**Topics: Normal distribution, Functions of Random Variables**

1. **The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?**
2. 0.3875
3. **0.2676**
4. 0.5
5. 0.6987

**Ans:**

We have a normal distribution with μ = 45 and σ = 8.0.

Let X be the amount of time it takes to complete the repair on a customer's car.

To finish in one hour, you must have X ≤ 50, so the question is to find Pr(X> 50).

Pr(X > 50) = 1 - Pr(X ≤ 50).

Z = (X -μ)/ σ = (X - 45)/8.0

Thus, the question can be answered by using the normal table to find Pr(X ≤ 50)

= Pr(Z ≤ (50 - 45)/8.0)

= Pr(Z ≤ 0.625)

=73.4%

Probability that the service manager will not meet his demand will be **0.2676**

= 100-73.4

= 26.6% or 0.2676

1. **The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.**
2. **More employees at the processing center are older than 44 than between 38 and 44.**
3. **A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.**

**Ans:**

Pr(X > 50). Pr(X > 50) = 1 - Pr(X ≤ 50).

Z = (X -μ)/ σ

= (X - 38)/6

We have a normal distribution with = 38 and = 6.

Let X be the number of employees. So according to question

1. Probability of employees greater than age of 44= Pr(X>44)

Pr(X > 44) = 1 - Pr(X ≤ 44).

Z = (X -μ)/ σ

= (X - 38)/6

Thus the question can be answered by using the normal table to find Pr(X ≤ 44) = Pr(Z ≤ (44 - 38)/6)

= Pr(Z ≤ 1)=84.1345%

Probability that the employee will be greater than age of 44

= 100-84.1345

**=15.86%**

So, the probability of number of employees between 38-44 years of age

= Pr(X<44)-0.5

=84.1345-0.5

**= 34.1345%**

Therefore the statement that “More employees at the processing center are older than 44 than between 38 and 44” is **TRUE**.

1. Probability of employees less than age of 30 = Pr(X<30).

Z = (X -μ)/ σ

= (30 -38)/6

Thus the question can be answered by using the normal table to find Pr(X ≤ 30)

= Pr(Z ≤ (30 -38)/6)

= Pr(Z ≤ -1.333)

**=9.12%**

So, the number of employees with probability 0.912 of them being under age 30.

= 0.0912\*400

**=36.48(or 36 employees).**

Therefore, the statement B of the question is also **TRUE.**

1. **If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.**

**Ans:**

As we know that if X ∼ N (µ1, σ1^2), and Y ∼ N (µ2, σ2^2) are two independent random variables then.

**X + Y ∼ N (µ1 + µ2, σ1^2 + σ2^2)**, and

**X − Y ∼ N (µ1 − µ2, σ1^2 + σ2^2).**

Similarly, if

Z = aX + bY, 1

where X and Y are as defined above,

i.e. Z is linear combination of X and Y, then

**Z ∼ N (aµ1 + bµ2, a^2σ1^2 + b^2σ2^2).**

Therefore, in the question

2X1~ N (2 u,4 σ^2) and X1+X2 ~ N (µ + µ, σ^2 + σ^2) ~ N (2 u, 2σ^2)

**2X1-(X1+X2) = N (4µ,6 σ^2)**

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. **48.5, 151.5**
6. 90.1, 109.9

**Ans:**

**The two values of a and b, symmetric about the mean, are such that the probability of the random variable taking a value between them is**

**0.99: [ 48.48 , 151.51]**

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

**Ans:**

1. Range is Rs **(99.00810347848784, 980.9918965215122) in Millions.**
2. 5th percentile of profit (in Million Rupees) is **169.875.**
3. Probability of Divisions making a loss
4. Probability of Division 1 making a loss is **0.0477903522728147**
5. Probability of Division 2 making a loss is **0.040059156863817086**