Monte Carlo Option Pricing – Research key points

Source 1: "Monte Carlo Simulation"

Source 2: "What is the Monte Carlo method? | Monte Carlo Simulation in Finance | Pricing Options"

Source 1: Introduction into Monte Carlo simulations

Covers: basic principles, how it can be useful, gave 2 examples of MC simulations

Thoughts: Solid basic introduction, not focused on finance applications

- **Monte Carlo Simulations are: simulations evolving randomly**
  - Fundamental concept
- **Law of Large Numbers**
  - Average values become closer to true value with larger sample size (increasing N)
  - Result confidence increases with sample size
- **Randomness removes bias**
  - Don't need to measure all outcomes, as its time and resource consuming
  - Take a random sample and average
    - Random sample removes selective bias
- **Fluctuations from true result get smaller with increased sample size**
  - Logical, follows the Law of large numbers
    - *Clean graphic can be including in final report*
- **Possible paths can be near infinite, and thus almost impossible to model**
  - For many processes, the number of outcomes is near infinite
  - Modelling all outcomes exactly can be impossible
    - Simulate reprehensive group (to mitigate bias, employ randomness)
- **When near Infinite Number of possibilities (outcomes) USE MONTE CARLO SIMULATIONS**
  - Take a random subset, extrapolate up

Source 2: Covers MC simulations in Finance, specifically for pricing options

Covers: (1) What is MC , (2) how they work , (3) when should you use MC, (4) Assumptions, (5) Disadvantages, (6) calculating the Greeks.

Thoughts: Excellent source: in depth, accurate, covers finance applications well.

(1) – What are Monte Carlo methods?

- **MC methods (in finance):**
  - Simulate many scenarios of uncertainty effecting asset or instrument
    - Then estimate it's value by averaging outcomes across simulations

- **Instead of using heavy financial theory:**
  - Simulate instruments key components
    - Run simulations many times (Law of Large Numbers) & average results

- **Monte Carlo options pricing method developed in 1977 by Phelim Boyle**

- **MC method often last resort**
  - MC methods useful for options with multiple uncertainties / complex features

- **Mc methods widely used in**
  - Look back, Asian options, (Both path dependant)
    - And Real Options Analysis

- **Historically MC methods too slow, but with faster computers this is less of a concern**
  - Strong opportunity to mention quantum computing background!!!

(2) - How do Monte Carlo methods work?

- **Example of 1 die vs 2 dice**
  - 1 die has uniform probability distribution, 2 dice produce bell-shaped distribution
    - Can determine 2 dice outcome probability 2 ways:
      - Mathematically - (analyse combinations)
      - Empirically - (Simulate many roles, plot results)

- **Monte Carlo models use empirical (Brute Force) approach**
  - Simulate underlying process & relevant risk factors
    - To estimate price of derivatives

- **Simulate price path for the underlying (based off random movements)**
  - Then calculate the derivatives payoff from the price path

- **Repeat simulation many times to generate future payoff samples**
  - Average to estimate the derivatives expected payoff
    - Under risk neutral measure

            - Discount that value at the risk-free rate to obtain fair value today

- **Number of iterations chosen by user**
    - Depends on the accuracy level needed

- **Uncertainty (Δ), is Standard deviation (σ) of discounted simulation payoffs**
    - Derivative uncertainty (Δ = σ) inversely proportionally to sqrt N. iterations
        - *Make clean equation for report*

(3) – When should you use Monte Carlo Methods

- **MC methods are highly flexible, allowing complex stochastic features:**
    - Jumps, mean reversion
        - & different distributions (including changing distributions) can be assumed

- **MC methods typically used when >/ 3 stochastic variables (>/ means x or more)**
    - Make PDE or Lattice approaches impractical
        - MC methods more efficient due to scaling
            - MC computational time scales linearly with N. variables
            - Others scale exponentially with N. variables (PDE, Lattice)

- **MC method is the brute force approach to pricing options**
    - **Doesn't use a lot of financial theory**
        - **Simply uses computer power to simulate thousands of possible price paths.**

(4) Assumptions made in Monte Carlo methods

- **Must specify asset distributions, Volatility structure, & absence or existence of Jumps**
    - In reference to the driving asset

- **Other methods (Black-Scholes & Lattice models)**
    - Provide fair value and imply a trading strategy
        - I.e. delta hedging – enabling you to hedge the options risk exposure

(5) Advantages of the Monte Carlo method

- **Options that require MC pricing are very hard to hedge accurately**
    - Open only being sold when buyers pay well above fair market value
        - Sellers typically keep options on books for full life of contract
            - Can only hedge them roughly

- **MC advantages include:**
    - Ability to price options whose payoff depends solely on the underlying's final value
    - As well as those whose payoff depends on the entire price path

- **MC methods can also value options whose payoff depends on multiple underlying assets**
    - i.e. basket or rainbow options

        - While incorporating correlations between asset returns

- **MC methods handle compounded uncertainties by using joint probability distributions**
    - Such as bivariate distribution when 2 random variables are involved

- **Above idea generalises to any N. of random variables, producing multivariate distributions**
    - E.g. pricing a stock option in foreign currency requires modelling both the stock & exchange rate paths
        - As well as their correlation

- **Monte Carlo metho cannot easily handle early exercise**
    - A least-squares MC method with backward induction is used in such cases

(6) How to calculate the Greeks

- **To computer Greeks with Monte Carlo**
    - First price the derivative
        - Then reprice it after making a small change to an input
        - (i.e., spot price for delta, volatility for Vega)
            - Same N. iterations must be used in both runs for consistency