Seismic waves

1 Assignment Description

These assignment sheets describe the propagation of seismic waves. They provide a brief description of the topic and set problems some of which must be solved using python programs.

The assignment consists in writing an essay that contains an answer to each question and which expand on the material included in these notes. The references are there to provide you with extra source of information and they are not exhaustive.

The essay must be readable on its own without reference to the assignment sheets. The essay must not necessarily follow the same order as the notes and does not need to answer the questions in the same order either. As a matter of fact using a different structure and order is a bonus. Do describe the interpretation or consequences of the results that you obtain. We also expect you to focus more on some aspects of the material presented in these notes. This could be a more detailed discussion of the derivation of the equations or algorithm described in the notes or to better explain the interpretation of the results. You can also solve problems which are not set and which answer some further questions that you might have.

Some parts of the assignment sheets will need to be included in your essay, but do not copy these section verbatim, use your own words. The equations do not need to be changed, but you can provide more detailed explanations or derivations when appropriate.

The template program `ray_IASP91_v1.py` contains comments like `### MARKING coding 1` and `### END MARKING`. Please write your code in between these comments. The coding number informs you which coding section the section refers to. All the functions you are asked to write are already partially defined, but commented out for coding 2 or 3 so that you can run and test your program after each coding task is completed.

You have to submit 2 files: 1 pdf file for the essay and 1 python program. We ask that you typeset your essay in LaTeX.

There is no page limit for the essay, but as an indication, the model essay is 9 pages long, including all the figures.

2 Introduction

The Earth being mostly a solid ball, waves can propagate inside it. The largest wave are produced by earthquakes and they are usually so large that they can be detected by seismographs around the globe. Here are some of their properties

- There are 2 types of seismic waves: P (primary) and S (secondary) waves.
- P wave are longitudinal compressions of rocks.
- S wave are deformations of rocks in the direction perpendicular to the travelling direction of waves.
- P and S waves travel at different speed, usually called V_p and V_s, and $V_p > V_s$.
- There are no S waves in liquids, only P waves.
- P and S waves satisfy the same wave equation which, in 3 dimensions, is given by

$$\begin{aligned}\frac{1}{V^2}\frac{\partial^2\phi}{\partial t^2} &= \boldsymbol{\nabla}^2\phi \\ &= \frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}\,, \qquad (1)\end{aligned}$$

where V, equal to V_p or V_S for the corresponding wave type, is the speed of propagation of the waves. (Remember also that $\boldsymbol{\nabla}\phi = (\frac{\partial\phi}{\partial x}, \frac{\partial\phi}{\partial y}, \frac{\partial\phi}{\partial z})$).

- In inhomogeneous media V is usually position-dependent and can be discontinuous at the interface between 2 different types of rocks. Waves are partially reflected and partially refracted (transmitted but in a different direction) in inhomogeneous material. P waves then generate reflected and refracted waves of both P and S types. So do S waves. When V changes slowly, the reflected waves are small and can be neglected. On discontinuities, every incident wave of a given type generates 4 different waves: 2 reflected and 2 refracted which all follow different paths and travel at different speeds. This all happens at the same time, but we will later use a method which follows one path at a time. With this method we will have to follow each path for each type of wave separately.

Seismic waves provide a large amount of information on the inside structure of the Earth and geophysicists record both the amplitudes and the arrival time of these waves generated by earthquakes. This is how we know the inner structure of our planet and its 4 main layers:

- The crust, on which we live, is between 3 km and 50 km deep.
- The mantel is solid and approximately 2850 km thick.
- The outer core is liquid and 2265 km deep
- The inner core is 1215 km deep and made out of solid iron nickel and other metals.

The aim of this essay is to study the propagation of seismic waves inside the Earth using a technique called *ray paths* which involves the computation of integrals that can easily be computed numerically.

3 Ray Theory

This section is largely inspired by N. Rawlinson's lecture notes [1].

Question 1:

Show that if V is constant,

$$\phi = A\exp\big(-i\omega(t-\mathbf{k}.\mathbf{x})\big)\,, \tag{2}$$

is a solution of (1) where $\mathbf{k}$ is a constant vector and $\mathbf{k}.\mathbf{x} = k_x x + k_y y + k_z z$. How is $\mathbf{k}$ related to V? How can one see that V is indeed the speed of the wave?

Notice that the period of the wave is $T = 2\pi/\omega$. Inside the Earth, V is a function of the position as the properties of rocks vary. If the speed of the wave varies very little over distances of the order of the wavelength $\lambda = V/T$, we can compute approximate solutions of the form

$$\phi = A(\mathbf{x})\exp\big(-i\omega(t+T(\mathbf{x}))\big)\,, \tag{3}$$

where A and T are both real functions.

Question 2:

Substitute (3) into (1). This leads to a complex equation. Show that the real and imaginary parts of the equation imply that

$$2\omega\boldsymbol{\nabla}A\cdot\boldsymbol{\nabla}T + \omega A\boldsymbol{\nabla}^2 T = 0\,, \tag{4}$$

$$\boldsymbol{\nabla}^2 A - \omega^2 A\boldsymbol{\nabla}T\cdot\boldsymbol{\nabla}T = -\frac{\omega^2}{V^2}A\,. \tag{5}$$

Dividing (5) by $-A\omega^2$ we obtain

$$-\frac{\boldsymbol{\nabla}^2 A}{A\omega^2} + |\boldsymbol{\nabla} T|^2 = \frac{1}{V^2}. \tag{6}$$

Our ansatz (3) assumes that V changes very little over a distance of the wavelength λ. Using a Taylor expansion, we can write

$$\frac{\mathrm{d}^2 A(x)}{\mathrm{d}x^2} \approx \frac{A(x+\mathrm{d}x) + A(x-\mathrm{d}x) - 2A(x)}{\mathrm{d}x^2}\,, \tag{7}$$

and take $\mathrm{d}x = \lambda$ so that

$$\begin{aligned} -\frac{\boldsymbol{\nabla}^2 A}{A\omega^2} &\approx \frac{A(x+\mathrm{d}x) + A(x-\mathrm{d}x) - 2A(x)}{\lambda^2}\frac{\tau^2}{A(x)4\pi^2} \\ &= \frac{A(x+\mathrm{d}x) + A(x-\mathrm{d}x) - 2A(x)}{A(x)}\frac{1}{4\pi^2 V^2} \ll \frac{1}{V^2}. \end{aligned} \tag{8}$$

where τ is the period of the wave. We use the fact that $V = \lambda/\tau$ and we can neglect the first term in (6) if A is nearly constant over a distance $\mathrm{d}x = \lambda$. Then (6) simplifies to

$$|\boldsymbol{\nabla} T|^2 = \frac{1}{V^2}. \tag{9}$$

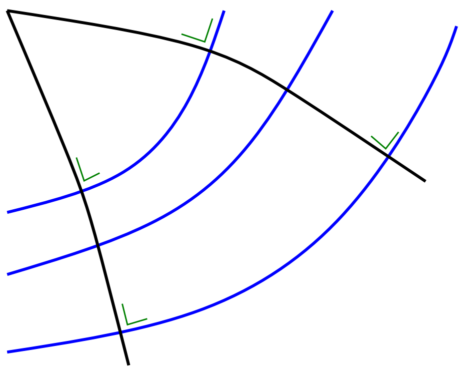


Figure 1: Wave-front (blue) and ray path (black).

Note that in (3) $T(\mathbf{x}) = T_0$, where T_0 is a constant, defines a surface corresponding to a wave front. This is the surface corresponding to all parts of the wave which have travelled a time T_0 since a reference point, usually taken as the source of the wave. Ray paths are the lines which are perpendicular to those surfaces at all points.

The derivatives of T in the direction parallel to the wave-fronts is zero (as T is constant on it). Equivalently

$$\boldsymbol{\nabla} T \cdot \mathbf{q}_{\text{wf}} = 0\,, \tag{10}$$

where $\mathbf{q}_{\text{wf}}$ is a vector tangent/parallel to the wave-front.

Indeed: we can take a system of coordinates around a point $\mathbf{r}$ such that x and y are parallel to the wave-front and z perpendicular to it: $\partial T/\partial x = \partial T/\partial y = 0$. We have $\boldsymbol{\nabla} T \cdot \mathbf{q}_{\text{wv}} = (\partial T/\partial x, \partial T/\partial y, \partial T/\partial z) \cdot (q_x, q_y, 0) = 0$ and as the result is independent of the system of coordinates, (10) holds.

A ray path can be parametrised as $\mathbf{r}(s)$ where s is a parameter satisfying $|\mathrm{d}\mathbf{r}/\mathrm{d}s| = 1$, and s is measured in the same units as $\mathbf{r}$. The vector $\mathrm{d}\mathbf{r}/\mathrm{d}s$ is tangent to a ray path: if we consider two close points $\mathbf{r}(s+\delta s)$ and $\mathbf{r}(s)$, the vector joining them is

$$\mathbf{b} = \mathbf{r}(s+\delta s) - \mathbf{r}(s) = \delta s\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}s} + \mathcal{O}(\mathrm{d}s^2)\,. \tag{11}$$

If we take δs very small then $\mathbf{b}$ is parallel to $\mathrm{d}\mathbf{r}/\mathrm{d}s$. We also notice that

$$T(\mathbf{r}+\mathbf{q}) - T(\mathbf{r}) \approx \mathbf{q}\cdot\boldsymbol{\nabla} T\,, \tag{12}$$

and if $\mathbf{q}$ is infinitesimal and parallel to the wave-front (i.e. proportional to $\mathbf{q}_{\mathrm{wf}}$), the right-hand side is zero as $\boldsymbol{\nabla}T$ is perpendicular to the wave-front. If $\mathbf{q}_{\mathrm{rp}}$ is parallel to the ray path, then $T(\mathbf{r}+\mathbf{q}_{\mathbf{rp}})-T(\mathbf{r})$ is the time taken by the wave to travel from $\mathbf{r}$ to $\mathbf{r}+\mathbf{q}_{\mathbf{rp}}$ which is $|\mathbf{q}_{\mathbf{rp}}|/V$ and so $|\mathbf{q}_{\mathbf{rp}}|/V = \mathbf{q}_{\mathbf{rp}}\cdot\boldsymbol{\nabla}T = |\mathbf{q}_{\mathbf{rp}}||\boldsymbol{\nabla}T|$. We thus have

$$V|\boldsymbol{\nabla}T| = 1\,. \tag{13}$$

As $V\boldsymbol{\nabla}T$ and $\mathrm{d}\mathbf{r}/\mathrm{d}s$ are vectors perpendicular to the wave front and of norm 1 we have

$$\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}s} = V\boldsymbol{\nabla}T\,, \tag{14}$$

as one can always orient s so that the equality holds with a positive sign.

The time δT needed to go from a point s on the path to a close point $s+\delta s$ on the same path is $\delta T = T(s+\delta s) - T(s) = \delta s\frac{dT}{ds}$. As $\delta s/\delta T = V$, $\frac{\mathrm{d}T}{\mathrm{d}s} = \frac{1}{V} = u$ where we have defined $u = 1/V$ which is called the slowness by geophysicists.

We now take the gradient of $\frac{\mathrm{d}T}{\mathrm{d}s} = u$ from both sides, we get

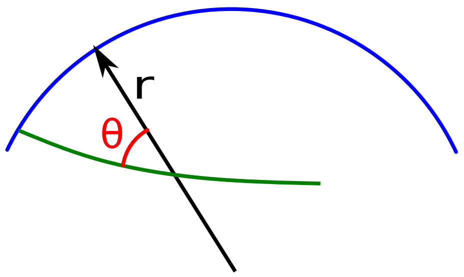
$$\frac{\mathrm{d}\boldsymbol{\nabla}T}{\mathrm{d}s} = \nabla u\,. \tag{15}$$

And inserting (14) into (15) we find

$$\frac{\mathrm{d}}{\mathrm{d}s}\left[u\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}s}\right] = \nabla u\,. \tag{16}$$

Knowing the underlying structure of the Earth, implies knowing $u(\mathbf{r})$ and we can solve (16) for $\mathbf{r}(s)$ corresponding to the ray followed by the wave.

For simplicity we assume that the Earth is radially symmetric and that the speed of seismic waves only depends on the radial coordinate r measured from the centre of the Earth. We actually use the polar coordinates (r,Δ,ϕ) where we use the letter Δ instead of the more common θ to avoid ambiguity with the wave incident angle.



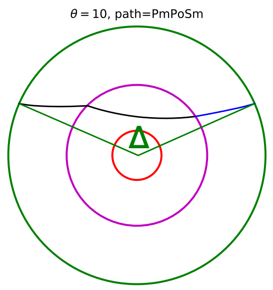


Figure 2: a) Incident angle θ between ray path (green) and radial direction (black). b) Ray path for a P wave propagating through the mantel and the inner core and re-entering the mantel as an S wave (path PmPoSm; the naming is explained in section 4.3). Δ is the total deflection angle.

Question 3:

Compute

$$\frac{d}{ds}\left[\mathbf{r}\times\left(u\frac{d\mathbf{r}}{ds}\right)\right] \tag{17}$$

and show that

$$\mathbf{r}\times\left(u\frac{d\mathbf{r}}{ds}\right) = \mathbf{K}. \tag{18}$$

where $\mathbf{K}$ is a constant vector.

We then write

$$\left| \mathbf{r} \times u \frac{d\mathbf{r}}{ds} \right| = |\mathbf{K}| = p = ru\sin(\theta_i) \tag{19}$$

where θ_i is the angle between the ray path direction $\frac{d\mathbf{r}}{ds}$ and the radial direction. Notice that p, called the ray parameter, is a constant. In other words, for any given path, p will remain the same for all times, even when the wave changes type.

When $\theta_i = \pi/2$, the ray path is perpendicular to the radial direction meaning it has reached its lowest point.

We use (13) to evaluate $c(\mathbf{r})^{-2} = u(\mathbf{r})^2$ by computing

$$\boldsymbol{\nabla} T \cdot \boldsymbol{\nabla} T = \left(\frac{\partial T}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial T}{\partial \theta}\right)^2 + \frac{1}{r^2\sin(\theta^2)}\left(\frac{\partial T}{\partial \phi}\right)^2 = u(\mathbf{r})^2. \tag{20}$$

For a radially symmetric Earth, waves travel along a great circle and the last term is zero. We then seek solution for T of the form

$$T(r,\theta) = f(\theta) + g(r) \tag{21}$$

and substitute it into (20) to get

$$r^2\left[\frac{dg(r)}{dr}\right]^2 - r^2 u(r)^2 = -\left[\frac{df(\theta)}{d\theta}\right]^2 = -k^2. \tag{22}$$

Both sides must be equal to the same constant (explain in your essay why a constant) which we call $-k^2$. As $df/d\theta = k$ we have

$$f(\theta) = \int_0^\theta k\,d\phi = k\theta. \tag{23}$$

From the left hand side of (22)

$$\frac{dg(r)}{dr} = \pm\left(u(r)^2 - \frac{k^2}{r^2}\right)^{1/2}. \tag{24}$$

and we have

$$T(r,\theta) = k\theta \pm \int_0^r \left(u(x)^2 - \frac{k^2}{x^2}\right)^{1/2} dx. \tag{25}$$

Question 4:

Use (14) to show that

$$\frac{d\mathbf{r}}{ds} = \left[\pm\frac{1}{ru(r)}\left(r^2u(r)^2 - k^2\right)^{1/2}, \frac{k}{u(r)r}, 0\right] \tag{26}$$

Hint: search the literature to find the expression for the gradient ($\boldsymbol{\nabla}$) in polar coordinates.

As $|d\mathbf{r}/ds| = 1$, the angle θ between the incident ray path and the radial direction is

$$\sin(\theta) = \frac{k}{ru(r)} \tag{27}$$

and so $k = ru(r)\sin(\theta) = p$ (see (19)). Then $T(r,\theta)$ can be obtained by computing the integral

$$T(r,\theta) = p\theta \pm \int_{r_s}^r \frac{\sqrt{x^2u(x)^2 - p^2}}{x}dx. \tag{28}$$

with r_s the radius at the starting point. As T is a travelling time it must increase and so we take the $+$ sign for ascending waves, $dr > 0$, and the $-$ sign for descending waves, $dr < 0$.

It can be shown [1] that the wave travelling time along the path is a local minimum (Fermat's principle). Mathematically:

$$\frac{\partial T}{\partial p} = \theta \pm \int_{r_s}^{r} \frac{-p}{x\sqrt{r^2u(x)^2 - p^2}}dx = 0 \tag{29}$$

and so

$$\theta = \pm p \int_{r_s}^{r} \frac{1}{x\sqrt{r^2u(x)^2 - p^2}}dx. \tag{30}$$

Substituting this back into (28):

$$\begin{aligned} T(r,\theta) &= \pm \int_{r_s}^{r} \left[\frac{p^2}{x\sqrt{x^2u(x)^2 - p^2}} + \frac{\sqrt{x^2u(x)^2 - p^2}}{x} \right] dx & (31) \\ &= \pm \int_{r_s}^{r} \frac{x^2u(x)^2}{x\sqrt{x^2u(x)^2 - p^2}}dx & (32) \end{aligned}$$

The total angle of propagation between 2 points can be determined from (30):

$$\Delta = \int_{r_s}^{r} \frac{p}{x\sqrt{x^2u(x)^2 - p^2}}dx. \tag{33}$$

4 Ray path inside the Earth

Computing (32) and (33) by hand in very hard or impossible in most cases. On the other hand it is very easy to evaluate these integrals numerically using

$$\begin{aligned} T &= \sum_{r_i} \frac{r_i^2u(r_i)^2}{r_i\sqrt{r_i^2u(r_i)^2 - p^2}}dr & (34) \\ \Delta &= \sum_{r_i} \frac{p}{r_i\sqrt{r_i^2u(r_i)^2 - p^2}}dr & (35) \\ r_i &= r_s + (i+\frac{1}{2})dr, \qquad i \in [0,N] \quad \text{such that } r_N = r - \frac{dr}{2}. & (36) \end{aligned}$$

In the example considered for this essay, we assume that the earthquake's epicentre is located on the Earth surface, or very near it, and we will compute the trajectory and the travel times of the wave generated by these earthquakes.

4.1 Numerical Ray path

To compute the ray path, we proceed as follows:

- Choose the wave incident angle at the surface, θ_i, for the path considered and compute $p = r_E \sin(\theta_i)/V(r_E)$, see (19). Here $V(r_E)$ is the velocity of the wave that we start with at the surface.
- $V_P(r)$ and $V_S(r)$ are known from the standard model of the earth [2]. Use the polynomial interpolations of [3] noticing that there is a missing decimal point on the 3rd line of table 2.1. (already encoded in the supplied file `ray_IASP91_v1.py`)
- Evaluate the integrals (34) and (35) step by step starting from $r = r_E$.
- Use negative dr (the wave is moving downwards) until $r^2u^2 < p^2$. When this condition is fulfilled, the wave has reached its turning point.
- Once the turning point has been reached, we continue the integration but using positive dr until $r = r_E$.
- The values of the integrals (34) and (35) are respectively the travelling time T and the deflection angle of the path Δ (see fig 2 b).

4.2 Boundary Crossing

When the wave crosses a boundary, no special measures need to be taken, except if we want to consider a wave type change across that boundary. In that case all we have to do is to use the speed corresponding to the type of wave we are considering (the type of transmitted wave). For example, if we want to consider an S wave travelling in the mantel and entering the outer core and consider the resulting P component in the outer core, we will use V_s in the mantel and V_p in the outer core when evaluating (34) and (35). We should emphasize here that as the outer core is liquid, there are no S wave in it ($V_s = 0$).

4.3 Reflection on a Boundary

To perform a reflection on a boundary, we have to check when r crosses the considered boundary. When this happens, all we have to do is to change the sign of dr so that the wave moves in the opposite direction. We can also change the type of wave when we consider the reflected wave of the other type.

We will then consider some of the simple paths followed by waves labelling them as a series of wave type, P and S, followed by the region where they propagate, using respectively 'm', 'o' and 'i' for the mantel, outer core and inner core. As an example, the path PmPoSm corresponds to a P wave in the mantel, transmitted to the outer core and then transmitted as an S wave inside the mantel, while SmPm correspond to an S wave in the mantel which is reflected on the mantel-outer core boundary as a P wave.

We will also ignore the reflections of seismic waves on the boundary between the crust and the mantel. The reason is that the crust is so thin that these reflected waves do not travel very far. The speed of the P and S wave in the crust, on the other hand, are included in the expression of V_P and V_s obtained from [2].

5 Ray path Analysis

We are now ready to study seismic wave ray paths by computing some of the most important ones and by generating figures showing their trajectories. To do this we will write some code which will perform the integration (34) and (35) for us and generate some figures to represent the Earth and its different layers as well as some ray paths. After generating some simple paths where the wave is not reflected and does not change type, we will improve our code to take into account reflections and wave type changes at boundaries. We will then generate figures where the rays of a given type will be plotted for all incident angles between 1 and 89 degrees by steps of 1 degree. This will illustrate the effect of the Earth core.

Coding task 1:

The program `ray_IASP91_v1.py` is our first attempt to compute the travel time and trajectory of waves inside the Earth. It defines a class called `raytrace` which already contains several class functions:

- `Vp(self,r)` and `Vs(self,r)` : returns the speed of respectively P and S waves at the specified position r (distance from the centre of the Earth).
- `u(self,r,wave)` : returns the slowness of the wave, $1/V$, at the specified position for the specified type of wave.
- `plot_circle(self,R,col)` : plot a circle of the specified radius in the specified colour. This is used to plot the Earth and the different layers boundaries.
- `trajectory(self,theta,wave,dr)` computes the trajectory of the wave by initialising several class variables used to store the coordinates of the rays and the wave type. It returns the travel time T and the deflection angle Δ. The function uses the class function `Int(dr,p,wave)` which is described below.

The class `raytrace` contains other functions which must be completed (in between the marked area):

- `plot_VpVs(self)` : the function must plot V_p (in blue) and V_s (in red) as a function of r using the supplied class function `Vp()` and `Vs()`. The horizontal and vertical axis label must be "r" and "`Vs, Vp`" respectively, using a 12pt font. The figure should also have labels `Vp` and `Vs` to describe the curves (use the `label` argument of the plot function and the `plt.legend` function to display them).
- `T_Delta_Int(self,r,p,wave)`: Complete the definition of the function so that it returns a 2 element array with the values of the integrand of (32) and (33), in that order.
- `Int(self,dr,p,wave)`: the function is nearly complete, but you must complete, and uncomment, the two lines starting with `###`: complete the `while` statement and update `r`. The function integrates (32) and (33) downward from r_E, checking for the turning point at every step. Once reached it changes the sign of dr and integrates the upward section of the path. The function must return the integrated values of T and Δ as a 2 component numpy array. This works for all P waves, even when they enter the core. For S waves, the program will fail if they enter the outer core because $V_s = 0$. In that case, the function must return the array `np.array([-1,0])`.
- `plot_trajectory(self)`: complete the function so that it plots the ray path. The class variables `list_r` and `list_th` contains a list of respectively the values of the polar coordinates r and of Δ for the trajectory. Δ start with the value 0 but symmetric figures look nicer, so you should thus subtract from Δ half of the final value of Δ.

The provided program `run_code1.py` loads the module `ray_IASP91_v1.py` to generate 2 figures: a figure of V_p and V_s as functions of r (which you should include in your essay) and the trajectory of an S wave with an incident angle of 15 degrees.

We will now compute more complex ray path involving reflections and change of wave type at boundaries.

Coding task 2:

Make a copy of `ray_IASP91_v1.py` and call it `ray_IASP91_v2.py`. You must then add 3 new class functions which will be modified version of `Int(self,dr,p,wave)`. They are already defined but commented out. Please leave their definitions in between the `MARKING` comments.

- `Int_mPm(self,dr,p,wave)`: make a copy of `Int(self,dr,p,wave)` and modify it so that it computes the ray path of a wave bouncing on the top of the outer core as a P wave. The most important changes are the following
  - The first `while` test must be changed to test when one has reached the outer core as well as when the wave is turning back.
  - If the wave turns back before reaching the mantel, the path is not of the desired type and the function must return an numpy array containing `[-1,0]` to represent an invalid path.
  - When the wave has reached the outer core, dr must change sign and the wave type set to `P` before computing the upper part of the path.
- `Int_mSm(self,dr,p,wave)`: is identical to `Int_mPm(self,dr,p,wave)` except that the wave type must be set to `S` after the wave has reflected.
- `Int_mPoSm(self,dr,p,wave)`: this is slightly more complex than `Int_mPm(self,dr,p,wave)`. It must also reach the outer core and return `[-1,0]` if it does not. It must become a P wave inside the outer core and when it leaves the outer core, it must become an S wave again. Notice that this also works for waves entering the inner core as P wave (effectively `PmPoPiPoSm` path). We don't need to exclude these paths.
- The class functions `Int(self,dr,p,wave)`, `Int_mPm(self,dr,p,wave)`, `Int_mSm(self,dr,p,wave)` and `Int_mPoSm(self,dr,p,wave)` should all return the numpy array `np.array([-1,0])` when the select type of path does not exist for the specified incident angle.

- The class functions `Int(self,dr,p,wave)`, `Int_mPm(self,dr,p,wave)`, `Int_mSm(self,dr,p,wave)` and `Int_mPoSm(self,dr,p,wave)` duplicate some fair amount of code. Can you add one or more class function to avoid these duplications? (It won't make the code faster but it could make it easier to read.)
- `trajectory(self,theta,path,dr)`: this is very similar to the previous function `trajectory(self,theta,wave,dr)` except that the second argument is a path (which can be `Pm`, `PmPm`, `PmSm`, `PmPoSm`, `Sm`, `SmPm`, `SmSm` or `SmPoSm` instead of the initial wave type. The initial wave type is now the first letter of the `path`. One must then test the path, except for the first letter, and call `Int` or one of the 3 functions described above, depending on the selected path.

The provided program `run_code2.py` can be used to test your code. It loads the module `ray_IASP91_v2.py` to generate 8 figures: the trajectories of the following paths and incident angles : `Pm`, $\theta_i = 15°$; `Sm`, $\theta_i = 15°$; `PmPm`, $\theta_i = 5°$; `SmPm`, $\theta_i = 5°$; `PmSm`, $\theta_i = 5°$; `SmSm`, $\theta_i = 5°$; `PmPoSm`, $\theta_i = 5°$ and `SmPoSm`, $\theta_i = 5°$. It also outputs on the screen the incident angle and the selected path type followed by the time taken by the wave to reach the surface as well as its total deflection angle Δ.

To have a better idea of how the different trajectories compare, we would like to plot all the rays of a given type of path for a large range of incident angles (between 1 and 89 degrees). This will be done by the program `run_code3`. To do this we must call the class function `plot_trajectory` several times, but for that we must plot the circles for the different layers only once and remove the value of θ from the title. To do this we will add a class function, called `plot_multi_trajectory`, to `ray_IASP91_v2.py`.

Coding task 3:

Add the class function `plot_multi_trajectory(self,nocircle=False)` to `ray_IASP91_v2.py` by copying the class function `plot_trajectory(self)` and making the following changes:

- Add a test before plotting the 3 circles so that it is only plotted when `nocircle` is false. (`run_code3` sets `nocircle` to `False` the first time it calls `plot_multi_trajectory` and then sets it to `True`).
- Remove the value of θ from the title, keeping only the path.
- Remove the line `plt.show()` as the program `run_code3.py` calls the `show()` function to display all the trajectories on the same figure.

The function `plot_multi_trajectory` is already defined but commented out. Please write your code in between the `MARKING` comments.

The program `run_code3.py` will be used to test your program. It generates a figure of V_p and V_s as a function of r, it also generate figures showing all the ray paths from 1 to 89 degrees by steps of 1 degrees for the following paths Pm, Sm, PmPm, SmPm, PmSm, SmSm, PmPoSm, and SmPoSm. You should use these in the essay and comment on the effect that the Earth core imprints on the trajectory of seismic waves.

It also outputs on the screen the incident angle and the selected path type followed by the time taken the wave to reach the surface as well as its total deflection angle Δ. for the following paths: Pm, $\theta_i = 15°$; Sm, $\theta_i = 15°$; PmPm, $\theta_i = 5°$; SmPm, $\theta_i = 5°$; PmSm, $\theta_i = 5°$; SmSm, $\theta_i = 5°$; PmPoSm, $\theta_i = 5°$ and SmPoSm, $\theta_i = 5°$.

Question 5:

How long does it take for an S wave and a P wave propagating in the mantel to reach Durham directly if they are emitted from an earthquake in Tokyo. The geographical coordinate of Durham are $\theta_D = 54.7753°$ N, $\phi_D = 1.5849°$ W and Tokyo, $\theta_T = 35.6895°$ N, $\phi_T = 139.6917°$ E. Use a modified copy of `run_code1.py` or `run_code2.py` taking $dr = 0.1km$. Determine T so that Δ is correct within half a degree (provide the detail of how you obtained your answer in a tables).

Question 6:

In the calculation above, we have not computed the amplitudes of the waves. What information can you draw from the figure you have generated on the relative amplitudes of the waves?

5 References

[1] Nick Rawlinson *Lecture notes on Seismology* http://rses.anu.edu.au/~nick/teaching.html

[2] Adam M. Dziewonski and Don L. Anderson *Preliminary reference Earth model* Physics of the Earth and Planetary Interiors, 25 (1981) 297356

[3] Peter Bormann *Global 1-D Earth models* DOI: http://doi.org/10.2312/GFZ.NMSOP_r1_DS_2.1

6 To submit:

- One pdf file for the essay, including the figures generated by `run_code3.py`. Give all your figures a caption describing their content and refer to them in the text by number.
- Python code 2: `ray_IASP91_v2.py`

