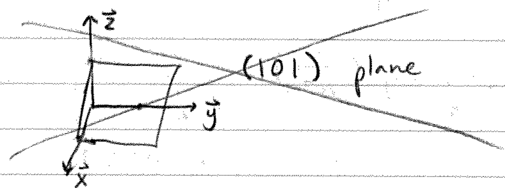
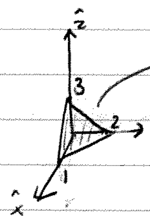
Alex Omid-Zohoor

Quals Notebook

Quals - Devices

- Crystal Structure :
    - primitive cell is the smallest unit cell
    - Miller Indices

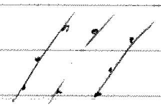
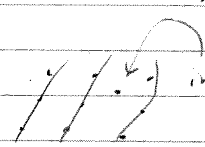




$(1, 2, 3) \rightarrow \left(\frac{1}{1}, \frac{1}{2}, \frac{1}{3}\right) \rightarrow (6, 3, 2)$ ← Miller Indices.

more localized effect

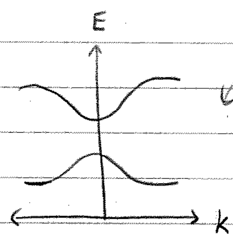
changes lattice around the defect.

- imperfections  vacancy  interstitial defect

- imperfections can be fixed by annealing.

- Energy Band

E-k diagram



$m_n^* = \left(\frac{1}{\frac{1}{\hbar^2}\frac{d^2E}{dk^2}}\right)(E)$

$F = m_n^* \frac{dv}{dt}$

↑ effective mass accounts for complex crystalline fields + quantum mechanics in crystal.

- Hole - $J = -e\sum_{filled} v_i = -e\left[\sum_{total} v_i - \sum_{empty} v_i\right] = +e\sum_{empty} v_i$

"0" "hole"

⇒ negative effective mass of electron in valence band. ← ??

$$\frac{1}{m_{xy}^*} = \frac{1}{\hbar^2}\frac{d^2E}{dk_x dk_y}$$

$T\uparrow \Rightarrow E_g \downarrow$ Lattice expands ⇒ bonding is weaker ⇒ easier for e^- to escap

- Semiconductor in Equilibrium - No external force applied.

- Density of states: $g(E) \propto (m^*)^{3/2} \sqrt{E}$

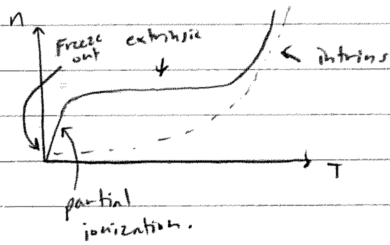
$$n(E) = g(E) \cdot f_F(E) \Rightarrow n = \int_{E_c}^{\infty} n(E)\,dE$$

- intrinsic carrier concentration: $n_i^2 = n_0 p_0 = N_c N_v \exp\left(\frac{-E_g}{kT}\right)$

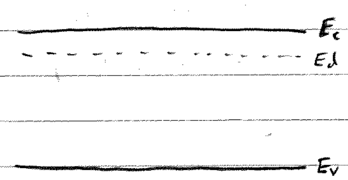
E_g, N_c, and N_v are all also related to temperature. $N_c, N_v \propto (m^* T)^{3/2}$

$T\uparrow \Rightarrow E_g \downarrow, N_c \uparrow, N_v \uparrow \Rightarrow n_i \uparrow$

- Carrier Concentration vs. Temperature



$T\uparrow \Rightarrow E_F \downarrow$



Degenerate: - if doping concentration is higher than N_c or N_v
- if $E_F \geq E_c$ or $E_F < E_v$
- behaves like a conductor.

★

Quals - Devices, Teaching Notes 10/7

Pierret, Ch 2 - Carrier Modeling

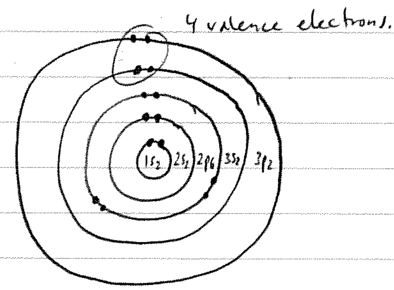
2.1 Quantization: $E_H = -\dfrac{m_0 q^4}{2(4\pi \varepsilon_0 \hbar n)^2} = -\dfrac{13.6\ eV}{n^2}$ $n = 1, 2, 3 \ldots$

$\uparrow$ electron binding energy. $\hbar = \dfrac{h}{2\pi}$, where h is plancks cons

$1\ eV = 1.6 \cdot 10^{-19}\ J$ $h = 6.62607 \cdot 10^{-34}\ J \cdot s$ $\varepsilon_0 = 8.85 \cdot 10^{-14}\ F/cm$

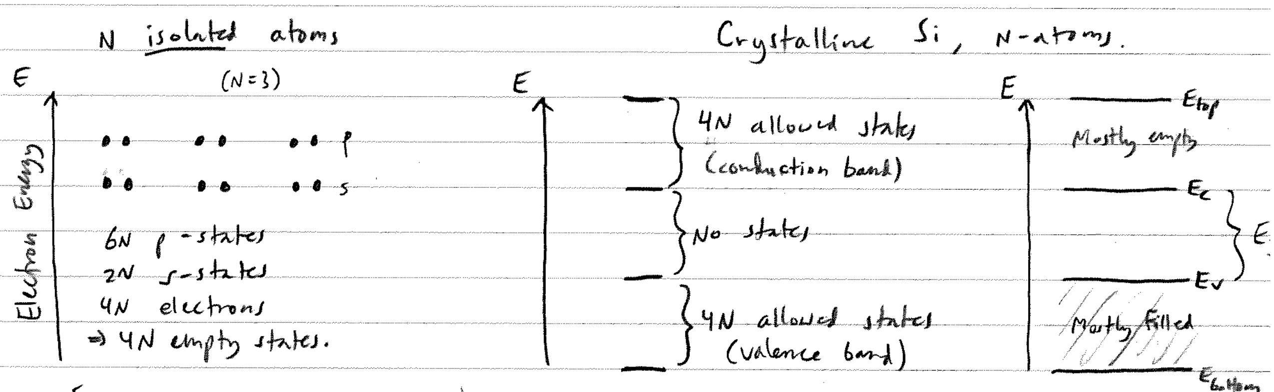
2.2 Semiconductor Models:

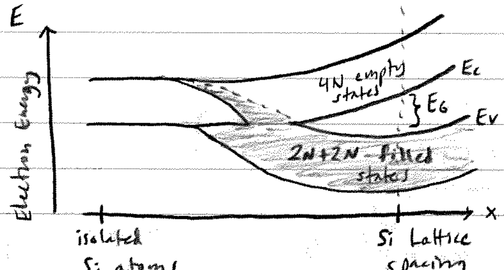
Si:



Bonding model → spatial
Energy band model → energy

figure 2.5:





(at 300K)

Band Gap - Insulator ~ 10eV $\quad SiO_2 \approx 8eV \quad$ Diamond $\approx 5eV$

Semiconductor ~ 1eV $\quad GaAs \approx 1.42eV \quad Si \approx 1.12eV \quad Ge \approx 0.66e$

Metal ~ 0eV

2.3 Carrier Properties

Charge : $q = 1.6 \cdot 10^{-19}\ C$

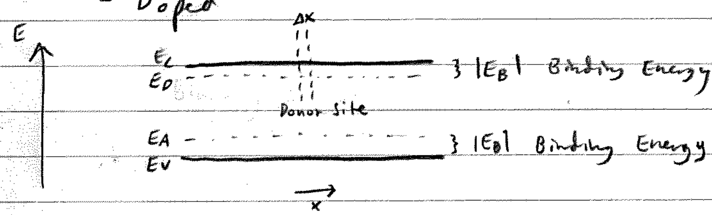
Effective mass : $F = -q\mathcal{E} = m_n^* \frac{dv}{dt}$

Carrier Concentration:

- Intrinsic $\quad n = p = n_i \quad$ at 300K:

	n_i
GaAs	$2 \cdot 10^6 /cm^3$
Si	$1 \cdot 10^{10} /cm^3$
Ge	$2 \cdot 10^{13} /cm^3$

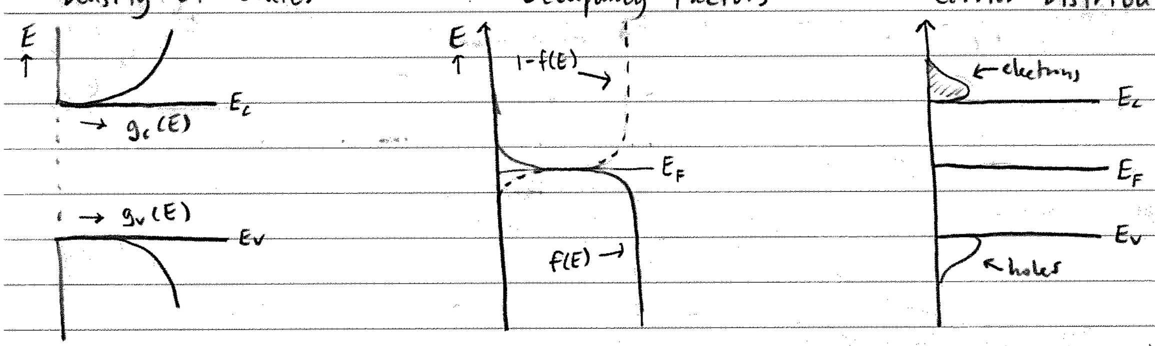
- Doped



2.4 State and Carrier Distributions.

Figure 2.16

Density of States $\qquad$ Occupancy Factors $\qquad$ Carrier Distributions



$$g_c(E) = \frac{m_n^* \sqrt{2m_n^*(E-E_c)}}{\pi^2 \hbar^3} \qquad E \geq E_c$$

$$g_v(E) = \frac{m_p^* \sqrt{2m_p^*(E_v-E)}}{\pi^2 \hbar^3} \qquad E \leq E_v$$

$$\boxed{f(E) = \frac{1}{1+e^{(E-E_F)/kT}}}$$

↑ probability that an available energy state at E will be occupied by an electron. (1-f(E) for hole)

What would happen if $m_n^* >$
where must E_i lie?

A: $n = p$, $g_c(E) > g_v(E)$

⇒ E_i must lie below midgap

Effective Density of States:

$$N_c = 2\left[\frac{m_n^* kT}{2\pi\hbar^2}\right]^{3/2} \qquad N_v = 2\left[\frac{m_p^* kT}{2\pi\hbar^2}\right]^{3/2}$$

$$\left.\begin{aligned} n &= N_c\, e^{(E_F - E_c)/kT} & p &= N_v\, e^{(E_v - E_F)/kT} \\ n &= n_i\, e^{(E_F - E_i)/kT} & p &= n_i\, e^{(E_i - E_F)/kT} \\ n_i &= \sqrt{N_c N_v}\, e^{-E_G/2kT} & np &= n_i^2 \end{aligned}\right\} \text{valid for nondegenerate, equilibrium.}$$

Charge Neutrality:

$$p - n + N_D^+ - N_A^- = 0 \quad \xRightarrow{\text{Assuming Total Ionization}} \quad \boxed{p - n + N_D - N_A = 0}$$

$$\left.\begin{aligned} n &= \frac{N_D - N_A}{2} + \left[\left(\frac{N_D - N_A}{2}\right)^2 + n_i^2\right]^{1/2} \\ p &= \frac{n_i^2}{n} = \frac{N_A - N_D}{2} + \left[\left(\frac{N_A - N_D}{2}\right)^2 + n_i^2\right]^{1/2} \end{aligned}\right\} \text{Assumes nondegenerate and total ionization.}$$

4 cases:

1) intrinsic : $N_A = N_D = 0 \Rightarrow \boxed{n = p = n_i}$

2) Doped : $\Rightarrow N_D - N_A \approx N_D >> n_i$ or $N_A - N_D \approx N_A >> n_i$ ← most common

$\Rightarrow \boxed{n \approx N_D,\ p \approx \frac{n_i^2}{N_D}}$ or $\boxed{p \approx N_A,\ n \approx \frac{n_i^2}{N_A}}$

3) Doped, high T : $n_i >> |N_D - N_A| \Rightarrow \boxed{n \approx p \approx n_i}$

4) Compensated semiconductor : $N_D \approx N_A \neq 0 \Rightarrow$ must use full equations.

Position of Fermi Level in intrinsic semiconductor:

$$\boxed{E_i = \frac{E_c + E_v}{2} + \frac{3}{4} kT \ln\left(\frac{m_p^*}{m_n^*}\right)}$$

$$E_F - E_i = kT \ln\left(\frac{N_D}{n_i}\right) \quad , \quad N_D \gg N_A, n_i \qquad n\text{-type}$$

$$E_i - E_F = kT \ln\left(\frac{N_A}{n_i}\right) \quad , \quad N_A \gg N_D, n_i \qquad p\text{-type.}$$

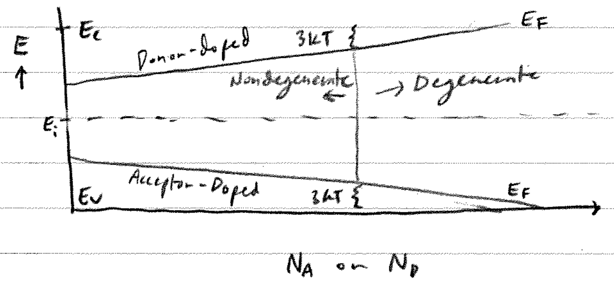
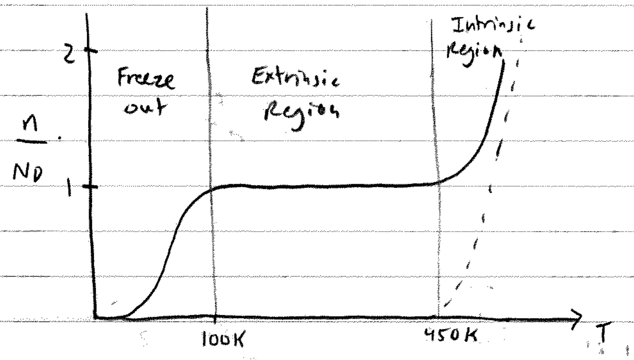


Figure 2.21

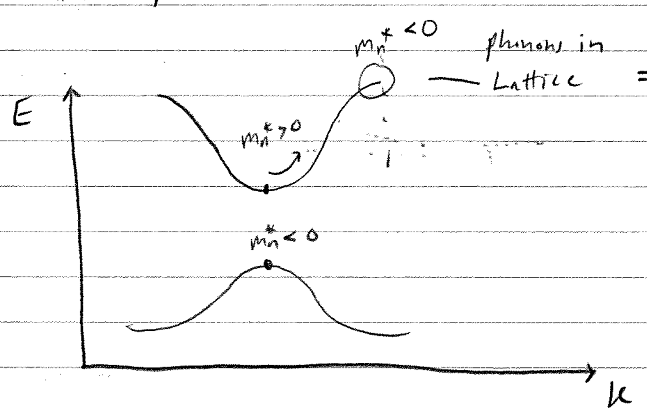


(For Si doped at $10^{15}\,cm^{-3}$)

Figure 2.22

$$E_G(T) = E_G(0) - \frac{\alpha T^2}{(T+\beta)}$$

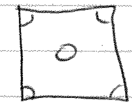
Effective Mass



phonons in Lattice $\Rightarrow$ Bragg

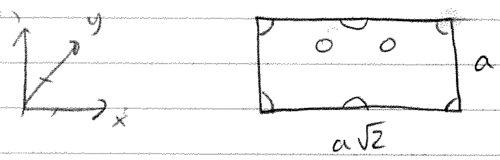
treat e^- in valence band as a hole.

1.5. a)



b) 2

c)



4

$$\boxed{\mu = \frac{q\tau}{m^*}}$$ mean free time

$$\cancel{f = m^* a = q\mathcal{E} = m^* a \frac{dv}{dt}}$$

$$\cancel{\Rightarrow q\mathcal{E} = m^* \frac{d\mu\mathcal{E}}{dt} \Rightarrow q\mathcal{E} = m^* \mu \frac{d\mathcal{E}}{dt}}$$

$$v = a \cdot \tau \qquad f = m^* a = q\mathcal{E}$$

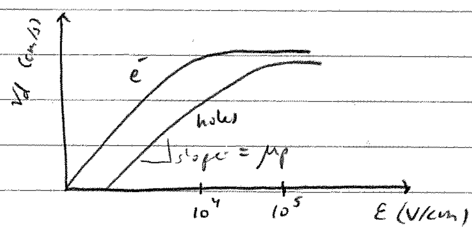
$$\Rightarrow v = \frac{q\mathcal{E}}{m^*}\tau \qquad \mu = \frac{v}{\mathcal{E}} \Rightarrow \boxed{\mu = \frac{q\tau}{m^*}}$$

Ch. 3 Carrier Action

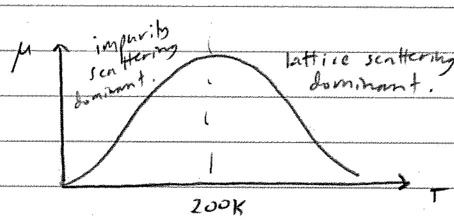
10/12/

Drift Current: $J_{drift_n} = n\mu_n \mathcal{E} q \qquad J_{drift_p} = p\mu_p \mathcal{E} q$

$v_{drift} = \mu\mathcal{E}$ for $\mathcal{E} < 10^5$ V/cm



$$\mu = \frac{\tau q}{m^*}$$



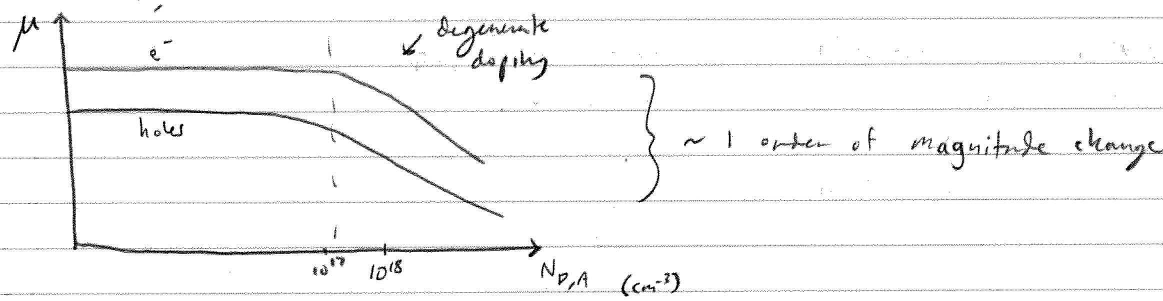
Low T: $\mu_I \propto \frac{T^{3/2}}{N_I}$

High T: $\mu_L \propto T^{-3/2}$

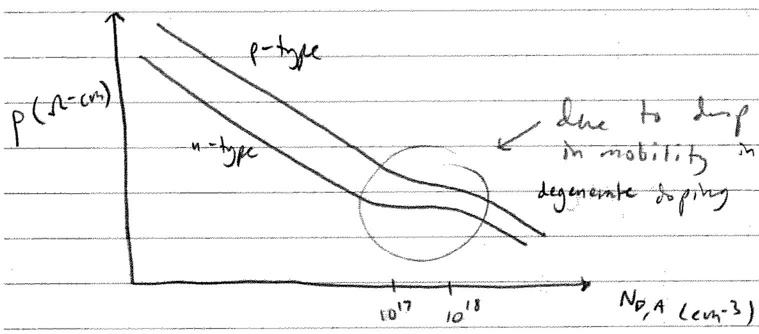
Shallow Impurities: require ~kT to ionize ⇒ most ionize

Deep Impurities: require energies larger than ~5kT to ionize ⇒ only a fraction ionize.
These impurities can be effective recombination centers and are called traps.

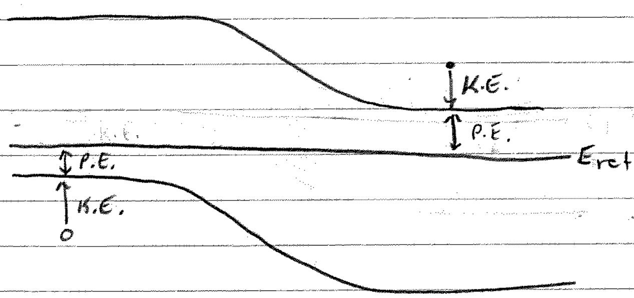
Si, 300K



Resistivity: $\rho = \frac{1}{\mu_n n q + \mu_p p q}$



Band Bending:



$$\frac{d\mathcal{E}}{dx} = \frac{\rho}{\varepsilon_0 \varepsilon_r}$$

$$\mathcal{E} = -\frac{dV}{dx} = \frac{1}{q}\frac{dE_c}{dx} = \frac{1}{q}\frac{dE_v}{dx} = \frac{1}{q}$$

$$V = -\frac{1}{q}(E_c - E_{ref})$$

Diffusion: $J_{diff_n} = D_n q \frac{dn}{dx}$ $J_{diff,p} = -D_p q \frac{dp}{dx}$

$$J_{Total} = \underbrace{\mathcal{E} q(\mu_n n + \mu_p p)}_{\text{drift}} + \underbrace{q\left(D_n \frac{dn}{dx} - D_p \frac{dp}{dx}\right)}_{\text{diffusion.}}$$

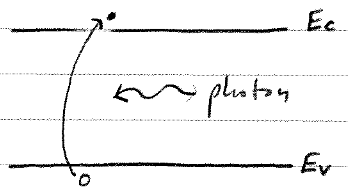
Einstein Relationship: $\frac{D_n}{\mu_n} = \frac{kT}{q}$ $\frac{D_p}{\mu_p} = \frac{kT}{q}$

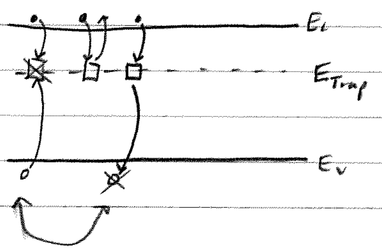
$$D_p = \mu_p \frac{kT}{q} \qquad \mu_p \propto \begin{cases} T^{3/2}, & T < 200K \\ T^{-3/2}, & T > 200K \end{cases}$$

$$\Rightarrow D_p \propto \begin{cases} T^{5/2}, & T < 200K \\ T^{-1/2}, & T > 200K \end{cases}$$

$\Rightarrow$ at room temp (300K), $D_p \propto T^{-1/2}$ $\Rightarrow$ as you increase T, diffusion rate (D_p) goes down.

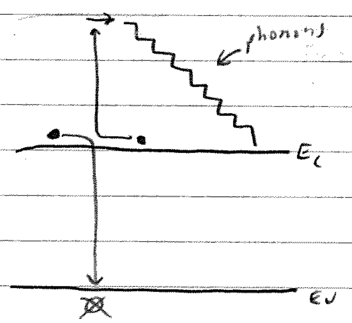
Recombination - Generation





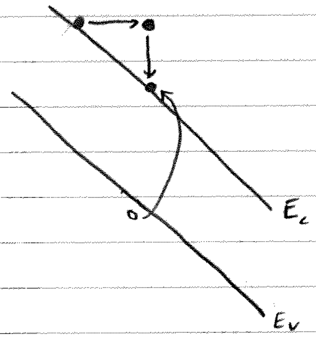
$\leftarrow$ Recombination / Generation center.

these two are the same thing.



Auger Recombination.

Impact Ionization



start with one $\bar{e}$ in conduction band. End with $2\bar{e}$ in conduction band and one hole in valence band. Highly energetic $\bar{e}$ breaks a bond in one of the atoms.

Minority Carrier Diffusion Equations.

$$\frac{\partial \Delta n_p}{\partial t} = D_N \frac{\partial^2 \Delta n_p}{\partial x^2} - \frac{\Delta n_p}{\tau_n} + G_L + \underbrace{\mu \left(\frac{n\, \partial \mathcal{E}}{\partial x} + \mathcal{E} \frac{\partial n}{\partial x} \right)}_{\text{due to applied external field.}}$$

Quasi-Fermi Level: when you are not in equilibrium (e.g. light shining, or voltage applied).

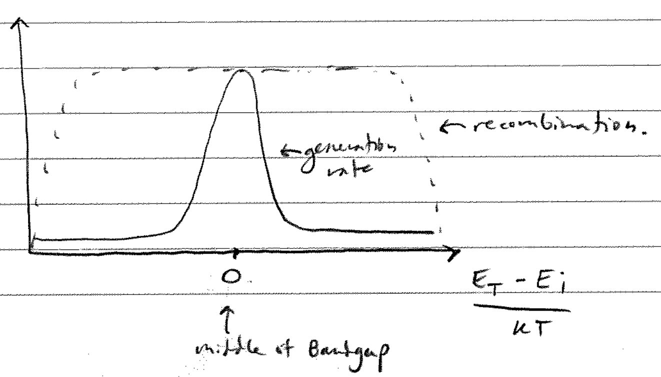
$\Rightarrow n \neq p \quad \Rightarrow \quad F_P \neq F_N$

$$F_N = E_i + kT \ln\left(\frac{n}{n_i}\right) \qquad J_N = \mu_n n \nabla F_N$$

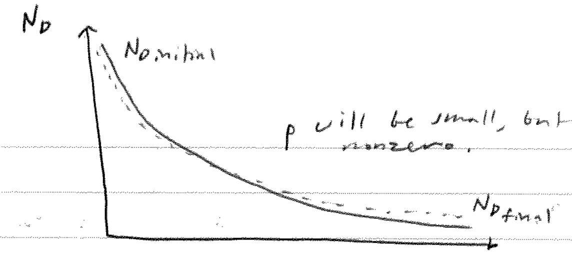
$$F_P = E_i - kT \ln\left(\frac{p}{n_i}\right) \qquad J_P = \mu_p p \nabla F_P$$

Shockley - Reed - Hall Recombination Rate:

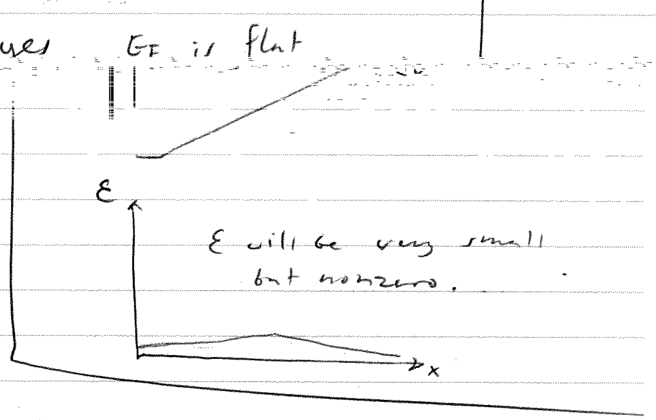
Net recombination rate $\quad U = \dfrac{v_{th}\, \sigma\, N_T (pn - n_i^2)}{n + p + 2n_i \cosh[(E_T - E_i)/kT]}$

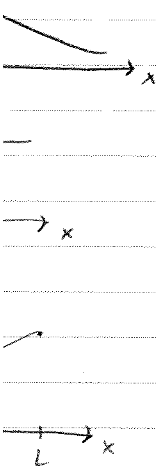


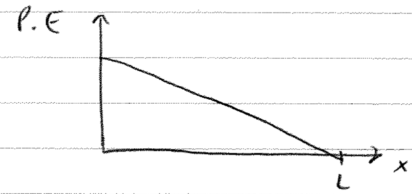
does $v_{sat} = v_{th}$? why?

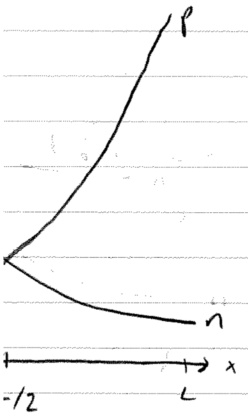


3.12 a) yes E_F is flat

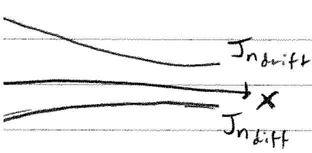


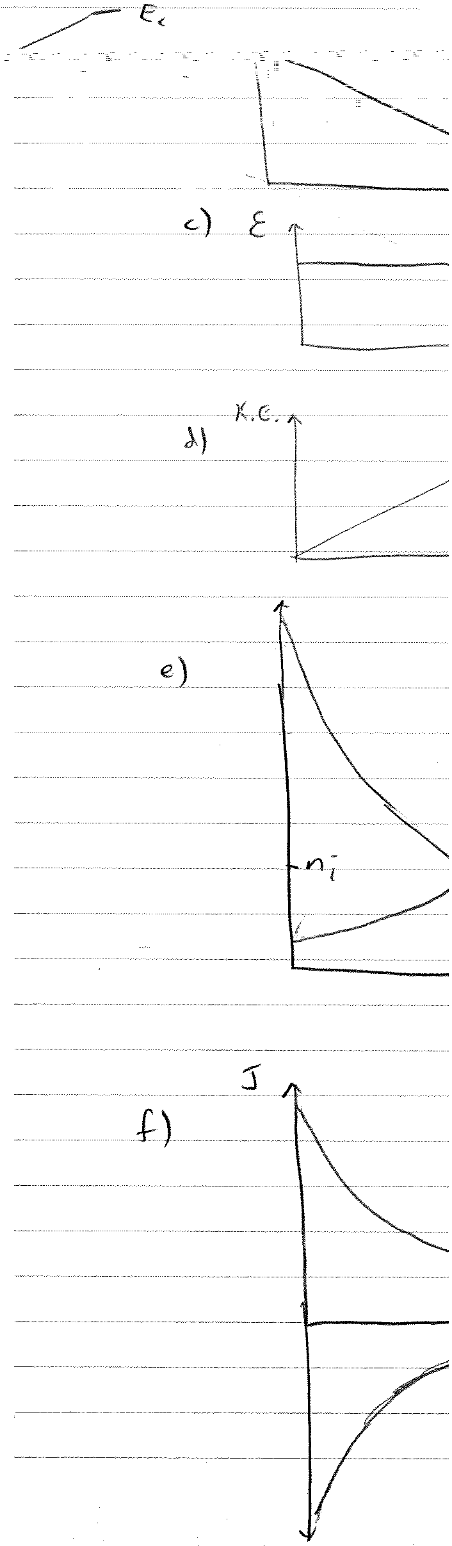




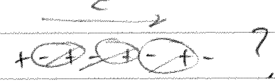


$$J_n = q\left(n\mu\mathcal{E} + D_n \frac{\partial n}{\partial x}\right) \underset{\text{equili}}{=} 0$$





- Ask Harris: does $\rho = 0$ uniformly in 3.12 a? if so, how is there a nonzero $\mathcal{E}$ field?



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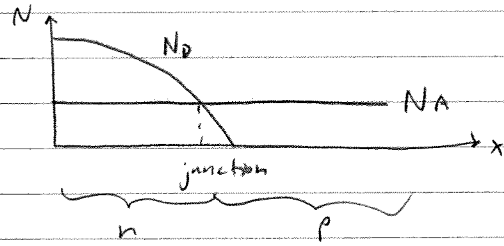
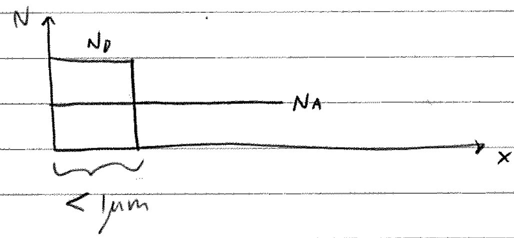
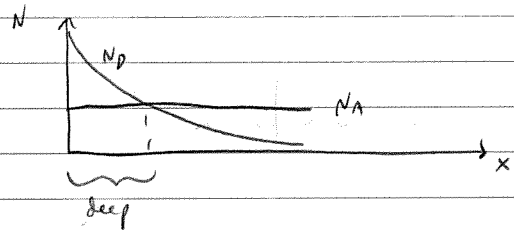
Ch. 5 - pn Junction Electrostatics

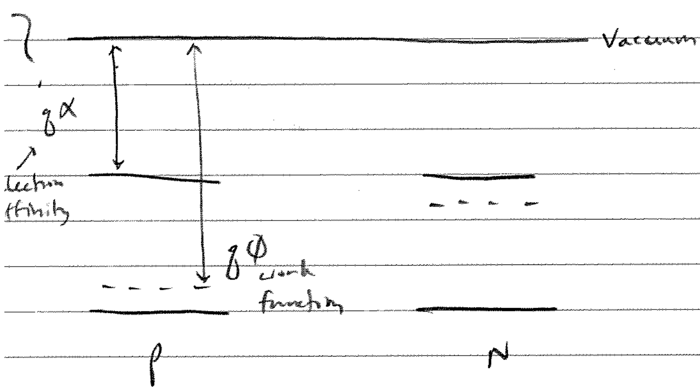




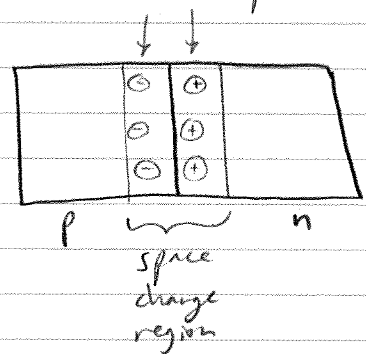
step junction - typically shallow doping

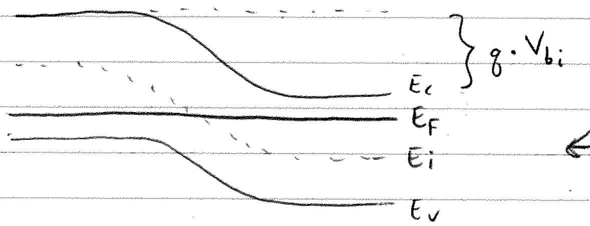


linearly graded junction - typically deep diffusion.



$\leftarrow E$ built-in (causes band bending)

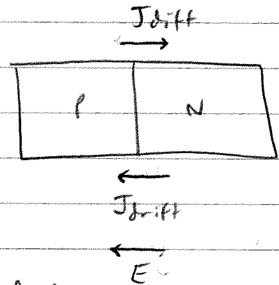


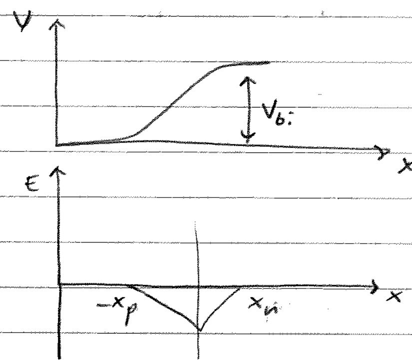


$\leftarrow$ equilibrium

in equilibrium:

$J_{diff} + J_{drift} = 0$





$$\Rightarrow q\mu_n n E + qD_N \frac{dn}{dx} = 0$$

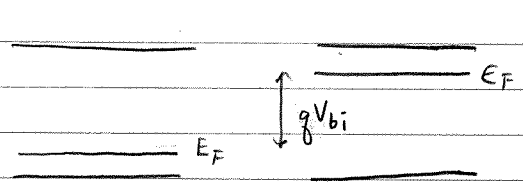
$$\Rightarrow E = -\frac{D_N}{\mu_n}\frac{dn/dx}{n} = -\frac{kT}{q}\frac{dn/dx}{n}$$

$$\Rightarrow V_{bi} = \int_{x_n}^{x_p} E(x)\,dx = \int_{x_n}^{x_p} -\frac{kT}{q}\frac{dn/dx}{n} = \frac{kT}{q}\ln\left(\frac{n(x_n)}{n(x_p)}\right)$$

$$\Rightarrow \boxed{V_{bi} = \frac{kT}{q}\ln\left(\frac{N_A N_D}{n_i^2}\right)}$$

$\leftarrow$ only works for nondegenerate case.
(when degenerate, dopants aren't totally ionized
$\Rightarrow$ can't use $n(-x_n) = N_D$ $n(-x_p) = \frac{n_i^2}{N_A}$)

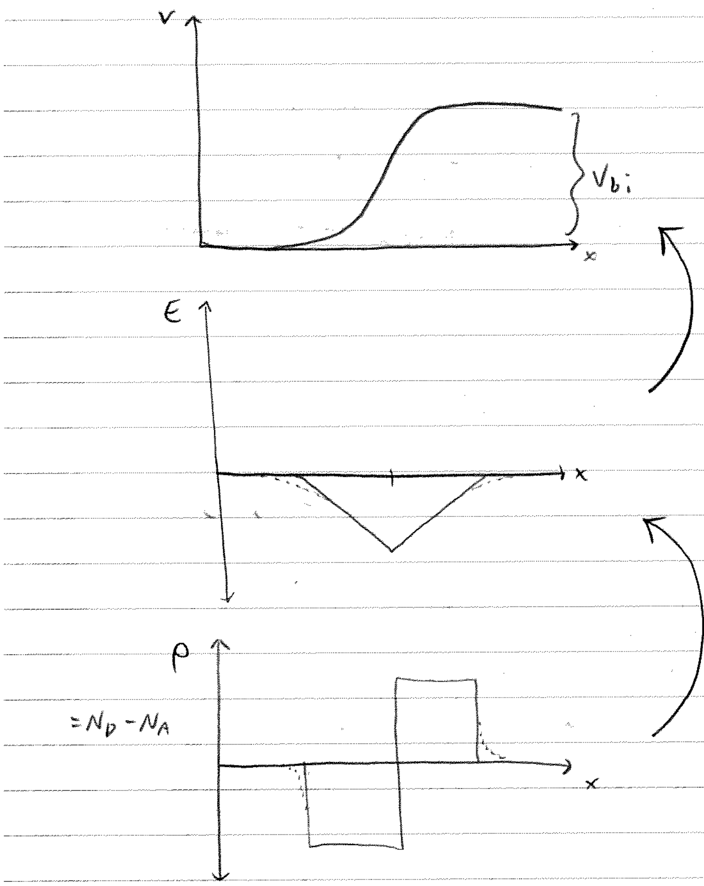
another way to find V_{bi}:



$$V_{bi} = \frac{1}{q}\left(E_{F_1} - E_{F_2}\right)$$

how to find:
E
x_n, x_p

Assumptions:
1. $\rho = 0$ in bulk
2. $n = p = 0$ in depletion region.
3. step junction.



Approximate ——
Actual - - - - -

$\Rightarrow V_{bi} = \text{function}(x_n, x_p) = \frac{kT}{q} \ln \frac{\cdots}{\cdots}$

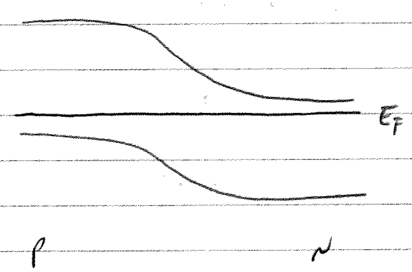
$$\frac{dV}{dx} = -E.$$

$$\frac{dE}{dx} = -\frac{\rho}{K_s \varepsilon_0}$$

$$x_n N_D = x_p N_A$$

plugging this in, we can figure out x_n and x_p.

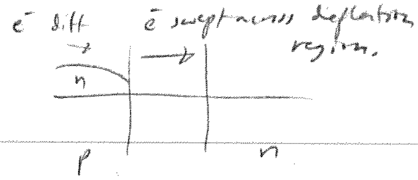
Ch. 6 pn Junction Diode: I-V Characteristics

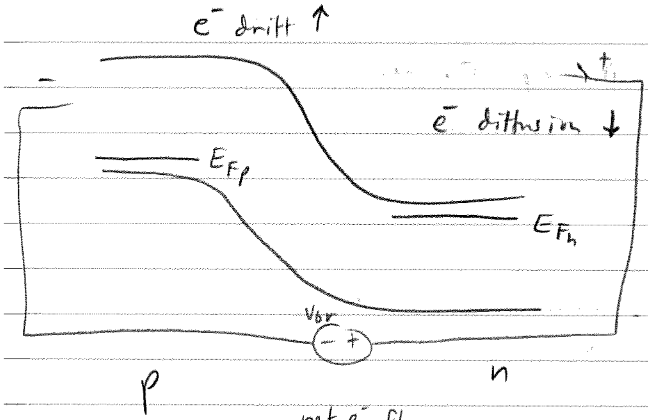


equilibrium

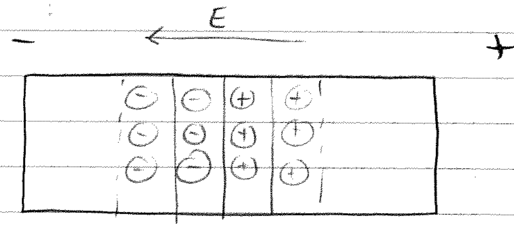
net e^- flow →

net current ←





Reverse bias

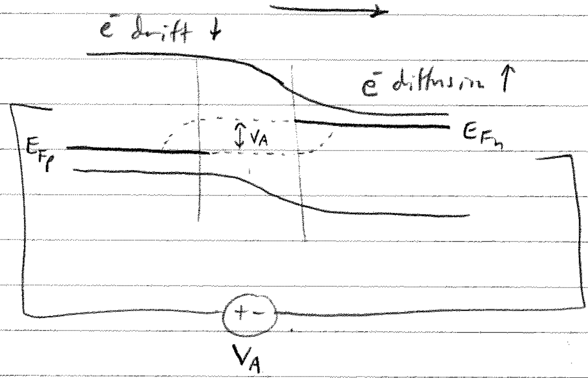


depletion region expands.

drops $V_{bi} + V_{br}$ across depletion region

net e^- flow ←

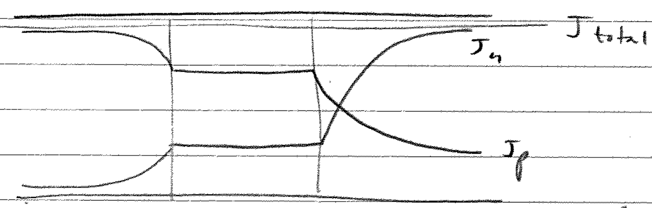
net current →



Forward bias

current determined mainly by diffusion of e^- from n to p and diffusion of holes from p to n but at the far left and far right, current is determined by drift of the majority ca.

Assume that there is no recombination or generation in the depletion region $\Rightarrow$ J_N and J_p are flat in the depletion region.



$J = D \frac{dn}{dx}$ need to know $n(x)$

can find $n(x)$ from the continuity equation.

$\Delta p = p - p_0$

$\Rightarrow \quad 0 = D \frac{d^2 \Delta n}{dx^2} - \frac{\Delta p_n}{\tau_p}$ need boundary conditions: Law of the junction:

allows you to calculate the concentration of the carriers at the boundary

$$\boxed{np = n_i^2 \exp\left(\frac{qV_A}{kT}\right)^n}$$

can use this to solve $n(x)$, then can find J_n and J_p $\quad (J_n + J_p = J_{total})$

Reverse bias current: $J_{total} = q\left(\frac{D_p}{L_p} \cdot \frac{n_i^2}{N_D} + \frac{D_N}{L_p}\frac{n_i^2}{N_A}\right)$

velocity — p on n-side — velocity — n on p-side:

$L = \sqrt{D\tau}$

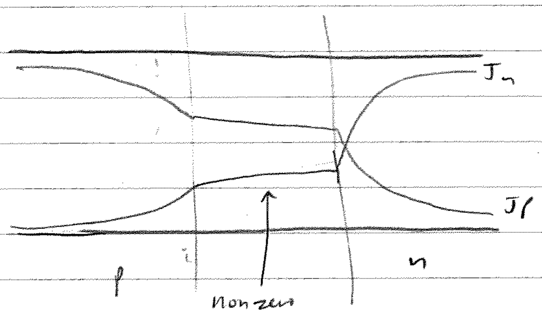
$$\boxed{I_o = qA\left(\frac{D_p}{L_p} \cdot \frac{n_i^2}{N_D} + \frac{D_N}{L_p}\frac{n_i^2}{N_A}\right)}$$

$J = q \cdot v \cdot n$

$v = \frac{L}{\tau} = \frac{\sqrt{D\tau}}{\tau} = \frac{D}{L}$

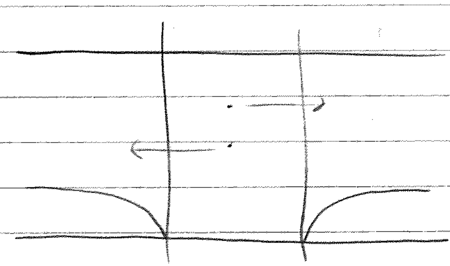
$$\boxed{I = I_o \cdot \left(e^{\left(\frac{qV_A}{kT}\right)} - 1\right)}$$

Non-idealistic Phenomenon



Non-zero recombination-generati current in depletion region.

e.g. for reverse bias:



almost zero carrier concentration in depletion region.

⇒ almost no recombination.

but there is generation in depletion region

⇒ causes generation currents

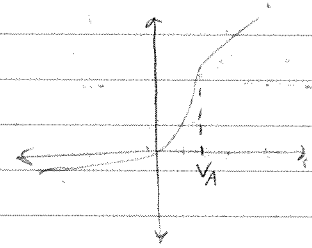
$I_{diff} \propto n_i^2$ $\quad I_G \propto n_i$

⇒ in reverse bias, generation current is greater than diffusion current in Si but not in Ge.

for small forward bias, there are significant carriers in the depletion region ⇒ recombination occurs more than generation.
⇒ we get a nonzero recombination current

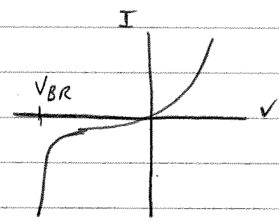
for very small forward bias, this recombination current dominates the diffusion current. But the opposite happens for large forward bias.

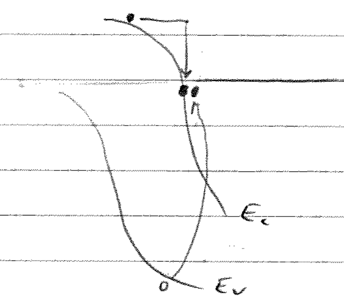
for $V_A > V_{bi}$ $\quad I \propto e^{\frac{qV_A}{2kT}}$



Avalanche Breakdown

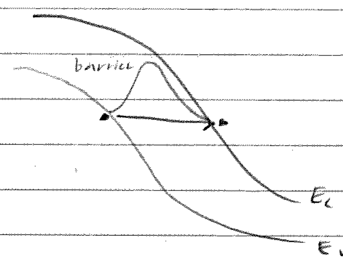
(due to impact ionization)





$V_{BR} \propto \frac{1}{N_B^{0.75}}$

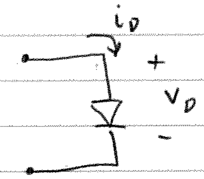
Zener Breakdown
(tunneling)



low V_{BR} ⇒ usually zener
high V_{BR} ⇒ usually avalanche

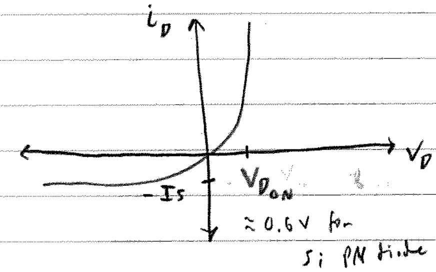
Quals - Circuits (Presenting on Diodes) 10/16/2011

Diode:

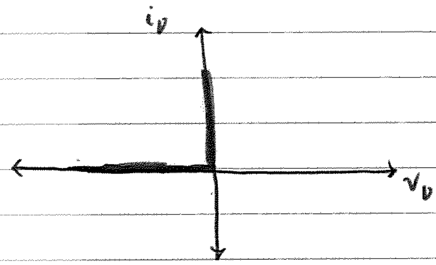


$$i_D = I_S\left(e^{\frac{V_D}{V_T}} - 1\right)$$

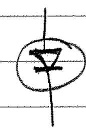
$$I_S = 10^{-12}A, \quad V_T = \frac{kT}{q} \approx 0.025V \text{ at } 300K.$$



Actual (Nonlinear)

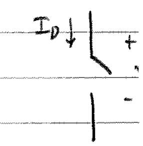


Ideal

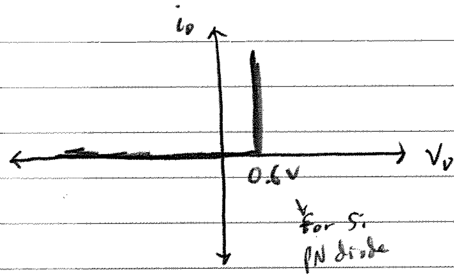


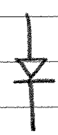
used when $V_D >> 0.6V$

Switch mo

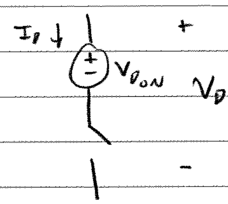


Piecewise Linear

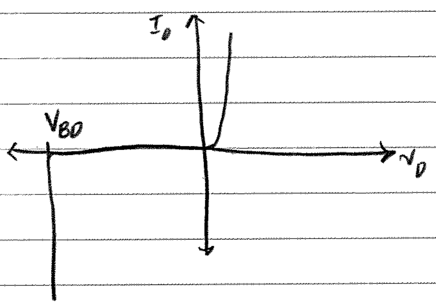




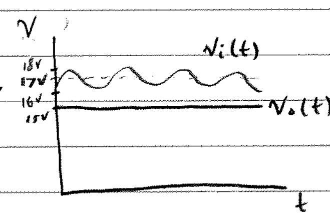
switch model

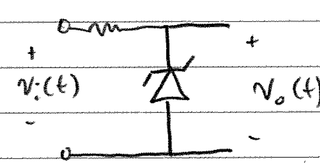


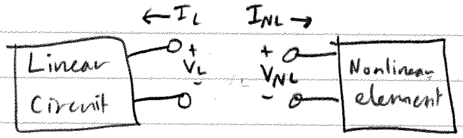
Zener Diode: Designed to operate in breakdown mode:



e.g.







$I_L = f_L(V_L)$

$I_{NL} = g_{NL}(V_{NL})$

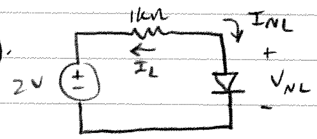
$I_{NL} = -I_L$

$V_{NL} = V_L$

$\Rightarrow I_{NL} = -f_L(V_{NL}) \leftarrow$ linear "loadline"

$I_{NL} = g_{NL}(V_{NL}) \Rightarrow$ solve for V_{NL}.

e.g.

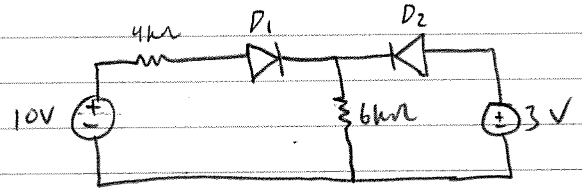


Given $I_s = 10^{-15}$ A, find V_{NL}

Diode Circuit Analysis by Assumed Diode States:

1) Specify ideal diode model or Piecewise Linear model
2) Each diode can be ON or OFF
3) Circuit containing n diodes will have 2^n states
4) Combo consistent with KVL + KCL will be the solution.

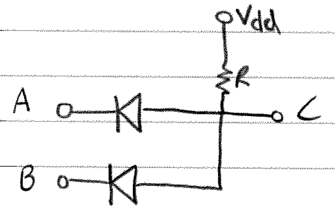
e.g.



D_1 on D_2 off.

4 possible states.

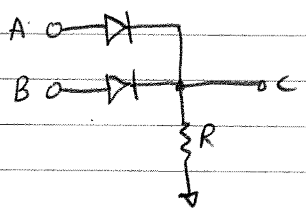
Diode Logic: AND Gate



$C = A \cap B$ A and B vary between 0V and Vdd

$\Rightarrow$ C varies between 0.6V and Vdd

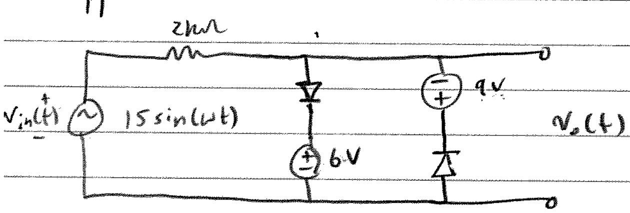
OR Gate:

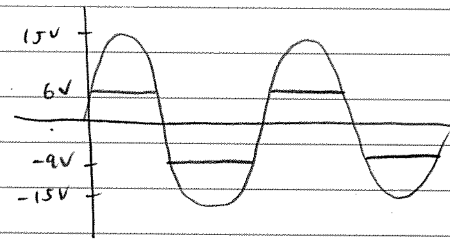


$C = A \cup B$ A and B vary between 0V and V_{DD}

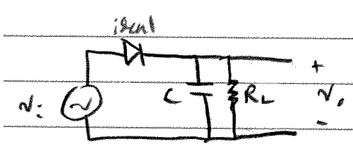
$\Rightarrow$ C varies between 0V and Vdd − 0.6V

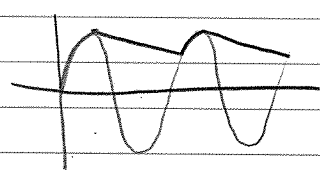
Clipper Circuit:



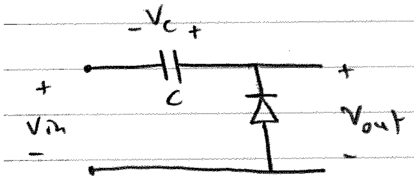


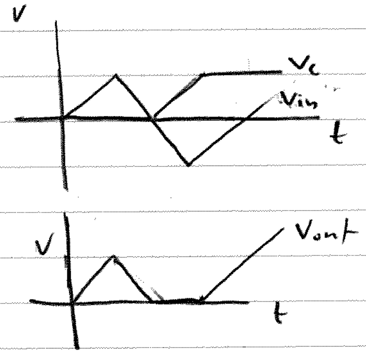
Peak Detector (Rectifier)





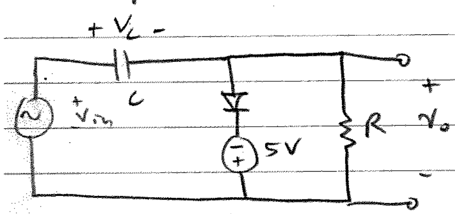
Level, shifter

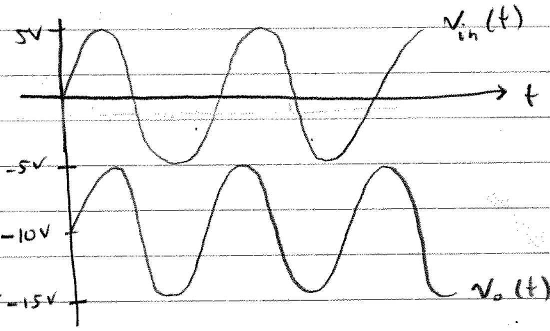




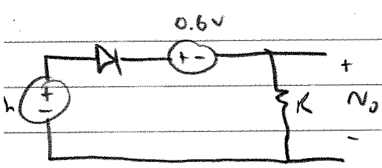
1) Diode = open, $V_c = 0$, $V_{out} = V_{in}$
2) Diode = short, $V_c = -V_{in}$, $V_{out} = 0$
3) Diode = open, $V_c = -V_{in}(min)$, $V_{out} = V_{in} + V_c$

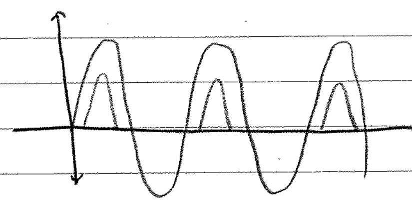
Clamp Circuit (level shifter)



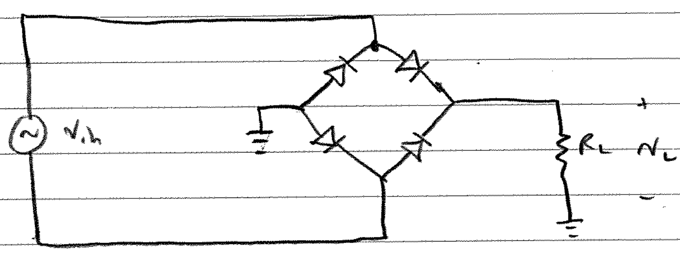


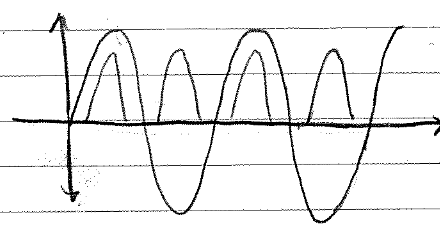
Half Wave Rectifier



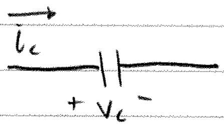


Full Wave Rectifier





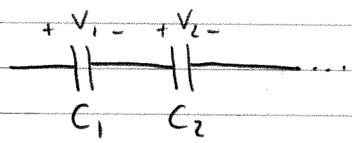
Capacitors:



$q = VC \qquad i_c = C\frac{dV}{dt} = \frac{dq}{dt} \Rightarrow V_c(t) = V_c(0) + \frac{1}{C}\int i_c(t)$

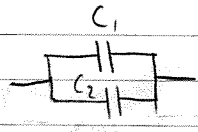
I-V characteristics.

$W = \frac{1}{2}CV^2 = \frac{Q^2}{2C} = \frac{1}{2}QV$



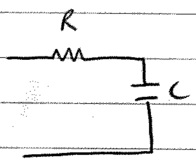
$C_t = \left(\frac{1}{C_1} + \frac{1}{C_2} + \cdots\right)^{-1}$

$V_1 = \frac{C_2}{C_1 + C_2} V_t$



$C_t = C_1 + C_2 + \cdots$

$q_1 = \frac{C_1}{C_1 + C_2} Q_{total}$

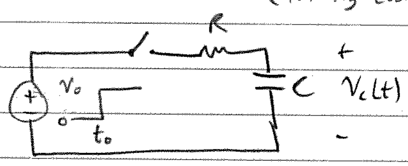


$\tau = RC$

(after $\sim 5\tau$, we assume cap is totally charged)

—||— DC - open circuit

high freq. AC - short circuit



1) $\tau = RC$

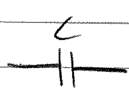
2) initial conditions at $t < 0$

$V_c(t) = V(0)\, e^{-t/\tau}$

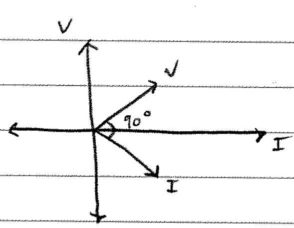
3) $t > 0$

$V_c(t) = V_c(\infty)\,(1 - e^{-t/\tau})$

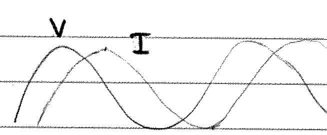
4) $V_c(t) = [V_c(0) - V_c(\infty)]\, e^{-t/\tau} + V_c(\infty)$



$Z = \frac{1}{sC} = \frac{1}{j\omega C}$



Voltage leads current by 90°



Quals Q

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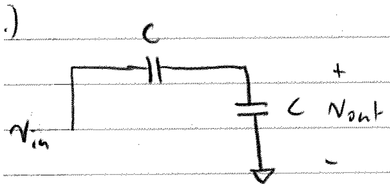
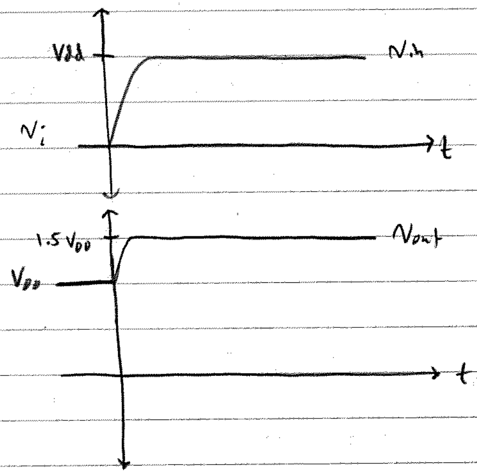
1.)





2.) Initial:

$\left(\frac{1}{2} C V_{DD}^2\right) \cdot 2 = C V_{DD}^2$

Final:

$\frac{1}{2} C (1.5 V_{dd})^2 = 1.25 C V_{dd}^2$

3.) $V_{out} = \frac{C_{left}}{C_{left}+C_{right}} V_{dd} + V_{dd}$ $\Rightarrow$ if $C_{left} \gg C_{right}$ $V_{out_{max}} = 2V$

4.) Modify the circuit to achieve final vout = 3Vdd ??

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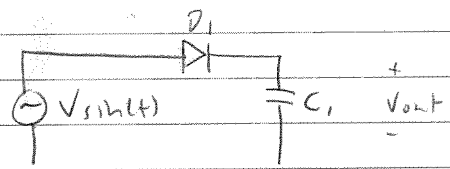
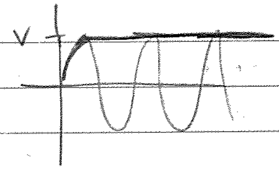
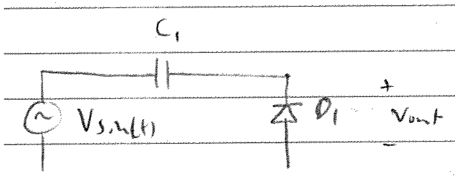
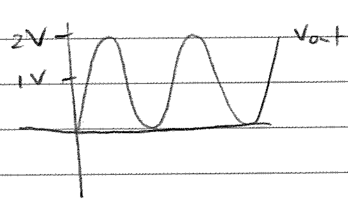


what is steady state Vout value? $\boxed{V}$





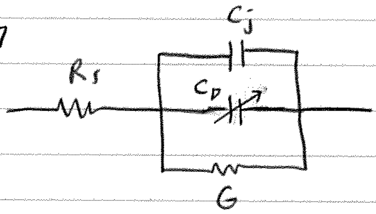
find steady state Vout (t)



Quals - Devices (Ch 7 + 8) 10/17/

PN Diodes Small signal Admittance and Transient Response

Ch. 7

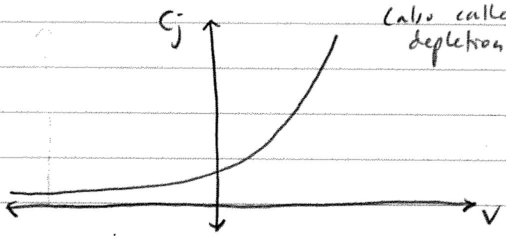


Model of PN junction

Reverse Bias: $G \approx 0 = \frac{dI}{dV}$ ← I is constant vs. V in reverse bias.

$C \approx C_j = \frac{K_s \epsilon_0 A}{W}$ ← depletion width

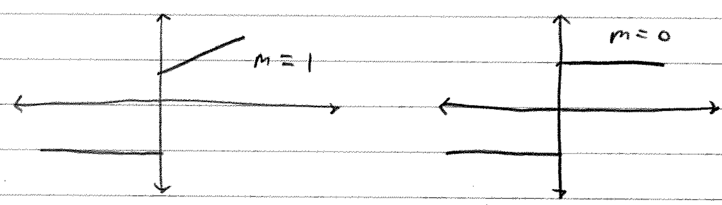
C_j caused by movement of majority carriers (also called depletion capacitance)



$= \frac{dQ}{dV}$ $Q = x_n N_D \cdot A$ ← function of applied voltage.

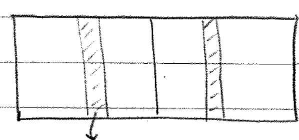
$= \frac{C_{j0}}{\left(1 - \frac{V}{V_{bi}}\right)^{\frac{1}{2+m}}}$

Linearly Graded Junction $N_B = N x^m$



step junction

Forward Bias:

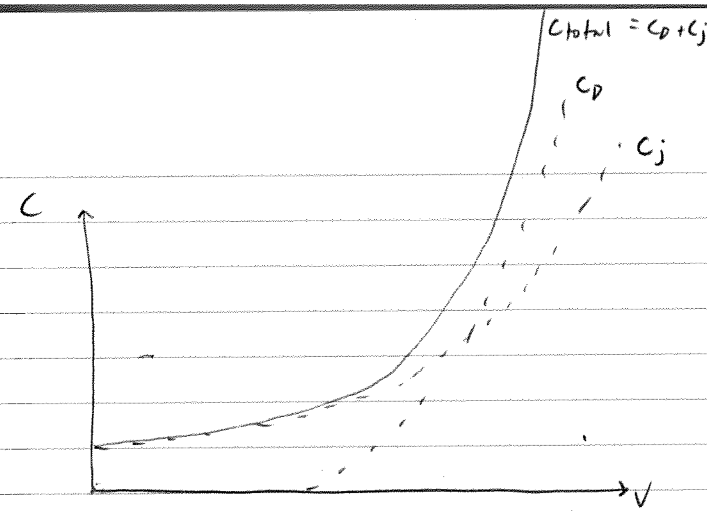


$C = C_D + C_j$

7.3.2

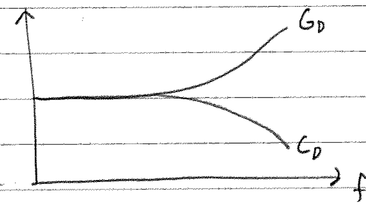
diffusion region ⇒ C_D diffusion capacitance.

$$\frac{\partial p}{\partial t} = D \frac{\partial^2 p}{\partial x^2} - \frac{\Delta p}{\tau_p} \quad \ldots$$

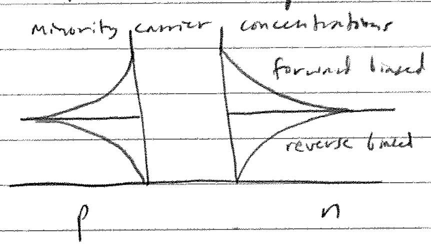


Reverse bias: C_j dominates
Forward bias: C_D dominates.

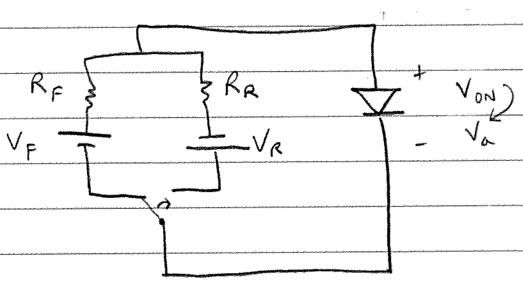
? pg. 320: Why does G_D ↑ as f ↑ ?

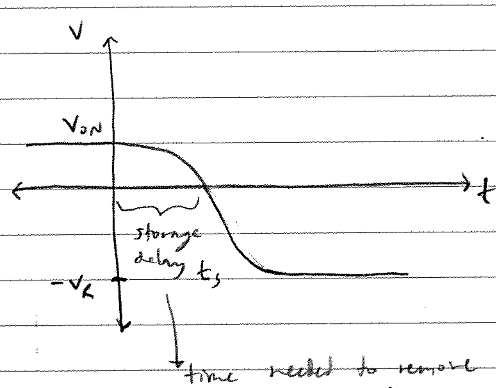


Ch. 8 Transient Response.



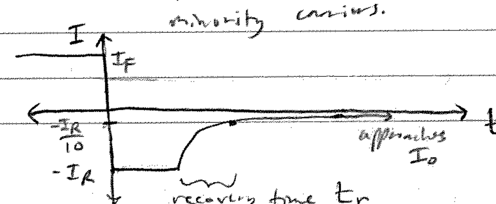
off → on Majorities diffuse
on → off Must remove minorities
1) recombination (slow)
2) drift current





$$I_F = \frac{V_F - V_{ON}}{R_F} \approx \frac{V_F}{R_F}$$

$$I_R = \frac{V_R + V_a}{R_R} \approx \frac{V_R}{R_R}$$



represents recombination process

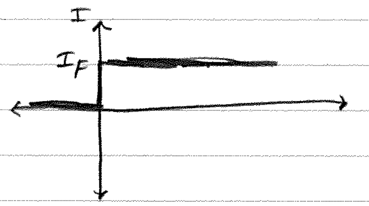
represents the number of minorities you have to remove.

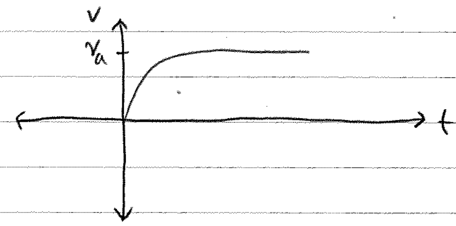
$$t_s = \tau_p \ln\left(1 + \frac{I_F}{I_R}\right)$$

represents drift process

Increasing # of R-G centers $N_t \uparrow$ will decrease t_s, but increase leakage current $\Rightarrow$ there is a tradeoff.

Turn ON: Due to movement of majority carriers (very quick process)





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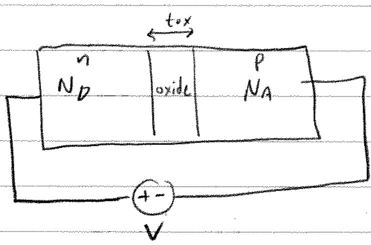
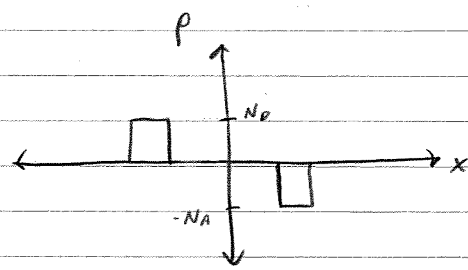
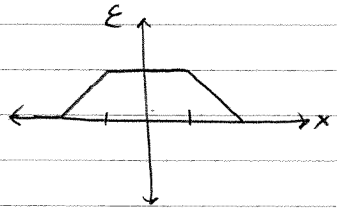


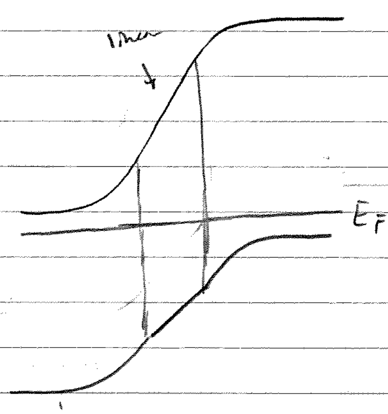
$N_D = N_A$

a) sketch $\rho(x)$ at $V=0$

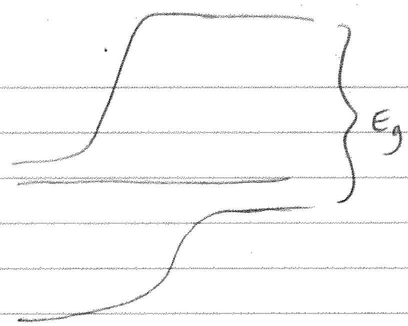


b)



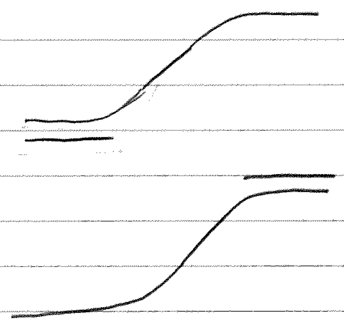


c)



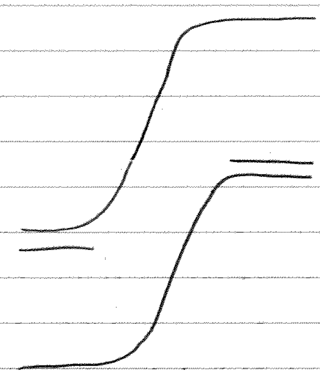
V_{bi} = same as for a PN

d)



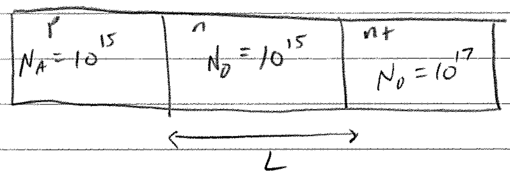
accumulation

e)



inversion

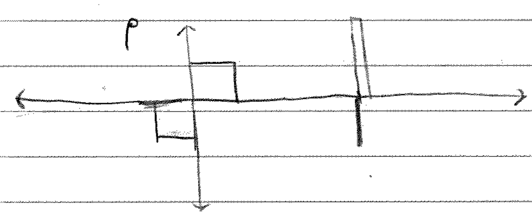
Question from [redacted] (old EE216 midterm question)

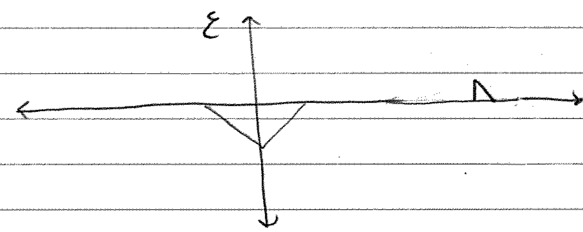


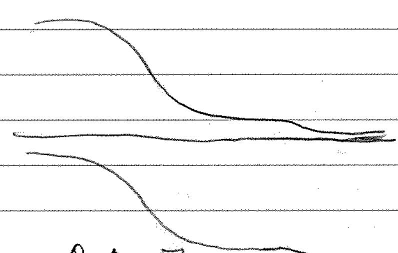
a) draw ρ, $\mathcal{E}$, V, band diagram.

b) if $L \ll x_n$, repeat part a)

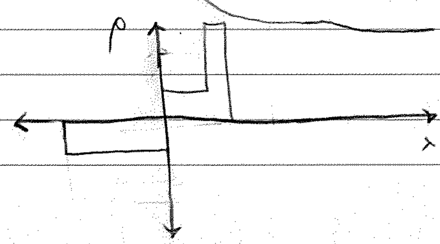
a)

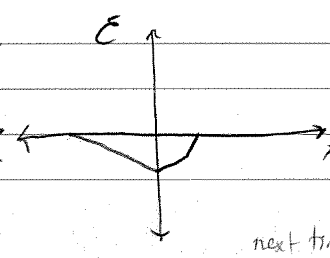


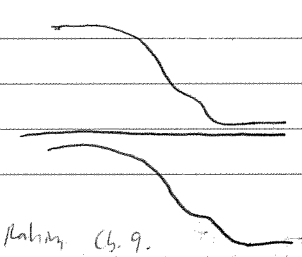




b)



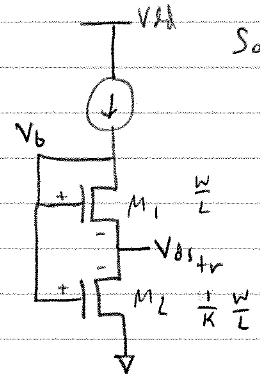




next time Rahim Ch. 9.

10/19/11

Sooch Cascode

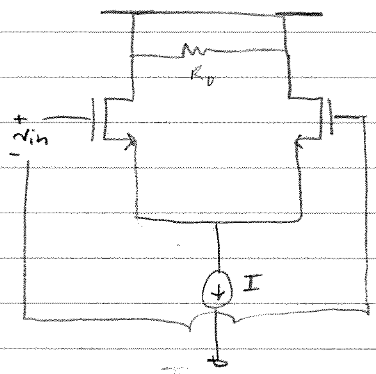


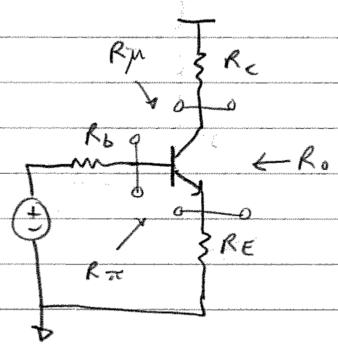
$V_{gs_1} = V_{gd_2}$ $\quad V_{gs_1} > V_T \Rightarrow V_{gd_2} > V_T \Rightarrow$ M2 is in Tri

$\frac{1}{2}\mu C_{ox}\frac{W_1}{L}(V_{ov})^2 =$

$$V_{ds_{tr}} = (\sqrt{K+1} - 1)V_{ov} \qquad V_b = \sqrt{K+1}\,V_{ov} + V_t$$

Diff pair





assume $r_\pi = \infty$

$$R_o = r_o \,||\, \frac{R_c + R_E}{1 + g_m R_E}$$

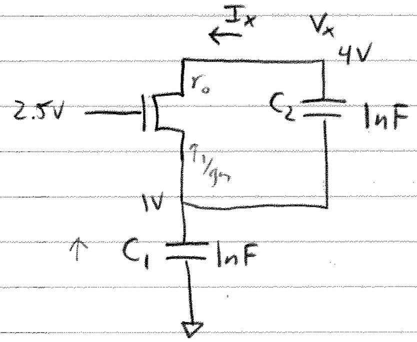
$$\frac{v_e}{R_E} = -(i_t + g_m v_e) = -\frac{v_e + v_t}{R_c}$$

$$v_e\left(\frac{1}{R_E} + \frac{1}{R_c}\right) = -\frac{v_t}{R_c} \Rightarrow v_e = -\frac{R_E}{R_c + R_E}v_t$$

$$\Rightarrow -\frac{v_t}{R_c + R_E} = -i_t - g_m\left(\frac{R_E}{R_c + R_E}\right)v_t \Rightarrow v_t\left(\frac{1 + g_m R_E}{R_c + R_E}\right) = i_t$$

$$\Rightarrow \frac{v_t}{i_t} = \frac{R_c + R_E}{1 + g_m R_E} \qquad \Rightarrow R_o = r_o \,||\, \frac{R_c + R_E}{1 + g_m R_E}$$

Murmann 2009

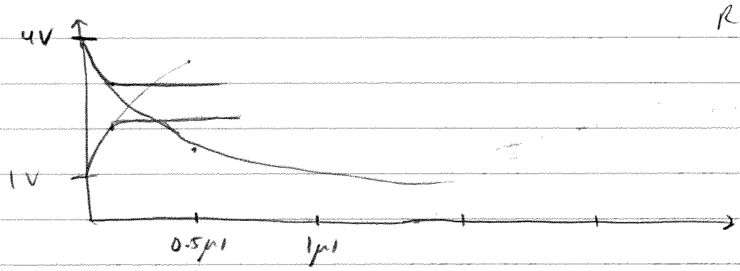


$V_t = 0.5V \quad \mu C_{ox} \frac{W}{L} = \frac{1mA}{V^2}$

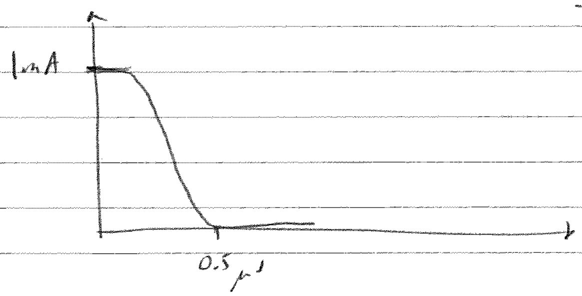
$V_{GS} = 3V \quad 1V = V_{ov}$ to start $\Rightarrow$ sat.

$g_m = \frac{2I_D}{V_{ov}} \approx 2mA \Rightarrow 500\Omega$

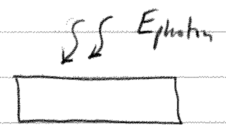
$RC = 0.5\mu s$



$-t/\tau$



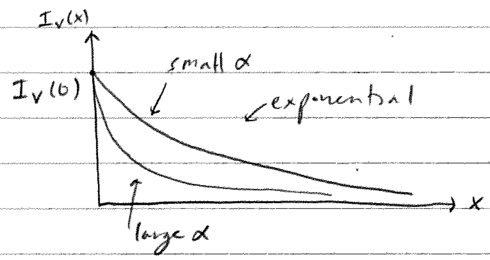
[redacted] - Solar Cells + Photodiodes (from Neaman) 10/21/11



if $E_{photon} > E_G$, photon absorbed.
otherwise, transparent $\Rightarrow$ not absorbed.

$$E = \frac{hc}{\lambda} = \frac{1.24}{\lambda} \Rightarrow \lambda \le \frac{1.24}{E_g} \text{ will be absorbed.}$$

light intensity.
$I_v(0)$
x=0



Assuming a uniform absorption coefficient (α independent of x)

α is a function of λ.

$$I_v(x) = I_v(0)\, e^{-\alpha x}$$

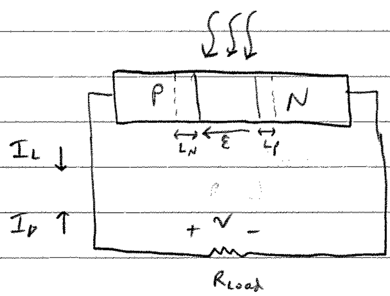
$$g' = \frac{\alpha I_v(0)}{E_{photon}} = \frac{\alpha I_v(0)}{h\nu} \leftarrow \frac{\text{number}}{cm^3 \cdot s}$$

generation rate.

reflection at surface

in general: $G_L = \alpha(\lambda)\, I_0(\lambda)\, [1 - R(\lambda)]\, e^{-\alpha(\lambda) x}$ $\leftarrow$ in general, these param are all functions of wavelength.

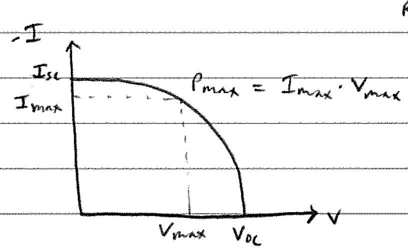
Solar cell:



$$I_{net} = I_L - I_D = I_L - I_s\left[e^{V/(kT/q)} - 1\right]$$

$$I_{sc} = I_L$$

$$V_{oc} = \frac{kT}{q} \ln\left(1 + \frac{I_L}{I_s}\right)$$

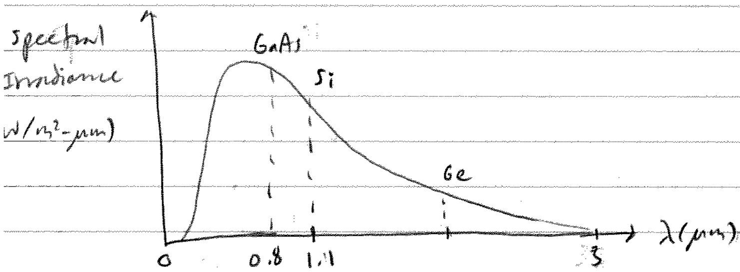


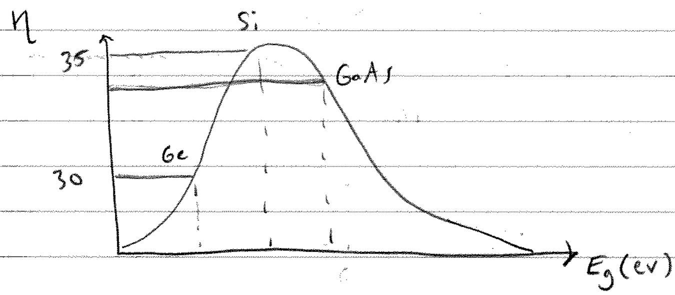
$$P = VI = V\left(I_L - I_s\left[e^{V/(kT/q)} - 1\right]\right)$$ to find P_{max}, set $\frac{\partial P}{\partial V} = 0$

$$\Rightarrow \left(1 + \frac{V_m}{(\frac{kT}{q})}\right) e^{V/(kT/q)} = 1 + \frac{I_L}{I_s}$$

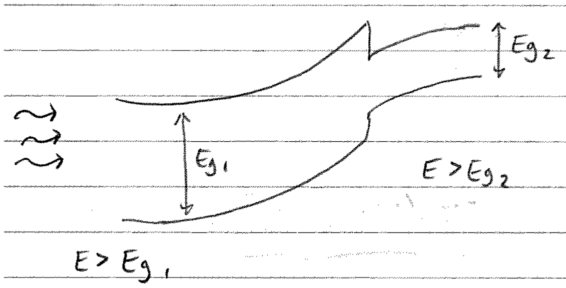
Conversion efficiency : $\eta = \frac{P_{max}}{P_{in}} \cdot 100$

fill factor : $ff = \frac{I_m V_m}{I_{sc} V_{oc}}$



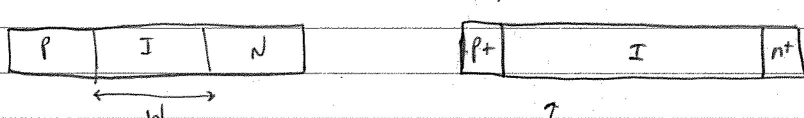


Heterojunction : can absorb a wider spectrum of light.



Amorphous Si : $\alpha \uparrow$ $\eta \downarrow$ but it's cheap.

PIN : Widen depletion region → more photocurrent :

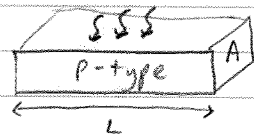


typical structure for a solar cell.

$$\tau_p = \frac{1}{c_p N_T} \qquad \tau_n = \frac{1}{c_n N_T}$$ where c_p and c_n are capture coefficients.

Photodetectors: Reverse Biased.

Photoconductors:



$$\sigma_0 = \mu_p q p_0 + \mu_n q n_0$$

When you shine light $\Rightarrow$ $\sigma_t = (\mu_p q(p_0+\Delta p) + \mu_n q(n_0+\Delta n))$ $\qquad \Delta p = \Delta n$

$$\Delta\sigma = q(\Delta n)(\mu_n+\mu_p) \qquad J = \sigma_t \mathcal{E} = (\sigma_0 + \Delta\sigma)\mathcal{E}$$

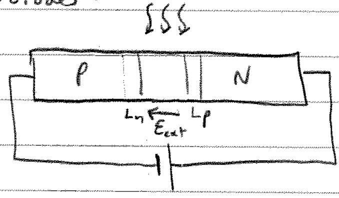
$$I_L = J_L A = \Delta\sigma A\cdot\mathcal{E} = q G_L \tau_p(\mu_p+\mu_n) A\cdot\mathcal{E} \qquad t_n = \frac{L}{\mu_n \mathcal{E}}$$

t_n ← electron transient time. time for electron to flow across photoconductor.

$$I_L = q G_L \left(\frac{\tau_p}{t_n}\right)\left(1+\frac{\mu_p}{\mu_n}\right) A L \qquad \Gamma_{ph} = \frac{I_L}{q G_L A\cdot L}$$

Γ_{ph} ↑ photoconductor gain (current measured divided by total charge generated)

photodiodes:



reverse biased

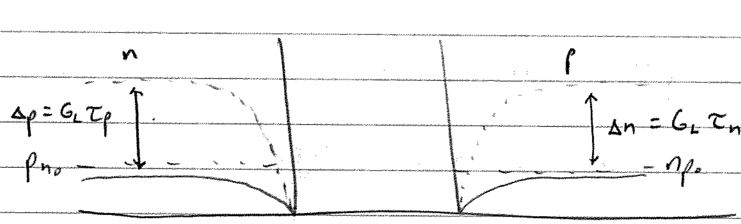
$$I_L = q\int G_L\cdot dx = q G_L(L_p + L_n + W)$$

$$I_p = q\frac{D_p p_{n0}}{L_p} + q G_L L_p$$

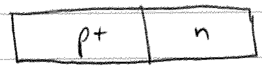
$$I_n = q\frac{D_n n_{p0}}{L_n} + q G_L L_n$$

$$I_W = q G_L W$$

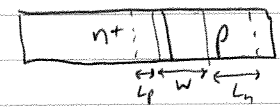
$$I_{total} = I_p + I_n + I_W = \underbrace{\left(\frac{q D_p p_{n0}}{L_p} + \frac{q D_n n_{p0}}{L_n}\right)}_{I_s} + \underbrace{q G_L(L_n+L_p+W)}_{I_L}$$



which is better?



$L_n > L_p$



$L_n = D_n \tau_n \quad\quad L_p = D_p \tau_p$

$\downarrow \quad\quad\quad\quad \downarrow$

$\frac{kT}{q} \mu_n \tau_n \quad\quad \frac{kT}{q} \mu_p \tau_p$

$\mu_n > \mu_p \Rightarrow L_n > L_p$

$\mu_n > \mu_p$
$\Rightarrow$ easier to collect e^- than holes.

+ bottom structure will generate more electrons that can diffuse into the depletion currents

---

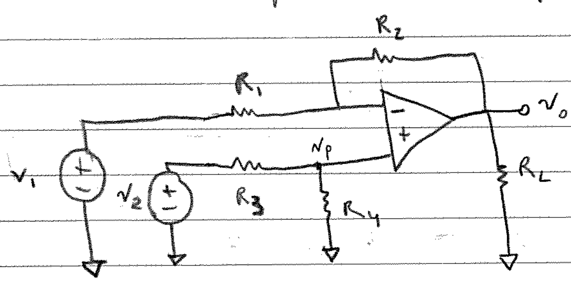
$Y = G + j\omega C$

as $f\uparrow$ $C\downarrow$ because the minority carriers cannot move fast enough to follow the voltage.

Harris says G is independent of frequency, which does not agree with Pierret.

Quals - Circuits Opamps 10/22/11

Difference Amplifier: express v_o in terms of v_1 and v_2



$v_p = \frac{R_4}{R_3 + R_4} v_2$

$\frac{v_1 - v_p}{R_1} = \frac{v_p - v_o}{R_2} \Rightarrow -\frac{v_o}{R_2} = \frac{v_1 - v_p}{R_1} - \frac{v_p}{R_2}$

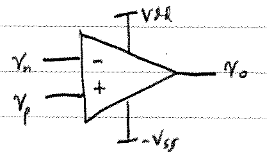
$\Rightarrow v_o = v_p + \frac{R_2}{R_1}(v_p - v_1) = \left(\frac{R_1 + R_2}{R_1}\right) v_p - \frac{R_2}{R_1} v_1$

$= \left(\frac{R_1+R_2}{R_1}\right)\left(\frac{R_4}{R_3+R_4}\right) v_2 - \frac{R_2}{R_1} v_1$ when $R_1 = R_3$ and $R_2 = R_4$,

this simplifies to $v_{out} = \frac{R_2}{R_1}(v_2 - v_1)$

Quals - Circuits, Opamps - [redacted] 10/22/11

OpAmp - Very high gain differential amp



$$\Rightarrow \quad v_o = A\cdot(v_p - v_n)$$

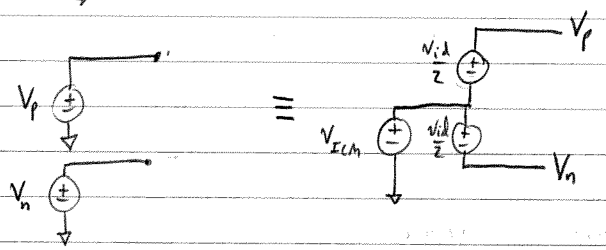
$$v_{ICM} = \frac{v_N + v_p}{2} \qquad v_{id} = v_p - v_n$$

$$v_p = v_{ICM} + \frac{v_{id}}{2}$$

$$v_n = v_{ICM} - \frac{v_{id}}{2}$$

Ideal Opamp

1. Infinite input impedance (zero input current)
2. Zero output impedance
3. Infinite open loop gain $(A = \infty)$ ← differential gain
4. Zero Common Mode gain
5. Infinite bandwidth

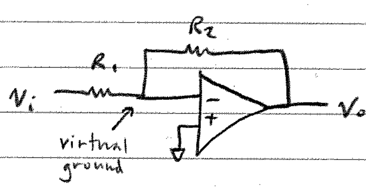


DC - direct coupled.

Opamps are DC amplifiers. They can amplify DC inputs.

Usually we use opamps in negative feedback configurations.

Inverting Amp:

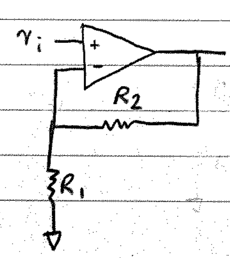


$$\frac{v_o}{v_i} = -\frac{R_2}{R_1}$$

$$\text{if } A < \infty, \quad \frac{v_o}{v_i} = -\frac{R_2}{R_1}\left(\frac{1}{1 + \left(1 + \frac{R_2}{R_1}\right)/A}\right)$$

☆ Input Impedance $= R_1$ ⇒ need to make R_1 and R_2 high if you want high input impedance.

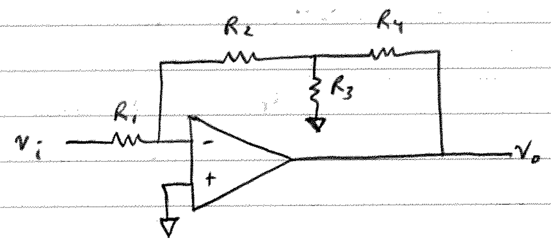
Non-inverting Amp:



$$\frac{v_o}{v_i} = \frac{R_1 + R_2}{R_1} = 1 + \frac{R_2}{R_1}$$

Very high input impedance.

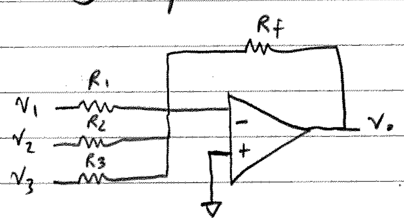
High Gain, High Input Impedance:



$$\frac{v_o}{v_i} = -\left(\frac{R_2}{R_1} + \frac{R_4}{R_1} + \frac{R_4 R_2}{R_1 R_3}\right)$$

Can increase $\frac{v_o}{v_i}$ by tuning R_3 and R_4.

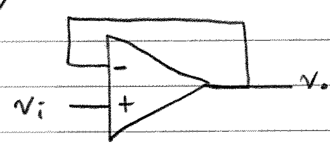
Summing Amp:



$$v_o = -\left(\frac{R_f}{R_1} v_1 + \frac{R_f}{R_2} v_2 + \frac{R_f}{R_3} v_3\right)$$

Can derive this from superposition.

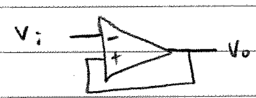
Voltage Buffer / Follower:



$$\frac{v_o}{v_i} = 1$$

Very high input impedance
Very low output impedance
Can sometime have faster response.

Latch:



Common Mode Rejection Ratio: CMRR

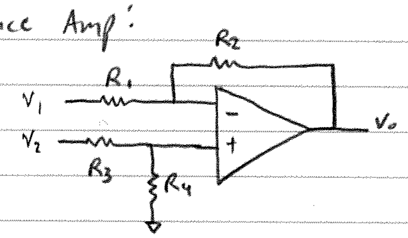
$$V_o = A_d v_{id} + A_{cm} v_{Icm}$$

$\downarrow$ $A_d \to \infty$ $\quad$ $\downarrow$ $A_{cm} \to 0$ for an opamp

$$\boxed{CMRR = 20\log\frac{|A_d|}{|A_{cm}|}}$$

want this to be as high as possible.

Difference Amp:



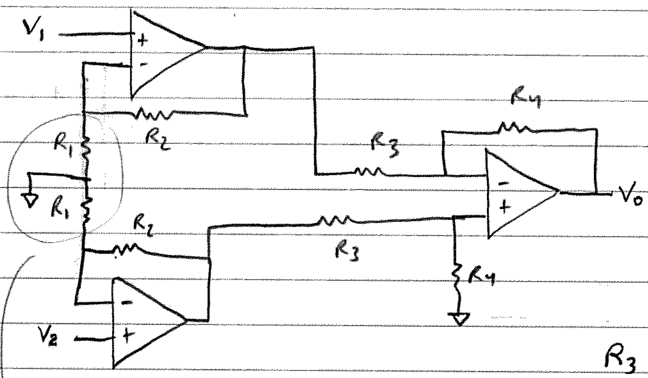
$$V_o = \left(\frac{R_1+R_2}{R_1}\right)\left(\frac{R_4}{R_3+R_4}\right)V_2 - \frac{R_2}{R_1}V_1$$

when $\frac{R_2}{R_1} = \frac{R_4}{R_3}$, $V_o = \frac{R_2}{R_1}(V_2 - V_1)$

typically, $R_1 = R_3$, $R_2 = R_4$

$R_{id} = R_1 + R_3$

Same problem as inverting amp: if you want high input resistance + high gain, must make R_2 and R_1 (and R_4 and R_3) very large.

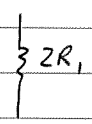


$V_o = \frac{R_2}{R_1}(V_2 - V_1)$

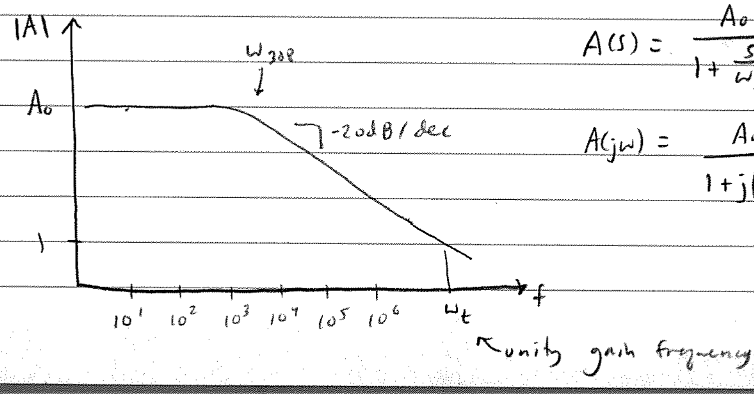
~ infinite R_{id}.

$R_3 = R_4$

to increase CMRR (reduce A_{cm})
replace this with:



frequency Response:



$$A(s) = \frac{A_o}{1 + \frac{s}{\omega_{3dB}}}$$

$$A(j\omega) = \frac{A_o}{1 + j\left(\frac{\omega}{\omega_{3dB}}\right)}$$

At high frequency: $|A(j\omega)| \approx A_0 \left(\frac{\omega_b}{\omega}\right) = \frac{\omega_t}{\omega}$

Gain Bandwidth Product: $\boxed{\omega_t = A_0 \omega_{3dB}}$

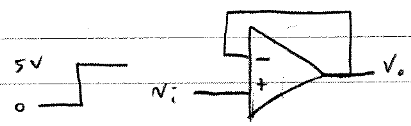
Nonidealities:

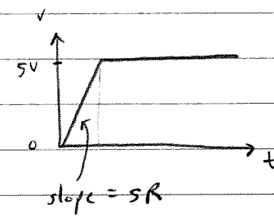
Output Voltage Saturation: Voltage limited by $\sim V_{dd} - 1V$, $\sim -V_{ss} + 1V$

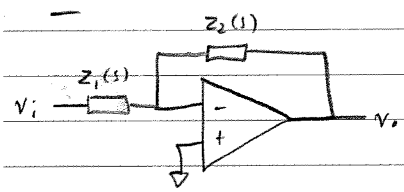
Output Current Limit: e.g. 20mA = max current output the opamp can supply.

Slew Rate: How fast the output of the opamp can change.

$$SR = \left.\frac{dV_o}{dt}\right|_{max}$$ units are typically V/µs in datasheets.

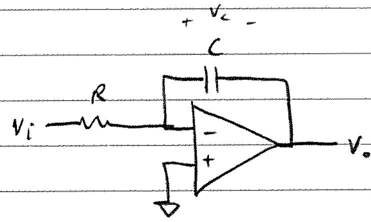






$$\frac{V_o(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)}$$

Integrator:

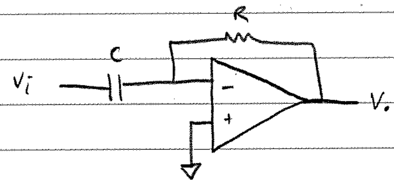


at $t=0$ $V_c(t=0) = V_c$

$$V_c(t) = V_c + \frac{1}{C}\int_0^t i_1(t)\,dt$$

$V_o = -V_c(t)$ $\Rightarrow$ $V_o(t) = \left(-\frac{1}{RC}\int_0^t V_i(t)\,dt\right) - V_c$

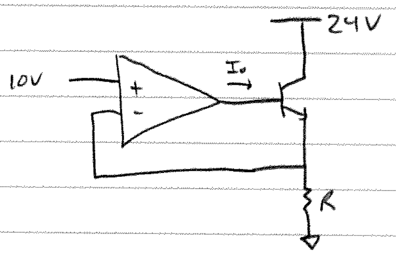
Differentiator



$$V_o(t) = -RC\,\frac{dV_i(t)}{dt}$$

· MIT question: 6.071, 2006 Final

1.



$I_o = 5mA$ $\quad \beta = 100$: calculate R:

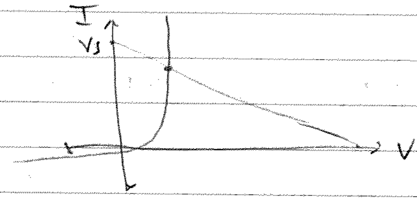
$I_c = \beta \cdot I_o = 500mA \Rightarrow 500mA \cdot R = 10V$

$\Rightarrow \boxed{R = 20\Omega}$

2. A. 0.6 V

B. 0.6 V

C. $i_L R_2 = 0.6 \Rightarrow i_L = \frac{0.6V}{R_2}$



$\Rightarrow V_L = i_L \cdot R_L \Rightarrow \frac{V_+}{R_2} \cdot R_L \Rightarrow V_L = \frac{R_L}{R_2} \cdot V_+$

⇒ the purpose of this circuit is to measure the IV characteristics of a diode

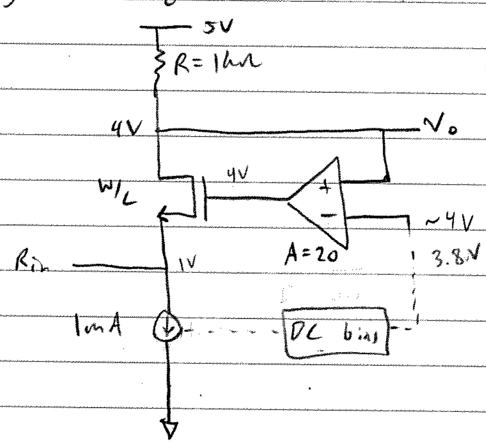
↘ the purpose of this circuit is to be a constant current source.

D. $i_D = I_s\left(e^{\frac{V_D}{V_T}} - 1\right) \approx \frac{V_s - 0.6}{R_1}$

$\Rightarrow V_D$ decreases by V_T

$\Rightarrow V_L$ decreases by $\frac{R_L}{R_2} \cdot V_T \Rightarrow i_L$ decreases by $\frac{V_T}{R_2}$

Wooley 2000 quals Q



$V_T = 1V \quad \mu C_{ox} = 100\mu A/V^2 \quad \frac{W}{L} = 5 \quad A = 20$

$R_{in} = ?$

$\frac{1}{2}\mu C_{ox}\frac{W}{L}(V_{GS} - 1V)^2 = 1mA \Rightarrow (V_{GS} - 1V)^2 = 4mA$

$\Rightarrow V_{GS} - 1 = 2 \Rightarrow V_{GS} = 3V$

$g_m = \frac{2I_D}{V_{ov}} = \frac{2mA}{2V} = 1mS$

Using Blackman's impedance formula: $\Rightarrow$ ~~$R_{in} = \frac{1}{g_m} = 1k\Omega$~~

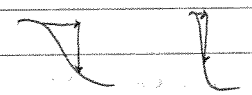
$Z_{(A=0)} = \frac{1}{g_m} \quad R(\text{port shorted}) = -\left(\frac{-g_m R V_t A}{V_t}\right) = g_m R A$

$R(\text{port open}) = 0$

$\Rightarrow Z = R_{in} = \frac{1}{g_m}\left(\frac{1 + g_m R A}{1 + 0}\right) = \frac{1}{g_m}(1 + g_m R A) = \frac{1}{g_m} + RA = 1k\Omega + 20k\Omega \Rightarrow \boxed{R_{in} = 2}$

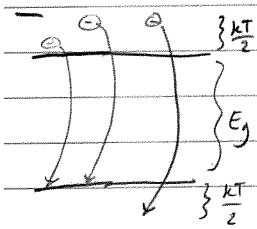
Quals - Devices [redacted] - LED's + Lasers

higher T ⇒ shorter mean free path.
⇒ need higher $\vec{E}$ and higher V_{br}



Avalanche breakdown: T ↑ V_{br} ↑

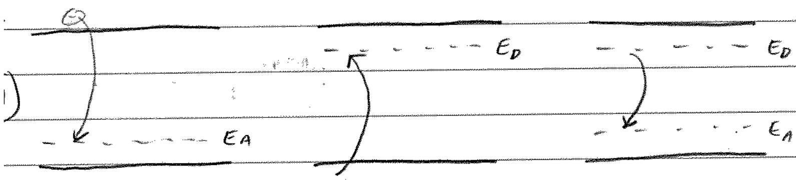
Zener breakdown: T ↑ V_{br} ↓ (V_{br} constant according to Harris)



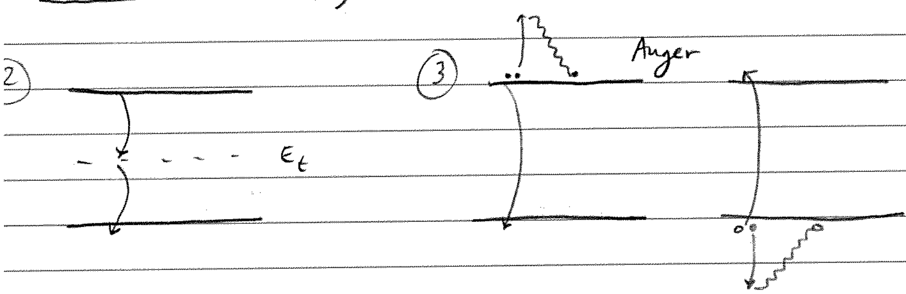
LED's:

range of E_{photon} ~ E_g to $E_g + kT$

$$\lambda = \frac{1.24}{E} \quad (E \text{ in eV}, \ \lambda \text{ in } \mu m)$$



1: Radiative Recombination (band to band)



2,3: Nonradiative Recombination

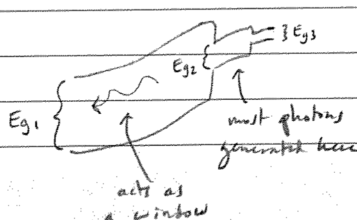
$$I(\nu) \propto \nu^2 (h\nu - E_g)^{1/2} e^{\left(\frac{-(h\nu - E_g)}{kT}\right)}$$ ← spontaneous emission rate.

Quantum Efficiency: $\eta = \dfrac{\text{radiative recombination rate}}{\text{total recombination rate (all processes)}} = \dfrac{R_r}{R_{nr} + R_r} = \dfrac{\tau_{nr}}{\tau_{nr} + \tau_r}$

$R_r \propto \dfrac{1}{\tau_r}$ $\quad R_r \propto n, p$ $\quad R_r = Bnp$ ← proportionality constant

$R_{r_{direct}} \sim 10^6 \cdot R_{r_{indirect}}$

- Want photons to be generated close to the surface so that it is less likely to be re-absorbed.
- Or you can use a heterojunction:



Laser/
LED materials: $Al_x Ga_{1-x} As$

$0 < x < 0.45 \rightarrow$ direct bandgap
$x > 0.45 \rightarrow$ indirect bandgap

$30\mu m < \lambda < 40 \mu m$ for typical LED's.

Internal Quantum Efficiency: $\eta_i = \underset{\text{injection efficiency}}{\gamma} \cdot \eta \leftarrow$ radiative efficiency

$$\gamma = \frac{J_n}{J_n + J_p + J_R}$$

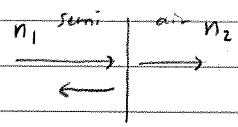
nonradiative recombination current

$$J_R = \frac{q n_i W}{2\tau} e^{\left(\frac{qV}{2kT} - 1\right)}$$

for n^+p, $J_n >> J_p, J_R \Rightarrow$ assume $\gamma = 1$

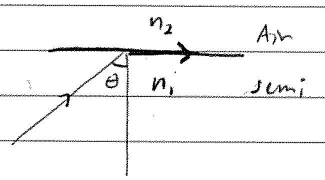
1) Re-absorb, if $h\nu \geq E_g$

2) Reflection $\rightarrow$ fresnel loss



$$\Gamma = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$$

3) Critical angle loss



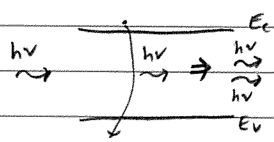
$$\theta = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

Visible range: $0.4\mu m < \lambda < 0.7\mu m \rightarrow 1.7eV < E_g < 3.1eV$

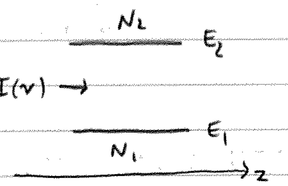
GaAs $\rightarrow \lambda = 0.871 \mu m$ (not in visible range).

$GaAs_{1-x}P_x$ for $x = 0.4$ $E_g = 1.9eV$

Laser:



Conditions:



$E_2 > E_1 \Rightarrow$ in thermal equilibrium, $N_2 < N_1$

$$\frac{dI(\nu)}{dz} \propto \frac{\#\text{ photon emitted}}{cm^3} - \frac{\#\text{ photon absorbed}}{cm^3}$$

$$\frac{dI(\nu)}{dz} = N_2 w_i h\nu - N_1 w_i h\nu$$ where w_i = induced transition probability.

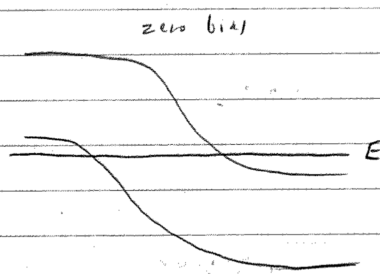
$$\frac{dI(\nu)}{dz} = \gamma(\nu) I(\nu)$$ where $\gamma(\nu) \propto N_2 - N_1$

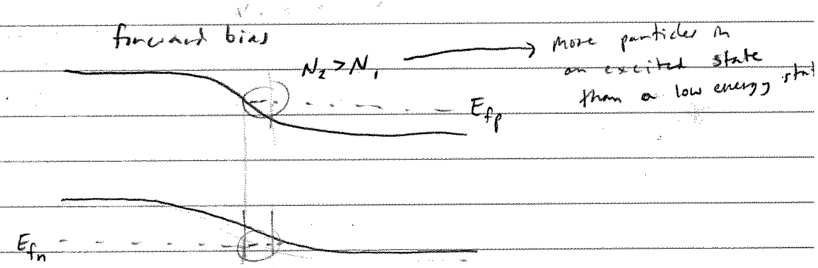
$$\boxed{I(\nu) = I(0)\, e^{\gamma(\nu) z}}$$

Conditions: need amplification and coherent emission.

amplification: $\gamma(\nu) > 0$ ($\gamma(\nu) < 0 \rightarrow$ absorption.)

$\gamma(\nu) \propto N_2 - N_1$ if $N_2 > N_1$ we call it population inversion.
this happens when both sides of a pn junction are degenerate.





optical cavity - $L = n\left(\frac{\lambda}{2}\right)$ where $n = 1, 2, 3, \ldots$ and L is the length of the cavity dimensions.

Quals - Circuits   Alex - Long Channel Model + CS   10/26/11

Subthreshold: $I_D = I_0 e^{(V_{GS}/\xi V_T)}$ where $V_T = \frac{kT}{q}$ (not threshold voltage) and $\xi > 1$.

MOS - IV : Triode: $I_D = \mu C_{ox} \frac{W}{L} (V_{GS} - V_T - \frac{V_{DS}}{2}) V_{DS}$ $V_{GD} > V_t$

Saturation: $I_D = \frac{1}{2} \mu C_{ox} \frac{W}{L} (V_{GS} - V_T)^2 (1 + \lambda V_{DS})$ $V_{GD} < V_t$

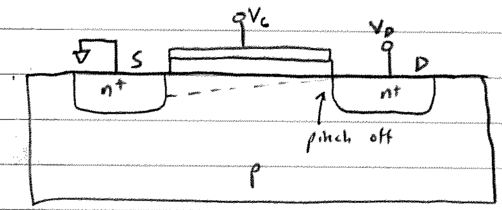
CLM! Assume $\frac{\Delta L}{L} = \lambda V_{DS} \Rightarrow \lambda \propto \frac{1}{L}$, $\Delta L \propto V_{DS}$

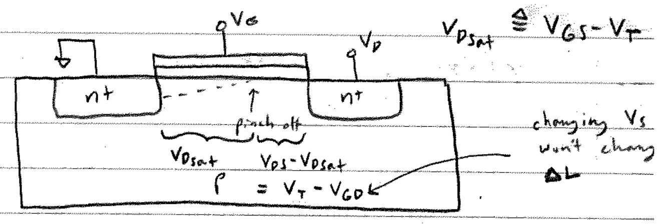
MOS - resistor: $V_{ds} << 2(V_{GS} - V_T) \Rightarrow I_D \approx \mu C_{ox} \frac{W}{L} (V_{GS} - V_T) V_{DS}$

$$\Rightarrow R_{on} = \frac{\partial I_D}{\partial V_{DS}} \approx \frac{1}{\mu C_{ox} \frac{W}{L} (V_{GS} - V_{TH})}$$

R_{on} controlled by V_{GS}.

$V_{Dsat} \triangleq V_{GS} - V_T$





$V_{GD} = V_T \Rightarrow$ Saturation: $V_{GD} < V_T \Rightarrow$ saturation.

Transconductance:

$$g_m = \left.\frac{\partial I_D}{\partial V_{GS}}\right|_{V_{DS} = \text{constant}} = \mu C_{ox} \frac{W}{L} V_{OV} = \sqrt{2\mu C_{ox} \frac{W}{L} I_D} = \frac{2 I_D}{V_{OV}}$$

with CLM: $g_m = \mu C_{ox} \frac{W}{L} V_{OV} (1 + \lambda V_{DS}) = \sqrt{\frac{2\mu C_{ox} \frac{W}{L} I_D}{1 + \lambda V_{DS}}} = \frac{2 I_D}{V_{OV}}$

Output Conductance: $g_{ds} = \left.\frac{\partial I_D}{\partial V_{DS}}\right|_{V_{GS} = \text{constant}} = \frac{\lambda I_D}{1 + \lambda V_{DS}} \approx \lambda I_D$ assuming $\lambda V_{DS} << 1$

$$r_o = \frac{1}{g_{ds}} \approx \frac{1}{\lambda I_D}$$

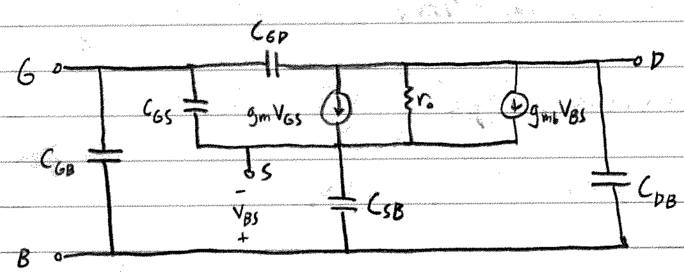
Second Order Effects:

Body Effect: $V_{TH} = V_{TH0} + \gamma (\sqrt{|2\Phi_F + V_{SB}|} - \sqrt{2\Phi_F})$ $V_{TH0} = \underbrace{\Phi_{MS}}_{\text{work function difference between gate + substrate}} + 2\Phi_F + \underbrace{\frac{Q_{d}}{C_{ox}}}_{\text{charge in depletion region}}$

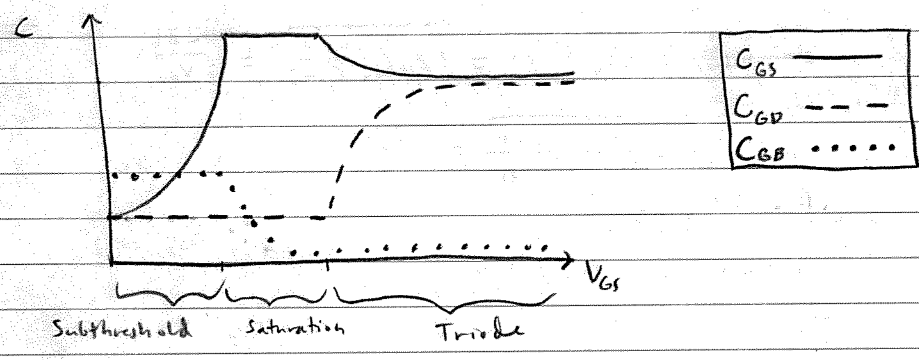
$\Phi_F = (\frac{kT}{q}) \ln (\frac{N_{sub}}{n_i})$ $\gamma = \sqrt{2 q \epsilon_{si} N_{sub}} / C_{ox}$

$$g_{mb} = \frac{\partial I_D}{\partial V_{BS}} = \mu C_{ox} \frac{W}{L} (V_{GS} - V_T) \left(-\frac{\partial V_T}{\partial V_{BS}}\right) = g_m \frac{\gamma}{2\sqrt{2\Phi_F + V_{SB}}} = \eta g_m.$$

MOS small signal model:

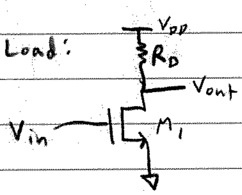


	Subthreshold	Triode	Saturation
C_{gs}	C_{ov}	$\frac{1}{2} WLC_{ox} + C_{ov}$	$\frac{2}{3} WLC_{ox} + C_{ov}$
C_{gd}	C_{ov}	$\frac{1}{2} WLC_{ox} + C_{ov}$	C_{ov}
C_{gb}	$\left(\frac{1}{C_{CD}} + \frac{1}{WLC_{ox}}\right)^{-1}$	0	0



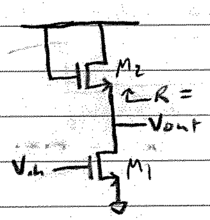
Common Source:

Resistive Load:



$A_v = -g_m (R_D \| r_o)$

Diode connected Load:



$R = \frac{1}{g_m + g_{mb}} \| r_o \approx \frac{1}{g_m + g_{mb}}$

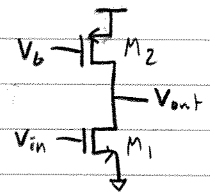
$$\Rightarrow A_v = -g_{m_1} \frac{1}{g_{m_2} + g_{mb}} = -\frac{g_{m_1}}{g_{m_2}} \left(\frac{1}{1+\eta}\right) = \sqrt{\frac{(W/L)_1}{(W/L)_2}} \left(\frac{1}{1+\eta}\right)$$

$$\approx -\frac{g_{m_1}}{g_{m_2}}$$

independent of bias currents + voltages $\Rightarrow$ Linear

But, gain $\propto \sqrt{\frac{(W/L)_1}{(W/L)_2}}$ $\Rightarrow$ gain $\xleftrightarrow{\text{Tradeoff}}$ Voltage Swing

Current Source Load: High gain and Voltage Swing:



$A_v = -g_{m_1}(r_{o_1} \| r_{o_2})$

How to increase gain by sizing? $\lambda \propto \frac{1}{L} \Rightarrow r_o \propto \frac{L}{I_D}$

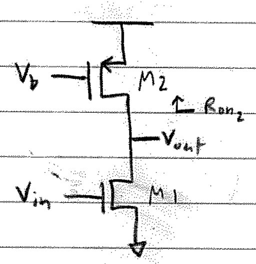
Fix W and I_D, $g_{m_1} r_{o_1} = \sqrt{2\left(\frac{W}{L}\right)_1 \mu C_{ox} I_D} \; \underbrace{\frac{1}{\lambda I_D}}_{r_{o_1}}$

$\Rightarrow$ $L \uparrow \Rightarrow$ gain $\uparrow$ since $r_o \propto L$ and $g_m \propto \frac{1}{\sqrt{L}}$

$L \uparrow \Rightarrow \left(\frac{W}{L}\right) \downarrow \Rightarrow V_{ov} \uparrow$ since $V_{ov} \propto \frac{1}{\sqrt{\frac{W}{L}}}$ for a fixed I_D $\Rightarrow$ Voltage Swing $\downarrow$

if $I_D \uparrow \Rightarrow$ gain $\downarrow$

Triode Load:



$A_v = -g_m(R_{on_2} \| r_{o_1} \| r_{o_2}) \approx -g_m R_{on_2}$

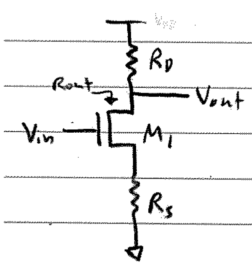
$R_{on_2} = \frac{1}{\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - V_b - |V_{TH_p}|)}$

$V_{out,MAX} = V_{DD} \Rightarrow$ Larger Voltage Swing

BUT Susceptible to process and temperature variations

(R_{on_2} is a function of μ_p, C_{ox}, V_{THp})

CS with Source Degeneration:



This is a trick. $\downarrow$ to simplify the gain calculation

$G_m = \frac{\partial I_D}{\partial V_{in}} = \frac{g_m}{1+g_m R_S}$ $R_{out} = (1+g_m R_S) r_o$

Resistance seen at drain

$A_v = -G_m R_D = -\frac{g_m R_D}{1+g_m R_S} = -\frac{R_D}{\frac{1}{g_{m_1}} + R_S}$

Resistance in source path.

$\approx -\frac{R_D}{R_S}$ for $g_m R_D > g_m R_S \gg 1$

Now, considering Body Effect:

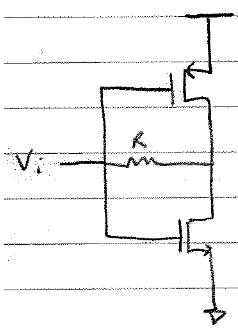
$$G_m = \frac{g_m r_o}{R_s + [1 + (g_m + g_{mb}) R_s] r_o}$$

$$R_{out} = [1 + (g_m + g_{mb}) r_o] R_s + r_o \approx [1 + (g_m + g_{mb}) R_s] r_o$$

Lemma: $A_V = -G_m R_{out}$

where G_m = transconductance when output is grounded.
R_{out} = output resistance when input is grounded.

[redacted] Questions from some Book.



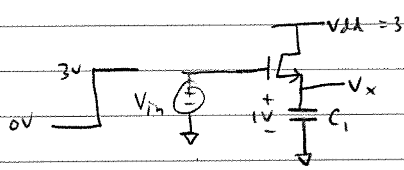
$V_A = 60V$ $|V_t| = 2V$ $R >> r_{ds}$ $\frac{V_o}{V_i} = ?$

without R: $\frac{V_o}{V_i} = -g_m (r_{ds} \| r_{ds}) = -\frac{1}{2} g_m r_{ds}$

R acts as feedback. ?

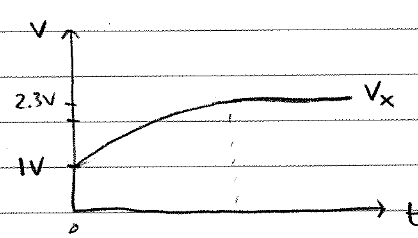
by KCL: $A_V = \frac{V_o}{V_i} = -\frac{(g_{m1} + g_{m2})}{(r_{o1} \| r_{o2} \| R)}$

Razavi 2.11 a) sketch V_x as a function of time



$V_{th} = 0.7V$

$i = C \frac{\delta V}{\delta t}$



$\frac{dV_x}{dt} = \frac{i}{C}$

$dV_x = \frac{i}{C} dt$

$dV_x = \left(\frac{1}{2}\mu C_{ox} \frac{W}{L} (3V - V_x - 0.7V)^2\right) dt / C$

let $K = \frac{1}{2C}\mu C_{ox}\frac{W}{L}$ $\Rightarrow$ $\frac{dV_x}{(2.3-V_x)^2} = K\,dt$ $\Rightarrow$ $\frac{1}{(2.3-V_x)} = Kt + A =$

Initial conditions at $t=0$, $V_x = 1$ $\Rightarrow$ $A = \frac{1}{1.3} \approx 0.77$

$\Rightarrow$ $2.3 - V_x = \frac{1}{Kt+0.77}$ $\Rightarrow$ $\boxed{V_x = \begin{cases} 2.3 - \frac{1}{Kt+0.77} & \text{for } 0<t<\frac{0.77}{K} \\ 2.3 & t > \frac{0.77}{K} \end{cases}}$

$V_{dd} = 3V$ $\Rightarrow$ MOSFET either in sat or cutoff.

$i_c = C\frac{dV_x}{dt} = \frac{1}{2}\mu C_{ox}\frac{W}{L}(3 - V_x - 0.7)^2 = K(2.3-V_x)^2$

$\cancel{\Rightarrow \frac{dV_x}{dt} = \frac{K}{C}(5.29 - 4.6V_x + V_x^2)}$

$\cancel{sV = \frac{K}{C}}$

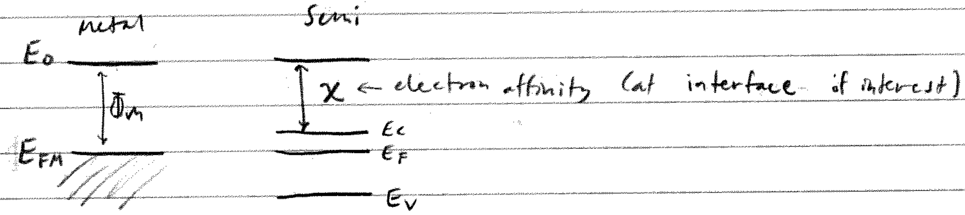
$\Rightarrow \int\frac{dV_x}{(2.3-V_x)^2} = \int K\,dt$ $\Rightarrow$ $2(2.3-V_x)^{-1} + C_1 = Kt + C_2$

$\Rightarrow \frac{1}{2.3-V_x} = Kt + C_3$ $\Rightarrow$ $V_x = 2.3 - \frac{1}{Kt+C}$

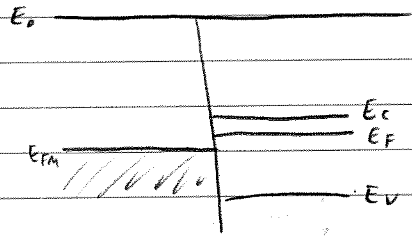
$V_x(0) = 1 = 2.3 - \frac{1}{Kt+C_3}$ $\Rightarrow$ $\frac{1}{C_3} = 1.3$ $\Rightarrow$ $C_3 \approx 0.77$

Quals Device - [redacted] - MS and MOS 10/29/

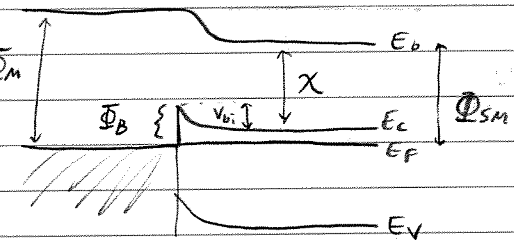
MS



⇓



⇒



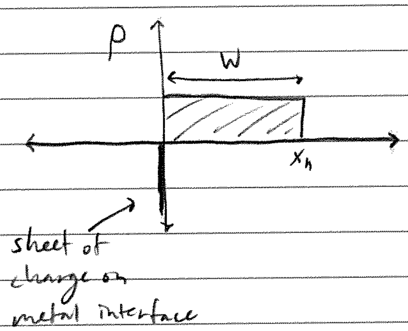
$\Phi_B \propto V_{bi}$ $\quad V_{bi} = \frac{1}{q}\left[\Phi_B - (E_c - E_F)\Big|_{FB}\right] = \Phi_M - \Phi_{SM}$

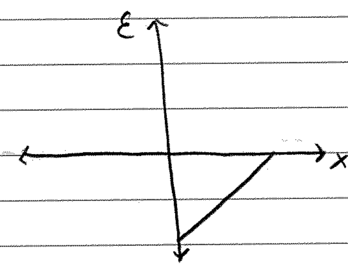
↖ flat band

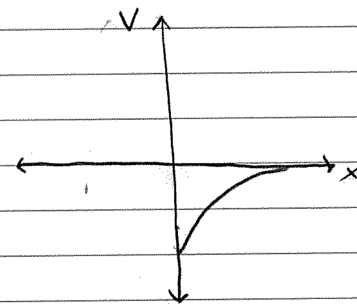
$\Phi_M = \Phi_B + \chi$

	rectifying	ohmic
n-type	$\Phi_M > \Phi_S$	$\Phi_M < \Phi_S$
p-type	$\Phi_M < \Phi_S$	$\Phi_M > \Phi_S$

rectifying: $I = I_0(e^{V/V_T} - 1)$



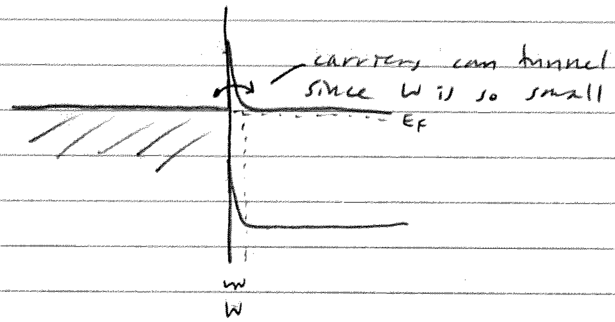




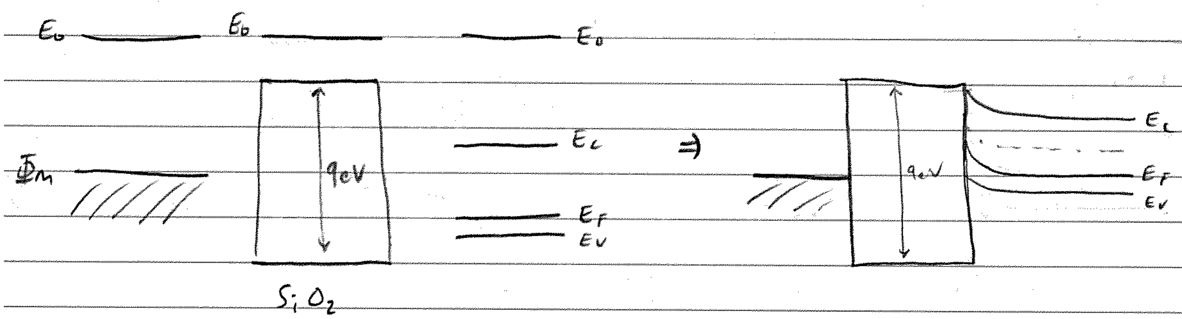
degenerate

How to get an ohmic contact in reality? Heavily dope semiconductor.
($>10^{18}\,cm^{-3}$ for silicon)

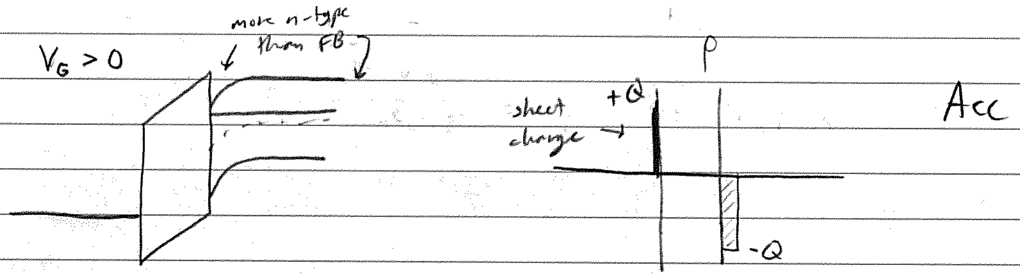
⇒



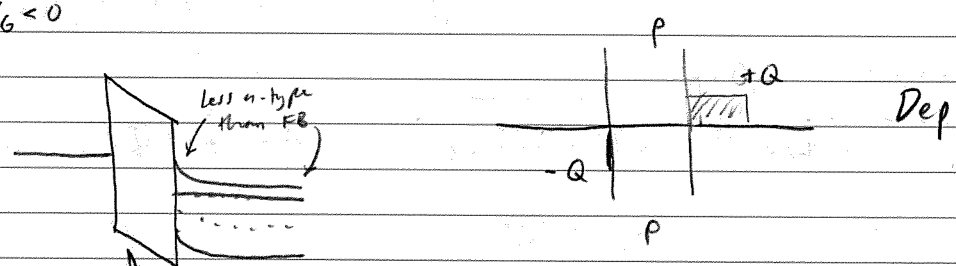
MOS



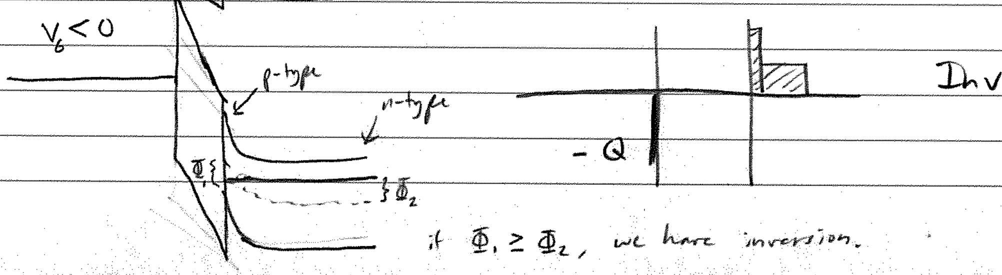
n-type $V_G > 0$



$V_G < 0$

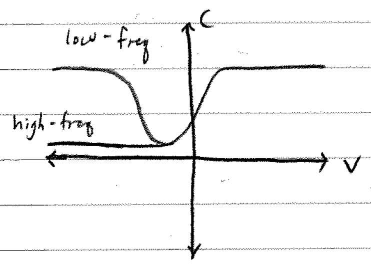


$V_G < 0$

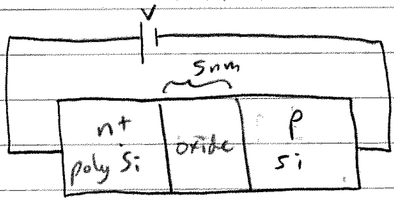


if $\Phi_1 \geq \Phi_2$, we have inversion.

C-V characteristics of MOS (n-type):



Question from MIT course.



$\chi_{gate} = 4.04 eV$

$C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}} = 7 \cdot 10^{-7} \frac{F}{cm^2}$

a) if $Q_G = -2 \cdot 10^{-7} \frac{C}{cm^2} \Rightarrow \mathcal{E}_{ox} = ?$

b) $Q_{semi} = -2 \cdot 10^{-7} \frac{C}{cm^2} \Rightarrow V = ?$

a) $Q = VC \Rightarrow V = \frac{Q}{C} = -\frac{2}{7} V \qquad \mathcal{E}_{ox} = \frac{-\frac{2}{7} V}{5 \cdot 10^{-7} m} = -\frac{2}{35} \cdot 10^{7}\ V/cm$

b) $W_{max} = \sqrt{\frac{2 \varepsilon_r \varepsilon_0}{q N_A} (2 \Phi_F)} \qquad \Phi_F = \frac{kT}{q} \ln\left(\frac{N_A}{n_i}\right) \Rightarrow W_{max} = 2.6 \cdot 10^{-6} cm$

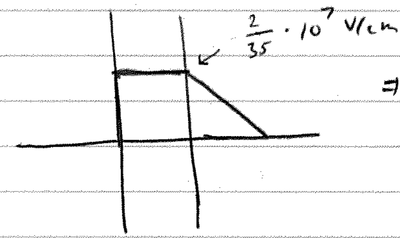
$\Rightarrow Q_{max} = q N_A W_{max} = 6 \cdot 10^{-19} C \cdot 6 \cdot 10^{17} cm^{-3} \cdot 2.6 \cdot 10^{-6} cm = 2.496 \cdot 10^{-7} \frac{C}{cm^2}$

$\Rightarrow$ no inversion.

$Q = q N_A \cdot W \Rightarrow W = \frac{Q}{q N_A} = \sqrt{\frac{2 K_s \varepsilon_0}{q N_A} \phi_s} \Rightarrow$ solve for $\Phi_s \leftarrow V$

$\Rightarrow W = 2.1 \cdot 10^{-6} cm \qquad \Rightarrow \Phi_s = \frac{W^2 q N_A}{2 K_s \varepsilon_0} =$

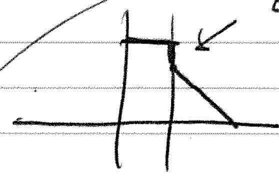
$C = \frac{Q}{V}$



$$\Rightarrow V = \left(5\,\text{nm} + \frac{2.1 \cdot 10^{-6}\,\text{cm}}{2}\right)\left(\frac{2}{35} \cdot 10^{7}\ \text{V/cm}\right) =$$

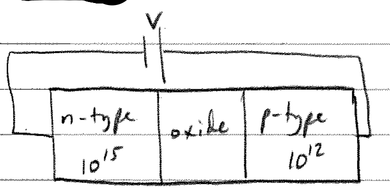
$$C = \frac{Q}{V}$$

$$E_1 - E_2 = \frac{Q}{K_s \varepsilon_0}$$

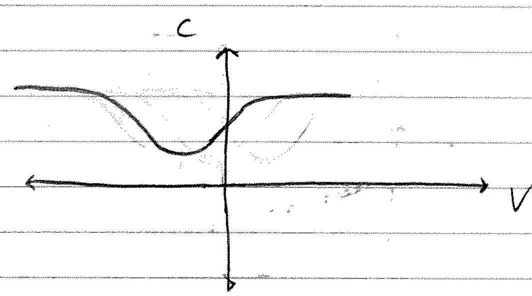


if there was an inversion layer.

[redacted] Question (from EE216)

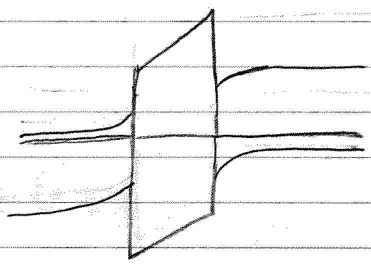


C-V



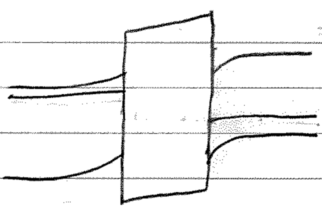
Draw band diagram for:

a) $V_G = 0$



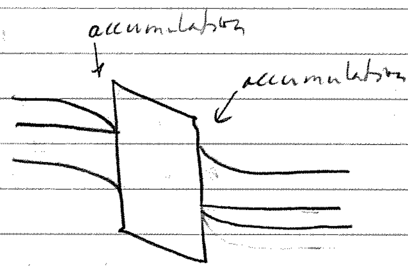
b) $V_G > 0$

$|V_G|$ small

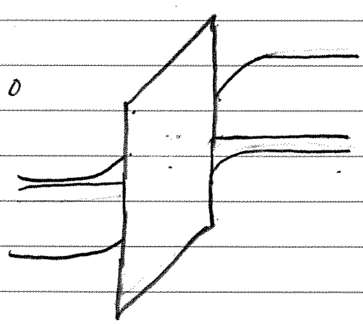


OR

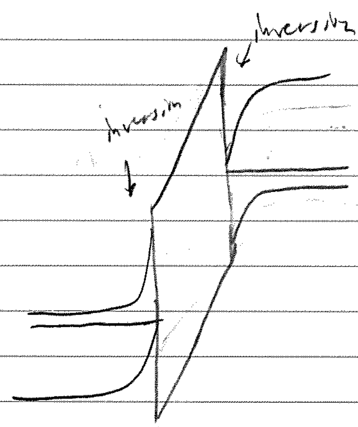
$|V_G|$ large



c) $V_G < 0$



OR



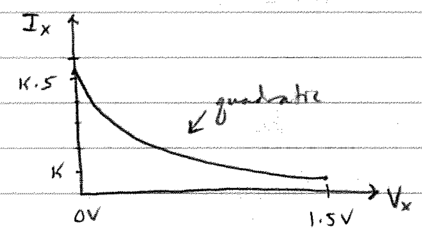
Circuits : Razavi

2.5 a) vary V_x from 0-1.5 V, plot I_x and g_m

stays in sat, since $V_{gd} = 0 < V_{TH} = 0.7V$ let $K = \frac{1}{2} \mu C_{ox} \frac{W}{L}$

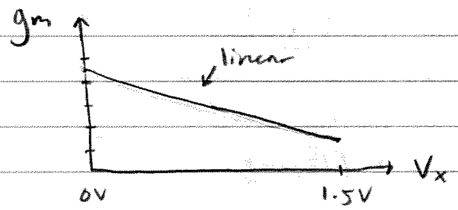
$I_{x_{max}} = \frac{1}{2} \frac{W}{L} \mu C_{ox} (3-0.7)^2 = K \cdot 2.3^2 = K \cdot 5.29$

$I_{x_{min}} = K(1.5-0.7)^2 = K \cdot 0.64$



$g_m = \mu C_{ox} \frac{W}{L} V_{ov}$ (neglecting CLM)

$\Rightarrow g_{m_{max}} = 2K(2.3)$ $g_{m_{min}} = 2K(0.8)$

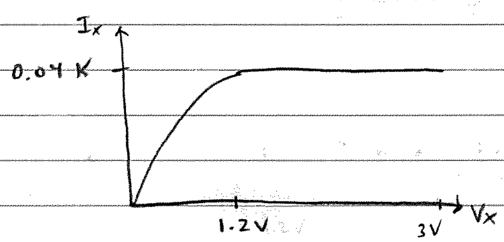


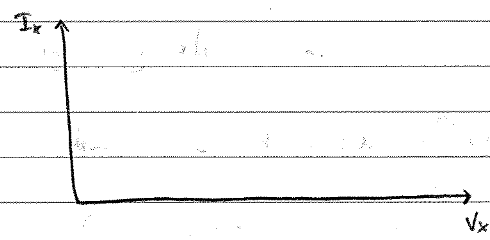
b) vary V_x from 0-3V, plot I_x and g_m $V_{ov} = 0.2V$

starts in triode, goes to saturation at $V_{gd} = V_t \Rightarrow V_x = 1.9V - 0.7V = 1.2V$

in triode: $I_x = \mu C_{ox} \frac{W}{L} \left(0.2V - \frac{V_x - 1}{2}\right) V_x$

in sat: $I_x = \frac{1}{2} \mu C_{ox} \frac{W}{L} (0.2V)^2 = K(0.04)$ (ignoring CLM)



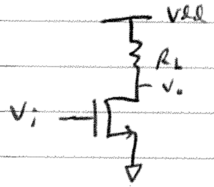


$g_m = \frac{\partial I_d}{\partial V_{gs}}$

what is g_m in triode ??

Circuits - [redacted] - EE114 L5 Gain + biasing + finite output R considerations.

Upper bound of gain on CS with resistive load

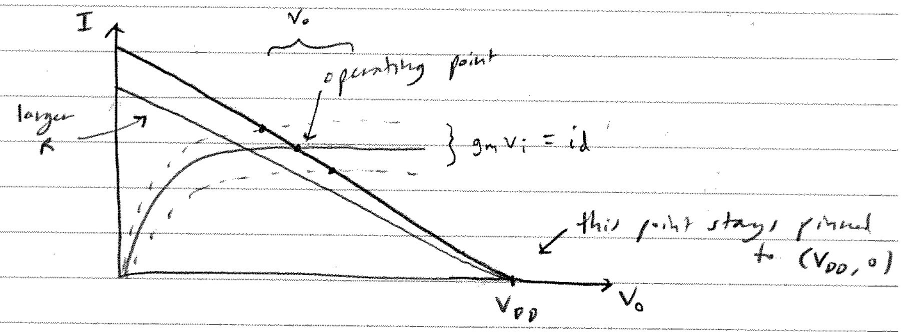


$$A_v = -g_m R_L = -\frac{2I_d}{V_{ov}} R_L \qquad I_D R_L < V_{dd}$$

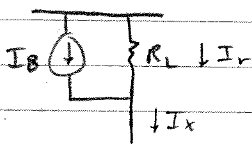
$$\Rightarrow |A_{v_{max}}| < \frac{2V_{DD}}{V_{ov}}$$

Load Line:

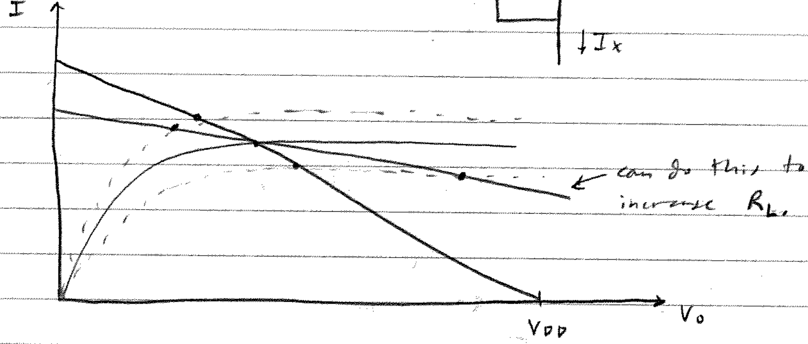
$$I_r = \frac{V_{dd} - V_o}{R_L} \qquad I_d$$



We can fix this by replacing R_L with:



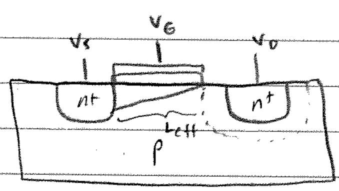
$$\Rightarrow I_x = \frac{V_{DD} - V_o}{R_L} + I_B$$



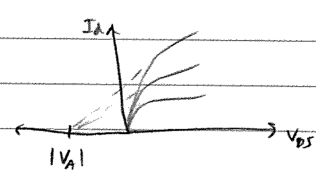
Why can't we get infinite gain?

$\underbrace{\text{Mismatch between } I_B \text{ and } I_d}_{\text{can use common mode feedback.}}$ and finite output resistance r_o.

CLM:

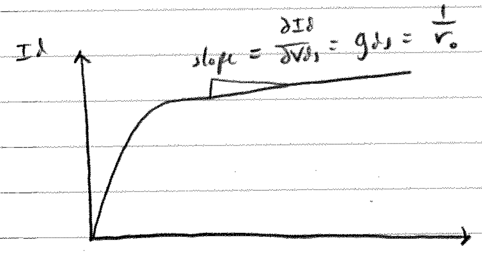


for $V_{DS} > V_{ov}$, a depletion region forms around drain $\Rightarrow$ channel shortens.



$$\lambda = \frac{\Delta L}{L} = \frac{1}{V_A} \qquad I_d = \frac{1}{2}\mu C_{ox}\frac{W}{L}(V_{GS} - V_t)^2(1 + \lambda V_{DS})$$

for EE114, $\lambda = 0.1\ V^{-1}$ for $L = 1\mu m$ and $\lambda \propto \frac{1}{L}$

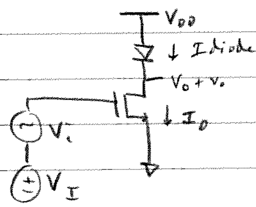


$$g_{ds} = \frac{\lambda I_D}{1+\lambda V_{ds}} \approx \lambda I_D \quad \text{(for long L)}$$

$$r_o \approx \frac{1}{\lambda I_D}$$

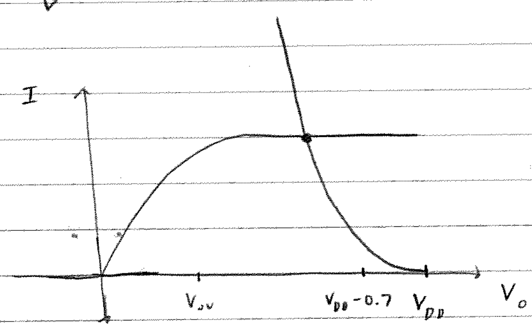
$$\boxed{|A_v|_{R_L\to\infty} = g_m r_o = \frac{g_m}{g_{ds}} = \frac{g_m}{\lambda I_D} = \frac{2}{\lambda V_{ov}}}$$ ← intrinsic gain.

Question - [redacted]

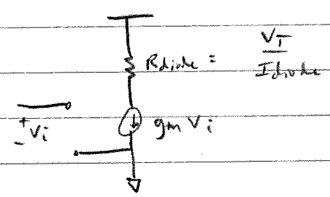


$I_{diode} = I_s e^{V_{diode}/V_T}$ $\left(V_T = \frac{kT}{q}\right)$

ignore r_o and γ.

① 

② $$\frac{\partial(-I_{diode})}{\partial V_o} = \frac{\partial}{\partial v_o} -I_s e^{(V_{DD}-V_o)/V_T} = \frac{I_s}{V_T} e^{(V_{DD}-V_o)/V_T} = \frac{I_{diode}}{V_T}$$

$\Rightarrow$ 

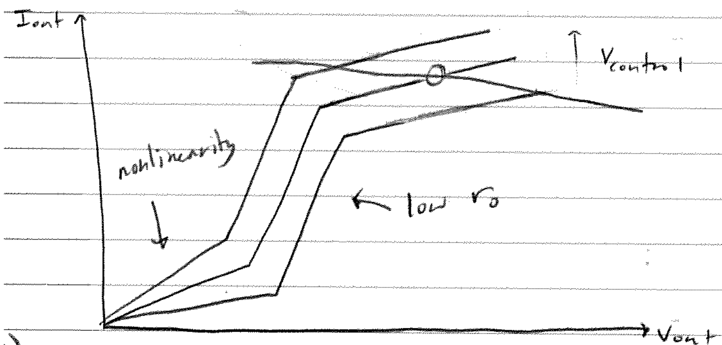
$\Rightarrow$ $A_v = -\frac{g_m V_T}{I_{diode}}$ $\left(V_T = \frac{kT}{q}\right)$

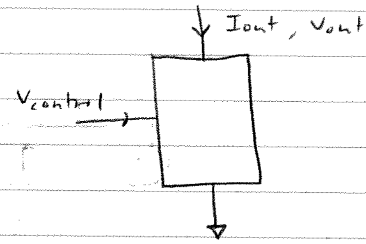
$g_m = \frac{2I_d}{V_{ov}}$, $I_d = I_{diode}$

$\Rightarrow$ $\boxed{A_v = -\frac{2V_T}{V_{ov}}}$

③ $A_{v_{max}} = -\frac{2V_T}{V_{ov_{min}}} \approx -\frac{2(0.026)V}{0.15V} = -\frac{0.052}{0.15} \approx \boxed{-0.35}$

Question - [redacted] from Quals - Dutton (2008 or 2007)





1)

a) Is this device useful for building and amplifier?

b) Where and how would you bias it?

c) what are the limitations?

a) Yes.

b) Bias it in far right region, by placing a resistive load || current source at the output and choosing $V_{control}$ such that the operating point is at point O

c) Limitations are finite output resistance $\left(\frac{\partial I_{out}}{\partial V_{out}} \neq 0\right)$, limited output swing to keep the gain high (must stay in far right region)
And high voltage operating point

) Do lower V_T devices have lower OFF currents for $V_{gs} = 0V$?
No. lower V_T ⇒ higher off current for same S (subthreshold slope)

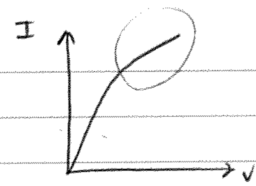
) Series resistance (emitter) degrades bipolar transconductance more than (source resistance) for MOS? (T/F) explain

$G_m = \frac{g_m}{1+g_m R_S}$ for MOS $\cancel{G_m = \frac{g_m}{1+g_m R_S + \frac{R_S}{R_\pi}}}$ $G_{m\,BJT} = \frac{g_m}{1+(1+\frac{1}{\beta})g_m R_E}$ ⇒ $\boxed{T}$

) Does decreasing C_{ox} improve gate delay independent of carrier mobility
No. gate delay depends on time it takes for minority carriers to form and inversion channel ⇒ carrier mobility matters.

Ideal MOSFET IV and Nonideal MOSFET

Devices - [redacted]

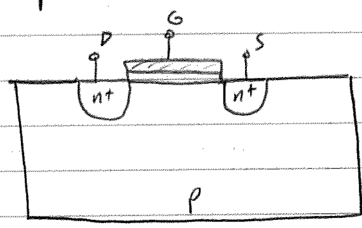


carrier concentration at boundary.

low level injection: $p = \frac{n_i^2}{N_D} \cdot e^{qV/kT}$

High level injection: $p = n \simeq n_i e^{qV/2kT}$

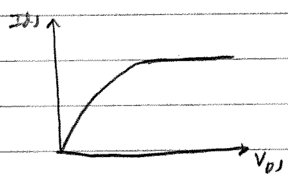
MOSFET:



inversion is quicker than with MOS, since electrons can come from the source + drain (n^+) rather than bulk.

Source is the one that provides the carriers.

Current is dependent on how many carriers you have in the channel + the voltage across it (V_{ds})



thickness of oxide

$$V_T = V_{(\Phi_S = 2\Phi_F)} = 2\Phi_F + \frac{K_S x_o}{K_o}\sqrt{\frac{4qN_A}{K_S \varepsilon_o} \cdot \Phi_F}$$

$\bar{\mu} < \mu_n$ effective mobility at the surface is ~40% - 60% of the mobility in the bulk silicon, due to surface scattering.

V_y is channel potential

$$I = q \underbrace{v}_{\bar{\mu}\mathcal{E} = \bar{\mu}\frac{dV_y}{dy}} n \underbrace{A}_{t_i \times W} = v(\underbrace{qn \cdot t_i}_{\text{charge density at a certain point}} \cdot W) = v \cdot W \cdot Q_d \qquad Q_d = C_{ox}(V_G - V_T - V_y)$$

$$\Rightarrow I = \bar{\mu}\frac{dV_y}{dy} \cdot C_{ox}(V_G - V_T - V_y)\, W \qquad \Rightarrow \int_0^L I dy = \int_0^{V_{ds}} dV_y\, C_{ox}(V_G - V_T - V_y)\, W Q_d$$

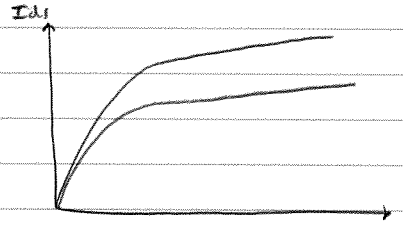
$$\Rightarrow I = \bar{\mu} C_{ox} \frac{W}{L}\left[(V_G - V_T)\cdot V_{DS} - \frac{1}{2}V_{DS}^2\right]$$

Why current doesn't depend on V_{ds} in Sat:

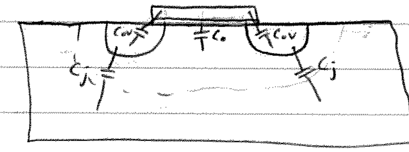
$V_{GS} - V_T$

this voltage doesn't change.

CLM:



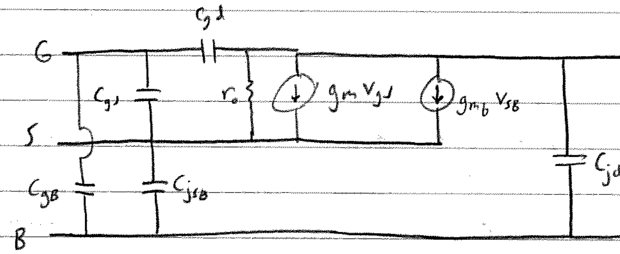
$$I = \bar{\mu} C_{ox} \frac{W}{L-\Delta L} \left[(V_G - V_T) \cdot V_{DS} - \frac{1}{2} V_{DS}^2 \right]$$



Capacitor: Intrinsic $C_o = C_{ox} \cdot L \cdot W$

Sat: $C_{gs} = \frac{2}{3} C_o \qquad C_{gd} = 0$

Linear: $C_{gs} \approx C_{gd} \approx \frac{1}{2} C_o$



Transit Frequency: when $i_{in} = i_{out}$ $\qquad i_{in} = j\omega C_o V_{gs} \qquad i_{out} = g_m V_{gs}$

$\Rightarrow f_T = \frac{g_m}{2\pi C_o}$

$g_m = \frac{\partial I_{ds}}{\partial V_{gs}} \Rightarrow$ in Linear: $g_m = \mu C_{ox} \frac{W}{L} V_{DS}$

Saturation: $g_m = \mu C_{ox} \frac{W}{L} (V_{GS} - V_T)$

$\Rightarrow$ in Linear, $f_T = \frac{\mu V_{DS}}{2\pi L^2}$. in Saturation $f_T = \frac{\mu (V_{GS} - V_t)}{2\pi L^2} = \frac{\mu V_{ov}}{2\pi L^2}$

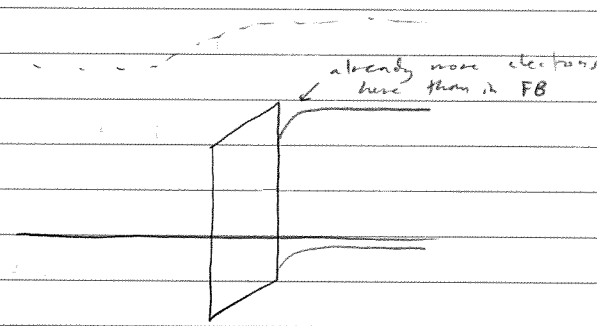
Nonidealities:

1) $\Phi_M \neq \Phi_S$ so even if $V_G = 0$,

$\Rightarrow$ we need less V_G to get inversion.

$\Rightarrow V_T' = V_T + \Delta V_T$

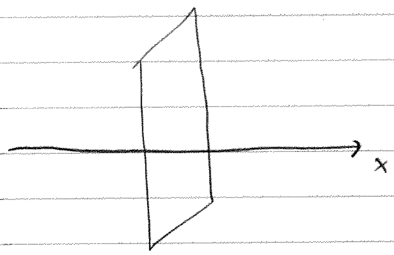
$\Delta V_T = \Phi_M - \Phi_S$



charge density per area = C/m^2

② Charge in oxide : $\Delta V_T = -\frac{1}{K_o \epsilon_o}\int_0^{x_o} x \cdot \rho_{ox}(x)\,dx$

uses Poisson eq. + eq. for $\vec{E}$ field & potential.



$x\rho_{ox}(x) \Rightarrow$ charge near semiconductor matters more.

if charge only exists at the boundary,

$\Delta V_T = -\frac{Q_F}{C_o} \rightarrow C/m^2$

different types of charge:

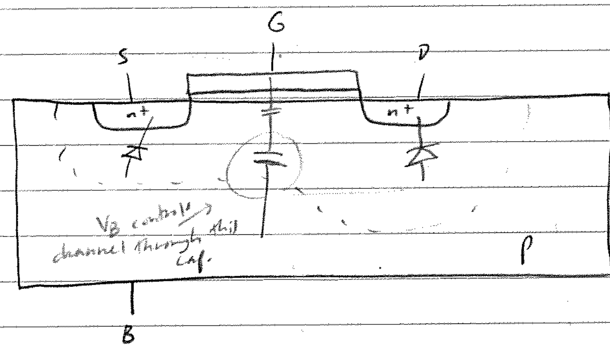
mobile Na^+, K^+ (comes from human body)

fixed: exists in oxide: can be reduced by annealing.

interface traps: (bias stress effect)

To change V_T, you can implant some charges to the surface of the Silicon.

③ Backgate Effect:

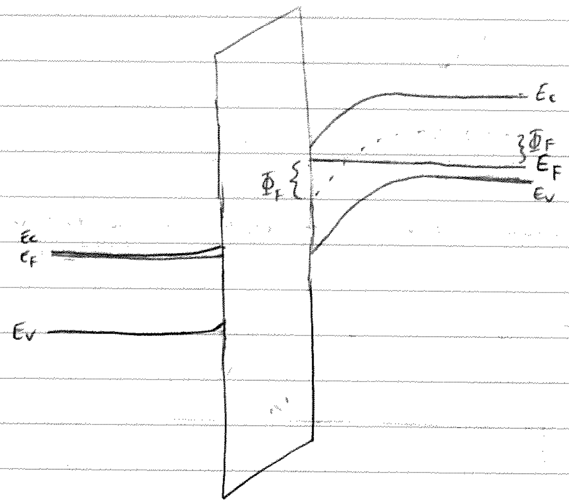


Normally want V_B = Lowest voltage such that diodes remain reverse biased. raising V_B reduces V_T, since it acts as a second gate (similar to raising V_G)

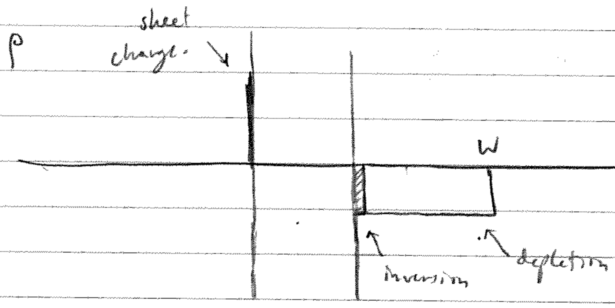
$$\Delta V_T = (V_T - 2\Phi_F)\left(\sqrt{1-\frac{V_{BS}}{2\Phi_F}} - 1\right) = \gamma\left(\sqrt{|2\Phi_F + V_{SB}|} - \sqrt{2\Phi_F}\right)$$

Question from Berkeley, pg. 65

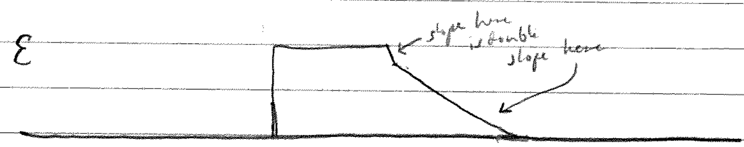
a) i)



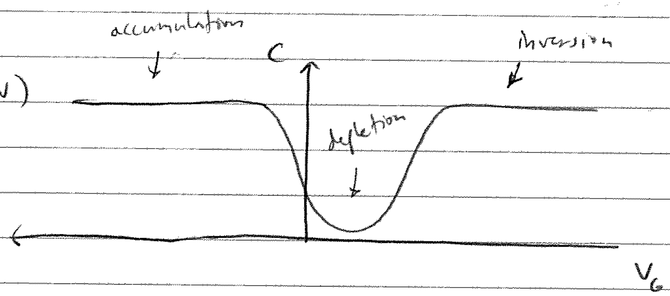
ii)



iii)



iv)



pg. 74. d)

$N_a = 10^{17} cm^{-3}$ $\qquad \varepsilon_{Si} = 10^{-12}$ F/cm $\qquad \varepsilon_{ox} = 3.45 \cdot 10^{-13}$ F/cm $\qquad T_{ox} = 3.45$ nm

$$\Phi_F = E_i - E_F = kT \ln\left(\frac{N_A}{n_i}\right) = 0.026 \cdot 2.3 \cdot 7 = 0.42\ V \approx 0.4V$$

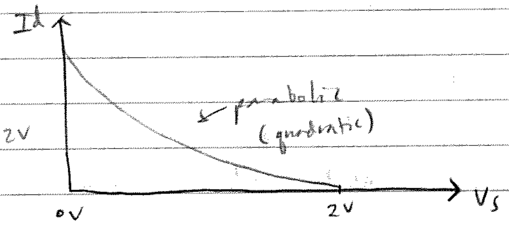
$$V_T = 2\Phi_F + \frac{\varepsilon_{Si} T_{ox}}{\varepsilon_{ox}} \sqrt{\frac{4 q N_A}{\varepsilon_{Si}\, \varepsilon_0} \cdot \Phi_F}$$

$$\sqrt{\frac{4 \cdot 1.6 \cdot}{10^{-10}} \cdot 0.4} \qquad \qquad \ln(10) = 2.3$$

$$V_T = 0.8V + 3.45 \cdot 10^{-6}_{cm} (1.6 \cdot 10^{5}) \approx 1.05\ V$$

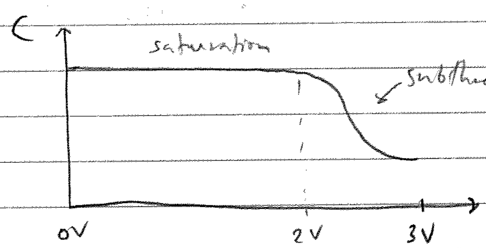
$\mu = 1500$ in Si for e^- $\qquad \mu = 500$ in Si for holes

i) sketch Id for $0 < V_s < 2V$



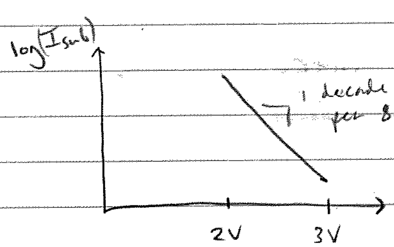
always in saturation.

ii) sketch C_g



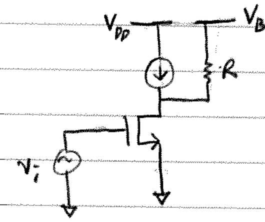
iii) $I_{sub} = I_o e^{V_{GS}/\zeta V_T}$ $\qquad \zeta > 1$

$\Rightarrow$



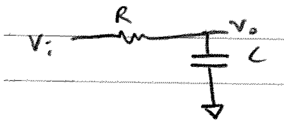
Circuits - [redacted] - Intrinsic + Extrinsic Capacitance.

Intrinsic :



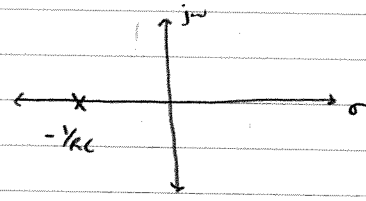
speed: Time domain - response to pulse
Freq domain - frequency where gain goes to 1.

Intrinsic caps: Required for MOSFET operation



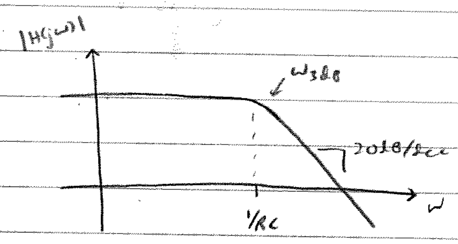
$$H(s) = \frac{V_i(s)}{V_o(s)} = \frac{\frac{1}{sC}}{R+\frac{1}{sC}} = \frac{1}{1+sRC} = \frac{1}{1-s/p}$$

$$\boxed{p = -\frac{1}{RC}}$$



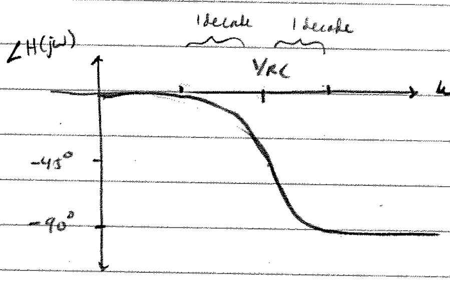
$$H(j\omega) = \frac{1}{1+j\omega RC}$$

$$|H(j\omega)| = \frac{1}{\sqrt{1+\omega^2R^2C^2}} \qquad \angle H(j\omega) = \tan^{-1}(-\omega RC)$$

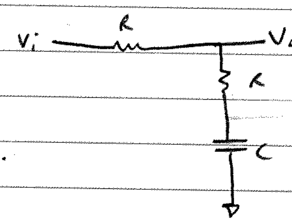


$$20\log|H(j\omega_{3dB})| = 20\log\left(\frac{1}{\sqrt{2}}\right) \approx -3$$

at ω_{3dB}, power is halved.

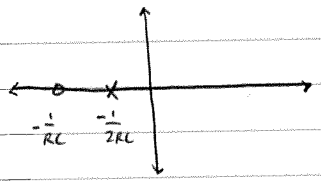


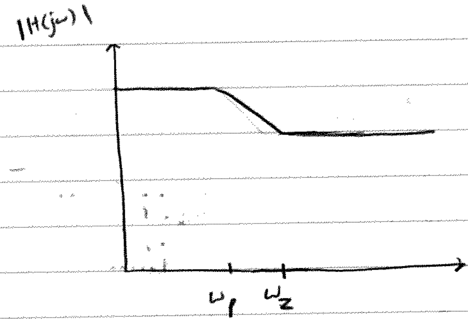
Zero:

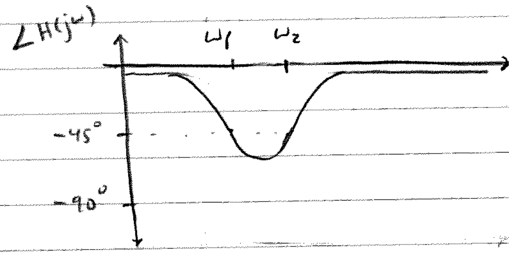


$$H(s) = \frac{\frac{1}{sC}+R}{\frac{1}{sC}+2R} = \frac{1+sRC}{1+2sRC} = \frac{1-\frac{s}{z}}{1-\frac{s}{p}}$$

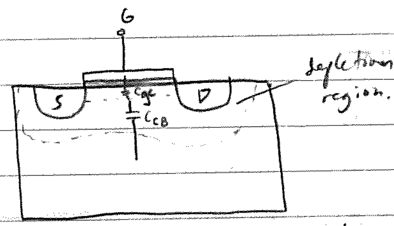
$$z = -\frac{1}{RC} \qquad p = -\frac{1}{2RC}$$



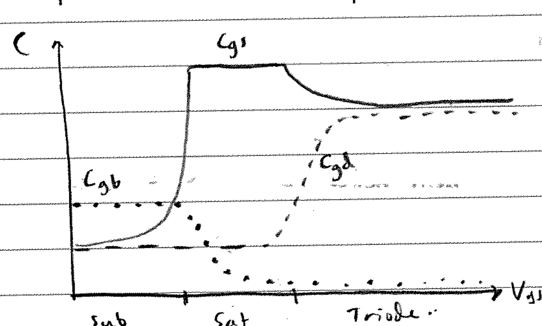




Capacitances:



	Subthreshold	Saturation	Triode
C_{sb}	C_{jsb}	C_{jsb}	C_{jsb}
C_{db}	C_{jdb}	C_{jdb}	C_{jdb}
C_{gs}	C_{ov}	$\frac{2}{3} WLC_{ox} + C_{ov}$	$\frac{1}{2} WLC_{ox} + C_{ov}$
C_{gd}	C_{ov}	C_{ov}	$\frac{1}{2} WLC_{ox} + C_{ov}$
C_{gb}	$\left(\frac{1}{C_{CB}} + \frac{1}{C_{GC}}\right)^{-1} = \left(\frac{1}{WLC_{ox}} + \frac{1}{\frac{\varepsilon_{si}}{x_d} WL}\right)^{-1}$	0	0



Power-Bandwidth Tradeoff. (CS)

$$\omega_{3dB} = \frac{1}{R_i C_{gs}} = \frac{1}{R_i \frac{2}{3} W L C_{ox}} = \underbrace{\frac{3}{4}\frac{\mu}{L^2}}_{\text{technology}} \underbrace{\frac{R}{|a_{v_0}| \cdot R_i}}_{\text{design constraints.}} \cdot V_{ov}$$

design knob

$$g_m = \frac{2 I_d}{V_{ov}} \qquad |a_{v_0}| = g_m R \quad \Rightarrow \quad I_d = \underbrace{\frac{1}{2}\frac{|a_{v_0}|}{R}}_{\text{design constraints}} V_{ov}$$

design knob.

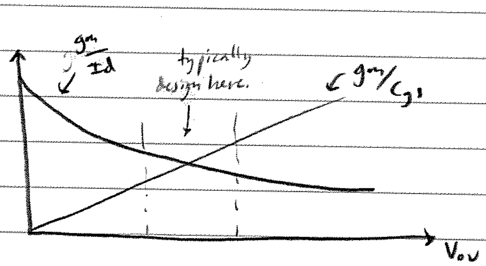
$\Rightarrow$ $V_{ov} \uparrow$ $\omega_{3dB} \uparrow$, but $I_d \uparrow$ $\Rightarrow$ more bandwidth requires more power.

Want high $\frac{g_m}{I_d}$ and high $\frac{g_m}{C_{gs}}$

$$\Downarrow \qquad\qquad \Downarrow$$

$$\frac{g_m}{I_d} = \frac{2}{V_{ov}} \qquad \frac{g_m}{C_{gs}} = \frac{3}{2}\frac{\mu V_{ov}}{L^2}$$

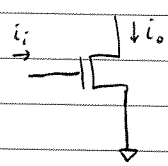
	V_{ov} small	V_{ov} large
$\frac{g_m}{I_d}$	Large :)	Small :(
$\frac{g_m}{C_{gs}}$	Small :(	Large :)



$$\frac{g_m}{I_d} \cdot \frac{g_m}{C_{gs}} = \frac{3\mu}{L^2}$$

gets better with tech scaling.

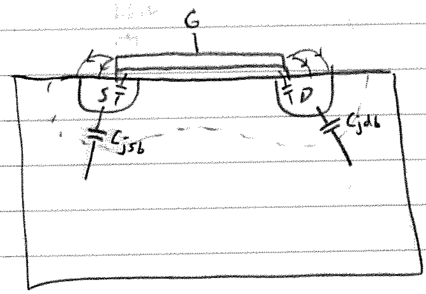
$\omega_t = \frac{g_m}{C_{gs}}$ = transit frequency :



ω at which $\frac{i_o}{i_i} = 1$

$$\omega_t = \frac{g_m}{C_{gs}}.$$

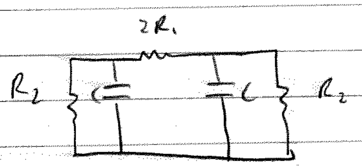
Extrinsic Cap: $C_{ov} = C_{ov}' \cdot W$ depends on both direct cap and from fringing field.

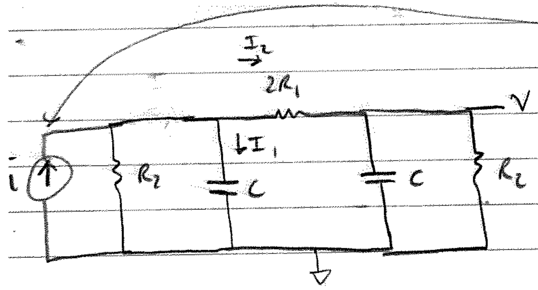


Q: 1999 Tom Lee

$$\frac{1}{R_2 \| (2R_1 + R_2)\, C}$$

2)





apply thevenin equivalent,

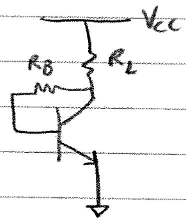
$$\Rightarrow \left(2R_1 + \frac{R_2}{1+sR_2C}\right)$$

current divider $$I_2 = \frac{\frac{1}{sC}}{\frac{1}{sC} + 2R_1 + \frac{R_2}{1+sR_2C}}\, I = \frac{I}{1 + 2sR_1C + \frac{sR_2C}{1+sR_2C}}$$

$$\Rightarrow \frac{V}{I} = \frac{2R_1 + \frac{R_2}{1+sR_2C}}{1 + 2sR_1C + \frac{sR_2C}{1+sR_2C}} = \frac{1}{\frac{1}{2R_1 + \frac{R_2}{1+sR_2C}} + sC}$$

$$= \frac{2R_1 + \frac{R_2}{1+sR_2C}}{1 + sC\left(2R_1 + \frac{R_2}{1+sR_2C}\right)}$$

1)



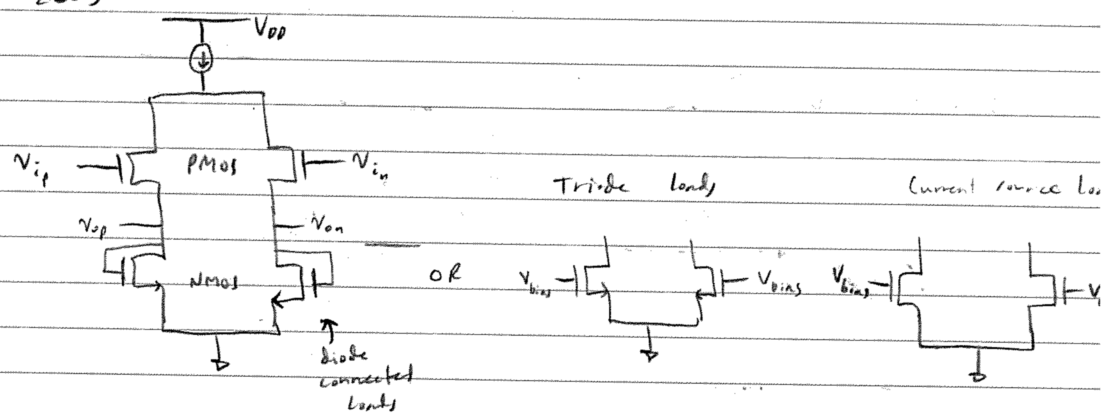
for saturation: $V_{BE} > V_t$ $\beta = \frac{I_c}{I_B}$

$V_{BC} > V_t$

$V_c = V_{cc} - I_c R_L$ $V_B = V_c - \frac{1}{\beta} I_c \cdot R_B \Rightarrow V_{BC} = -\frac{1}{\beta} I_c \cdot R_B < 0$ if $I_c > 0$

$\Rightarrow$ contradiction. $\Rightarrow$ Transistor will never be in saturation.

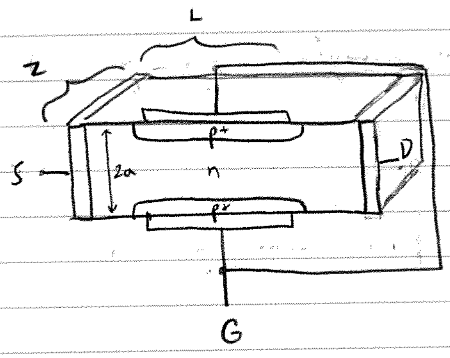
Simon Wong 2003



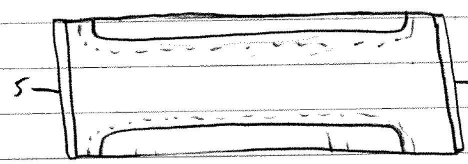
Devices - Alex Ch 15 + 19 (JFET, MESFET, and Modern FET Structures)

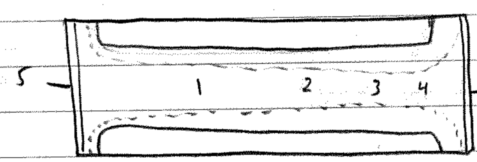
Ch. 15

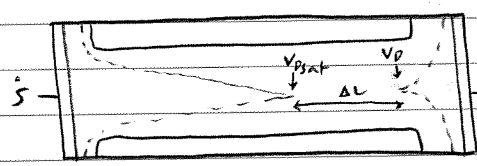
JFET -



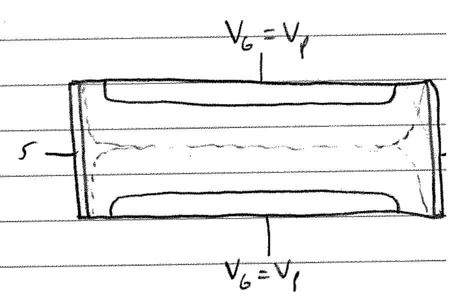
if $V_G = 0V$, $V_S = 0V$, $V_D > 0V$

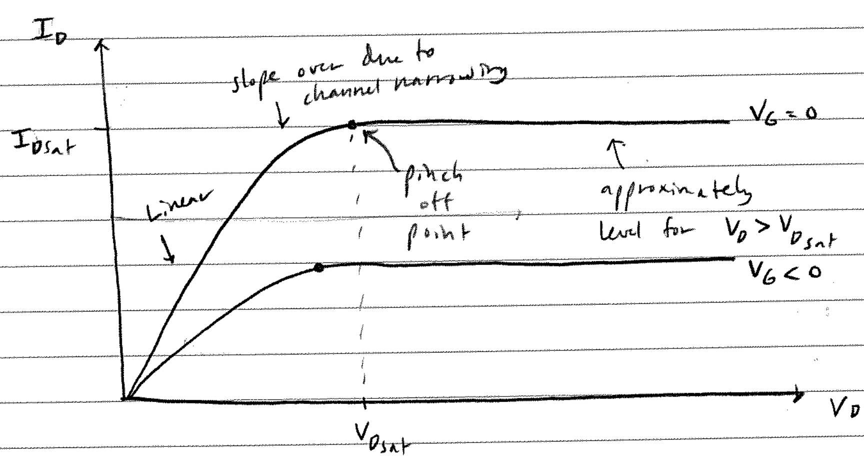






Gate pinch off





$$I_D = \frac{2qZ\mu_n N_D a}{L}\left\{V_D - \frac{2}{3}(V_{bi} - V_P)\left[\left(\frac{V_D + V_{bi} - V_G}{V_{bi} - V_P}\right)^{3/2} - \left(\frac{V_{bi} - V_G}{V_{bi} - V_P}\right)^{3/2}\right]\right\}$$

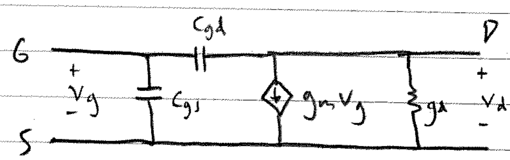
for $0 \le V_D \le V_{Dsat}$; $V_P \le V_G \le 0$

$$I_{Dsat} = \frac{2qZ\mu_n N_D a}{L}\left\{V_G - V_P - \frac{2}{3}(V_{bi} - V_P)\left[1 - \left(\frac{V_{bi} - V_G}{V_{bi} - V_P}\right)^{3/2}\right]\right\}$$

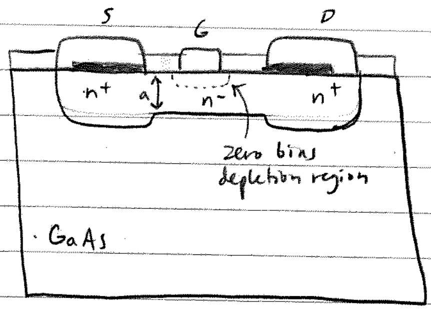
$$= I_{D_0}\left(1 - \frac{V_G}{V_P}\right)^2 \qquad I_{D_0} = I_{Dsat}\Big|_{V_G = 0}$$

↑
"Square Law"

Small Signal Model:



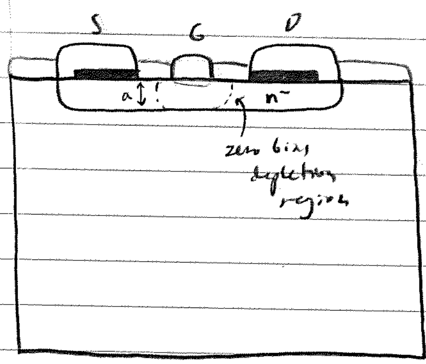
MESFET - GaAs MESFET - good for high freq. ICs (MMICs)



D-MESFET

(Digital logic circuits use both D + E
Analog MMICs use D's.)

Depletion Mode - Normally ON



E-MESFET

Enhancement Mode - Normally OFF

Can use same equations as for JFET with the following mods:

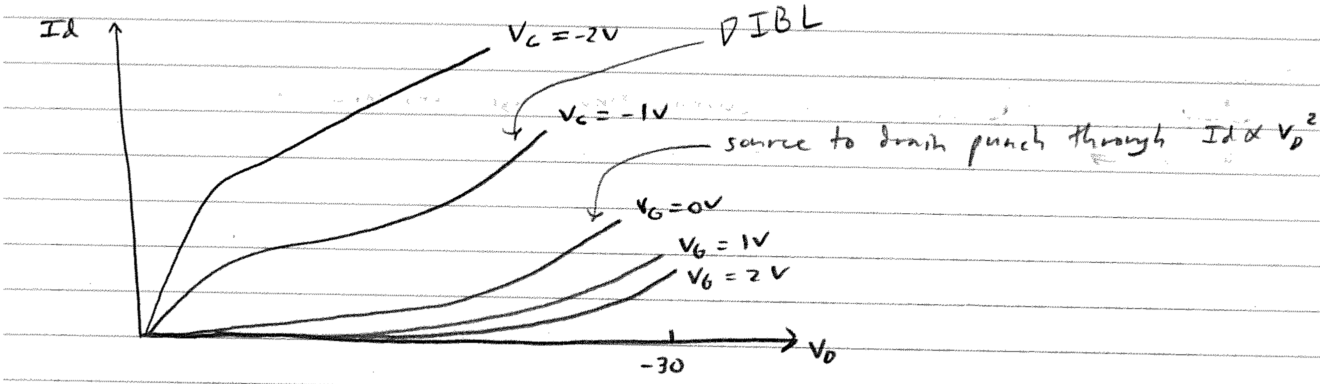
$2a \to a$

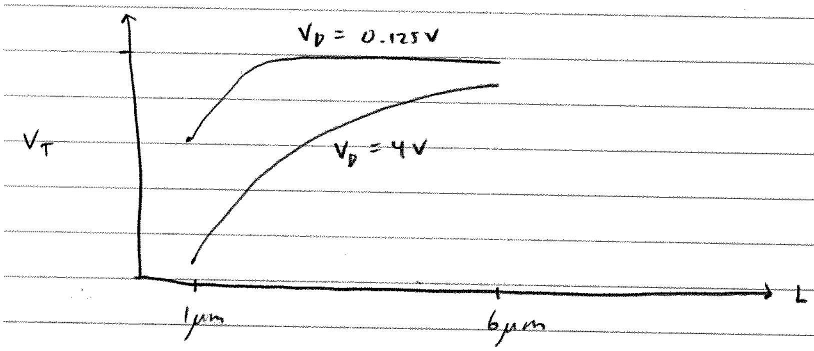
short channel $\Rightarrow$ gradual channel approx doesn't hold $\to$ variable mobility.

short channel $\Rightarrow$ velocity saturation.

Ch. 19 Modern FET Structures

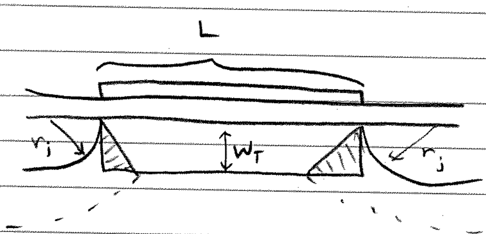
- Short channel effects:

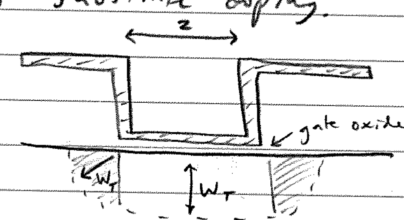




$$L_{min} = 0.4\left[r_j x_o (W_s + W_D)^2\right]^{1/3} \qquad x_o \text{ in } Å$$

L_{min} can be made smaller by reducing depth of source drain islands, reducing oxide thickness, or increasing substrate doping.



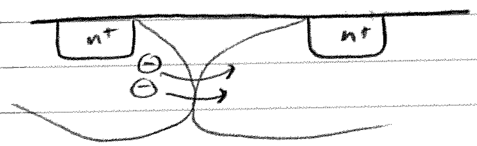


in short channel devices, the source and drain assist in depleting the region under the gate.

$$\Delta V_T(\text{short channel}) = -\frac{q N_A W_T}{C_o}\frac{r_j}{L}\left(\sqrt{1+\frac{2W_T}{r_j}} - 1\right)$$

$$\Delta V_T(\text{narrow width}) = \frac{q N_A W_T}{C_o}\frac{\pi W_T}{2z}$$

Parasitic BJT Action : 1) source to drain punch-through

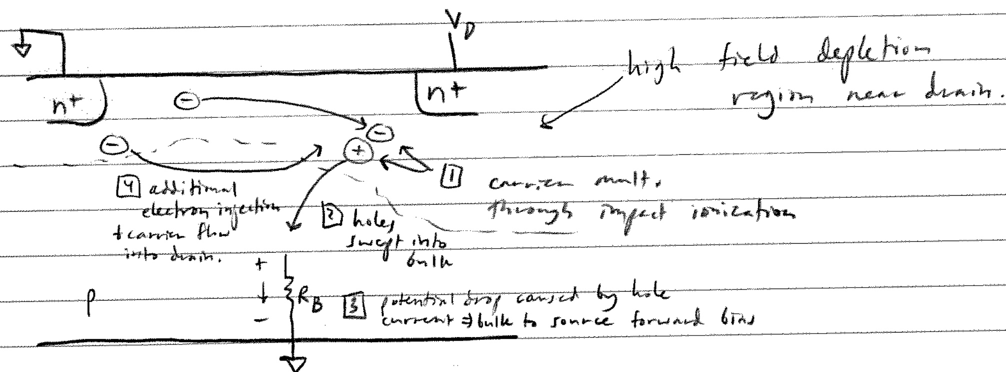


current no longer constrained to surface channel.

$I_d \propto V_D^2$

- Can fix by increasing doping of subgate region,
- but increasing substrate doping increases parasitic caps.
- ⇒ selectively increase subgate doping through deep-ion implantation.

2) carrier multiplication + regenerative feedback



Hot Carrier Effects :

1) Oxide charging (BIAS STRESS)
- channel carriers + carriers entering drain depletion region gain sufficient energy to surmount Si-SiO₂ surface barrier.
→ enter oxide → neutral centers in oxide trap carriers → charge build up
→ changes V_T and g_m over time.
- solution : LDD (lightly doped drain)

2) Velocity Saturation

- high $\mathcal{E}$ field ($> 3\cdot 10^4$ V/cm for e^-, $> 10^5$ V/cm for holes)
- $I_{D_{sat}}$ reduced : $I_{D_{sat}} \approx Z C_o (V_G - V_T) v_{sat}$
- I_{sat} linearly dependent on $(V_G - V_T)$ rather than square law.

3) Velocity overshoot / Ballistic Transport

- if $L \lesssim l \Rightarrow$ ballistic transport. (l is avg. distance between scattering event (mean free path))
- could lead to faster devices
- Average velocity can exceed v_{sat}. this is called velocity overshoot.
- $L \leq 0.3\mu m$ for GaAs

Circuits - Alex   L8, L9, L10 Miller Approximation and ZVTC, + Backgate Effect.

Razavi: $V_t = 0.7V$ $V_{DD} = 3V$

3.10

a) $\left(\frac{W}{L}\right)_1 = 100$ $\left(\frac{W}{L}\right)_2 = 20$ $V_{DS} = V_{GS} - V_t$

$Id_2 = Id_1 \Rightarrow 20(V_{GS_2} - V_t)^2 = 100(V_{GS_1} - V_t)^2$

$\Rightarrow (2.3 - V_o)^2 = 5(V_i - 0.7)^2$ Edge of Triode $\Rightarrow V_{DS_1} = V_{GS_1} - V_t$
$\Rightarrow V_o = V_i - V_t$

$\Rightarrow 5.29 - 4.6V_o + V_o^2 = 5V_o^2 \Rightarrow 5.29 - 4.6V_o - 4V_o^2 = 0$

$\Rightarrow V_o = 0.71\ V \quad \Rightarrow V_i = 1.41\ V$

$g_{m_1} = \frac{2Id}{V_{ov_1}}$ $g_{m_2} = \frac{2Id}{V_{ov_2}}$ $a_{v_o} = -\frac{g_{m_1}}{g_{m_2}} = -\frac{V_{ov_2}}{V_{ov_1}} = -\frac{3 - 0.71 - 0.7}{1.41 - 0.7} = \frac{1.59}{0.71} =$

$\Rightarrow \boxed{a_{v_o} = -2.23}$

b) $V_o = V_i - V_t - 0.05\ V$

$\Rightarrow (2.3 - V_o)^2 = 10\left(V_i - V_t - \frac{V_o}{2}\right)V_o$

$\Rightarrow (2.3 - V_o)^2 = 10\left(V_o + \cancel{V_t} + 0.05V - \cancel{V_t} - \frac{V_o}{2}\right)V_o = 10\left(0.05V_o + \frac{V_o^2}{2}\right)$

$\Rightarrow 5.29 - 4.6V_o + V_o^2 - 0.5V_o - 5V_o^2 = 0 \Rightarrow 5.29 - 5.1V_o - 4V_o^2 = 0$

$\Rightarrow V_o = 0.6774\ V \quad \Rightarrow V_i = 1.4274$

$\Rightarrow g_{m_1} = \frac{\partial}{\partial V_{gs}}\left(\mu C_{ox}\frac{W}{L}\left(V_{gs} - V_t + \frac{V_{DS}}{2}\right)V_{DS}\right) = \mu C_{ox}\frac{W}{L}V_{DS} = \frac{Id}{\left(V_{ov} + \frac{V_{DS}}{2}\right)}$

$g_{ds} = \frac{\partial}{\partial V_{DS}}\left(\mu C_{ox}\frac{W}{L}\left(V_{gs} - V_t + \frac{V_{DS}}{2}\right)V_{DS}\right) = \mu C_{ox}\frac{W}{L}\left[(V_{gs} - V_t) + V_{DS}\right]$

$\Rightarrow \text{gain} = -\frac{g_{m_1}}{g_{m_2} + g_{ds}} = -\frac{\cancel{\mu C_{ox}}\left(\frac{W}{L}\right)_1 V_o}{\cancel{\mu C_{ox}}\left(\left(\frac{W}{L}\right)_2 V_{ov_2} + (V_{ov_1} + V_o)\left(\frac{W}{L}\right)_1\right)}$

$= -\frac{100\ V_o}{20(3 - V_o) + 100(V_i - 0.7 + V_o)} = -\frac{100(0.6774)}{20(3 - 0.6774 - 0.7) + 100(1.4274 - 0.7 + 0.6774)}$

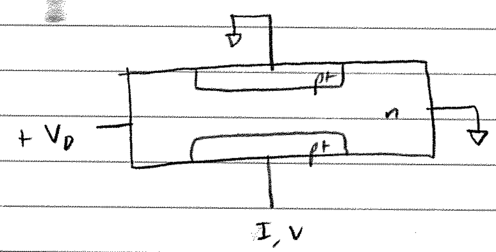
$= -0.3917$

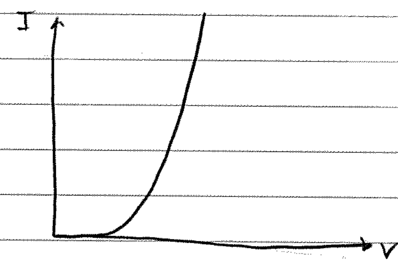
$\Rightarrow$ $\boxed{a_{v_0} = -0.3917}$ neglecting g_{ds}, $a_{v_0} = -2.0874$

3.11

Quals-Dutton 2009

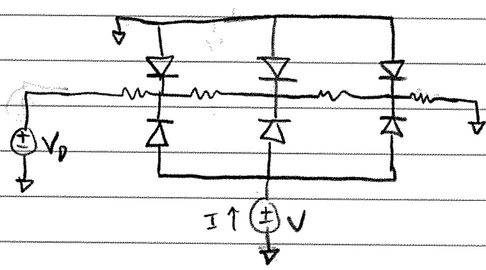
1. a)



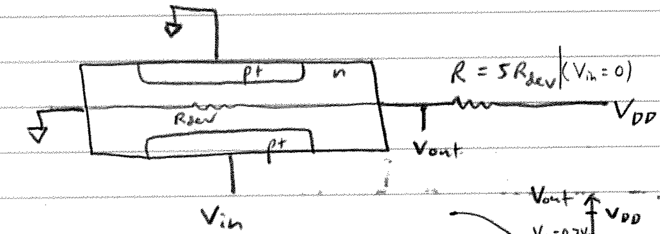


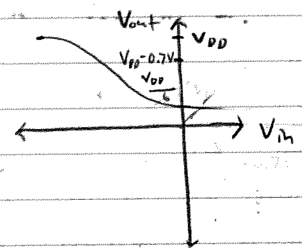
sketch I, V for $V_D > 0$.

diode forward biased



b)





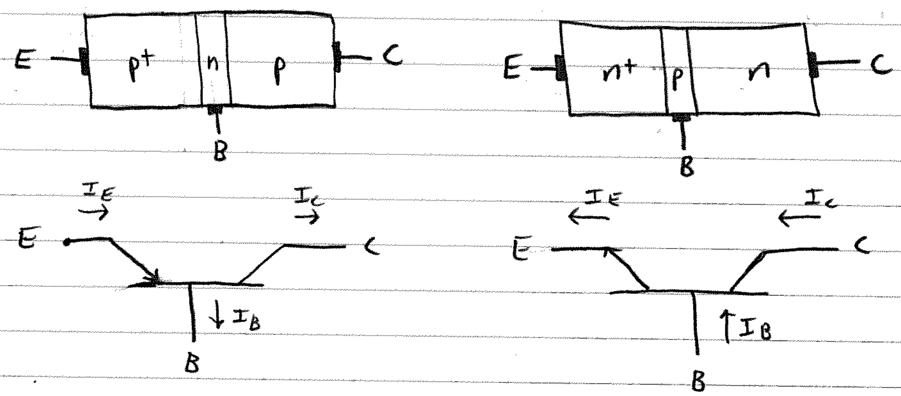
sketch V_{out} vs V_{in}

for $V_{in} < 0$

11/7/11

Devices - Alex CH 10 - 11.1 BJT Fundamentals and Static Characteristics.

10 - BJT Fundamentals

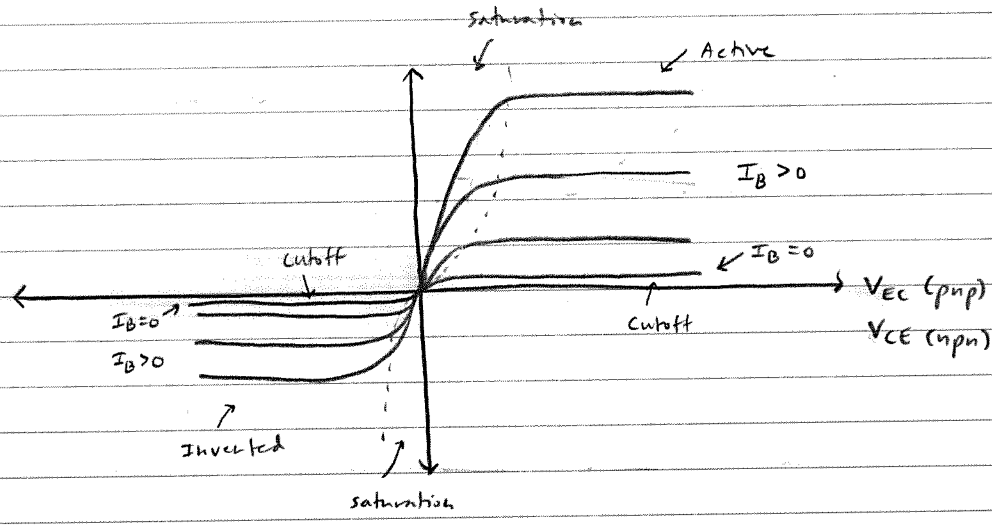


$I_E = I_B + I_c$ $V_{EB} + V_{BC} + V_{CE} = 0$

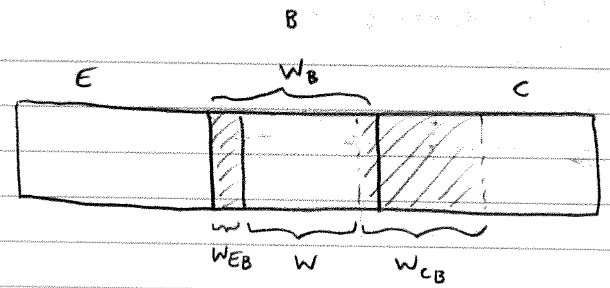
In the limit of $\frac{W}{L_B} \to 0$ (narrow base)

$I_c = I_s (e^V$

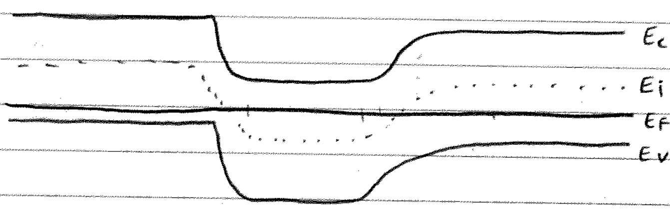
Biasing Mode	E-B Bias	C-B Bias
Saturation	Forward	Forward
☆ Active	Forward	Reverse
Inverted	Reverse	Forward
Cutoff	Reverse	Reverse



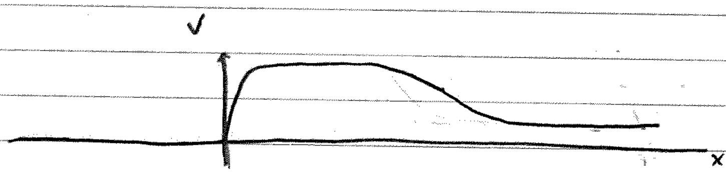
BJT in Equilibrium Conditions



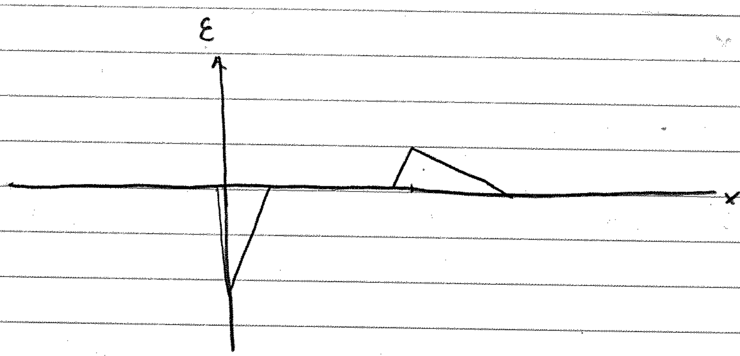
Depletion regions



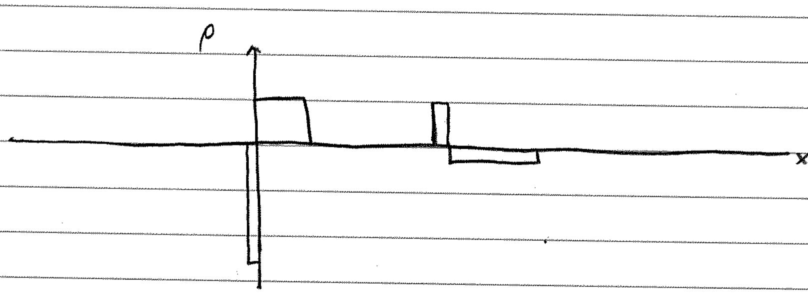
Band diagram



Potential

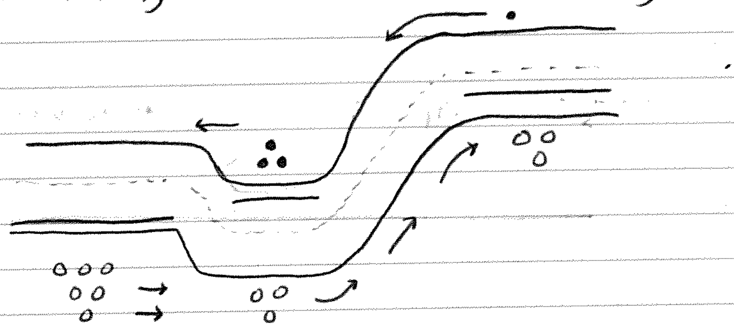


Electric Field



Charge Density

Carrier Activity under Active mode biasing



W is narrow compared to minority carrier diffusion length.

$\Rightarrow$ most holes diffuse completely through quasineutral base region.

Performance Parameters

Emitter Efficiency : $\boxed{\gamma = \frac{I_{Ep}}{I_E} = \frac{I_{Ep}}{I_{Ep}+I_{En}}}$ for pnp

← ideally unity

Base Transport Factor : $\boxed{\alpha_T = \frac{I_{Cp}}{I_{Ep}}}$ ← ideally unity

Common Base DC current gain : $\boxed{\alpha_{dc} = \gamma\alpha_T} = \frac{I_{Cp}}{I_E}$ ideally unity

Common Emitter DC current gain : $\boxed{\beta_{dc} = \frac{I_C}{I_B} = \frac{\alpha_{dc}}{1-\alpha_{dc}}}$ ideally infinite

CH 11 : BJT Static Characteristics.

Solve using diffusion equations and boundary equations.

get complicated eqs for I_{En}, I_{Cn}, I_{Ep}, I_{Cp}, $\Delta p_B(x)$

Can make some simplifications if $W \ll L_B$

for $W << L_B$;

perturbed carrier concentration is

$$\Delta p_B(x) = \Delta p_B(0) + [\Delta p_B(W) - \Delta p_B(0)]\frac{x}{W}$$ ← Linear

Performance Parameters:

$$\gamma = \frac{1}{1 + \frac{D_E}{D_B}\frac{N_B}{N_E}\frac{W}{L_E}}$$ ← $N_E >> N_B \Rightarrow$ better emitter efficiency (near unity)

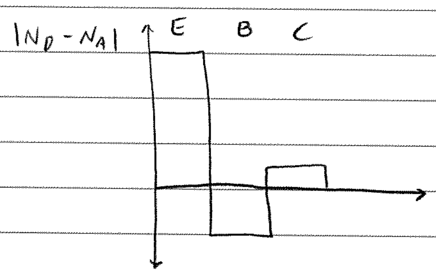
$$\alpha_T = \frac{1}{1 + \frac{1}{2}\left(\frac{W}{L_B}\right)^2}$$ ← $W << L_B \Rightarrow$ better base transport fac (near unity)

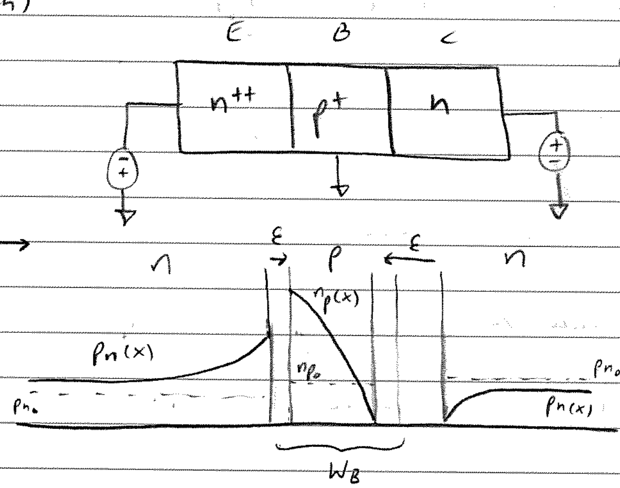
$$\alpha_{dc} = \frac{1}{1 + \frac{D_E}{D_B}\frac{N_B}{N_E}\frac{W}{L_E} + \frac{1}{2}\left(\frac{W}{L_B}\right)^2}$$

$$\beta_{dc} = \frac{1}{\frac{D_E}{D_B}\frac{N_B}{N_E}\frac{W}{L_E} + \frac{1}{2}\left(\frac{W}{L_B}\right)^2}$$

[redacted] - BJT (Neaman)

Bias in Active Mode.





choose $W_B << L_p, L_n$

in base

cross-sectional Area between base and emitter junction.

$$\cancel{i_c = I_s \exp\left(\frac{V_{BE}}{V_T}\right)} \qquad I_s = q D_n A \frac{d n_B(x)}{\partial x} = \frac{-q D_n A_{BE}}{W}$$

width of quasineutral base region

$i_E \Rightarrow$ $i_{E_1} = i_c$

i_{E_2}

↑ due to diffusion of holes from B to E.

$$\cancel{I_E = I_{SE} \exp\left(\frac{V_{BE}}{kT}\right) = i_{E_1} + i_{E_2}}$$

$i_B \Rightarrow$ $i_{B_1} = i_{E_2}$ ← diffusion of holes from B to E.

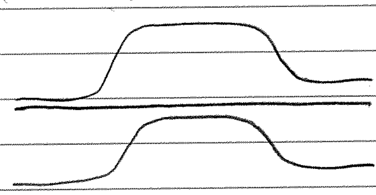
i_{B_2} ← recombination current. Some electrons entering the base actually recombine with holes. Thus, we need a hole current to replace these holes.

We want $i_c = i_E \Rightarrow \alpha_{dc} \approx \frac{i_c}{i_E} \approx 1$

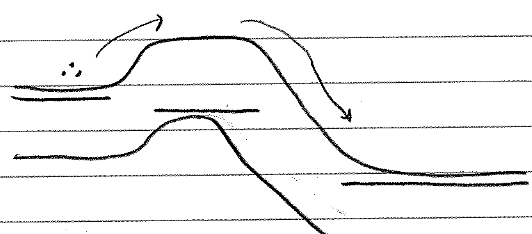
↑ common base current gain.

$\beta = \frac{i_c}{i_B} = \frac{\alpha_{dc}}{1-\alpha_{dc}}$ ← common base current gain.

Band Diagram: eg.



Active



Berkeley Quals

Fall 2004 Bokor

BJT:

3. a) $\beta = \frac{I_c}{I_B} = \frac{2mA}{10\mu A} = 0.2 \cdot 1000 = 200$

$I_c = I_s \exp\left(\frac{V_{BE}}{V_T}\right) \Rightarrow g_m = \frac{\partial I_c}{\partial V_{BE}} = \frac{I_c}{V_T}$

MOSFET: $g_m = \frac{2I_d}{V_{ov}} = \frac{16mA}{(5-0.7)V} = \frac{16mA}{4.3V} \approx 4mS$

BJT: $g_m = \frac{8mA}{0.026V} \approx 300mS$

b) Discuss the comparison of the two g_m values. If one is much larger than the other, give some physical explanation.

$g_{m_{BJT}} \gg g_{m_{MOS}}$. For BJT, the concentration of carriers is exponentially dependent on V_{BE}, but for MOS, the carriers in the inversion layer are linearly dependent on V_{GS} (like a capacitor).

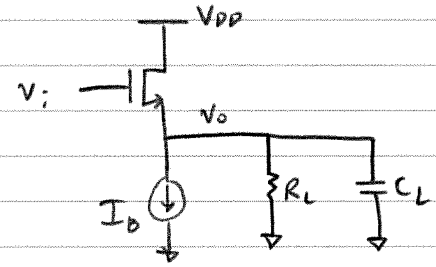
As far as the input-output characteristics are concerned, in what ways does the transistor with lower g_m outperform the transistor with higher g_m?

MOS: higher input impedance (no r_π). lower power. better scaling. compatible with digital process.

I read 4.1 - 4.2 of Gray + Meyer

Circuits - [redacted] L 11-13, CD, PVT, Current Mirrors

L11 - Common Drain



$V_o = g_m(V_i - V_o) R_{L_{tot}}$

$\Rightarrow V_o(1 + g_m R_{L_{tot}}) = V_i \, g_m R_{L_{tot}}$

$\Rightarrow \dfrac{V_o}{V_i} = \dfrac{g_m R_{L_{tot}}}{1 + g_m R_{L_{tot}}}$

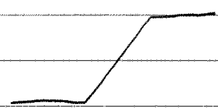
$C_{L_{tot}} = C_L + C_{sb} \qquad R_{L_{tot}} = R_L \| r_o \| \dfrac{1}{g_{mb}}$

$$\Rightarrow A_v = \frac{v_o}{v_i} = \underbrace{\frac{g_m}{g_m + \frac{1}{R_{L_{tot}}}}}_{a_{v_o}} \left(\frac{1 + \frac{sC_{gs}}{g_m}}{1 + \frac{s(C_{gs} + C_{L_{tot}})}{g_m + \frac{1}{R_{L_{tot}}}}} \right)$$

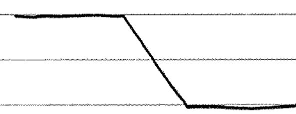
← 1 zero, 1 pole

if $R_L \to \infty$, $r_o \to \infty$, $g_{mb} = 0 \Rightarrow a_{v_o} = 1$

if $|z| < |p|$



$|z| > |p|$



if $|z| = |p|$ infinite bandwidth ?

CD Input Impedance: NMOS

$Y_{in} = s(C_{gd} + C_{gb}) + sC_{gs}(1 - a_v(s))$

for PMOS with Body - Source Tie,

$Y_{in} = sC_{gd} + s(C_{gs} + C_{gb})(1 - a_v(s)) \approx sC_{gd}$

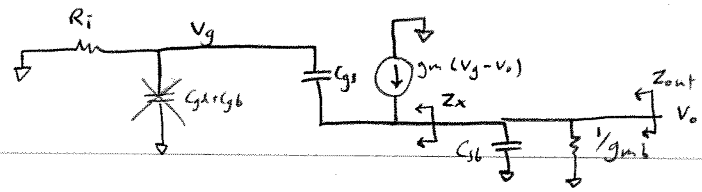
$\Rightarrow$ very small input capacitance.

$\Rightarrow$ capacitive impedance up to moderate frequencies

~ $C_{gd} + C_{gb}$ plus a fraction of C_{gs}.

$Z_{out} = \dfrac{1}{g_m + g_{mb}} \Big\| \dfrac{1}{s(C_{gs} + C_{sb})}$ low output impedance up to high freq.

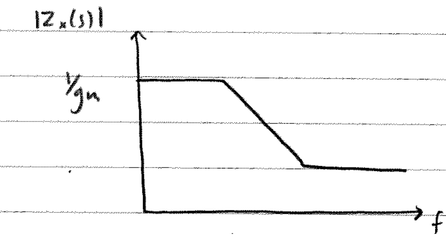
↑ ignoring R_i.

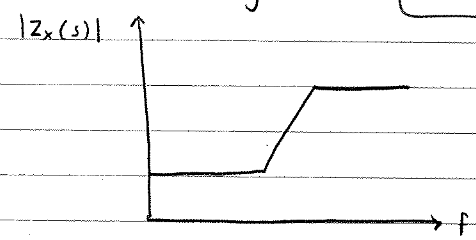


if we consider R_i:

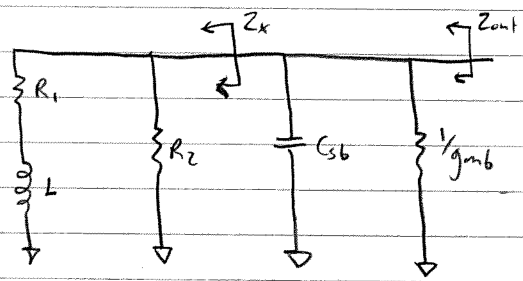
$$Z_x \approx \frac{1}{g_m} \frac{1+sR_iC_{gs}}{\left(1+\frac{sC_{gs}}{g_m}\right)}$$

Two cases: $R_i < \frac{1}{g_m}$ $\qquad$ $R_i > \frac{1}{g_m}$ $\qquad$ $\boxed{\text{Inductive!!}}$





Equivalent circuit for $R_i > \frac{1}{g_m}$



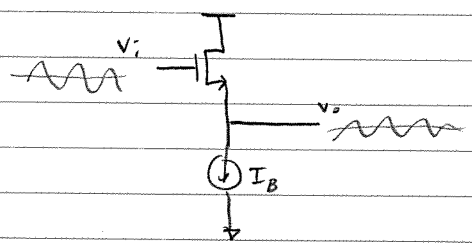
$$R_1 \| R_2 = \frac{1}{g_m}$$

$$R_2 = R_i$$

$$L = \frac{R_i^2 C_{gs}}{g_m R_i - 1}$$

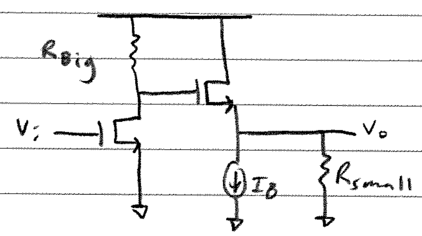
This circuit is prone to ringing! Forms an LC tank with any capacitance at the output.

Application 1: Level Shifter:



$\} \; V_t + V_{ov} \approx$ constant.

Application 2: Voltage Buffer

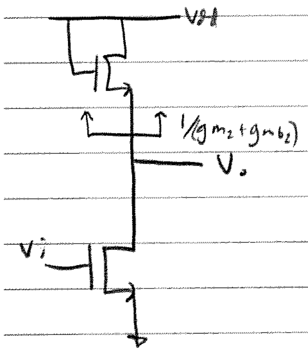


Issues: Nonlinearity - V_t, function of V_o (Body Effect)

I_D and thus V_{ov} changes with V_o, (gets worse for small R_L)

Reduced input + output swing.

Application 3: Load Device:



Advantages:

- Ratiometric (reduced PVT)
- First order cancellations of nonlinearities.

Disadvantage:

- Reduced swing.

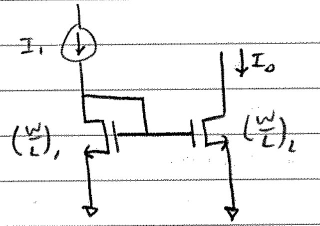
L12 - PVT

if Temperature ↑, R_{nwell} ↑, R_{poly} ↑, μC_{ox} ↓, C_{poly} ↓, V_t ↓

Pelgroms Rule:

$$\sigma_{\Delta x} = \frac{A_x}{\sqrt{WL}}$$

L13 - Current Mirrors



if $(\frac{W}{L})_1 = (\frac{W}{L})_2$, $I_o = I_1$

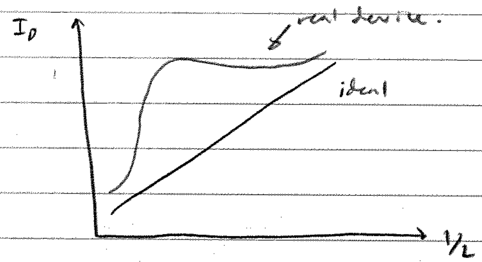
Want to:

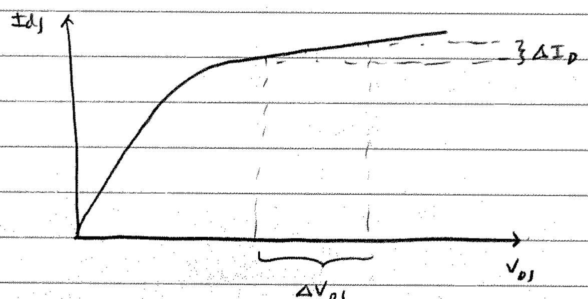
- Minimize error between I_o and I_i
- Maximize R_{out}
- Minimize $V_{o_{min}}$, V_i
- Minimize C_o

Can scale by changing ratio of $\left(\frac{W}{L}\right)_1$ and $\left(\frac{W}{L}\right)_2$ $\ni$ if $I_o = k I_i$,

$$k = \frac{\left(\frac{W}{L}\right)_2}{\left(\frac{W}{L}\right)_1}$$

usually, we scale W, not L. Since $L_{eff} \neq L$ and it is more variable.

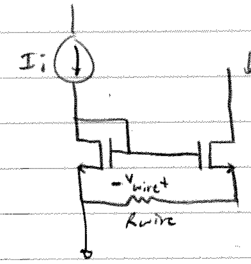




$$\varepsilon = \frac{I_o - I_i}{I_i} = \frac{\Delta V_{ds} \cdot g_{ds}}{I_i} \approx \lambda V_{DS}$$

$$\Rightarrow \boxed{\varepsilon = \lambda V_{DS}}$$

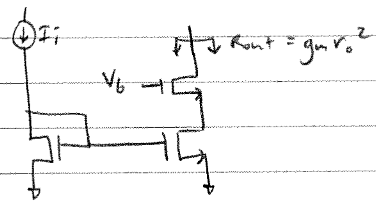
$\lambda \propto \frac{1}{L}$ $\ni$ to reduce ε use large L.



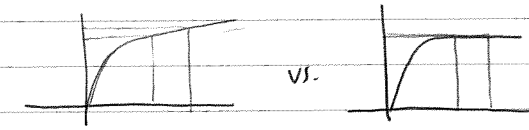
$$\varepsilon = \frac{\Delta I}{I_i} \simeq \frac{g_m V_{wire}}{I_i} \simeq \boxed{\frac{2V_{wire}}{V_{ov}} = \varepsilon}$$

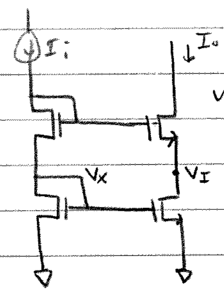
$\boxed{\text{to reduce } \varepsilon \text{ increase } V_{ov}}$, but this reduces output swing (increase $V_{o_{min}}$).

Can use Cascode to increase R_o:



want high R_o to minimize ε due to ΔV_{ds}



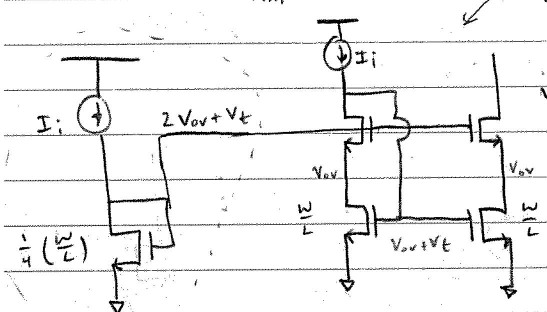


$V_{o_{min}} = 2V_{ov} + V_t$

$V_X = V_I \Rightarrow \varepsilon \simeq 0$

To reduce $V_{o_{min}}$:

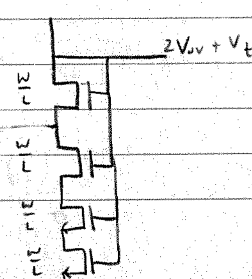
requires that $V_t > V_{ov}$ (which is usually true)



$V_{o_{min}} = 2V_{ov}$

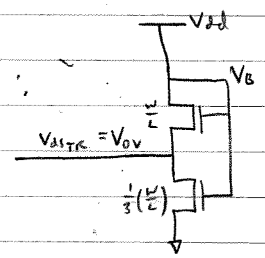
problem: sensitive to backgate effect.

To reduce backgate effect replace this with:



but requires a lot of headroom (high V_{in})

Typically, people will use this instead:

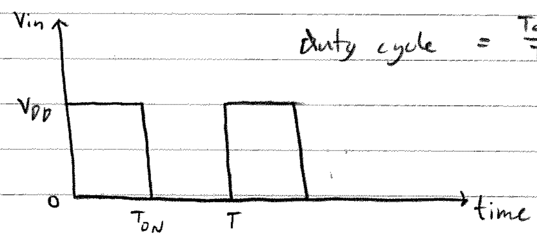


sensitive to backgate effect, but
requires less headroom at input.
(lower V_{in}).

$$V_{ds_{TR}} = \left(\sqrt{k+1} - 1\right) V_{ov} \qquad V_B = \sqrt{k+1}\, V_{ov} + V_T = V_{ds_{TR}} + V_{ov} + V_T$$

Quals Wong 2009

$$\frac{1}{\sqrt{LC}}$$

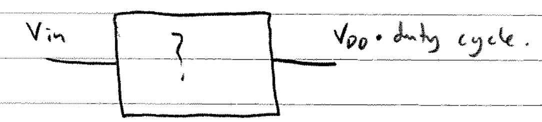


duty cycle $= \frac{T_{ON}}{T}$

F.S. $C_n = \frac{1}{T}\int_0^T f(t)\, e^{-2\pi i n t/T}\, dt$

$$= \frac{V_{dd}}{T}\int_0^{DT} e^{-2\pi i n t/T}\, dt = \frac{V_{dd}}{T}\left(-\frac{T}{2\pi i n} e^{-2\pi i n t/T}\Big]_0^{DT}\right)$$

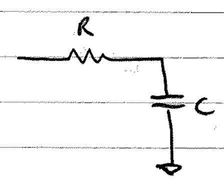
$$= V_{dd}\left(-\frac{1}{2\pi i n}\left(e^{-2\pi i n D} - 1\right)\right)$$



$$= \frac{V_{dd}}{2\pi i n}\left(1 - e^{-2\pi i n D}\right) \quad \text{for } n \neq 0$$

for $n = 0$, $C_0 = \frac{1}{T}\int_0^T f(t)\, dt = V_{dd} \cdot D$ ← DC component (This is what we want)

Design a circuit such that for the periodic input waveform shown, the output is approximately a DC voltage of V_{DD} · Duty cycle.

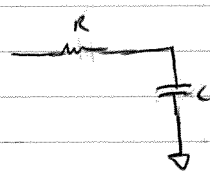


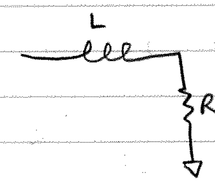
$$\frac{1}{2\pi RC} \ll \frac{1}{T} \Rightarrow RC \gg \frac{T}{2\pi}$$

If the output has to drive a heavy load, $(I = 1A)$, how would you modify your circuit?

for RC

need R_{out} small $\Rightarrow$ need huge C.





$$\frac{R}{R+j\omega L} = \frac{1}{1+j\omega\frac{L}{R}}$$

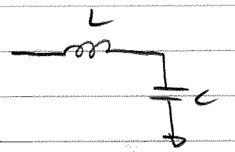
$$\Rightarrow \omega_{3dB} = \frac{R}{2\pi L}$$

for LR, need $\frac{R}{2\pi L} \ll \frac{1}{T} \Rightarrow$ need $\frac{L}{R} \gg \frac{T}{2\pi}$

$\Rightarrow$ can use small R. $\Rightarrow$ lower R_{out} $\Rightarrow$ can drive heavy load.

~~Answer:~~

Could also use LC filter (adding R_{Load} will reduce peaking)

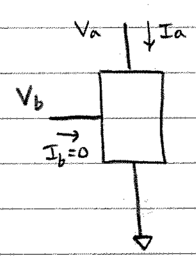


$$\frac{\frac{1}{sC}}{\frac{1}{sC}+sL} = \frac{1}{1+s^2LC} \qquad 2 \text{ poles at } \frac{1}{2\pi\sqrt{LC}} \ll \frac{1}{T}$$

$$\Rightarrow LC \gg \left(\frac{T}{2\pi}\right)^2$$

Or could add a unity gain voltage buffer.

Quals - Dutton 2007



$$I_a = \left(V_b + \frac{V_a}{10}\right)^2 = V_b^2 + \frac{V_aV_b}{5} + \frac{V_a^2}{100}$$

$$g_m = \frac{\partial I}{\partial V_b} = 2V_b + \frac{V_a}{5}$$

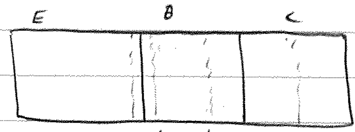
$$g_{ds} = \frac{\partial I}{\partial V_a} = \frac{V_b}{5} + \frac{V_a}{50} = \frac{10V_b+V_a}{50} \Rightarrow r_o = \frac{50}{10V_b+V_a} = \frac{50}{5g_m} = \frac{10}{g_m}$$

$$\Rightarrow \text{intrinsic gain} = g_m r_o = g_m\frac{10}{g_m} = \boxed{10}$$

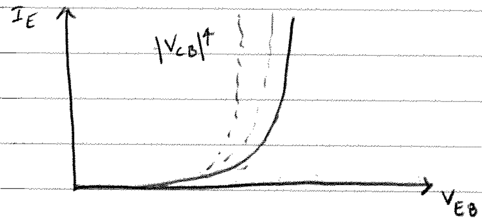
Device - [redacted] Ch 11 + ch 12 BJT Nonidealities

Ch. 11 - BJT Nonidealities.

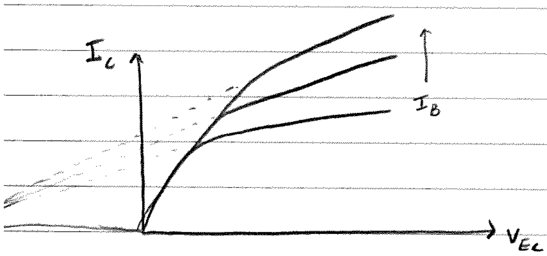
1) Base Width Modulation ("Early Effect")

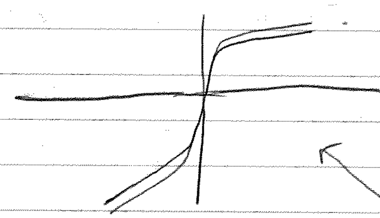


if we change V_{CB} so as to increase the reverse bias, W will change.



input characteristic of CB.

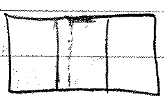


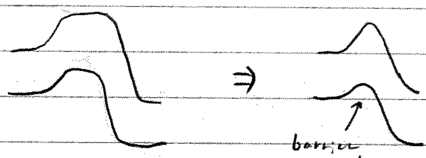


To reduce early effect (Base width modulation), increase N_B (doping of Base).

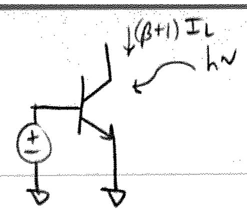
This effect is also much worse in Inverted mode, since B-E becomes reverse biased. + $N_E >> N_B$ $\Rightarrow$ depletion region is larger.

2) Punch Through - when depletion regions touch eachother ($W \to 0$) we observe punch through.



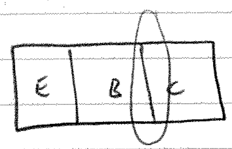


can cause breakdown if some other breakdown mechanism doesn't occur first.



3) Breakdown

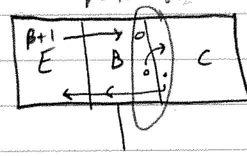
- Common Base



this junction typically breaks down V_{CBO} (avalanche or punch through) since it is the junction that is reverse biased in active mode.

- Common Emitter

positive feedback.



breakdown voltage

Thus, V_{CEO} smaller than for common base. (V_{CBO})

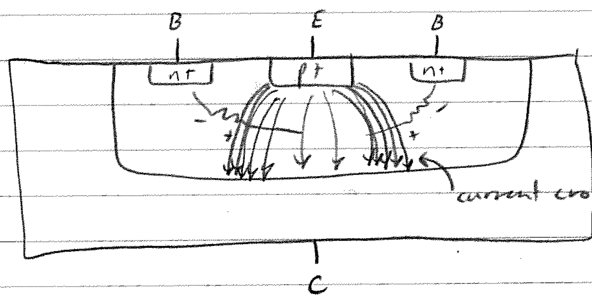
(carrier multiplication & internal feedback mechanism. or punch through)

in active mode,
B-E is forward biased
$\Rightarrow V_{BE}$ is small
$\Rightarrow V_{CE} = V_{CB} + V_{BE} \approx V_{CB}$

Base current is constant. for CE configuration.

Can be used as a photodetector if you shine light on the depletion region of B-C. Should multiply photocurrent by $\beta+1$.

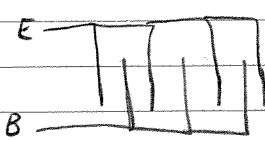
4) Geometrical Effects

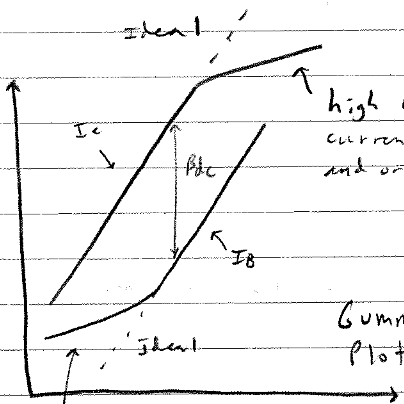


current crowding - can cause local heating, reduces current and

solutions: - increase $N_B \uparrow$ to reduce parasitic resistance r_b.

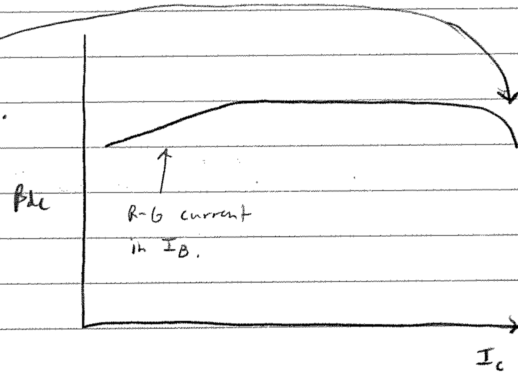
- interdigitate E and B.



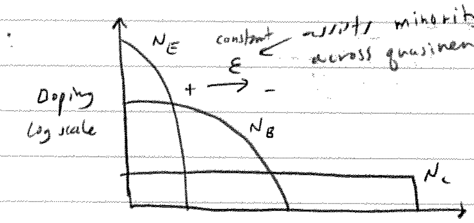


high level injection, current crowding, and or series resistance.

Gummel Plot

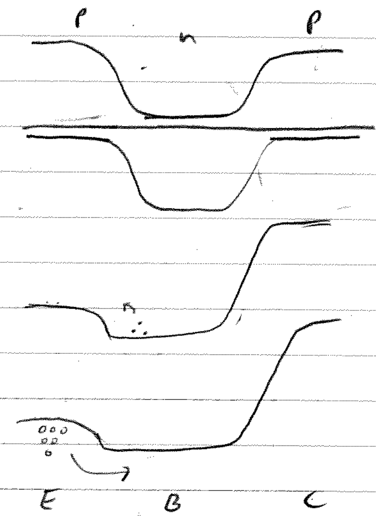


Graded Base:

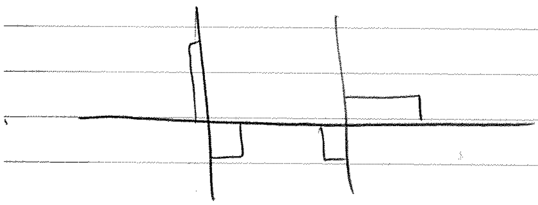


← assists minority carrier transport across quasineutral region of the base (W)

⇒ increases high frequency response, speeds up transient, reduces recombination in base.



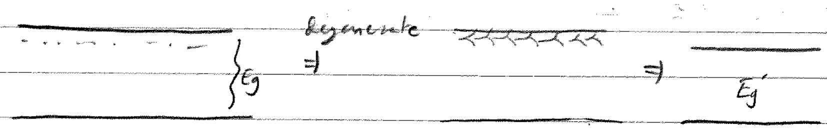
Kirk Effect (Base Expansion): ???

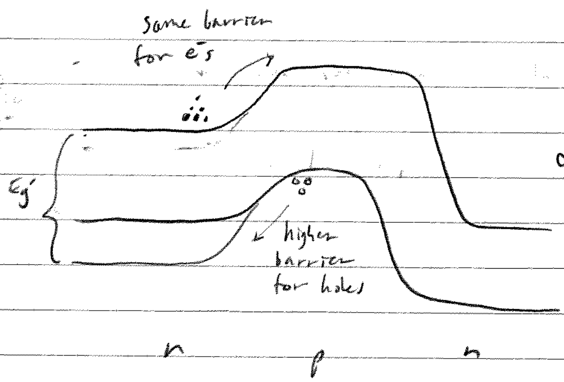


if Emitter is degenerately doped, $\gamma = \frac{I_e}{I_h} \downarrow$ (emitter efficiency is reduced).

↓

⇒ bandgap is reduced. ⇒ e^- current is reduced.





$$\alpha_{DC} = \alpha_T \cdot \gamma \qquad \beta = \frac{\alpha_{DC}}{1-\alpha_{DC}} = \frac{I_c}{I_b}$$

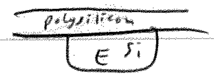
$$\alpha_T = \frac{I_{Cn}}{I_{En}} \qquad \gamma = \frac{I_{E_n}}{I_{E_n} + I_{E_p}} \quad \text{for npn}$$

$$\alpha_T = \frac{I_{C_p}}{I_{E_p}} \qquad \gamma = \frac{I_{E_p}}{I_{E_p} + I_{E_p}} \quad \text{for pnp}$$

Heterojunction Bipolar Transistors (HBT)

- using a base material with smaller E_g than the emitter increases emitter efficiency and allows higher doping of the base.
  ↑ reduces parasitic resistance and early effect.
- Has best high frequency performance
- semiconductor materials must be lattice-matched to minimize defects at the interface (e.g. AlGaAs/GaAs)

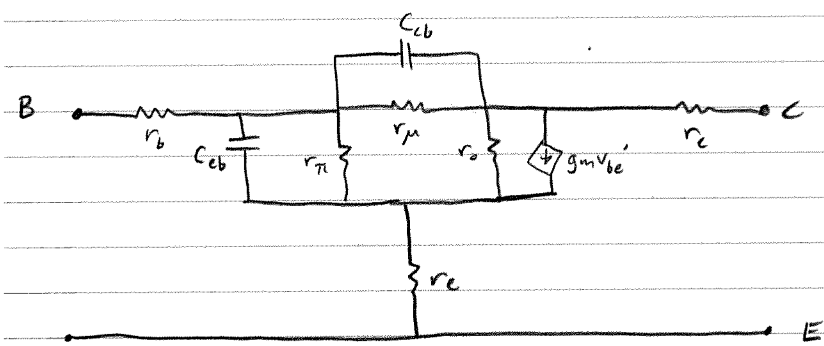
- Polysilicon emitter BJT.



Increases emitter efficiency $\gamma \uparrow$

Ch. 12 - BJT Dynamic Response Modeling.

Small signal model (Hybrid Pi)

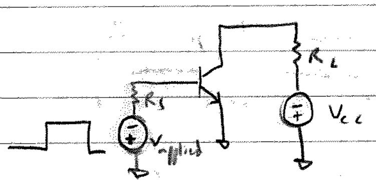


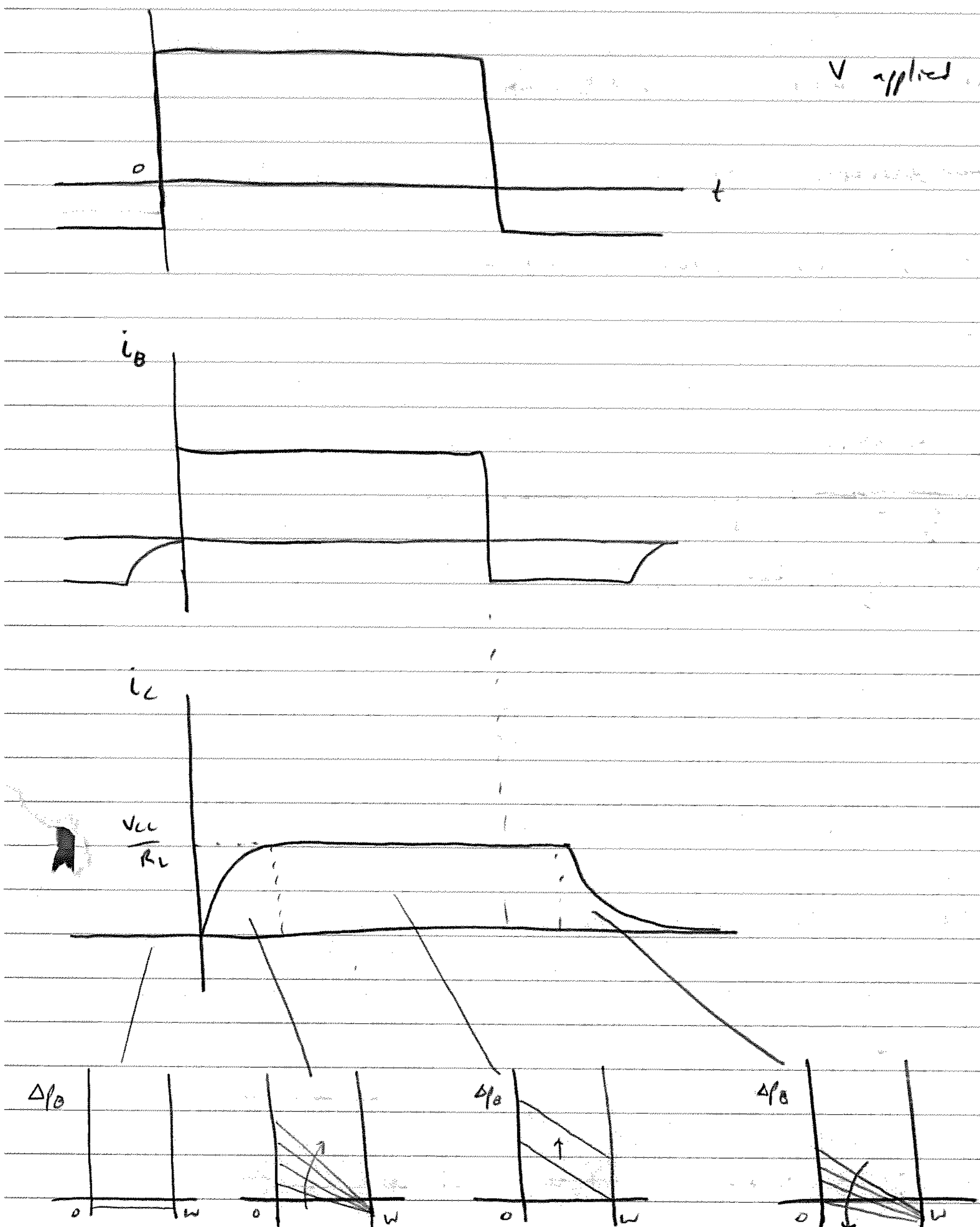
$$g_m = \frac{I_c}{V_T} \qquad \left(V_T = \frac{kT}{q} \right)$$

$$r_\pi = \left(\frac{I_B}{V_T} \right)^{-1} = \frac{\beta_{dc}}{g_m}$$

How to increase speed: $W \downarrow$, $N_B \uparrow$ ($\Rightarrow r_b \downarrow$),
reduce size of PN junctions $\Rightarrow$ $C \downarrow$

Transient (switching) Response (Between Sat and Cutoff)

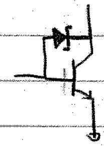




To speed up transient Turn ON: Place cap in parallel with R_S. The discharge of this cap provides a pulse of current that accelerates the build up of stored charge and reduces the time to saturation.

To speed up transient Turn OFF:

1) introduce trap states (Au, for example) to increase recombination rate of minority carriers.



2) use schottky diode clamped npn transistor

clamps C-B junction to a low positive voltage.

⇒ limits number of minority carriers in base during on state. Also, schottky diode itself has very little minority carrier charge storage.

11.3.1 MOS vs. BJT for switching.

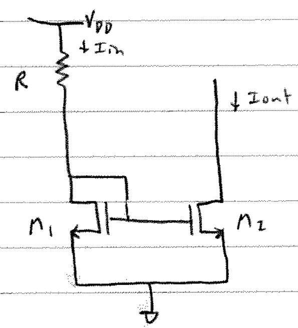
CMOS - Low power dissipation, high packing density

But BJT - can supply several times the drive current of an equal size MOS transistor

$\Rightarrow$ faster switching of large capacitive loads.

BiCMOS - used when you need BJTs for high-load, high-speed portions of predominantly CMOS circuitry.

Circuits - Alex L14 - L15 Supply insensitive bias current generation
voltage biasing considerations.

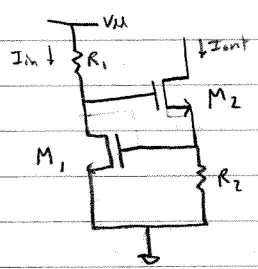


$$I_{out} \approx \frac{V_{DD} - V_t - V_{ov}}{R} \quad \overset{\text{almost}}{\cancel{\propto}}\ V_{DD}$$

$$\Rightarrow \quad S_{V_{DD}}^{I_{out}} \approx 1$$

Sensitive to V_{DD} variations.

V_t - referenced bias:



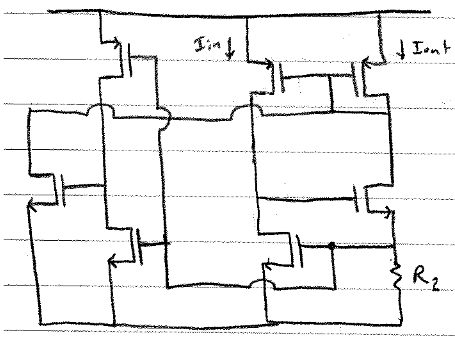
$$I_{out} = \frac{V_{gs_1}}{R_2} = \frac{V_t + V_{ov}}{R_2} = \frac{V_t + \sqrt{\frac{2 I_{in}}{\mu C_{ox} \left(\frac{W}{L}\right)_1}}}{R_2}$$

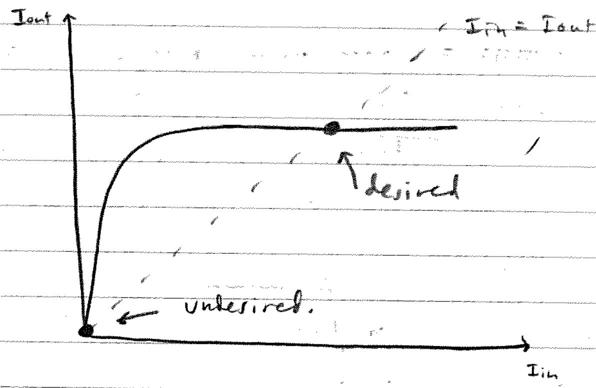
by choosing a large $\left(\frac{W}{L}\right)_1$, we get $V_{ov_1} < V_t$

$$\Rightarrow \quad I_{out} \approx \frac{V_t}{R_2}$$

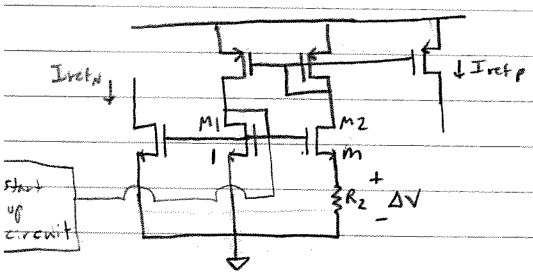
$$S_{V_{DD}}^{I_{out}} = S_{V_{DD}}^{I_{in}} \cdot S_{I_{in}}^{I_{out}} \approx 1 \cdot \frac{I_{in}}{I_{out}} \cdot \frac{\partial I_{out}}{\partial I_{in}} = \frac{1}{2} \frac{V_{ov_1}}{V_t + V_{ov_1}} \approx 11.5\% \quad \text{for} \quad V_{ov} = 0.15\,V,\ V_t = 0.5\,V$$

Self Biasing / Start-up Circuits: 2 stable operating points.





ΔV_{GS} Reference (constant g_m reference)



$$\Delta V = I_{ref} \cdot R_2 = V_{gs_1} - V_{gs_2} = V_{ov_1} - V_{ov_2}$$

$$= V_{ov_1}\left(1 - \frac{1}{\sqrt{m}}\right)$$

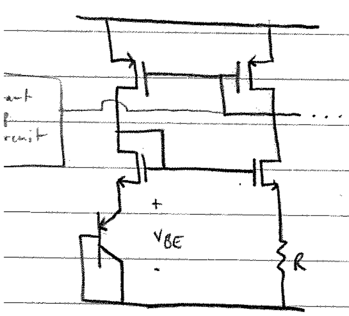
$$\Rightarrow I_{ref} \approx \frac{V_{ov_1}\left(1 - \frac{1}{\sqrt{m}}\right)}{R_2}$$

Why is this useful? $g_{m_1} = \frac{2 I_{ref}}{V_{ov_1}} \Rightarrow g_{m_1} = \frac{2\left(1 - \frac{1}{\sqrt{m}}\right)}{R_2}$

$\Rightarrow$ g_m independent of T and Vdd variations.

- use large V_{ov} to reduce mismatch errors.
- minimize $I \cdot R_2$ to reduce sensitivity to backgate effect.

V_{BE} Reference



$$I_{out} = \frac{|V_{BE}|}{R}$$

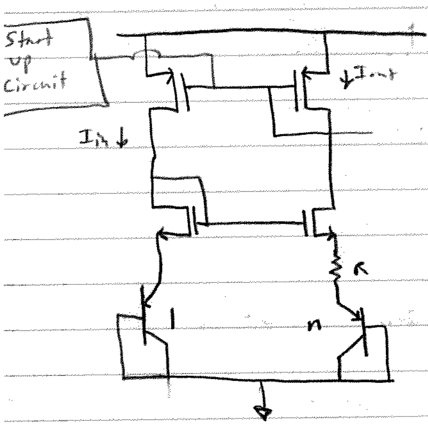
$$TC_F = \frac{\left(\frac{\partial I_{out}}{\partial T}\right)}{I_{out}} \qquad \frac{\%}{K} \qquad \frac{\partial V_{BE}}{\partial T} \approx -2mV/K$$

same as °C

$$\approx -0.53\% / K$$

not very good (~53% at $\Delta T = 100K$)

ΔV_{BE} Reference

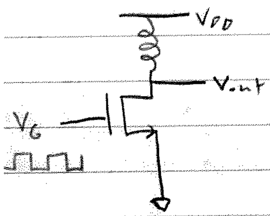


$I_{out} = \frac{1}{R} \frac{kT}{q} \ln(n)$

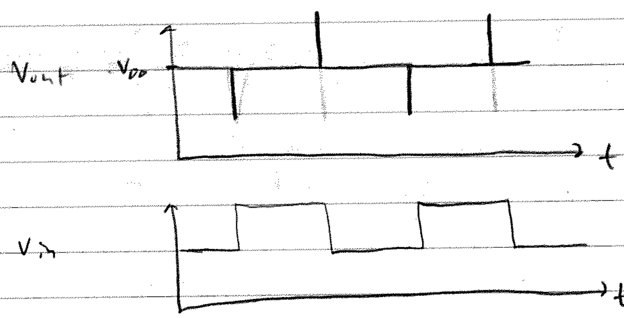
$TC_F \approx 0.13\% / K$ ← a little better.

TC of resistor and $\frac{kT}{q}$ partially cancel.

Questions - Quals Kovacs 2006

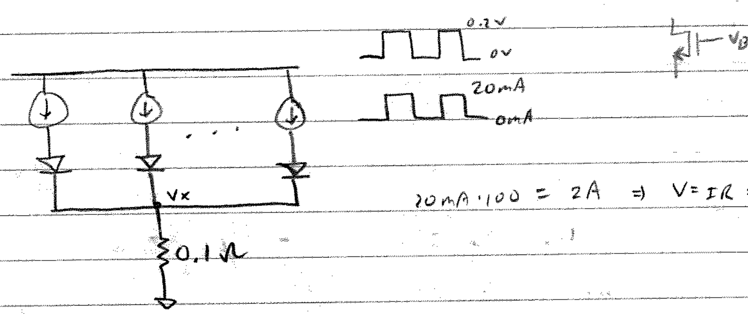


$V = L \frac{\partial I}{\partial t}$

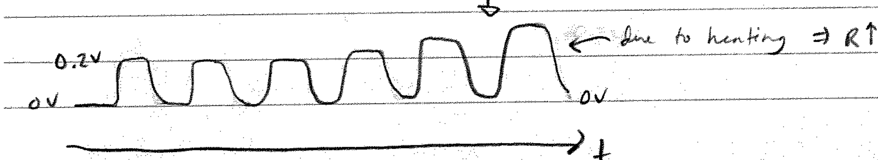


$\frac{V_{out}}{V_{in}} = -g_m s L$ ⇒ differentiator.

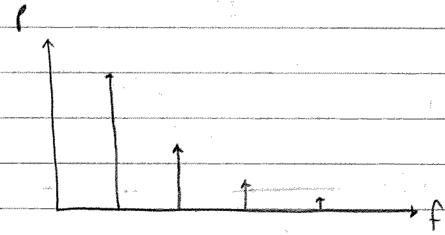
Kovacs - 2005



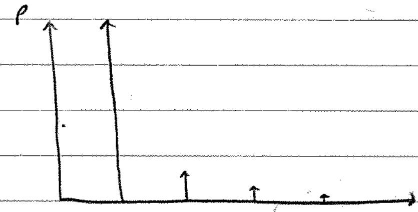
$20mA \cdot 100 = 2A \Rightarrow V = IR = 0.2V$



power spectrum: initial:



after 1 minute:



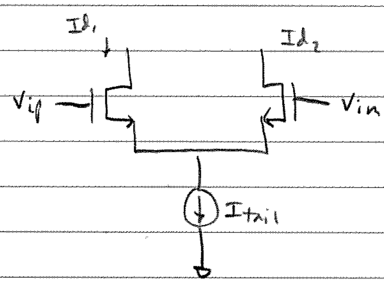
more total power.

but perhaps slightly less square, due to increased RC time constant

⇒ more lowpass filtering.

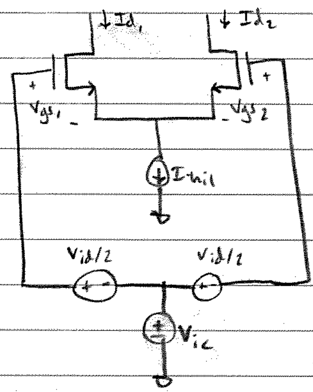
Circuits: [redacted] L16 - L18 differential pair + multi-stage amps.

L16 - Differential Pair



$V_{id} = V_{ip} - V_{im}$

$V_{ic} = \frac{V_{ip} + V_{im}}{2}$



$$V_{ip} - V_{gs_1} = V_{im} - V_{gs_2}$$

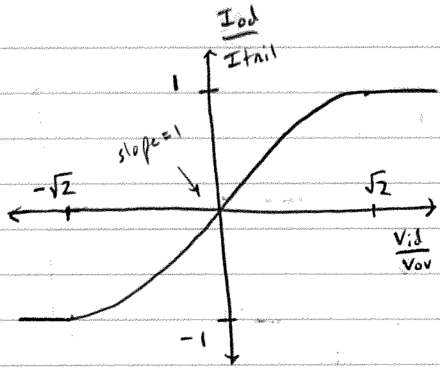
$$I_{d_1} + I_{d_2} = I_{tail}$$

$$V_{gs_1} = V_t + \sqrt{\frac{2 I_{d_1}}{\mu C_{ox} \frac{W}{L}}} \qquad V_{gs_2} = V_t + \sqrt{\frac{2 I_{d_2}}{\mu C_{ox} \frac{W}{L}}}$$

$$I_{od} = I_{d_1} - I_{d_2} = \frac{1}{2} \mu C_{ox} \frac{W}{L} V_{id} \sqrt{\frac{4 I_{tail}}{\mu C_{ox} \frac{W}{L}} - V_{id}^2}$$

for $V_{id} = 0$, $\frac{I_{tail}}{2} = \frac{1}{2} \mu C_{ox} \frac{W}{L} V_{ov}^2$

$$\Rightarrow \frac{I_{od}}{I_{tail}} = \frac{V_{id}}{V_{ov}} \sqrt{1 - \left(\frac{V_{id}}{2 V_{ov}}\right)^2}$$

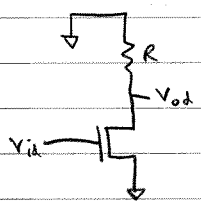


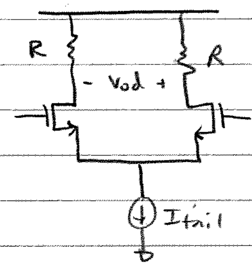
$$G_m = \frac{dI_{od}}{dV_{id}} = \frac{I_{tail}}{V_{ov}}$$

$$g_{m_1} = g_{m_2} = \frac{I_{tail}}{V_{ov}}$$

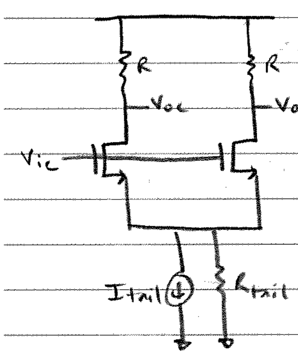
$$V_x = V_{ic} - V_t - V_{ov}\sqrt{1 - \frac{1}{4}\left(\frac{V_{id}}{V_{ov}}\right)^2} \quad \Rightarrow V_x \text{ is an AC ground}$$

Small Signal Half Circuit:

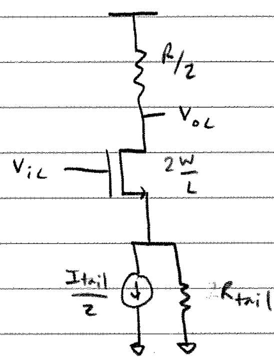




Common Mode Gain



$\Rightarrow$



$$A_{cm} = \frac{V_{oc}}{V_{ic}}$$

$$\Rightarrow A_{cm} \approx -\frac{2g_m}{1+2g_m R_{tail}}\left(\frac{R}{2} \| r_o[1+g_m 2R_{tail}]\right)$$

$$\approx -\frac{g_m R}{1+2g_m R_{tail}}$$

$CMRR = \left|\frac{A_{dm}}{A_{cm}}\right|$ ideally ∞. $\approx 1+2g_m R_{tail}$ for this circuit.

(Alternate definitions include $CMRR = \left|\frac{A_{DM}}{A_{CM-DM}}\right| = \frac{g_m}{\Delta g_m}(1+2g_m R_{tail})$ for a circuit with Δg_m mismatch between M1 + M2

Power Supply Rejection Ratio: $A_+ = \frac{v_{od}}{v_{dd}}$ $A_- = \frac{v_{od}}{v_{ss}}$

$PSRR_+ = \left|\frac{A_{dm}}{A_+}\right|$ $PSRR_- = \left|\frac{A_{dm}}{A_-}\right|$

$\mu C_{ox} \frac{W}{L} = 4 mA/V^2$ $V_t = 1V$

$\frac{1}{2} \mu C_{ox} \frac{W}{L} (V_{gs} - V_t)^2 = 0.5 mA$

$V_{gs} =$

$2 - 1.5 = 0.5$

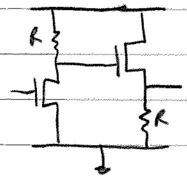
$\downarrow 1mA$

$1 + \sqrt{0.5}$ $1 + \frac{\sqrt{2}}{2}$

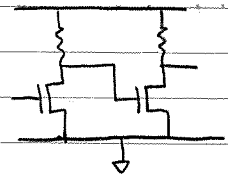
$2 - 1.707$ 2.707

0.3

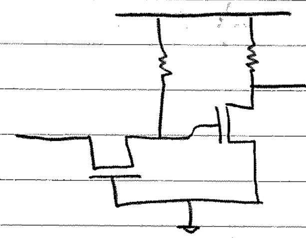
L17 - Multi-Stage Amplifiers (Single Ended)



CS - CD



CS - CS



CG - CS

CS - CD	CS - CS	CG - CS
High R_{in}	High R_{in}	Low R_{in}
Low R_{out}	High R_{out}	High R_{out}
Medium A_v	High A_v	I to V gain (transresistance)
	(Limits for Cascading)	

capacitance at source.

CG Transfer Function: $\frac{i_o}{i_i} \approx \frac{1}{1 + s\frac{C_S}{g_m'}}$ for $g_m' R_s >> 1$

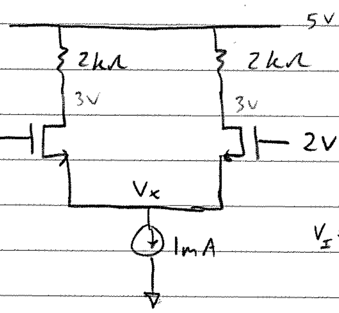
$\frac{v_o}{v_i} \approx g_m' R_D$ $R_i = \frac{R_D}{g_m' r_o} + \frac{1}{g_m'}$ $R_o = R_D \| (1 + g_m' R_S) r_o$

L18 Multi-Stage Amplifiers (Differential)

$BW_{cascade} = BW_{stage} \sqrt{2^{1/N} - 1}$ ← Bandwidth Shrinkage

Quals Questions

Murmann 2010



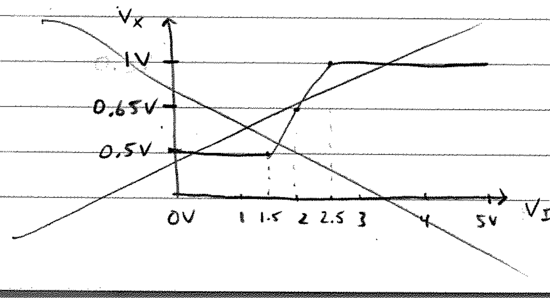
$\mu C_{ox} \frac{W}{L} = 4mA/V^2$

$V_t = 1V$

sketch V_x vs. V_I as V_I ranges from 0-5V

$V_I = 0$: $4mA/V^2 \cdot (V_{GS} - 1)^2 = 1$

$V_{GS} = \sqrt{\frac{1}{4}} + 1 = 1.5V$



$V_I = 2$: $V_{GS} = \sqrt{\frac{0.5}{4}} + 1 \approx 1.35V$

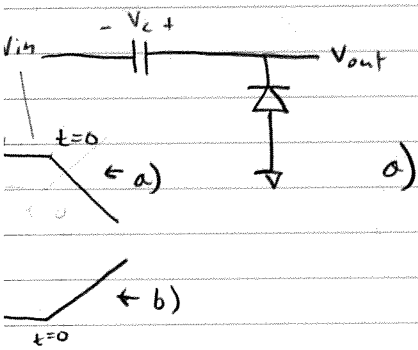
$V_{ov} = \frac{1}{2\sqrt{2}}$

$\Rightarrow \sqrt{2}\, V_{ov} = \frac{1}{2}$

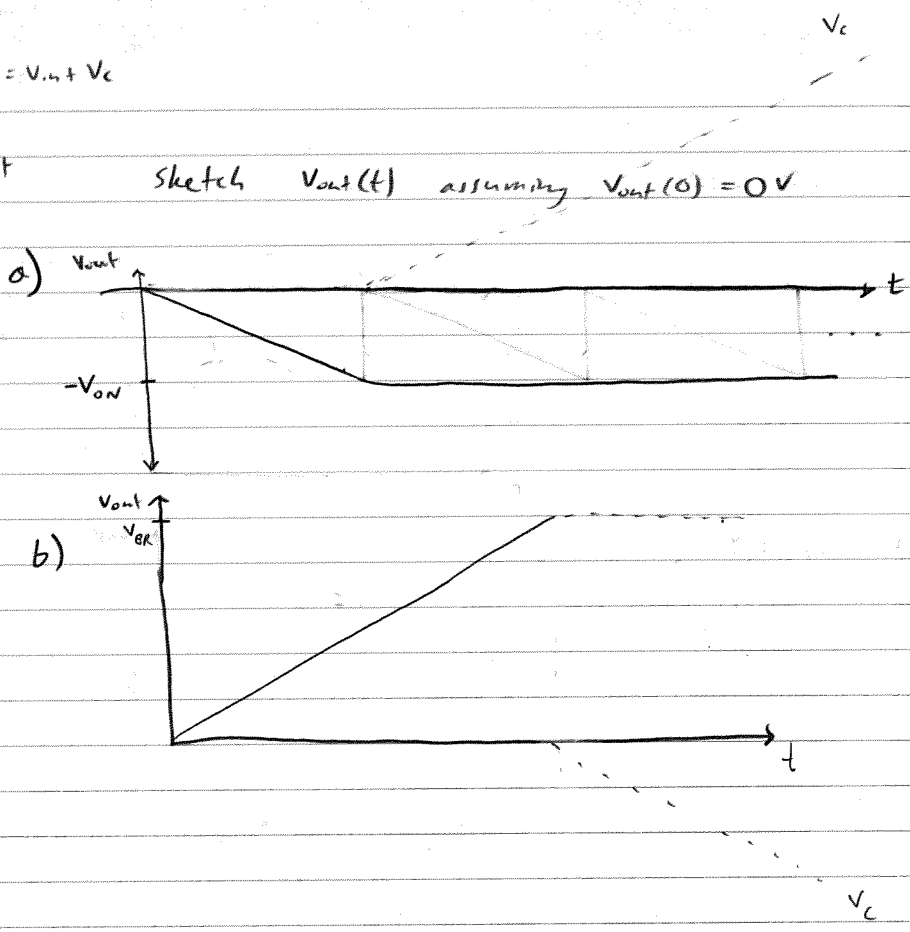
$V_I = 2.5$: $V_{GS} = \sqrt{\frac{1}{4} + 1} = 1.5V$

$\Rightarrow V_x = 2.5 - 1.5 = 1V$

$V_{out} = V_{in} + V_c$

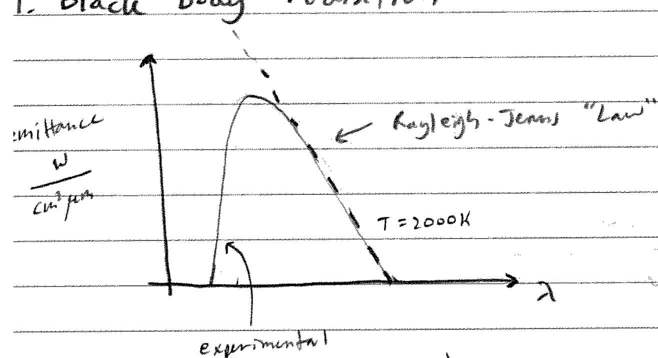


Sketch $V_{out}(t)$ assuming $V_{out}(0) = 0\,V$



Devices - [redacted] - Quantum Mechanics (Ch. 2 Pierret Advanced.)

1. Black body radiation



Planck:

$$E_n = nh\nu = n\hbar\omega, \quad n = 0, 1, 2, \ldots$$

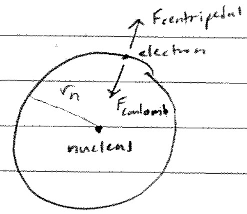
Motivated by discrete spectral emission lines of heated gases. (H, He, etc).

2. Bohr Atom

$$\boxed{L_n = m_o v r_n = n\hbar}$$

L_n ← angular momentum; v ← electron velocity

$n = 1, 2, 3 \ldots$



$F_{centripetal} = F_{coulomb}$

$$\frac{m_o v^2}{r_n} = \frac{q^2}{4\pi\varepsilon_o r_n^2} \Rightarrow \boxed{r_n = \frac{4\pi\varepsilon_o (n\hbar)^2}{m_o q^2}}$$

$$E_n = PE + KE = -\frac{q^2}{4\pi\varepsilon_o r_n} + \frac{1}{2}\frac{q^2}{4\pi\varepsilon_o r_n} = -\frac{1}{2}\frac{q^2}{4\pi\varepsilon_o r_n}$$

$$\Rightarrow \boxed{E_n = -\frac{m_o q^4}{2(4\pi\varepsilon_o n\hbar)^2} = -\frac{13.6}{n^2}\ eV}$$ ← applies for Hydrogen atom.

3. Wave-Particle Duality

deBroglie:

$$p = mc = \frac{E}{c^2}\cdot c = \frac{E}{c} = \frac{h\overset{\text{freq.}}{\nu}}{c} = \frac{h\nu}{\lambda\nu} = \frac{h}{\lambda}$$

$$\Rightarrow \boxed{p = \frac{h}{\lambda}}$$

4. a) Wave mechanics → schroedinger
   b) Matrix mechanics → heisenberg

} quantum mechanics.

5. 5 postulates of quantum Mechanics: (time dependent)

① $\psi = \psi(x, y, z, t)$

② $-\frac{\hbar^2}{2m} \nabla^2 \psi + U\psi = -\frac{\hbar}{i} \frac{\partial \psi}{\partial t}$

time dependent schroedinger equation

③ ψ, $\nabla \psi$ must be finite (bounded), continuous, and single valued for all x, y, z, t.

④ $\psi^* \psi \, dV$ is probability a particle will be found in dV.

$\Rightarrow \iiint_V \psi^* \psi \, dV = 1$

⑤ $\underset{\text{variable}}{\underset{\uparrow}{\langle a \rangle}} = \int_V \psi^* \underset{\text{operator}}{\underset{\uparrow}{a_{op}}} \psi \, dV$ must use the lookup table to find the operator for a given a.

	a		a_{op}
e.g.	x, y, z	$\longrightarrow$	x, y, z
	$f(x,y,z)$	$\longrightarrow$	$f(x,y,z)$
	p_x	$\longrightarrow$	$\frac{\hbar}{i}\frac{\partial}{\partial x}$
	E	$\longrightarrow$	$\frac{\hbar}{i}\frac{\partial}{\partial t}$

momentum in x direction.

Time Independent: particle has constant energy.

$$\boxed{\psi(x, y, z, t) = \psi(x, y, z)\, e^{-iEt/\hbar}}$$

$$\boxed{\nabla^2 \psi + \frac{2m}{\hbar^2}[E - U]\psi = 0}$$

1) Free Particle.

$F = 0$, $U = \text{constant}$, $(F = \nabla U)$ 1 dimension.

$\downarrow$ set $U = 0$

continuous $\Rightarrow$ all wavelengths can exist. E-k plot is continuous.

$$\frac{d^2\psi}{dx^2} + \frac{2mE}{\hbar^2}\psi = 0$$

$$\Rightarrow \psi = A_+ e^{ikx} + A_- e^{-ikx}$$

$$\boxed{k = \sqrt{\frac{2mE}{\hbar^2}} = \frac{2\pi}{\lambda}}$$

$$\Rightarrow \psi(x,y,z,t) = \left(A_+ e^{ikx} + A_- e^{-ikx}\right) e^{-iEt/\hbar}$$

$$\boxed{\psi(x,y,z,t) = \underbrace{A_+ e^{i(kx - Et/\hbar)}}_{\text{wave } +x} + \underbrace{A_- e^{-i(kx - Et/\hbar)}}_{-x \text{ wave}}}$$

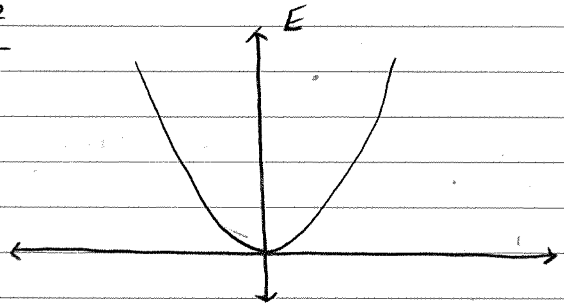
$$\langle p \rangle = \langle p_x \rangle = \int_{-\infty}^{\infty} \psi^* \frac{\hbar}{i}\frac{d\psi}{\cancel{dx}}\cancel{dx} = \int_{-\infty}^{\infty} \psi^* \frac{\hbar}{i} d\psi$$

$$= \hbar k \int_{-\infty}^{\infty} \psi^* \psi \, dx = \hbar k \quad \Rightarrow \quad \langle p \rangle = \hbar k$$

$$E = \frac{\hbar^2 k^2}{2m} = \frac{\langle p \rangle^2}{2m}$$

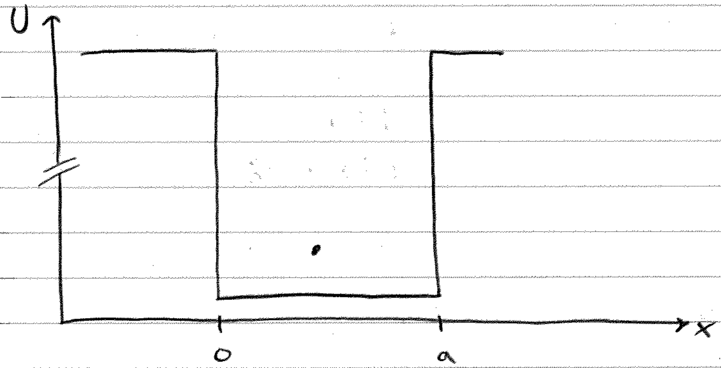
quadratic.

E-k diagram.



$\Rightarrow$ Energy is continuous (not quantized) for an isolated free particle.

2) Particle in an infinite potential well.



m

E

$0 < x < a$

$U(x) = \text{constant} = 0$

$\psi = A\sin(kx) + B\cos(kx)$

$\psi(0) = \psi(a) = 0$ (boundary conditions)

$\Rightarrow B = 0$ due to

$\psi = A\sin(ka) = 0 \Rightarrow \boxed{k = \frac{n\pi}{a}} = \frac{2\pi}{\lambda}$ $n = \pm1, \pm2, \pm3, \ldots$

only discrete wavelengths that depend on the dimensions of the well can exists

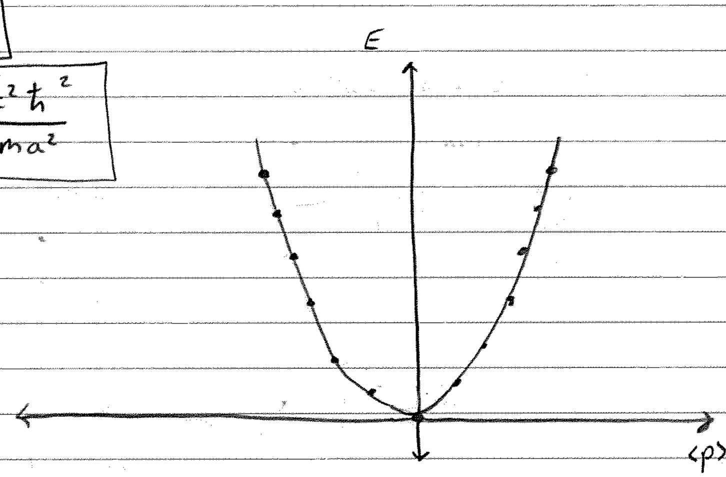
But since the well is infinitely deep, an infinite number of different wavelengths can exist. E-k plot is discrete.

$\Rightarrow \boxed{\psi_n(x) = A_n \sin\left(\frac{n\pi x}{a}\right)}$

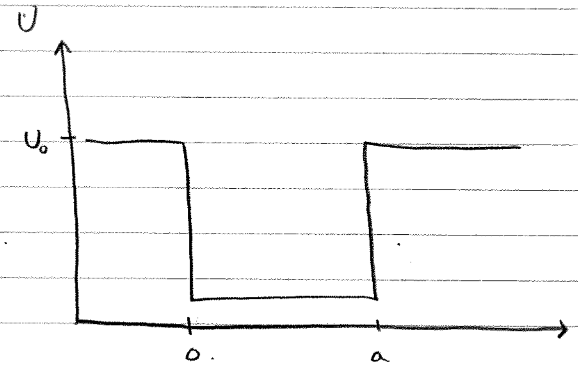
$E = \frac{\hbar^2 k^2}{2m} \Rightarrow \boxed{E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}}$

E is quantized for a particle in a box.

(but follows the same exact parabolic shape as for a free particle)



3) Particle in a finite potential well



m

E

U = 0

$0 < E < U_0$

$0 < x < a$: $\frac{d^2\psi}{dx^2} + k^2\psi_0 = 0 \longrightarrow \psi = A_0 \sin(kx) + B_0 \cos(kx)$

$k = \sqrt{\frac{2mE}{\hbar^2}}$

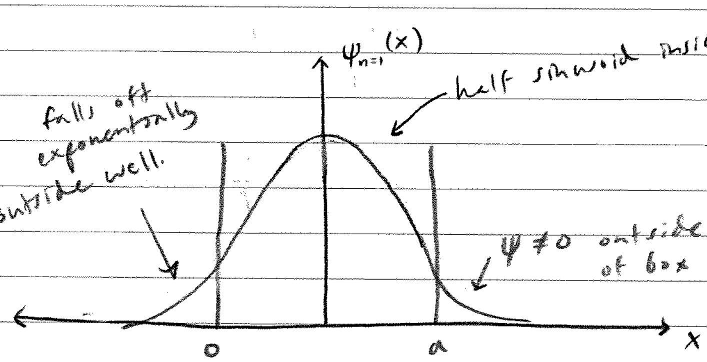
$x < 0$, $x > a$: $\frac{d^2\psi}{dx^2} - \alpha^2\psi = 0 \longrightarrow \psi = A_- e^{\alpha x} + B_- e^{-\alpha x}$

$\alpha = \sqrt{2m(U_0 - E)/\hbar^2}$

$\psi_-(-\infty) = 0$; $\psi_+(+\infty) = 0$

$\psi_-(0) = \psi(0)$; $\psi_+(a) = \psi(a)$

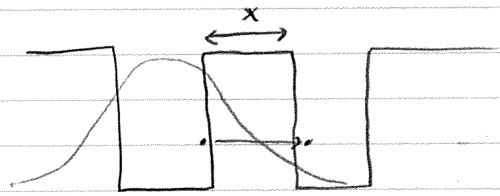
$\frac{d\psi_-}{dx}\Big|_0 = \frac{d\psi}{dx}\Big|_0 \qquad \frac{d\psi}{dx}\Big|_a = \frac{d\psi_+}{dx}\Big|_a$ Solve.



$\Rightarrow |\psi|^2 \neq$ outside box.

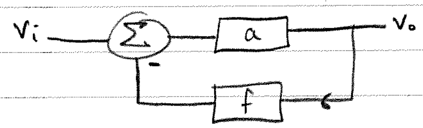
$\Rightarrow$ there is a finite probability of finding the particle outside of the well.

This explains tunneling:



if x is small, there is significant probability of tunneling.

Circuits - [redacted] L19-21 Feedback



$$v_o = a(v_i - f v_o)$$

$$\Rightarrow \frac{v_o}{v_i} = \frac{a}{1+af} = A$$

$a \leftarrow$ Open Loop Gain

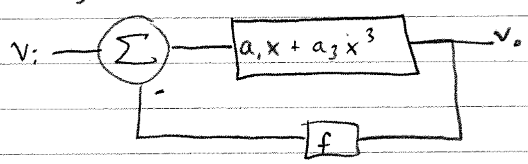
$af \leftarrow$ Loop Gain (T)

$\frac{a}{1+af} \leftarrow$ Closed Loop Gain

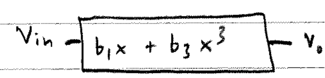
Gain Sensitivity

$$\frac{\Delta A}{A} = \frac{\frac{\Delta a}{a}}{1+af} = \frac{\frac{\Delta a}{a}}{1+T} \Rightarrow \text{gain sensitivity reduced by } 1+T$$

Linearity:



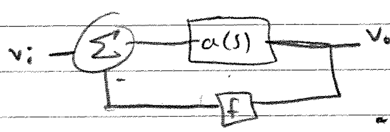
$\Rightarrow$



$$\Rightarrow b_1 = \frac{a_1}{1+a_1 f} \qquad b_3 = \frac{a_3}{(1+a_1 f)^4}$$

$\leftarrow$ cubic nonlinearity reduced by $(1+T)^4$!

Bandwidth:

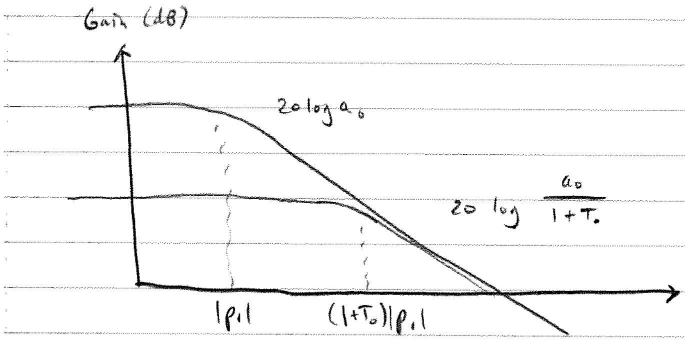


$$a(s) = \frac{a_0}{1 - \frac{s}{p_1}}$$

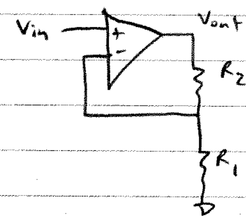
$$A(s) = \frac{a_0}{1+a_0 f} \cdot \frac{1}{1 - \frac{s}{p_1}\left(\frac{1}{1+a_0 f}\right)}$$

$\Rightarrow$ Bandwidth increases by $(1+T)$

Gain decreases by $(1+T)$ GBW Product is constant.



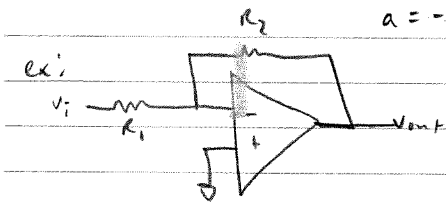
ex: Non-Inverting Opamp.



$$f = \frac{R_1}{R_1+R_2} \qquad a = g$$

$$\frac{V_{out}}{V_{in}} = \frac{a}{1+af} = \frac{g}{1+g\frac{R_1}{R_1+R_2}} \approx \frac{R_1+R_2}{R_1} = 1 + \frac{R_2}{R_1}$$

ex:

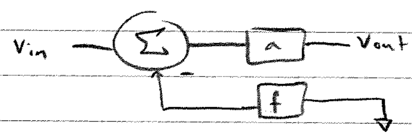


$$a = -\frac{R_2}{R_1+R_2}\,g \qquad af = g\,\frac{R_1}{R_1+R_2} \qquad \frac{a}{1+af} = -\frac{\left(\frac{R_2}{R_1+R_2}\right)g}{1+\frac{R_1}{R_1+R_2}\,g}$$

$$\Rightarrow \lim_{g\to\infty} \frac{a}{1+af} = -\frac{R_2}{R_1} \quad ✓$$

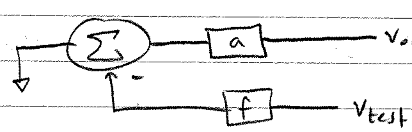
solve with superposition

Find open loop gain



$$\Rightarrow \frac{V_{out}}{V_{in}} = a$$

Find loop gain



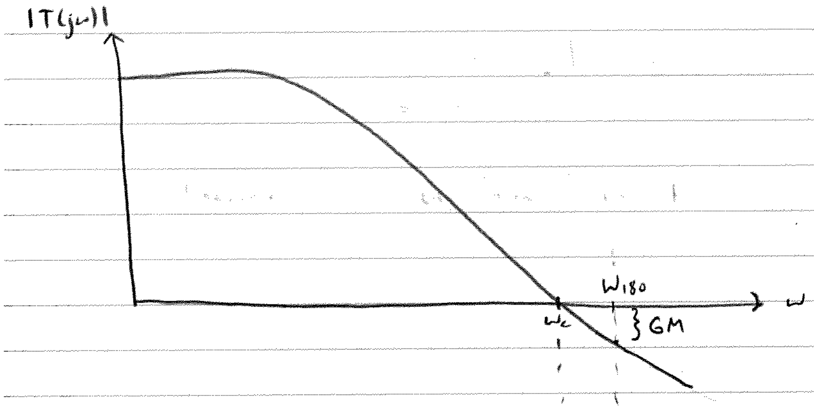
$$\Rightarrow \frac{V_{out}}{V_{test}} = -af$$

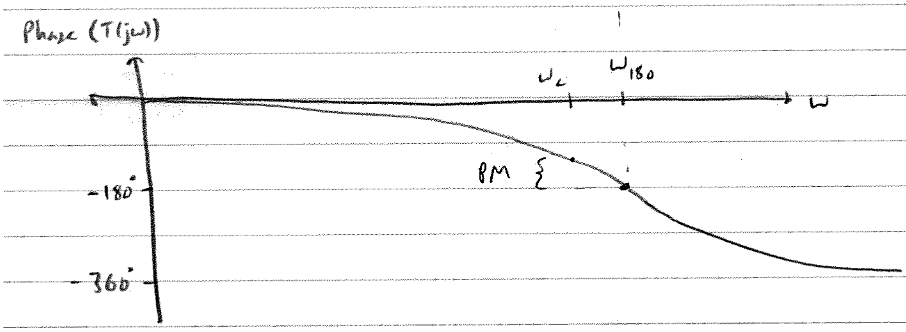
Stability: All poles must be in left half plane.

$$GM = \left.\frac{1}{|T(j\omega)|}\right|_{\omega=\omega_{180^\circ}} \qquad PM = 180^\circ + \text{Phase}[T(j\omega)]_{\omega=\omega_c}$$

Typically want GM ≥ 3...5 $\qquad$ Typically want PM ≥ 60...70°

Bode Criterion: if $|T(j\omega)| > 1$ at ω where $|Phase(T(j\omega))| = -180°$ system is unstable.





Return Ratio Analysis:

1. Set all independent sources to zero.
2. Disconnect the dependent source from the rest of the circuit which introduces a break in the feedback loop.
3. On the side of the break that is not connected to the dependent source, connect an independent test source S_t. of the same sign and type as the dependent source.
4. Find the return signal S_r generated by the dependent source.
5. Return ratio $R = -\frac{S_r}{S_t}$.

A_∞ is transfer function when gain element becomes infinite..

d is transfer function when gain element becomes zero.

$$A = A_\infty \frac{R}{1+R} + \frac{d}{1+R}$$

Blackman's Impedance Formula

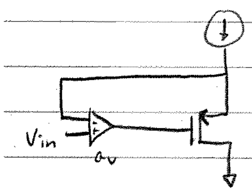
$$Z_{port} = Z_{port}(k=0)\,\frac{1+R(\text{port shorted})}{1+R(\text{port open})}$$

① Find port impedance with feedback loop broken
- e.g. set $g_m = 0$ ($k=0$ in model; can also be a_v, etc).

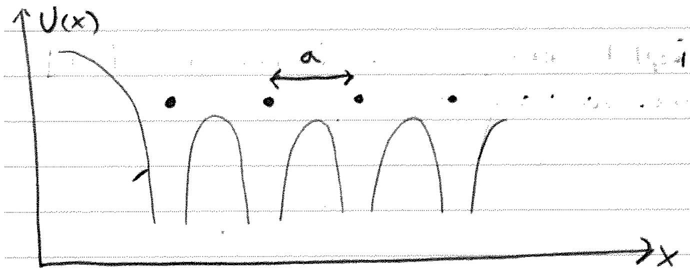
② Calculate loop gain in circuit with port under consideration shorted.

③ Calculate loop gain in circuit with port under consideration open.

ex: Bootstrapped Source Follower



Device - [redacted] Quantum (Pierret Advanced Ch. 3)



if potential is periodic function of x in infinite crystal lattice

Bloch Theorem: $u(x+a) = u(x)$ — unit cell wavefunction with same periodicity as the potential.

$$\psi(x+a) = e^{ika}\psi(x)$$ a is interatomic distance.

$$\psi(x) = e^{ika}u(x)$$

$\psi(\mathbf{r}+\mathbf{a}) = e^{i\mathbf{k}\mathbf{a}}\psi(\mathbf{r})$ $\quad U(\mathbf{r}+\mathbf{a}) = U(\mathbf{r})$

$\psi(\mathbf{r}) = e^{i\mathbf{k}\mathbf{a}}u(\mathbf{r})$ For 3-D

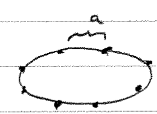
$u(\mathbf{r}+\mathbf{a}) = u(\mathbf{r})$

For 1-D system, 2 (and only 2) values of k exist for each value of E.

for a given E, values of k differing by a multiple of $\frac{2\pi}{a}$ give rise to the same wave function solution

If the periodic potential or crystal is assumed to be infinite in extent then k must be real, and can assume a continuum of values.

—

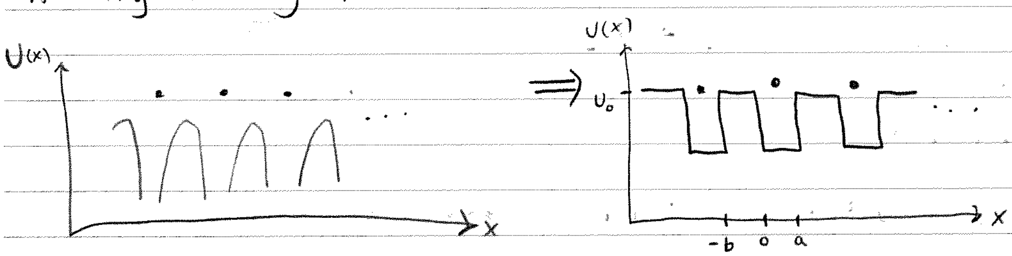


$$\psi(x) = \psi(x+Na) = e^{ikNa}\psi(x)$$

$$\Rightarrow e^{ikNa} = 1 \Rightarrow k = \frac{2\pi n}{Na} \qquad n = 0, \pm 1, \pm 2, \ldots, \pm \frac{N}{2} \qquad -\frac{\pi}{a} \leq k \leq \frac{\pi}{a}$$

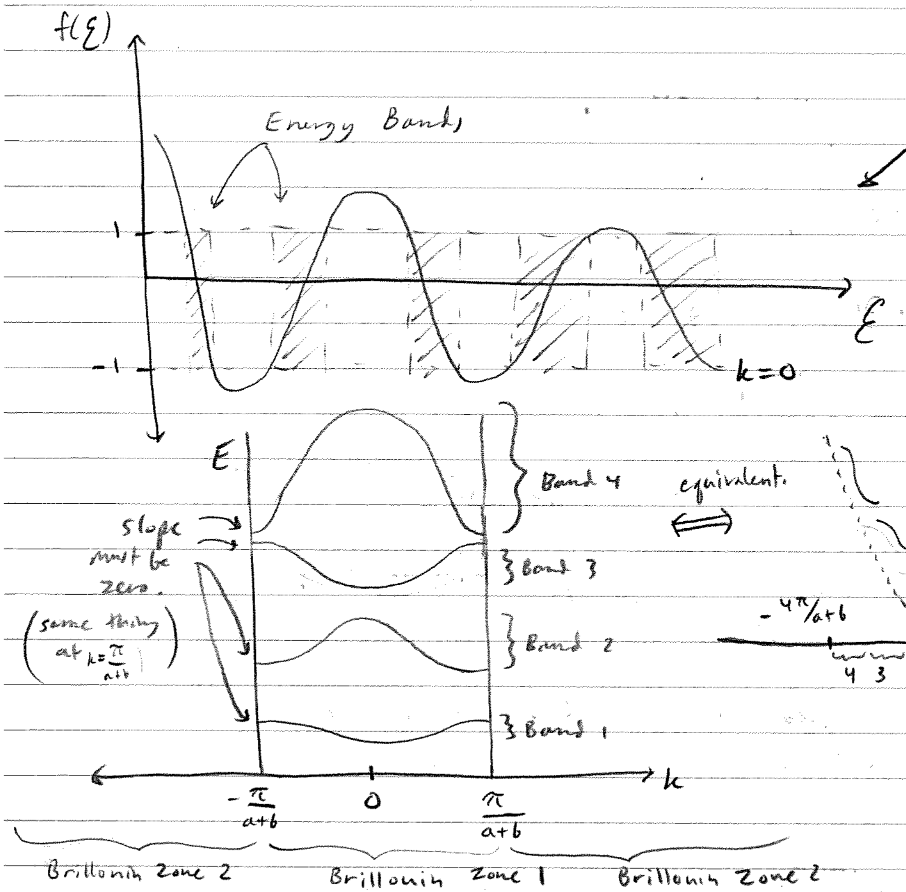
As $N \to \infty$, E-k becomes approximately continuous.

Kronig-Penney Model:

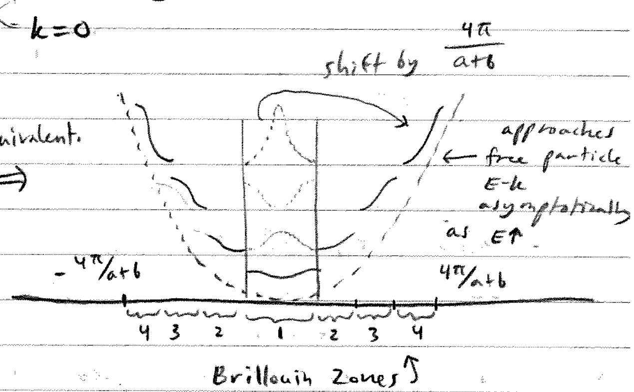


$$\mathcal{E} = \frac{E}{u_0}$$

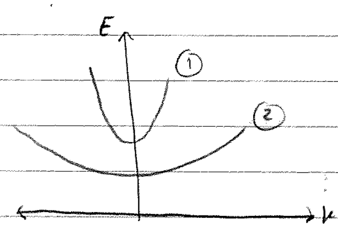
$$f(\mathcal{E}) = \cos(k(a+b)) \Rightarrow$$ acceptable energy levels E correspond to $|f(\mathcal{E})| \le 1$



do this for all possible values of k to generate the E-k plots



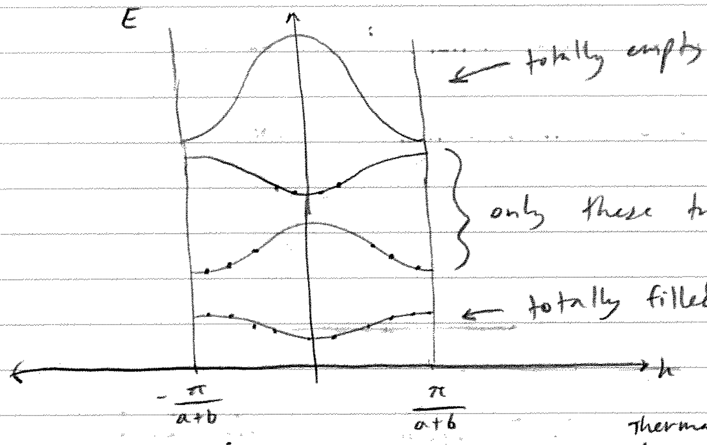
$$v_g = \frac{1}{\hbar}\frac{dE}{dk} \quad \leftarrow \text{group velocity} \qquad m^* = \frac{1}{\hbar^2 \cdot \frac{d^2E}{dk^2}} \quad \leftarrow \text{effective mass.}$$



$$\Rightarrow m_1^* < m_2^*$$

effective mass is positive near bottom of band and negative near top of band.

Charge Transport: Totally filled or empty bands do not contribute to currents



Thermal expansion ⇒ lattice constant ↑ ⇒ weakening of interatomic bonds ⇒ E_G ↓

$$E_G(T) = E_G(0) - \frac{\alpha T^2}{(T+\beta)} \qquad T\uparrow \Rightarrow E_G\downarrow$$

11/23/11

Circuits - Alex 2-Port Feedback Ch. 8 Gray + Meyer

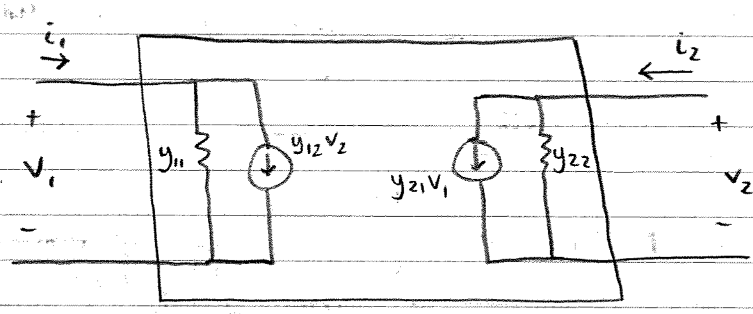
	Shunt	Series
Input	I	V
Output	V	I

Name	Amplifier Type	Z_i'	Z_o'	Feedback Parameter
Series-Shunt	Voltage	$Z_i(1+T)$	$\frac{Z_o}{(1+T)}$	h
Shunt-Series	Current	$\frac{Z_i}{(1+T)}$	$Z_o(1+T)$	g
Shunt-Shunt	Transimpedance	$\frac{Z_i}{(1+T)}$	$\frac{Z_o}{(1+T)}$	y
Series-Series	Transconductance	$Z_i(1+T)$	$Z_o(1+T)$	z

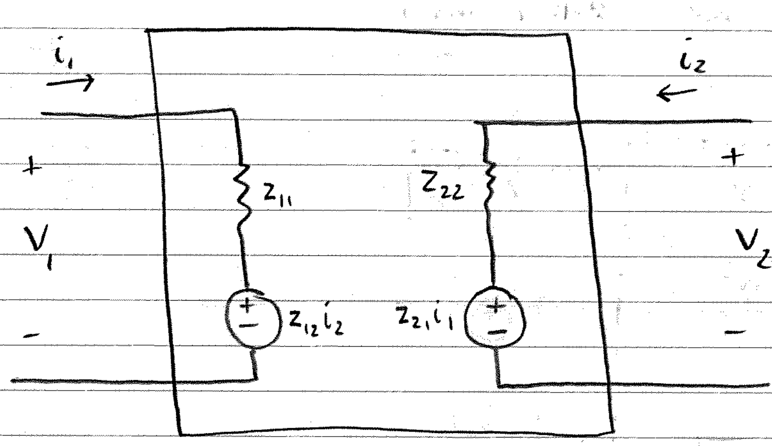
Assumptions for 2-port Model:

- neglect feed forward (i.e. $|y_{21_f}| \ll |y_{21_a}|$)
- neglect feedback through a (i.e. $|y_{12_f}| \gg |y_{12_a}|$)
- add $(y,z,h,g)_{11}$ and $(y,z,h,g)_{22}$ to input and output (loading effects)

Shunt-Shunt:

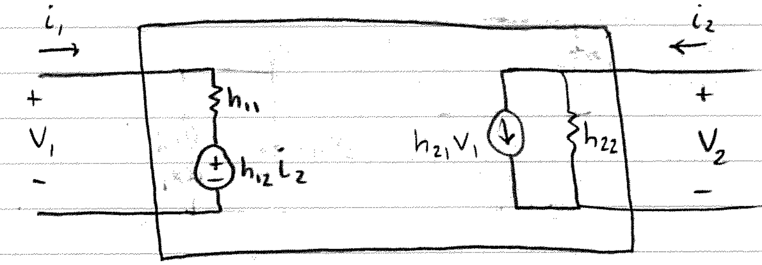


Series-Series:

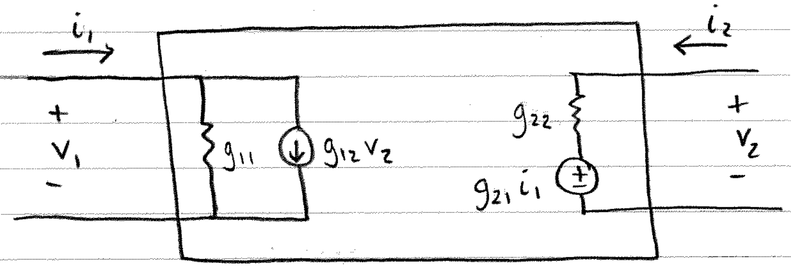


see next page

Series - Shunt :



Shunt - Series :



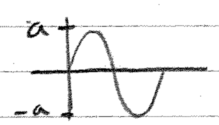
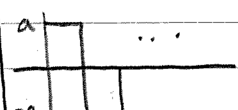
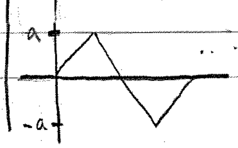
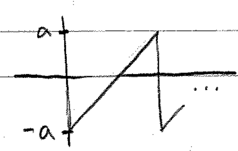
Circuits : MIT 6.002 L1-L3 12/3/11

RMS of a periodic function: $f_{rms} = \sqrt{\frac{1}{T}\int_0^T (f(t))^2 dt}$

e.g. sine wave : $f(t) = a\sin(t)$

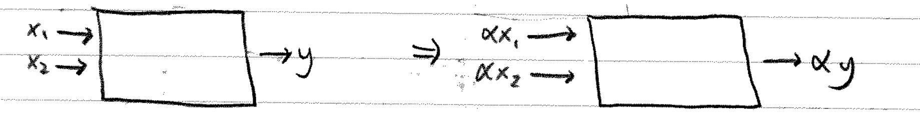
$$f_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} a^2 \sin^2(t)\,dt} = \sqrt{\frac{a^2}{2\pi}\int_0^{2\pi}\frac{1}{2}(1-\cos(2t))\,dt} = \sqrt{\frac{a^2}{4\pi}\int_0^{2\pi} 1-\cos(2t)\,dt}$$

$$= \sqrt{\frac{a^2}{4\pi}\left(t - \tfrac{1}{2}\sin(t)\right]_0^{2\pi})} = \sqrt{\frac{a^2}{4\pi}\left[(2\pi - 0) - (0-0)\right]} = \sqrt{\frac{a^2}{2}} = \frac{a}{\sqrt{2}}$$

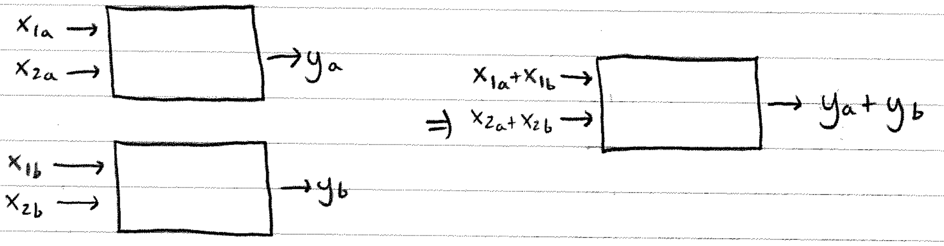
Periodic function		
$a\sin(t)$		$\frac{a}{\sqrt{2}}$
square wave		a
pulse train		$a\sqrt{D}$
triangle or saw	 OR 	$\frac{a}{\sqrt{3}}$

Linearity $\Rightarrow$ Homogeneity, Superposition

Homogeneity:

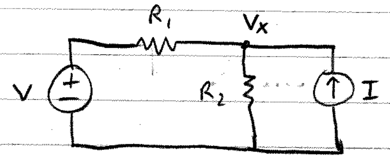


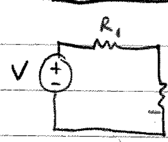
Superposition:



Superposition for circuit analysis: The output of a circuit is determined by summing the responses to each <u>independent</u> source acting alone

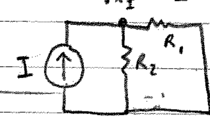
e.g.





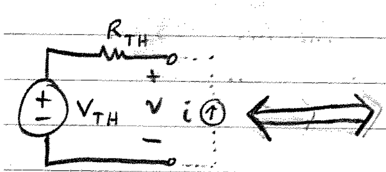
$V_{x_V} = \frac{R_2}{R_1+R_2} V$

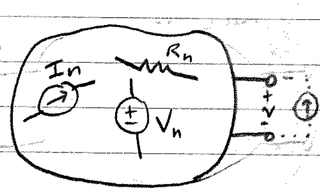
$V_{x_I} = I \frac{R_1 R_2}{R_1+R_2}$



$$\Rightarrow V_x = V_{x_V} + V_{x_I} \Rightarrow V_x = \frac{R_2}{R_1+R_2} V + \frac{R_1 R_2}{R_1+R_2} I$$

Thevenin and Norton Equivalent:





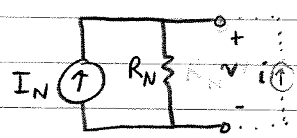
can replace arbitrary (linear) network of R, I, V with a thevenin equivalent:

$v = v_{TH} + i R_{TH}$

V_{TH} = open circuit voltage.

R_{TH} = resistance of network seen from port (Independent sources set to 0)

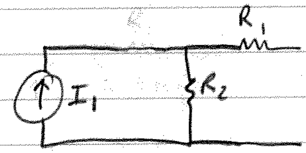
Norton:



$I_N = \frac{V_{TH}}{R_{TH}}$

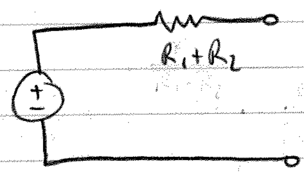
$R_N = R_{TH}$

e.g. find Thevenin equivalent of :

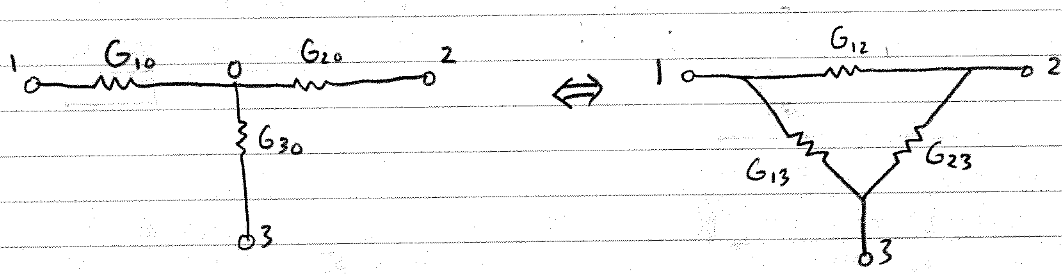


$V_{TH} = V_{oc} = I_1 R_2$ $\qquad R_{TH} = R_1 \| R_2$

$\Rightarrow$ $I_1 R_2$



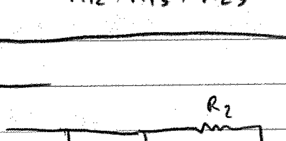
Star to Delta Conversion:



$$\boxed{G_{12} = \frac{G_{10} G_{20}}{G_{10}+G_{20}+G_{30}} \qquad G_{23} = \frac{G_{20}+G_{30}}{G_{10}+G_{20}+G_{30}} \qquad G_{13} = \frac{G_{10} G_{30}}{G_{10}+G_{20}+G_{30}}}$$

Delta to Star Conversion:

$$\boxed{R_{10} = \frac{R_{12} R_{13}}{R_{12}+R_{13}+R_{23}} \qquad R_{20} = \frac{R_{12} R_{23}}{R_{12}+R_{13}+R_{23}} \qquad R_{30} = \frac{R_{13}+R_{23}}{R_{12}+R_{13}+R_{23}}}$$



Find Thevenin + Norton Eq.

by superposition. $V_1 = I\,(R_1 \| R_2)$ $\qquad V_2 = \frac{R_1}{R_1+R_2} V$

$\Rightarrow V_{TH} = V_1 + V_2 = \frac{R_1}{R_1+R_2}\,(V + I R_2)$ $\qquad R_{TH} = R_1 \| R_2$

$\Rightarrow I_N = \frac{V_{TH}}{R_{TH}} = I + \frac{V}{R_2}$

BJT Current Equations: Pierret 11.1.2 - 11.1.4

$$I_E = qA\left[\left(\frac{D_E}{L_E} n_{E_0} + \frac{D_B}{L_B} p_{B_0} \frac{\cosh(W/L_B)}{\sinh(W/L_B)}\right)\left(e^{V_{EB}/V_T} - 1\right) - \left(\frac{D_B}{L_B} p_{B_0} \frac{1}{\sinh(W/L_B)}\right)\left(e^{V_{CB}/V_T} - 1\right)\right]$$

$$I_C = qA\left[\left(\frac{D_B}{L_B} p_{B_0} \frac{1}{\sinh(W/L_B)}\right)\left(e^{V_{EB}/V_T} - 1\right) - \left(\frac{D_C}{L_C} n_{C_0} + \frac{D_B}{L_B} p_{B_0} \frac{\cosh(W/L_B)}{\sinh(W/L_B)}\right)\left(e^{V_{CB}/V_T} - 1\right)\right]$$

$$I_B = I_E - I_C$$

Ebers-Moll Equations + Model:

$$I_{F0} = qA\left(\frac{D_E}{L_E} n_{E0} + \frac{D_B}{L_B} p_{B0} \frac{\cosh(W/L_B)}{\sinh(W/L_B)}\right)$$

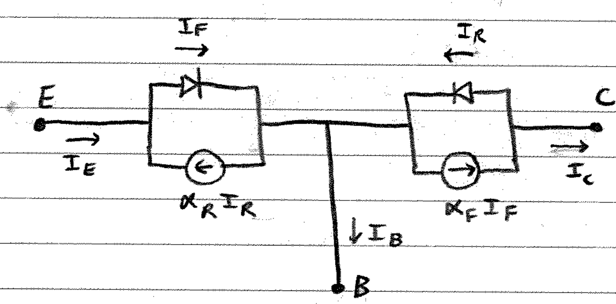
$$I_{R0} = qA\left(\frac{D_C}{L_C} n_{C_0} + \frac{D_B}{L_B} p_{B0} \frac{\cosh(W/L_B)}{\sinh(W/L_B)}\right)$$

$$\alpha_F I_{F0} = \alpha_R I_{R0} = qA \frac{D_B}{L_B} \frac{p_{B0}}{\sinh(W/L_B)}$$ note: $(\alpha_F = \alpha_{dc})$

$$\Rightarrow I_E = \overbrace{I_{F0}\left(e^{V_{EB}/V_T} - 1\right)}^{I_F} - \alpha_R I_{R0}\left(e^{V_{CB}/V_T} - 1\right)$$

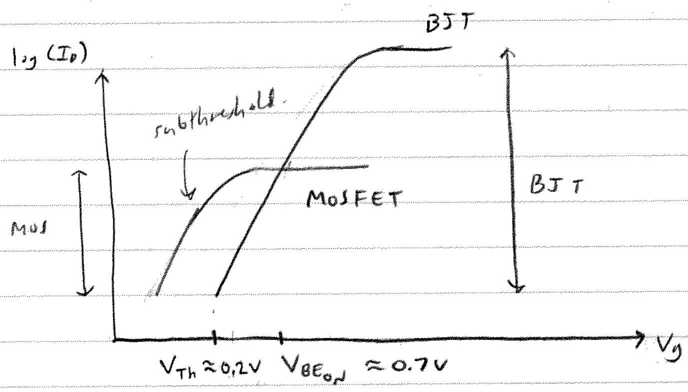
$$I_C = \alpha_F I_{F0}\left(e^{V_{EB}/V_T} - 1\right) - \overbrace{I_{R0}\left(e^{V_{CB}/V_T} - 1\right)}^{I_R}$$

Model:

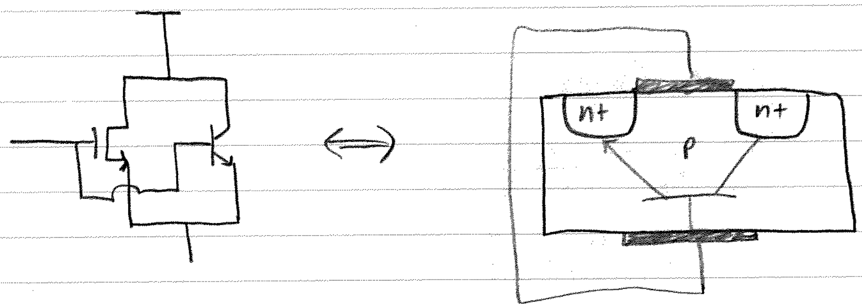


for pnp BJT

Dutton Quals Question.

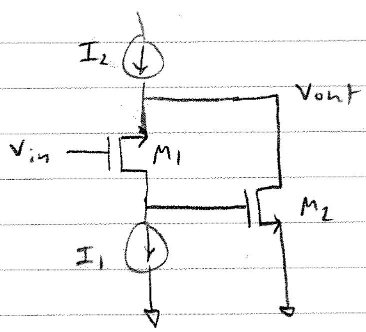


range is higher for BJT. why might this be helpful or important?



last 2 lectures of EE214A.

Return Ratio Example: Blackman's for Super Source Follower

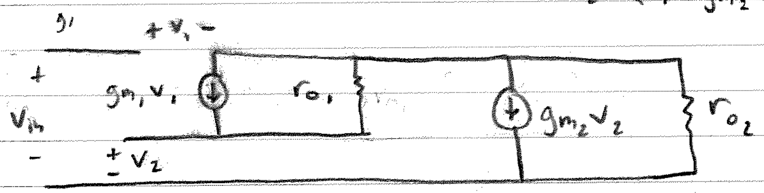


$$R_{out} = R_{out_{(k=0)}} \frac{1 + R(\text{short})}{1 + R(\text{open})}$$

$$R_{out_{k=0}} = R_{out_{(g_{m_2}=0)}} = r_{o2}$$

$$R(\text{short}) = 0 \qquad R(\text{open}) = -\frac{v_r}{v_t} = g_{m_2} r_{o_1} (1 + g_{m_1} r_{o_2})$$

$$\Rightarrow R_{out} = r_{o_2} \left(\frac{1+0}{1 + g_{m_2} r_{o_1} (1 + g_{m_1} r_{o_2})} \right) \approx \boxed{\frac{1}{g_{m_2} g_{m_1} r_{o_1}}}$$



can also use $\frac{1}{\beta g_{m_2}}$ trick where $\beta = g_{m_1} r_{o_1}$

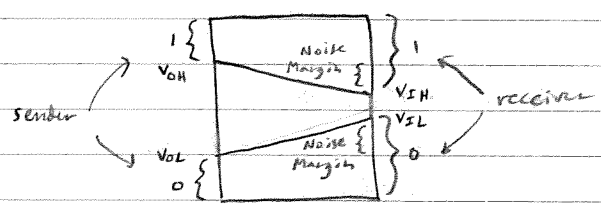
$$\Rightarrow R_{out} = \frac{1}{g_{m_2} g_{m_1} r_{o_1}}$$

Circuits

MIT - 6.002 L4 - 12/5/11

4. The Digital Abstraction

Digital: better noise immunity (lots of noise margin)

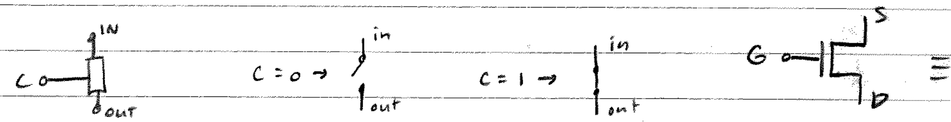


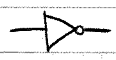
hold

Static Discipline: If inputs to the digital system meet valid input thresholds, then the system guarantees its outputs will meet valid output thresholds.

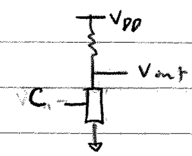
5. Inside The Digital Gate

Switch device:

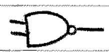




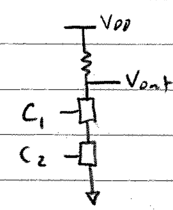
Inverter:



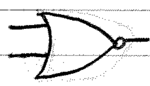
$V_{out} = \bar{C}$



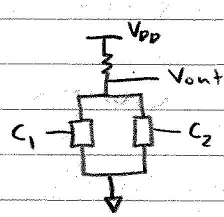
NAND:



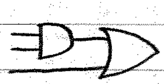
$V_{out} = \overline{C_1 \cap C_2}$

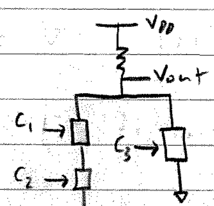


NOR:



$V_{out} = \overline{C_1 \cup C_2}$





$V_{out} = \overline{(C_1 \cap C_2) \cup C_3}$

Systems $\boxed{EE102A}$ $\boxed{L1}$ 12/18

Systems : Take a signal and convert it to another signal.

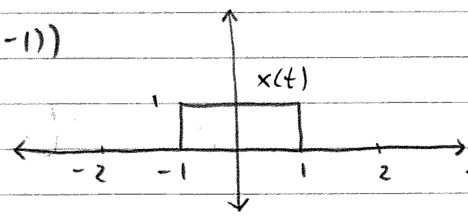
$\boxed{L2}$ Time Scaling: $x(bt)$ scales $x(t)$ $0<b<1$ expands
$b>1$ compresses

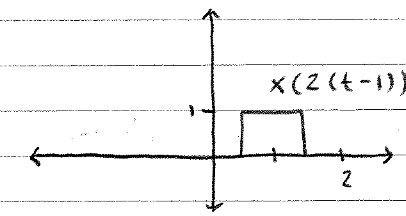
$x[nk]$ compresses, downsamples, or decimates $x[n]$
$x[n/m]$ expands, upsamples, or interpolates $x[n]$
This specifies every mth sample. Intermediate samples must be synthesized (set to zero or interpolated).

Time Reversal: $x(-t)$ $x[-n]$

Time Shift: $x(t-t_1)$ delayed, $x(t+t_1)$ advanced
$x[n-n_1]$ delayed, $x[n+n_1]$ advanced

E.g. $x(2(t-1))$





Even : $x(t) = x(-t)$

Odd : $x(t) = -x(-t)$ $\Rightarrow$ $x(0) = 0$

Decomposition: $X_e(t) = \frac{1}{2}(x(t) + x(-t))$

$X_o(t) = \frac{1}{2}(x(t) - x(-t))$

Periodic: $x(t)$ is periodic if $\exists\ T_0 > 0$ s.t.

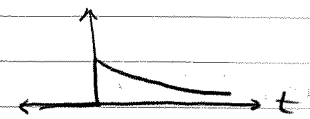
$x(t+T_0) = x(t)$ $\forall t$, T_0 is the period of $x(t$

$x[n]$ is periodic if $\exists\ N_0 > 0$ s.t.

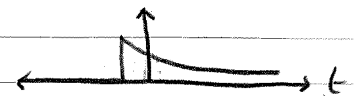
$x[n+N_0] = x[n]$ $\forall n$ N_0 is the period of $x[n]$

☆ The smallest T_0 or N_0 is the <u>fundamental period</u> of the periodic signal.

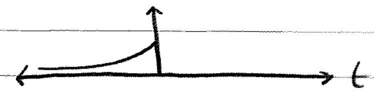
Causal: non-zero only for $t \geq 0$



Noncausal: non-zero for some $t < 0$



Anticausal: non-zero only for $t \leq 0$



Complex Signals: $z(t) = x(t) + j\,y(t)$ $\quad x, y \in \mathbb{R} \quad j = \sqrt{-1}$

Complex Numbers: $z = x + jy$

$x = \mathrm{Re}(z)$ in-phase component (in phase with a cosine)

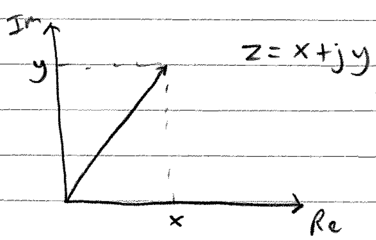
$y = \mathrm{Im}(z)$ quadrature component (delayed 90° from cosine)
(sine lags cosine by 90°)

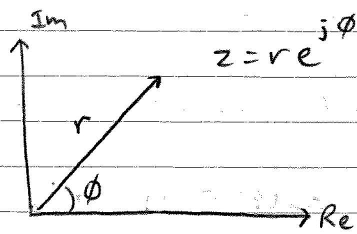
Polar Form: $z = re^{j\phi}$

r: modulus or magnitude of z $\quad r = |z| = \sqrt{(\mathrm{Im}(z))^2 + (\mathrm{Re}(z))^2}$

ϕ: angle or phase of z $\quad \phi = \tan^{-1}\left(\frac{\mathrm{Im}(z)}{\mathrm{Re}(z)}\right)$

$$\boxed{e^{j\phi} = \cos\phi + j\sin\phi}$$





Complex Exponential: $z = x + jy$

$$e^z = e^{x+jy} = e^x e^{jy} = e^x(\cos y + j\sin y)$$

Energy: Signal energy for a (possibly complex) $x(t)$ is:

$$\boxed{E_x = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 \, dt}$$

$$\boxed{\text{Energy signal: } 0 < E_x < \infty \quad P_x = 0}$$

Power: The average of signal energy over time is the signal power:

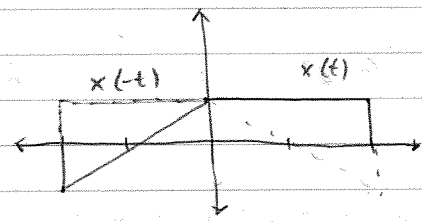
$$\boxed{P_x = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt}$$

$$\boxed{\text{Power Signal: } 0 < P_x < \infty \quad E_x = \infty}$$

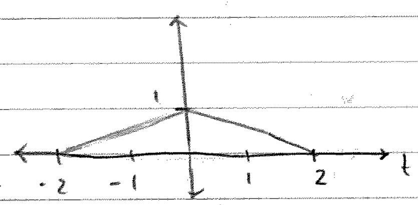
EE102A PSET 1

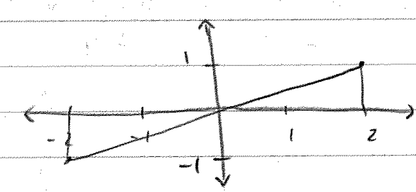
. $x_e(t) = \frac{1}{2}(x(t) + x(-t))$

$x_o(t) = \frac{1}{2}(x(t) - x(-t))$



$x_e(t) =$





Trick: Since the graph consists of straight lines, we can compute the values at $-2, 0,$ and 2 and connect them.

3. $y(t) = x_1(t) + x_2(t) = y(t+T_0)$ must $x_1(t) + x_2(t)$ be periodic?

No. e.g. if $a(t)$ is not periodic and $b(t)$ is periodic,

let $x_1(t) = b(t) + a(t)$ $\quad x_2(t) = b(t) - a(t)$

$\Rightarrow$ neither $x_1(t)$ or $x_2(t)$ are periodic, but:

$y(t) = x_1(t) + x_2(t) = 2b(t)$ is periodic.

4. $x(t)$ periodic and $x(t)$ is odd. What is $x(T_0)$?

$x(t) = -x(-t) \Rightarrow x(0) = 0 \qquad x(0+T_0) = x(0)$

$\Rightarrow x(T_0) = 0$

5. $x_1(t)$ and $x_2(t)$ are periodic with periods T_1 and T_2
Find values of T_1 and T_2 such that $x_1(t) + x_2(t)$ is aperiodic

$y(t) = x_1(t) + x_2(t)$

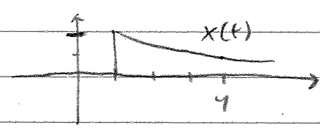
$\Rightarrow y(t+T) = x_1(t+T) + x_2(t+T) = y(t)$ iff $T = nT_1 = mT_2$

where $n, m \in \mathbb{Z} \quad \Rightarrow \frac{T_1}{T_2} = \frac{m}{n}$.

$\Rightarrow x_1(t) + x_2(t)$ is aperiodic if $\frac{T_1}{T_2}$ is an irrational number

e.g. let $T_1 = \pi \quad T_2 = 1$

6. $x(t) = \frac{1}{\sqrt{t}} u(t-1)$



$$E_x = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 dt = \int_1^\infty \frac{1}{t} dt = \ln(t)\Big]_1^\infty = \infty - 0 = \infty$$

$$P_x = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt = \lim_{T\to\infty} \frac{1}{2T} \int_1^T \frac{1}{t} dt = \lim_{T\to\infty} \left(\frac{\ln(T)}{2T} - \cancel{\frac{\ln(1)}{2T}}^{\,0} \right)$$

$$= \lim_{T\to\infty} \frac{\ln(T)}{2T} = \lim_{T\to\infty} \frac{(1/T)}{2} = 0$$

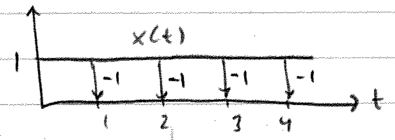
← L'hopital's rule $\quad \lim_{x\to c} \frac{f(x)}{g(x)} = \lim_{x\to c} \frac{f'(x)}{g'(x)}$

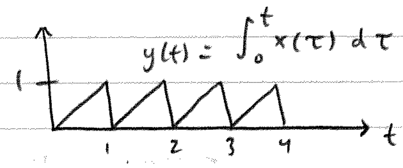
$E_x = \infty \qquad P_x = 0$

Energy: $0 < E_x < \infty$, $P_x = 0$
Power: $0 < P_x < \infty$, $E_x = 0$

$\Rightarrow$ $x(t)$ is neither an Energy or a Power signal.

7.





8. a) $\int_{-\infty}^{\infty} f(t+1)\delta(t+1)\, dt = f(t+1)\big|_{t=-1} = \boxed{f(0)}$

b) $\int_{-\infty}^{\infty} e^{j\omega T}\delta(t)\, dt = e^{j\omega T}\big|_{t=0} = e^{j\omega T}$

c) $\int_{0}^{\infty} f(t)(\delta(t-1)+\delta(t+1))dt = f(t)\big|_{t=1} = \boxed{f(1)}$

d) $\int_{-\infty}^{\infty} f(\tau)\delta(t-\tau)\,\delta(t-2)\, d\tau = \delta(t-2)\int_{-\infty}^{\infty} f(\tau)\,\delta(t-\tau)\, d\tau$

$= \delta(t-2)\; f(\tau)\big|_{\tau=t} = \delta(t-2)\, f(t) = \boxed{\delta(t-2)\, f(2)}$

Devices - $\boxed{\text{EE311}}$ $\boxed{\text{L1}}$

Moores Law - # of transistors that can be placed inexpensively on an IC doubles every 2 years. (exponential growth)

Scaling: Constant $\vec{E}$ field $\Rightarrow$ all device parameters are scaled by same factor α: $t_{ox}\downarrow$, $L\downarrow$, $x_j\downarrow$, $V_{DD}\downarrow$, Channel Doping $\uparrow$

Why Scale? 1. increase packing density $\sim \alpha^2$ $\quad f_t = \frac{3}{2}\frac{2\pi\mu}{L^2}$

2. improve frequency response (speed) $\sim \alpha$ (digital) $\sim \alpha^2$ (analog)

3. power/ckt : $\sim \frac{1}{\alpha^2}$

4. improve current drive (transconductance g_m)

$g_{m_{sat}} = \mu C_{ox}\frac{W}{L}(V_{gs}-V_t) \qquad g_{m_{Triode}} = \mu C_{ox}\frac{W}{L} V_{ds}$

Want to maximize I_{ON} and minimize I_{OFF}

$I_D \propto g_m \propto \frac{K}{t_{ox}}$

historically C_{ox} has been increased by $t_{ox}\downarrow$. Can also increase C_{ox} by $K\uparrow$. $\Rightarrow$ can use $t_{ox}\uparrow$ $\Rightarrow$ reduced gate leakage.

Ge Nanowire and Carbon Nanotube FETs.
- Key challenge: Controlled Growth.

Interconnect Scaling:
- Bigger Chip $\Rightarrow$ Longer Interconnects
- scaling to smaller dimensions $\Rightarrow$ reduced cross-section
- $\Rightarrow$ Larger R, L, C

- Need Better materials
    - lower resistivity metals (Current metals are Al and Cu)
    - lower K dielectrics
    - other solutions (3D or optical interconnects)

Systems $\boxed{\text{EE102A}}$ $\boxed{\text{L3}}$ Signal Models. 12/21

Sinusoidal: $x(t) = A\cos(\omega t + \theta) = A\cos(2\pi f t + \theta)$ $\quad T = \frac{1}{f} = \frac{2\pi}{\omega}$

Euler: $e^{j\phi} = \cos\phi + j\sin\phi$ $\quad \Rightarrow \cos\phi = \frac{1}{2}(e^{j\phi} + e^{-j\phi})$ $\quad \sin\phi = \frac{1}{2j}(e^{j\phi} - e^{-j\phi})$

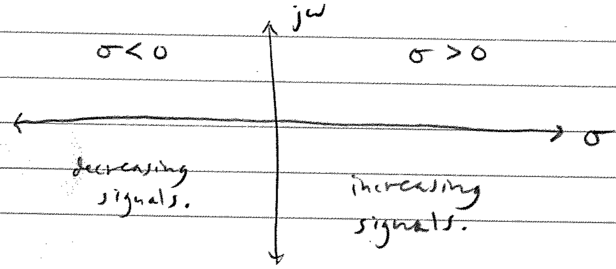
$$Ae^{j(\omega t + \theta)} = A\cos(\omega t + \theta) + jA\sin(\omega t + \theta)$$

Exponential: $x(t) = e^{\sigma t}$ $\quad \sigma < 0 \Rightarrow$ decay $\quad \sigma > 0 \Rightarrow$ growth.

damped or growing sinusoids: $x(t) = e^{\sigma t}\cos(\omega t + \theta)$

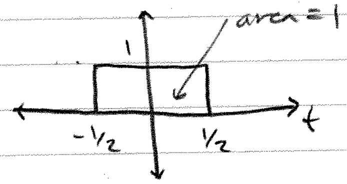
Complex exponential signal: $e^{(\sigma + j\omega)t + j\theta} = e^{\sigma t}(\cos(\omega t + \theta) + j\sin(\omega t + \theta$

$s = \sigma + j\omega$



unit step: $u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$

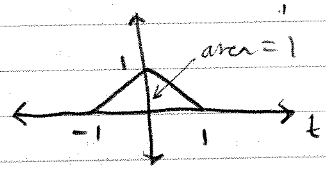
rect: $\text{rect}(t) = \begin{cases} 1 & |t| \leq \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$



unit ramp: $r(t) = \begin{cases} t & t \geq 0 \\ 0 & t < 0 \end{cases}$

$$r(t) = \int_{-\infty}^{t} u(\tau)\, d\tau$$

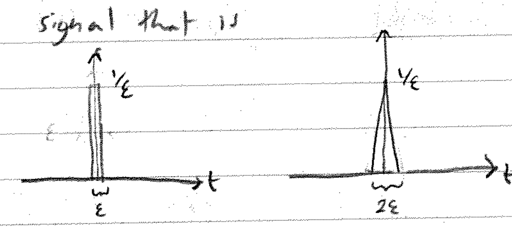
unit triangle: $\Delta(t) = \begin{cases} 1 - |t| & , |t| < 1 \\ 0 & , \text{otherwise} \end{cases}$



delta function (impulse): idealization of a signal that is

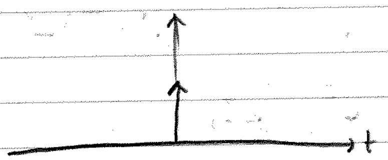
- very large near $t = 0$
- very small away from $t = 0$
- has integral 1

e.g.



for ε very small.

$\delta(t)$



Properties of δ:

$$\int_{-\infty}^{\infty} f(t)\, \delta(t)\, dt = f(0)$$

$$\int_{-\infty}^{\infty} \alpha\, \delta(t)\, f(t)\, dt = \alpha f(0)$$

$$f(t)\, \delta(t) = f(0)\, \delta(t)$$

sifting: $$\int_{-\infty}^{\infty} f(t)\, \delta(t-T)\, dt = f(T)$$

$u'(t) = \delta(t)$

Derivatives of δ : $\int_{-\infty}^{\infty} \delta'(t) f(t)\, dt = \delta(t) f(t) \Big]_{-\infty}^{\infty} - \int_{-\infty}^{\infty} \delta(t) f'(t)\, dt$

$= 0 - f'(0) = f'(0)$

in general : $\int_{-\infty}^{\infty} \delta^{(k)}(t)\, f(t)\, dt = (-1)^k f^{(k)}(0)$ if $f^{(k)}(0)$ is continuous at

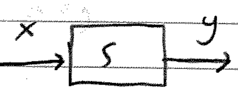
$\delta(t)$ is not a signal in the ordinary sense.
It only makes mathematical sense when inside an integral sign.
Some innocent looking expressions such as $\delta(t)^2$ or $\delta(t^2)$ don't make any sense at all.

$\boxed{L4}$ Systems characteristics and Models

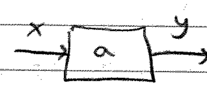
A system is a function mapping input signals into output signals.

this does not necessarily mean multiplication.

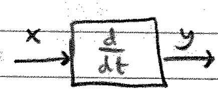
$y = Sx$ or $y = S(x)$ meaning the system S acts on x



scaling system : $y(t) = a\, x(t)$ $a \in \mathbb{R}$

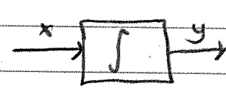


differentiator : $y(t) = x'(t)$



integrator : $y(t) = \int_a^t x(\tau)\, d\tau$

often $-\infty$ or 0

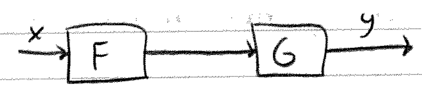


time shift system : $y(t) = x(t-T)$, called delay system if $T > 0$
called predictor system if $T < 0$

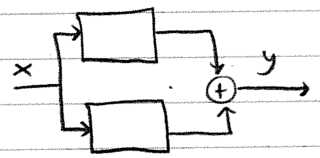
convolution system : $y(t) = \int x(t-\tau) h(\tau)\, d\tau$

Interconnection of Systems:

Cascade (series): $y = G(F(x)) = GFx$

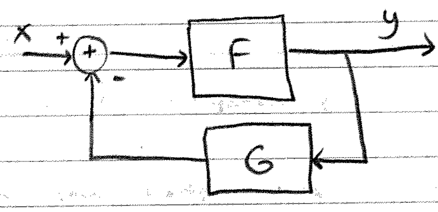


Sum (parallel):

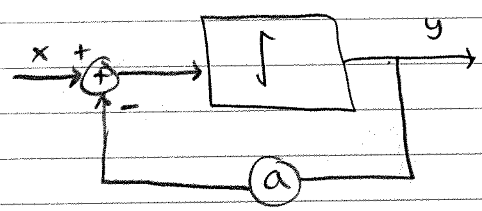


$$y = F(x) + G(x) = Fx + Gx$$

Feedback: $y = F(x - Gy)$



e.g.



$$y(t) = \int_a^t (x(\tau) - a y(\tau))\, d\tau$$

OR $y'(t) = x(t) - a y(t)$

Linearity: A system F is linear if the following 2 properties hold:

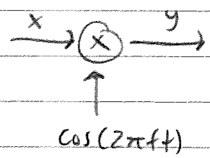
1) Homogeneity: $F(ax) = a F(x) \quad a \in \mathbb{R}$

2) Superposition: $F(x+y) = F(x) + F(y)$

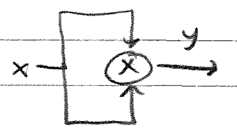
$\Rightarrow$ Scaling before or after the system is the same
summing before or after the system is the same

Linear Systems: Scaling, differentiator, integrator, running average, time shift, convolution, Summer, difference.

Nonlinear Systems: Sign detector, multiplier (sometimes), comparato



Multiplier as modulator is linear



Multiplier as squaring system is nonlinear.

System Memory

A system is memoryless if output depends only on present input

A system with memory has an output signal that depends on inputs in the past or future.

A causal system has an output that depends only on past or present inputs.

Time Invariance:

A system is time invariant if a time shift in the input only produces the same time shift in the output.

$y(t) = F x(t) \Rightarrow y(t-\tau) = F x(t-\tau)$ if F is time invariant

System Stability: Bounded Input Bounded Output (BIBO) Stable if:

Any bounded input $|x(t)| \leq M_x < \infty$ always results in

a bounded output: $|y(t)| \leq M_y < \infty$

$\int_{-\infty}^{\infty} |h(t)|\, dt < \infty \Rightarrow$ BIBO Stable

EE102A PSET 2

1. State whether systems are linear or time invariant:

a) $y(t) = x(t)\sin(\omega t + \phi)$ Yes, No.

b) $y(t) = t x'(t)$ Yes, No.

c) $y(t) = 1 + x(t)\cos(\omega t)$ No, No.

d) $y(t) = \cos(\omega t + x(t))$ No, No.

e) $y(t) = \int_{-t}^{t} x(\tau)\,d\tau$ Yes, No.

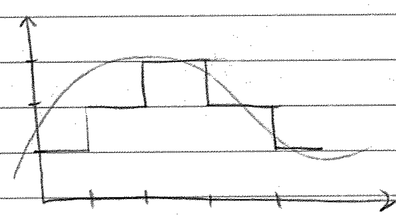
f) $y(t) = \int_{-\infty}^{t/2} x(\tau)\,d\tau$ Yes, No.

2. $y(t) = \dfrac{x^2(t)}{\int_{-\infty}^{t} x(\tau)\,d\tau}$

a) does the system obey homogeneity? Yes. $\dfrac{a^2 x^2(t)}{a\int_{-\infty}^{t} x(\tau)\,d\tau} = a\,y(t)$

b) does the system obey superposition? No.

c) is the system linear? No.

3.  is it linear? No. due to quantization error. (solutions given say it is linear...)

4. $x_e(t) = \frac{1}{2}(x(t) + x(-t))$ $\Rightarrow$ homogeneity ✓ superposition ✓ $\Rightarrow$ $\boxed{\text{Linear}}$

$\frac{1}{2}(x(t-\tau) + x(-(t-\tau))) \neq \frac{1}{2}(x(t-\tau) + x(-t-\tau))$

$\Rightarrow$ time variant.

5. a) $x(t) = 4\Delta\left(\frac{t}{2}\right) - 6\Delta(t) + 4\Delta(2t)$

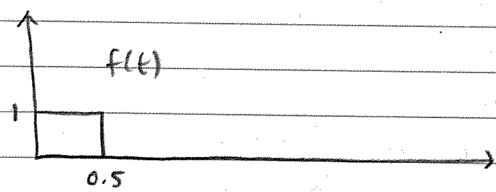
$= 4\Delta\left(\frac{t}{2}\right) - 2\Delta(t) - 2\Delta\left((t-\tfrac{1}{2})/2\right) - 2\Delta\left((t+\tfrac{1}{2})/2\right)$

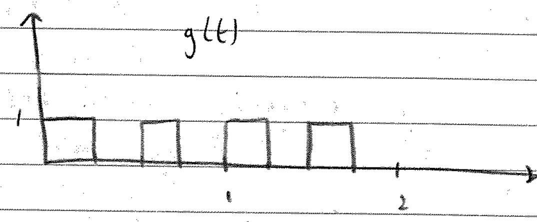
b) $x(t) = 2\Delta\left(\frac{t}{2}\right) - 2\Delta(t) + \mathrm{rect}\left(\frac{t}{2}\right)$

6. $y' = z - ay \Rightarrow z = y' + ay \Rightarrow z' = y'' + ay'$

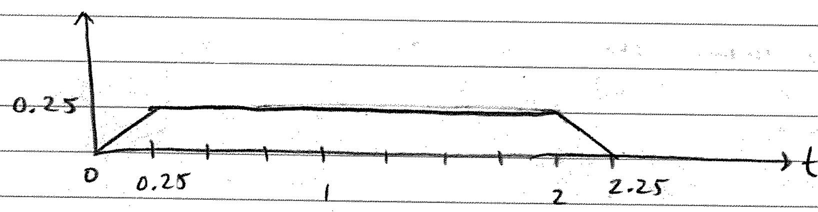
$z' = x - by \Rightarrow y'' + ay' = x - by \Rightarrow y'' + ay' + by = x$

7.





graphically compute $h(t) = (f * g)(t)$



Devices $\boxed{EE311}$ $\boxed{L2\_1}$ MOS Gate Dielectrics 12/23/11

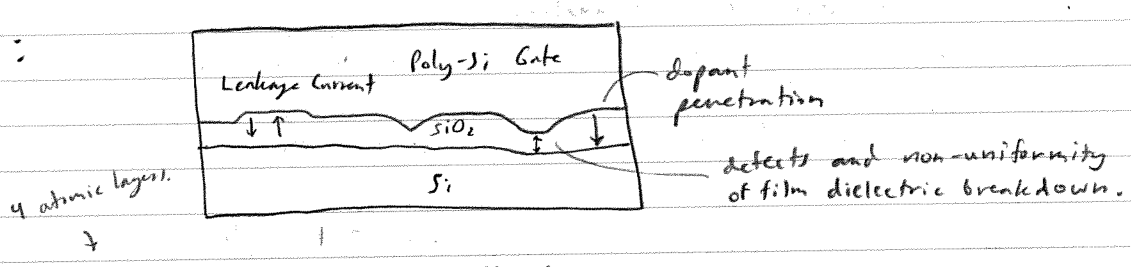
Scaling: $I_D \propto$ Charge $\times$ velocity

$\propto C_{ox}(V_{GS}-V_T) \times$ velocity

$\propto \frac{k}{t_{ox}}(V_{GS}-V_T) \times$ velocity

$\Rightarrow$ scale down t_{ox} to increase I_D.

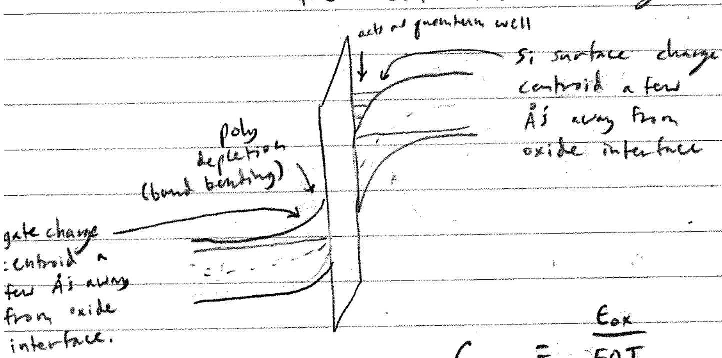
Problems:

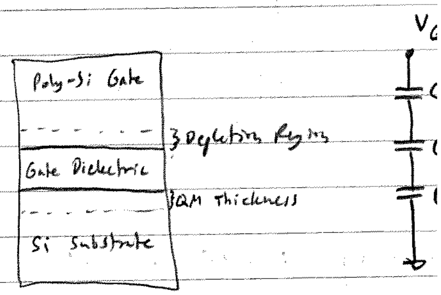


Below 20 Å (2nm), problems with SiO_2:

- Gate leakage $\Rightarrow$ circuit instability, power dissipation.
- Performance degradation due to t_{ox}(electrical) > t_{ox}(physical)
  - Carrier quantization in channel and depletion in poly-Si gate.
- Degradation and breakdown.
- Dopant penetration through gate oxide
- Defects.

As $t_{ox} \downarrow$, I_G (gate leakage due to direct tunneling) dominates the OFF-state leakage current.





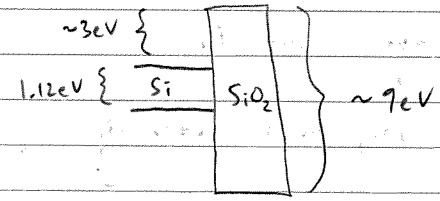
$$C_{ox} = \frac{\varepsilon_{ox}}{EOT}$$

Equivalent Oxide Thickness $\rightarrow$ $EOT = EOT_{DR} + EOT_{GD} + EOT_{QM}$

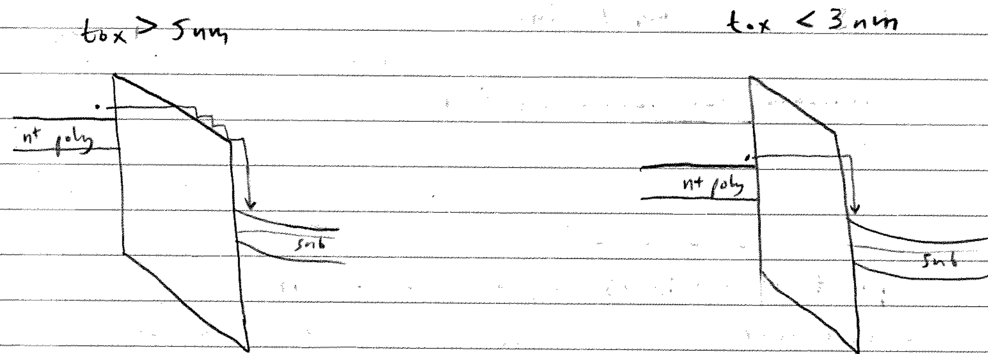
Thus increased t_{ox} reduces C_{ox} which reduces g_m and I_d(on)

Technology: Atomic Layer CVD (chemical vapor deposition).
- Deposition is done one monolayer at a time: excellent control
- Being investigated for high k dielectrics like ZrO_2 and HfO_2

Reliability of SiO_2:

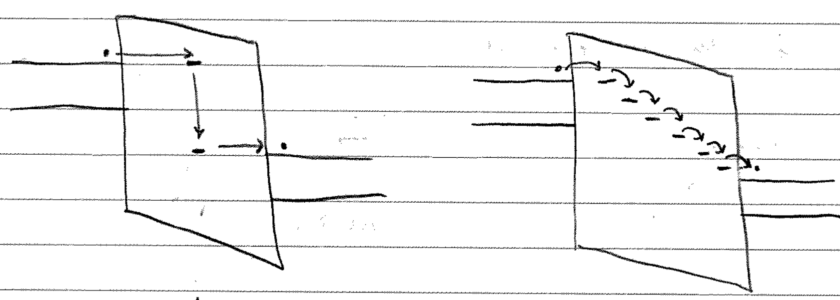


Conduction in Dielectrics: Tunneling



Fowler-Nordheim (FN) Tunneling   Direct Tunneling

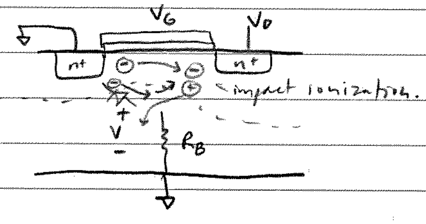
Conduction in Dielectrics: Leakage (Trap assisted tunneling)



- Trap assisted tunneling resulting in leakage at low gate voltage.
- Increase in traps will cause more leakage.
- Traps are present in as-grown dielectric
- Traps can be generated by electrical stress.

Hot Carrier Effects:

- change in V_t, g_m
- leakage
- junction breakdown



Systems $\boxed{\text{EE 102 A}}$ $\boxed{\text{L5}}$ 12/24/11

Time Domain Analysis of Continuous Time Systems.

zero input response: system response to zero input. This is due only to initial conditions. (This is zero for a linear system)

zero state response: system response to zero initial conditions. This is due only to input.

output = zero state response + zero input response

Impulse response: $h(t,\tau) = H(\delta(t-\tau))$ ← for time variant system,

$$\boxed{h(t) = H(\delta(t))}$$ for LTI system

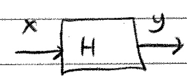
Extended Linearity: if $y_n = Sx_n$ $\quad \sum_n a_n y_n = S\left(\sum_n a_n x_n\right)$

⇒ summation and system operator commute

if $y_n = Sx_n$ $\quad \int_{-\infty}^{\infty} a(\tau)\, y(t-\tau)\, d\tau = S\left(\int_{-\infty}^{\infty} a(\tau)\, x(t-\tau)\, d\tau\right)$

⇒ integration and system operator commute.

Can write $x(t) = \int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\, d\tau$



$$\Rightarrow \; y(t) = H(x(t)) = H\left(\int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\, d\tau\right) = \int_{-\infty}^{\infty} x(\tau)\, H(\delta(t-\tau))\, d\tau$$

$$= \int_{-\infty}^{\infty} x(\tau)\, h(t,\tau)\, d\tau$$ if H is LTI, $y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau = x(t) * h(t$

⇒ The response of an LTI system is completely characterized by its impulse response $h(t)$.

EE102A | L6 | Convolution

$y(t) = (x * h)(t)$ for an LTI system

Properties of Convolution:

Commutative: $f*g = g*f$

Associative: $f*(g*h) = (f*g)*h$

Distributive: $f*(g+h) = f*g + f*h$

Properties of Convolution Systems

- The properties of the convolution integral have important consequences for systems described by convolution:

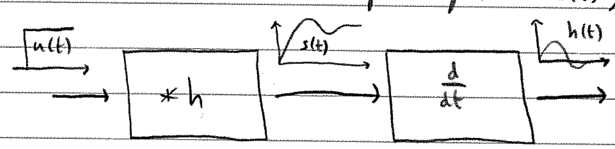
Convolution systems are linear: $h*(\alpha x_1 + \beta x_2) = \alpha(h*x_1) + \beta(h*x_2)$

Convolution systems are time invariant: if $x_1(t) = x(t-T)$, $y_1(t) = y(t-T)$

i.e. convolution systems commute with delay.

Measuring the impulse response of an LTI system:

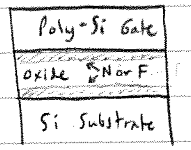
Since it can be practically difficult to measure the impulse response of an LTI system directly (input amplitude is often limited, so a very short pulse then has very little energy), A common alternative is to measure the step response $s(t)$, the response to a unit step input $u(t)$.



Device | EE311 | L 2-2 | MOS Gate Dielectrics

Nitrided SiO_2:

- Incorporating N or F instead of Hydrogen strengthens the Si/SiO_2 interface and increases the gate dielectric lifetime because Si-F and Si-N bonds are stronger than Si-H bonds.



Nitroxides:
- improves reliability
- barrier to dopant penetration from poly-Si gate into Si substrate (esp. if gate is doped with Boron).
- marginal increase in K.
- used extensively.

High K Dielectrics:

$$I_D \propto C_{ox} \propto \frac{K}{t_{ox}}$$

Historically, C_{ox} has been increased by decreasing t_{ox}, but it can also be increased by using a higher K dielectric. For higher K, we can use larger t_{ox}.

$\Rightarrow$ Higher thickness $\Rightarrow$ reduced gate leakage $\quad \underset{\text{direct tunneling current density}}{J_{DT}} \propto e^{-t_{ox}}$

20Å SiO_2 $K=4$ 40Å Si_3N_4 $K=8$

$$C_{ox} = \frac{K\epsilon_0 A}{t_{ox}} \Rightarrow t_{high\text{-}K} = \left(\frac{K_{high\text{-}K}}{K_{SiO_2}}\right) t_{SiO_2}$$

	K	Conduction Band Barrier Height
SiO_2	4	3.2 eV
Si_3N_4	8	2.2 eV
	↑ good	↑ bad (reduction in bandgap $\Rightarrow$ increased gate leakage)

Requirements for MOS Gate Dielectrics

- high K $\Rightarrow$ higher charge induced in channel
- wide bandgap $\Rightarrow$ higher barriers $\Rightarrow$ lower leakage
- ability to grow high purity films on Si with clean interface
  - high resistivity and breakdown voltage
  - low bulk and interfacial trap densities.
- Compatibility with substrate + top electrode (minimal interdiffusion + reaction)
- Thermal stresses - most oxides have larger thermal expansion coefficients than Si.
- Good Si fabrication processing compatibility
  - stability at high T, ability to be cleaned, etched, etc.

Candidates for high K:

	K	E_g	ΔE_c to Si
SiO_2	4	9	3.5
Si_3N_4	7	5.3	2.4
HfO_2	25	6	1.5
ZrO_2	25	5.8	1.4

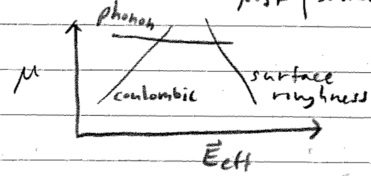
Higher K materials have lower bandgap.

Atomic Layer Deposition → then annealing.

Issues with High K Dielectrics:
- How good is interface with Si? → mobility
- Compatibility with gate electrode ⇒ metal gate
- Device reliability + lifetime.
- Minimum EOT achievable.

	T ↑
μ_c	↑
μ_{ph}	↓
μ_{sr}	same

Reduced Mobility in high K Gate stacks



$$\frac{1}{\mu_{eff}} = \frac{1}{\mu_c} + \frac{1}{\mu_{ph}} + \frac{1}{\mu_{sr}}$$

interface traps cause coulombic scattering ⇒ reduces mobility.

High-K / Poly-Si gate transistors: high V_T, degraded channel mobility (phonon scatter
Metal gate screens surface phonon scattering + improves channel mobility.

Systems $\boxed{\text{EE102A}}$ $\boxed{\text{L7}}$ Continuous Time Fourier Series

Frequency domain representation of continuous time signals in general means a Fourier Series or Fourier Transform.

Fourier Series: Time limited signals and periodic signals.
Fourier Transforms: Any energy signal, many power signals.

Fourier Series: If $f(t)$ is a well behaved periodic signal, or it is defined for only a finite interval, then $f(t)$ can be written as a Fourier series:

$$f(t) = \sum_{n=-\infty}^{\infty} D_n e^{jnw_0 t} \qquad \text{where} \quad w_0 = \frac{2\pi}{T_0}$$

$$D_n = \frac{1}{T_0}\int_{\tau}^{\tau+T_0} f(t)\, e^{-jnw_0 t}\, dt \qquad \text{for } \forall\ n \in \mathbb{Z}$$

The sequence $\{D_n\}$ is called the Fourier coefficients of $f(t)$.

Fourier Transform: If $f(t)$ is a well-behaved aperiodic signal, then $f(t)$ can be written as:

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(jw)\, e^{jwt}\, dw \qquad \leftarrow \text{inverse transform}$$

where $F(jw) = \int_{-\infty}^{\infty} f(t)\, e^{-jwt}\, dt$ $\leftarrow$ $F(jw)$ is the Fourier Transform of $f(t)$.

In either case $f(t)$ is represented as a weighted average or linear combination of complex exponentials.

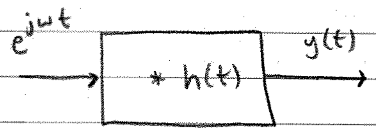
⊗ Very important for LTI systems, since complex exponentials are eigenfunctions:
If $x(t)$ is an eigenfunction of a system, then $y(t) = a\,x(t)$ and a is a complex constant (eigenvalue)

Fourier Series of Cosine: $f(t) = A\cos(w_0 t + \theta) = A\left(\frac{e^{j(w_0 t+\theta)} + e^{-j(w_0 t+\theta)}}{2}\right)$

$$= \frac{A}{2}e^{j\theta}e^{jw_0 t} + \frac{A}{2}e^{-j\theta}e^{-jw_0 t}$$

$\Rightarrow D_1 = \frac{A}{2}e^{j\theta} \quad D_{-1} = \frac{A}{2}e^{-j\theta} \quad D_n = 0 \text{ for } \forall\ n \neq 1, -1$

LTI system:



$$y(t) = \int_{-\infty}^{\infty} h(\tau) e^{j\omega(t-\tau)} d\tau = e^{j\omega t} * h(t)$$

$$= e^{j\omega t} \int_{-\infty}^{\infty} h(\tau) e^{-j\omega \tau} d\tau = H(j\omega) e^{j\omega t}$$

$\Rightarrow$ Complex exponential is an eigenfunction of LTI system with eigenvalue $H(j\omega)$

Parseval's Theorem: $\lim\limits_{N\to\infty} \sum\limits_{n=-N}^{N} |D_n|^2 = \frac{1}{T_0} \int_{t_0}^{t_0+T_0} |f(t)|^2 dt$

i.e. the power of the signal in the time domain equals the sum of the powers of the frequency components.

$D_0 = \frac{1}{T_0} \int_{t_0}^{t_0+T_0} f(t) dt$ ← DC coefficient or time-average mean of the signal.

Symmetry: Fourier coefficients for a real waveform have the following symmetry properties:

$\mathcal{R}(D_{-n}) = \mathcal{R}(D_n)$ even

$\mathrm{Im}(D_{-n}) = -\mathrm{Im}(D_n)$ odd

$D_{-n} = D_n^*$

$|D_{-n}| = |D_n|$

$\angle D_{-n} = -\angle D_n$ odd

$$\mathrm{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$

EE102A PSET 3

1. a) $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$

$\Rightarrow \boxed{h(t) = u(t)}$

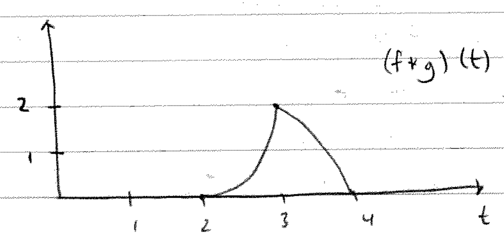
b) $y(t) = \int_{t-T}^{t} x(\tau)\, d\tau = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$

$\Rightarrow \boxed{h(t) = u(t) - u(t-T)}$

c) $y(t) = x(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau \Rightarrow \boxed{h(t) = \delta(t)}$

d) $y(t) = x(t-1) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau \Rightarrow \boxed{h(t) = \delta(t-1)}$

2.



3. $\cancel{\int_{-\infty}^{\infty} u(\tau)\, rect(t-1-\tau)\, d\tau = \int_{0}^{\infty} rect(t-1-\tau)\, d\tau}$

$\int_{-\infty}^{\infty} rect(\tau-1)\, u(t-\tau)\, d\tau = \int_{-\infty}^{t} rect(\tau-1)\, d\tau = \begin{cases} 0, & t < \frac{1}{2} \\ t - 1/2, & \frac{1}{2} < t < \frac{3}{2} \\ 1, & t > 3/2 \end{cases} \leftarrow \int_{1/2}^{t} 1\, d\tau$

4. $(\delta * \delta)(t) = \delta(t)$

5. $x'(t) = (x * \delta')(t)$ if $f * g = y$ show that $f' * g' = y''$

$y' = y * \delta' = (f*g) * \delta' = f * (g * \delta') = f * g'$

$y'' = y' * \delta' = f * g' * \delta' = f * \delta' * g' = f' * g'$ ✓

6. $x \star y = z \Rightarrow \int_{-\infty}^{\infty} x(\tau)\, y(t+\tau)\, d\tau = z(t)$

$y \star x = \int_{-\infty}^{\infty} y(\tau)\, x(t+\tau)\, d\tau$ let $\tau' = t + \tau \Rightarrow \int_{-\infty}^{\infty} y(\tau'-t)\, x(\tau')\, d\tau'$

$= \int_{-\infty}^{\infty} x(\tau')\, y(-t+\tau')\, d\tau' = z(-t)$

7. a) $f(t) = \cos(3\pi t) + \frac{1}{2}\sin(4\pi t)$

$$T_1 = \frac{2}{3} \qquad T_2 = \frac{1}{2}$$

$\Rightarrow T_0 = LCM\left(\frac{2}{3}, \frac{1}{2}\right) = \boxed{2} \qquad \Rightarrow \omega_0 = \frac{2\pi}{T} = \pi$

b) $f(t) = \cos(3\pi t) + \frac{1}{2}\sin(4\pi t)$

$$= \frac{1}{2}\left(e^{j3\pi t} + e^{-j3\pi t}\right) + \frac{1}{4j}\left(e^{j4\pi t} - e^{-j4\pi t}\right)$$

$$\Rightarrow \boxed{D_3 = \frac{1}{2} \quad D_{-3} = \frac{1}{2} \quad D_4 = -\frac{1}{4}j \quad D_{-4} = \frac{1}{4}j}$$

8. $f(at) \Rightarrow$ period is $\frac{T_0}{a} \Rightarrow \omega_0 = \frac{2\pi}{T_0/a} = a\frac{2\pi}{T_0}$

$\Rightarrow f(at) = \sum_{n=-\infty}^{\infty} D_{n,a} e^{jna\omega_0 t} \qquad D_{n,a} = \frac{1}{T_0/a}\int_{t_0}^{t_0+T_0/a} f(at)\, e^{-ja\omega_0 n t}\, dt$

let $\tau = at \Rightarrow D_{n,a} = \frac{1}{T_0/a}\int_{t_0}^{t_0+T_0} f(\tau)\, e^{-j\omega_0 n \tau}\, \frac{d\tau}{a}$

$$= \frac{1}{T_0}\int_{t_0}^{t_0+T_0} f(\tau)\, e^{-j\omega_0 n \tau}\, d\tau = D_n$$

$\Rightarrow$ The fourier series coefficients are the same as in the unscaled case.

$\Rightarrow f(at) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0(at)}$ which is the original fourier series evaluated at at.

9. $f(t) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t} \qquad D_n = \frac{1}{T_0}\int_{t_0}^{t_0+T_0} f(t)\, e^{-j\omega_0 n t}\, dt$

$$\hat{D}_n = \frac{1}{T_0}\int_{t_0}^{t_0+T_0} f(t-\tau)\, e^{-j\omega_0 n t}\, dt = \frac{1}{T_0}\int_{t_0}^{t_0+T_0} f(\tau')\, e^{-j\omega_0 n(\tau'+\tau)}\, d\tau'$$

$$= e^{-j\omega_0 n\tau} D_n \Rightarrow \boxed{f(t-\tau) = \sum_{n=-\infty}^{\infty} D_n e^{-j\omega_0 n\tau} e^{jn\omega_0 t}}$$

Systems $\boxed{\text{EE102A}}$ $\boxed{\text{L8}}$ The Continuous Time Fourier Transform 12/30/20

$$\boxed{\begin{aligned} F(j\omega) &= \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt \\ f(t) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} F(j\omega)\, e^{j\omega t}\, d\omega \end{aligned}}$$

$\text{rect}(t) \Leftrightarrow \text{sinc}\left(\frac{\omega}{2\pi}\right)$

$\text{rect}\left(\frac{t}{T}\right) \Leftrightarrow T\,\text{sinc}\left(\frac{\omega T}{2\pi}\right)$

$e^{-at}u(t) \Leftrightarrow \frac{1}{a+j\omega}$

Properties of odd and even functions:

let $e_1(t)$ and $e_2(t)$ be even functions and $o_1(t)$ and $o_2(t)$ be odd functions

$e_1(t) \pm e_2(t) = \text{even}$

$e_1(t)\, e_2(t) = \text{even}$

$o_1(t) \pm o_2(t) = \text{odd}$

$o_1(t)\, o_2(t) = \text{even}$

$e_1(t)\, o_2(t) = \text{odd}$

Symmetry Properties

For any real, imaginary, or complex signal:
- an even signal has an even transform $f(t) = f(-t) \Rightarrow F(j\omega) = F(-j\omega)$
- an odd signal has an odd transform $f(t) = -f(-t) \Rightarrow F(j\omega) = -F(-j\omega)$

A real signal has Hermitian symmetry: $F(-j\omega) = F^*(j\omega)$

An imaginary signal has anti-Hermitian symmetry: $F(-j\omega) = -F^*(j\omega)$

$f(t)$	$F(j\omega)$
real + even	real + even
real + odd	imag + odd
imag + even	imag + even
imag + odd	real + odd

Systems $\boxed{\text{EE102A}}$ $\boxed{\text{L9}}$ Fourier Transform Theorems

Linearity: $a f_1(t) + b f_2(t) \Leftrightarrow a F_1(j\omega) + b F_2(j\omega)$

Scaling: $f(at) \Leftrightarrow \frac{1}{|a|} F\left(\frac{j\omega}{a}\right)$

Complex Conjugation: $f^*(t) \Leftrightarrow F^*(-j\omega)$

Duality: $F(j\omega)\Big|_{\omega \to t} \Leftrightarrow 2\pi f(-t)\Big|_{t \to \omega}$

Shift: $f(t-\tau) \Leftrightarrow e^{-j\omega\tau} F(j\omega)$

Modulation: $f(t)\, e^{j\omega_0 t} \Leftrightarrow F(j(\omega-\omega_0))$

$f(t)\cos(\omega_0 t) \Leftrightarrow \frac{1}{2}\left(F(j(\omega-\omega_0)) + F(j(\omega+\omega_0))\right)$

$f(t)\sin(\omega_0 t) \Leftrightarrow \frac{1}{2}\left(F(j(\omega-\omega_0)) - F(j(\omega+\omega_0))\right)$

Derivative: $f'(t) \Leftrightarrow j\omega F(j\omega)$

$f^n(t) \Leftrightarrow (j\omega)^n F(j\omega)$

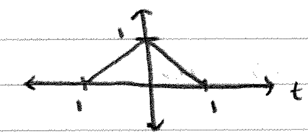
$(-jt) f(t) \Leftrightarrow F'(j\omega)$ by duality

Parsevals Theorem: $\mathcal{E}_f = \int_{-\infty}^{\infty} |f(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |F(j\omega)|^2 d\omega$

Convolution: $(f_1 * f_2)(t) \Leftrightarrow F_1(j\omega)\, F_2(j\omega)$

↓

example: $\Delta(t) = \begin{cases} 1-|t|, & |t|<1 \\ 0, & \text{otherwise} \end{cases}$



$= \text{rect}(t) * \text{rect}(t)$

since $\text{rect}(t) \Leftrightarrow \text{sinc}\left(\frac{\omega}{2\pi}\right)$

$\Delta(t) \Leftrightarrow \text{sinc}^2\left(\frac{\omega}{2\pi}\right)$

EE102A PSET 4

1. $D_n = \frac{1}{2}\int_{-1}^{1} |t|\, e^{-jn\omega_0 t} dt \qquad \omega_0 = \frac{2\pi}{T} = \pi$

$\Rightarrow D_n = \frac{1}{2}\int_{-1}^{0} -t e^{-jn\pi t} dt + \frac{1}{2}\int_{0}^{1} t e^{-jn\pi t} dt = \frac{1}{2}\int_{0}^{1} (t e^{jn\pi t} + t e^{-jn\pi t}) dt$

$= \frac{1}{2}\left(\frac{j e^{-jn\pi}}{n\pi} + \frac{e^{-jn\pi}-1}{(n\pi)^2} - \frac{j e^{jn\pi}}{n\pi} + \frac{e^{jn\pi}-1}{(n\pi)^2}\right)$

$= \frac{1}{2}\left(\cancel{\frac{j(-1)^n}{n\pi}} + \frac{(-1)^n-1}{n\pi^2} - \cancel{\frac{j(-1)^n}{n\pi}} + \frac{(-1)^n-1}{(n\pi)^2}\right) = \frac{(-1)^n-1}{(n\pi)^2}$

since $e^{j\pi} = e^{-j\pi} = -1$

$\Rightarrow D_n = \begin{cases} \frac{1}{2}, & n=0 \\ -\frac{2}{(n\pi)^2}, & n \text{ odd} \\ 0, & n \text{ even} \end{cases}$

2. a) even

b) odd

c) real

d) real and even

e) imag and odd

f) imag and even

3. a) $D_0 = \frac{1}{T}\int_0^T f(t)\,dt = \frac{\alpha T A}{T} = \alpha A$

$a = \alpha A \Rightarrow \alpha = \frac{a}{A}$

b) $D_1 = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\, e^{-j2\pi t/T}\,dt = \frac{A}{T}\int_{-\alpha T/2}^{\alpha T/2} e^{-j2\pi t/T}\,dt = \frac{A}{T}\left(\frac{T}{-j2\pi}\, e^{-j2\pi t/T}\Big]_{-\alpha T/2}^{\alpha T/2}\right)$

$= \frac{A}{T}\left(\frac{T}{-j2\pi}\left(e^{-j\pi\alpha} - e^{j\pi\alpha}\right)\right) = \frac{A}{\pi}\left(\frac{1}{2j}\left(e^{j\pi\alpha} - e^{-j\pi\alpha}\right)\right)$

$= \frac{A}{\pi}\sin(\pi\alpha) = A\alpha\,\text{sinc}(\pi\alpha)$

c) $\alpha = \frac{1}{2}$

1. a) $f(t) = \Delta(t+1) + \Delta(t) + \Delta(t-1)$

$F(j\omega) = (e^{-j\omega} + 1 + e^{j\omega})\,\text{sinc}^2(\frac{\omega}{2\pi}) = (1 + 2\cos(\omega))\,\text{sinc}^2(\frac{\omega}{2\pi})$

b) $f(t) = 2\Delta(t/2) - \Delta(t)$

$F(j\omega) = 4\,\text{sinc}^2(\frac{\omega}{\pi}) - \text{sinc}^2(\frac{\omega}{2\pi})$

c) $f(t) = \text{rect}(t) * \text{rect}(t/3)$

$F(j\omega) = 3\,\text{sinc}(\frac{\omega}{2\pi})\,\text{sinc}(\frac{3\omega}{2\pi})$

real + even $\Leftrightarrow$ real + even

real + odd $\Leftrightarrow$ imag + odd $\qquad f*g = F(j\omega)\,G(j\omega)$

even x odd = odd $\qquad$ odd $\Leftrightarrow$ odd $\qquad \Rightarrow \boxed{\text{true}}$

imag x real = imag $\qquad$ real + odd $\Leftrightarrow$ imag + odd $\quad \Rightarrow$ h(t) is real and odd

b) odd $*$ odd $\iff$ odd $\cdot$ odd $=$ even $\Rightarrow$ $\boxed{\text{false}}$

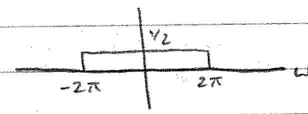
c) $f(t) * f(-t) \iff F(j\omega)\,F(-j\omega)$ even $\Rightarrow$ $\boxed{\text{true}}$

6. a) $f_1(t) = \text{sinc}(2t) \qquad f_2(t) = \text{sinc}(t)\cos(3\pi t) \qquad f(t) = (f_1 * f_2)(t)$

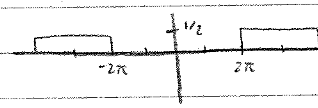
$\text{rect}(t) \iff \text{sinc}\left(\frac{\omega}{2\pi}\right) \Rightarrow \text{sinc}\left(\frac{t}{2\pi}\right) \iff 2\pi\,\text{rect}(-\omega) = 2\pi\,\text{rect}(\omega)$

$\Rightarrow \text{sinc}(t) = \text{rect}\left(\frac{\omega}{2\pi}\right)$

$\Rightarrow F_1(j\omega) = \frac{1}{2}\text{rect}\left(\frac{\omega}{4\pi}\right)$



$F_2(j\omega) = \frac{1}{2}\left(\text{rect}\left(\frac{\omega-3\pi}{2\pi}\right) + \text{rect}\left(\frac{\omega+3\pi}{2\pi}\right)\right)$



$F(j\omega) = F_1(j\omega)\,F_2(j\omega) = 0 \Rightarrow F(j\omega) = 0$

b) $f(t) = 0$ since $0 \iff 0$

$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(j\omega)\,e^{j\omega t}\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} 0 \cdot e^{j\omega t}\,dt = 0$

7. $\int_{-\infty}^{\infty} \text{sinc}^2(t)\cos(2\pi t)\,dt \qquad \cos(2\theta) = 2\cos^2(\theta) - 1$

$\downarrow$

$= \int_{-\infty}^{\infty} \text{sinc}^2(t)\left(2\cos^2(\pi t) - 1\right)dt = 2\int_{-\infty}^{\infty}\left(\text{sinc}(t)\cos(\pi t)\right)^2 dt - \int_{-\infty}^{\infty}\text{sinc}^2(t)\,dt$

$\text{sinc}(t)\cos(\pi t) \iff \frac{1}{2}\left(\text{rect}\left(\frac{(\omega-\pi)}{2\pi}\right) + \text{rect}\left(\frac{\omega+\pi}{2\pi}\right)\right)$

$\text{sinc}(t) \iff \text{rect}\left(\frac{\omega}{2\pi}\right)$

$\Rightarrow \frac{2}{2\pi}\int_{-2\pi}^{2\pi}\left(\frac{1}{2}\right)^2 d\omega - \frac{1}{2\pi}\int_{-\pi}^{\pi} 1^2\,d\omega = \frac{8\pi}{8\pi} - \frac{2\pi}{2\pi} = 1 - 1 = \boxed{0}$

Systems $\boxed{\text{EE102A}}$ $\boxed{\text{L 10}}$ Fourier Theorems + Generalized Fourier Transforms

$\mathcal{F}[f(t)] = F(j\omega) \qquad \mathcal{F}^{-1}[F(j\omega)] = f(t)$

Frequency Domain Convolution Theorem:

$$\mathcal{F}[f_1(t) f_2(t)] = \frac{1}{2\pi}(F_1 * F_2)(j\omega)$$

e.g. $\mathcal{F}[\text{sinc}^2(t)] = \frac{1}{2\pi}\left(\text{rect}\left(\frac{\omega}{2\pi}\right) * \text{rect}\left(\frac{\omega}{2\pi}\right)\right) = \Delta\left(\frac{\omega}{2\pi}\right)$

Generalized Fourier Transforms:

$$\delta(t) \Leftrightarrow 1$$

$$\delta(t-\tau) \Leftrightarrow e^{-j\omega\tau}$$

$$1 \Leftrightarrow 2\pi\,\delta(\omega)$$

$$e^{j\omega_0 t} \Leftrightarrow 2\pi\,\delta(\omega-\omega_0)$$

$$\cos(\omega_0 t) \Leftrightarrow \pi\left(\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\right)$$

$$\sin(\omega_0 t) \Leftrightarrow j\pi\left(\delta(\omega-\omega_0) - \delta(\omega+\omega_0)\right)$$

$$\text{sgn}(t) \Leftrightarrow \begin{cases} \frac{2}{j\omega}, & \omega \neq 0 \\ 0, & \omega = 0 \end{cases} \qquad \text{where} \quad \text{sgn}(t) = \begin{cases} -1, & t < 0 \\ 0, & t = 0 \\ 1, & t > 0 \end{cases}$$

$$u(t) \Leftrightarrow \pi\,\delta(\omega) + \frac{1}{j\omega}$$

Integral Theorem:

$$\int_{-\infty}^{t} f(\tau)\,d\tau \Leftrightarrow \pi F(0)\,\delta(\omega) + \frac{F(j\omega)}{j\omega}$$

Systems $\boxed{\text{EE102A}}$ $\boxed{\text{L11}}$ Frequency Response of LTI Systems 12/31/20

For an LTI system: $x(t) \longrightarrow \boxed{*h(t)} \longrightarrow y(t)$ $\qquad y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$

$$Y(j\omega) = H(j\omega)\, X(j\omega)$$

Scaled by system amplitude response $|H(j\omega)|$: $|Y(j\omega)| = |H(j\omega)|\,|X(j\omega)|$

Phase shifted by system phase response $\angle H(j\omega)$: $\angle Y(j\omega) = \angle H(j\omega) + \angle X(j\omega)$

If input is complex exponential: $x(t) = e^{j\omega_0 t}$, $\quad X(j\omega) = 2\pi\,\delta(\omega - \omega_0)$

$Y(j\omega) = H(j\omega)\, 2\pi\,\delta(\omega-\omega_0) = H(j\omega_0)\, 2\pi\,\delta(\omega-\omega_0)$

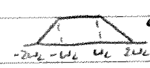
$\Rightarrow y(t) = H(j\omega_0)\, e^{j\omega_0 t} = |H(j\omega_0)|\, e^{j(\omega_0 t + \angle H j\omega_0)}$

Thus a sinusoidal input $e^{j\omega_0 t}$ to an LTI system produces a sinusoidal output at the:

- Same frequency
- Scaled in amplitude
- Phase shifted
- This corresponds to multiplication by a complex number $H(j\omega_0)$

Note that magnitudes multiply and phases add.

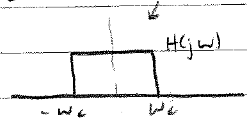
Ideal Filters: decays as $\frac{1}{t}$



$\leftarrow$ decays as $\frac{1}{t^2}$

in general $h(t)$ will decay as $\frac{1}{t^n}$ where n is the number of times you ~~can differentiate $H(j\omega)$ before you~~ get impulses.

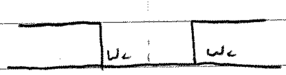
Lowpass:



$$H(j\omega) = \text{rect}\left(\frac{\omega}{2\omega_c}\right)$$

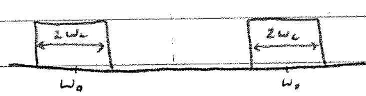
$= \text{rect}\left(\frac{\pi}{\omega_c}\frac{\omega}{2\pi}\right) = \left(\frac{\pi}{\omega_c}\right)\left(\frac{\omega_c}{\pi}\right)\text{rect}\left(\frac{\pi}{\omega_c}\frac{\omega}{2\pi}\right) \Rightarrow h(t) = \frac{\omega_c}{\pi}\,\text{sinc}\left(\frac{\omega_c}{\pi}t\right)$

Highpass:



$H(j\omega) = 1 - \text{rect}\left(\frac{\omega}{2\omega_c}\right) \Rightarrow h(t) = \delta(t) - \frac{\omega_c}{\pi}\,\text{sinc}\left(\frac{\omega_c}{\pi}\right.$

Ideal Bandpass:



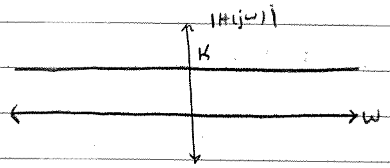
$H(j\omega) = \text{rect}\left(\frac{\omega - \omega_0}{2\omega_c}\right) + \text{rect}\left(\frac{\omega+\omega_0}{2\omega_c}\right)$

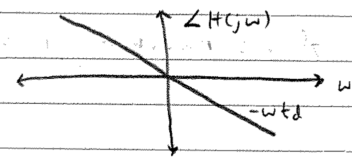
$\Rightarrow h(t) = 2\cos(\omega_0 t)\, \frac{\omega_c}{\pi}\,\text{sinc}\left(\frac{\omega_c}{\pi}t\right)$

Distortionless LTI Systems:

ideally we want: $y(t) = \underbrace{K}_{\text{constant gain}} x(t - \underbrace{t_d}_{\text{constant delay}}) \Rightarrow H(j\omega) = K e^{-j\omega t_d}$

such that $|H(j\omega)| = K$ $\quad \angle H(j\omega) = -\omega t_d$ ← negative linear phase as a function of frequency ⇒ distortionle

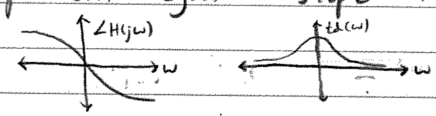




Phase distortion and group delay:

Group delay is the frequency dependent negative slope of $\angle H(j\omega)$:

$t_d(\omega) = -\frac{d}{d\omega} \angle H(j\omega)$



Importance of Amplitude and Phase Distortion depends on Application:

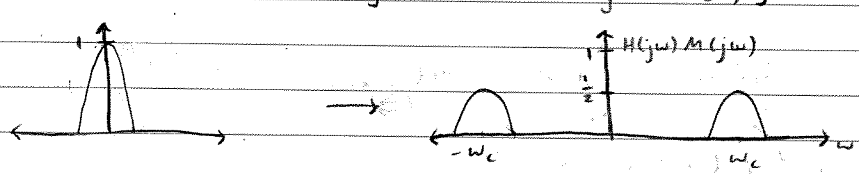
Audio or Speech: - Amplitude distortion is very important
- Humans are relatively insensitive to phase distortion

Images or Video: - Phase distortion is very important. Nonlinear phase ⇒ blurry images.
- Amplitude distortion is relatively unimportant as long as it is slowly varying.

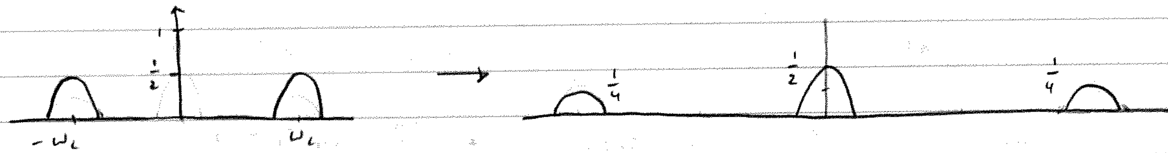
Systems $\boxed{\text{EE102A}}$ $\boxed{\text{L12}}$ Modulation and Demodulation

Double-Sideband, Suppressed Carrier (Multiply by a cosine)

$m(t)\cos(\omega_c t) \Longleftrightarrow \frac{1}{2}\left[M(j(\omega+\omega_c)) + M(j(\omega-\omega_c))\right]$



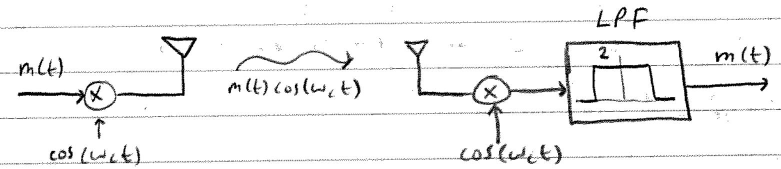
To demodulate, we multiply again by $\cos(\omega_c t)$



$$\mathcal{F}(m(t)\cos^2(\omega_c t)) = \frac{1}{2\pi}\left[\frac{1}{2}M(j(\omega+\omega_c)) + \frac{1}{2}M(j(\omega-\omega_c))\right] * [\pi\delta(\omega+\omega_c) + \pi\delta(\omega-\omega_c)]$$

$$= \frac{1}{4}M(j(\omega+2\omega_c)) + \frac{1}{2}M(j\omega) + \frac{1}{4}M(j(\omega-2\omega_c))$$

Lowpass filtering extracts $M(j\omega)$



This is a <u>synchronous</u> receiver meaning that the transmitter and receiver must be in phase. Synchronizing the receiver requires a more complex system (PLL).

One solution is to demodulate with a complex exponential.

$$e^{j\omega_c t} \Longleftrightarrow 2\pi\,\delta(\omega-\omega_c)$$

Then for an arbitrary phase shift θ between in the modulated signal:

$$\underset{\text{carrier}}{\mathcal{F}(\cos(\omega_c t-\theta))} = \mathcal{F}[\cos(\theta)\cos(\omega_c t) + \sin(\theta)\sin(\omega_c t)]$$

$$= \cos\theta\,(\pi\delta(\omega+\omega_c) + \pi\delta(\omega-\omega_c)) + \sin\theta\,(\pi j\delta(\omega+\omega_c) - \pi j\delta(\omega-\omega_c))$$

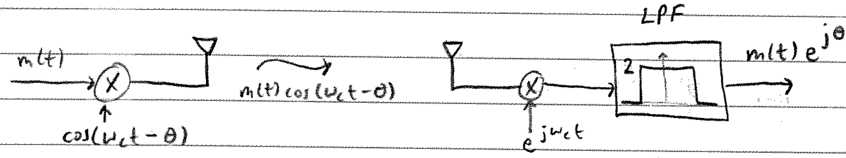
$$= \pi(\cos\theta + j\sin\theta)(\delta(\omega+\omega_c)) + \pi(\cos\theta - j\sin\theta)(\delta(\omega-\omega_c))$$

$$= \pi e^{j\theta}\delta(\omega+\omega_c) + \pi e^{-j\theta}\delta(\omega-\omega_c)$$

modulated

$$\Rightarrow \mathcal{F}(m(t)\cos(\omega_c t-\theta)) = \frac{1}{2}e^{j\theta}M(j(\omega+\omega_c)) + \frac{1}{2}e^{-j\theta}M(j(\omega-\omega_c))$$

$$\Rightarrow \underset{\text{demodulated}}{\mathcal{F}(m(t)\cos(\omega_c t-\theta)e^{j\omega_c t})} = \underbrace{\frac{1}{2}e^{j\theta}M(j\omega)}_{\text{recovered signal.}} + \frac{1}{2}e^{-j\theta}M(j(\omega-2\omega_c))$$



This is a quadrature receiver.
Cost is that the receiver has to be implemented for complex signals. This is done by keeping track of two real signals, the real part and the imaginary part (the I and Q channels).

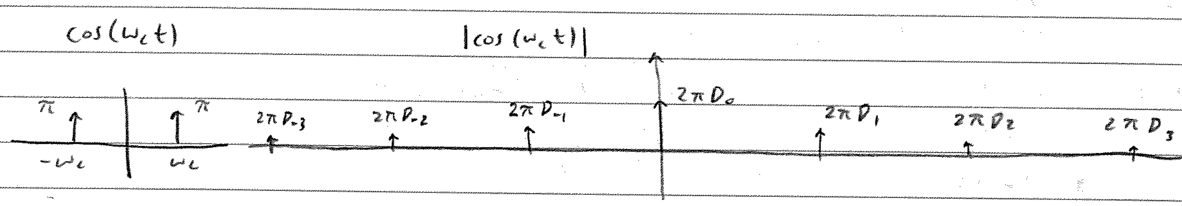
A cheap alternative that avoids the need for synchronization is AM modulation.
We add a constant before multiplying:

$$x_{AM}(t) = [A + m(t)] \cos(\omega_c t) \quad \text{such that} \quad A + m(t) > 0$$

Simple receivers for these signals are envelope detectors, (rectifier → LPF)
Choose LPF such that $\omega_{3dB_m} < \omega_{3dB_{LPF}} < \omega_c$

$$\mathcal{F}\left[|\cos(\omega_c t)|\right] = \mathcal{F}\left[\sum_{n=-\infty}^{\infty} D_n e^{j2n\omega_c t}\right] = \sum_{n=-\infty}^{\infty} D_n 2\pi\,\delta(\omega - 2n\omega_c)$$

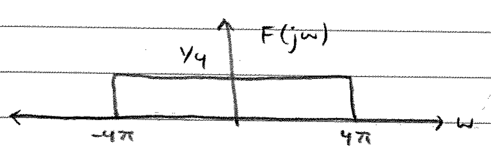
These are an array of δ's separated by $2\omega_c$



EE102A PSET 5

1, a) $f(t) = \text{sinc}(2t)\cos(2\pi t) \Leftrightarrow \frac{1}{2\pi}\left(\frac{1}{2}\text{rect}\left(\frac{\omega}{4\pi}\right) * \left(\pi\delta(\omega+2\pi) + \pi\delta(\omega-2\pi)\right)\right)$

$$= \frac{1}{4}\text{rect}\left(\frac{\omega+2\pi}{4\pi}\right) + \frac{1}{4}\text{rect}\left(\frac{\omega-2\pi}{4\pi}\right) = \frac{1}{4}\text{rect}\left(\frac{\omega}{8\pi}\right)$$



b) $F(j\omega) = \frac{1}{4}\text{rect}\left(\frac{\omega}{8\pi}\right) \Rightarrow \boxed{f(t) = \text{sinc}(4t)}$

2. a) $\int_{-\infty}^{\infty} f_1(t)\, f_2^*(t)\, dt = \int_{-\infty}^{\infty} f_1(t) \left(\frac{1}{2\pi}\int_{-\infty}^{\infty} F_2(j\omega)\, e^{j\omega t} d\omega\right)^* dt \qquad \left(e^{j\omega t^*} = e^{-j\omega t}\right)$

$= \frac{1}{2\pi}\int_{-\infty}^{\infty} f_1(t) \int_{-\infty}^{\infty} F_2^*(j\omega)\, e^{-j\omega t}\, d\omega\, dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} F_2^*(j\omega) \int_{-\infty}^{\infty} f_1(t)\, e^{-j\omega t} dt\, d\omega$

$= \frac{1}{2\pi}\int_{-\infty}^{\infty} F_2^*(j\omega)\, F_1(j\omega)\, d\omega$ ✓

b) real signals have Hermitian symmetry : $F(-j\omega) = F^*(j\omega)$

$\Rightarrow \int_{-\infty}^{\infty} f_1(t)\, f_2^*(t) = \int_{-\infty}^{\infty} f_1(t)\, f_2(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F_1(j\omega)\, F_2^*(j\omega) = \int_{-\infty}^{\infty} F_1(j\omega)\, F_2(-j\omega)$ ✓

c) $\mathrm{sinc}(t-n) \Leftrightarrow e^{-j\omega n}\, \mathrm{rect}\left(\frac{\omega}{2\pi}\right)$

$\mathrm{sinc}(t-m) \Leftrightarrow e^{-j\omega m}\, \mathrm{rect}\left(\frac{\omega}{2\pi}\right)$

$\Rightarrow \int_{-\infty}^{\infty} \mathrm{sinc}(t-n)\, \mathrm{sinc}(t-m)\, dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{j\omega(m-n)}\, \mathrm{rect}\left(\frac{\omega}{2\pi}\right) d\omega$

$= \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j\omega(m-n)}\, d\omega$ period of $e^{j\omega(m-n)}$ is $T_0 = \frac{2\pi}{(m-n)}$

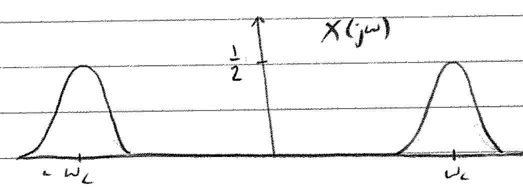
$\Rightarrow 2\pi = T_0\,(m-n)$ integral of a complex exponential over an integer number of periods is zero.

$= \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j\omega 0}\, d\omega = \frac{1}{2\pi}\int_{-\pi}^{\pi} 1\, d\omega = \frac{2\pi}{2\pi} = 1$

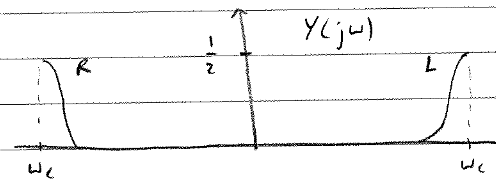
$\Rightarrow \int_{-\infty}^{\infty} \mathrm{sinc}(t-n)\, \mathrm{sinc}(t-m)\, dt = \begin{cases} 1, & n=m \\ 0, & n\neq m \end{cases}$ ✓

$\Rightarrow$ the set of sincs shifted by integer delays are an orthogonal set of signals.

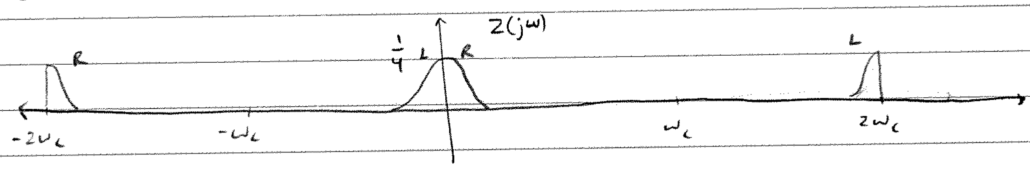
3. a)



b)



c)

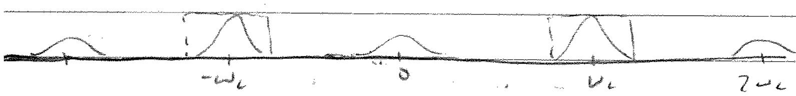


Yes, the system recovers $m(t)$.

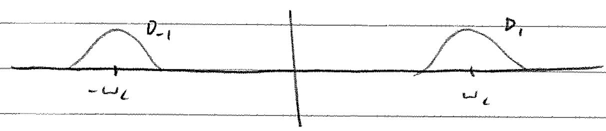
assume $T_0 = \frac{2\pi}{\omega_c}$

4. F.S. of $f(t)$: $f(t) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t}$ $\quad \omega_0 = \frac{2\pi}{T_0}$ $\quad D_n = \frac{1}{T_0}\int_{\tau}^{\tau+T_0} f(t)\, e^{-jn\omega_0 t}\, dt$

$\Rightarrow f(t) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_c t}$ $\quad D_0 = \int_0^T f(t)\,dt \neq 0$ $\quad D_1, D_{-1} \neq 0$



$\downarrow$



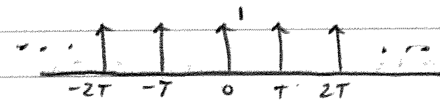
b) $T = \frac{2\pi}{\omega_c}$ bandwidth is $2\pi B$ need $\omega_c = \frac{2\pi}{T} \geq 2\pi B$

$\Rightarrow T \leq \frac{1}{B}$ Longest possible period is $\frac{1}{B}$

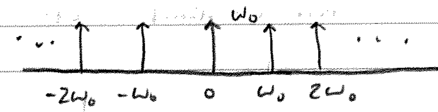
Systems $\boxed{\text{EE102A}}$ $\boxed{\text{L13}}$ Impulse Trains, Periodic Signals, + Sampling 1/4/12

Impulse Trains - Sampling Functions

$$\delta_T(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT)$$



$$\delta_T(t) \Leftrightarrow \omega_0 \delta_{\omega_0}(\omega) \qquad \omega_0 = \frac{2\pi}{T}$$



Ideal Sampling:

$$f(t)\,\delta_T(t) = \sum_{n=-\infty}^{\infty} f(nT)\,\delta(t-nT)$$

Fourier Transform of Periodic Signals:

if $f_1(t)$ is one cycle of a periodic function with period T,

$$f(t) = f_1(t) * \delta_T(t) \Rightarrow F(j\omega) = \omega_0 F_1(j\omega)\,\delta_{\omega_0}(\omega) \qquad \omega_0 = \frac{2\pi}{T}$$

$\Rightarrow$ The Fourier transform of a periodic signal is the sampled Fourier transform of one period.

Sampling Theorem: F.T. of time-sampled signal:

$$\bar{f}(t) = f(t)\,\delta_T(t) \qquad \bar{F}(j\omega) = \frac{1}{T} F(j\omega) * \delta_{\omega_0}(\omega) = \frac{1}{T}\sum_{n=-\infty}^{\infty} F(j(\omega - n\omega_0$$

sampled signal

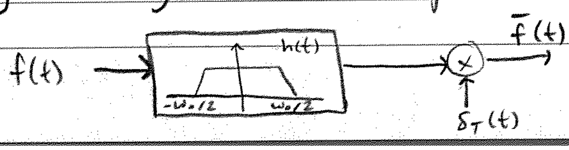
$\Rightarrow$ The spectrum of the sampled signal consists of shifted replicas of the original spectrum scaled by $\frac{1}{T}$.

Nyquist Rate:

$\frac{1}{T} > 2B \leftarrow$ Nyquist rate.

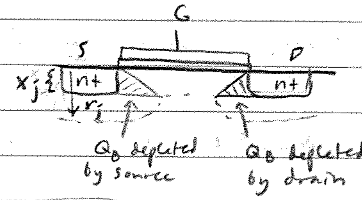
The signal can be recovered exactly (no aliasing) if sampling rate $\frac{1}{T}$ Hz is greater than the Nyquist rate ($2B$).

Minimizing Aliasing: First lowpass filter the signal, then sample:



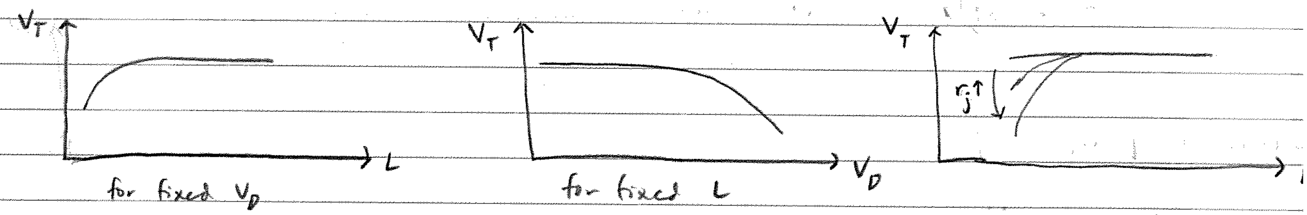
Devices $\boxed{EE311}$ $\boxed{L3}$ Shallow Junctions + Contacts

Short Channel Effects of Shallow Junctions:



$$V_T = V_{FB} - 2\phi_F - \frac{Q_B}{C_{ox}}\left[1 - \left(\sqrt{1 + \frac{2W}{r_j}} - 1\right) \cdot \frac{r_j}{L}\right]$$

$\Rightarrow V_T$ is a function of junction depth, depletion width, + channel length



Rolloff in V_T as $L\downarrow$ and $V_d\uparrow$

To minimize V_T rolloff: $r_j\downarrow$, $C_{ox}\uparrow$ (to increase gate control)

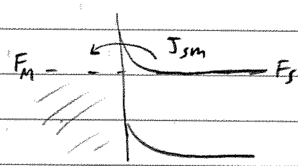
Sheet resistance increases as junction depth is reduced. (2009: $x_j \simeq 15$nm

$R_{sh} \propto \frac{1}{N_{sd} x_j}$ $\quad N_{sd_{MAX}}$ limited by solid solubility $\Rightarrow$ does not scale.

- Contact resistance is one of the dominant components for future technology. (L<
- Silicidation of junctions reduces parasitic resistance.

Tunneling - Ohmic Contacts

$$X_d = \sqrt{\frac{2K\epsilon_0 \phi_i}{q N_d}}$$



when $x_d \leq 2.5-5$nm, é's can tunnel directly through the barrier.

Required doping is: $N_{d_{min}} = 6 \cdot 10^{19}\,cm^{-3}$ for $x_d = 2.5$nm

$P(E)$ is tunneling probability: $P(E) \propto \exp\left(-\frac{2\Phi_B}{\hbar}\sqrt{\frac{\epsilon_s m^*}{N}}\right)$ doping density

$$\Rightarrow J_{sm} \propto \exp\left(-2x_d\sqrt{2m^*(q\Phi_B - qV)/\hbar^2}\right)$$

Specific contact resistivity $\rho_c = \rho_{co} \exp\left(\frac{2\Phi_B}{\hbar}\sqrt{\frac{\epsilon_s m^*}{N}}\right)$ Ω-cm^2

$\Rightarrow \rho_c\downarrow$ as doping density $N\uparrow$, barrier height $\Phi_B\downarrow$, effective mass $m^*\downarrow$

For a given doping density, contact resistance is higher for n-type than p-type Si since $\Phi_{B_n} > \Phi_{B_p}$

Systems $\boxed{\text{EE102A}}$ $\boxed{\text{L14}}$ The Laplace Transform 1/5/2012

Fourier Transform:

- must exist or be defined in a generalized sense.
- usually exists for communications, optics, image processing.

BUT - For many signals + systems Fourier Transform doesn't exist
  - signals that grow with time (e.g. bank account)
  - unstable systems (many mechanical or electrical systems)
- For these cases, we must use the Laplace Transform.

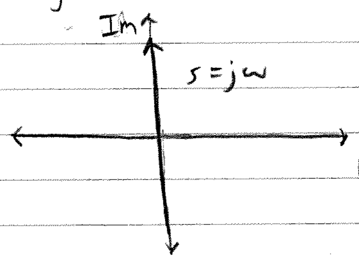
Bilateral Laplace Transform:

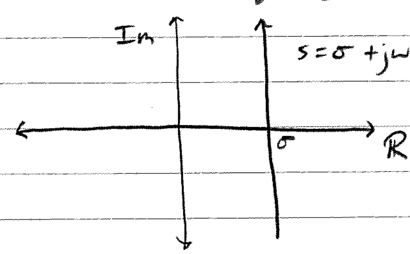
$$F(s) = \int_{-\infty}^{\infty} f(t)\, e^{-st}\, dt \qquad \text{where} \quad s = \sigma + j\omega$$

$$f(t) = \frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} F(s)\, e^{st}\, ds \qquad \text{where} \quad c > \sigma_0 \qquad \text{(but we will never use this)}$$

Notation: $\mathcal{L}[f(t)] = F(s)$ $\quad f(t) = \mathcal{L}^{-1}[F(s)]$ $\quad f(t) \Leftrightarrow F(s)$

Fourier Transform is a special case of the Laplace Transform: $F(j\omega) = F(s)\big|_s$
provided that the $j\omega$ axis is in the region of convergence.
$\Rightarrow$ The Fourier transform is the Laplace transform evaluated along the $j\omega$ axis. It is also called the frequency response.





Main motivation for Laplace Transform:

- It converts integral + differential equations into algebraic equations.
- Handles growing exponentials + unstable systems
- Easily includes non-steady-state conditions.

Unilateral Laplace Transform: The Bilateral Laplace Transform for causal sign

$$F(s) = \int_{0^-}^{\infty} f(t)\, e^{-st}\, dt$$

Table of Laplace Transforms

function	Laplace Transform	Property
General:		
$f(t)$	$F(s) = \int_0^\infty f(t)\, e^{-st}\, dt$	definition
$f(t) + g(t)$	$F(s) + G(s)$	superposition } Lineari
$\alpha f(t)$	$\alpha F(s)$	homogeneity } Lineari
f'	$sF(s) - f(0^-)$	differentiation
f''	$s^2 F(s) - sf(0^-) - f'(0^-)$	2nd derivative
$g(t) = \int_0^t f(\tau)\, d\tau$	$G(s) = \frac{1}{s} F(s)$	running integral
$f(\alpha t)$, $\alpha > 0$	$\frac{1}{\alpha} F\left(\frac{s}{\alpha}\right)$	time scaling
$e^{at} f(t)$	$F(s-a)$	frequency shifting
$t\, f(t)$	$-\frac{dF(s)}{ds}$	multiplication by t
$t^k f(t)$	$(-1)^k \frac{d^k F(s)}{ds^k}$	" "
$\frac{f(t)}{t}$	$\int_s^\infty F(s)\, ds$	division by t
$g(t) = \begin{cases} 0 & 0 < t < T \\ f(t-T) & t \geq T \end{cases}$	$G(s) = e^{-sT} F(s)$	time shift (delay)

function	Laplace Transform	comment
specific:		
$u(t)$ or 1	$\frac{1}{s}$	
δ	1	
δ^k	s^k	kth derivative of δ
t	$\frac{1}{s^2}$	ramp $= u(t) * u(t)$
t^k, $k \geq 0$	$\frac{k!}{s^{k+1}}$	powers of t
e^{at}	$\frac{1}{s-a}$	
$\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$	$= \frac{(1/2)}{s-j\omega} + \frac{(1/2)}{s+j\omega}$
$\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$	$= \frac{(1/2j)}{s-j\omega} - \frac{(1/2j)}{s+j\omega}$
$e^{-at}\cos(\omega t)$	$\frac{s+a}{(s+a)^2+\omega^2}$	frequency shifting
$e^{-at}\sin(\omega t)$	$\frac{\omega}{(s+a)^2+\omega^2}$	" "

Systems $\boxed{\text{EE102A}}$ $\boxed{\text{L15}}$ Inversion of the Laplace Transform

$$f(t) = \frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} F(s)\, e^{st}\, dt \quad \leftarrow \text{complicated}$$

Simpler approach: rewrite rational Laplace transform in simple terms we recognize & can invert by inspection (partial fraction

Partial Fraction Expansion:

residues.

$$F(s) = \frac{b(s)}{a(s)} = \frac{r_1}{s-\lambda_1} + \cdots + \frac{r_n}{s-\lambda_n}$$

roots of $a(s)$, poles of F

Heaviside "cover-up" procedure: $r_k = (s-\lambda_k)\,F(s)\Big|_{s=\lambda_k}$

e.g. $$\frac{s^2-2}{s(s+1)(s+2)} = \frac{r_1}{s} + \frac{r_2}{s+1} + \frac{r_3}{s+2}$$

$$\Rightarrow r_1 = \frac{0-2}{(1)(2)} = -1 \qquad r_2 = \frac{1-2}{-1\,(1)} = 1 \qquad r_3 = \frac{4-2}{-2(-1)} = 1 \qquad \checkmark$$

Another Method: $r_k = \dfrac{b(\lambda_k)}{a'(\lambda_k)}$

e.g. $$\frac{s^2-2}{s(s+1)(s+2)} = \frac{s^2-2}{s^3+3s^2+2s} \Rightarrow a'(s) = 3s^2+6s+2$$

$$\Rightarrow r_1 = \frac{0-2}{2} = -1 \qquad r_2 = \frac{1-2}{3-6+2} = 1 \qquad r_3 = \frac{4-2}{12-12+2} = 1 \qquad \checkmark$$

Last resort: Clear Fractions, solve linear equations:

$$\frac{s^2-2}{s(s+1)(s+2)} = \frac{r_1}{s} + \frac{r_2}{s+1} + \frac{r_3}{s+2}$$

$$\Rightarrow s^2-2 = r_1(s+1)(s+2) + r_2(s)(s+2) + r_3(s)(s+1)$$

equate coefficients of s^2, s, s^0. solve for r_1, r_2, r_3.

Nonproper Rational Functions: if order of $a(s) \le$ order of $b(s)$

$\Rightarrow$ divide a into b using long division.

Systems EE102A PSET 5 (woops. I've already done this PSET)

a) $f(t) = \text{sinc}(2t)\cos(2\pi t)$

$\text{rect}(t) \Leftrightarrow \text{sinc}\left(\frac{\omega}{2\pi}\right) \Rightarrow$ by duality $\text{sinc}\left(\frac{t}{2\pi}\right) \Leftrightarrow 2\pi\,\text{rect}(\omega)$

$\Rightarrow \text{sinc}(t) \Leftrightarrow \text{rect}\left(\frac{\omega}{2\pi}\right) \Rightarrow \text{sinc}(2t) \Leftrightarrow \frac{1}{2}\text{rect}\left(\frac{\omega}{4\pi}\right)$

$\cos(2\pi t) \Leftrightarrow \pi\left(\delta(\omega-2\pi) + \delta(\omega+2\pi)\right)$

$\Rightarrow F(j\omega) = \frac{1}{2\pi}\left(\frac{1}{2}\text{rect}\left(\frac{\omega}{4\pi}\right)\right) * \left(\pi\left(\delta(\omega-2\pi)+\delta(\omega+2\pi)\right)\right) = \frac{1}{4}\text{rect}\left(\frac{\omega}{8\pi}\right.$

b) $f(t) = \mathcal{F}^{-1}\left(\frac{1}{4}\text{rect}\left(\frac{\omega}{8\pi}\right)\right) = \text{sinc}(4t)$

Systems EE102A PSET 6

1. $f(t) = \text{sinc}^2\left(\frac{t}{3}\right)\left(\Delta(3t) * \delta_1(t)\right)$

$\text{sinc}(t) \Leftrightarrow \text{rect}\left(\frac{\omega}{2\pi}\right) \Rightarrow \text{sinc}^2(t) \Leftrightarrow \Delta\left(\frac{\omega}{2\pi}\right) \Rightarrow \text{sinc}^2\left(\frac{t}{3}\right) \Leftrightarrow 3\Delta\left(\frac{3\omega}{2\pi}\right)$

$\text{rect}(t) \Leftrightarrow \text{sinc}\left(\frac{\omega}{2\pi}\right) \Rightarrow \Delta(t) \Leftrightarrow \text{sinc}^2\left(\frac{\omega}{2\pi}\right) \Rightarrow \Delta(3t) \Leftrightarrow \frac{1}{3}\text{sinc}^2\left(\frac{\omega}{6\pi}\right)$

$\delta_T(t) \Leftrightarrow \omega_0\,\delta_{\omega_0}(\omega) \Rightarrow \delta_1(t) \Leftrightarrow 2\pi\,\delta_{2\pi}(\omega)$

$\Delta(3t) * \delta_1(t) \Leftrightarrow \frac{1}{3}\text{sinc}^2\left(\frac{\omega}{6\pi}\right)\cdot 2\pi\,\delta_{2\pi}(\omega)$

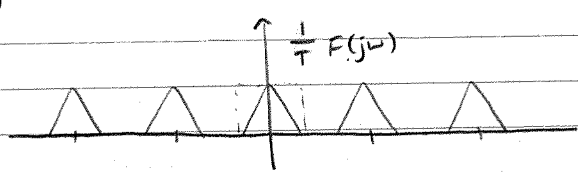
$\text{sinc}^2\left(\frac{t}{3}\right)\left(\Delta(3t) * \delta_1(t)\right) \Leftrightarrow \Delta\left(\frac{3\omega}{2\pi}\right) * \left(\text{sinc}^2\left(\frac{\omega}{6\pi}\right)\delta_{2\pi}(\omega)\right)$

$= \sum_{n=-\infty}^{\infty} \text{sinc}^2\left(\frac{n}{3}\right)\Delta\left(\frac{3(\omega - 2\pi n)}{2\pi}\right)$

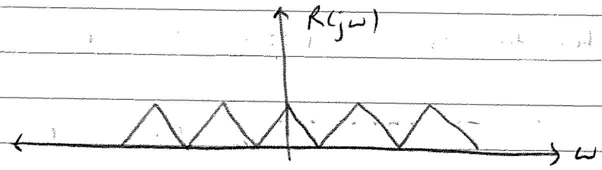
2. a) $r(t) = f(t)\cos(\omega_c t)\,\delta_{2\pi/\omega_c}(t)$

$\Rightarrow R(j\omega) = \frac{1}{2\pi}\mathcal{F}\left(f(t)\cos(\omega_c t)\right) * \omega_c\,\delta_{\omega_c}(\omega) = \frac{\omega_c}{4\pi}\left(F(j(\omega+\omega_c)) + F(j(\omega-\omega_c))\right) * \delta_{\omega}$

$\Rightarrow$



b) $\omega_{min} = \frac{\omega_c}{n}$ with the largest $n \in \mathbb{Z}$ such that $\frac{\omega_c}{n} \geq 2(2\pi B) = 4\pi B$ (Nyquist)



3. a) $\tilde{f}(t) = \bar{f}(t) * p(t) = \bar{f}(t) * \text{rect}\left(\frac{4t}{T}\right)$ $\quad \text{rect}(t) \Leftrightarrow \text{sinc}\left(\frac{\omega}{2\pi}\right)$

b) $\tilde{F}(j\omega) = \frac{T}{4} \text{sinc}\left(\frac{\omega T}{8\pi}\right) \bar{F}(j\omega)$

c) No. The filter will extract $F(j\omega)\left(\frac{T}{4}\text{sinc}\left(\frac{\omega T}{8\pi}\right)\right)$

d) $H_r(j\omega) = \dfrac{\text{rect}\left(\frac{\omega T}{2\pi}\right)}{\left(\frac{T}{4}\text{sinc}\left(\frac{\omega T}{8\pi}\right)\right)}$

4. a) $f(t) = \delta_2(t) + \delta_2(t-1-\tau)$

b) $F(j\omega) = \pi\delta_\pi(\omega) + e^{-j\omega(1+\tau)}\pi\delta_\pi(\omega) = \pi\left(1+e^{-j\omega(1+\tau)}\right)\delta_\pi(\omega)$

$= \sum_{n=-\infty}^{\infty} \pi\left(1+e^{-j\omega(1+\tau)}\right)\delta(\omega-n\pi) = \pi\sum_{n=-\infty}^{\infty}\left(1+e^{-jn\pi(1+\tau)}\right)\delta(\omega-n\pi)$

$= \pi\sum_{n=-\infty}^{\infty}\delta(\omega-n\pi)\left(1+e^{-jn\pi}e^{-jn\pi\tau}\right) = \pi\sum_{n=-\infty}^{\infty}\delta(\omega-n\pi)\left(1+(-1)^n e^{-jn\pi\tau}\right)$

c) $F(j\omega)\big|_{\tau=0} = \pi\sum_{n=-\infty}^{\infty}\delta(\omega-n\pi)\left(1+(-1)^n\right) = 2\pi\sum_{n=-\infty}^{\infty}\delta(\omega-2\pi n) = 2\pi\delta_{2\pi}(\omega)$

d)

e) $g(t)$ is real + even $\quad$ real $\Rightarrow$ hermitian $\quad G(-j\omega) = G^*(j\omega)$

even $\Rightarrow$ even. $\quad G(-j\omega) = G(j\omega)$

$\Rightarrow G(j\omega)$ is real + even.

Yes. We can reconstruct $G(j\omega)$. We just need to LPF $\bar{G}(j\omega)$ and take the real part.

Systems $\boxed{\text{EE102A}}$ $\boxed{\text{L16}}$ Applications of the Laplace Transform 1/6/2012

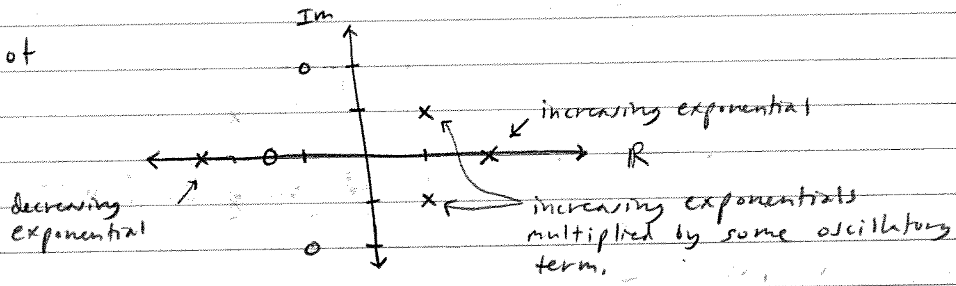
Laplace Transforms allow us to solve LCCODE's.

LCCODE's have rational transfer functions $H(s) = \frac{b(s)}{a(s)}$ of the form:

$$H(s) = \frac{b_0 + b_1 s + \cdots + b_m s^m}{a_0 + a_1 s + \cdots + a_n s^n} = k \frac{(s-z_1)\cdots(s-z_m)}{(s-p_1)\cdots(s-p_n)} \begin{matrix} \leftarrow \text{zeros} \\ \leftarrow \text{poles} \end{matrix} \qquad k = \frac{b_m}{a_n}$$

When coefficients of $a(s)$ + $b(s)$ are all real, the poles and zeros of $H(s)$ (roots of $b(s)$ and $a(s)$) are real or come in complex pairs.

Pole-Zero Plot



If the poles of $H(s)$ are $p_1, \ldots, p_n$ the asymptotic growth (or decay) rate of $h(t)$ is determined by the maximum real parts

$$\alpha = \max\{\mathcal{R}(p_1), \ldots, \mathcal{R}(p_n)\}$$ poles which achieve this max real part are called dominant.

e.g. $H(s) = \frac{100}{s+2} + \underbrace{\frac{1}{s+1}}_{}$ (dominant) $\Rightarrow h(t) = 100e^{-2t} + e^{-t}$

System is <u>stable</u> only when all poles of $H(s)$ have negative real part (roots of $a(s)$ in left half plane).

Characteristic Equation: The poles of a system depend only on the coefficients of:

$$a_n y^{(n)} + a_{n-1} y^{(n-1)} + \cdots + a_0 y = 0$$

With zero initial conditions, the Laplace transform of this equation is:

$$Y(s)\left[a_n s^n + a_{n-1} s^{n-1} + \cdots + a_0\right] = 0$$

The polynomial is the characteristic polynomial for the system:

$$\mathcal{X}(s) = a_n s^n + a_{n-1} s^{n-1} + \cdots + a_0$$

First Order System: $X(s) = a_1 s + a_0$

has single pole at $\lambda = -a_0/a_1$ and corresponds to a single exponent

Stable if pole is in left half plane $\Rightarrow -\frac{a_0}{a_1} < 0$

$\Rightarrow$ Stable iff both $\boxed{a_1 > 0 \text{ and } a_0 > 0}$ (coefficients all positive)

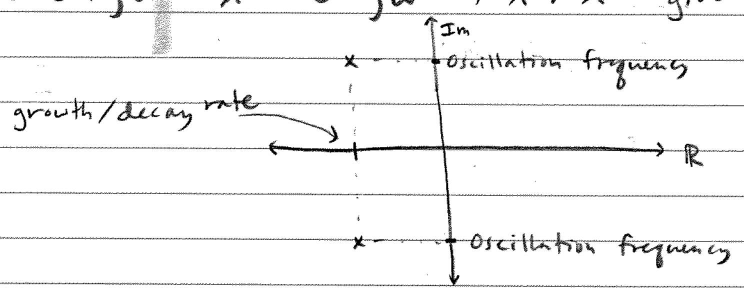
Second Order System: $X(s) = a_2 s^2 + a_1 s + a_0 = s^2 + bs + c$

has 2 poles: $\lambda_{1,2} = -\left(\frac{b}{2}\right) \pm \sqrt{\left(\frac{b}{2}\right)^2 - c}$

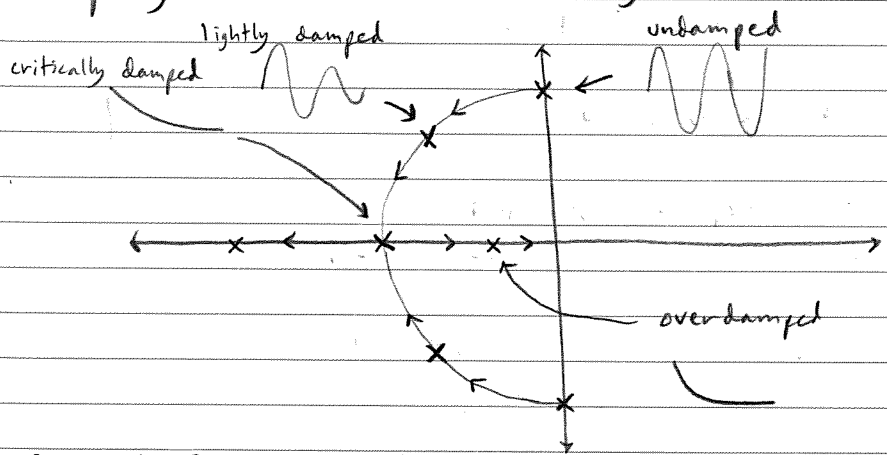
Stable iff $\boxed{a_2 > 0,\ a_1 > 0, \text{ and } a_0 > 0}$ (coefficients all posit

Properties of Complex Pole Pair

$\lambda = \sigma + j\omega \quad \lambda^* = \sigma - j\omega \Rightarrow \lambda + \lambda^*$ give the time-domain term $ae^{\sigma t}\cos($



Damping of Second Order Systems



Critically damped case has fastest decay rate. This is often a good design choice.

For a complex system, typically either a single pole or complex pair will be dominant.

You can tell a lot about the system by the characteristics of this dominant pole: (without having to explicitly solve for the time response

- Stability, Type of solution, Rate of Decay.

Systems $\boxed{EE102A}$ $\boxed{L17}$ Frequency Response, Bode Plots, and Filters

The Laplace transform $H(s)$ exists for any $s = \sigma + j\omega$ in the region of convergence.

One special case is when $\sigma = 0$ and $s = j\omega$. This corresponds to the Fourier transform, and $\underline{H(j\omega)}$ is called the <u>Frequency Response</u> of the system.

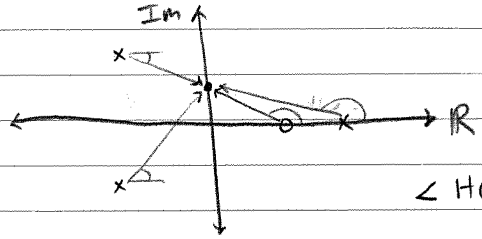
The frequency response characterizes the system in steady state (after the transients have died out).

2 applications of frequency response: Filter Design + Feedback Control.

Bode Plots: $$20\log_{10}|H(j\omega)| = 20\log_{10}|k| + \sum_{i=1}^{m} 20\log_{10}|j\omega - z_i| - \sum_{i=1}^{n} 20\log_{10}|j\omega - p_i|$$

$$\angle H(j\omega) = \angle k + \sum_{i=1}^{m}\angle(j\omega - z_i) - \sum_{i=1}^{n}\angle(j\omega - p_i)$$

Graphical Interpretation: $|H(j\omega)|$, $\angle H(j\omega)$



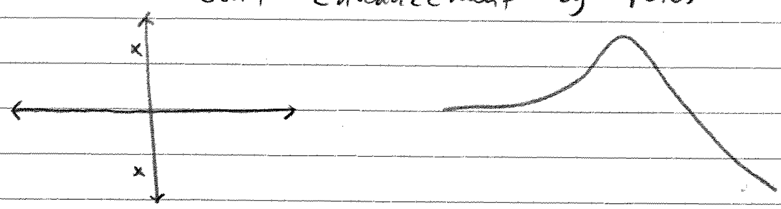
$$|H(j\omega)| = |k| \frac{\prod_{i=1}^{m}|s - z_i|}{\prod_{i=1}^{n}|s - p_i|}$$

$$\angle H(j\omega) = \angle k + \sum_{i=1}^{m}\angle(j\omega - z_i) - \sum_{i=1}^{n}\angle(j\omega - p_i)$$

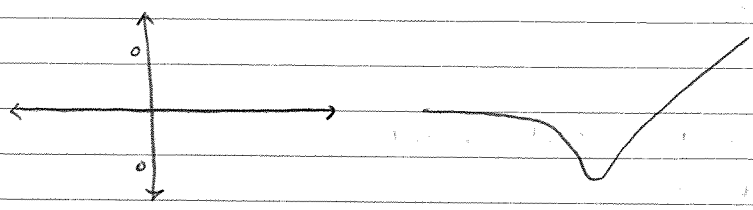
$\Rightarrow$ $|H(j\omega)|$ gets big when $j\omega$ is near a pole
$|H(j\omega)|$ gets small when $j\omega$ is near a zero

$\angle H(j\omega)$ changes rapidly when $j\omega$ is near a pole or zero.

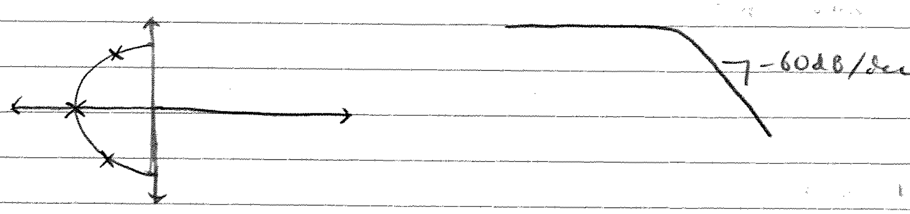
Gain Enhancement by Poles



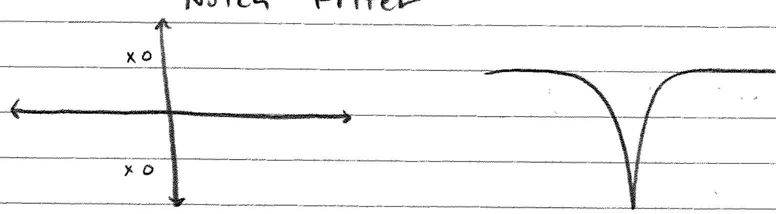
Gain Suppression by Zeros



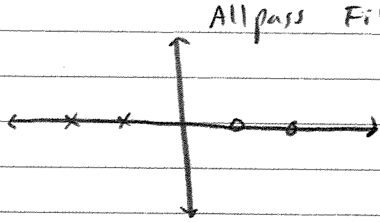
Butterworth Filter



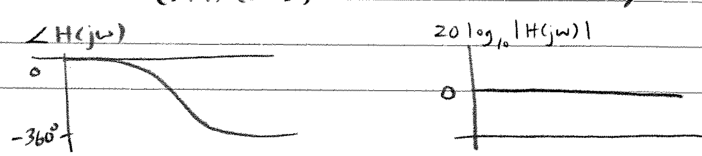
Notch Filter



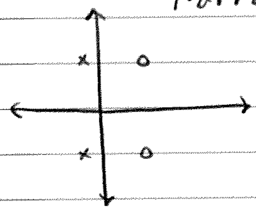
Allpass Filter

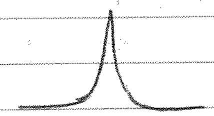


$$H(s) = \frac{(s-1)(s-3)}{(s+1)(s+3)} \qquad |H(j\omega)| = 1, \ \forall \omega$$

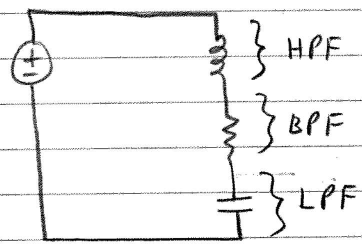


Narrow band band pass



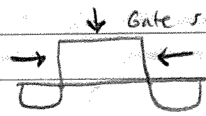


RLC Circuit Examples.



Devices $\boxed{\text{EE311}}$ $\boxed{\text{L4}}$ Silicides + Metal Gates 1/7/2012

MOS Gate Electrode



⇒ As channel length is scaled, gate resistance increases.

- Gate electrode is also used as an interconnect layer in many applications.

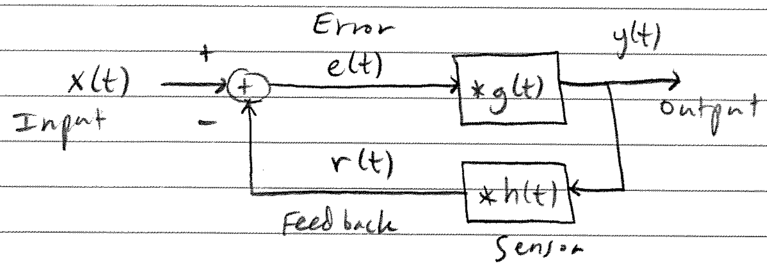
Silicidation of junctions is necessary to minimize the impact of junction parasitic resistance

Why we use silicides:

- Silicides are Si-metal compounds.
- Low resistance
- Good process compatibility with Si.
- Little or no electromigration.
- Easy to dry etch.
- Good contacts to other materials.

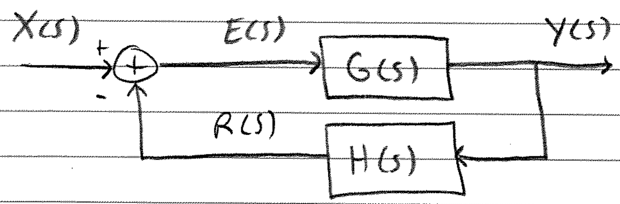
Systems $\boxed{EE102A}$ $\boxed{L18}$ Feedback 1/8/201

Feedback :
- reduce sensitivity to parameter variation
- reduces sensitivity to external interference,
- reduces nonlinearity



time domain

$\Downarrow$



transfer function

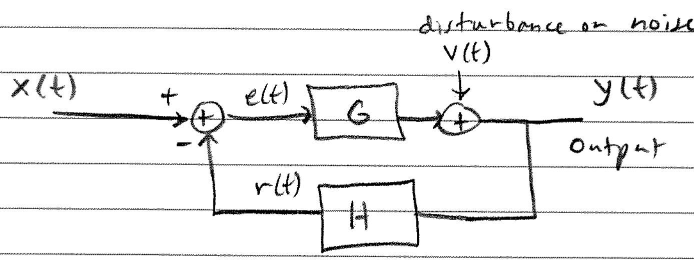
$$Y(s) = (X(s) - H(s)Y(s))\,G(s) \Rightarrow T(s) = \frac{G(s)}{1 + G(s)H(s)}$$

$G(s)H(s)$ is <u>Loop Gain</u>

Can trade gain for sensitivity.

$$S = \frac{\Delta T/T}{\Delta G/G} = \frac{1}{1+GH}$$ reduced by $1 + GH$ ← loop gain.

- Reducing Effects of Disturbances with Feedback:



$y(t)\Big|_{v(t)=0} = \frac{G}{1+GH} x(t) \qquad y(t)\Big|_{x(t)=0} = \frac{1}{1+GH} v(t)$

$\Rightarrow y(t) = \frac{G}{1+GH} x(t) + \frac{1}{1+GH} v(t)$

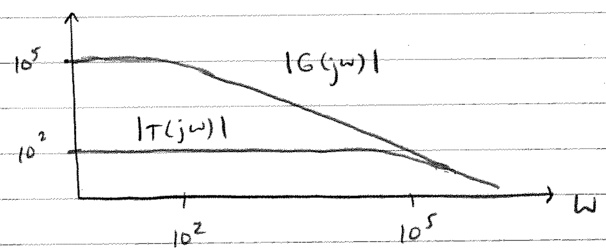
if $GH \gg 1$, $y(t) \simeq \frac{1}{H} x(t) + \frac{1}{GH} v(t)$

$\Rightarrow$ interference is suppressed by the reciprocal of the loop gain, while the closed loop transfer function has a gain of the reciprocal of the feedback gain.

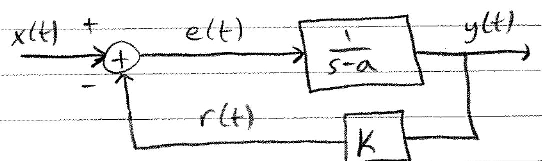
Can also trade gain for improved linearity.

For dynamic systems:

Can trade gain for bandwidth.



Stabilization of Unstable Systems:

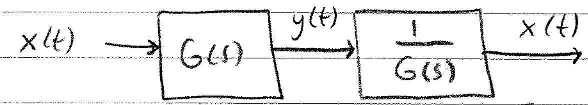


The plant has a transfer function that is an increasing exponential $(a>0)$. It is unstable.

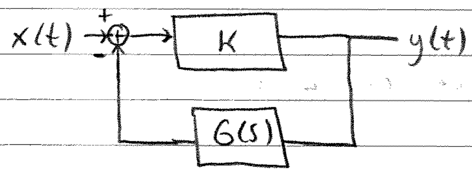
$$T(s) = \frac{G(s)}{1+G(s)H(s)} = \frac{\frac{1}{s-a}}{1+\frac{K}{s-a}} = \frac{1}{s-a+K}$$

this is stable for $K>a$. $\Rightarrow$ we can get any decay rate with enough feedback gain.

Inverse Systems



One way to implement this inverse system is with feedback:



$$T(s) = \frac{K}{1+KG(s)}$$

for frequencies where $KG(s) \gg 1$

$$T(s) \approx \frac{1}{G(s)}$$

Feedback Summary:

Can be used to

- Reduce sensitivity to system variations
- Suppress the effect of disturbances or noise
- Improve the linearity of a non-linear system

The costs of feedback are

- Increased complexity
- Reduced gain
- Potential instability.

Systems $\boxed{\text{EE102A}}$ $\boxed{\text{L19}}$ Overview and Conclusions

Key ideas:

- Linearity & Time Invariance
- Convolution systems
- Complex exponentials, and transfer functions
- Representation of signals by linear combos of complex exponentials
    - Fourier Series
    - Fourier Transform
- Convolution becomes multiplication (and vice versa)
- Communications and modulation

- Sampling and reconstruction
- Even unstable signals can have transforms
    - Laplace Transform
- Solving for the evolution of dynamic systems
- Steady state frequency response
- Feedback and automatic control.

Complex exponential $e^{j\omega t}$ is an eigenfunction of LTI system with eigenvalue $H(j\omega)$

Since complex exponentials are so easy to analyze for LTI systems, we'd like to represent arbitrary signals as linear combinations of complex exponentials.

- If $f(t)$ is periodic or of finite duration: <u>Fourier Series</u>

$$f(t) = \sum_{n=-\infty}^{\infty} D_n e^{j\omega_0 n t} \qquad D_n = \frac{1}{T_0}\int_{\tau}^{\tau+T_0} f(t)\, e^{-j\omega_0 n t}\, dt$$

where T_0 is the fundamental period and $\omega_0 = \frac{2\pi}{T_0}$

- If $f(t)$ is aperiodic, but is an energy or power signal: <u>Fourier Transform</u>

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(j\omega)\, e^{j\omega t}\, dt \qquad F(j\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$$

- If $f(t)$ is not a power signal, we use the <u>Laplace Transform</u>

$$f(t) = \frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} F(s)\, e^{st}\, dt \qquad F(s) = \int_{-\infty}^{\infty} f(t)\, e^{-st}\, dt$$

Although the specifics differ, all of these have similar characteristics:
- Expanding a signal in time compresses it in frequency.
- Time shift $\rightarrow$ multiplication by complex exponential in frequency.
- Multiplication by a complex exponential in time $\rightarrow$ frequency shift.
- Convolution in time domain $\rightarrow$ multiplication in frequency
- Multiplication in time domain $\rightarrow$ convolution in frequency domain

Convergence:

Frequency domain representations:

- Converge to the midpoint of a discontinuity
- Oscillate at either side of discontinuity (Gibbs Effect)

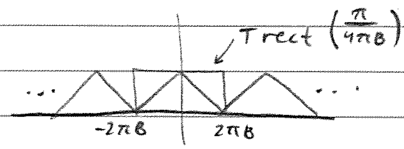
Impulse Trains: $\delta_T(t) \Leftrightarrow \omega_0 \delta_{\omega_0}(\omega)$ relates F.S. to F.T.

If $f_1(t)$ is one cycle of a periodic function $f(t)$ with period T:

$$f(t) = f_1(t) * \delta_T(t) \Rightarrow F(j\omega) = \omega_0 F_1(j\omega)\,\delta_{\omega_0}(\omega)$$

$\Rightarrow$ F.T. of periodic signal is sampled F.T. of one period.

Sampling:



$\Rightarrow$ LPF with $H(j\omega) = T\,\mathrm{rect}\left(\frac{\pi}{4\pi B}\right)$ $\quad h(t) = \mathrm{sinc}(2Bt)$

to recover $f(t)$ from $\bar{f}(t)$

Reconstructed signal is:

$$\bar{f}(t) * h(t) = \left(\sum_{n=-\infty}^{\infty} f(nT)\,\delta(t-nT)\right) * h(t) = \sum_{n=-\infty}^{\infty} f(nT)\,\mathrm{sinc}(2Bt-n)$$

This is the Shannon sampling theorem

Periodic signals have sampled spectra.

Sampled signals have periodic spectra.

Dynamic Systems: Laplace Transform

Differentiation: multiply by s : $f'(t) \longrightarrow sF(s) - f(0)$

Integration: multiply by $1/s$: $\int_0^t f(\tau)\,d\tau \longrightarrow \frac{1}{s}F(s)$

EE102A PSET 7

1. a) $f(t) = (1-t^2)e^{-2t} = e^{-2t} - t^2 e^{-2t}$

$t^k f(t) \iff (-1)^k \frac{d^k F(s)}{ds^k}$

$\Rightarrow t^2 e^{-2t} \iff (-1)^2 \frac{d^2}{ds^2}\left(\frac{1}{s+2}\right) = \frac{d}{ds}\left(\frac{-1}{(s+2)^2}\right) = \frac{2(s+2)}{(s+2)^4} = \frac{2}{(s+2)^3}$

$\Rightarrow F(s) = \frac{1}{s+2} - \frac{2}{(s+2)^3} = \frac{s^2+4s+2}{(s+2)^3}$

b) $f(t) = \begin{cases} 0 & 0 \le t < 1 \\ 1 & 1 \le t < 2 \\ e^{-2(t-2)} & 2 \le t \end{cases}$

$= u(t-1) - u(t-2) + e^{-2(t-2)}$

$\Rightarrow F(s) = e^{-s}\frac{1}{s} - e^{-2s}\frac{1}{s} + e^{-2s}\left(\frac{1}{s+2}\right)$

$= \frac{e^{-s}}{s} + e^{-2s}\left(-\frac{2}{s(s+2)}\right)$

$= \frac{(s+2)e^{-s} - 2e^{-2s}}{s(s+2)} = \frac{se^{-s} + 2(e^{-s} - e^{-2s})}{s(s+2)}$

2. $f(t) = \begin{cases} \sin(2\pi t) & 0 \le t < 1 \\ 0 & 1 \le t \end{cases}$

$f(t) = \sin(2\pi t) - \sin(2\pi(t-1))$

$= \frac{2\pi}{s^2+4\pi^2} - \frac{2\pi}{s^2+4\pi^2} e^{-s} = \frac{2\pi}{s^2+4\pi^2}(1-e^{-s})$

3. $f(t) = \left(\frac{1}{2} + \frac{1}{2}\cos(4\pi t)\right)e^{-t} \iff \frac{1}{2}\left(\frac{s+1}{(s+1)^2+16\pi^2} + \frac{1}{(s+1)}\right)$

4. a) $\frac{s^2}{(s+1)^2}$

$$\begin{array}{r|l} & 1 \\ s^2 & s^2+2s+1 \\ & -s^2 \\ \hline & 2s+1 \end{array}$$

$\Rightarrow 1 - \frac{2s+1}{(s+1)^2}$

$\frac{2s+1}{(s+1)^2} = \frac{r_1}{(s+1)^2} + \frac{r_2}{(s+1)} \Rightarrow r_1 = -1 \quad r_2 = 2$

$\Rightarrow 1 + \frac{1}{(s+1)^2} - \frac{2}{s+1} \Rightarrow f(t) = (\delta(t) + te^{-t} - 2e^{-t})u(t)$

$\Rightarrow f(t) = (\delta(t) + (t-2)e^{-t})u(t)$

b) $$\frac{4}{s^3+4s} = \frac{4}{s(s^2+4)} = \frac{r_1}{s} + \frac{r_2 s + r_3}{s^2+4} \Rightarrow r_1 = 1$$

$$r_2 = \lim_{s\to\infty}\left(\frac{4}{(s^2+4)} - r_1\right) = -r_1 = -1 \qquad r_2 + r_3 = -1 \Rightarrow r_3 = 0$$

$$\Rightarrow \frac{1}{s} - \frac{s}{s^2+4} \Rightarrow f(t) = (1-\cos(2t))u(t)$$

c) $$\cancel{\frac{s+3}{(s+1)^2(s+2)} = \frac{r_1}{s+2} + \frac{r_2 s + r_3}{(s+1)^2} \qquad r_1 = 1 \qquad r_2 = -r_1 = -1}$$

$$\cancel{r_2 + r_3 = 2 \Rightarrow r_3 = 3 \Rightarrow \frac{1}{s+2} + \frac{-s+3}{(s+1)^2}}$$

$$\frac{s+3}{(s+1)^2(s+2)} = \frac{r_1}{s+2} + \frac{r_2}{s+1} + \frac{r_3}{(s+1)^2} \qquad r_1 = 1,\ r_3 = 2,\ r_2 = -1$$

$$\Rightarrow \frac{1}{s+2} - \frac{1}{s+1} + \frac{2}{(s+1)^2} \Rightarrow f(t) = e^{-2t} - e^{-t} + 2te^{-t}$$

$$\Rightarrow f(t) = e^{-2t} + (2t-1)e^{-t}$$

d) $$\frac{10}{(s+1)(s^2+4s+13)} = \frac{10}{(s+1)((s+2)^2+9)} = \frac{r_1}{s+1} + \frac{r_2 s + r_3}{((s+2)^2+9)}$$

$$r_1 = 1 \qquad r_2 = -r_1 = -1 \qquad r_3 = -3$$

$$\Rightarrow \frac{1}{s+1} - \frac{s+3}{(s+2)^2+9} = \frac{1}{s+1} - \frac{s+2}{(s+2)^2+9} - \frac{1}{3}\frac{3}{(s+2)^2+9}$$

$$\Rightarrow f(t) = e^{-t} - e^{-2t}\cos(3t) - \frac{1}{3}e^{-2t}\sin(3t)$$

$$\Rightarrow f(t) = e^{-t} - e^{-2t}\left(\cos(3t) + \frac{1}{3}\sin(3t)\right)$$

5. a) i) $L\,\theta''(t) = g\,\theta(t) - k_0\,\theta(t)$

ii) $\cancel{s^2 P(s) = 10P(s) - 9P(s) = P(s) \Rightarrow s^2}$

$\theta''(t) + (k_0 - g)\,\theta(t) = 0$ no damping $\Rightarrow$ unstable.

$s^2 P(s) - s\,\theta(0) - \theta'(0) - P(s) = 0$

$$\Rightarrow s^2 P(s) - P(s) - 1 = 0 \Rightarrow P(s) = \frac{1}{s^2-1} = \frac{1}{(s+1)(s-1)}$$

$$= \frac{r_1}{s+1} + \frac{r_2}{s-1}, \quad r_1 + r_2 = 0 \Rightarrow r_2 = \frac{1}{2},\ r_1 = -\frac{1}{2}$$

$$\Rightarrow \theta(t) = \frac{1}{2}(e^{t} - e^{-t})$$ which does not converge to 0 $\Rightarrow$ unstable.

iii) $s^2 P(s) - s\theta(0) - \theta'(0) + P(s) = 0$

$s^2 P(s) - 1 + P(s) = 0 \Rightarrow P(s) = \frac{1}{s^2+1}$

$\Rightarrow \theta(t) = \sin(t) \quad \Rightarrow$ unstable.

b) i) $a(t) = k_1 \theta'(t) + k_0 \theta(t)$

$\theta''(t) + k_1 \theta'(t) + (k_0 - 10)\theta(t) = 0$ will be stable if $k_1 > 0$ and $k_0 > 10$

ii) We want a critically damped solution with roots at $s = -1$.

$\Rightarrow X(s) = (s+1)^2 = s^2 + 2s + 1$

$\frac{1}{\text{time constant}}$

for $\theta(t)$, $X(s) = s^2 + k_1 s + (k_0 - 10) \Rightarrow k_1 = 2 \quad k_0 = 11$

iii) $s^2 P(s) - s\theta(0) - \theta'(0) + 2(sP(s) - \theta(0)) + P(s) = 0$

$s^2 P(s) + 2sP(s) + P(s) = 1 \Rightarrow P(s) = \frac{1}{s^2+2s+1} = \frac{1}{(s+1)^2}$

$\Rightarrow \theta(t) = te^{-t}$ which is stable

6. a) $Y(s) = \frac{1}{s}\left(\frac{1}{s+1}\right) = \frac{1}{s} - \frac{1}{s+1} \Rightarrow y(t) = (1 - e^{-t})u(t)$

b) $a(X(s) - Y(s))\left(\frac{1}{s+1}\right) = Y(s) \Rightarrow \frac{a}{s+1} X(s) = \left(1 + \frac{a}{s+1}\right) Y(s)$

$\Rightarrow \Rightarrow \frac{Y(s)}{X(s)} = \frac{\frac{a}{s+1}}{1 + \frac{a}{s+1}} = \frac{a}{s+1+a}$

c) $1 + a = \frac{1}{\tau} = \frac{1}{100m} = \frac{1}{0.1} = 10 \Rightarrow \boxed{a = 9}$

$\Rightarrow \frac{1}{s} \frac{9}{s+10} = \frac{r_1}{s} + \frac{r_2}{s+10} \Rightarrow r_1 = \frac{9}{10} \quad r_2 = -\frac{9}{10}$

$\Rightarrow \frac{9}{10}\left(\frac{1}{s} - \frac{1}{s+10}\right) \Rightarrow y(t) = \frac{9}{10}(1 - e^{-10t})$

d) $H(0) = \frac{a}{1+a} = \frac{9}{10} \Rightarrow$ steady state error is 10%.

7. a) $h(t) \Leftrightarrow H(s)$ $\quad \frac{d}{dt} h(t) \Leftrightarrow \underset{\swarrow}{}$ $sH(s) - h(0)$ $\boxed{\text{True}}$

adds zero, doesn't affect stability

b) $h(t) \Leftrightarrow H(s)$ $\quad \int_{-\infty}^{t} h(\tau)d\tau \Leftrightarrow \frac{1}{s}H(s)$ $\Rightarrow$ adds a pole at zero

which would seem to result in an unstable system by the definition given above (no poles on $j\omega$ axis or in right half plane), but $H(s)$ may have a zero at 0 in which case the two cancel and the system remains stable.

$\Rightarrow$ $\boxed{\text{False}}$

c) $\boxed{\text{False}}$ $\quad H_{inv} = \frac{1}{H(s)}$ $\Rightarrow$ zeros of $H(s)$ become poles of $H_{inv}(s)$

1/9/2012

Systems $\boxed{\text{EE261}}$ $\boxed{\text{HW 1}}$

1. a) ✓

b) $\sum_{k=0}^{n-1} e^{2\pi i k/n}$ $\qquad \sum_{k=0}^{n-1} r^k = \frac{1-r^n}{1-r}$ $\qquad r = e^{2\pi i/n}$

$$\Rightarrow \sum_{k=0}^{n-1} e^{2\pi i k/n} = \frac{1-e^{2\pi i}}{1-e^{2\pi i/n}} = \frac{1-1}{1-e^{2\pi i/n}} = \boxed{0}$$

c) $$\sum_{k=-N}^{N} e^{2\pi i k t} = \frac{\sin(2\pi t(N+1/2))}{\sin(\pi t)}$$

2. a) $f(x) = \sin(2\pi m x) + \sin(2\pi n x)$

$f(x+T) = \sin(2\pi m(x+T)) + \sin(2\pi n(x+T)) = f(x)$

$\Rightarrow$ is is the smallest T such that $mT \in \mathbb{Z}$ and $nT \in \mathbb{Z}$

$\Rightarrow$ Let $T = \frac{1}{\tau}$, we need the largest τ such that $\frac{m}{\tau} \in \mathbb{Z}$ and $\frac{n}{\tau} \in \mathbb{Z}$

$\Rightarrow$ $\tau = GCD(m,n)$ $\Rightarrow$ $\boxed{T = \frac{1}{GCD(m,n)}}$

b) $g(x) = \sin(2\pi p x) + \sin(2\pi q x)$ $\quad p = \frac{m}{r}$ $\quad q = \frac{n}{s}$ $\Rightarrow$ T is smallest T

s.t. $\frac{m}{r}T \in \mathbb{Z}$ and $\frac{n}{s}T \in \mathbb{Z}$ $\Rightarrow T = \frac{a}{b} \Rightarrow \frac{m}{r}\frac{a}{b} \in \mathbb{Z}$ and $\frac{n}{s}\frac{a}{b} \in \mathbb{Z}$

$\Rightarrow$ let $a = LCM(r,s)$ $\quad b = GCD(m,n)$ $\Rightarrow$ $\boxed{T = \frac{LCM(r,s)}{GCD(m,n)}}$

c) $f(t) = \cos(t) + \cos(\sqrt{2}t)$

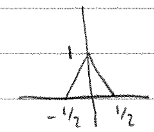
Assume $f(t) = f(t+T) \Rightarrow \cos(t+T) + \cos(\sqrt{2}t + \sqrt{2}T) = \cos(t) + \cos(\sqrt{2}t)$

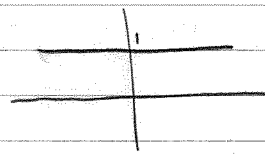
$\Rightarrow \frac{T}{2\pi} \in \mathbb{Z}$ and $\frac{\sqrt{2}T}{2\pi} \in \mathbb{Z}$

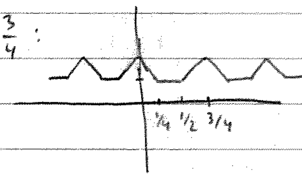
let $T = k\cdot 2\pi,\ k \in \mathbb{Z} \Rightarrow \frac{\sqrt{2}T}{2\pi} = \frac{k\cdot 2\pi}{\sqrt{2}\pi} = \frac{k}{\sqrt{2}} \notin \mathbb{Z}$

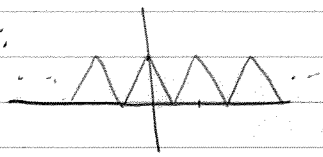
d) ✓

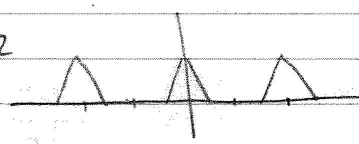
3. a) $g(t+T) = \sum_{n=-\infty}^{\infty} f(t+T-nT) = \sum_{n=-\infty}^{\infty} f(t-(n-1)T) = \sum_{m=-\infty}^{\infty} f(t-mT) = g(t$

b) 

$\Lambda(t)$ $T = \frac{1}{2}$: 

$T = \frac{3}{4}$: 

$T = 1$: 

$T = 2$ 

c) No. Especially if T = period of $f(t)$.

Then $g(t) = \sum_{n=-\infty}^{\infty} f(t-nT) = \sum_{n=-\infty}^{\infty} f(t)$ ← doesn't converge unless $f(t)=($

4. Parsevals: $\int_0^1 |f(t)|^2 dt = \sum_{n=-\infty}^{\infty} |\hat{f}(n)|^2$

$f(t) = \sum_{n=-\infty}^{\infty} D_n e^{j\omega_0 nt}$ $\quad D_n = \frac{1}{T}\int_0^T f(t)\, e^{-j\omega_0 nt} dt$

$g(t) = f(tT)$ which is period 1.

$\Rightarrow \hat{g}(t) = \int_0^1 g(t)\, e^{-j2\pi nt} dt = \int_0^1 f(tT)\, e^{-j2\pi nt} dt$ $\quad$ let $\tau = tT \quad d\tau = T dt$

$\Rightarrow \hat{g}(t) = \frac{1}{T}\int_0^T f(\tau)\, e^{-2\pi n\tau/T} d\tau = D_n$

$\int_0^1 |g(t)|^2 dt = \int_0^1 |f(tT)|^2 dt = \frac{1}{T}\int_0^T |f(\tau)|^2 d\tau$

$\Rightarrow \sum_{n=-\infty}^{\infty} |D_n|^2 = \frac{1}{T}\int_0^T |f(t)|^2 dt$

5. ✓

Devices EE316 L1 1/10/2012

Wong OH - Th - 3-4 CISX - 312

ee316ta@gmail.com

- finfet ☆
- strained silicon
- ultrathin body SOI ☆ used in research, not yet commercialized.

Key Points
- Scaling - how it affects circuit/system performance.
- Today we are 22nm
- Develop an intuitive feel in addition to solving equations

Textbook: Taur & Ning "Fundamentals of Modern VLSI Devices" 2nd. Ed.
(not required)

CMOS performance - consider how it affects digital performance, not analog performance.

- BJTs are coming back possibly. Complementary lateral BJTs from IBM. (they used to be vertical).

- Now lithography is good enough that you can pattern a short base to make a lateral BJT.

2011 - $300 Billion revenue (worldwide). US GDP is $15 Trillion
2% of US GDP

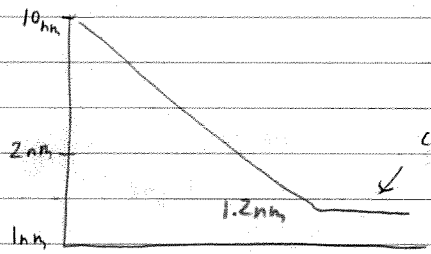
- Computer - Billions of transistors. GB of storage must have at least 1B transisto[rs] if its DRAM or SRAM (doesn't count for hard disk)
- Data center consumes ~1% of energy in US.
- CPU consumes
  - ~50% of server power (other 50% goes to cooling, etc).
  - ~50% of CPU power consumed in active devices. (other 50% in wires)

- Two classes of applications:
    - High performance applications - Data Centers (don't care as much about power)
    - Power constrained applications - mobile devices (should be low power).

- (~1 Billion transistors per chip. (per die)

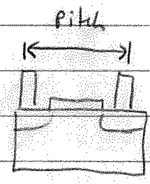
- Transistor count doubles about every 2 years (Moores Law)

- physical tox



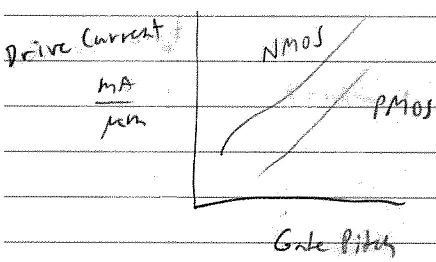
can't scale further due to gate leakage from tunneling.

- gate pitch



smaller pitch → better packing density
→ more chips / wafer
→ more profit since the cost of a wafer is relatively constant.

scales by 0.7 for every generation.



- 90nm technology → 50nm L 1.2nm tox

- 65nm technology → 35nm L 1.2nm tox

- 45nm technology → 34nm L 1.2nm tox
(can't decrease much more, because tox is not scaled down)

45nm High K with Metal Gate, 2007

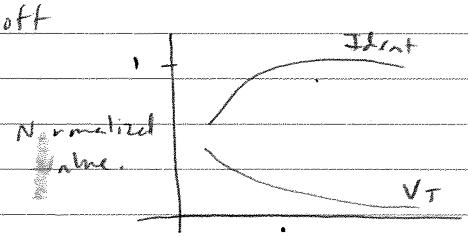
32 nm technology - 28nm L 2008

- SRAM occupies about half of the chip area for a CPU.
  ⇒ being able to make a small SRAM cell is important.

32 nm technology, high K metal gate. 2009. (Intel)

intrinsic variation - due to underlying physics.
extrinsic variations - due to imperfections in manufacturing.

Tradeoff



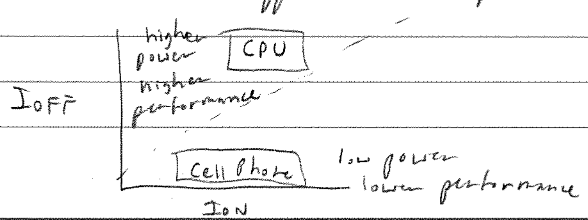
Future:

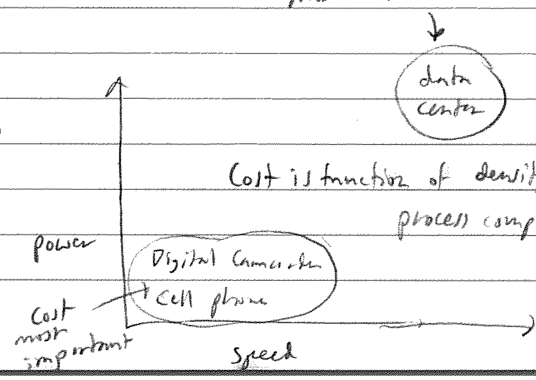
- Transport enhanced FET
- Multigate / FinFET (EE316)
- Nanoelectromechanical relays (high subthreshold slope)
- Nanotubes, Nanowires (EE320)
- Molecular devices, spintronics, quantum cascade.

time

- high k metal gate
- ultrathin body SOI
- strained silicon.
- FinFET
- Device design is application dependent

performance most important. Then cost





Cost is function of density & process complexity

In Strong Inversion,

Large change in carrier concentration with small changes in surface potential (ψ_s) → "pinning" of surface potential at $2\psi_B$.

Inversion Layer Thickness Estimate : $\frac{Q_i}{q n(0)} = 2\varepsilon_{si} kT/(qQ_i)$

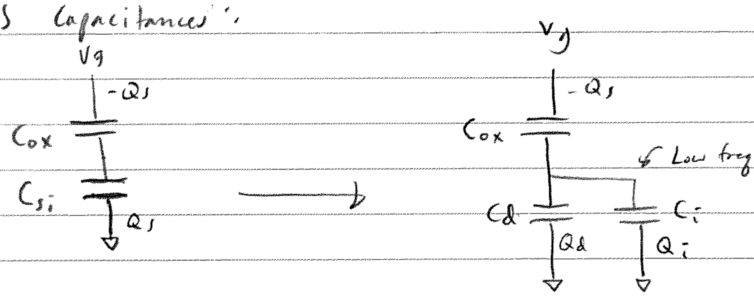
↑ ~ few Å s (0.4nm)

Gate Voltage Equation ; $\boxed{V_g = V_{ox} + \psi_s = -\frac{Q_s}{C_{ox}} + \psi_s}$

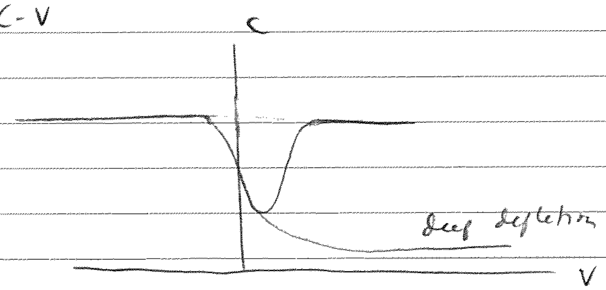
$C_{ox} = \varepsilon_{ox}/t_{ox}$ and $\varepsilon_{ox} E_{ox} = \varepsilon_{si} E_s$ and $Q_s = -\varepsilon_{si} E_s$

and $E_{ox} = \frac{V_{ox}}{t_{ox}}$

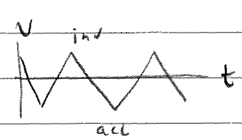
MOS Capacitances :

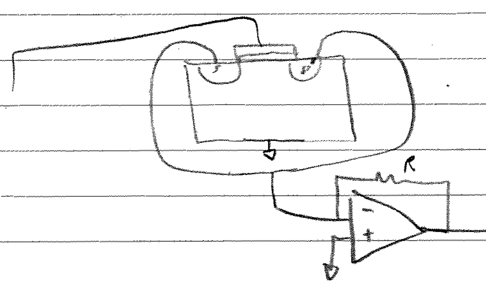


C-V



- Split C-V measurement





- start from inversion, then go to acc.
this is faster.

should shift the C-V curve left or right.

Flatband Voltage: $V_g = V_{FB} + \psi_s + V_{ox} = V_{FB} + \psi_s - \frac{Q_s}{C_{ox}}$

$\sim 4eV$

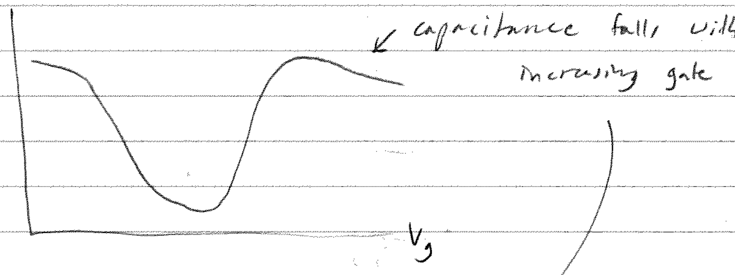
$\phi_m = \chi$ (n^+poly) — used for pmos (n-type sub) $\phi_{Aluminum} \approx 4eV$

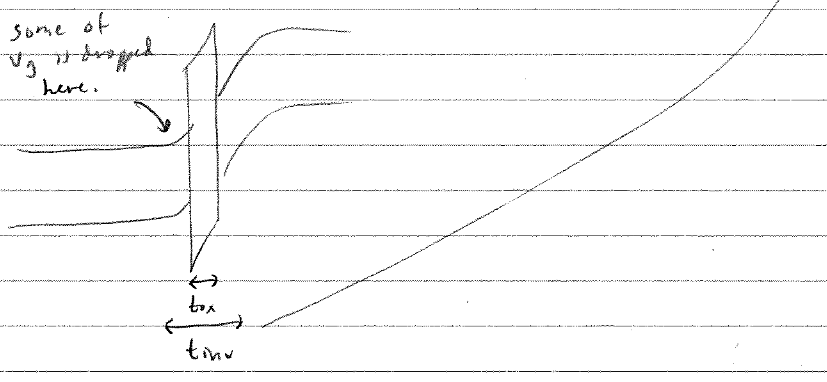
$\phi_m = \chi + \frac{E_g}{2q}$ (midgap)

$\phi_m = \chi + \frac{E_g}{q}$ (p^+poly) — used for nmos (p-type sub)

for p-type need a workfunction close to that of p^+ poly.

Polysilicon Gate Depletion Effect.





You can have a fermi level higher than the conduction band. (called degenerate)

for 10^{20} doped Si, E_F sits ~10mV above E_c.

$$\int t e^{at} = \left(\frac{t}{a} - \frac{1}{a^2}\right) e^{at}$$

Systems EE261 HW 2 $\qquad 1 - \cos(2A) = 2\sin^2(A) \qquad$ 1/12/12

1\. a) $\hat{f}(n) = \frac{1}{1}\int_{-1/2}^{1/2} f(t)\, e^{-2\pi j n t/1}\, dt$

$$= \int_{-1/2}^{0} (2t+1)\, e^{-2\pi j n t}\, dt = 2\int_{-1/2}^{0} t\, e^{-2\pi j n t}\, dt + \int_{-1/2}^{0} e^{-2\pi j n t}\, dt$$

$$= 2\left(\frac{t}{-2\pi j n} + \frac{1}{4\pi^2 n^2}\right) e^{-2\pi j n t}\Big]_{-1/2}^{0} + \left(\frac{1}{-2\pi j n}\, e^{-2\pi j n t}\Big]_{-1/2}^{0}\right)$$

$$= \frac{1}{2\pi^2 n^2} - \left(\left(\frac{1}{2\pi^2 n^2} + \frac{1}{2\pi j n}\right) e^{j\pi n} + \left(\frac{1}{-2\pi j n} + \frac{1}{2\pi j n}\, e^{j\pi n}\right)\right)$$

$$= \frac{1}{2\pi^2 n^2}\left(1 - (-1)^n\right) - \frac{1}{j2\pi n}(-1)^n + \frac{1}{j2\pi n}\left((-1)^n - 1\right)$$

$$\boxed{\hat{f}(n) = \frac{1}{2\pi^2 n^2}\left(1 - e^{j\pi n}\right) - \frac{1}{j2\pi n}}$$

b) $g(t) = f(-t) \Rightarrow \hat{g}(n) = \hat{f}(-n) \qquad \mathcal{F}f^- = (\mathcal{F}f)^-$

$$\Rightarrow \hat{g}(n) = \frac{1}{2\pi^2 n^2}\left(1 - e^{j\pi n}\right) + \frac{1}{j2\pi n}$$

c) $h(t) = f(t) + g(t) \Rightarrow \hat{h}(n) = \hat{f}(n) + \hat{g}(n)$

$$\Rightarrow \hat{h}(n) = \frac{1}{\pi^2 n^2}\left(1 - \frac{1}{2}\left(e^{j\pi n} + e^{-j\pi n}\right)\right) = \frac{1}{\pi^2 n^2}\left(1 - \cos(\pi n)\right)$$

$$= \frac{2}{\pi^2 n^2}\sin^2\left(\frac{\pi n}{2}\right) = \frac{1}{2}\operatorname{sinc}^2\left(\frac{n}{2}\right)$$

$$\Rightarrow h(t) = \sum_{n=-\infty}^{\infty} \hat{h}(n)\, e^{j2\pi n t} = \boxed{\sum_{n=-\infty}^{\infty} \frac{1}{2}\operatorname{sinc}^2\left(\frac{n}{2}\right) e^{j2\pi n t} = h(t)}$$

2\. a) $\|f+g\|^2 + \|f-g\|^2 \qquad$ Note: $\|f\|^2 = (f,f) \qquad (f,g) = \overline{(g,f)}$

$$(f+g, f+g) + (f-g, f-g) = (f+g, f) + (f+g, g)$$

$$+ (f-g, f) - (f-g, g) = (f,f) + \cancel{(g,f)} + \cancel{(f,g)} + (g,g)$$

$$+ (f,f) - \cancel{(g,f)} - \cancel{(f,g)} + (g,g) = 2(f,f) + 2(g,g)$$

$$= 2\|f\|^2 + 2\|g\|^2$$

b)

c)

$\mathrm{sinc}(x) \Leftrightarrow \mathrm{rect}\, s$

3. a) $f(x) = a\,\mathrm{sinc}(b(x-c))$

$\mathcal{F}f(s) = \frac{a}{|b|}\,\mathrm{rect}\left(\frac{s}{b}\right) e^{-2\pi j s c}$

b) $g(x) = 3\Delta\left(\frac{x+2}{3}\right) + 3\Delta\left(\frac{x-2}{3}\right)$

$\mathcal{F}g(s) = 3\left(3\,\mathrm{sinc}^2(3s)\, e^{-4\pi j s} + 3\,\mathrm{sinc}^2(3s)\, e^{4\pi j s}\right)$

$= 18\,\mathrm{sinc}^2(3s)\left(\frac{1}{2}\left(e^{-4\pi j s} + e^{4\pi j s}\right)\right)$

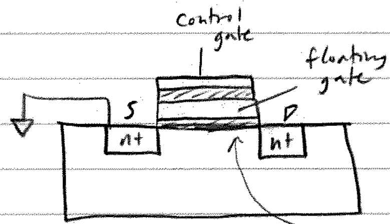
$= 18\,\mathrm{sinc}^2(3s)\cos(4\pi s)$

4. $f(t) = \Pi(t) - \frac{1}{2}\Delta(2t)$

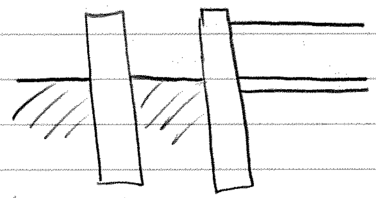
$\mathcal{F}f(s) = \mathrm{sinc}(s) - \frac{1}{4}\,\mathrm{sinc}^2\left(\frac{s}{2}\right)$

Devices, Different Structures + Devices 1/14/12

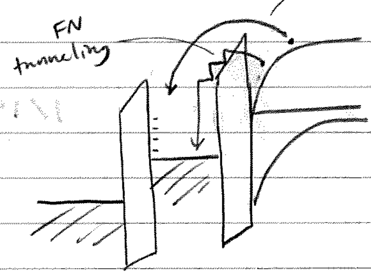
Flash Memory:



tox thin but not too thin (don't want direct tunneling)



Write: $V_{control\,gate} >> V_{th}$



- bands bend
- e's tunnel or are injected (hot carriers) into floating gate
- as e's build up in floating gate, band bending decreases, e's stop entering floating gate.

Read: $V_{t_0} < V_{control} < V_{t_1}$

(1) shielding ⇒ $R_{channel}$ is big $V_{t_1} > V_{th_{original}}$ (V_{th} increases)

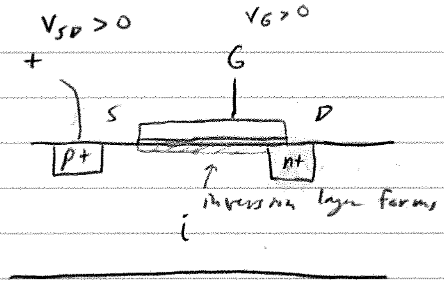
(0) no shielding ⇒ $R_{channel}$ is smaller. $V_{t_0} = V_{th_{original}}$

$$I_{channel} \approx \frac{V_{DS}}{R_{channel}}$$

Erase: $V_{control\,gate} << 0$ (or apply $V_S >> 0$)

⇒ bands bend the other way, e's tunnel into the source.

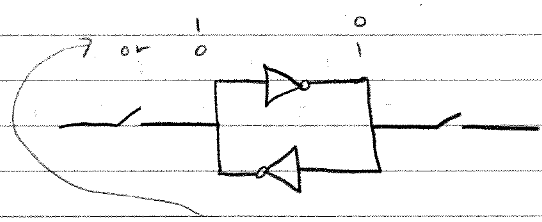
IMOS:



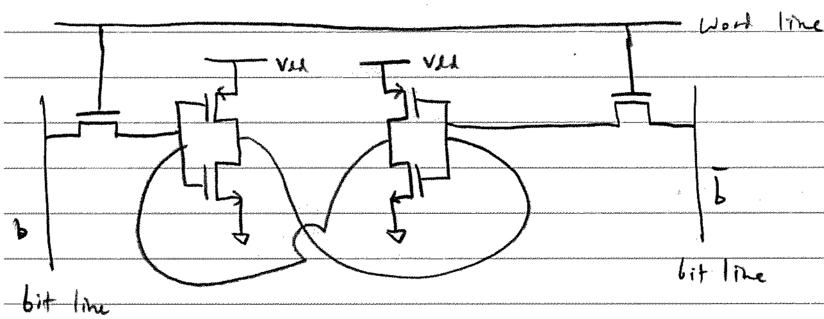
Trigate / FinFET:

- I_{ON} increased (3 channels)
- I_{OFF} decreased (better control of channel.)
  - SCE suppressed
  - Subthreshold slope increased ⇒ $I_{sub} \downarrow$
  - Can use thicker oxide while still achieving same performance ⇒ $I_{gate} \downarrow$
- For scaled MOSFETs, 70% of I_{OFF} comes from gate leakage.

SRAM:
(latch)



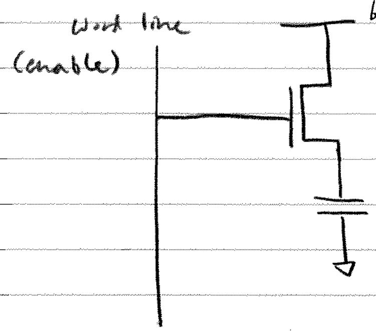
Bistable system (two stable states)



To Write: - make word line high to turn on switches.
- make $b = 1$, $b' = 0$

To Read: - make word line high
- read from one of the bit lines.

DRAM :



bit line → where you read out or write the data.
word line → control

	word	bit
Read	1	read out.
Write	1	1 or 0 (depending on what you want to write)

Since the cap is leaky, you need to rewrite every few cycles.

↓

read → if 1, write 1
read → if 0, do nothing.

$$\int_{-\infty}^{\infty} |f(t)|^2 dt = \int_{-\infty}^{\infty} |Ff|^2 ds$$

$$\int_{-\infty}^{\infty} f(t)\,\overline{g(t)}\,dt = \int_{-\infty}^{\infty} F(s)\,\overline{G(s)}\,ds$$

Systems EE261 HW 3 1/13/12

1. a) $\int_{-\infty}^{\infty} \text{sinc}^4(t)\,dt = \int_{-\infty}^{\infty} |F\,\text{sinc}^2|^2\,ds = \int_{-1}^{1} |\Lambda(s)|^2\,ds$

$= 2 \cdot \int_0^1 s^2 dt = 2\left(\frac{1}{3}s^3\Big]_0^1\right) = \boxed{\frac{2}{3}}$

b) $\int_{-\infty}^{\infty} \frac{2}{1+(2\pi t)^2}\,\text{sinc}(2t)\,dt$ $\quad e^{-a|t|} \Leftrightarrow \frac{2a}{a^2+4\pi^2 s^2}$

$\Rightarrow \frac{2a}{a^2+4\pi^2 t^2} \Leftrightarrow e^{-a|-s|} = e^{-a|s|}$

$\text{sinc}(2t) \Leftrightarrow \frac{1}{2}\Pi\left(\frac{s}{2}\right)$

$\Rightarrow \int_{-\infty}^{\infty} e^{-|s|}\,\frac{1}{2}\Pi\left(\frac{s}{2}\right)ds = \frac{1}{2}\int_{-1}^{1} e^{-|s|}\,ds = \int_0^1 e^{-s} = -e^{-s}\Big]_0^1 = -e^{-1}+1$

$= \boxed{1-\frac{1}{e}}$

2. a) $G(s) = \int_{-\infty}^{\infty} f(t)\cos(2\pi s_0 t)\,e^{-2\pi j s t}\,dt$ $\quad \cos(2\pi s_0 t) = \frac{1}{2}\left(e^{j2\pi s_0 t} + e^{-j2\pi s_0 t}\right)$

$\Rightarrow G(s) = \frac{1}{2}\int_{-\infty}^{\infty} f(t)\left(e^{-2\pi j(s-s_0)t} + e^{-2\pi j(s+s_0)t}\right)dt$

$= \frac{1}{2}\int_{-\infty}^{\infty} f(t)\,e^{-2\pi j(s-s_0)t}\,dt + \frac{1}{2}\int_{-\infty}^{\infty} f(t)\,e^{-2\pi j(s+s_0)t}\,dt$

$= \frac{1}{2}F(s-s_0) + \frac{1}{2}F(s+s_0)$

b) $f(t) = 4\,\text{sinc}^2(2t)\cos(2\pi\cdot 4t) = \boxed{4\,\text{sinc}^2(2t)\cos(8\pi t)}$

3. a) $\int_{-\infty}^{\infty} t^2 e^{-\pi t^2}\,dt = \left(\frac{j}{2\pi}\right)^2 F(e^{-\pi t^2})''(0)$

$F(e^{-\pi t^2}) = e^{-\pi s^2} \Rightarrow \frac{d}{ds}e^{-\pi s^2} = -2\pi s\,e^{-\pi s^2}$

$\frac{d^2}{ds^2}e^{-\pi s^2} = (-2\pi s)(-2\pi s)\,e^{-\pi s^2} - 2\pi\,(e^{-\pi s^2}) = 4\pi^2 s^2 e^{-\pi s^2} - 2\pi e^{-\pi s^2}$

$\Rightarrow \int_{-\infty}^{\infty} t^2 e^{-\pi t^2}\,dt = \left(\frac{j}{2\pi}\right)^2(0-2\pi) = -\frac{1}{(2\pi)^2}(-2\pi) = \boxed{\frac{1}{2\pi}}$

OR integrate by parts: let $u=t$ $\quad dv = te^{-\pi t^2}$ $\quad du = dt$ $\quad v = \frac{1}{-2\pi}e^{-\pi t^2}$

$\Rightarrow uv - \int v\,du = -\frac{t}{2\pi}e^{-\pi t^2}\Big]_{-\infty}^{\infty} + \frac{1}{2\pi}\int e^{-\pi t^2}dt = (0-0) + \frac{1}{2\pi}(1) = \boxed{\frac{1}{2\pi}}$

b) $\int_{-\infty}^{\infty} t^4 e^{-\pi t^2} dt$ let $u = t^3$ $dv = te^{-\pi t^2}$ $du = 3t^2 dt$ $v = -\frac{1}{2\pi} e^{-\pi t^2}$

$uv - \int v\,du = -t^3 \frac{1}{2\pi} e^{-\pi t^2} \Big]_{-\infty}^{\infty} - \frac{3}{2\pi} \int t^2 e^{-\pi t^2} dt = (0-0) - \frac{3}{2\pi}\left(\frac{1}{2\pi}\right) = \boxed{\frac{3}{(2\pi)^2}}$

c)

4. $f(t) = f_0 + f_1 t + f_2 t^2 + \dots + f_n t^n = f_0 - f_1 t + f_2 t^2 + \dots + f_n (-t)^n$

$\Rightarrow$ $f_1 t + f_3 t^3 + \dots + f_{2n+1} t^{2n+1} = -(f_1 t + f_3 t^3 + \dots + f_{2n+1} t^{2n+1})$

Assume f is even. let f_e denote the sum of even power terms in f. let f_o denote the sum of odd power terms in f.

$f - f_e = f_o$ since even $\pm$ even $=$ even, this contradicts the hypothesis that f is even. Hence f is an even function iff $f_k = 0$ for all odd values of k.

5. a) $f(t) = 2\Delta\left(\frac{t-2}{2}\right) + 2.5\,\Delta\left(\frac{t-4}{2}\right)$

$F(s) = 4\,\text{sinc}^2(2s)\, e^{-j4\pi s} + 5\,\text{sinc}^2(2s)\, e^{-j8\pi s}$

b) $\mathcal{F}f(s) = \sum_{k=1}^{n-1} \mathcal{F}\left(f(kT)\,\Delta\left(\frac{t-nT}{T}\right)\right)$

$= \sum_{k=1}^{n-1} f(kT)\; T\,\text{sinc}^2(sT)\, e^{-j2\pi kTs}$

$= \boxed{T\,\text{sinc}^2(sT) \sum_{k=1}^{n-1} f(kT)\, e^{-j2\pi kTs}}$

Systems EE261 HW 4 1/14/2012

1. mean = expected value = center of mass

$$E(X) = \int_{-\infty}^{\infty} x f(x)\,dx = \frac{i}{2\pi}(\mathcal{F}f)'(0)$$

$$f' \Leftrightarrow 2\pi i s\, \mathcal{F}f \qquad -2\pi i t f(t) \Leftrightarrow \mathcal{F}f'$$

$$\Rightarrow t f(t) \Leftrightarrow \frac{i}{2\pi}\mathcal{F}f'$$

$$\Rightarrow \int_{-\infty}^{\infty} t f(t) e^{-j2\pi s t}\,dt = \frac{i}{2\pi}\mathcal{F}f'(s)$$

$$\Rightarrow \boxed{\int_{-\infty}^{\infty} t f(t)\,dt = \frac{i}{2\pi}\mathcal{F}f'(0)}$$

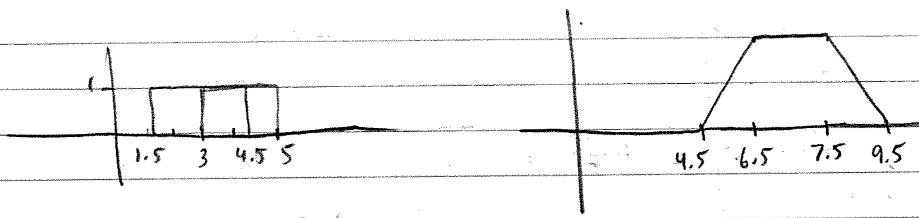
now let $g(t) = t f(t) \Rightarrow -2\pi i t g(t) \Leftrightarrow \mathcal{F}g' = \frac{i}{2\pi}\mathcal{F}f''$

$$\Rightarrow t^2 f(t) \Leftrightarrow \left(\frac{i}{2\pi}\right)^2 \mathcal{F}f''$$

$$\Rightarrow \int_{-\infty}^{\infty} t^2 f(t) = \left(\frac{i}{2\pi}\right)^2 \mathcal{F}f''(0)$$

2.

3. a)



b) $f(x) = \Lambda\left(\frac{1}{5}(x-7)\right) - \Lambda\left(2(x-7)\right)$ directly

$$\Rightarrow \mathcal{F}f = e^{-j14\pi s}\, 5\,\mathrm{sinc}^2(5s) - e^{-j14\pi s}\,\frac{1}{2}\,\mathrm{sinc}^2\left(\frac{s}{2}\right)$$

$$= e^{-j14\pi s}\left(5\,\mathrm{sinc}^2(5s) - \frac{1}{2}\,\mathrm{sinc}^2\left(\frac{s}{2}\right)\right)$$

or by convolution theorem: $\Pi\left(\frac{1}{3}(x-3)\right) \Leftrightarrow e^{-j6\pi s}\, 3\,\mathrm{sinc}(3s)$

$$\Pi\left(\frac{1}{2}(x-4)\right) \Leftrightarrow e^{-j8\pi s}\cdot 2\,\mathrm{sinc}(2s) \Rightarrow \mathcal{F}f = e^{-j14\pi s}\, 3\,\mathrm{sinc}(3s)\cdot 2\sin(2s)$$

$$\Rightarrow \mathcal{F}f = e^{-j14\pi s}\, 6\, \frac{\sin(3\pi s)\sin(2\pi s)}{6\pi^2 s^2} \Rightarrow \mathcal{F}f = e^{-j14\pi s}\frac{\sin(3\pi s)\sin(2\pi s)}{\pi^2 s^2}$$

F(s) falls off as $\frac{1}{s^2}$ because f(x) is continuous, but does not have continuous derivative.

4. $g(t) = (f * h)(t) \Rightarrow G(s) = F(s)\,H(s)$

$g(t) = h(t) = \text{sinc}(t) \Rightarrow G(s) = H(s) = \Pi(s)$

$\Rightarrow F(s) = 1$ for $-\frac{1}{2} \leq s \leq \frac{1}{2}$.

5.

6. a) $W_H(t) = \Pi(t)\cos^2(\pi t)$

$\Rightarrow W(s) = \text{sinc}(s) * \frac{1}{2}\left(\delta(s-\frac{1}{2}) + \delta(s+\frac{1}{2})\right) * \frac{1}{2}\left(\delta(s-\frac{1}{2}) + \delta(s+\frac{1}{2})\right)$

$\Rightarrow W(s) = \text{sinc}(s) * \left(\frac{1}{4}\delta(s-1) + \frac{1}{2}\delta(s) + \frac{1}{4}\delta(s+1)\right)$

$\Rightarrow W(s) = \frac{1}{4}\left(\text{sinc}(s-1) + 2\,\text{sinc}(s) + \text{sinc}(s+1)\right)$

b) $W(s)$ has a slightly wider peak than $\text{sinc}(s)$, so it will have smoothing effects on $F(s)$ but it dies off faster than sinc, so we should expect less weird artifacts such as ringing.

7. Autocorrelation $f \star f = f(t) * \overline{f(-t)} = \int_{-\infty}^{\infty} f(\tau) f(\tau - t)\,d\tau$

for real f, $f \star f = \int_{-\infty}^{\infty} f(\tau) f(\tau - t)\,d\tau$

a) $(f \star f)(0) = \int_{-\infty}^{\infty} f(\tau) f(\tau)\,d\tau = \int_{-\infty}^{\infty} f^2(\tau)\,d\tau$?

$\boxed{|(f \star f)(x)| \leq (f \star f)(0)}$ by Cauchy Schwartz inequality.

b) $W_{Ff} = \frac{1}{Ff(0)} \int_{-\infty}^{\infty} Ff(s)\,ds \Rightarrow W_f W_{Ff} = \left(\frac{1}{f(0)} \int_{-\infty}^{\infty} f(t)\,dt\right)\left(\frac{1}{Ff(0)} \int_{-\infty}^{\infty} Ff(s)\,ds\right)$

$= \left(\frac{Ff(0)}{f(0)}\right)\left(\frac{f(0)}{Ff(0)}\right) = 1$

c) $f \star f = f(t) * f(-t) \Leftrightarrow Ff \cdot Ff^{-} = Ff\,\overline{Ff}$ since $Ff^{-} = \overline{Ff}$ (f is real $\Rightarrow Ff$ has hermitian symmetry)

assuming f is real. $= |Ff|^2 \Rightarrow W_{f \star f} W_{|Ff|^2} = 1$

Devices $\boxed{\text{Graphene}}$ 1/15/2012

Nobel Prize in Physics
- 2009 - CCD
- 2010 - Graphene. (Geim + Novoselov)

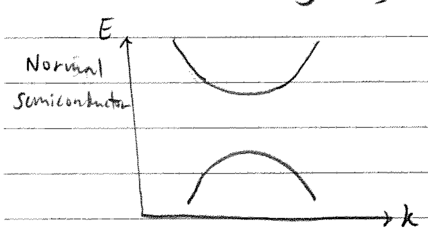
Important Properties of Graphene:

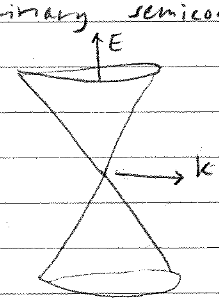
High Mobility:

Material	Graphene	CNT	GaAs	Si	Organics
e^- Mobility (cm^2/Vs)	200,000	100,000	8,500	1,400	~ 1

What is Graphene:

- Graphene is a 2-D crystalline form of carbon: a single layer of carbon atoms arranged in hexagons. (Can be used to make bucky balls (0-D), CNTs (1-D) and graphite (3-D pencil lead).
- Graphite consists of layers of carbon atoms (graphene sheets) tightly bonds in plane, but only loosely bonded between planes
- As a crystal, 2-D graphene is quite different from 3-D materials like Si. In Si, charge carriers (e^-'s and h^+'s) interact with the periodic field of the atomic lattice to form quasiparticles (excitations that act like classical particles with mass m^*). But quasiparticles in graphene don't look anything like an ordinary semiconductor's.





Graphene. $m^* = 0$ for both e^- & h^+
$\Rightarrow$ quasiparticles behave as if they were massless electrons (Dirac Fermions), which travel at a constant speed (some fraction of $c = 3\cdot10^8 m/s$

Graphene's unusual electronic properties arise from the fact that the Carbon atom has 4 e^-'s, 3 of which are tied up in bonding with its neighbors. But the unbound 4th e^-'s are in orbitals extending vertically above and below the plane, and the hybridization of these spreads across the whole graphene sheet. Thus, electrons are sort of free and can move ballistically — without collisions — over great distances, even at T = 300K. $\Rightarrow$ σ of graphene is 10 - 100x higher than Si
By applying a gate voltage, one can continually control the carrier density by varying the voltage and thus the conductivity.

Why is Graphene Useful:

- Super small transistors - 2008 - 1nm graphene transistor, 1 atom thick, 10 atoms long.
- Energy storage - can be used to create ultra capacitors.
- Optical Devices - Solar cells + Flexible touch screens. (It's also 97% transparent)
- High Energy Particle Physics: $m^* = 0 \Rightarrow$ electrons can move very fast through the lattice. In fact they behave as Heisenberg's relative particles.

Systems EE261 HW 5 1/15/2012

1. a) $\langle g\delta', \varphi\rangle = \langle \delta', g\varphi\rangle = -\langle \delta, (g\varphi)'\rangle$

$= -\langle \delta, g\varphi' + \varphi g'\rangle = -\langle \delta, g\varphi'\rangle - \langle \delta, \varphi g'\rangle$

$= -g(0)\varphi'(0) - \varphi(0)g'(0) = -g(0)\langle \delta, \varphi'\rangle - g'(0)\langle \delta, \varphi\rangle$

$= g(0)\langle \delta', \varphi\rangle - g'(0)\langle \delta, \varphi\rangle = \langle g(0)\delta' - g'(0)\delta, \varphi\rangle$

b) $f' * g = f * g'$

2. $f(t) = u(t-1) - u(-(t+1))$ $\qquad u(t) \Leftrightarrow \frac{1}{2}\left(\delta(s) + \frac{1}{\pi i s}\right)$

$\mathrm{sgn}(t) \Leftrightarrow \frac{1}{\pi i s}$

$\Rightarrow \mathcal{F}f = e^{-2\pi i s}\left(\frac{1}{2}\delta(s) + \frac{1}{2\pi i s}\right) - e^{2\pi i s}\left(\frac{1}{2}\delta(-s) - \frac{1}{2\pi i s}\right)$

$= e^{-2\pi i s}\left(\frac{1}{2}\delta(s) + \frac{1}{2\pi i s}\right) - e^{2\pi i s}\left(\frac{1}{2}\delta(s) - \frac{1}{2\pi i s}\right)$

$= \delta(s)\,\frac{1}{2}\left(e^{-2\pi i s} - e^{2\pi i s}\right) + \frac{1}{2\pi i s}\left(e^{2\pi i s} + e^{-2\pi i s}\right)$

$= -i\,\delta(s)\sin(2\pi s) + \frac{1}{\pi i s}\cos(2\pi s) = \boxed{\frac{1}{\pi i s}\cos(2\pi s)}$

real + odd $\Leftrightarrow$ imag + odd ✓ $\qquad$ odd $\times$ even = odd

3. a) $\cancel{g(t) = f(t) \sum_{k=-\infty}^{\infty} Tp(t-kT) = f(t) \cdot p(t) * T\,Ш_T}$

$\cancel{\Rightarrow G(s) = F(s) * (T\,P(s)\ \tfrac{1}{T}\,Ш_{1/T})}$

~~$\Rightarrow$ let bandwidth of $p(t)$ be B_p~~

$\cancel{\Rightarrow \text{if } 2(B+B_p) < \tfrac{1}{T} \text{ then } \Pi_{1/T}(s)\,G(s) = F(s) * P(s)}$

$\cancel{\Rightarrow \text{if } \tfrac{1}{T} > 2B, \quad \Pi_{1/T}(s)\,G(s) = P(0)\,F(s)}$

$\cancel{\Rightarrow f(t) = \tfrac{1}{P(0)}\,\mathcal{F}^{-1}(\Pi_{1/T}\,G)(t)}$

$$g(t) = f(t)\left(\sum_{k=-\infty}^{\infty} Tp(t-kT)\right)$$

$$= f(t) \cdot T\,p(t) * Ш_T$$

$$\Rightarrow G(s) = \cancel{T}\,F(s) * \left(P(s)\ \cancel{\tfrac{1}{T}}\,Ш_{1/T}\right) = F(s) * \left(P(s) \sum_{k=-\infty}^{\infty} \delta\left(s-\tfrac{k}{T}\right)\right)$$

$$= F(s) * \sum_{k=-\infty}^{\infty} P\left(\tfrac{k}{T}\right)\delta\left(s-\tfrac{k}{T}\right) = \sum_{k=-\infty}^{\infty} P\left(\tfrac{k}{T}\right) F\left(s-\tfrac{k}{T}\right)$$

$\Rightarrow$ just like with ideal sampling, the fourier transform of the sampled signal is an infinite sum of shifted copies of the original signals Fourier transform. The only difference is in the case when realistic pulses are used, each of these copies is scaled by the value of the pulses fourier transform at the center frequency of that copy. But a scaling factor can easily be taken out by adjusting the gain of the lowpass filter we use to reconstruct $f(t)$ from its samples. The answer is yes, under certain conditions it's possible to recover $f(t)$ from $g(t)$

b) $\frac{1}{T} > 2B$.

4. $f(t) = \cos(2\pi t)$ $\quad \frac{1}{T} = \frac{2}{3} \Rightarrow T = \frac{3}{2}$

$\Rightarrow \cancel{\hat{f}(t) = \cos(2\pi t)\,\text{Ш}_{3/2}}$ $\quad \cos(2\pi t) \Longleftrightarrow \frac{1}{2}(\delta(s-1) + \delta(s+1))$

$\Rightarrow \hat{F}(s) = \frac{1}{2}(\delta(s-1) + \delta(s+1)) * \text{Ш}_{2/3}$

$= \sum_{k=-\infty}^{\infty} \delta(s - (\frac{1}{3} + \frac{2}{3}k))$

$\Rightarrow \Pi_{4/3}\hat{F}(s) = (\delta(s-\frac{1}{3}) + \delta(s+\frac{1}{3})) = G(s)$

$\Rightarrow g(t) = 2\cos(\frac{2}{3}\pi t)$

5. a) $f_0(t) = f(t)\,\text{Ш}_{2T}(t) \Rightarrow F_0(s) = F(s) * \frac{1}{2T}\text{Ш}_{\frac{1}{2T}}(s)$

$\Rightarrow F_0(s) = F(s) * \frac{1}{2T}\sum_{k=-\infty}^{\infty} \delta(s - \frac{k}{2T}) = \frac{1}{2T}\sum_{k=-\infty}^{\infty} F(s) * \delta(s - \frac{k}{2T})$

$= \boxed{\frac{1}{2T}\sum_{k=-\infty}^{\infty} F(s - \frac{k}{2T})}$

b) $f_1(t) = f(t)\,\text{Ш}_{2T}(t - (T+\alpha))$

$\Rightarrow F_1(s) = F(s) * \left(e^{-2\pi i(T+\alpha)s}\frac{1}{2T}\text{Ш}_{1/2T}\right)$

$= F(s) * \left(\frac{1}{2T}\sum_{k=-\infty}^{\infty} e^{-2\pi i(T+\alpha)s}\delta(s - \frac{k}{2T})\right)$

$= F(s) * \left(\frac{1}{2T}\sum_{k=-\infty}^{\infty} e^{-2\pi i k(\frac{T+\alpha}{2T})}\delta(s - \frac{k}{2T})\right)$

$= \frac{1}{2T}\sum_{k=-\infty}^{\infty} e^{-\pi i k(1+\frac{\alpha}{T})} F(s - \frac{k}{2T})$

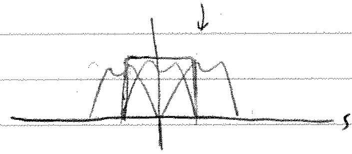
c) Yes, if $0 \le \alpha < T$. To see how, lets look at the band from 0 to $1/T$ Hz. Let $G_0(s)$ be the portion of $F_0(s)$ in $s = [0, 1/T]$ and let $G_1(s)$ be the portion of $F_1(s)$ in $s = [0, 1/T]$

$G_0(s) = \frac{1}{2T}F(s) + \frac{1}{2T}F(s - \frac{1}{2T})$

$G_1(s) = \frac{1}{2T}F(s) + \frac{1}{2T}e^{-\pi i(1+\alpha/T)}F(s - \frac{1}{2T})$

if $\alpha = 0$, $F(s) = G_0(s) + G_1(s)$

if $\alpha = 1$, $G_0(s) = G_1(s)$ $\Rightarrow$ we have aliasing and can't recover $F(s)$



if $0 < \alpha < 1$, we have 2 equations and 2 unknowns ($F(s)$ and $F(s - \frac{1}{2T})$) and can recover $F(s)$.

6. Orthogonality: $f \perp g$ iff $(f,g) = \int_{-\infty}^{\infty} f \cdot \bar{g} \, dt = 0$

$$\int_{-\infty}^{\infty} \operatorname{sinc}(t-a) \overline{\operatorname{sinc}(t-b)} \, dt = \int_{-\infty}^{\infty} e^{-2\pi i s a} \Pi \cdot \overline{e^{-2\pi i s b} \Pi} \, ds$$

$$= \int_{-1/2}^{1/2} e^{-2\pi i s a} e^{2\pi i s b} \, ds = \int_{-1/2}^{1/2} e^{-2\pi i s (a-b)} ds = -\frac{1}{2\pi i (a-b)} e^{-2\pi i s (a-b)} \Big]_{-1/2}^{1/2}$$

$$= -\frac{1}{2\pi i (a-b)} \left(e^{-\pi i (a-b)} - e^{\pi i (a-b)} \right) = \frac{e^{\pi i (a-b)}}{2\pi i (a-b)} \left(1 - e^{-2\pi i (a-b)} \right)$$

$= 0$ for $a-b \in \mathbb{Z}$ and $a - b \neq 0$ $\Rightarrow$ sinc$(t-a)$ and sinc$(t-b)$ are orthogonal

if $a-b = 0$, $\int_{-\infty}^{\infty} \operatorname{sinc}(t-a) \overline{\operatorname{sinc}(t-b)} \, dt = \int_{-\infty}^{\infty} |\operatorname{sinc}(t-a)|^2 = \int_{-\infty}^{\infty} |e^{-2\pi i s a} \Pi|^2 ds$

$$= \int_{-\infty}^{\infty} (|e^{-2\pi i s a}| \, |\Pi|)^2 ds = \int_{-\infty}^{\infty} (1 \cdot |\Pi|)^2 ds = \int_{-1/2}^{1/2} 1 \, ds = 1$$

$\Rightarrow$ sinc$(t-a)$ and sinc$(t-b)$ are not orthogonal for $a - b = 0$.

$$W = e^{2\pi i/N}$$

Systems EE261 HW 6 1/16/2012

1. $$F[m] = \sum_{k=0}^{N-1} f[k]\, e^{-2\pi i k m/N} \qquad G[m] = \sum_{k=0}^{2N-1} g[k]\, e^{-2\pi i k m/(2N)}$$

$$\cancel{= \sum_{k=0}^{N-1} g[k] w^{-km} + \sum_{k=N}^{2N-1} g[k] w^{-km} = \sum_{k=0}^{N-1} f[k] w^{-km} + \sum_{k=0}^{N-1} f[k+N] w^{-(k+N)m}}$$

$$\cancel{= \sum_{k=0}^{N-1} f[k] w^{-km} + \sum_{k=0}^{N-1} f[k] w^{-km} = 2F[m]}$$

$$= \sum_{k=0}^{N-1} f[k]\, e^{-2\pi i k m/(2N)} + \sum_{k=N}^{2N-1} f[k]\, e^{-2\pi i k m/(2N)}$$

$$= \sum_{k=0}^{N-1} f[k]\, e^{-2\pi i k m/(2N)} + \sum_{k=0}^{N-1} f[k+N]\, e^{-2\pi i m(k+N)/(2N)}$$

$$= \sum_{k=0}^{N-1} f[k]\, e^{-2\pi i k m/(2N)} + e^{-\pi i m} \sum_{k=0}^{N-1} f[k]\, e^{-2\pi i k m/(2N)}$$

$$= \begin{cases} 2F[m/2], & m \text{ even} \\ 0, & m \text{ odd} \end{cases}$$

2. $$\cancel{F[m] = \sum_{k=0}^{N-1} f[k] w^{-km} = \sum_{k=0}^{N-1} f[k]\, e^{-2\pi i k m/N}}$$

$$\cancel{G[m] = \sum_{k=0}^{N/2-1} g[n] w^{-km} = \sum_{k=0}^{N-1} g[n]\, e^{-2\pi i k m/(N/2)}}$$

$$\cancel{= \frac{1}{2} \sum_{k=0}^{N/2-1} f[2k]\, e^{-2\pi i 2km/N}} \qquad \cancel{\text{let } n = 2k}$$

$$\cancel{= \frac{1}{4}\left(\sum_{k=0}^{N-1} f[k]\, e^{-2\pi i k m/N} + e^{-i\pi m} \sum_{k=0}^{N-1} f[k]\, e^{-2\pi i k m/N} \right)}$$

$$\cancel{= \frac{1}{4}\left(F[m] + (-1)^m F[m]\right) = \frac{1}{4} F[m]\,(1 + (-1)^m)}$$

First lets derive the formula for downsampling:

$$g[n] = f[2n] \qquad n = 0, \ldots, \frac{N}{2} - 1$$

$$F[m] = \sum_{k=0}^{N-1} f[k]\, e^{-2\pi i k m/N}$$

$$G[m] = \sum_{k=0}^{\frac{N}{2}-1} g[k]\, e^{-2\pi i k m/(N/2)} = \sum_{k=0}^{\frac{N}{2}-1} f[2k]\, e^{-2\pi i (2k) m/N}$$

$$= \frac{1}{2}\left(\sum_{k=0}^{N-1} f[k]\, e^{-2\pi i k m/N} + \sum_{k=0}^{N-1} f[k]\, e^{-\pi i k}\, e^{-2\pi i k m/N} \right)$$

$$= \frac{1}{2}\left(\sum_{k=0}^{N-1} f[k]\, e^{-2\pi i k m/N} + \sum_{k=0}^{N-1} f[k]\, e^{-2\pi i k (m+\frac{N}{2})/N} \right)$$

$$= \boxed{\frac{1}{2}\left(F[m] + F[m+\tfrac{N}{2}]\right)} \quad \text{for even downsampling.}$$

Now let $g[n] = f[2n+1] \qquad n = 0, \dots, \frac{N}{2}-1$

$$F[n] = \sum_{k=0}^{N-1} f[k]\, e^{-2\pi i k m/N}$$

$$G[n] = \sum_{k=0}^{\frac{N}{2}-1} g[k]\, e^{-2\pi i k m/(N/2)} = \sum_{k=0}^{\frac{N}{2}-1} f[2k+1]\, e^{-2\pi i (2k) m/N} = e^{2\pi i m/N} \sum_{k=0}^{\frac{N}{2}-1} f[2k+1]\, e^{-2\pi i (2k}$$

$$= \frac{1}{2} e^{2\pi i m/N} \left(\sum_{k=0}^{N-1} f[k]\, e^{-2\pi i k m/N} - \sum_{k=0}^{N-1} f[k]\, e^{-\pi i k} e^{-2\pi i k m/N} \right)$$

$$= \frac{1}{2} e^{2\pi i m/N} \left(\sum_{k=0}^{N-1} f[k]\, e^{-2\pi i k m/N} - \sum_{k=0}^{N-1} f[k]\, e^{-2\pi i k (m+\frac{N}{2})/N} \right)$$

$$= \boxed{\frac{1}{2} e^{2\pi i m/N} \left(F[m] - F[m+\tfrac{N}{2}] \right)} \quad \text{for odd downsampling.}$$

2. By linearity, $G[m] = \frac{1}{2}\left(\frac{1}{2}(F[m] + F[m+\frac{N}{2}]) + \frac{1}{2} e^{2\pi i m/N} (F[m] - F[m+\frac{N}{2}]) \right)$

$$= \boxed{\frac{1}{4}\left((1 + e^{2\pi i m/N}) F[m] + (1 - e^{2\pi i m/N}) F[m+\tfrac{N}{2}] \right)}$$

Another method: First, let $h[n] = (\frac{1}{2}, 0, 0 \cdots, 0, 0, \frac{1}{2})$ (length N)

$\Rightarrow h[n] = \frac{1}{2}(\delta_0[n] + \delta_{N-1}[n])$

$$X[n] = f[n] * h[n] = \frac{1}{2}(f[n] + f[n+1]), \quad n = 0, \dots, N-1$$

$$\mathcal{F}\delta_k = \omega^{-k} \Rightarrow X[m] = F[m] \cdot H[m] = F[m] \cdot \frac{1}{2}\left(1 + e^{-2\pi i m (N-1)/N}\right)$$

$$= F[m] \cdot \frac{1}{2}\left(1 + e^{-2\pi i m} e^{-2\pi i m/N}\right) = \frac{1}{2} F[m] \left(1 + e^{2\pi i m/N}\right)$$

Now perform even downsampling to get $G[m]$

$$G[m] = \frac{1}{2}\left(X[m] + X[m+\tfrac{N}{2}]\right) = \frac{1}{4} F[m] \left(1 + e^{2\pi i m/N}\right)$$

$$+ \frac{1}{4} F[m+\tfrac{N}{2}] \left(1 + e^{2\pi i [m+N/2]/N}\right)$$

$$= \frac{1}{4} F[m] \left(1 + e^{2\pi i m/N}\right) + \frac{1}{4} F[m+\tfrac{N}{2}] \left(1 - e^{2\pi i m/N}\right) \quad \checkmark$$

3. a) $F[m] = \sum_{k=0}^{N-1} f[k]\, e^{-2\pi i k m/N}$

$G[m] = \sum_{k=0}^{2N-1} g[k]\, e^{-2\pi i k m/(2N)} = \sum_{k=0}^{N-1} f[k]\, e^{-2\pi i k (m/2)/N}$

$\Rightarrow G[2m] = \sum_{k=0}^{N-1} f[k]\, e^{-2\pi i k m/N} = F[m]$

b) $\cancel{G[m] = \sum_{k=0}^{N-1} f[k]\, e^{-2\pi i k (m/2)/N}}$

$\cancel{\sum_{k=-\infty}^{\infty} F\left[\tfrac{1}{2}(m-1)-k\right] \operatorname{sinc}\left(k+\tfrac{1}{2}\right)}$

$\cancel{= \sum_{k=-\infty}^{\infty}\left(\sum_{n=0}^{N-1} f[n]\, e^{-2\pi i n\left(\frac{1}{2}(m-1)-k\right)/N}\right) \operatorname{sinc}\left(k+\tfrac{1}{2}\right)}$

$\cancel{= \sum_{k=-\infty}^{\infty}\left(\sum_{n=0}^{N-1} f[n]\, e^{-\pi i n m/N}\, e^{\pi i n/N}\, e^{2\pi i n k/N}\right) \operatorname{sinc}\left(k+\tfrac{1}{2}\right)}$

? Look up linear interpolation in book.

4. a) Linear, Time variant.

b) Nonlinear, Time invariant

c) $w(t) = \int_{-\infty}^{\infty} v(\tau)\, e^{-2\pi i t\tau}\, d\tau = \mathcal{F}v$

$\int_{-\infty}^{\infty} (\alpha v_1(\tau) + \beta v_2(\tau))\, e^{-2\pi i t\tau}\, d\tau = \alpha w_1(t) + \beta w_2(t) \Rightarrow$ Linear

$\int_{-\infty}^{\infty} v(\tau - \tau_0)\, e^{-2\pi i t\tau}\, d\tau = \int_{-\infty}^{\infty} v(T)\, e^{-2\pi i t(T+\tau_0)}\, dT$

$= \int_{-\infty}^{\infty} v(T)\, e^{-2\pi i t\tau_0} e^{-2\pi i t T}\, dT \neq w(t-\tau_0) = \int_{-\infty}^{\infty} v(\tau)\, e^{2\pi i t\tau_0} e^{-2\pi i t\tau}\, d\tau \Rightarrow$ Time Varian

d) $\frac{d}{dt}(\alpha v_1(t) + \beta v_2(t)) = \alpha \frac{d}{dt} v_1(t) + \beta \frac{d}{dt} v_2(t) = \alpha w_1(t) + \beta w_2(t) \Rightarrow$ Linear

$\frac{d}{dt} v(t-\tau) = w(t-\tau) \Rightarrow$ Time invariant

e) Nonlinear, Time variant

5. a) $\delta(t)\, e^{-i\pi t^2} = \delta(t)$ $\quad \delta(t) * e^{i\pi t^2} = e^{i\pi t^2}$ $\quad e^{i\pi t^2} e^{-i\pi t^2} = e^{0} = \boxed{1 = w(t)}$

$\delta(t) f(t) = f(0)\, \delta(t)$ $\quad \delta(t) * f(t) = f(t)$

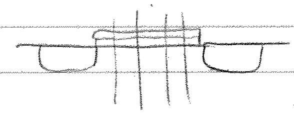
b) $\delta(t-\tau)\, e^{-i\pi t^2} = \delta(t-\tau)\, e^{-i\pi\tau^2}$ $\quad e^{i\pi t^2} * \delta(t-\tau)\, e^{-i\pi\tau^2} = e^{-i\pi\tau^2} e^{i\pi(t-\tau)^2}$

$e^{-i\pi\tau^2} e^{i\pi(t-\tau)^2} e^{-i\pi t^2} = e^{i\pi(t^2 - 2\tau t + \tau^2 - \tau^2 - t^2)} = \boxed{e^{-2i\pi\tau t} = w(t)}$

c) $\boxed{w(t) = \int_{-\infty}^{\infty} h(t,\tau)\, v(\tau)\, d\tau = \int_{-\infty}^{\infty} e^{-2\pi i t\tau}\, v(\tau)\, d\tau}$ This is the Fourier Transform.

Devices EE316 L3 - Long Channel MOSFET

- Long Channel Behavior :
  - The electric field in channel is essential 1-D. (Normal to the semiconductor surface)
  - $E_x >> E_y$ ⟶



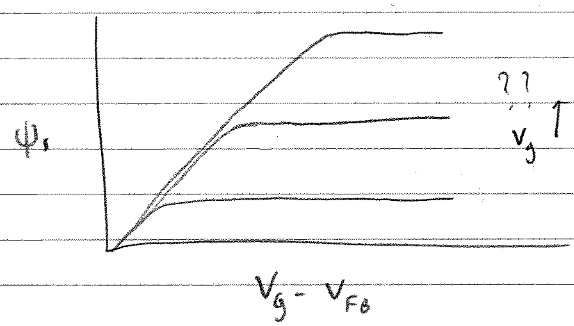
    ⇒ we can solve poisson's eq. in slices.

- Drain Current Model:
  - $n(x) = \frac{n_i^2}{N_a} e^{q(\psi - V)/kT}$ ← boltzmann approx.
  - $\psi(0,y) = V(y) + 2\psi_B$
  - $W_{dm}(y) = \sqrt{\frac{2\epsilon_{si}(V(y) + 2\psi_B)}{qN_a}}$ ← depletion region width at drain.

$\Rightarrow J_n(x,y) = -q\mu_n n(x,y) \frac{dV(y)}{dy}$ ← gradient of quasi fermi level along y-direction.

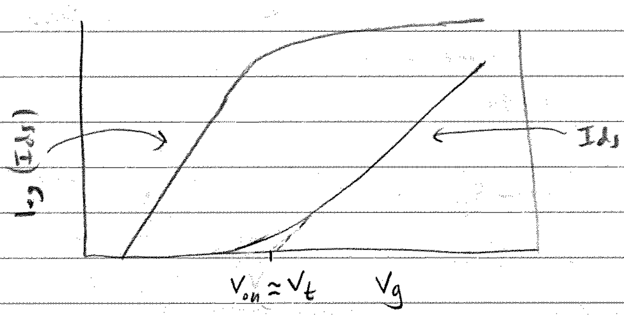
↓

Gradual Channel Approx.



surface potential becomes pinned.

$$V_t = V_{fb} + 2\psi_B + \frac{\sqrt{4\epsilon_{si} q N_a \psi_B}}{C_{ox}}$$



Experimental Determination of the Threshold Voltage

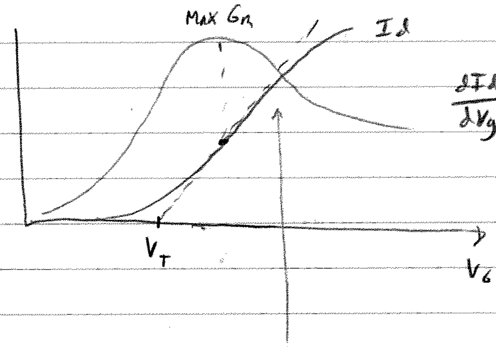
LE – Linear Extrapolation at maximum G_m point. ← used for modeling devic

CC – Constant Current Method ← used in industry

TC – Transconductance Change

- All devices are now non-uniformly doped along x-direction, so $2\psi_B$ point doesn't make sense (it depends on x)

LE

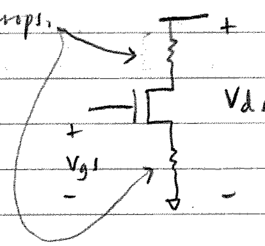


$$\frac{\partial I_d}{\partial V_g} = G_m$$

bends over because at high V_G, V_y increases, $\Rightarrow \mu_{eff} \downarrow$

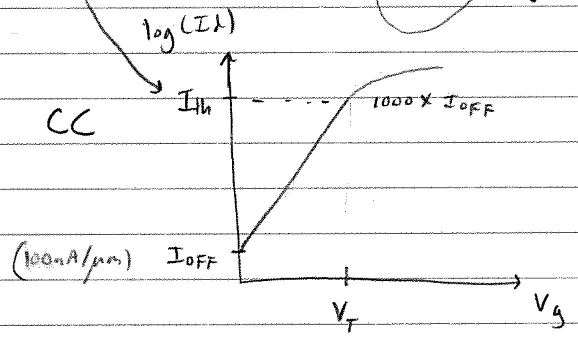
also, bends over due to drain, source, and contact resistances.

causes voltage drops.



define arbitrarily.

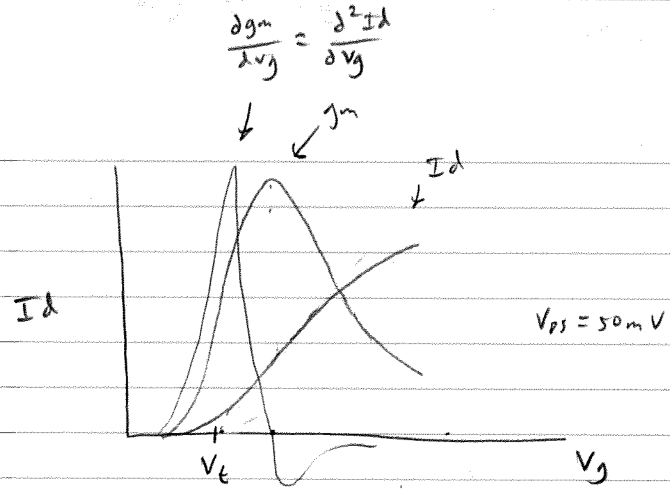
CC



← review this.

Used in industry because it's very fast. Don't need to sweep, just measure a point, so you can do it in a tester.

TC – Used for things like Thin SOI, Finfet, which have no doping ($\Phi_B = 0$)



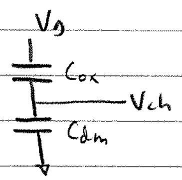
$V_{DS} = 50\,mV$

- For organic TFTs, Poisson's eq. doesn't apply, so we can't use the same approximations. But we generally use the same methods to extract "V_{th}"

$$m = 1 + \frac{C_{dm}}{C_{ox}} = 1 + \frac{3t_{ox}}{W_{dm}} \qquad C_{dm} = \frac{\epsilon_{si}}{W_{dm}} \qquad \text{typically, } m \approx 1.2$$

body-effect coefficient.

$$I_{ds} = I_{dsat} = \frac{1}{2}\mu_{eff} C_{ox} \frac{W}{L} \frac{(V_{gs}-V_t)^2}{m}$$

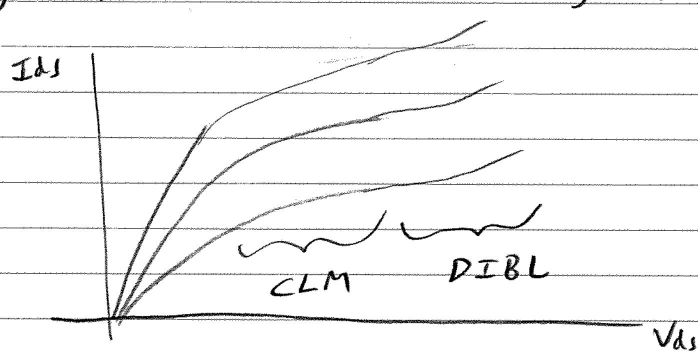


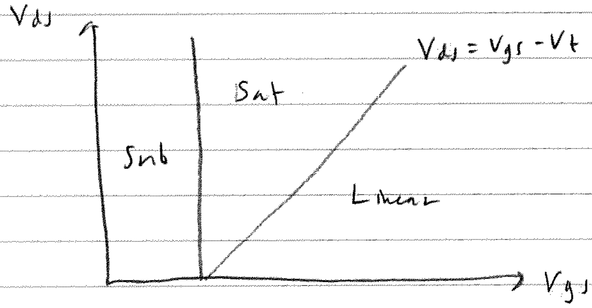
voltage divider.

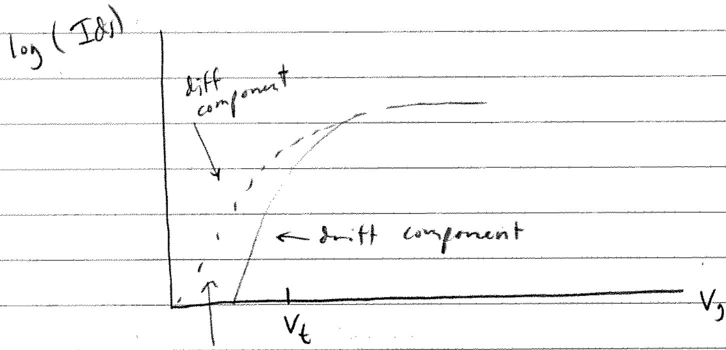
$$V_{ch} = \frac{C_{ox}}{C_{ox}+C_{dm}} V_g = \frac{V_g}{m}$$

Pinch OFF: $V_{ds} = V_{dsat} = \frac{V_g - V_t}{m}$

Beyond Pinch OFF: Channel Length Modulation.







In Subthreshold, I_{ds} is dominated by diffusion current.

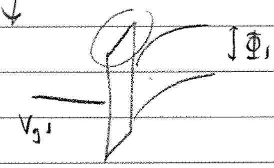
Subthreshold:

$$I_{ds} = \mu_{eff} C_{ox} \frac{W}{L} (m-1) \left(\frac{kT}{q}\right)^2 e^{q(V_g - V_t)/mkT} \left(1 - e^{-qV_{ds}/kT}\right)$$

Inverse Subthreshold Slope:

$$S = \left(\frac{\partial \log(I_d)}{\partial V_g}\right)^{-1} = m \frac{kT}{q} \ln(10) = \underbrace{\frac{\partial V_G}{\partial \psi_s}}_{m} \underbrace{\frac{\partial \psi_s}{\partial \log I_d}}_{\frac{kT}{q}\ln(10)}$$

$$= 2.3 \frac{mkT}{q} \qquad (60mV/dec \text{ for } m = 1,\ T = 300K)$$

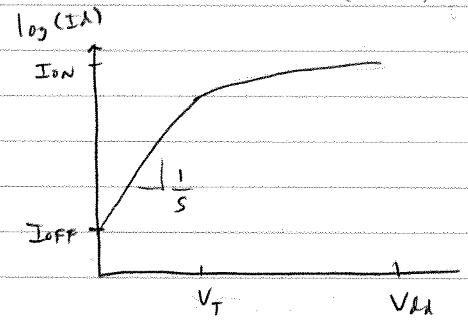


not all of applied V_{gs} goes to band bending Φ_s. Some goes to bending in the oxide.

for low m, we want $C_{ox} >> C_{dm}$

$\Rightarrow$ we want to minimize t_{ox}.

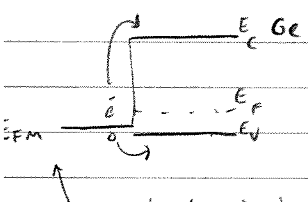
We want S small (slope steep) so we can get a maximum $\left(\frac{I_{ON}}{I_{OFF}}\right)$



Can get S lower than 60 mV/dec, if you have gain (rather than capacitive division) between gate and channel.
Or, if you have some different mechanism of current. (e.g. IMOS)

Meeting With Professor Saraswat

- 1998 - Q2
- 2003 - Leaky Cap — gate leakage does not change Cap value.
- 2004 - Ultimate Limit to Scaling - ~0.5nm, beyond that, é keeps moving back + forth.
- 2007 - Current Source into gate of MOSFET
- SOI



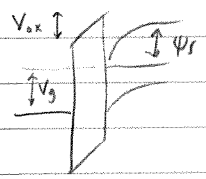
fermi level pinning, doesn't matter what type of metal you use.
If this is close to midgap, you can have an ambipolar Id-Vg characteristic.

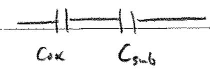
Devices EE316 L4 Device Scaling + Short Channel Effects

$$S = \left(\frac{\partial \log I_D}{\partial V_g}\right) = \frac{\partial V_g}{\partial \psi_s} \frac{\partial \psi_s}{\partial \log I_d}$$

capacitive divider

60 mV per decade in MOSFETs due to Fermi Dirac distribution. $n \propto e^{q\psi_s/kT}$





Gate to channel potential coupling : >1 in MOSFETs.

$S \downarrow \Rightarrow \frac{I_{on}}{I_{off}} \uparrow$ ⇒ we want steeper subthreshold slope.

Body Effect : Dependence of V_{th} on V_{bs}

$$V_{th} = V_{fb} + 2\psi_B + \frac{\sqrt{2 \varepsilon_{si} q N_a (2\psi_B + V_{bs})}}{C_{ox}}$$

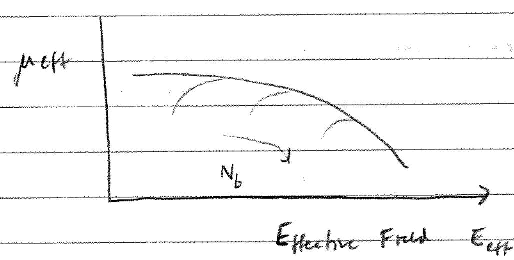
$V_{bs} > 0$ $V_{th} \uparrow$ $V_{bs} < 0$ $V_{th} \downarrow$

$\boxed{\frac{dV_{th}}{dT} \text{ is typically } -1\,mV/K}$ for $N_a \sim 10^{16}\,cm^{-3}$ (for diode, $V_{be} \sim -2\,mV/K$)

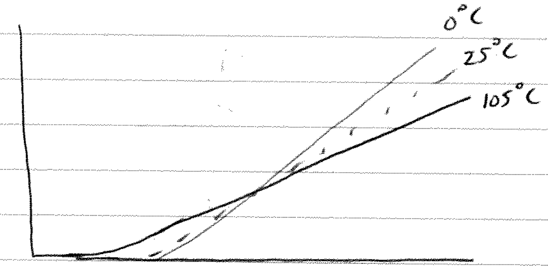
Temperature dependence.

In Consumer electronics, devices run at 85°C.

Electron Mobility:



Temperature Dependence of I_d



$I \propto \frac{1}{T}$ since $\mu \propto T^{-3/2}$ $I \propto \mu$

But $T\downarrow \Rightarrow V_{th}\uparrow$

Intrinsic MOSFET Capacitance:

Sub: $C_g = WL\, C_{dep} = \boxed{WL\left(\frac{1}{C_{ox}} + \frac{1}{C_d}\right)^{-1}}$

C_{ox}, C_{dep}

Linear: $\boxed{C_g = WL\, C_{ox}}$ (depl. cap is shielded)

Sat: $Q_i(y) = -C_{ox}(V_g - V_t)\sqrt{1 - \frac{y}{L}}$

$$\boxed{C_g = \frac{2}{3} WL\, C_{ox}}$$

MOSFET Scaling

Goals: achieve density and performance gains, and power reduction in VLSI.

Issues: Short channel effect, power density, switching delay, reliability.

Constant-Field Scaling: Scale the device voltages + device dimensions (both horizontal + vertical) by the same factor ($\kappa > 1$) such that the electric field remains unchanged.
(also increase substrate doping by κ)

$\frac{V}{\kappa}$, $\frac{L}{\kappa}$, $\frac{W}{\kappa}$, $\frac{t_{ox}}{\kappa}$, κN_a, $\frac{x_j}{\kappa}$

Derived Scaling Behavior of device parameters.	E	1
	V_0	1
	W_d	$1/K$
	C	$1/K$
	Q_i	1
	I_{ds}	$1/K$
	R_{ch}	1

Derived Scaling behavior of circuit parameters.	Circuit delay $(\tau = \frac{CV}{I})$	$1/K$
	(ON) Power Dissipation $(P \sim VI)$	$1/K^2$
	(ON) Power Delay Product $(P \cdot \tau)$	$1/K^3$
	Circuit Density $(\propto 1/A)$	K^2
	(ON) Power Density (P/A)	1

ION implantation – makes shallower but sharper junction compared to (gas) diffusion based doping.

Generalized scaling:

↓

allow $\vec{E}$ to scale up by α $(\vec{E} \to \alpha \vec{E})$ $\alpha > 1$

while device dimension scale down by K. $K > 1$

i.e. voltage scales by $V \to (\alpha/K)\, V$

$N_a, N_d \to (\alpha K) N_a, \ (\alpha K) N_d$

More flexibility than constant field scaling, but has reliability + power concerns.

↓

Generalized scaling circuit parameters.		Long Ch.	Vel. Sat
	Circuit delay time $(\tau = CV/I)$	$1/(\alpha K)$	$1/K$
	Power dissipation (IV)	α^3/K^2	α^2/K^2
	Power delay product	α^2/K^3	
	Circuit Density	K^2	
	Power Density	α^3	α^2

Scaling of Depletion Width (Constant Field Scaling)

$$W_{dm} = \sqrt{\frac{4\varepsilon_{si} kT \ln(N_a/n_i)}{q^2 N_a}}$$

$\Rightarrow$ $\underbrace{\text{maximum gate depletion width}}_{}$ scales even less than $\frac{1}{\sqrt{K}}$.

determines minimum channel length.
you can get without suffering short
channel effects.

Constant Voltage Scaling ($\alpha = K$)

$N_a \to K^2 N_a \quad \Rightarrow \quad W_{dm} \to \frac{1}{K} W_{dm}$

But $\quad \vec{E} \to kE \quad$ (reliability issue due to high $\vec{E}$ field)

$P/A \to K^3 P/A$

Scaling in practice. (rising oxide field)

Feature size	Power Supply	Gate Oxide	Oxide Field
$2\mu m$	5 V	35 nm	1.4 MV/cm
0.7 65 nm	1.2V	1.2nm	10MV/cm
45 nm	1 V	1 nm	10MV/cm
0.7 32 nm	0.9V	0.95 nm	9.5 MV/cm

$$\begin{array}{c|c} & \xrightarrow{0.7} \\ 0.7 \downarrow & \sim 0.5 \end{array}$$

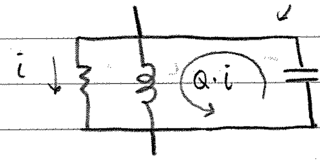
$0.7 \sim \sqrt{2}$

Scaling in Practice : Field + Power Density have gone up.

by physics, we managed to cope with reliability requirements
at higher fields.

Circuits - RLC Review

Parallel RLC:

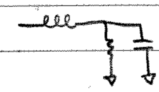


$$\omega_o = \frac{1}{\sqrt{LC}} \qquad Z_o = \sqrt{\frac{L}{C}}$$

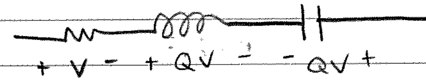
(ω_o for 1nH + 1pF is 5 GHz
⇒ for example for 9nH and 25pF ⇒ 0.333 GHz

$$Q = \omega \frac{\text{Energy Stored}}{\text{Avg. Power Dissipated}} = \frac{\omega_o}{BW} = \text{\# cycles ringing}$$

$$Q = \frac{R}{\sqrt{L/C}} = \frac{R}{\omega_o L} = \omega_o RC$$ ← also applies to



Series RLC:

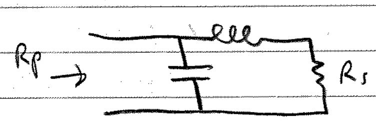


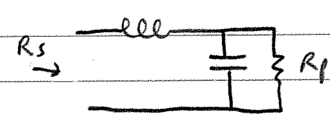
$$Q = \frac{\sqrt{L/C}}{R} = \frac{\omega_o L}{R} = \frac{1}{\omega_o RC}$$

$$R_p = R_s(Q^2+1)$$
$$X_p = X_s\left(\frac{Q^2+1}{Q^2}\right)$$

} holds over a narrow bandwidth around ω_o.

Impedance Transformations: L-Match





Upward Impedance Transformer   Downward Transformer.

$$Q \approx \sqrt{\frac{R_p}{R_s}} \quad \text{for } Q \gg 1.$$

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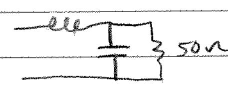
e.g. $R_2 = 5\Omega$ $\quad R_1 = 50\Omega$ $\quad \omega_o = 2\pi \cdot 1GHz$



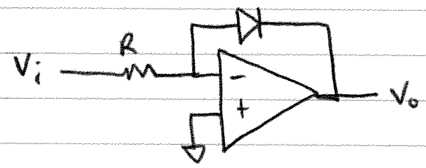
$$\frac{1}{\sqrt{LC}} = 2\pi \cdot 1GHz \qquad \frac{R_2}{R_1} = Q^2+1 \Rightarrow Q = 3 = \frac{50}{\sqrt{L/C}} = \omega_o RC \Rightarrow C = \frac{3}{2\pi \cdot 1GHz \cdot 50}$$

$$\Rightarrow \boxed{C \approx 10pF} \qquad \frac{1}{2\pi\sqrt{1nH \cdot 1pF}} = 5GHz \Rightarrow \frac{1}{2\pi\sqrt{2.5nH \cdot 10pF}} = 1GHz$$

$$\Rightarrow \boxed{L \approx 2.5nH}$$

Circuits — Clever Opamp Circuits

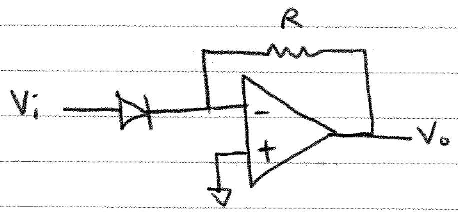
Log Output



$$V_{out} \approx -V_T \ln\left(\frac{V_{in}}{I_s R}\right)$$

(for $V_{in} > 0$)

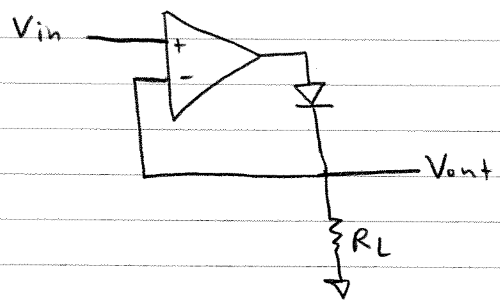
Exponential Output



$$I = I_s\left(e^{V_i/V_T} - 1\right) \approx I_s e^{V_i/V_T}$$

$$\Rightarrow V_{out} = -I_s R\, e^{V_i/V_T}$$

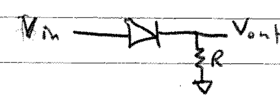
Precision Rectifier



$$V_{out} = \begin{cases} V_{in}, & V_{in} > 0 \\ 0, & V_{in} \leq 0 \end{cases}$$

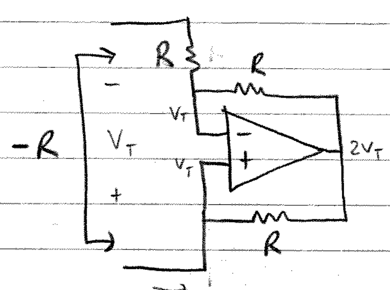
(as opposed to regular rectifier

$$V_{out} = \begin{cases} V_{in} - 0.7, & V_{in} > 0.7 \\ 0, & V_{in} \leq 0.7 \end{cases}$$

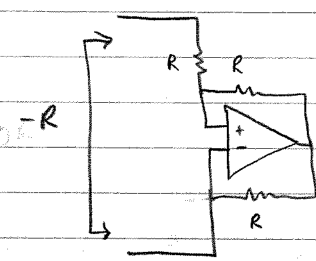


)

Negative Resistor



OR



$$\vec{i_t} = -\frac{V_T}{R} \Rightarrow \frac{V_T}{R_t} = -R$$

Devices Pease - Random Facts

- 1 kg = 2.2 lbs
- 1 m = 3.3 ft
- 1 km = 0.62 mi
- 1 Cal = 4 kJ

1 lbs of bullets, vs. batteries, vs. beans.

Bullets: $KE = \frac{1}{2} m v^2$ $m \approx 10g = 10^{-2} kg$ $v \approx 300 m/s \approx 333 m/s$

$\Rightarrow KE \approx \frac{1}{2}(10^{-2})(3.33 \cdot 10^2)^2 \approx 0.5 \cdot 10^3 J = 0.5 kJ$

$1 lb \approx \frac{1}{2.2} kg = 0.45 kg \Rightarrow 0.45 kg \cdot \frac{0.5 kJ}{0.01 kg} = \boxed{22.5 kJ}$

Batteries: Assume 1 9V lasts 2 hours powering a circuit that draws ~100mA.

$= E = P \cdot t = IV \cdot t = (9V)(0.1A) \cdot (7200s) \approx 6.5 kJ / battery.$

assume ~ 10 batteries / lb $\Rightarrow \boxed{65 kJ}$

Beans: 1 lb of beans. Should be enough to feed a man for a day. $\Rightarrow$ about 2000 Cal.

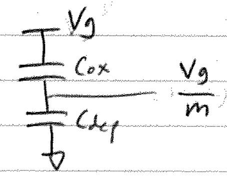
$(2000 Cal)(4kJ/cal) = \boxed{8000 kJ}$! huge compared to batteries + bullets

Subthreshold Current:

(diffusion current $J_D = q D \frac{dn}{dx}$)

$$I_{dsub} = \mu C_{ox} \frac{W}{L} (m-1) \left(\frac{kT}{q}\right)^2 e^{(V_g - V_t)/(m V_T)} \left(1 - e^{-qV_{ds}/kT}\right)$$

$m = 1 + \frac{C_{dm}}{C_{ox}}$ ← voltage division of V_g



want m small.

$$S = \left(\frac{d(\log I_{ds})}{dV_g}\right)^{-1} = \overset{\ln(10)}{2.3}\, m \frac{kT}{q} = 60\ \text{mV/dec for } m = 1.$$

($C_{dep} = 0$)

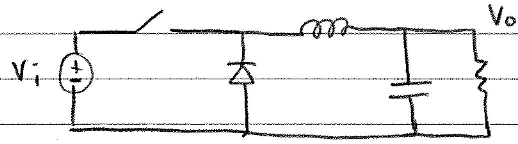
want S small. so slope is large.

$$S = \underbrace{\frac{dV_g}{d\psi_s}}_{\substack{\text{gate to}\\ \text{channel}\\ \text{coupling}\\ >1}} \underbrace{\frac{d\psi_s}{d(\log I_d)}}_{\substack{\text{60 mV/dec}\\ \text{in MOSFETs}\\ \text{due to}\\ \text{Fermi Dirac}\\ \text{distribution.}}}$$

Simplified version from Razavi $I_{sub} = I_s e^{V_g/SV_T}$ $\quad S > 1$

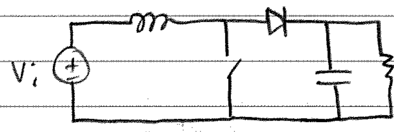
Power Electronics: DC-DC converters

Buck:



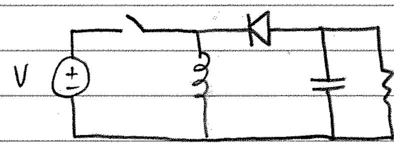
$$\frac{V_o}{V_i} = D$$ step down

Boost:



$$\frac{V_o}{V_i} = \frac{1}{1-D}$$ step up

Buck Boost:



$$\frac{V_o}{V_i} = -\left(\frac{D}{1-D}\right)$$ inverte step ... or st down

① $(V_i - V_o) D - V_o D' = 0 \Rightarrow V_i D = V_o (D' + D) \Rightarrow \frac{V_o}{V_i} = \frac{D}{D'+D} = \frac{D}{1-D+D} = D$

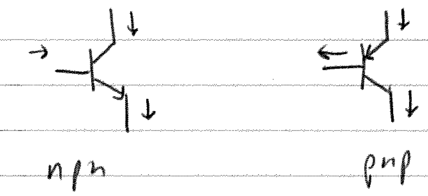
② $V_i D + (V_i - V_o) D' = 0$

$V_i (D + D') = V_o D'$

$$\frac{V_o}{V_i} = \frac{D + D'}{D'} = \frac{D + 1 - D}{1-D} = \frac{1}{1-D}$$

③ $V_i D + V_o D' = 0 \Rightarrow \frac{V_o}{V_i} = -\frac{D}{D'} = -\frac{D}{1-D}$

BJT Review



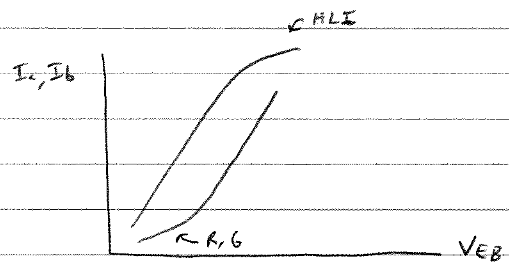
for pnp:

$$\gamma = \frac{I_{Ep}}{I_{Ep}+I_{En}} = \frac{1}{1+\frac{D_E}{D_B}\frac{N_B}{N_E}\frac{W}{L_E}}$$

$$\alpha_T = \frac{I_{Cp}}{I_{Ep}} = \frac{1}{1+\frac{1}{2}\left(\frac{W}{L_B}\right)^2}$$

$$\alpha_{DC} = \gamma\alpha_T$$

$$\beta = \frac{\alpha_{DC}}{1-\alpha_{DC}} = \frac{1}{\frac{D_E}{D_B}\frac{N_B}{N_E}\frac{W}{L_E}+\frac{1}{2}\left(\frac{W}{L_B}\right)^2} = \frac{I_c}{I_b}$$



Law of junction: $np = n_i^2 \exp\left(\frac{qV_A}{kT}\right)$

$$\Delta n_p\Big|_{-x_p} = \frac{n_i^2}{N_A}\left(e^{qV_A/kT}-1\right)$$

$$\Delta p_n\Big|_{x_n} = \frac{n_i^2}{N_D}\left(e^{qV_A/kT}-1\right)$$

F.S. $\quad f(t) = \sum_{n=-\infty}^{\infty} c_n e^{2\pi i n t/T}$

$$c_n = \frac{1}{T}\int_0^T f(t)\, e^{2\pi i n t/T}\, dt$$

F.T. $\quad f(t) = \int_{-\infty}^{\infty} F(s)\, e^{2\pi i s t}\, ds$

$$F(s) = \int_{-\infty}^{\infty} f(t)\, e^{-2\pi i s t}\, dt$$

$$\mathcal{F} f^{-} = (\mathcal{F} f)^{-}$$

$$\mathcal{F}\mathcal{F} f = f^{-}$$

$$\overline{\mathcal{F} f} = (\mathcal{F} f)^{-}$$

$$\int_{-\infty}^{\infty} f(x)\, \overline{g(x)}\, dx = \int_{-\infty}^{\infty} F(s)\, \overline{G(s)}\, ds$$

$$\mathcal{F}\, g'(x) = 2\pi i s\, G(s)$$

$$\mathcal{F}\, g^{(n)}(x) = (2\pi i s)^n\, G(s)$$

$$\mathcal{F}(x^n g(x)) = \left(\frac{i}{2\pi}\right)^n G^{(n)}(s)$$

D.F.T $\quad F[m] = \sum_{k=0}^{N-1} f[n]\, e^{-2\pi i k m/N}$

$$f[m] = \frac{1}{N} \sum_{k=0}^{N-1} F[k]\, e^{2\pi i k m/N}$$

EE316

TCAD Tool Tutorial - Monday, Jan 30th, 5-6PM Packard 101

V_t is exponentially dependent on channel length L.

Turn HW into bin in 332X

EE316 - Sentaurus Tutorial

emacs.cshrc

ctrl x s to save
ctrl x c to quit
structure editing: 2
output file is .trd (not .grd anymore)

default units in Sentaurus are in μm.

tecplot_sv ← to visualize script.

↓
open files that end in _msh.dat

Device Simulation

Initial Step refers to percent of total sweep. e.g. 0.01 means 1%.

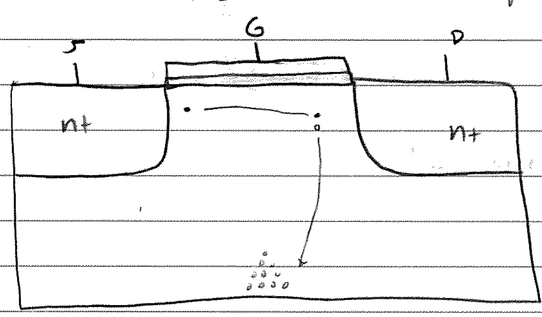
inspect ← to visualize data from a simulation.

file → export → csv

[redacted]

EE316

PDSOI - main benefit: reduces junction capacitance at S and D.
- Downside: History effect (causes problem for circuit designers)



holes built up. This causes a change in V_t.

⇒ V_t depends on how long the device has been on (history effect).

Parasitic BJT action tends not to occur since L is typically longer than the base width of a BJT.

FDSOI - scale length no longer set by doping. set by thickness of bulk.
⇒ shorten scale length
- reduced junction capacitance
- Thin BOX (Buried oxide) has less short channel effects than a Thick BOX for a given t_{Si} (bulk thickness)

EE316 - Device Simulations 2/7/12

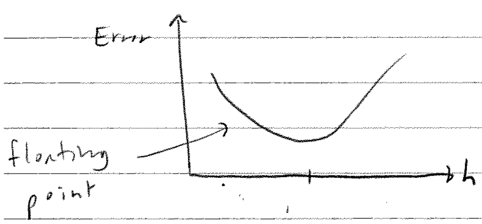
Gradual Channel Approximation: $E_{vert} \gg E_{horiz.}$ (analytically)
allows us to solve 1-D Poisson's equation ∧
This is not true for short channel devices.
⇒ we must use TCAD to solve the 2-D Poisson's equation numerically.

Quantum Effect: Sheet charge approximation (that all inversion charges are located at the surface like a sheet of charge) no longer holds since quantum confinement causes charges to be located a finite distance away from the interface, which effectively decreases gate capacitance.

Recombination / Generation:

- Shockley-Read-Hall (SRH) recombination/generation (indirect semiconducto
- Direct recombination / thermal excitation (direct semiconductors)
- Auger recombination / impact ionization
- Band to band tunneling (acts as a generation process)

Which physics to include: see slide 23 of Ximeng Guan's lecture 1.



← discretization step size. (mesh size)

⇒ Smaller mesh isn't always better (more accurate).
Also smaller mesh ⇒ longer simulation time.

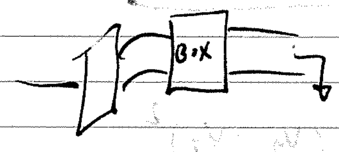
EE316 2/16/2012

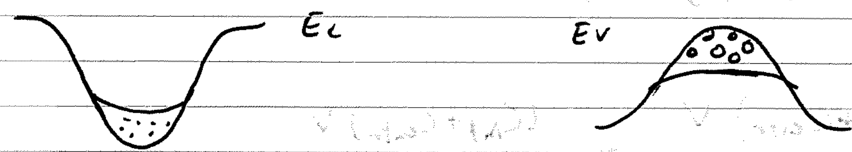
What makes a good source + drain

- a good reservoir of $\bar{e}$'s
- always in equilibrium (has established fermi level)

7-48 - Nondegenerate $E_F < E_c$

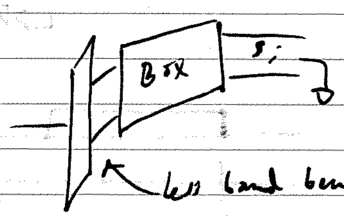
EE316 3/1/2012





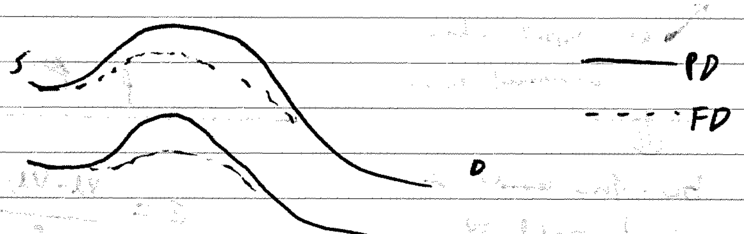
⇒ as you collect holes in the back of the substrate in a PDSOI transistor, $V_T \downarrow$ (history effect).

Why does FD SOI not have history effect?



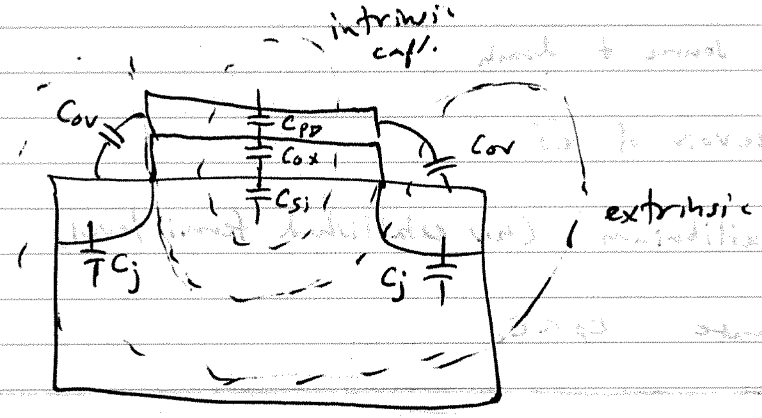
less band bending than for PDSOI.

⇒



barrier is lower.

⇒ holes can flow out easier ⇒ less pronounced history effect.



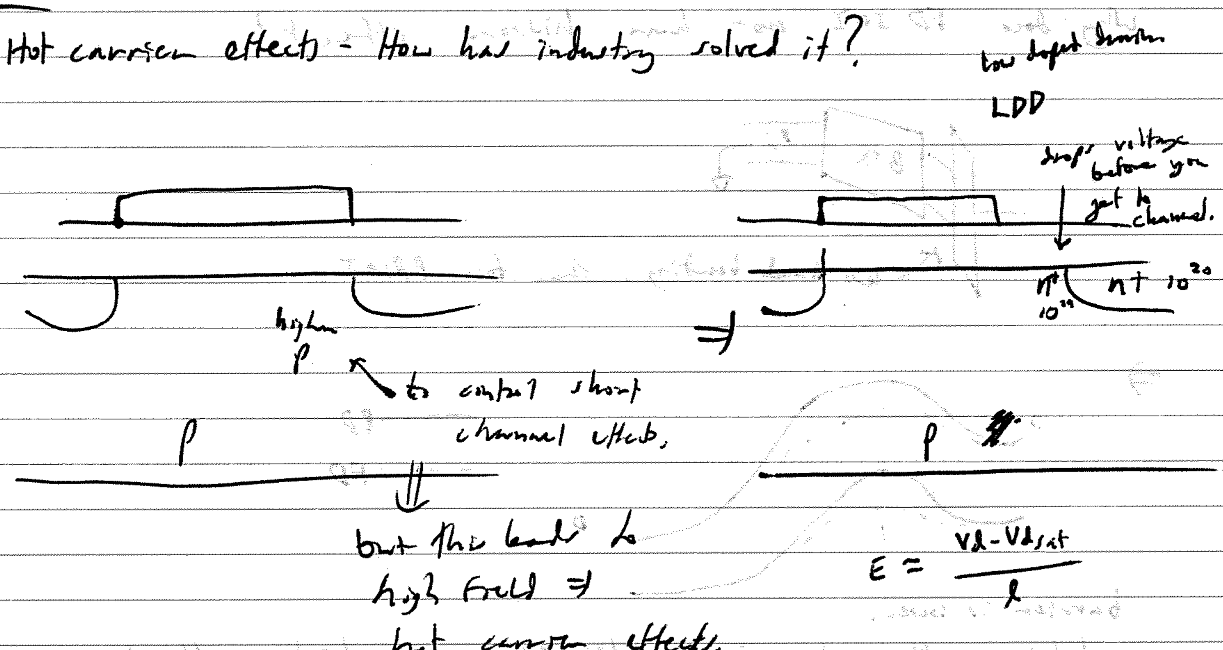
C_{int}

$$I = \mu C_{ox} \frac{W}{L} (V_{gs} - V_T)^2$$

$$\tau = \frac{(C_{int} + C_{extr}) V}{I} = \frac{(C_{int} + C_{extr}) V}{f(C_{int})}$$

$\Rightarrow$ want C_{int} ~~large~~ large, but C_{extr} small.

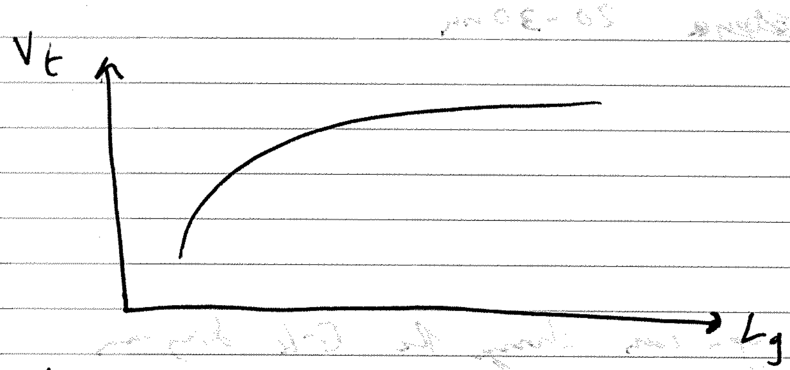
Hot carrier effects - How has industry solved it?

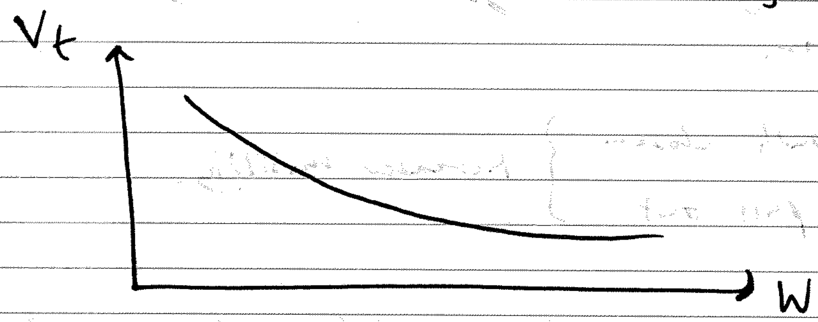


$\Rightarrow$

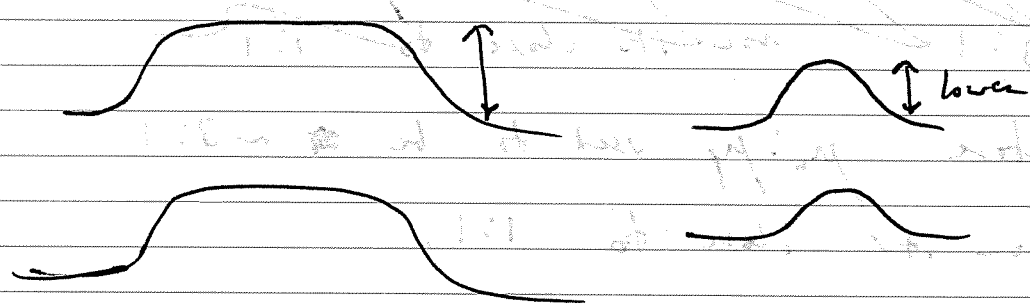
$$E = \frac{V_d - V_{dsat}}{l}$$

Narrow Width effect:





SCE - ~~DIBL~~ Proximity effect.



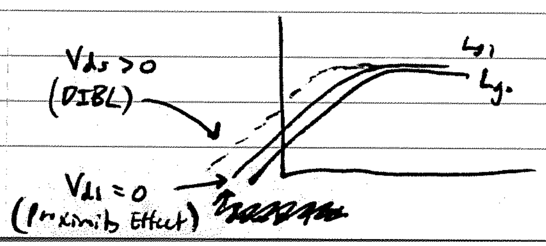
DIBL

related to SC

~ W_{dm}

SS: depends on C_{ox} + C_{dep}

because ? has less control over chan



$L_{g1} < L_{g0}$.

to first order SS does not change.

(but in fact it does due to the impact of fringing caps (gets worse)) used in past when $L_G \approx 100nm$

↓ before ↓ now

Thick BOX: 200 - 400nm , 70 - 100nm

Thin BOX: ~~[illegible]~~ 20 - 30nm

(currently more common)

Strained Si

By straining Si, you can change the E-k diagram of semiconductor.

p-channel → push closer } increases mobility.
n-channel → pull out }

for similar amount of stress, hole mobility improves more than electron mobility.

~~3:1 now its close to 1:1~~

before $\mu_n : \mu_p$ used to be ~~a~~ $\sim 3:1$

now its close to 1:1.