0$

$_g\langle\beta=0|\hat{x}^2|\beta=0\rangle_g = \frac{1}{4}e^{-r}$

$_g\langle\beta=0|\hat{p}^2|\beta=0\rangle_g = \frac{1}{4}e^{r}$

2. 物理量（演算子）

— 位置，運動量　$\hat{x}, \hat{p}$

— 生成，消滅　$\hat{a} = \hat{x} + i\hat{p}$, $\hat{a}^\dagger = \hat{x} - i\hat{p}$

— 電場　$\hat{E}(r,t) = \frac{i}{2}\left(\hat{a}e^{i(k\cdot r - \omega t)} - \hat{a}^\dagger e^{-i(k\cdot r - \omega t)}\right)$

"New" — 強度　$\hat{I} = \hat{a}^\dagger\hat{a}$

3. 時間発展

$$\begin{pmatrix}\hat{a}_1' \\ \hat{a}_2'\end{pmatrix} = B\begin{pmatrix}\hat{a}_1 \\ \hat{a}_2\end{pmatrix}$$

$$B = \begin{pmatrix} e^{i\Psi/2} & 0 \\ 0 & e^{-i\Psi/2}\end{pmatrix}\begin{pmatrix}\cos\left(\frac{\Theta}{2}\right) & \sin\left(\frac{\Theta}{2}\right) \\ -\sin\left(\frac{\Theta}{2}\right) & \cos\left(\frac{\Theta}{2}\right)\end{pmatrix}\begin{pmatrix} e^{i\Phi/2} & 0 \\ 0 & e^{-i\Phi/2}\end{pmatrix}$$

$\frac{\Theta}{2} = -\frac{\pi}{4}$, $\Psi = \Phi = 0$

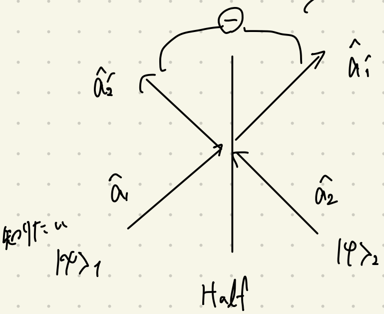
$$B_{half} = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & -1 \\ 1 & 1\end{pmatrix}$$

$$\hat{L}_2 = \frac{1}{2i}\left(\hat{a}_1^\dagger\hat{a}_2 - \hat{a}_1\hat{a}_2^\dagger\right)$$

$$e^{-i\Theta\hat{L}_2}\begin{pmatrix}\hat{a}_1\\ \hat{a}_2\end{pmatrix}e^{i\Theta\hat{L}_2}$$

$$=\begin{pmatrix}\cos\left(\frac{\Theta}{2}\right) & \sin\left(\frac{\Theta}{2}\right)\\ -\sin\left(\frac{\Theta}{2}\right) & \cos\left(\frac{\Theta}{2}\right)\end{pmatrix}\begin{pmatrix}\hat{a}_1\\ \hat{a}_2\end{pmatrix}$$

1.6 バランスホモダイン測定



・スペクトルアナライザ
・オシロスコープ

2つの強度

1. 光強度 : $\hat{I}=\hat{a}^\dagger\hat{a}$
2. $(\hat{I}_2-\hat{I}_1)^2$

$$\hat{a}_1'=\frac{1}{\sqrt{2}}(\hat{a}_1-\hat{a}_2)$$

$$\hat{a}_2'=\frac{1}{\sqrt{2}}(\hat{a}_1+\hat{a}_2)$$

$$\hat{I}_1=\hat{a}_1'^\dagger\hat{a}_1'=\frac{1}{2}(\hat{a}_1^\dagger\hat{a}_1+\hat{a}_2^\dagger\hat{a}_2-\hat{a}_1^\dagger\hat{a}_2-\hat{a}_2^\dagger\hat{a}_1)$$

$$\hat{I}_2=\hat{a}_2'^\dagger\hat{a}_2'=\frac{1}{2}(\hat{a}_1^\dagger\hat{a}_1+\hat{a}_2^\dagger\hat{a}_2+\hat{a}_1^\dagger\hat{a}_2+\hat{a}_2^\dagger\hat{a}_1)$$

$$\hat{I}_2-\hat{I}_1=\hat{a}_1^\dagger\hat{a}_2+\hat{a}_2^\dagger\hat{a}_1$$

$$\Big\Downarrow\ \frac{\Theta}{2}=-\frac{\pi}{4}$$

$$e^{-i\Theta\hat{L}_2}(\hat{a}_2^\dagger\hat{a}_2-\hat{a}_1^\dagger\hat{a}_1)e^{i\Theta\hat{L}_2}\ \cdots\ ☆$$

(∵)

$$e^{-i\Theta\hat{L}_2}\hat{a}_2^\dagger\hat{a}_2 e^{i\Theta\hat{L}_2}$$

$$= e^{-i\Theta\hat{L}_2}\hat{a}_2^\dagger e^{i\Theta\hat{L}_2} e^{-i\Theta\hat{L}_2}\hat{a}_2 e^{i\Theta\hat{L}_2}$$

$$= \left(e^{-i\Theta\hat{L}_2}\hat{a}_2 e^{i\Theta\hat{L}_2}\right)^\dagger \left(e^{-i\Theta\hat{L}_2}\hat{a}_2 e^{i\Theta\hat{L}_2}\right)$$

$$= \left\{\left(-\sin\frac{\Theta}{2}\right)\hat{a}_1^\dagger + \left(\cos\frac{\Theta}{2}\right)\hat{a}_2^\dagger\right\}\left\{\left(-\sin\frac{\Theta}{2}\right)\hat{a}_1 + \left(\cos\frac{\Theta}{2}\right)\hat{a}_2\right\}$$

$$= \sin^2\frac{\Theta}{2}\hat{a}_1^\dagger\hat{a}_1 + \cos^2\frac{\Theta}{2}\hat{a}_2^\dagger\hat{a}_2 - \sin\frac{\Theta}{2}\cos\frac{\Theta}{2}\left(\hat{a}_1^\dagger\hat{a}_2 + \hat{a}_1\hat{a}_2^\dagger\right)$$

$$e^{-i\Theta\hat{L}_2}\hat{a}_1^\dagger\hat{a}_1 e^{i\Theta\hat{L}_2}$$

$$= \cos^2\frac{\Theta}{2}\hat{a}_1^\dagger\hat{a}_1 + \sin^2\frac{\Theta}{2}\hat{a}_2^\dagger\hat{a}_2 + \underbrace{\cos\frac{\Theta}{2}\sin\frac{\Theta}{2}}_{\frac{1}{2}\sin\Theta}\left(\hat{a}_1^\dagger\hat{a}_2 + \hat{a}_1\hat{a}_2^\dagger\right)$$

$$\star \Leftrightarrow \quad -\sin\Theta\left(\hat{a}_1^\dagger\hat{a}_2 + \hat{a}_1\hat{a}_2^\dagger\right) = \hat{a}_1^\dagger\hat{a}_2 + \hat{a}_1\hat{a}_2^\dagger$$

$$\Theta = -\frac{\pi}{2} \quad \left(\because \frac{\Theta}{2} = -\frac{\pi}{4}\right)$$

$$\langle I_2 - I_1\rangle = {}_2\langle\varphi| \otimes {}_1\langle\psi|\hat{I}_2 - \hat{I}_1|\psi\rangle_1 \otimes |\varphi\rangle_2$$

$$= {}_1\langle\psi|\hat{a}_1^\dagger|\psi\rangle_1 \cdot {}_2\langle\varphi|\hat{a}_2|\varphi\rangle_2 + {}_1\langle\psi|\hat{a}_1|\psi\rangle_1 \cdot {}_2\langle\varphi|\hat{a}_2^\dagger|\varphi\rangle_2$$

$\downarrow$ $|\varphi\rangle_2 \to |\alpha\rangle_2$, $\alpha = |\alpha|e^{i\theta}$

$$= {}_1\langle\psi|\hat{a}_1^\dagger|\psi\rangle_1 \cdot {}_2\langle\alpha|\underbrace{\hat{a}_2|\alpha\rangle}_{\alpha}{}_2 + {}_1\langle\psi|\hat{a}_1|\psi\rangle_1 \cdot \underbrace{{}_2\langle\alpha|\hat{a}_2^\dagger}_{\alpha^*}|\alpha\rangle_2$$

$$= |\alpha|\left(\underbrace{{}_1\langle\psi|\hat{a}_1^\dagger|\psi\rangle_1}_{\hat{x}_1 - i\hat{p}_1} e^{i\theta} + \underbrace{{}_1\langle\psi|\hat{a}_1|\psi\rangle_1}_{\hat{x}_1 + i\hat{p}_1} e^{-i\theta}\right)$$

$$= |\alpha| \left(2\cos\theta \, {}_1\langle\psi|\hat{x}_1|\psi\rangle_1 + 2\sin\theta \, {}_1\langle\psi|\hat{p}_1|\psi\rangle_1 \right)$$

○ 入力：スクイーズド光　，スペクトルアナライザーでの測定

$$|\psi\rangle_1 = |p=0\rangle_g = \hat{S}(r)|0\rangle$$

$$\langle I_2 - I_1\rangle = 2|\alpha| \left(\cos\theta \, {}_1\langle\psi|\hat{x}_1|\psi\rangle_1 + \sin\theta \, {}_1\langle\psi|\hat{p}_1|\psi\rangle_1 \right)$$

$$= {}_1\langle\psi| \, 2|\alpha| \left(\hat{x}_1\cos\theta + \hat{p}_1\sin\theta \right) |\psi\rangle_1$$

$$\langle (I_2 - I_1)^2\rangle = {}_1\langle\psi| \, 4|\alpha|^2 \left(\hat{x}_1\cos\theta + \hat{p}_1\sin\theta \right)^2 |\psi\rangle_1$$

$$|\psi\rangle_1 = \hat{S}(r)|0\rangle_1$$

$$= 4|\alpha|^2 \Big({}_1\langle 0| \underbrace{\hat{S}^\dagger(r)\hat{x}_1^2\hat{S}(r)}_{\hat{x}_1^2 e^{-2r}} |0\rangle_1 \cos^2\theta + {}_1\langle 0| \underbrace{\hat{S}^\dagger(r)\hat{p}_1^2\hat{S}(r)}_{\hat{p}_1^2 e^{2r}} |0\rangle_1 \sin^2\theta$$

$$+ {}_1\langle 0| \hat{S}^\dagger(r) \left(\hat{x}_1\hat{p}_1 + \hat{p}_1\hat{x}_1 \right) \hat{S}(r) |0\rangle \cos\theta\sin\theta \Big)$$

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$$\hat{S}^\dagger(r)\hat{x}_1\hat{p}_1\hat{S}(r) + \hat{S}^\dagger(r)\hat{p}_1\hat{x}_1\hat{S}(r)$$

$$= \hat{S}^\dagger(r)\hat{x}_1\hat{S}(r)\hat{S}^\dagger(r)\hat{p}_1\hat{S}(r) + \hat{S}^\dagger(r)\hat{p}_1\hat{S}(r)\hat{S}^\dagger(r)\hat{x}_1\hat{S}(r)$$

$$= \hat{x}_1 e^{-r}\hat{p}_1 e^{r} + \hat{p}_1 e^{r}\hat{x}_1 e^{-r}$$

$$= \hat{x}_1\hat{p}_1 + \hat{p}_1\hat{x}_1$$

$$= \frac{\hat{a}_1 + \hat{a}_1^\dagger}{2} \cdot \frac{\hat{a}_1 - \hat{a}_1^\dagger}{2i} + \frac{\hat{a}_1 - \hat{a}_1^\dagger}{2i} \cdot \frac{\hat{a}_1 + \hat{a}_1^\dagger}{2}$$

$$= \frac{1}{4i}\left(2\hat{a}_1^2 - 2\hat{a}_1^{\dagger 2} \right) = \frac{1}{2i}\left(\hat{a}_1^2 - \hat{a}_1^{\dagger 2} \right)$$

$$= \frac{1}{2i}\left({}_1\langle 0|\hat{a}_1^2|0\rangle_1 - {}_1\langle 0|\hat{a}_1^{\dagger 2}|0\rangle_1 \right) = 0$$

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$$= 4|\alpha|^2 \left(e^{-2r} {}_1\langle 0| \hat{x}_1^2 |0\rangle_1 \cos^2\theta + e^{2r} {}_1\langle 0| \hat{p}_1^2 |0\rangle_1 \sin^2\theta \right)$$

$$= |\alpha|^2 \left(e^{-2r} \cos^2\theta + e^{2r} \sin^2\theta \right)$$

$$\langle (I_2 - I_1) \rangle = 2|\alpha| \left({}_1\langle 0| \underbrace{\hat{S}_1^\dagger(r) \hat{x}_1 \hat{S}_1(r)}_{x e^{-r}} |0\rangle_1 \cos\theta + {}_1\langle 0| \underbrace{\hat{S}_1^\dagger(r) \hat{p}_1 \hat{S}(r)}_{p e^{r}} |0\rangle_1 \sin\theta \right)$$

$$= 0$$

$$\therefore \langle \Delta (I_2 - I_1) \rangle^2 = \langle (I_2 - I_1)^2 \rangle$$

○ オシロスコープでの測定

$$\langle I_2 - I_1 \rangle = 2|\alpha| {}_1\langle \psi | \hat{x}_1 \cos\theta + \hat{p}_1 \sin\theta) |\psi\rangle_1$$

$\downarrow \theta = 0$

$$= 2|\alpha| \langle \psi | \hat{x}_1 | \psi \rangle_1$$

$$= 2|\alpha| \langle \psi | \hat{x}_1 \int dx_1 |x\rangle_1 {}_1\langle x| |\psi\rangle_1$$

$$= 2|\alpha| \int dx_1 \; x_1 \langle \psi | x \rangle_1 {}_1\langle x | \psi \rangle_1$$

$$= 2|\alpha| \int dx_1 \; x_1 |\psi(x)|^2$$

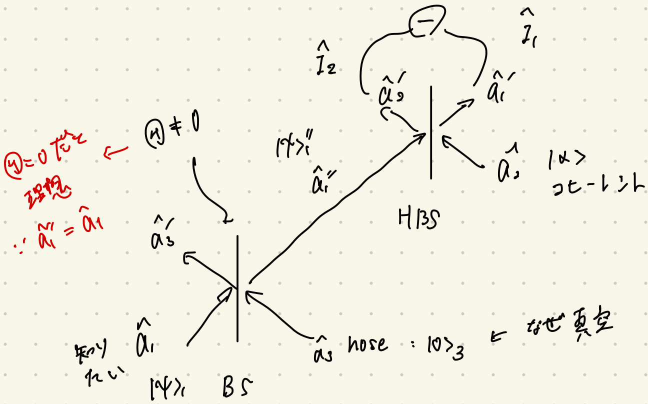
← この測定で $|\psi(x)|$ が求まり、

ここで

$\psi(x) =$

$\psi(x)$ が予測可

1.9 ロスと真空場



$$\hat{L}_2 = \frac{1}{2i}\left(\hat{a}_1^\dagger \hat{a}_3 - \hat{a}_1 \hat{a}_3^\dagger\right)$$

$$\langle (I_2 - I_1)^2 \rangle = 4|\alpha|^2 \, {}_{1''}\langle\psi| (\hat{x}_1 \cos\theta + \hat{p}_1 \sin\theta)^2 |\psi\rangle_{1''} \quad \text{“} \underbrace{e^{i\Theta\hat{L}_2} |\psi\rangle_1 \otimes |0\rangle_3}_{} \text{”}$$

$\theta = 0$ $|\psi\rangle_{1''}$ に “〜” を代入

$$= 4|\alpha|^2 \, {}_3\langle 0| \otimes {}_1\langle\psi| e^{-i\Theta\hat{L}_2} \hat{x}_1^2 e^{i\Theta\hat{L}_2} |\psi\rangle_1 \otimes |0\rangle_3$$

$$e^{-i\Theta\hat{L}_2} \hat{x}_1^2 e^{i\Theta\hat{L}_2}$$

$$= e^{-i\Theta\hat{L}_2} \hat{x}_1 e^{i\Theta\hat{L}_2} e^{-i\Theta\hat{L}_2} \hat{x}_1 e^{i\Theta\hat{L}_2}$$

$$= \left(e^{-i\Theta\hat{L}_2} \frac{\hat{a}_1 + \hat{a}_1^\dagger}{2} e^{i\Theta\hat{L}_2} \right)^2$$

$$= \frac{1}{4}\left(\cos\frac{\Theta}{2} \hat{a}_1 + \sin\frac{\Theta}{2} \hat{a}_3 + \cos\frac{\Theta}{2} \hat{a}_1^\dagger + \sin\frac{\Theta}{2} \hat{a}_3^\dagger \right)^2$$

$$= \left(\hat{x}_1 \cos\frac{\Theta}{2} + \hat{x}_3 \sin\frac{\Theta}{2} \right)^2$$

$$= \cos^2\frac{\Theta}{2}\hat{x}_1^2 + \sin^2\frac{\Theta}{2}\hat{x}_3^2 + 2\cos\frac{\Theta}{2}\sin\frac{\Theta}{2}\hat{x}_1\hat{x}_3$$

$\because [\hat{x}_1, \hat{x}_3] = 0$

$$= 4|\alpha|^2 \Big({}_1\langle\psi|\hat{x}_1^2|\psi\rangle_1 \cdot \underbrace{{}_3\langle 0|0\rangle_3}_{=1} \cos^2\frac{\Theta}{2} + \underbrace{{}_1\langle\psi|\psi\rangle_1}_{=1} \cdot {}_3\langle 0|\hat{x}_3^2|0\rangle_3 \sin^2\frac{\Theta}{2}$$

$$+ 2\,{}_1\langle\psi|\hat{x}_1|\psi\rangle_1 \cdot \underbrace{{}_3\langle 0|\hat{x}_3|0\rangle_3}_{0} \cos\frac{\Theta}{2}\sin\frac{\Theta}{2} \Big)$$

$$\frac{\hat{a}^2 + \hat{a}^{\dagger 2} + \hat{a}\hat{a}^\dagger + \hat{a}^\dagger\hat{a}}{4}$$

$$= 4|\alpha|^2 \Big({}_1\langle\psi|\hat{x}_1^2|\psi\rangle_1 \cos^2\frac{\Theta}{2} + \frac{1}{4}\sin^2\frac{\Theta}{2} \Big)$$

ロスを受けた分，真空場が侵入?

1.10 ウィグナー関数