Poetry

Structure engineering: *The art and science of designing and making structures with economy and elegance so they can safely resist the forces to which they may be subjected.*

Three principles of engineering

- $F = ma$
- You cannot push or a rope
- To get the answer you must know the answer

3.5 sig figs, engineering notation.

Equilibrium

$\sum F = 0$, $\sum M = 0$. FBD.

Uniform loads (W); Moment = moment calculated from all forces acting on the centroid; $M = WL(L/2) = \frac{1}{2}WL^2$.

Vertical load on a hanging cable: $WL/2$
Horizontal load on a hanging cable: $\frac{WL^2}{8h}$
Don't forget a sus bridge has 2 cables.

Deformation

Hooke's law: $F = k\Delta L$, k is axial stiffness.
Stress: $\sigma = F/A$; Strain: $\epsilon = \Delta l/l_0$;
$\sigma = E\epsilon$, E is Young's modulus (or material stiffness);
$F = EA\Delta l/l_0$, $k = AE/l_0$.

Stress-strain curve of low-alloy steel

- Small strain: linearly elastic, Hooke's law applies
- Yield: plastic deformation occurs, σ_{yield} changes little as ϵ increases
- Strain hardening, necking, fracture

Strain energy: area under stress-strain curve, energy per unit volume

- Resilience: energy of the elastic part
- Toughness: energy until breaks (elastic+plastic)

Compressive strength;
Ductility: strain at fracture;
Coefficient of thermal expansion (thermal strain) $10^{-6}/K$;

Allowable stress design: $\sigma_{allowed} = \sigma_{yield}/\mathrm{FoS}$; $\sigma_{applied} < \sigma_{allowed}$ = safe.

Limit states design: probability of failure is low enough.

Dynamics

A mass on a spring being pulled downward

- $m\ddot{x} + kx = 0$
- $x(t) = A\sin(\omega t + \phi) + \Delta_0$
- $\omega = \sqrt{k/m}$, $f = \frac{1}{2\pi}\sqrt{k/m}$, $T = f^{-1} = 2\pi\sqrt{m/k}$, $\Delta_0 = mg/k$, A and ϕ depends on initial conditions
- $f \approx 15.76/\sqrt{\Delta_0}$, f in Hz and Δ_0 in mm

Free vibration

- $m\ddot{x} + c\dot{x} + kx = mg$
  - $c = 2\beta\sqrt{mk}$, β is the fraction compared to critical damping
- $x(t) = Ae^{-\beta\omega_n t}\sin(\omega_n t\sqrt{1-\beta^2} + \phi) + \Delta_0$
- Single DoF: $f \approx 15.76/\sqrt{\Delta_0}$
- Multi-DoF: $f_n \approx 17.76/\sqrt{\Delta_0}$, use Δ_0 in the middle

Forced vibration

- Force $F(t) = F_0\sin(\omega_t) + mg$
- Steady state: $x(t) = DAF \cdot F_0/k\sin(\omega t + \phi) + \Delta_0$
- $DAF = 1/\mathrm{hypot}(1 - (f/f_n)^2, 2\beta f/f_n)$
- f is based on ω, f_n is resonance frequency
- $f/f_n = 0$: DAF = 1; $f/f_n = 1$: goes to infinity for $\beta = 0$
- Experienced force: $mg + DAF \cdot F_0$

Geometry

Centroidal axis: $\bar{y} = \sum Ay/\sum A$

Second moment of area I: $\int y^2 dA$ where y is related to $\bar{y}$

Second moment of area of a rectangle with width b and height h: $bh^3/12$

- Add/subtract primitives
- After translation: $I = I_{\bar{y}} + Ad^2$

Bending of beams: $\phi = d\theta/dl$, $r = \phi^{-1}$, $\epsilon = \phi y$, $M = EI\phi$

First moment of area $Q(y)$: $\int y dA$, y is related to $\bar{y}$, integral starts from the top/bottom

- Maximized at $y = \bar{y}$

Buckling

$\mathrm{FoS} = 3.0$.

Euler buckling load: $P = \dfrac{\pi^2 EI}{L^2}$

- Use the direction with the smallest I

Buckling of thin plates: $\sigma = \dfrac{k\pi^2 E}{12(1-\mu^2)}\left(\dfrac{t}{b}\right)^2$

- Free edges: use the Euler buckling formula
- Two fixed edges + uniform stress: $k = 4$
- One fixed edge + uniform stress: $k = 0.425$
- Two fixed edges + "triangle" stress: $k = 6$

Shear buckling of webs: $\tau = \dfrac{5\pi^2 E}{12(1-\mu^2)}\left(\left(\dfrac{t}{h}\right)^2 + \left(\dfrac{t}{a}\right)^2\right)$

Truss

Truss design iteration

- Geometry
- Determine applied loads
- Analyze internal forces
- Select members
- Determine maximum displacement
- Check dynamic properties

Applied loads: Uniformly distributed load equally distributed on each joint by area/length

Solve for reaction forces using equilibrium;
Method of sections: isolate, F/M equlibrium
Method of joints: start at one end and solve for reactions for each member
Positive for tension, negative for compression

Slenderness ratio $r = \sqrt{I/A}$
Select members: $\mathrm{FoS} = 2.0$, $L/r < 200$, $\sigma_y = 350\mathrm{MPa}$ if not given;

Wind pressure: $1/2\rho v^2 c_D$, $c_D = 1.5$ for bridge;
$W_{wind} = 2.0\mathrm{KPa}$; Area to consider: truss, handrail, etc.; Only consider one face.

Truss deflection

- Solve the truss, F for each member
- Each member's elongation $\Delta L = \epsilon L = FL/EA$
- Apply a dummy load P^* in the same direction of deflection to solve for
- Solve the truss under the dummy load, each F^*
- Virtual work $P^*\delta = \sum F^*\Delta L$, solve for deflection δ

Beam

Axial $N(x)$, shear $V(x)$, bending $M(x)$, deflection $\delta(x)$

Solve for reaction forces;
SFD: integrate applied loads (including reaction forces, up is positive), endpoints are zero; positive y up;
BMD: integrate SFD, endpoints are zero; Bottom tension = positive, positive y down;

Flexural stress: $\sigma = My/I$

Beam deflection

- $\phi = M/EI$, integrate ϕ -> $\theta = \frac{dy}{dx}$, integrate θ -> δ
- Moment area theorem 1: $\Delta\theta$ = area under ϕ
- Moment area theorem 2: tangential deviation equals the area under the Mx/EI graph

Shear: $\tau = VQ/Ib$

- Definition: force divided by parallel area
- In the material vs. At glue/nail joints

Concrete

$\mathrm{FoS} = 2.0$ concrete and $1/0.6$ for steel. $E_s = 200000\mathrm{MPa}$ and $\sigma_y = 400\mathrm{MPa}$ for steel.

Reinforced concrete beam, maximum width b, minimum width b_w, height h, distance from max compression to the centroid of tensile reinforcement steels d, stirrups (shear reinforcement) spacing s.

Concrete compressive strength in MPa f_c', $E_c = 4730\sqrt{f_c'}$ in MPa, tensile $f_t' = 0.33\sqrt{f_c'}$ in MPa.

$n = E_s/E_c$, $\rho = A_s/bd$, $k = \sqrt{(n\rho)^2 + 2n\rho} - n\rho$, $j = 1 - k/3$, all dimensionless.

Calculate jd. Experienced stress $\sigma_s = \dfrac{M}{A_s jd}$, $\sigma_c = \dfrac{M}{A_s jd}\dfrac{k}{1-k}\dfrac{1}{n}$. Steel yield at $M_y = A_s f_y jd$.

$d_v = 0.9d$. Yield strength $f_y = \sigma_y$.

A_v: for example, two 10M stirrups in the cross section means a A_v of $2 \times 100\mathrm{mm}$.

Failure shear: crushing $V_{max} = 0.25 f_c' b_w jd$; without stirrups $V_c = \dfrac{230\sqrt{f_c'}}{1000 + 0.9d} b_w jd$, with stirrups $V_c = 0.18\sqrt{f_c'} b_w jd$;
$V_s = \dfrac{A_v f_y jd}{s}\cot 35^\circ$, failure $V_t = V_c + V_s$, safe $V_r = 0.5V_c + 0.6V_s$.

Safe s: $s = \dfrac{0.6 \cdot A_v f_y jd \cot 35^\circ}{V - 0.5 \cdot 0.18\sqrt{f_c'} b_w jd}$

Evaluating concrete:

- SFD, BMD
- Evaluate flexural + rebars
- Use V_c without stirrups when $\dfrac{A_v f_y}{b_w s} < 0.06\sqrt{f_c'}$ and with stirrups otherwise

- Calculate V_s, V_t, V_{max}, capacity $V = \min(V_{max}, V_t)$

Concrete design

- SFD, BMD
- Check if $V_{max}/2.0$ works, else change b_w and/or d
- Check if $V_c/2.0$ without stirrups works, good => design complete
- Calculate V_s with minimum s, calculate V_c and V_r and see if it works
  - OR: directly calculate s from the V_r equation
- Not working: reiterate the previous step with safe s

Timber

Anisotropic material, stiffer + higher strength at "vertical" direction in a tree.

Use the 5th-percentile strength in design with $\mathrm{FoS} = 1.5$.

Use the 50th-pencentile strength in deflection calculation.