Traffic Flow

Abstract

The purpose of this essay is to analyse the flow of traffic within road networks throughout the Earth's surface. We will be exploring both discrete and deterministic ODE models of traffic flow in order to gain an understanding of how we can best analyse and accurately model traffic flow on our roads. When analysing the activity of individual vehicles we will consider factors such as velocity, headway and 'car density' of the road in which the vehicle is travelling on and the distances it has between adjacent cars; we will investigate how these parameters affects traffic flow in a real-life scenario.

1 Introduction

Traffic flow has been a topic of interest in both Mathematics and general life for many years. Due to this it has been studied from many different view points and from these view points there have been two essential independent approaches macroscopic and microscopic (dynamical models). Throughout this essay we will be discussing and analysing the microscopic approach (dynamical models), exploring the results and gaining an understanding of our observations when modelled using certain modelling assumptions which we will explain later.

We are concerned with the latter (dynamic) approach. When considering the dynamic approach the global features of traffic flow are to be explained from the collective motion of individual vehicles. We shall base this model on the fact that each driver controls his vehicle according to his preceding vehicle's motion. Most models take the assumption that the acceleration of a vehicle is proportional to the velocity of the successive vehicle(s), and therefore have essentially used the first-order differential equation of motion of an individual vehicle. These models often have difficulty in explaining the specific property of traffic flow, which has two distinct behaviours free and congested flow - it is necessary to model both in a complete model.

The importance of understanding traffic flow, and at least having a basic model is paramount in every day society. Whether this is for general convenience when judging when best to leave the house to meet friends at the cinema, or judging how quickly an the nearest ambulance can reach someone in need under current traffic conditions. However, whilst congestion is obvious in roads, it also extends to various other networks such as the Internet - so it is obviously important to understand the characteristics of traffic jams and gauge how traffic flow is likely to be under given circumstances.

2 Road Networks

The pattern of the flow of traffic through a network, and the behaviour of traffic flow in general is the consequence of a subtle and complex interaction between drivers. For example, in a road network we would normally expect each driver to attempt to choose the most convenient route, and this choice will depend upon the choices of routes made by others. This mutual interdependence makes it difficult to predict the effects of changes to the road network, such as the construction of a new road or the introduction of tolls in certain places.

Due to this, and the unique characteristics of each road (and many other factors) road networks in the UK and other countries are often extremely complex and unpredictable, and it is exceedingly difficult to accurately model them completely. The fluctuation of car density, varying speed limits and number of carriageways are just a few of what is a multitude of factors we would have to accurately record and consider in order to provide a precise model.

Thus throughout the following investigation, and analysis we shall assume a simplified model of a road network. However, we will provide pertinent data for the network in order to provide an accurate insight into the activity of individual vehicles and traffic as a whole on our simplified network, and by extrapolating this data gain a reasonable insight into the flow of traffic on more complicated road networks.

2.1 Traffic flow in a simple road network

In the simplest models of traffic flow, we model traffic flow on a uniform straight road, with arbitrary length. The characteristic of the traffic flow would be especially difficult to model if the number of cars on the network was to differ thus we shall ensure that this is constant, consequently we also ensure the car density p on the network is also constant $p = \frac{N}{L} = \frac{total\, number\, of\, cars}{length\, of\, road\, (or\, road\, network)}$. In order to keep N consistent when a car drives past a detector at the end of the road we place that car at the start of the road, this essentially equates to cars driving on a circular road - this is not realistic but allows for far easier modelling. Whilst this seems far removed from activity on a real road, this greatly simplifies the act of modelling the road network and we are still able to obtain useful data about the flow of traffic.

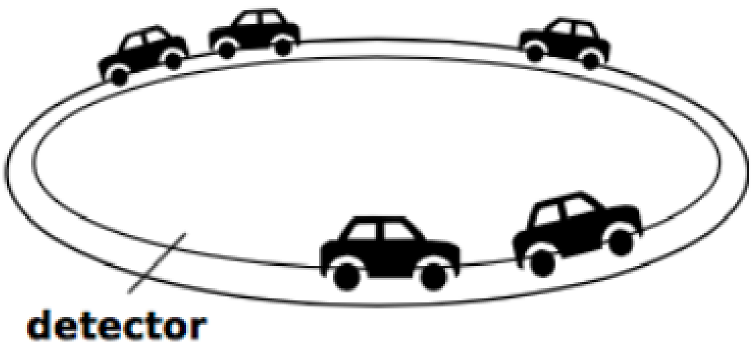

Figure 1: Cars on a road with periodic boundary conditions (circular road) and a detector to measure the flow. Cars drive in one direction only, and cannot overtake. Source: Assignment document

In this simple system the flow of traffic is simply the number of cars travelling past a set point in the system per unit of time, this equates to $q = \frac{\Delta N}{\Delta T} = \frac{vehicles\, that\, have\, passed\, the\, point}{time\, passed}$.

Whilst this is certainly a simple system we are still able to gain approximations to the two quantities which are considered most important when modelling traffic flow - from which we can make somewhat accurate guesses on how traffic flow is when the values of car density and flow of traffic are at certain points.

3 Traffic Flow Properties and Time-space Diagrams

Traffic flow is generally constrained along a one-dimensional pathway (e.g. a travel lane). A time-space diagram can be constructed from studying the movement and interaction of vehicles from point to point, and assigning a vehicle a distance value x for each time value t. It shows graphically the flow of vehicles along a pathway over time, with time being displayed along the horizontal axis, and distance along the vertical axis. Traffic flow is given by the individual traffic lines of each individual vehicle, the gradient of the tangent to the trajectory line is given by $grad = \frac{\Delta x}{\Delta t}$ which by considering dimensional analysis gives $grad_{units} = \frac{m}{s} = ms^{-1}$ so the gradient is a speed. Clearly, the greater the gradient (steeper) the trajectory lines - the faster the flow of traffic. Parallel trajectories indicate vehicles are following one another, and intersections on the graph occur when a vehicle overtakes another, the diagrams are useful tools for analysing the traffic flow characteristics of a given roadway segment over time.

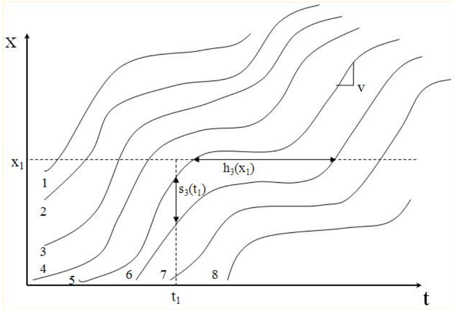

Figure 2: Time space diagram. Source: https://en.wikipedia.org/wiki/File:Time_Space_Diag_Figure_1.JPG

There are three main variables that are necessary to visualise a traffic stream: speed, car density and flow. The following are graphs which exhibit relationships between the 3 variables - following the diagram we shall elaborate on each property individually. The following is the fundamental diagram of traffic flow,

This is simply a diagram that presents the relation between traffic flux and traffic density. The basis of which is formed by a macroscopic model involving traffic flux, traffic density and speed. Using this as a basis for our own diagrams later it will be useful in predicting the capability of a road system.

3.1 Speed (v)

Speed is simply the distance covered per unit time for the vehicle in question.

For practical applications time mean speed is often used to calculate the mean speed. Time mean speed is 'the average speed of a traffic stream passing a fixed point along a roadway measured over a fixed period of time'. Time mean speed can be calculated by recording velocities using loop detectors, summing the speeds recorded and dividing the total by the number of recordings. In mathematical terms, time mean speed v_t is given by,

$$v_t = \frac{1}{n}\sum_{i=1}^{n} v_i$$

where n is the number of recordings and each v_i is an individual recording of a cars speed. Time mean speeds do not provide reasonable travel time estimates unless the speed of the point sampled is representative of the speed of all other points along a roadway segment, or there are a large number of closely-spaced detectors along the segment. Thus it does not account for all the speeds of each individual vehicle across the roadway.

Another method of providing the average speed of a traffic stream is space mean speed. Space mean speed is 'the average speed of a traffic stream computed as the length of roadway segment divided by the total time required to travel the segment'. Space mean speed is required in order to compute accurate travel times. Space mean speed may be sampled using data from an AVL (automatic vehicle location) equipped vehicle, or a probe vehicle. Space mean speed is given by,

$$v_s = \frac{1}{t_s}$$

where t_s is the average travel time, travel time may be expressed as the sum of all travel time recordings divided by the number of recording i.e. $t_s = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{v_i}$. Thus space mean speed is given by,

$$v_s = \frac{n}{\sum_{i=1}^{n}}$$

If for our simple system, we consider a car driving at constant speed then the above is not necessary. We may denote this constant speed as v, and the maximal rate at which it accelerates

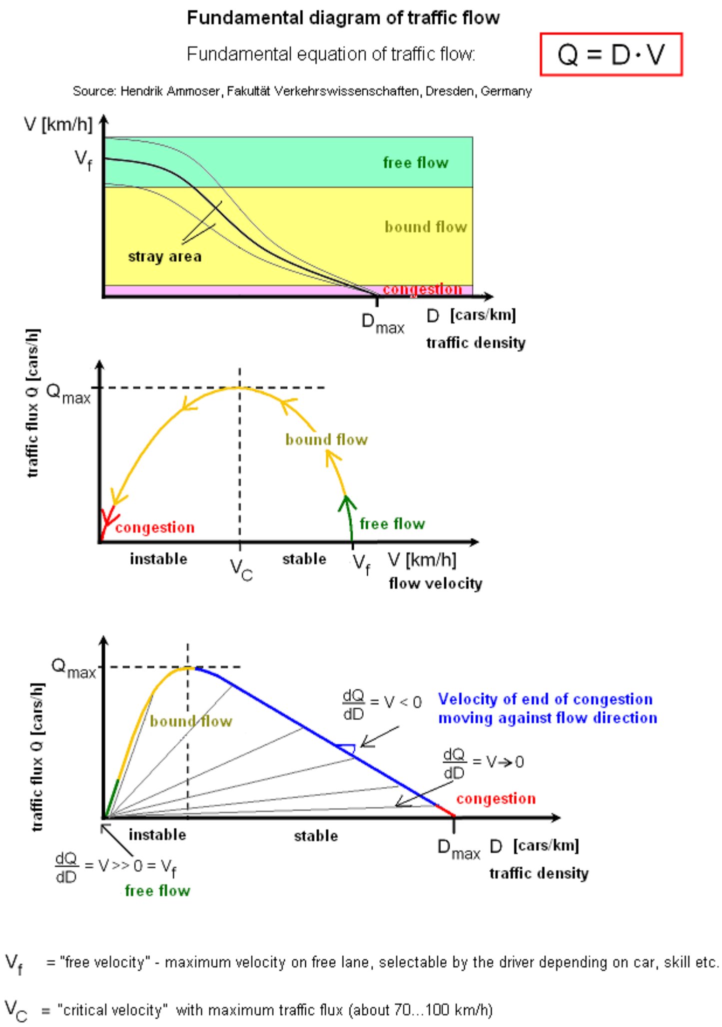

Figure 3: The Fundamental diagram of traffic flow. Source: https://upload.wikimedia.org/wikipedia/commons/5/59/Fundamental_Diagram.PNG

as a_{max}, from this we can conclude various information about the cars motion. We have, **Q1**

$$v_{final} = at + v_{initial} \tag{1}$$

where v_{final} is the final velocity, a is the acceleration (or deceleration), t is the time over which this occurs and $v_{initial}$ is the initial velocity.

From 1 substituting in the values for acceleration ($a = a_{max}$), initial velocity ($v_{initial} = v$), final velocity is 0 ($v_{final} = 0$) and time for which it occurs ($t = \Delta t$) we can derive the following equality,

$$0 = a_{max}\Delta t + v$$

from which we can rearrange to derive the time it will take for the car to come to a halt under these circumstances, which is given by,

$$\Delta t = -\frac{v}{a_{max}}. \tag{2}$$

We also have the following equation:

$$s = v_{initial}t + \frac{1}{2}at^2 \tag{3}$$

where v_{final} is again the final velocity, a is the acceleration (or deceleration), t remains the time over which this occurs, $v_{initial}$ is still the initial velocity and s represents the displacement or distance travelled by the car.

This is useful as it allows us to form an expression for the distance the car will travel in these circumstances - and hence given parameters we can determine whether the car collides with a car ahead (if this where in the scenario that a car ahead immediately stopped), which is useful to know as it gives each driver an idea of the braking distance to employ when driving at certain speeds. This expression for the braking distance that the car will travel is given by the following derivation, from substitution into 3 (specifically using $s = \Delta x$ for the distance travelled before the car stops $t = \Delta t = -\frac{v}{a_{max}}$ for time, $v_{initial} = 0$, $v_{final} = v$ and $a = a_{max}$. From both 3 and 2 we can derive an expression for Δx,

$$\begin{aligned}\Delta x &= 0\,(\Delta t) + \frac{1}{2}\,(a_{max})\,(\Delta t)^2 \\ &= \frac{1}{2}\,(a_{max})\left(-\frac{v}{a_{max}}\right)^2 \\ &= \frac{v^2}{2a_{max}}\end{aligned}$$

$$\Delta x = \frac{v^2}{2a_{max}} \tag{4}$$

Clearly speed is useful to consider regardless of the model we are considering. We will use the equations derived thus far in order to formulate an expression for the flow q past a fixed detector (in the simple model we have discussed) given that the road length is given by the value of l_c.

Flow of traffic is simply the number of cars travelling past a fixed point per unit time, we can use this to simplify the equation for flow in our simple system.

Currently the equation from above is $q = \frac{\Delta N}{\Delta T}$ by considering the flow for one second this simplifies to $q = \Delta N$ where q is flow, ΔN is the number of cars passing the detector and ΔT is the time in which this occurs.

The number of cars passing per second will be $\frac{s}{\Delta L}$ where s is the distance the car moves and $\Delta L = l_c +\ Deltax$ which is the sum of the car length and the distance between each car - thus ΔL is the space for each car.

Clearly $s = v_0$ where v_0 is the speed the car is travelling at, this is because speed is simply the distance moved per second.

Thus we have the following derivation,

$$q = \frac{\Delta N}{\Delta t}$$

considering traffic flow in one second and using 4 we have

$$\begin{aligned}q &= \frac{\Delta N}{\Delta t} \\ &= \Delta N \\ &= \frac{v_0}{\Delta L} \\ &= \frac{v_0}{\Delta x + l_c} \\ &= \frac{v_0}{\frac{v_0^2}{2a_{max}} + l_c} \\ &= \frac{v_0 2a_{max}}{v_0^2 + l_c 2a_{max}}\end{aligned}$$

which is a representation of the flow of traffic using v_0, a_{max} and l_c alone.

By using this representation of traffic flow we can gain an idea of how traffic flow will differ in our simple system for various values of v_0 - providing we provide some typical values of a_{max} and l_c - in this case we will set $a_{max} = 5$ and $l_c = 5m$ to see how traffic flow differs for various speeds. Below is a visual representation of the fluctuation of traffic flow

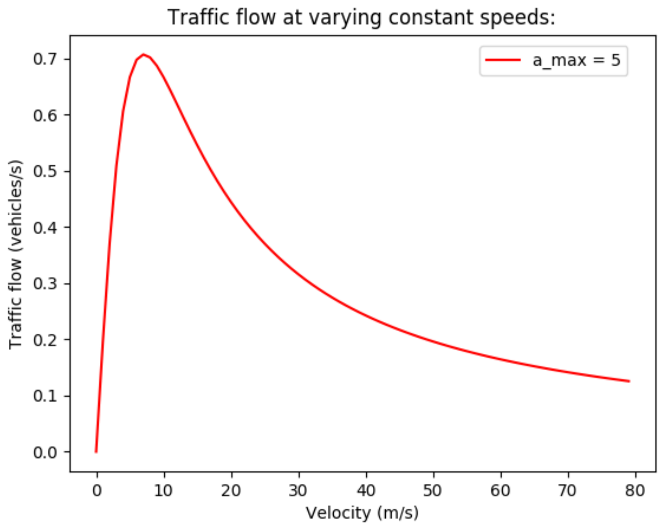

Figure 4: The effect of speed on traffic flow, with car length set to $5m$, and maximum acceleration set to $5ms^{-2}$.

From this graph, it is clear to see that as velocity increase, up until the velocity corresponding with maximum flow, flow also increases. In this graph the velocity associated with the global maximum is approximately 10m/s. After this point, velocity continues to increase, flow decreases.

This leads us on nicely to our next equation $q = pv$ which emphasises blatantly the importance of car density and speed in traffic flow. We will discuss this in further detail later however, for now this provides us with a clear relationship. 'Observation on limited access facilities suggests that up to a maximum flow, speed does not decline while density increases. However, above a critical threshold, increased density reduces speed. Additionally, beyond a further threshold, increased density reduces flow as well.'

Clearly we see that whilst the size of the flow differs at the majority of point, the characteristics of the graph are practically identical regardless of the value of a_{max}

Clearly from the above we can see that average speed of vehicles has a huge impact on flow of traffic of said vehicles. We endeavoured to explain why this is the case earlier.

From 6 we see there is a clear relationship between flow and speed (at least in the case where a_max is set to $5ms^{-1}$ and l_c and there is nothing about these values to suggest a similar relationship wouldn't hold for other 'typical' (by this we mean realistic) values. In order to display this we shall compare multiple plots of traffic flow against velocity, where the values of a_{max} (maximums acceleration) and l_c (the length of the car) are altered. This is an important side-note as it means that whilst the acceleration is often an important factor, we do not need to consider it when meticulously when modelling the traffic flow, as the general trend will remain the same regardless of parameter for maximum acceleration.

3.2 Car Density (p)

Elaborating from earlier vehicle density is defined as the 'number of vehicles per unit length of roadway'. The two most important densities are the critical density p_c and the jam density p_j

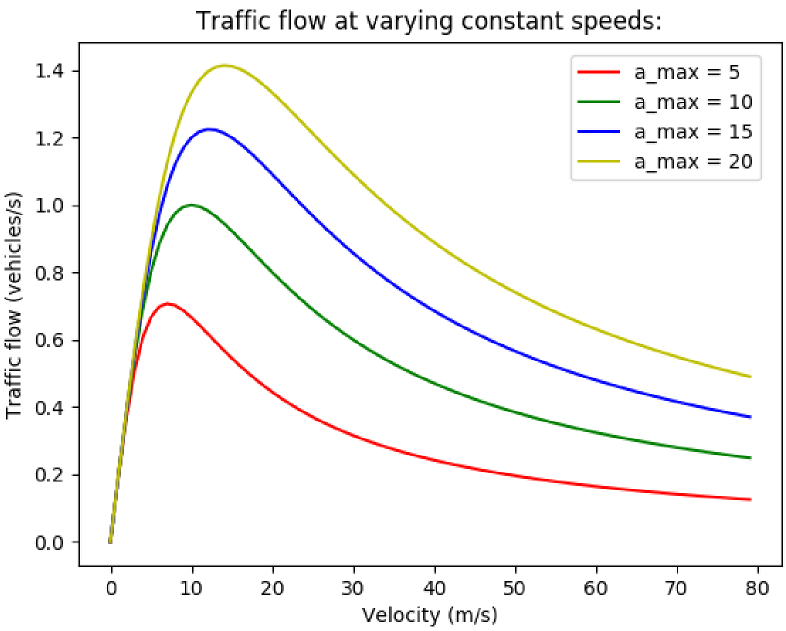

Figure 5: The effect of speed on traffic flow, with car length set to $5m$, and maximum acceleration being taken in the range $5ms^{-2}$ to $20ms^{-2}$.

No.	speed range	average speed (v_i)	volume of flow (q_i)	$v_i q_i$	$\frac{q_i}{v_i}$
1	2-5	3.5	1	3.5	2.29
2	6-9	7.5	4	30.0	0.54
3	10-13	11.5	0	0	0
4	14-17	15.5	7	108.5	0.45
	total		12	142	3.28

Figure 6: Flow Speed Relationship. Source: http://nptel.ac.in/courses/105101087/downloads/Lec-31.pdf

- the critical density being the highest achievable density under free flow, whilst the jam density is the highest achievable density. We can equate density to the $p = \frac{1}{s}$ where s is spacing, the distance between two vehicles or in our model from earlier and any model in general it may simply be put as $p = \frac{n}{L}$ where n is the number of cars and L is the length of the roadway.

3.3 Flow

Here we explore flow, and try to gain an understanding of the relationship between density and flow. Flow (q) is the number of vehicles passing a fixed point per unit of time. The inverse of flow is headway, another important factor in traffic flow - this is the time taken between two consecutive cars passing the same fixed point. In congestion, headway will remain constant but if a traffic jams form, headway h approaches infinity. $q = kv$ by dimensional analysis we see the unit is $m^{-1} \cdot ms^{-1} = s^{-1}$ as expected as this is a number of occurrences per unit time. **Q2**

When modelling traffic we will need to generate random roads, on which a modelling scenario can be established. When doing so the above properties will all be essential parameters in ensuring the model is accurate or at least provides us with useful information which we can work with. An example of which in our later model (that exhibits the importance of multiple parameters) is that when calculating the flow the flow is obviously be returned by the function doing so - and density is provided as a parameter. However, perhaps rather counter-intuitively it is essential we also return the density, despite the fact density is already passed as a parameter to this functions. This is because another function is implemented called fill road randomly, it will come as no surprise the purpose of this function is to formulate a representation of randomly filled road. This function begins by filling the road with random cars at approximate density which will be the density that was provided to us as a parameter, the function then later adjusts the value of this density to be the exact value of the density as calculated by our earlier equation $p = \frac{n}{L}$ where n is the number of cars and L is the length of the road.

The relationship discussed earlier is $q = pv$ often quoted as $q = kv$ with k replacing p for density. This relationship is simply that between flow of traffic (vehicles per unit time), space mean speed ($\frac{m}{s}$ and density p (vehicles per metre). So clearly given any 2 values for the above properties we can use them in order to calculate the third property.

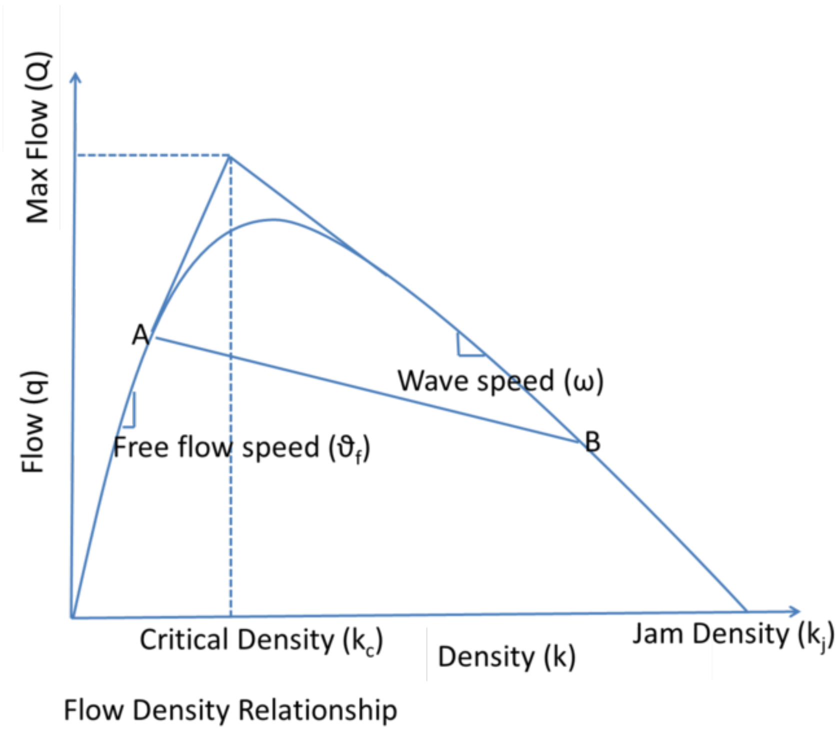

Figure 7: Flow Density Relationship. Source: https://upload.wikimedia.org/wikipedia/en/b/bc/Flow_Density_Relationship.png

The above diagram exhibits sudden major behavioural change in flow as the density changes. This provides us with further evidence for the property we have outlined earlier that as car density surpasses a critical point the speed of cars is reduced significantly.

4 Discrete Traffic Models

Was the previous model sufficient or too simple? Whilst it was clearly easy to implement and did provide us with some pertinent data, however it can clearly be improved perhaps without increasing difficulty of implementation too much as this is the main benefit of the original model.

How shall we do so?

A discrete mathematical model can provide a more accurate model and can be produced simply by extending from our basis of the original simple model. Again we will utilise the concept of a consistent car density and in doing so make use of the circular road mentioned earlier, in order to ensure cars never leave or join the road (the number of cars remains consistent). The road is split up into segments, each of length ΔL where $\Delta L = l_c + \Delta x$, this equates to saying that the distance assigned to each segment is the length of the car plus the minimum space necessary so that if a vehicle in front were to stop immediately the vehicles behind would have time to stop themselves and a collision would not occur. Each segment either contains a single car or is empty. The following is a representation of the discrete model in question.

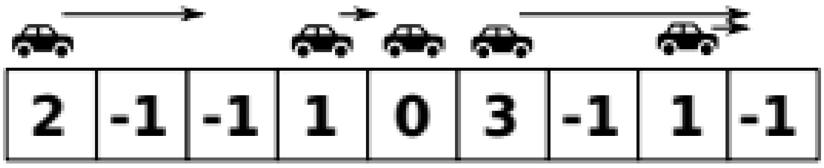

Figure 8: Discrete model road representation.

Each segment here contains a value, in the case that this value is -1 it means there is not a vehicle present in the segment, otherwise the value represents the vehicles speed v measured in segments per second we shall denote the position and speed of the n^{th} car on the road as x_n and v_n - again v_n the number of segments travelled per second. Whilst this model shall be a vast improvement on the previous model we shall still employ certain assumptions in order to avoid any unnecessary complexity. Thus in this model vehicles will not be able to overtake one another nor can they reverse.

Earlier we mentioned that there are many interdependences involved in traffic flow, as each driver considers the actions of drivers around them for example they may take a different route if they believe a certain route will be busy (as other drivers have decided to use that route) and there are many other examples. An obvious is example is that a driver is very unlikely to continue to driving at the same speed if they believe this speed will cause them to crash into the driver ahead. If we analyse the above diagram we can see an example of a driver (the 4^{th} car travelling at a speed of 3 segments per second) that is they continue at their current speed with crash into the car ahead. The fastest car on any circular system like ours will always crash into the ahead car if they do not alter their speed. Thus we should introduce some rules to the motion of each vehicle which mimics the nature of human drivers and thus prevents the crashing of cars in circumstances such as these.

In order to prevent these mishaps we will provide the following system of rules, these rules should be implemented on each time step to make sure no situation arises in which the cars can crash:

1. Each car accelerates as long as it has not reached the maximal legal speed vmax yet, $v_n \to min(v_n + 1, v_{max})$.
2. If a car gets too close to the next car, it decelerates, $v_n \to min(v_n, h_n - 1)$. Here $h_n = x_{n+1} - x_n$ is the so-called headway, the distance to the next car. This, together with rule 4 below, implies that cars will never hit cars in front of them, even if those cars brake to standstill instantaneously.

   (We vaguely discussed headway earlier).
3. Each car slows down randomly, $v_n \to max(v_n - 1, 0)$ with probability $p_{slowdown}$. This models, in a very crude way, random driver behaviour.
4. The car then moves as $x_n \to x_n + v_n$.

These rules do not necessarily capture a lot of realistic car driver behaviour, but they are simple to implement and can easily be extended to more complicated rules. Also, due to the countless amount of factors involved it is very difficult to generate realistic car driver behaviour as there will be an element of randomness to this - thus this model will suffice at least for the observations we will be making currently.

5 Time-space diagrams continued in the Discrete Model

Q3

Earlier we discussed time-space graphs and briefly explained how we can interpret their shape in order to obtain information about the characteristics of traffic flow. We also briefly discussed their role and usefulness for presenting data clearly on the 3 main variables in traffic flow - density, speed and flow. We will now provide some position - time graphs for the system modelled above, in doing so and using an analogous method to our analysis of the earlier space-time graph we should be able to formulate some general idea of the features of the new discrete model when modelling traffic. A position-time graph is simply a time-space graph with the axis roles swapped.

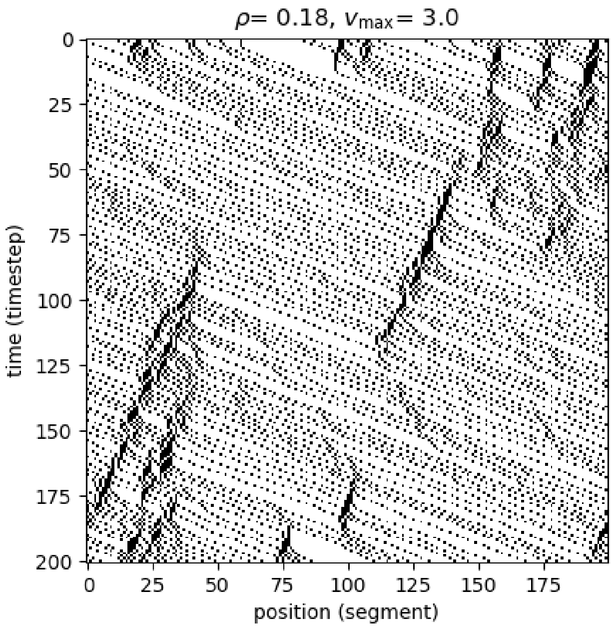

Figure 9: A position-time graph for our discrete model when $v_{max} = 3seg/s$ (segments per second) and the vehicle density of the entire system is $p = 0.18$.

The graph above 9 plots the position of each vehicle travelling on our circular road as time progresses, we have implemented the rules and modelling of the road using the discrete model we discussed earlier.

As mentioned earlier the position of an individual vehicle may be viewed by the 'trajectory lines' of the graph. When these trajectory lines are spaced out and we can clearly distinguish one from the other we can draw the conclusion that traffic is flowing more smoothly as this indicates that there aren't multiple vehicles around the same position thus we can conclude there is less congestion on the road. When the positions of vehicles are further apart the graph appears lighter. We see these regions for some of the times, such as at positions around segment 50 to 110 whereas we see darker regions more prominently for position segment 0 to 50 (at certain times).

An alternative method of considering congestion on whole rather than considering individual vehicles is to consider the horizontal lines on the graph. Each horizontal line in these 3 graphs (9, 10 and 11) represents a state of the road at a given point in time. The more periods of black each line has in the more congestion there is present in the road at that specific point in time. Then by considering how each horizontal line has changed from the previous one we can consider how the state of the road is progressing with time.

Conversely, to what we mentioned above the darker patches on the graph represent points in our circular road where congestion occurs and hence where traffic jams are likely to form, this is because for whatever reason multiple vehicles in the circular road have similar positions. From around segment 30 to 65 there at timesteps 75 to 150 and 0 to 50. The question we ask ourselves now is why has this congestion occurred? And can it be easily avoided?

For optimum traffic flow in this model each car would travel with a consistent distance between the car ahead and behind and they would each be able to continue travelling at the same speed as one another without interfering with one another. In terms of the position-time graph this would appear as a graph with each trajectory line appearing to be straight and have a consistent distance with the lines above and below.

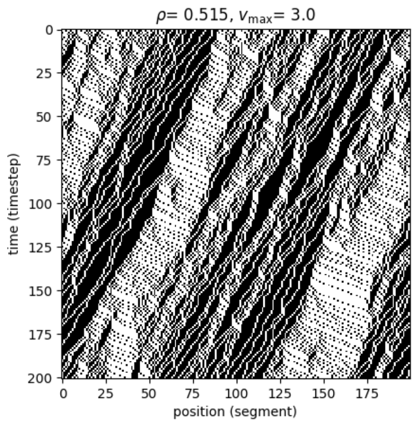

Figure 10: A position-time graph for our discrete model when $v_{max} = 3seg/s$ (segments per second) and the vehicle density of the entire system is $p = 0.515$.

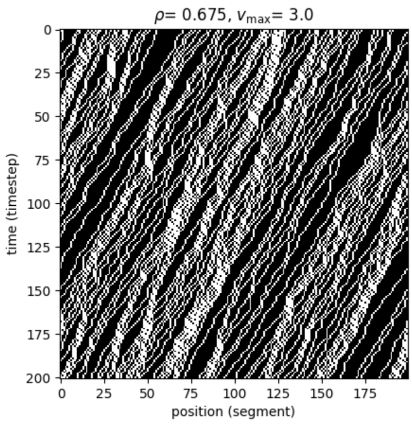

Figure 11: A position-time graph for our discrete model when $v_{max} = 3seg/s$ (segments per second) and the vehicle density of the entire system is $p = 0.675$.

Diagrams 10 and 11 displays the same information about the traffic (in principle) as the previous diagram 9 for different value of p (in these cases $p = 0.515$ and $p = 0.675$ respectively) however as we can see the change in car density p has huge repercussions when it comes to the behaviour of the traffic. Thus the plots are hugely different and display huge dark and massively frequent dark patches where traffic jams are formed - as in parts these dark patches are solid black across several positions (at the same timestep value) this suggests multiple vehicles are queueing up in back-to-back segments. Clearly the diagrams show us that the change in density across the entire system consequently causes huge changes in density in smaller regions of the system across multiple time-steps.

If we look closer at 10 and 11 (and even 9 to some extent) we see that when a traffic jam occurs (indicated by periods of darkness in horizontal lines) the next state of the road (for the next horizontal lines as this indicates a change in time) a traffic jam forms earlier than it did in the line previous to it. The reasoning for this is the fact that a traffic jam ahead is likely to cause congestion prior to the position of the traffic jam thus its likely to cause a traffic jam behind it. As time goes on we can see that the darker period gets earlier and earlier, so as time goes on the traffic jam is occurring on earlier on in the road.

Why are these traffic jams present? If each car was travelling at the same velocity it would consequently keep a constant distance between itself and the other cars on the system throughout its motion. When we explained the discrete model we mentioned a probability $p_{slowdown}$ this is the probability (in this case $p_{slowdown} = 0.1$) of a car slowing down to $max(v_n - 1, 0)$, when a driver slows down this will almost always cause another vehicle to slow down which will spark a chain affect of cars slowing down. In a traffic jam the speed of the car at the front dictates that of the cars behind it and one car slowing down can easily cause a leading car to have a slow speed, and once this traffic jam is present it is very difficult (at least in a system when car density is reasonably high) for the traffic jam to dissipate.

This probability $p_{slowdown}$ is essentially responsible for formation of spontaneous traffic jams in this case. The above only considered the cause of traffic congestion when traffic is running smoothly and then a car 'spontaneously' slows down. However, we can see that there is often a large amount of congestion in the initial state of the road or circumstances present (i.e. speeds of cars) that will lead on to the formation of congestion. This is because our road is filled randomly and the road can be filled with congestion present - this becomes more likely as car density is increased. The essential process of filling this road randomly in our model is assigning segments a random car with random speed until we have assigned all of the cars present. In order to make clear how this effects traffic flow in the following we will consider a small empty road, and explain how our simulation could fill this in a way that led to traffic (without a car randomly slowing down). If we were to randomly fill the road and this led to assigning each car in each segment speeds equal to the following $(3, 4, 3, 3, -1, 5, -1, -1)$ it is clear that the car with $speed = 4$ would have to slow down in order to prevent a collision occurring with the car that has car in segment 3 (with $speed = 3$) and similarly the car with $speed = 5$ will later have to slow down to avoid hitting the car in segment 1 (with $speed = 3$). Eventually this would lead to a situation where a car with $speed = 3$ is leading (specifically the car initially in segment 4) and other each of the cars behind it have been forced to a maintain a speed of $speed = 5$. The road model may appear as something like $(-1, -1, 3, 3, 3, 3, 3, -1)$ now if a were to slow down the it would the force others cars into slowing down immediately (unless it was the car on the rear) - clearly we can see in this example that a car slowing down has a massive affect on the flow of traffic. If we think about the previous in plain speech it simply accounts for cars getting stuck behind slower cars. In the aforementioned example we used a model with car density $p = 0.625$ which I would consider a high density model however the conclusions we drew on the causes of congestion are still present in lower density models to a lesser extent.

Thus clearly in the model the formation of 'spontaneous' traffic jams is largely dependent on the values of car density p and the probability of a car slowing down $p_{slowdown}$. Once congestion occurs the entire system is likely to exhibit consequences of this traffic jam throughout the motion of traffic.

If we consider the three graphs as a whole we notice that whilst they all differ in appearance they definitely share some characteristic. For example, we see the same feature of congestion ahead causing traffic jams behind them. If we look at each of the graphs we see that the traffic jam ahead must dissipate before the traffic behind its jam can shift this is obviously what we'd expect in real life. As a result each of the graphs has dark lines which map backwards as the time progresses. To be clear in 9 the density is much lower so these traffic jams dissipate relatively quickly and thus the trend of traffic jams going backwards in terms of position is far less noticeable as it dark region is not present in the graph for nearly as long. However, if we consider the graphs 10 and 11 we see solid black trajectories representing traffic jams, consistently going closer to the start of the origin in position as time progresses. There is some traffic that occurs at the end of the road later on that wasn't there initially, this is caused by congestion at the start of the road which again is caused by traffic ahead of it - as mentioned earlier this is present in 10 and 11 most evidently. This clearly indicates that greater car density p causes more frequent traffic jams, as explained in the previous paragraphs this is because it greatly amplifies the consequences of a vehicle slowing down or if the road that's initialised has slower cars ahead of faster cars. Whilst in real life road states aren't initialised this could simulates circumstances such as traffic lights leading onto a road turning or green or cars joining a road from an adjoining road.

At some points the traffic jams may appear to cause more congestion at the front of the jam this is visible with the when solid black patches wider, this could be cause by another vehicle randomly slowing down but also if one traffic jam was affected by another traffic jam this would have more or less the same effect. An example of this is the oval-shaped black patch which gets thicker in the middle at the point around $(160, 75)$.

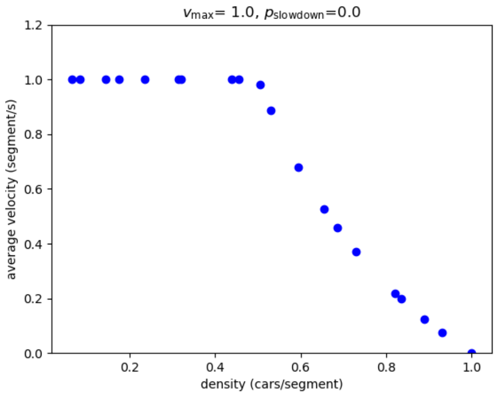

Figure 12: A density-velocity graph for our discrete model in purely deterministic traffic.

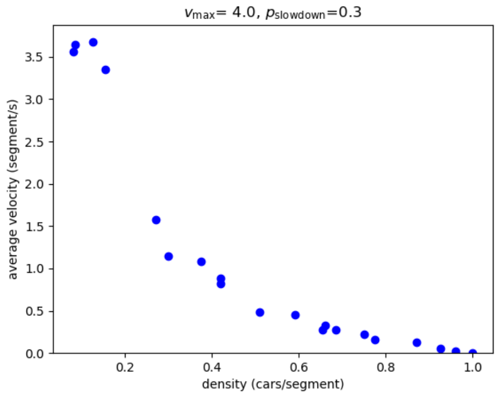

Figure 13: A density-velocity graph for our discrete model incorporating a probability of a driver randomly slowing down.

The figures above 12 and 13 display two alternative methods of modelling traffic flow, one which attempts to mimic real driver behaviour (using a probability that each driver randomly slows down) and another deterministic model which models the traffic as if each car were driven by a computer and the velocity of the car is dictated by rules $1, 2$ and 4 for our discrete model. Both of these graphs evidently show that there is direct relationship between increasing car density in the system and average speed being slower in the system.

We can see that in 12 there is no probability for slowing down, and that the maximum speed is set to $v_{max} = 1.0$. When car density is satisfactorily low $p < 0.41$ the flow of traffic is optimum and each car can travel at maximum speed As we increase the car density past this critical density of $p = 0.41$ the average speed begins to lower this is because the headway of some vehicles does not correspond to a sufficient length for the purpose of headway thus the car ahead were to stop immediately this would cause a collision so to prevent this some cars are forced into reducing there velocity. The cars then continue to reduce in velocity to the maximum velocity that provides a sufficient headway to cars ahead until the point when density is equal to 1 ($p = 1.0$) at which point every segment contains a car and the each car is forced to be stationary.

In 13 we display model that is fundamentally much the same as that in 12 however we introduce a probability to mimic the behaviour of actual drivers this is the probability that a driver randomly slows down. Also in this figure the maximum speed is much greater at $v_{max} = 4.0$ (segments per second). Clearly due to this high maximum speed, the speed is initially somewhat lower than this even at a low density in order to maintain a reasonable headway, as density is increased this again causes velocity to decrease as mentioned above. However, there is no point at which the velocity levels off at a constant amount, even for car density far less than $p << 0.41$ - this is due to the probability of slowing down we introduced, the likelihood of the average speed being similar for consecutive time periods is very low. Also, due to the random probability in slowing down being just that 'random' the transition of slowing down gradually as the density increases is not as smooth as it appears in the deterministic model, this is because the random deceleration will not be a gradual 'smooth' process it is likely to be 'sharp' and unpredictable.

Regardless of whether the model is discrete or not, the critical density at which flow is no longer optimum (and average velocity is no longer maximum) occurs far earlier if the maximum velocity is higher. In a model with a probability of slowing down there is never likely to be a consistent average velocity, due to the fact a number of vehicles are likely to slow down (specifically 30% chance of slowing down) whilst the other cars will accelerate providing they are able to (if there is sufficient headway and they're not at max speed yet).

Clearly we see again that velocity is greatly dependent on density and that the flow is massively dependent on both factors, this is one of the key themes in our essay and something that is very useful in order for us to be able to identify some of the key characteristics of a system at times when it is likely to experience congestion.

Below we further investigate differences between the deterministic model and the model that attempts to be more 'realistic' in the sense that it has a method of modelling driver behaviour by having a probability of slowing down $p_{slowdown}$.

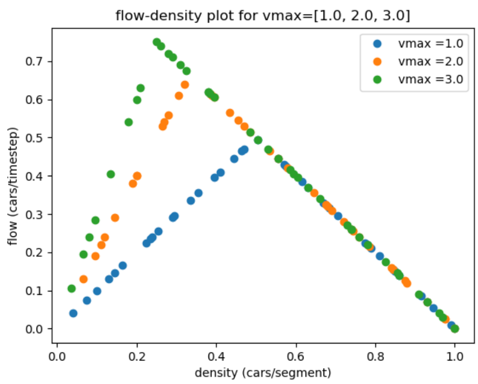

Figure 14: A flow-density graph for our discrete model plotting dlow against density for maximum velocities of $v_{max} = 1.0, 2.0$ and 3.0 - this is in purely deterministic traffic.

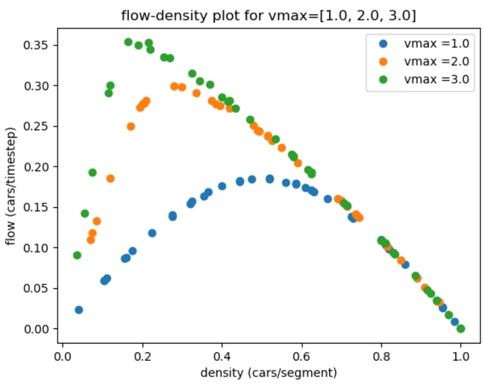

Figure 15: A flow-density graph for our discrete model plotting dlow against density for maximum velocities of $v_{max} = 1.0, 2.0$ and 3.0 - this is in purely deterministic traffic. This incorporates a probability of a driver randomly slowing down.

Much of the analysis of the above figures 14 and 15 will be repeated from the analysis of figures 12 and 13 however for clarity we will make this more clear.

Both graphs display an increase in the rate of flow as the car density in the road increases from its initial value of 0 ($p = 0$). However, in each case a critical density is reached which corresponds to a maximum rate of flow. If we consider 14 for each of the plots we see that this occurs when the car density reaches around $p = 0.35, 0.4$ and 0.45 for $v_{max} = 3.0, 2.0$ and 1.0 respectively. As the characteristics of the graph are massively similar excluding the location of this peak flow, this suggests some kind of horizontal translation corresponding with a change in the maximum velocity. It is also clear from the graph that increasing the maximum speed corresponds to an increase in the maximum flow - this is what we would expect as if we disregard headway and congestion, it would mean more vehicles are able to pass the detector per unit of time as they are travelling more distance per second.

The reasoning behind lower maximum flow occurring at lower densities for higher velocities (given by the horizontal translation when v_{max} is increased) is that a car travelling fastest will reach the car ahead faster under such circumstances, at this point the flow will be dictated by the slower car - even at low densities at which point the maximal flow will be reached for this velocity and will be dictated by the speed of the slowest car. As we increase the car density past the critical density due to the increase in number of cars the lowest speed is likely to be lower as well as the headway not being satisfactory for the faster cars and thus the flow is likely to be lower (past this critical density) thus the gradient of the graph is negative and continues to decrease until the road is full and the flow along the road is 0.

At lower car densities the gradient of the graph is positive this is due to the fact that the headway between cars is satisfactory for a longer period so faster cars can maintain their speed and even accelerate under these circumstances for a longer period until the headway is no longer satisfactory at which point they decelerate. Although, at this point the flow has already been significantly higher than it will be for other (higher) densities. When there is low car density even if cars travel in convoy, the leading car will be able to accelerate (due to the empty space ahead of it) and thus the other cars can accelerate leading to a greater average speed which consequently means more cars pass the detector in a given time frame which is greater flow.

Whilst the above is applicable to both graphs 14 and 15 we see that in the deteministic model, the flow increases relatively uniformly up until the critical density is reached. This is as expected as the flow should only depend on the congestion on the road and the speeds of the cars initially - as the cars do not slow down randomly and only do so to avoid a collision. Thus we expect the flow to increase we velocity initially and fall when velocity is forced into decreasing (cars are forced into decelerating) after the critical density has been reached. Whilst 15 is not such a uniform increase in flow followed by a uniform decrease, this is because the probability of slowing down is ever present and like to interfere with the acceleration and deceleration of each car which in turn affects the speed of cars surrounding it - thus whilst the trend of the flow is to increase and then decrease it does not do so at such a steady rate.

If we compare values at set points on the each graph 14 and 15 we see that peak values of flow occur for lower densities and are less in magnitude in 15 compared to in 14. This is not unusual, as the factor of cars randomly slowing down will invitably mean that maximum velocities are not reached, we can see this in 13 and thus if the maximum velocity is not reached neither cant he maximum flow - from $q = pv$ an equation that we explained in the Traffic Flow Properties section. Clearly, driver error is a huge factor in traffic flow being limited, as when only the headway between cars is considered the traffic flow seems consistently higher in the deterministic model compared to that in the more 'realistic' model at the same densities.

The discrete model we have modelled thus far has clearly been able to present us with some pertinent data, and is a vast improvement on our initial simple model. We have clearly been able to draw some basic initial conclusions surrounding the nature of traffic flow and how our key 3 variables speed v, car density p and flow of traffic q are related. At least to a general level we can now predict how the 3 variables are likely to change with time under realistic conditions. It has also been useful to incorporate the random nature of drivers (although somewhat crudely) and see how this further alters the flow of traffic, The deterministic version of this discrete model has provided a useful comparison in this sense as it does not involve random driver behaviour unlike the standard discrete model and this contrast presents to us the huge effect driver behaviour has on traffic flow. Despite the vast improvement in the model, it has been relatively easy to implement and draw conclusions from. However, this model is by no means perfect. Whilst been able to identify vague trends with for many of the figures we have produced and form an idea on how variables relate - its difficult for us to produce consistent, repeatable data. Whilst the current model is useful for individuals to learn about how traffic flow can be effected if we were to use it in an application such as 'Google Maps' the times for travel would vary greatly even at the same time of day under the same conditions. This huge variation between results is caused by an overly random nature of driver behaviour and makes it difficult to make a practical use of our mode despite the model identifying clear trends. For this reason, we shall consider a further model.

6 Deterministic ODE Traffic Models

Whilst we have drawn on some information surrounding deterministic traffic models, and we have certainly benefited from comparing our discrete model to the deterministic model (in terms of understanding both models), it makes sense if we now consider the deterministic model further as there is still much we can learn from it.

A deterministic model could simply be modelled using the same rules as our discrete model, however ignoring rule 3 in the process - this means the probability of slowing down is constantly 0 ($p_{slowdown} = 0$). This means the model of traffic flow is hugely dependent on the initial formation of the road, as the minimum distances and headways are defined here. The only reason a vehicle should slow down is to avoid a collision with another vehicle.

Whilst the model we will now explore is similar we shall make it a continuous-time model, that is, all car positions are given by functions of a continuous time variable t. **Q4**

You may be asking yourself why is it useful to have a fully deterministic traffic model, despite it not providing an accurate representation of real driver behaviour. To be clear a deterministic model does not account for random behaviour and simply models the situation using an initial road state and parameter values for variable such as maximum speed. This differs greatly from our stochastic model which would produce varying results from a single set of parameters, in the sense that we expect a deterministic model to produce the same result exactly given the same initial condition and parameter values. This is suitable in the sense that when we wish to provide a solution for a problem we wish to provide a consistent solution when presented with the exact same scenario. As random behaviour is very difficult to model the solutions to a scenario involving random behaviour vary greatly. Removing this random element presents us with a model which we are able to compare solutions to scenarios with different parameters and initial conditions and draw conclusions on how the parameters and initial conditions affect the solution. Also, if we require a numeric value for further calculations, for example we may need an approximation for flow q in order to calculate an estimated time of arrival it is no use having a system where flow varies between many values and in this case a deterministic model is more suitable.

In a deterministic model we begin similarly to the stochastic discrete model by forming a random initial road state - this is the only element of randomness present. The road state will again be on the same road state as that in 8 where the road is one-dimensional and circular where there no-side roads and overtaking cannot take place. Thisnext model is a continuous-time model, and each cars position is given by functions of a continuous time variable t. The equation of motion for the n^{th} car is given by Newton's law,

$$M\ddot{x_n} = F(t),$$

where M is the car mass and all the driver behacious is encoded in the force $F(t)$. $F(t)$ is vague and we shall specify in order to assume that the driver of each car will be aiming to achieve optimal speed (this may not always be the case real-life as many driver drive 'too slow' or 'too fast'). In doing so we form the following 'optimal velocity model',

$$\ddot{x_n} = s\left(v_{goal}\left(x_{n+1} - x_n\right) - v_n\right), \tag{5}$$

we are already familiar with many of these parameters, v_{goal} is the optimal speed, s is sensitivity which determines how quickly drivers adjust their speed. The difference $x_{n+1} - x_n$ is the difference between a cars position and the successive cars position otherwise known as headway. The driver behaviour in this deterministic model is entirely encoded in the choice of v_{goal} - and a reliable and much-studied choice is given by,

$$v_{goal}(\Delta x) = v_o\left(\tanh\left[m\left(\Delta x - b_f\right)\right] - \tanh\left[m\left(b_c - b_f\right)\right]\right) \tag{6}$$

The parameter b_c is an effective car length, once cars are separated by this distance, they are bumper-to-bumper and are forced into standstill. Here v_0 is related to maximal velocity and Δx corresponds to the distance between the front bumpers of adjacent cars.

If we consider a simple, stationary solution to 5 by distributing the cars at equal distance Δx of each other, and giving them a velocity which is equal to $v_{goal}\left(\Delta x\right)$. This is called free flow: the acceleration is (and remains) zero. The flow can be calculated analytically and is given by

$$q = v_{goal}\left(\frac{L}{N}\right)\frac{N}{L} = v_{goal}\left(p\right)p^{-1}, \tag{7}$$

where L is the length of the road, N is the number of cars and p is the car density respectively.

Knowing the relationship between flow q and speed v_{goal} will be fundamental to our calculations and if we know the optimum speed we are able to formulate the fundamental diagram of q against p as in 3 this is important as knowing how the three key variables relate in the model provides us with a strong basis from which we can draw conclusions about the accuracy and usefulness of the model. **Q6**

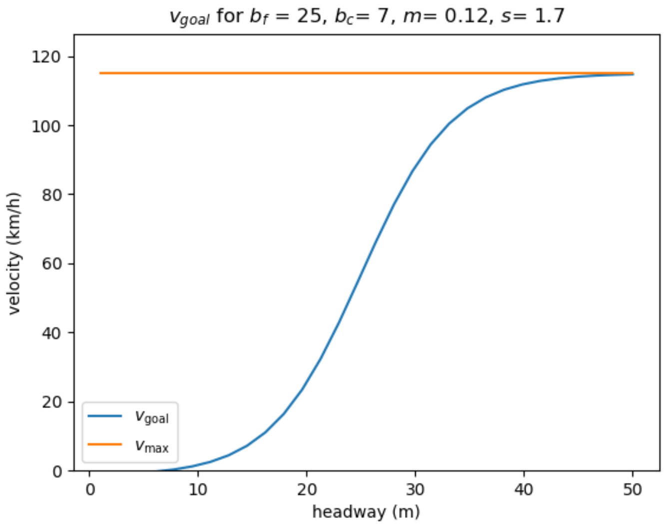

Figure 16: A graph of v_{goal} against headway.

If we analyse 5 we see that this double derivative with respect to time will provide us with an equation corresponding to acceleration - we have already established that the scenario we are considering is a standstill (read above) thus acceleration is 0 so $\ddot{x_n} = 0$. If we use this equality in 5 we are given the following,

$$0 = s\left(v_{goal}\left(x_{n+1} - x_n\right) - v_n\right)$$

this situation is described simply as free-flow which accounts for a scenario in the optimal velocity model where acceleration is 0. In this situation, we can use the homogeneous flow sloution, where the spacing between each vehicle is constant and equal thus it is defined as the length of the road divided by the number of vehicles $\frac{L}{N}$ as this is the only value that satisfies such criteria. In order to maintain this situation as in all previous deterministic models, the speed must be equal across all vehicles. Using $\Delta x = \frac{L}{N}$ across the system with equal spacing through the circular road we can substitute this into one of our original equation for flow in the optimal velocity model: In the following we make obvious use of the function for v_{goal} (6) and our original equation for density $p = \frac{N}{L}$ as well as the relation $q = pv$.

$$\begin{aligned} q &= pv \\ &= \frac{N}{L} v \\ &= v_{goal} \left(\frac{L}{N} \right) \frac{N}{L} \\ &= v_{goal} \left(\left(\frac{N}{L} \right)^{-1} \right) \frac{N}{L} \\ &= v_{goal} \left(p^{-1} \right) p \end{aligned}$$

Clearly using the fact that the distance separating cars is constant and consistent throughout the system as well as the formula for optimal velocity we see that in free-flow flow the above equation holds.

Q8

If we consider 13 further we can see that free-flow will become more stable as the value of v'_{goal} becomes closer to 0, as this means that v_{goal} will be closer to a constant value and vary a lot less. From the fact $q = v_{goal} \left(p^{-1} \right) p$ it means that flow will be closer to a constant value $v_{goal} \cdot p$ and hence free flow will be stable.

It can be proven analytically that the homogeneous free flow we consider is stable when $\frac{2v'_{goal}(h)}{s} < 1$. If we wish to the headway and sensitivity that this occur for, and solve the inequality - we can take the following equality,

$$\frac{2v'_{goal}(h)}{s} = 1.$$

Through rearranging this we arrive at,

$$v'_{goal}(h) = \frac{s}{2}. \tag{8}$$

An expression for v_{goal} we encountered earlier is present in 6 using $\Delta x = h$ we formulate the following equation,

$$v_{goal}(h) = v_o \left(\tanh \left[m \left(h - b_f \right) \right] - \tanh \left[m \left(b_c - b_f \right) \right] \right).$$

By differentiation we can find an expression for $v'_{goal}(h)$, namely this expression is

$$v_{goal}(h) = m v_0 \operatorname{sech}^2 \left(m(h - b_f) \right). \tag{9}$$

Equating 9 and 8 we arrive at,

$$m v_0 \operatorname{sech}^2 \left(m(h - b_f) \right) = \frac{s}{2}.$$

From here we should be able to do a series of rearrangements in order to obtain an expression for h.

$$\begin{aligned} m v_0 \operatorname{sech}^2 \left(m(h - b_f) \right) &= \frac{s}{2} \\ \operatorname{sech}^2 \left(m(h - b_f) \right) &= \frac{s}{2 m v_0} \\ \operatorname{sech} \left(m(h - b_f) \right) &= \sqrt{\frac{s}{2 m v_0}} \\ \left(m(h - b_f) \right) &= \pm \operatorname{sech}^{-1} \left(\sqrt{\frac{s}{2 m v_0}} \right) \end{aligned}$$

$$(h - b_f) = \pm\frac{1}{m}\,\mathrm{sech}^{-1}\left(\sqrt{\frac{s}{2mv_0}}\right)$$
$$h = \pm\frac{1}{m}\,\mathrm{sech}^{-1}\left(\sqrt{\frac{s}{2mv_0}}\right) + b_f$$

Now we have obtained an equation for h, and in order to calculate the h values we must first calculate v_0 using the parameters of the 16 figure for which we projected the optimal velocity. Thus we can equate the values of $b_f = 25$, $s = 1,7$ and $m = 0.12$.

We then calculate:

$$v_0 = \frac{v_{max}}{1 - \tanh(m(b_c - b_f))}.$$

Finally we obtain the $h = 33.10595$ or $h = 16.89405$ using the aforementioned parameters as well as the value of v_0 giving solutions to a rounded accuracy.

Here we exhibited the ability for this new improved deterministic model to provide values from which we can calculate, this was not present in the previous stochastic model as the values which we would perform our calculations varied hugely leading to confusion and inaccuracy when decided which value to use in the calculation. **Q5**

We expect that the function for optimal velocity v_{goal} will be as close as possible to the maximum possible velocity when headway is sufficient, this is because we expect that the flow will incrase as we increase v_{goal} and we should be able to achieve optimum flow for the specified density. The optimal velocity should be given by a function which is continuous as its a function of time t or can be written as one. We can see that 6 is continuous and hence satisfies this requirement. Also, as we know from differential equations and basic dynamic, $\frac{d}{dx}(v) = a$ where v is velocity and a is acceleration this displays that the derivative of velocity is acceleration and it is necessary for v to be differentiable. We also know that v must have a condition such that $v(0) = 0$ as cars must be able to come to a halt. If cars are instantiated with a distance to the next vehicle that is smaller than the minimum headway h then we know there is an issue, and this will lead to v_{goal} taking a negative value and being capable of driving backwards (as $tanhx$ can take negative values) and cars are unable to overtake so a car will do this to prevent a crash. A crash however, can occur as there is no condition preventing cars from being instantiated as crashing. **Q7**

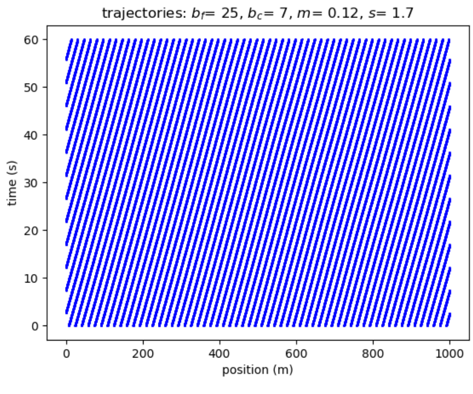

Figure 17: A trajectory-time graph for deterministic traffic, this models 60 cars per km.

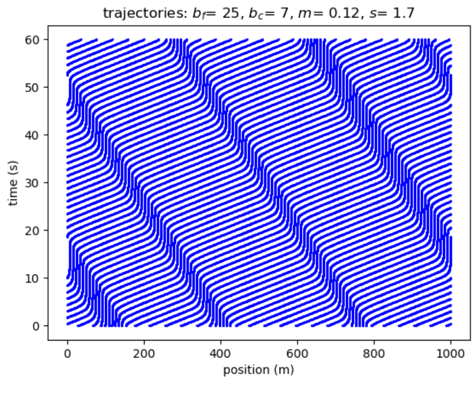

Figure 18: A trajectory-time graph for deterministic traffic, this models 40 cars per km.

In figure 17 we see a that the vehicles are relatively the there is consistent distance between each trajectory. Also, there is a consistent gradient which represents velocity for each trajectory thus each vehicle is travelling at the same constant speed. The density of the road again is consistent and relatively high however, due to the deterministic nature of the model no congestion occurs as the congestion is only effected by the initial filling of the road. This differs hugely from what we would expect a real-life trajectory-time graph to appear like as the nature of the graph is far too uniform. Prior to this when analysing the discrete stochastic model of traffic we saw a frequently fluctuating and worsening situation which is a polar opposite in that sense to this model. This is because the car density is at an appropriate level for traffic to flow with no hindrance and thus the graph seems smooth with not interference visible. This differs slightly from the figure to the right of it 18 which exhibits similar characteristics in that the state of the graph is dictated only by what is initialised as however we can clearly some congestion. This is because the density is substantially higher ($p = 60$) and the likelihood of each car being assigned a position and no congestion being present is slim-to-none thus we can see that the road is initialised with congestion present and this congestion remains within the graph throughout - as the graphs nature is dictated only by the rules of motion and the random initialisation of the road. Thus if congestion occurs but the distance between cars is still larger than or equal to the headway then this will not cause a car to decelerate and the motion will not alter as we progress with time - so the congestion remain the same cars throughout the entire graph. This shares some similarities to the discrete model in the sense that once congestion occurs in a discrete model it often remains in the model throughout the entire time the road state is being modelled (at least over the short times we used). However, it differs in the sense that in the discrete model the congestion propagated backwards throughout the road, whereas the congestion stays amongst the same cars it started with in the deterministic model in 18.

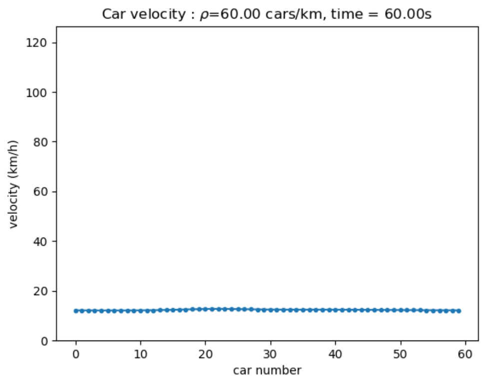

Figure 19: A velocity-time graph displaying individual velocities for deterministic traffic, this models 60 cars.

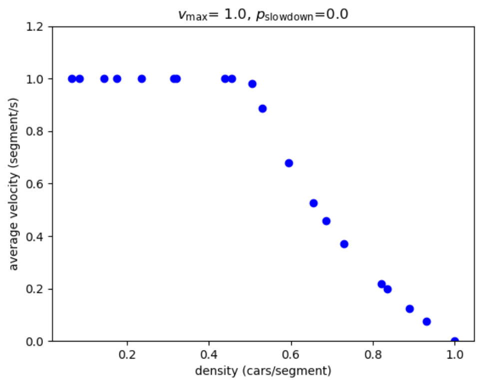

Figure 20: A density-velocity graph for deterministic traffic, this models 40 cars.

19 displays the individual velocities of 60 cars - we see that the cars each have nearly identical velocities as they are calculated using the same equation. This equation is designed for systems such as this with the fact that greatly differing speeds causes congestion in mind. Thus it makes sense that the velocities would be similar, this differs tremendously from our discrete model where there only an upper bound for velocity to dictate it.

In 18 we compare how the average velocity is affect the by the density this is somewhat similar to that in the discrete model (and very similar to that of the deterministic discrete model) where the average velocity is consistent until a critical density is reached at which point as density is increased average velocity decreased. This is exactly what we would expect from such a graph - its velocity decreased smoothly as density increases at exactly the right rate.

In 21 the simulate flow matches the free flow very accurately until the critical density is reached at which point free flow decreases far more appropriately and optimally than the simulated flow. Eventually at a density $p = 55$ approximately the flow gets back on track to almost exactly what the free flow is.

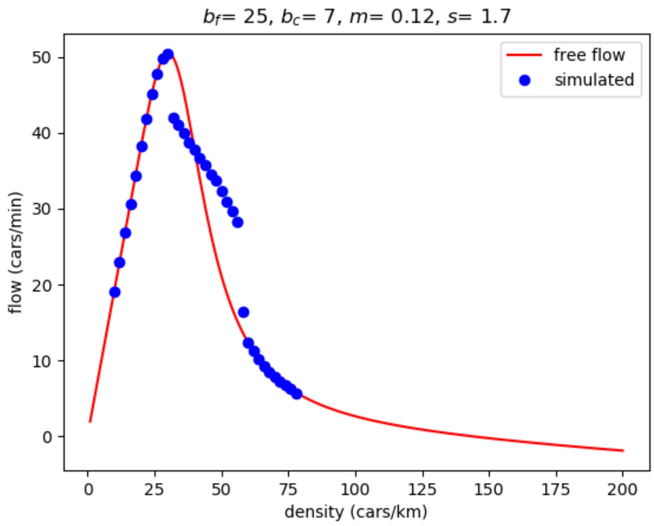

Figure 21: A graph graphing flow against in the deterministic model, comparing the simulated flow by my program to that of the expected free flow.

7 Conclusions

7.1 Observations

Throughout the investigations we found that when using either model the flow of traffic was greatly impacted by the initial speeds and positions of other cars, in fact in a deterministic model this was the only factor. However, in a stochastic model there was also a crude attempt to model driver error that often caused unrealistic breaking and traffic jams within the model. The huge element of randomness in the stochastic model meant that the results obtained from each run of the model varied greatly, making them useless for further numeric calculations. However, this does not negate the usefulness of the model entirely. Considering this, I believe the attempt to model driver error is something that was of use in identifying trends in the cause of spontaneous traffic jams forming and given time an improved model could provide a more realistic model of traffic in real-life scenarios than that of a deterministic model. In our model a driver with a lot of empty space ahead was just as likely to break as a driver with any distance greater than the headway of free space ahead - this is unrealistic and in an improved model I would not simply base random braking on a probability. When displayed graphically the model did provide a basic presentation of the formation of spontaneous traffic jams, and indicated that the driver error (or random breaking in this model) had a huge effect on this and these effects were greatly amplified when applied to a system with a high car density as we would expect in real life. For spotting realistic trends I believe the stochastic model had its own benefits and featured a realistic aspect usefully that was not present in the deterministic model. The deterministic model also provided some useful information, unlike the stochastic model I would not credit this to aiding us in spotting realistic trends. However, when using it to base other calculations on such as when we calculated the necessary maximum headway for free flow to occur the numerical information was very useful. However, in terms of the realism of the model, a deterministic model would not be satisfactory to analyse traffic alone and it would be necessary to incorporate some other model to do so - as we know not all drivers drive perfectly. I believe that both models had positive aspects that would aid us when analysing traffic, yet both of which with the right resources and a suitable time-frame in which to implement improvements could be far better.

Whilst the conclusion thus far seems to focus largely on the negatives of the models I do believe that they offered a great compromise between ease of implementation and providing some key data that would not be so easy to identify or so clear without the models. The relationship $q = pv$ was clear from the graphical representations and many of the graphs provided an insight into how altering one of the variables could vastly affect another. The graphs certainly displayed that whilst an optimum traffic flow was achievable for relatively low car densities (this was evident in the deterministic model) driver error could easily disrupt this. We also saw that at certain car densities traffic jams were unavoidable even in the deterministic model with no driver error - this suggests that often drivers are not responsible for the formation of traffic jams but it is in fact the poor design of road networks not accounting for high car densities on their roads. As from our simple model we saw car density is simply $p = \frac{N}{L}$ by increasing road length we can reduce car density and prevent congestion to some extent.

7.2 Improvements

A finite circular road is not a realistic model for a road system, whilst it is satisfactory to display how some traffic variables affect others it is not sufficient to model traffic on large road networks. If I were to improve from this, I believe that compromising ease of implementation is necessary in order to provide a model which is more accurate for real-life scenarios. I believe that we could enforce rules that at certain points in the network, for example at a specific segment on a road we could force cars to wait as this may help to simulate a junction or a set of traffic lights. This is because I believe that the current models did not contain any realistic obstructions or scenarios from real-life which would have a great bearing on the flow of traffic and in order to supply a significantly better model this may be necessary. I think if we are to just consider the flow of traffic when 'normal' driving is occurring with simply a speed limit alone this may be more practical for a motorway scenario. However, for normal road use we need to provide representations of A-roads and other small roads as this is where a lot of traffic occurs. The stochastic model of traffic made an attempt to model driver error by implementing a random probability of breaking. Whilst I agree that driver error is one of the main causes of traffic jams this does not seem a realistic method of simulating it. I believe that this probability could be dependent on how close you are to a car, as when the car gets closer to another car the driver is likely susceptible to making an error - thus I believe this probability does not constant but could be a variable which takes into account other factors such as distance from cars and the road on which it is driving. I also believe that whilst a headway is necessary in a model, especially in a deterministic one in which driver error is impossible it is also good to have a model which collisions could occur as collisions frequently cause traffic jams in real life. In order to do so I believe with a very small probability a collision could occur, in the case this is a major collision congestion should arise behind it - after a set time the car should be removed and normal traffic activity should resume.

References

Masako Bando, Katsuya Hasebe, Ken Nakanishi, Akihiro Nakayama, Akihiro Shibata, et al.. Phenomenological Study of Dynamical Model of Traffic Flow. Journal de Physique I, EDP Sciences, 1995, 5 (11), pp.1389-1399. < 10.1051/jp1:1995206 > . <jpa-00247145>

Kelly, F. (n.d.). The Mathematics of Traffic in Networks. [ebook] Available at: http://www.statslab.cam.ac.uk/ frank/PAPERS/PRINCETON/pcm0052.pdf [Accessed 27 Nov. 2017].

En.wikipedia.org. (2017). Traffic engineering (transportation). [online] Available at: https://en.wikipedia.org/wiki/Traffic engineering (transportation) [Accessed 29 Nov. 2017].