Monte Carlo Simulations in Finance | Pricing Options

1/12/25

Video covers: "What Monte Carlo method is" (1), "How it works" (2), "How it differs from Traditional Methods for pricing options" (3) "When should you use it, when not." (4)

(1) "What is Monte Carlo"

- Monte Carlo methods used in Finance to value & analyze complex financial instruments
  - by simulating the various sources of uncertainty effecting their value
    - then determine their average value over a large range of resultant outcomes

=> MC methods (Finance) =
- simulate many scenarios of uncertainty effecting asset or instrument
- then estimate it's value by averaging outcomes across simulations.

- Rather than using a lot of financial theory to price complex financial instruments
  - Instead build computer simulation of moving parts
    - Run model multiple times
      - Average to get "fair value" of the financial instrument.

=> Instead of heavy financial theory;
- simulate instruments key components
- run many times (law large numbers), & average result

- Monte Carlo option pricing method, 1977 - Phelim Boyle
  - often considered method of last resort by market practisioners
    - useful for valuing options with multiple sources of uncertainty or complex features (difficult / impossible to value through black Scholes style P.D.E / lattice based approach)

=> MC often Last Resort, MC methods useful for options w. multiple uncertainties / complex features

- widely used in valuing path dependant structures like look back & Asian options
  - & in "Real Options Analysis"

=> widely used in: $\underbrace{\text{Look back, Asian options}}_{\text{path dependant}}$, Real Options Analysis

Historically: MC methods too slow, faster computers = less of a concern

IMPORTANT NOTE: Quantum computers could be used, Q enviroment is random!

② "How does it Work"

- Gambling example: 2 dice, range of results: 2 -> 12
  - 1 dice, even probability distribution, 1 -> 6 each 1/6 probability (16.7%)
    - 2 dice, most likely to get 7
      - (1 way to get '2', 1 way to get '12': 6 ways to get '7') ('7' prob = 16.7%)
        - have to consider each dice seperately (1, 3 ≠ 3, 1)
          - 2 dice = bell curve distribution
            - 2 ways to work out probability:
              1) mathematical - Analyze problem, numerically solve
              2) Brute force - roll dice 5,000 times, plot results

- 1 die has uniform probability distribution, 2 dice produce bell-shaped distribution
  - can determine mathematically (analyze combinations) or empirically (simulate many roles, plot results)

- MC model use 2nd (Brute force/empirical) approach to price derivatives
  - MC method = simulating underlying process
    - followed by various risk factors affecting the price of the derivative you are trying to price

Monte Carlo models use empirical (brute force) approach
- simulate the underlying process & relevant risk factors - to estimate price of derivative

- First you generate a price path for the underlying?
  - based upon the random movements of various risk factors
    - & calculate the pay off from the derivative based on that path

- simulate price path for the underlying (based off random movements)
  - then calculate the derivatives payoff from the price path

- repeate previous steps, generating numerous sample values of the payoff
  - from the derivative in the future
    - calculate average of sample payoffs
      - giving an estimate of the derivatives expected payoff (in risk neutral world)
        - discount the payoff at risk-free rate
          - results in fair value of option today

- Repeate simulations many times to generate future payoff samples
  - average to estimate the derivates expected payoff (under risk neutral measure)
    - Discount that value @ risk free rate to obtain fair value today

- N. iterations carried out is at Discretion of operator
  - depends on required accuracy • (more = more accurate)

N. iterations chosen by user, depends on the accuracy level needed

- used to calculate standard deviation of the discounted payoff generated by the simulation
  - uncertainty of derivative is inversely proportional to $\sqrt{N.\text{iterations}}$: $\sigma(\$) \propto \frac{1}{\sqrt{N}}$

- Uncertainty = Standard deviation (σ) of discounted simulated payoffs;
  - derivative uncertainty inversely proportional to $\sqrt{N.\text{iterations}}$: $\sigma(\$) \propto \frac{1}{\sqrt{N}}$

(4) "When should you USE MC method"

- Monte Carlo method can have great flexibility
  - complex stochastic processes including; jumps, mean reversion, or both can be accomodated
  - different distributions, including changing distributions can be assumed

MC methods are highly flexiable, allowing complex stochastic features: jumps, mean reversion
- & different distributions (including changing distributions) can be assumed

- MC method generally used when there are 3 or more stochastic variables
  - (make a P.D.E or Lattice based approach difficult/impossible)
    - MC in these situations can be more efficient than other approaches
      - time taken for MC simulation ↑ in linear manner w N. variables
        - other methods, time taken ↑ exponentially w N. variables

- MC methods typically used when ≥ 3 stochastic variables make PDE or Lattice impractical
  - MC more efficient due to scaling. • MC computational time scales linearly with N. variables
    - others scale exponentially with N. variables

MC method is brute force approach to pricing options
- Doesn't use a lot of financial theory
  - simply uses computer power to simulate thousands of possible price paths.

⑤ Assumptions

- have to assume distribution for driving asset
  - & structure for it's volitility & absense or existance of Jumps

- must specify assets' distribution, volitility structure & if jumps are present or not

- other methods, Black-Scholes & lattice based approaches
  - give fair value for the option
    - specify trading strategy (i.e. Delta heging)
      - which allows you to hedge your risk exposure

other methods (Black-Scholes & lattice models) provide fair value & an implied trading strategy (i.e, delta hedging) - enabling you to hedge the option's risk exposure.

⑥ Advantages

- options requiring MC method to price are close to impossible to hedge (v. difficult to hedge accurately)
  - due to this, options usually only sold when buyer pays well above fair value
    - (seller usually keeps option on their books for its entire lifespan ∴ only roughly hedge it)

Options that require MC pricing v. hard to hedge accurately - often only sold when buyers pay well above fair value. Sellers typically keep options on books for full life of contract & can only hedge them roughly

- advantages of MC method are:
  - price options where payoff depends on the final price of underlying asset on expiration date
  - also when payoff depends on the price path followed by the under-lying

MC advantages include:
- ability to price options whose payoff depends solely on the underlying's final value
- as well as those whose payoff depends on the entire price path.

MC method can simerly be used to value options where the payoff depends on value of :
- multiple underlying assets
  - "basket options" or "rainbow option"
    - in pricing those deriates, correlation between asset returns is also incoperated

MC methods can also value options whose payoff depends on multiple underlying assets (basket or rainbow options) - while incoperating correlation between asset returns

- MC methods allow for compounding in the uncertainties such as when joint probability distribution is used.
  - in the case of 2 random variables : bivariate distribution tot.

Monte Carlo methods handel compounded uncertainties by using joint probability distributions - such as bivariate distribution when 2 random variables are involved

=> above, concept generalises to any number of random variables
- giving a multi-variable distribution
  - example : pricing an option on a stock in a foreign currency
    - paths followed by underlying stocks & the exchange rate has to be modelled
      - also the correlation between these two scources of risk must be incoperated

This idea generlises to any number of random variables, producing multivariate distributions
For example : pricing stock options in foreign currency requires modelling both the stock & exchange rate paths, as well as their correlation

- The monte carlo method cannot easily handel situations where there is early exercise
  - (in these simulation)
    - a least squares MC method w backward induction approach is used.

Monte Carlo method cannot easily handel early exercise
=> a least-squares MC method with backward induction is used for such cases

7 "How to calculate the greeks"

- calculating the greeks using Monte Carlo method
  - done by first pricing the derivative
    - then recalculating the price of the derivative
      - after making a small change in the inputs
        - such as spot price
          - to calculate Delta or Volitility (if calculating Vega)
            - (whose price sensitivity we are trying to find)
              - Same N. itteration should be run in calculating new price as when used when initally pricing the derivative

To compute Greeks with Monte Carlo
- first price the derivative
  - then reprice it after making a small change to an input (i.e, spot price for delta, volatility for vega)
    - Same N. iterations must be used in both runs for consistency

"What is the Monte Carlo method? | Monte Carlo Simulation in Finance | Pricing Options"

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