Mathematical Modelling

Week 1

Homework 2.1:

In the simplest model above, we have assumed that rabbits live forever. Which value must we take for R if each rabbit lives on average 5 years and if each rabbit couple has on average 4 youngs every year? You can assume a balanced number of male and female rabbits.

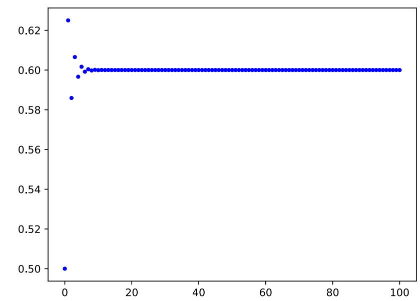
Solution:

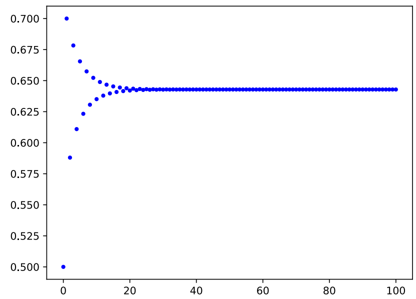
As for each rabbit there are 2 young per year its logical to say that R = 3 as it seems 3 rabbits will remain alive after each year, at least when considering the next year. However, if rabbits live for 5 years we want there to be 1 less rabbit after 5 years so by subtracting $\frac{1}{5}$of a rabbit each year we can achieve this so R = 2.8.

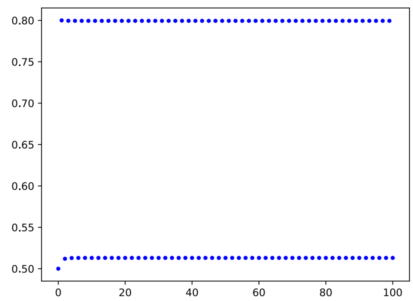
Homework 2.2:

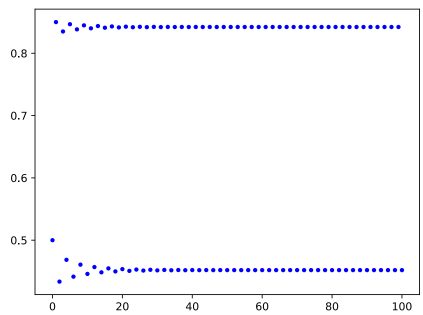
Run your program Compare the figure (a so-called bifurcation plot) with the numerical results you obtained in coding task 2.2 for the asymptotic behaviour, and explain how they match.

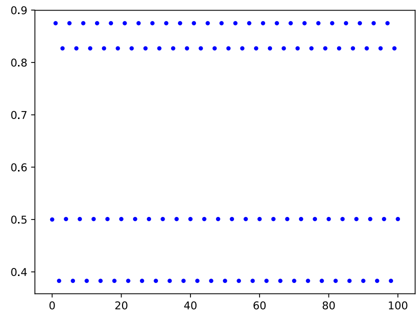
The bifurcation plot and the log plots all seem to differ greatly in appearance. However, when investigating further we notice that they all have asymptotes at the same level (Around 0.64). This is depicted by the various images below. The images are in the following order N = 2.5, 2.8, 3.2, 3.4, 3.5, 3.9 followed by the image of the bifurcation plot.

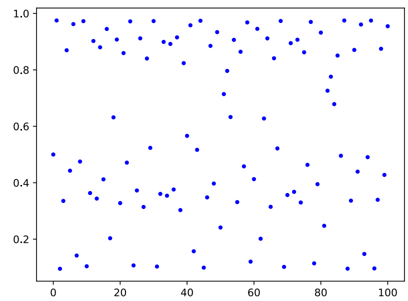


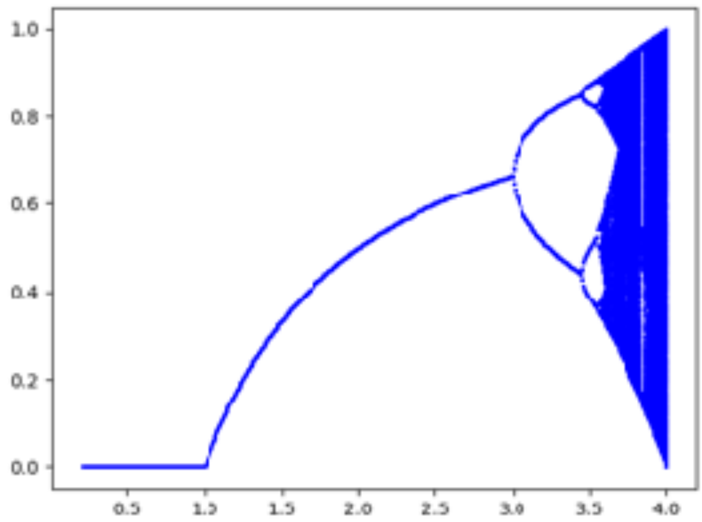












Homework 2.3:

If the Ricker Model is such that $R < 0$ this means that the population is decreasing. Thus if the parameter Q is not sufficient then the population will go extinct.

$$N_{stat} = N_{stat}exp[R(1 - N_{stat})] + Q$$

$$\implies N_{stat} - Q = N_{stat}exp[R(1 - N_{stat})]$$

$$\implies 1 - \frac{Q}{N_{stat}} = exp[R(1 - N_{stat})]$$

$$\implies ln(1 - \frac{Q}{N_{stat}}) = R(1 - N_{stat})$$

$$\implies \frac{ln(1 - \frac{Q}{N_{stat}})}{R} = 1 - N_{stat}$$

$$\implies \frac{ln(1 - \frac{Q}{N_{stat}})}{R} - 1 = -N_{stat}$$

$$\implies N_{stat} = 1 - \frac{ln(1 - \frac{Q}{N_{stat}})}{R}$$

Thus for a steady state to exist we must have that $\frac{Q}{N_{stat}} < 1$, and thus $Q < N_{stat}$. $\implies$

Homework 10.1:

- Less repeated code
- Modularity
- Encapsulation

This was completed on my home (Windows) computer and when I attempted to complete the homework today on my MacBook pyCharm claimed that my packages were not installed - although I am certain they are. Also, I originally tried to complete this in LaTex but again LaTex is not working properly on my MacBook - I will make sure to fix this as soon as possible. This prolonged the time it took me to complete this and unfortunately I have not completed the final task - apologies.