Week 3

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Prof: Ohm

Attendance (zoom)

Agenda

1. Introductions & icebreaker
2. The dot product continued: Projections
3. Determinants & the cross product
4. Intro to Planes
5. Closer

Opener

- Introduce yourself (name, major, preferred pronouns)
- What is your favorite discontinued food item or food that is hard to get in San Diego?

Projections

Formula(s)?

What does it look like?

Note: this has applications in physics (forces), linear algebra (Gram-Schmidt process), and more!

Let's play a quick kahoot to practice!

Determinants and the Cross Product

How do we take a determinant?

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} =$$

1. a) Calculate $\begin{vmatrix} 1 & 3 \\ 3 & 2 \end{vmatrix}$

b) Calculate $\begin{vmatrix} 1 & 3 & 2 \\ 0 & 1 & 4 \\ 2 & -1 & 1 \end{vmatrix}$

The cross product

How do we set up the cross product using determinants?

Most important property: $V \times W = \quad (W \times V)$

Examples

1. Calculate the cross product of $\langle 1, 2, 2 \rangle \times \langle 1, 1, 0 \rangle$

2. Calculate $\langle 2, -1, 3 \rangle \times \langle 6, -3, 9 \rangle$

3. Calculate the area of the parallelogram formed by $\langle 1, 2, 2 \rangle$ and $\langle 3, 1, 0 \rangle$

4. Calculate the volume of the parallelepiped formed by $\langle 1, 0, 2 \rangle$, $\langle 3, 2, 1 \rangle$, $\langle -1, 2, 2 \rangle$

I recommend doing many cross product problems on your own!

Intro to planes

Equation?

Common types of plane problems?

Ex 1: Find the plane which passes through $(2, 3, 1)$, $(1, 1, 2)$, and $(-1, 1, 3)$

Ex 2: Find the plane which passes through $(2, 1, 2)$ and the line $\langle 1, 4, -1 \rangle + \langle 2, 2, 2 \rangle t$

Closer: clear skies, muddy waters

What do you feel confident about? What do you need more practice with?

Solutions

Below!

kahoot problems

1. Calculate $\langle 2, 3\rangle \cdot \langle -3, 2\rangle$

$$-6+6=\underline{0}$$

2. Calculate $\langle 1, 2, 3\rangle \cdot \langle 3, 2, 2\rangle$

$$3+4+6=\underline{13}$$

3. Calculate the magnitude of $4i+2j-k$

$$\sqrt{4^2+2^2+(-1)^2}=\sqrt{16+4+1}=\underline{\sqrt{21}}$$

4. Calculate the projection of $\langle 2, 1, 1\rangle$ onto $\langle 1, 2, 2\rangle$

$$u_{\parallel v}=\text{proj}_u v=\frac{u\cdot v}{v\cdot v}v=\frac{6}{9}v=\frac{2}{3}v=\langle 2/3, 4/3, 4/3\rangle$$

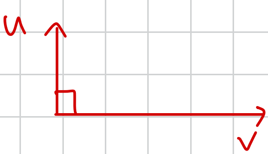
$u=\langle 2, 1, 1\rangle$

$v=\langle 1, 2, 2\rangle$

5. Calculate the projection of $\langle 4, 1, 1\rangle$ onto $\langle -1, 2, 2\rangle$

$$u_{\parallel v}=\frac{u\cdot v}{v\cdot v}v=\frac{-4+2+2}{9}v=\frac{0}{9}v=0v=\langle 4\cdot 0, 1\cdot 0, 1\cdot 0\rangle=\langle 0,0,0\rangle$$

Note: these vectors are orthogonal, so geometrically it makes sense for projection to be 0 vector



Projecting u down onto v has 0 length

6. Calculate the perpendicular component of u with respect to v, where $u = \langle 2, 1, 4 \rangle$, $V = \langle -1, 4, -1 \rangle$

$$u_{\perp v} = u - u_{\parallel v} = u - \frac{u \cdot v}{v \cdot v} v = \langle 2, 1, 4 \rangle - \frac{-2+4-4}{18} v$$

$$= \langle 2, 1, 4 \rangle + \frac{1}{9} \langle -1, 4, -1 \rangle$$

$$= \langle {}^{17}/_9, {}^{13}/_9, {}^{35}/_9 \rangle$$

7. Calculate the perpendicular component of $\langle -5, 3/2, 3/2 \rangle$ along $\langle 5/3, -1/2, -1/2 \rangle$

Tired : compute algebraically

$$u_{\perp v} = u - u_{\parallel v} = u - \frac{u \cdot v}{v \cdot v} v$$

$$u \cdot v = -5 \cdot 5/3 + 3/2 \cdot (-1/2) + 3/2 \cdot (-1/2)$$

$$= -25/3 - 3/4 - 3/4$$

$$= -100/12 - 9/12 - 9/12 = -118/12 = -59/6$$

$$v \cdot v = 25/9 + 1/4 + 1/4 = 100/36 + 9/36 + 9/36 = 118/36 = 59/18$$

$$u + \frac{59/6}{59/18} v = u + 3v = \langle -5 + 3(\tfrac{5}{3}), \tfrac{3}{2} - 3(\tfrac{1}{2}), \tfrac{3}{2} - 3(\tfrac{1}{2}) \rangle$$

$$= \langle -5 + 5, \tfrac{3}{2} - \tfrac{3}{2}, \tfrac{3}{2} \cdot \tfrac{3}{2} \rangle$$

$$= \underline{\langle 0, 0, 0 \rangle}$$

Inspired: just note that they are parallel so it will be 0

Scalar multiples

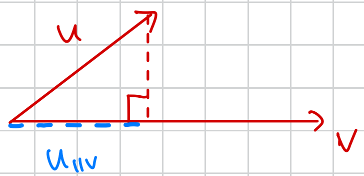
Projections

Formula(s)?

$$u_{\parallel v} = \text{proj}_v u = \frac{u \cdot v}{v \cdot v} v$$

$$u_{\perp v} = u - u_{\parallel v}$$

What does it look like?



Note: this has applications in physics (forces), linear algebra (Gram-Schmidt process), and more!

Let's play a quick kahoot to practice!

Determinants and the Cross Product

How do we take a determinant?

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc \qquad \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a(ei - fh) - b(di - fg) + c(dh - eg)$$

1. a) Calculate $\begin{vmatrix} 1 & 3 \\ 3 & 2 \end{vmatrix} = 1\cdot 2 - 3\cdot 3 = 2 - 9 = -7$

b) Calculate $\begin{vmatrix} 1 & 3 & 2 \\ 0 & 1 & 4 \\ 2 & -1 & 1 \end{vmatrix}$

$$1(1-(-4)) - 3(0-8) + 2(0-2)$$

$$= 1(5) - 3(-8) + 2(-2)$$

$$= 6 + 24 - 4 = 26$$

The cross product

How do we set up the cross product using determinants?

Most important property: $v \times w = -(w \times v)$

Examples

1. Calculate the cross product of $\langle 1, 2, 2\rangle \times \langle 1, 1, 0\rangle$

$$\begin{vmatrix} i & j & k \\ 1 & 2 & 2 \\ 1 & 1 & 0 \end{vmatrix} = \langle -2, 2, -1\rangle$$

2. Calculate $\langle 2, -1, 3 \rangle \times \langle 6, -3, 9 \rangle$

$$\begin{vmatrix} i & j & k \\ 2 & -1 & 3 \\ 6 & -3 & 9 \end{vmatrix} = \langle 0, 0, 0 \rangle$$

(they are scalar multiples)

I recommend doing many cross product problems on your own!

<u>Intro to planes</u>

Equation?

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

Common types of plane problems?

3 points, point and line, 2 lines, check coplanarity

Ex 1: Find the plane which passes through $(2,3,1)$, $(1,1,2)$, and $(-1,1,3)$
$\quad P \qquad\qquad Q \qquad\qquad R$

$$\overrightarrow{PQ} = \langle -1, -2, 1\rangle \qquad \overrightarrow{PR} = \langle -3, -2, 2\rangle$$

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} i & j & k \\ -1 & -2 & 1 \\ -3 & -2 & 2 \end{vmatrix} = \langle -4+2, -(-2+3), 2-6\rangle$$

$$= \langle -2, -1, -4\rangle$$

$$-2(x-2) - 1(y-3) - 4(z-1) = 0$$

Ex 2: Find the plane which passes through $(2,1,2)$ and the line $\langle 1,4,-1\rangle + \langle 2,2,2\rangle t$
$\quad P \qquad\qquad Q \qquad\qquad \overrightarrow{QR}$

$$\overrightarrow{PQ} = \langle -1, 3, -3\rangle$$

$$\overrightarrow{QR} = \langle 2, 2, 2\rangle$$

$$\overrightarrow{PQ} \times \overrightarrow{QR} = \langle 6-(-6), -2-(-6), -2-6\rangle$$

$$n = \langle 12, 4, -8\rangle$$

$$12(x-2) + 4(y-1) - 8(z-2) = 0$$