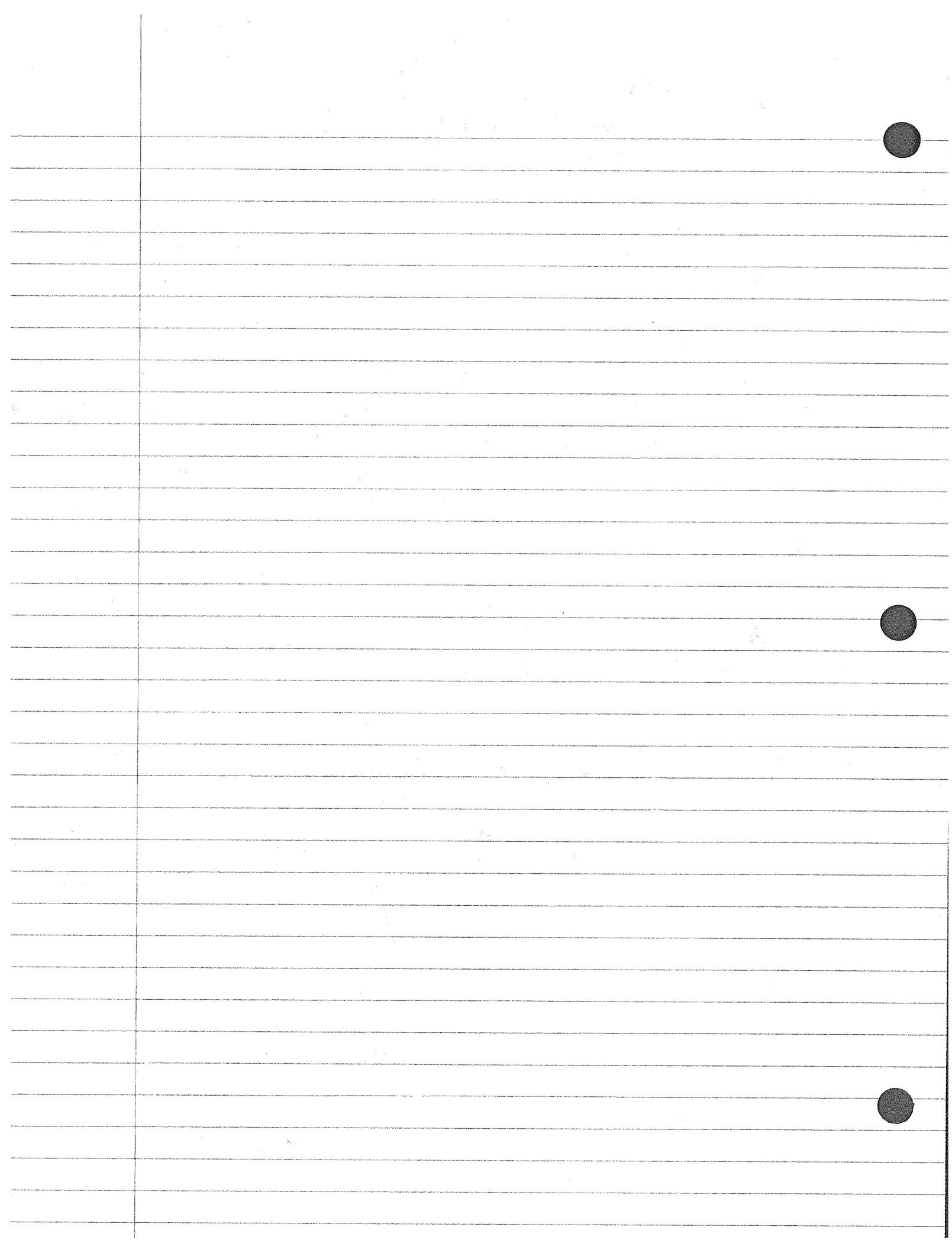
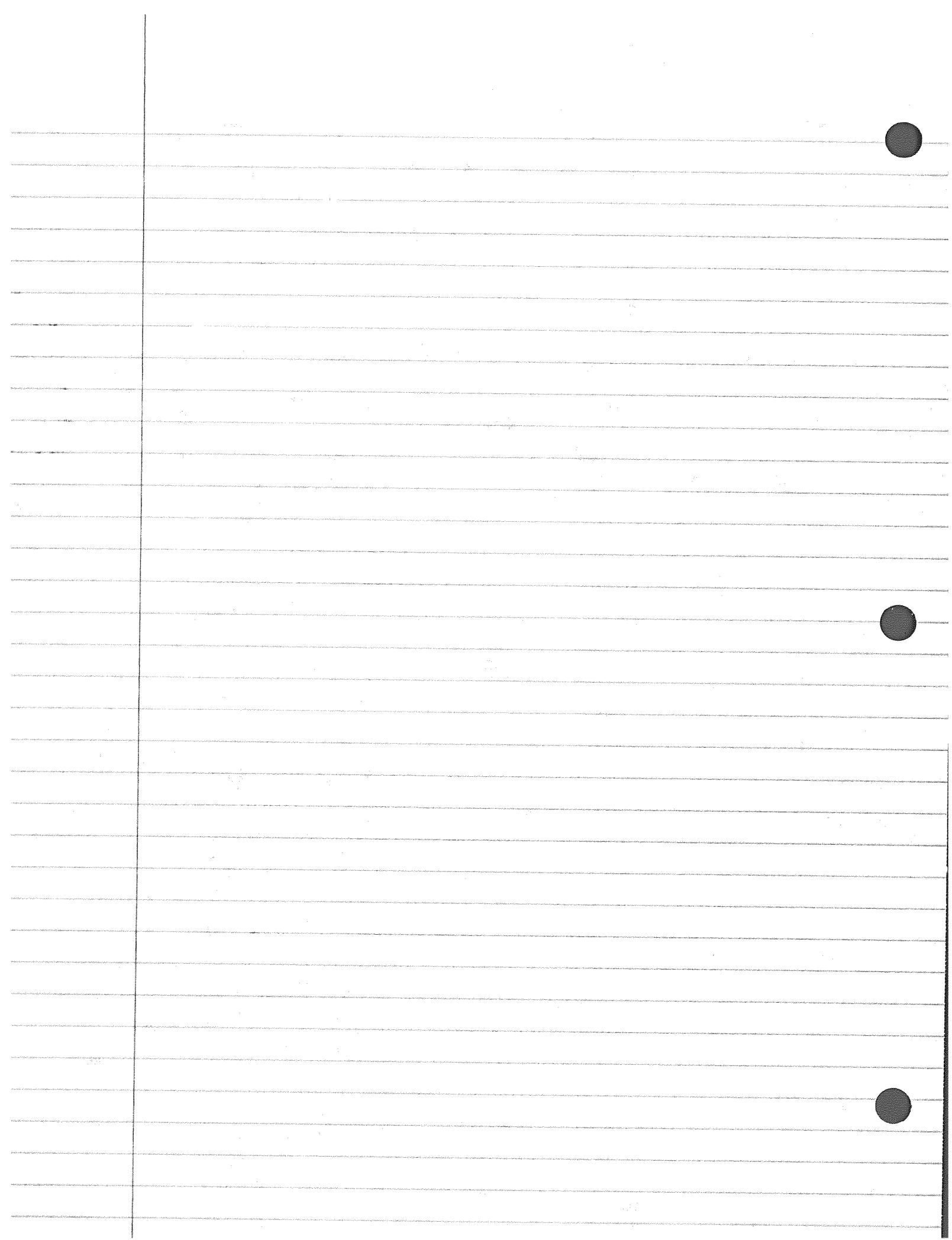


Alex Omid-Zohoor

Quals Solutions

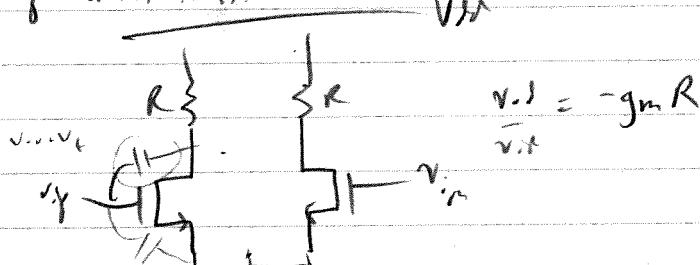


Circuits



2000 - Wong

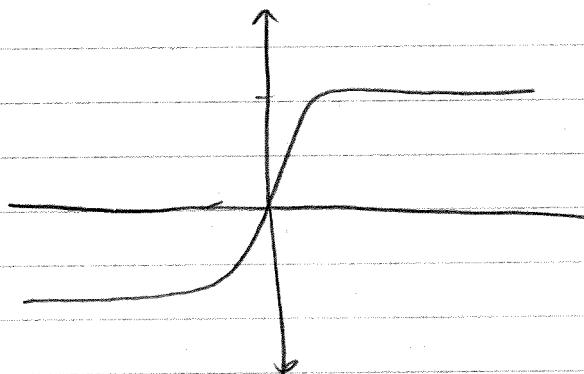
- voltage gain diff pair.
- CM input range
- active load
- freq limitation.



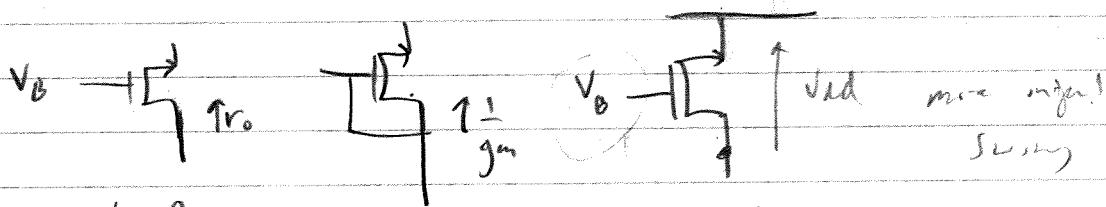
$$C = (1 - KR)$$

$$k = -gm$$

$$\pm \sqrt{2} V_{ov}$$



$$V_t < V_{cm} < V_{dd} + V_t$$



high R
high gain
relativ swing

low R
ratiometric

medium R

kT

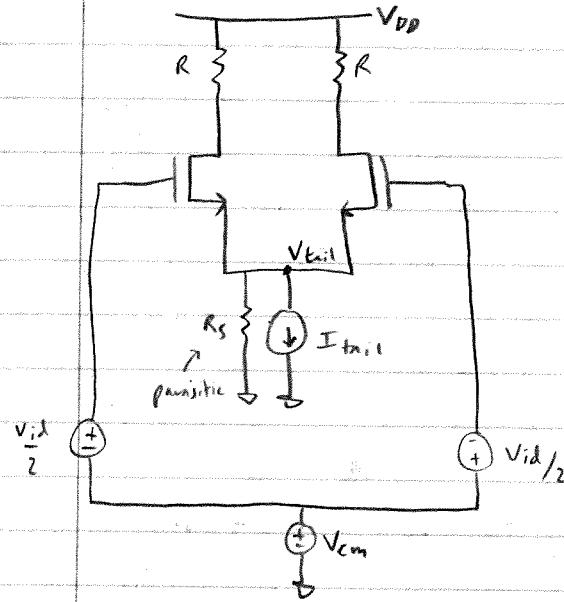
kT

kT

C_f

C_f

C_L



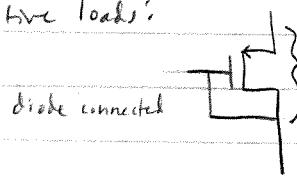
$$A_v = -g_m R$$

$$A_{cm} = -\frac{g_m R}{1 + 2g_m R_s}$$

on $\min\{V_{dd}, V_{dd} - \frac{I_{tail}}{2}R + V_t\}$
if you assume V_{dd} is the max voltage available.

$$CM \text{ input range: } V_{tail} + V_t < V_{cm} < V_{dd} - \frac{I_{tail}}{2}R + V_t$$

Active loads:

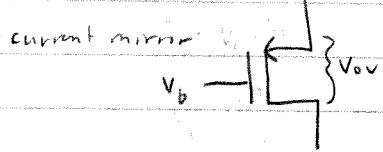


$$R_o = (\frac{1}{g_m} \parallel r_o) \approx \frac{1}{g_m}$$

$V_{out} + V_t \Rightarrow \text{low gain}$

High bandwidth, High linearity.

(bootstrapping) $C = (1 - kC_{gd})$ small since $k \approx 1$
very low output swing

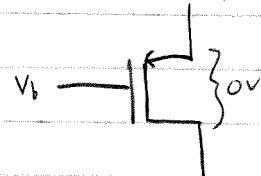


$$R_o = (r_o \parallel r_o) \approx \frac{r_o}{2}$$

$\Rightarrow \text{high gain}$

low output swing

triode load



R_o medium

\Rightarrow medium gain

high output swing

sensitive to process variations, (R depends on μC_{ox} , etc..)

2001 - Kovacs

Table : has sensor sensing something about a glowing cylinder.

- ruler, toilet paper, book, copper clad PCB, plastic ruler, full beverage can, paper, pen, hard cover books.

- Figure out what sensor was sensing

Exp 1: Sensor value vs. distance. Use ruler + pen

to mark off points on toilet paper. 1, 2, 4, 6, 8
(or could actually just count squares)

See if readings fall off as $\frac{1}{r}$, $\frac{1}{r^2}$ or $\frac{1}{r^3}$

point source of EM radiation.

Exp 2: Place hard cover book between sensor + cylinder.

(see if it's sensing visible light).

Exp 3: Place copper clad pcb between sensor + cylinder.

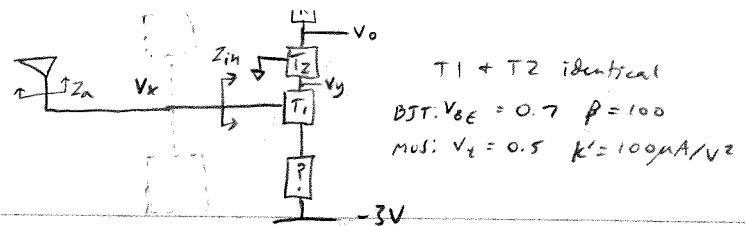
See if it's sensing EM radiation with low

frequency ($< 6\text{ GHz}$). PCB spacing $\sim 1\text{ mm} \Rightarrow f < \sim 3 \cdot 10^{11} = 300\text{ GHz}$.

To determine if it's sensing EM radiation:

Exp 2: If allowed, empty out beverage can + place it
over the sensor (or surround cylinder with it). See
if sensor reading drops sharply; Essentially
surrounding the sensor with a Faraday cage.

current mirror bias on V_{T1}
for T1 + T2



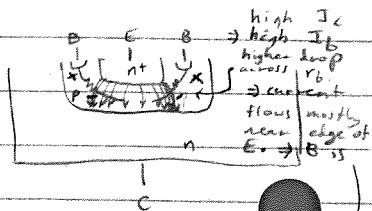
2000 Dutton

- Explain strategy in choosing bias and select V_x and components to get small signal gain from antenna to V₀. For the box with "?" you can choose any component or short it.
- What is small signal gain? How could you change circuit to improve it?
- What is impedance seen by antenna looking into transistor?
- What can you do to make it a good match to antenna?
- What are the dominant noise sources in the amplifier?

BJT: 1) LNA \Rightarrow want low noise, high gain.
 r_b may decrease 50% as I_c is increased from 0.1mA to 10mA. Due to current crowding.

To minimize noise, minimize r_b \Rightarrow maximize I_c.

$$A_V = g_m R = \frac{I_c}{V_T} \cdot R = \frac{V_{DD} - V_0}{V_T} \cdot R = \frac{V_{DD} - V_0}{V_T} = \text{constant.}$$



for npn = forward active: $V_{BE} \geq 0.7$ $V_{BC} \leq V_{BC(\text{sat})}$

\Rightarrow for DC set $V_{BC} = 0 \Rightarrow V_o = 0V$ To keep T₂ on,

need $V_y < -0.7V$ let ? be a short.

$$\Rightarrow I_c = \frac{3V - 0V}{R} \Rightarrow \text{choose } I_c = 3 \text{ mA} \Rightarrow R = 1 \text{ k}\Omega$$

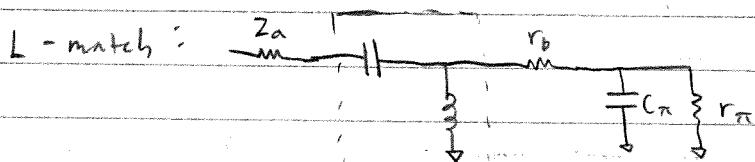
$$I_B = \frac{I_c}{\beta} = 30 \mu\text{A} \Rightarrow \frac{V_x - (-3V + 0.7V)}{r_b} = 30 \mu\text{A} \Rightarrow \text{solve for } V_x.$$

$$2) \text{ small signal gain is } -\frac{Z_{in}}{Z_a + Z_{in}} g_m R = -\frac{Z_{in}}{Z_a + Z_{in}} \frac{3V}{V_T} = -\frac{Z_{in}}{Z_a + Z_{in}} \frac{3V}{0.025V}$$

$$\Rightarrow A_V = -\frac{Z_{in}}{Z_a + Z_{in}} (120)$$

could improve this by using an active load instead of R.

$$3) Z_{in} = r_b + \left(\frac{1}{sC_\pi} \parallel r_\pi \right) \quad (C_\pi \text{ is bootstrapped completely by cascode}).$$



$$\frac{V^2}{I^2} = 4kTR$$

$$\frac{I^2}{R^2} = \frac{4kT}{R}$$

4) r_b is the dominant noise source in the amplifier. To first order, $F = \frac{r_b}{r_b + R_a} = 1 + \frac{r_b}{R_a}$

For bias networks, choose large Resistors. $\Rightarrow \frac{1}{R} = \frac{1}{k} = \text{small}$,
 \Rightarrow doesn't degrade noise performance of LNA.

Impedance Matching - review CE314

High Freq. Small Signal Model:

$$Z_i = \frac{1}{sc} + (\beta + 1) Z = \frac{1}{sc} + (-j \frac{w_T}{L} + 1) Z$$

$$\Rightarrow \frac{1}{sc} \Rightarrow \frac{1}{\beta + 1} \Rightarrow \frac{1}{\frac{1}{sc}} \Rightarrow -j \frac{w_T}{L} \text{ negative } R!$$

MOS 1) To minimize noise, maximize g_m (without changing W) \Rightarrow maximize V_{ov} \Rightarrow maximize V_x .

EE314 \rightarrow $(F_n = 1 + \frac{R_g}{R_a} + \frac{1}{\alpha} R_a g_m \left(\frac{W}{w_T} \right)^2)$ $w_T = \frac{g_m}{C_{gs}} \propto \frac{g_m}{W}$

Intro to Noise
Slide 55

\Rightarrow if we increase g_m by keeping w_T constant, we degrade our noise factor.

$g_m = \mu C_{ox} \frac{W}{L} V_{ov} \Rightarrow$ must increase V_{ov} while keeping W constant.

for MOS: saturation: $V_{gs} \geq V_t \quad V_{gd} \leq V_t \Rightarrow$ set $V_o = 0V$

need $V_g < -0.5V$ let ? be an inductor.

$-2.5V \leq V_x \leq 0V$ to keep T_1 and T_2 in saturation, want to maximize V_x .

To be safe let $V_x = -0.5V$ $\Rightarrow V_{ov} = V_x - V_{ss} - V_t = 2V$

$$\Rightarrow I_d = k' \frac{W}{L} V_{ov}^2 \quad \text{set } \frac{W}{L} = 5 \quad \Rightarrow I_d = 100 \mu A/V^2 \cdot 5 \cdot 4V^2 = 2mA = I_d$$

$$\Rightarrow \frac{V_{DD} - V_o}{R} = 2mA \Rightarrow R = \frac{3V}{2mA} \Rightarrow R = 1.5 k\Omega \quad g_m = \frac{2mA}{V_{ov}} = \frac{4mA}{2V} = 2mS$$

2) $A_v = -\frac{Z_{in}}{Z_{in} + Z_a} g_m R \Rightarrow A_v = -\frac{Z_{in}}{Z_{in} + Z_a} (3) \quad \leftarrow \text{lower gain}$

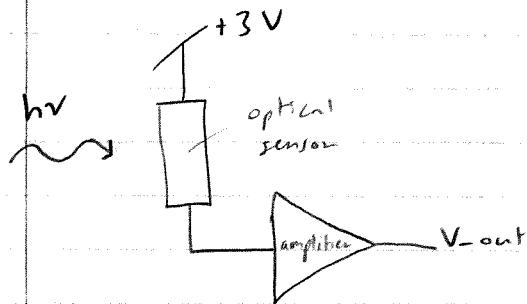
could greatly improve this by using an active load instead of R .

3) $Z_{in} = \frac{1}{sc} + w_T L + jwL \quad (C_{gd} \text{ is bootstrapped completely by cascode}).$

can choose degeneration inductance L such that $w_T L = R_a$ to get a good power match.

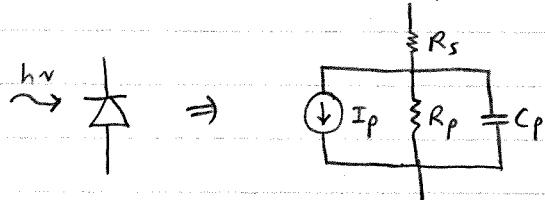
4) The dominant noise sources are gate noise and drain noise.

2002 - Dutton

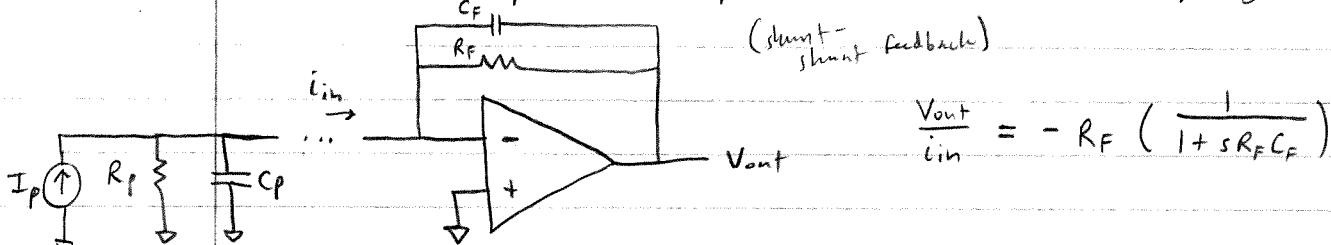


- Use reverse biased photodiode as sensor.
photons striking the depletion region cause e^- , e^+ pairs to form through photo generation. These carriers are swept across the depletion region, causing photocurrent I_p .

Circuit model:



- Use a transimpedance amplifier to detect and amplify signal.



$$\frac{V_o - V_i}{R_F \parallel C_F} = \frac{V_i}{R_p \parallel C_p}$$

$$A_{CL} \approx \frac{1}{f} = \frac{V_o}{V_i} = \left(\frac{1 + s R_p C_p}{R_p} + \frac{1 + s R_F C_F}{R_F} \right) \left(\frac{R_F}{1 + s R_F C_F} \right)$$

$$= \frac{R_F + s R_p R_F C_p + R_p + s R_p R_F C_F}{R_p + s R_p R_F C_F}$$

$$= \frac{\frac{R_F + R_p}{R_p} + s R_p (C_p + C_F)}{1 + s R_F C_F}$$

$$\Rightarrow A_{CL} = \frac{\frac{R_F + R_p}{R_p}}{\left(1 + s \frac{R_F R_p}{R_F + R_p} (C_p + C_F) \right)} \frac{1}{1 + s R_F C_F}$$

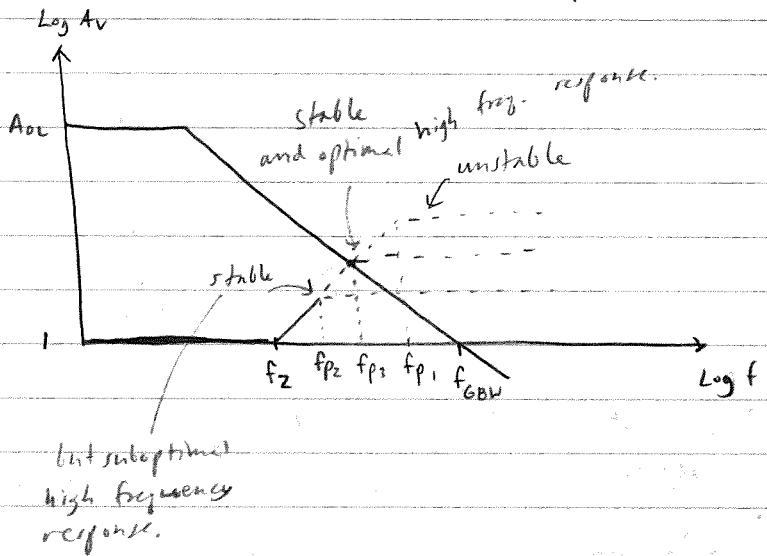
$$= \frac{R_F + R_p}{R_p} \frac{1 + j \frac{\omega}{\omega_p}}{1 + j \frac{\omega}{\omega_p}}$$

$$\omega_2 = \frac{R_F + R_p}{R_F R_p (C_F + C_p)}$$

$$\omega_p = \frac{1}{R_F C_F}$$

typically $R_p \gg R_F \Rightarrow A_{cl} (f \ll f_p) \approx 1$, $\omega_2 < \omega_p$
since $R_p \parallel R_F \approx R_p$.

$$A_{cl} (f \gg f_p) \approx \frac{C_F + C_p}{C_F}$$



$$\Rightarrow \frac{GBW}{f_p} = \frac{C_F + C_p}{C_F}$$

$$C_F = \frac{1}{4\pi R_F GBW} \left[1 + \sqrt{1 + 8\pi R_F C_p GBW} \right]$$

$$f_p \approx \sqrt{\frac{GBW}{2\pi R_F C_p}}$$

↑ in this analysis from TI, it seems that they assume $A_{cl} = \frac{1}{f}$.

Here is my solution: find f : DC: $\frac{R_p}{R_p + R_F}$

$$\text{zero: } Z_F \rightarrow \infty \Rightarrow R_F \parallel \frac{1}{sC_F} = \frac{R_F}{1 + sR_F C_F} \rightarrow \infty \Rightarrow 1 + sR_F C_F = 0$$

$\Rightarrow N(s) = 1 + sR_F C_F \leftarrow \text{this is numerator of transfer function.}$

$$f = \frac{R_p \parallel C_p}{R_p \parallel C_p + R_F \parallel C_F} = \frac{\frac{R_p}{1 + sR_p C_p}}{\frac{R_p}{1 + sR_p C_p} + \frac{R_F}{1 + sR_F C_F}} = \frac{1 + sR_F C_F}{1 + sR_F C_F + \frac{R_F (1 + sR_p C_p)}{R_p}}$$

$$= \frac{1 + sR_F C_F}{1 + \frac{R_F}{R_p} + sR_F C_F + sR_F C_p} = \frac{1 + sR_F C_F}{\frac{R_F + R_p}{R_p} + sR_F (C_F + C_p)}$$

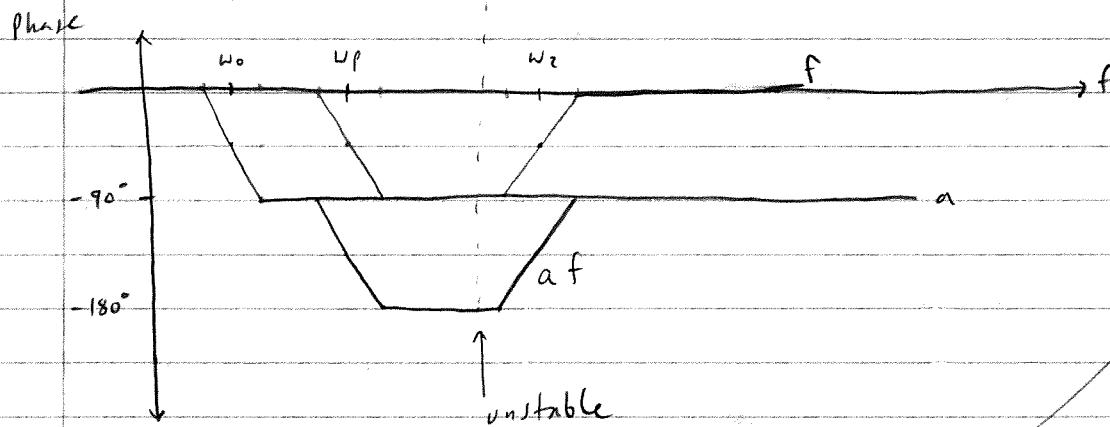
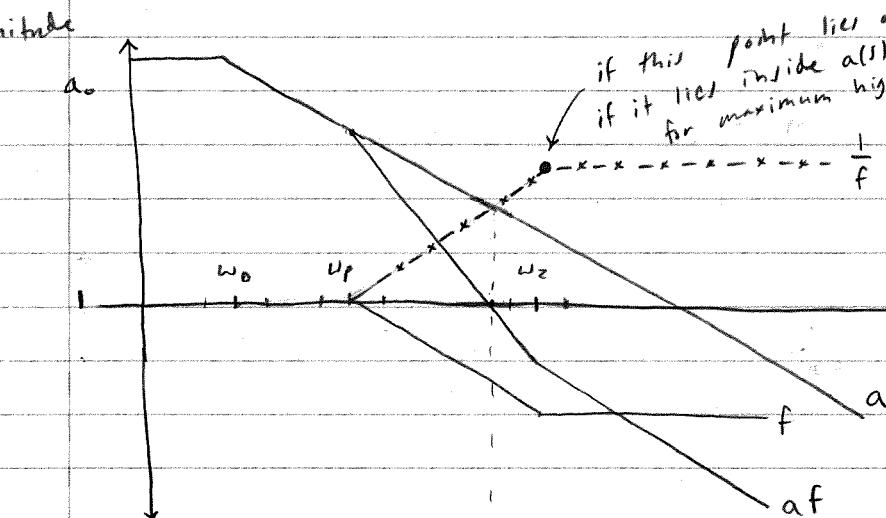
$$\Rightarrow f = \frac{R_p}{R_F + R_p} \left(\frac{1 + sR_F C_F}{1 + s \frac{R_p R_F}{R_F + R_p} (C_F + C_p)} \right)$$

$$\omega_p = \frac{1}{(R_p \parallel R_F)(C_F + C_p)} \quad \omega_2 = \frac{1}{R_F C_F}$$

$$\text{typically } R_p \gg R_F \Rightarrow f(0) = \frac{R_p}{R_p + R_F} \approx 1, \quad R_p \parallel R_F \approx R_F$$

$$\Rightarrow w_p < w_2 \quad f(\infty) = \frac{C_F}{C_F + C_P}$$

if this point lies outside of $a(s)$ \Rightarrow unstable.
 if it lies inside $a(s)$ \Rightarrow stable.
 for maximum high freq performance want the point to lie exactly on $a(s)$.
 $\Rightarrow \frac{C_F + C_P}{C_F} = \frac{GBW}{w_2}$
 $= GBW R_F C_F$
 \Rightarrow solve for C_F .



$$C_F = \frac{1}{4\pi R_F GBW} \left[1 + \sqrt{(1 + 8\pi R_F C_P GBW)} \right]$$

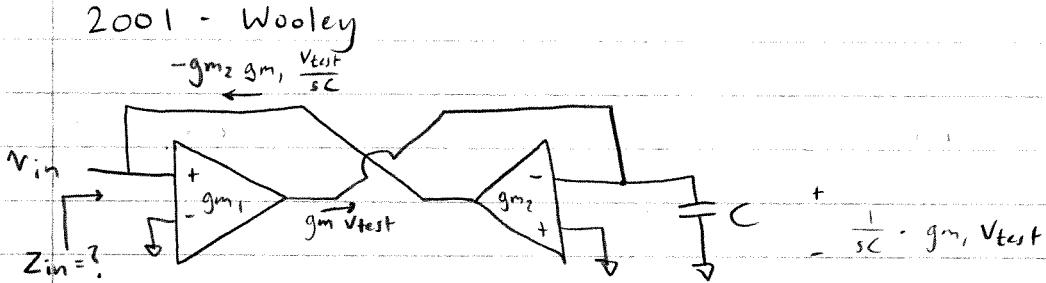
$$f_2 \approx \sqrt{\frac{GBW}{2\pi R_F C_P}} \quad \leftarrow \text{zero in feedback is pole in closed loop.}$$

\Rightarrow limits closed loop TIA bandwidth.

$\Rightarrow R_F \uparrow \Rightarrow \text{gain} \uparrow \Rightarrow \text{BW} \downarrow$

gain-bandwidth tradeoff.

2001 - Wooley



Use Blackman's. gm_2 is feedback element.

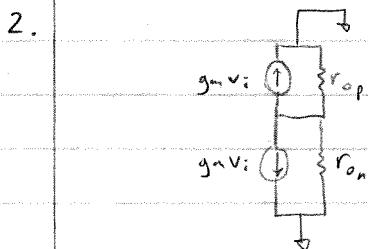
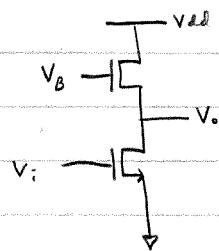
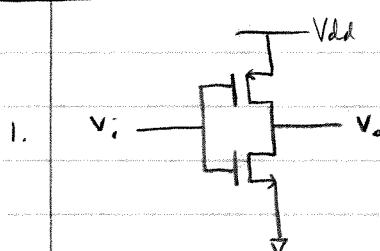
$$\Rightarrow Z_{(k=0)} = \infty \quad X$$

$$i_{test} = gm_1 gm_2 \frac{V_{test}}{sC} \Rightarrow \frac{V_{test}}{i_{test}} = Z_{in} = \frac{sC}{gm_1 gm_2}$$

$$\Rightarrow \text{looks inductive with } L_{eff} = \frac{C}{gm_1 gm_2}$$

2001 - Wong

1. Sketch schematic of cmos inverter + CS with active load
2. Compare small signal voltage gain assuming device sizes and bias currents are similar
3. Compare the bandwidth of the two circuits.



$$a_v = -(g_{m_n} + g_{m_p})(r_{op} \parallel r_{on}) \approx -g_{m_n} r_o$$

CS: $a_v = -g_{m_n} (r_{op} \parallel r_{on})$ for current mirror load.
 $\approx -\frac{1}{2} g_{m_n} r_o$

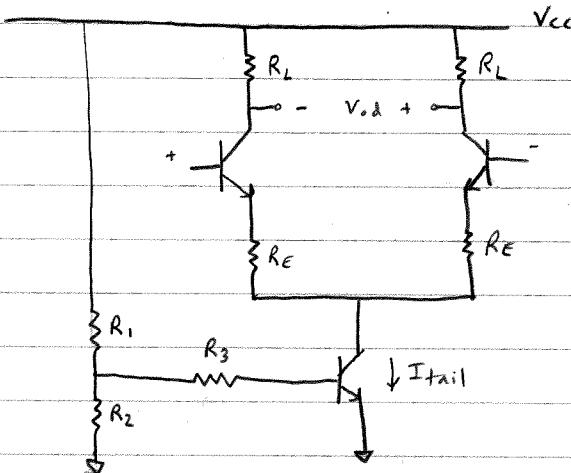
$$C_{gd} \approx 0.2 C_{gs}$$

3. Inverter: $C_{in} = (C_{gd} + C_{gs})(1-a_v) + 2C_{gs} \approx 2g_{m_n} r_o C_{gd}$

CS: $C_{in} = (C_{gd})(1-a_v) + C_{gs} \approx 0.5 g_{m_n} r_o C_{gd}$

Assume same $R_s \Rightarrow BW_{inverter} \approx \frac{1}{4} BW_{CS}$

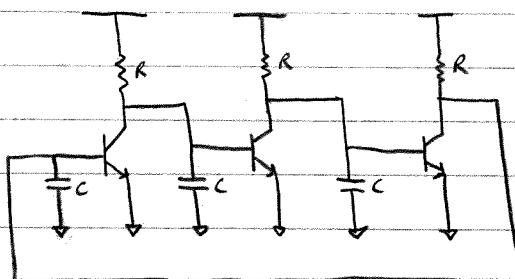
1. Amplifier



Advantages: Common mode rejection, linearity and bandwidth (Miller) can be traded off with gain by sizing R_L and R_E , high output swing, simple.

Disadvantages: Medium bandwidth, unable to drive resistive loads.

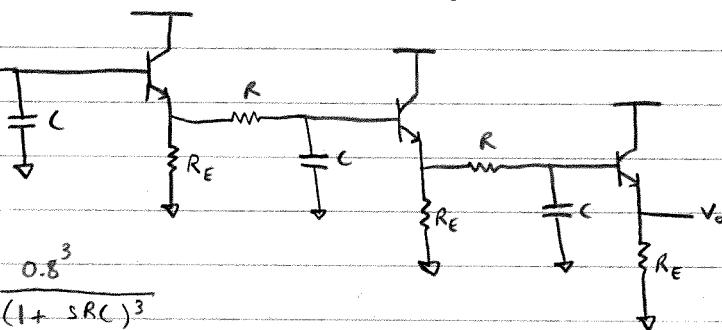
2. Oscillator



Advantages: simple, high tuning range (tune f by changing C or R).

Disadvantages: Worse phase noise performance than high Q tuned oscillators, sensitive to PVT variations (unlike crystal oscillators, for example)

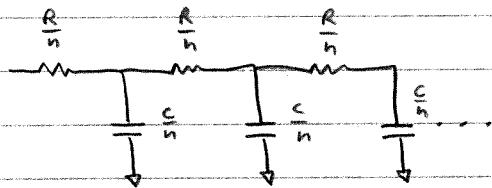
3. Filter:



$$\text{if } R_E \gg R, H(s) = \frac{0.8}{(1 + sRC)^3}$$

\Rightarrow 3rd order pole at $f \approx \frac{1}{2\pi RC}$

Compare to simple passive RC ladder:



$$\text{by } ZVTC: \quad \sum \tau = \frac{1}{n} \sum_{i=1}^n \frac{R}{n} = \frac{RC}{n^2} \frac{n(n+1)}{2} = \frac{RC}{2} \left(\frac{n+1}{n} \right)$$

$\omega_{3dB} = \frac{2}{RC} \left(\frac{n}{n+1} \right) \Rightarrow \lim_{n \rightarrow \infty} V_{3dB} = \frac{2}{RC}$ ← frequency limited due to loading effects...
poles don't occur at same frequency ⇒ non-ideal roll-off.

With active buffers (Common collector), loading effects are removed.

So far active cascaded implementation with Common Collector buffers:

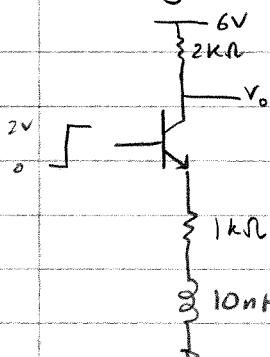
Advantages: simple to design, steep roll-off.

Disadvantages: possible instability, reduced headroom at output

$$(V_{Ocm} = V_{Icm} - (0.7V) \cdot 3)$$

flexible, could instead choose high pass or low pass for each stage
so that overall, you could get a Bandpass, highpass, or notch filter.

Wooley 2002



Sketch $V_o(t)$

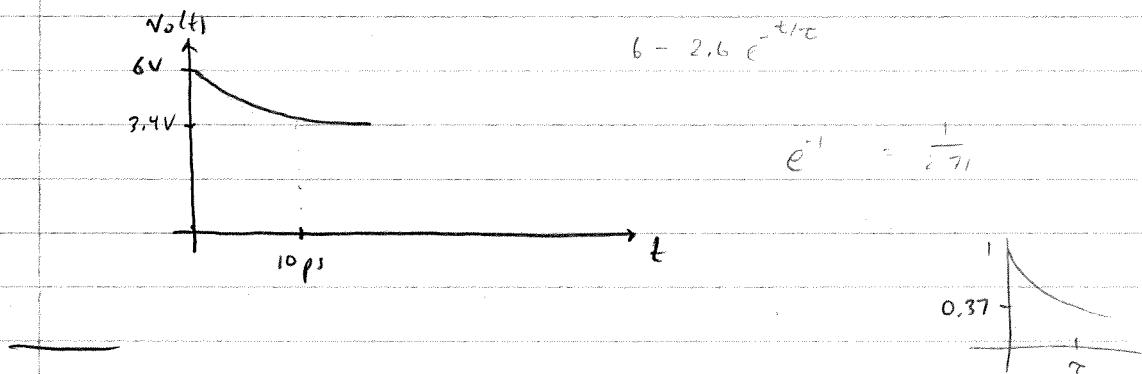
steady state: $V_L = 0$

$$\Rightarrow V_E = 2V - 0.7V = 1.3V \Rightarrow I_c = 1.3mA$$

$$\Rightarrow V_o = 6V - (1.3 \cdot 2)V = 3.4V$$

Initially $V_o = 6V$, $V_E = 0V$

$$V_L = L \frac{di}{dt} \quad \frac{L}{R} = \frac{10n}{1k} = 10 \cdot 10^{-12} = 10ps.$$



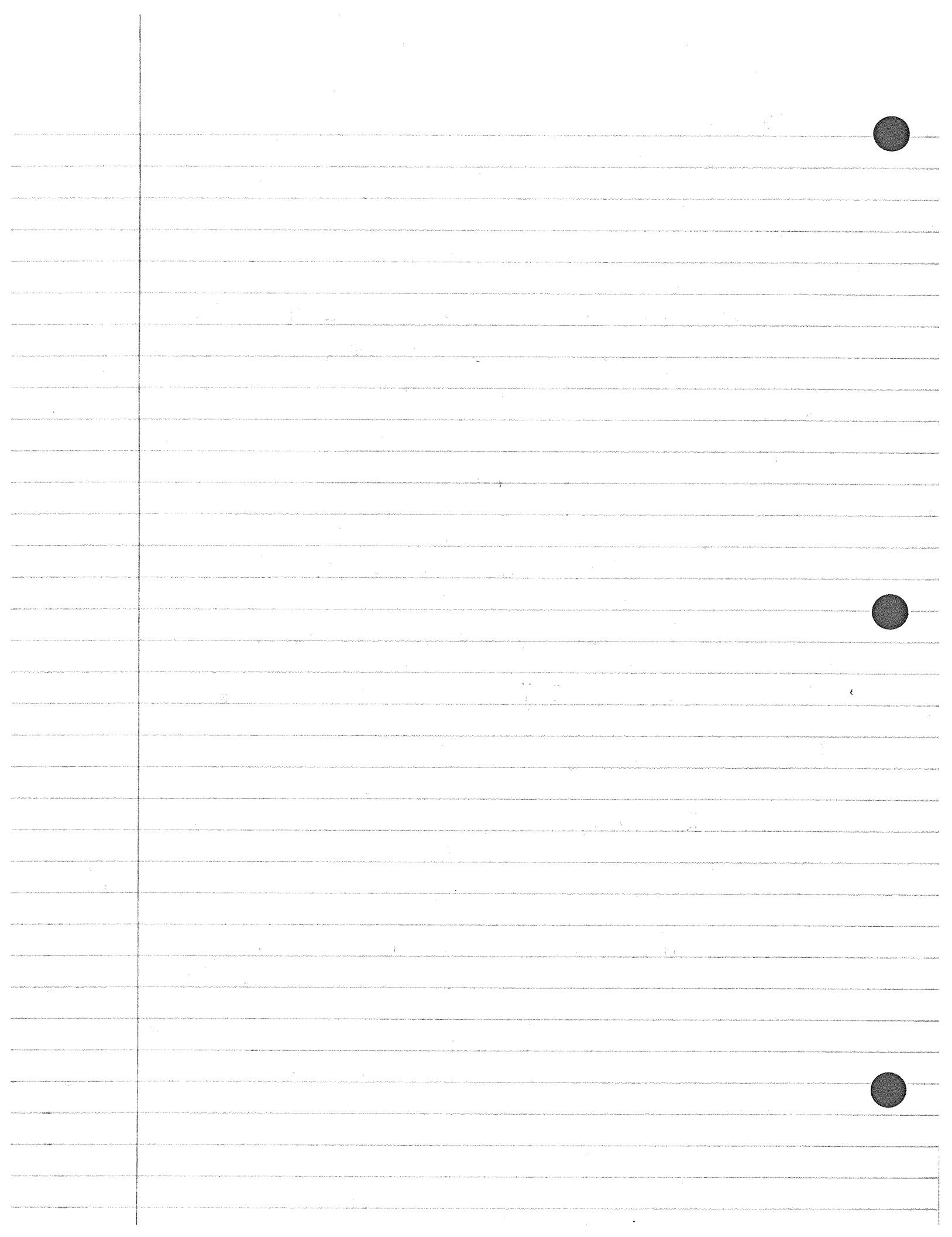
$$t < 0 \quad V_o = 6V, \quad i_c = 0A$$

$$V(t) = L \int_0^t i_L dt + V_o \quad \Rightarrow \quad i_L = 1.3mA - 1.3mA e^{-t/\tau}$$

$$\tau = \frac{L}{R} = 10ps.$$

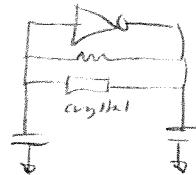
$$V_o = 6 - 2.6 e^{-t/\tau}$$

$$i_L = 1.3 (1 - e^{-t/\tau}) \quad \Rightarrow \quad 0.63 \text{ after } \tau$$

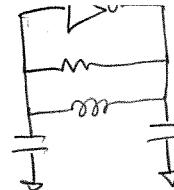


???

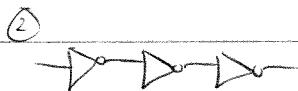
Pierce oscillator



OR



Wong 2002



If the output is connected to the input how will the circuit behave?

③

$\rightarrow \leftarrow$ will this circuit oscillate if output is connected to input?

Bonus: what components will you add to make it oscillate?

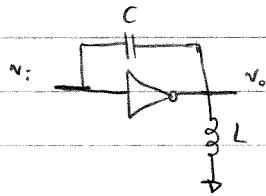
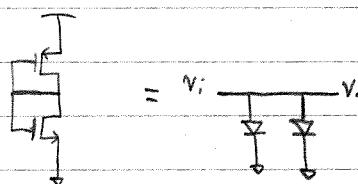
1. Buffer, (digital) latch

2. Ring oscillator $f = \frac{1}{2 \cdot 3 T_d} = \frac{1}{6 T_d}$ where T_d is the delay of each inverter.

3.

No, feedforward through wire. $\Rightarrow \frac{V_o}{V_i} = 1$

is instantaneous (compared to T_d of inverter)



1. Latch (holds an input value) $V_{DD} \xrightarrow{\oplus} \text{inverter} \rightarrow \text{buffer} \rightarrow \text{buffer}$ then open input: $V_{DD} \xrightarrow{\oplus} \text{Do} \xrightarrow{\oplus} \text{Do}$ V_{DD} (value is held).

2. Ring oscillator

inversion pole at input

?? 3. Loop gain phase is: $180^\circ + 90^\circ = 270^\circ \leftarrow$ not sufficient to cause instability.

Wong 2004

$$1. A_v = -g_{m_n} (r_{op} \parallel r_{on})$$

2. Assume dominant pole at output:

$$A_v(s) = -g_{m_n} Z_{out} = -g_{m_n} (r_{op} \parallel r_{on} \parallel \frac{R}{2} \parallel \frac{1}{2sC})$$

$$= -g_{m_n} (r_{op} \parallel r_{on} \parallel \frac{R}{2}) \left(\frac{1}{1 + s(r_{op} \parallel r_{on} \parallel \frac{R}{2})2C} \right) \Rightarrow \text{pole at } w_p = \frac{1}{(r_{op} \parallel r_{on} \parallel R)2C}$$

\Rightarrow lowpass response.

$$3. A_v(s) = -g_{m_n} Z_{out} = -g_{m_n} (r_{op} \parallel r_{on} \parallel \frac{R}{2} \parallel s\frac{L}{2}) = -g_{m_n} \frac{s\frac{L}{2}}{1 + s\frac{L}{2(r_{op} \parallel r_{on} \parallel \frac{R}{2})}}$$

$$\Rightarrow w_z = 0 \quad w_p = \frac{2(r_{op} \parallel r_{on} \parallel R/2)}{L} \Rightarrow \text{high pass response.}$$

$$4. A_v(s) = -g_{m_n} (r_{op} \parallel r_{on} \parallel \frac{R}{2} \parallel \frac{1}{2sC} \parallel s\frac{L}{2}) \quad \text{let } r_{op} \parallel r_{on} \parallel \frac{R}{2} = R_o$$

$$\text{let } R_o \parallel \frac{1}{sC} \parallel sL = Z_{out} = \left(\frac{1}{R_o} + sC + \frac{1}{sL} \right)^{-1}$$

$$= \left(\frac{sL}{sR_o L} + \frac{s^2 R_o C}{sR_o L} + \frac{2R_o}{sR_o L} \right)^{-1} = \frac{sR_o L}{s^2 R_o C L + sL + 2R_o} = \frac{sR_o}{s^2 R_o C + s + \frac{R_o}{L}}$$

$$\Rightarrow A_v(s) = -g_{m_n} (r_{op} \parallel r_{on} \parallel \frac{R}{2}) \frac{s}{s^2 R_o C + s + \frac{R_o}{L}}$$

$$w_z = 0 \quad w_{p,z} = -1 \pm \sqrt{1 - 4R_o^2 \frac{C}{L}} \quad w_o = \frac{1}{\sqrt{LC}}$$

$$Q = \frac{R_o}{\sqrt{LC}} = w_o R_o C$$

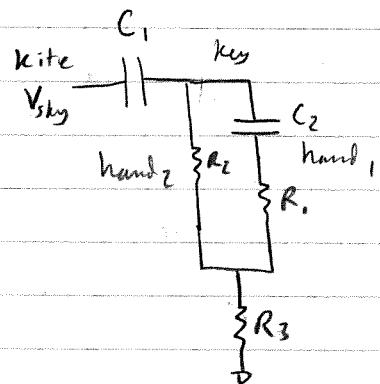
$$\Rightarrow \text{at } w = w_o = \frac{1}{\sqrt{LC}} \quad A_v(jw_o) = -g_{m_n} (r_{op} \parallel r_{on} \parallel \frac{R}{2})$$

and $A_v(jw) < A_v(jw_o)$ for $w < w_o$ and $w > w_o$

\Rightarrow band pass response.

Dutton 2003

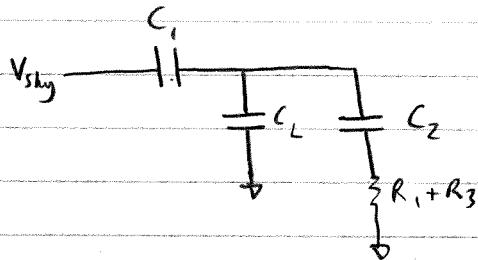
1) $Q = VC$



$C_2 \gg C_1$, Q is conserved.

$$Q = V_1 C_1 = V_2 C_2 \Rightarrow V_2 = \frac{C_1}{C_2} V_1 \Rightarrow V_2 \ll V_1$$

2)

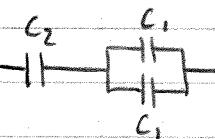


$$Q_L = Q_1 \cdot \frac{C_L}{C_L + C_2}$$

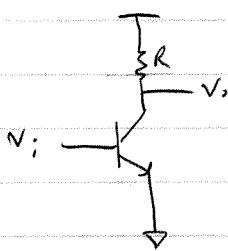
will charge with RC time
constant $\tau = (R_1 + R_3) \left(\frac{C_2(C_1 + C_L)}{C_1 + C_2 + C_L} \right)$

simulated in LT spice ✓.

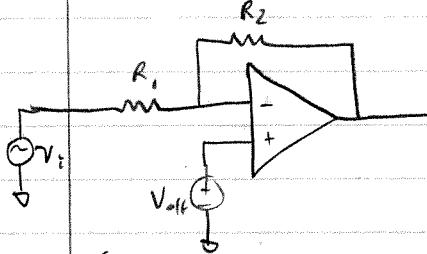
effective capacitance seen by $(R_1 + R_3)$ is



Kovacs 2003



$$Av = -gmR = -\frac{I_c}{V_T} R$$



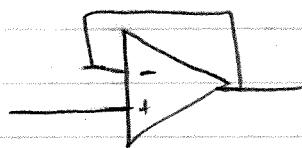
$$V_{out} = V_{off} + \frac{R_2}{R_1} V_i$$

$$\frac{V_{out} - V_{off}}{R_2} = -\frac{V_i - V_{off}}{R_1}$$

$$V_{out} - V_{off} = -\frac{R_2}{R_1} V_i + \frac{R_2}{R_1} V_{off}$$

$$V_{out} = \left(1 + \frac{R_2}{R_1}\right) V_{off} - \frac{R_2}{R_1} V_i$$

reduces max voltage swing at output.



voltage buffer

$$V_{out} = (V_{out} - V_{in}) A$$

Acts as a latch.

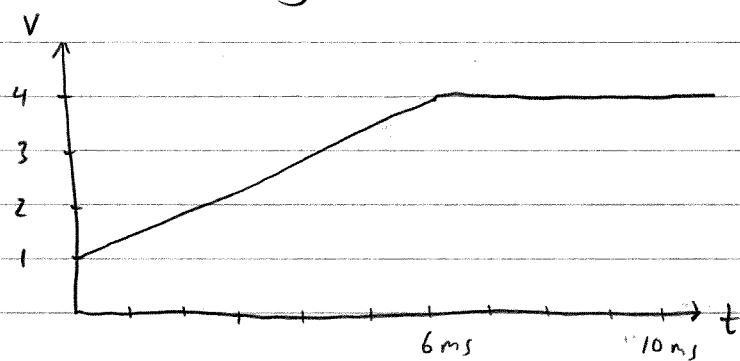
By superposition:

$$\text{set } V_i = 0 \Rightarrow V_{o_1} = \left(1 + \frac{R_2}{R_1}\right) V_{off} \quad (\text{Non-inverting configuration})$$

$$\text{set } V_{off} = 0 \Rightarrow V_{o_2} = -\frac{R_2}{R_1} V_i \quad (\text{Inverting configuration})$$

$$\Rightarrow V_o = V_{o_1} + V_{o_2} = \left(1 + \frac{R_2}{R_1}\right) V_{off} - \frac{R_2}{R_1} V_i$$

2004 - Wooley



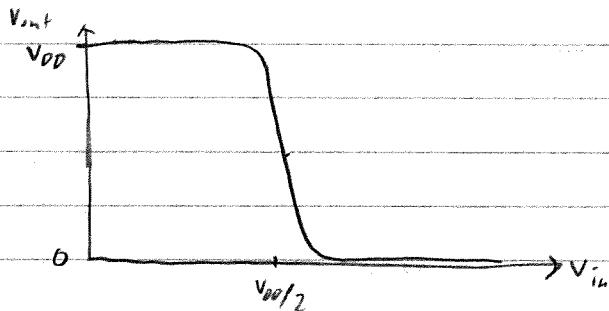
$$i = \frac{IV}{20\Omega} = 50\text{mA}$$

$$i = C \frac{dV}{dt} \Rightarrow V = \int i \cdot \frac{1}{C} = \int \frac{50\text{mA}}{100\mu\text{F}} = t \cdot 500 \text{ V/s} = t \cdot 0.5 \text{ V/ns}$$

$$\frac{4V - 1V}{0.5 \text{ V/ns}} = 6\text{ms} \quad \text{After } 6\text{ms}, V_0 \text{ remains constant, since}$$

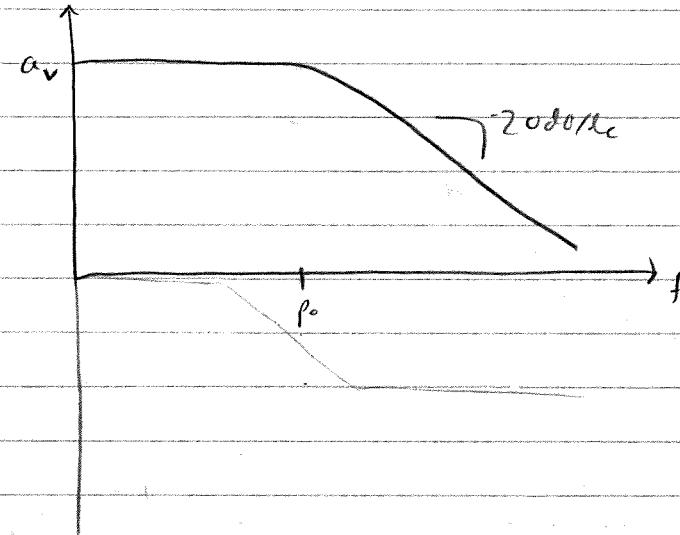
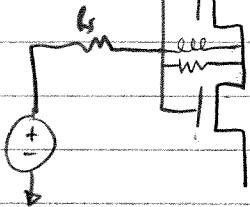
it acts as a charge conservation node.

2005 - Wong

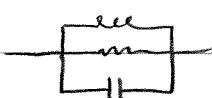


$$a_v = -(g_{m_n} + g_{m_p})(r_{op} \parallel r_{on})$$

$$|a_v| = a_v \left(\frac{1}{(Cg_d + Cg_a)(1 + a_v) R_s} \right)$$

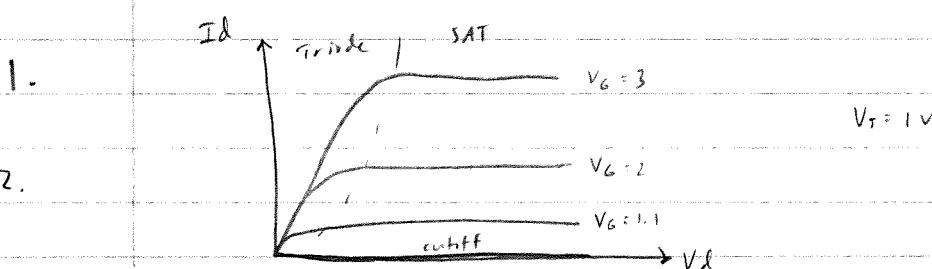


?



$$\omega_o = \frac{1}{\sqrt{LC}}$$

2002 - Shenoy

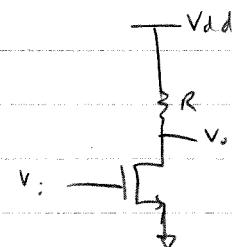


$$3. \quad Id = \mu C_{ox} \frac{W}{L} \left(V_{gs} - V_T - \frac{V_{ds}}{2} \right) V_{ds}$$

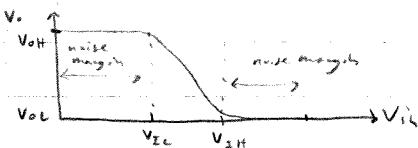
$$4. \quad g_m = \mu C_{ox} \frac{W}{L} (V_{gs} - V_T) = \frac{2Id}{V_{ds}} = \sqrt{2\mu C_{ox} \frac{W}{L} Id}$$

5. (Yes, if you assume V_{ov} is constant.) other wise no, $g_m \propto \sqrt{Id}$

6.



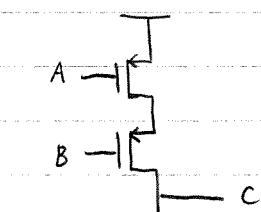
choose high R . \Rightarrow more gain
 \Rightarrow smaller "forbidden region" \Rightarrow higher noise margin.



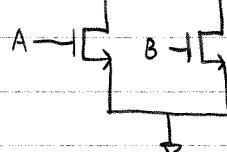
7. replace R with pmos load to improve speed? Yes. with R , max current is $\frac{Vdd}{R}$. with pmos, max current can be higher \Rightarrow speed increases \rightarrow can only increase this by decreasing R . \Rightarrow noise margin, speed tradeoff.

8. PMOS. \Rightarrow either n or p will be off when output is high or low \Rightarrow no static current.

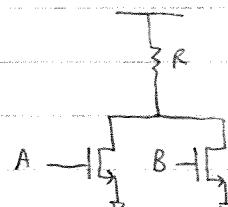
9. NOR



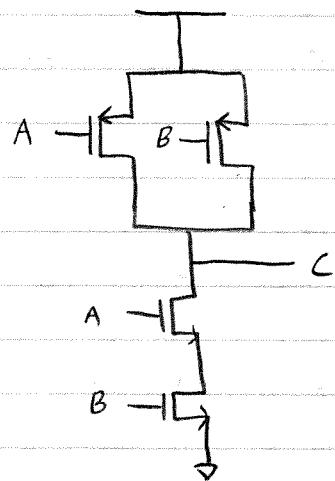
A	B	C
0	0	1
0	1	0
1	0	0
1	1	0



alternatively:

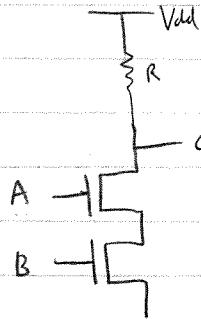


NAND :



A	B	C
0	0	1
0	1	1
1	0	1
1	1	0

alternatively:



1999 - Dutton

- 1) Impedance Match - Max Power Transfer
Narrowband Match - Low noise
LNA - Mixer - Filter \leftarrow Need some combination of these

2) MOS $I_{\text{out}} \sim (V_{\text{in}})^2$: small 3rd order nonlinearity and moderate gain.

Main considerations are gain and 3rd order nonlinearity:

$$\text{for BJT: } I = I_s \left(\exp\left(\frac{V_g}{V_T}\right) - 1 \right) = I_s \left(\frac{V_g}{V_T} + \frac{1}{2} \left(\frac{V_g}{V_T}\right)^2 + \frac{1}{3!} \left(\frac{V_g}{V_T}\right)^3 + \dots \right)$$

high gain ↑
3rd order term

for MOS $I_{out} \sim (V_{in})^2$ mostly just have 2nd order term.
moderate gain

for Mol Int ~ mostly just have 1st order term,
small gm.

Nonlinearity: 2nd Order - HD - largely suppressed by differential topologies.
IM - doesn't matter for nearby signals.

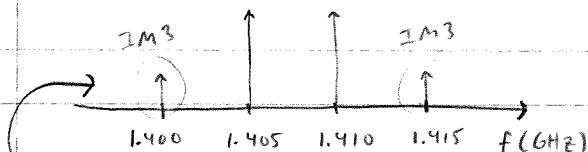
3rd order - HD - not cancelled by differential topologies.

IM - nearby signals can have IM products that fall on the signal of interest.

$$v_0 = a_1 v_i(t) + a_2 v_i(t)^2 + a_3 v_i(t)^3 \quad \text{if } v_i(t) = v_{i_0} \cos(\omega_i t) + v_{i_1} \sin(\omega_i t)$$

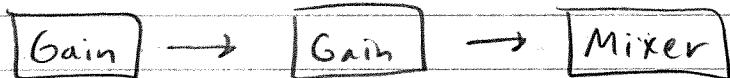
$$a_3 v_i(t)^3 = a_{32} \cos(\omega_1 t \pm 2\omega_2 t) + a_{33} \cos(2\omega_1 t \pm \omega_2 t)$$

$$a_{32} = \frac{3a_3}{4} V_{1a} V_{2a}^2 \quad a_{33} = \frac{3a_3}{4} V_{1a}^2 V_{2a}$$



this IM3 product will land on our signal at 1.400 GHz.

3)



- Linearity is worse since input to mixer is larger
⇒ higher HD.
- Potentially lower NF, if Mixer NF is high.
- Requires very high Q bandpass filter since filtering is done at ~1.4 GHz, and bandwidth is ~10 MHz
⇒ $Q \approx 141$

Better Solution:



- Linearity is better since input to mixer is small.
- Potentially higher NF (but not by much so long as first gain stage has reasonably high gain).
- Can use moderate Q filtering in first gain stage (LNA) and high performance lowpass filter in final gain stage.

Friis Equation:

$$F_{\text{total}} = 1 + (F_1 - 1) + \frac{(F_2 - 1)}{G_1} + \dots + \frac{(F_n - 1)}{G_1 \cdot G_2 \cdots G_n}$$

Where F is noise factor : $F = \frac{\text{SNR}_{\text{in}}}{\text{SNR}_{\text{out}}}$

Where $\text{SNR} = \frac{\text{signal power}}{\text{noise power}}$

$G = \text{power gain} = A_v^2$

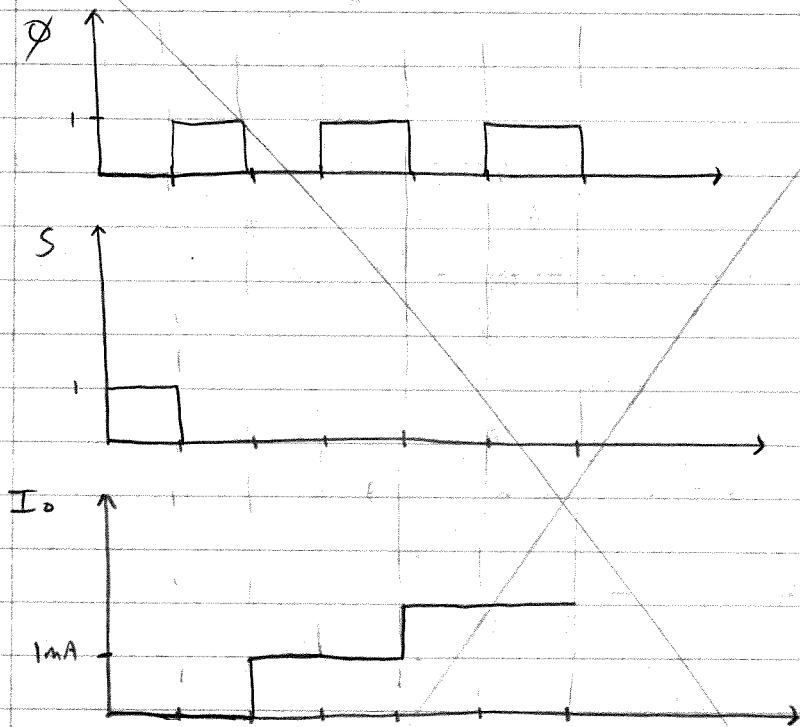
$$\text{Noise Figure } NF = 10 \cdot \log(F) \Rightarrow F = 10^{\frac{NF}{10}}$$

1999 - Wooley

Answer

J is wrong.

This circuit is an integrator. S resets the whole system.

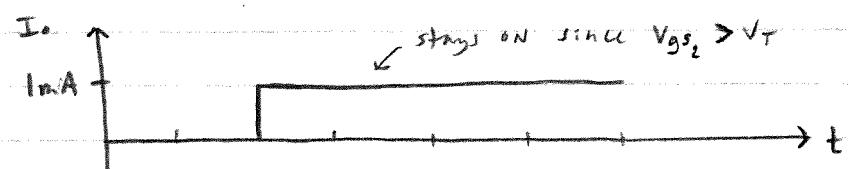
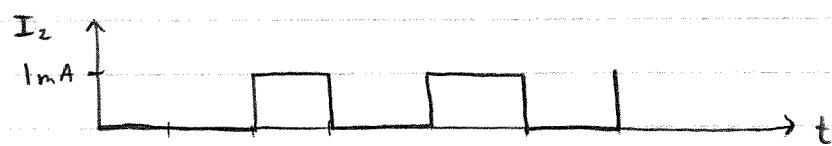
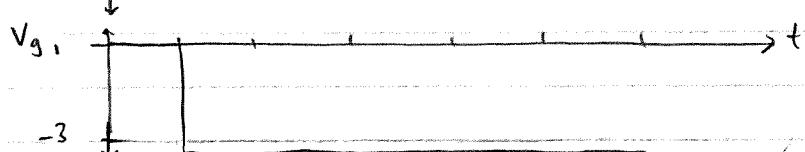
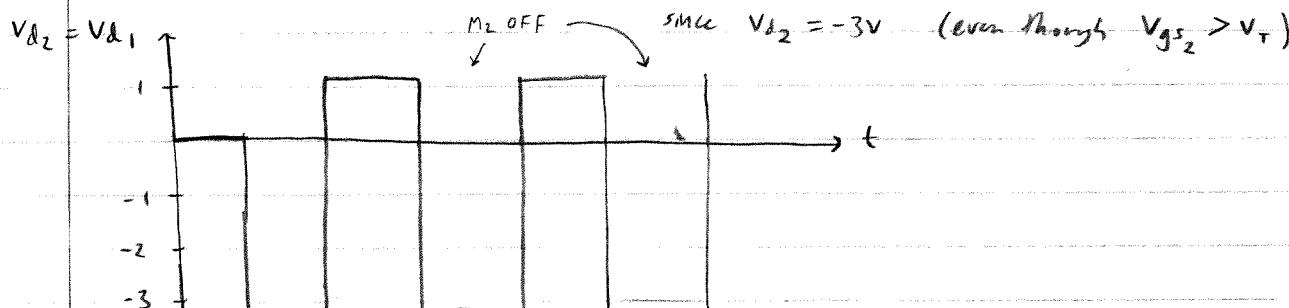
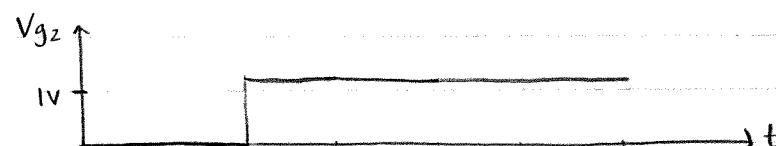
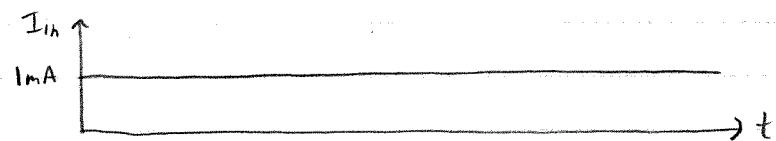
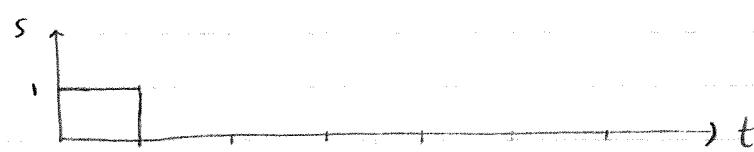
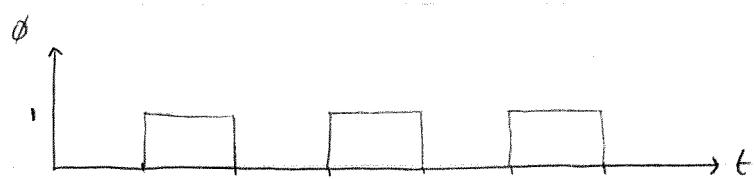


Φ cycle	I_o	I_1
1	0	9
2	1	8
:	:	:
9	8	1
10	9	0
11	10	0
12	10	-1
13	11	-1
14	11	-2

? What happens here.

See next page

	I _S	0	0	0	0	0
I ₁	0	9	9	9	9	9
I ₂	0	0	1	0	1	0



In this case $I_o \neq I_2$ due to the switching.

But since V_{gs2} is held on C_{gs2} , I_o remains constant at 1mA after the first cycle.

2003 - Shenoy

1. a) Triode: $I_{ds} = \mu C_o \frac{w}{L} (V_{gs} - V_t - \frac{V_{ds}}{2}) V_{ds}$

Sat: $I_{ds} = \frac{1}{2} \mu C_o \frac{w}{L} (V_{gs} - V_t)^2$

2. a) $V_1 = \frac{V}{2}$ both will be on

b) Saturation

c) $V_1 = \frac{V}{2}$??

3.

$w_2 = \frac{1}{3} w$

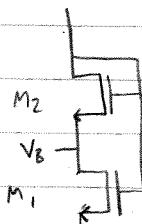
?

$M_3 \rightarrow$ sat, $M_2 \rightarrow$ triode

$$\frac{k'}{2} \left(\frac{w}{L} \right)_1 V_{ov}^2 = \frac{k'}{2} \left(\frac{w}{L} \right)_2 [2(2V_{ov}) V_{ov} - V_{ov}^2] \Rightarrow \left(\frac{w}{L} \right)_1 = 3 \left(\frac{w}{L} \right)_2 \Rightarrow w_2 = \frac{1}{3} w_1$$

$V_{dsstr} = (\sqrt{k+1} - 1) V_{ov}$

Sooch Cascade:



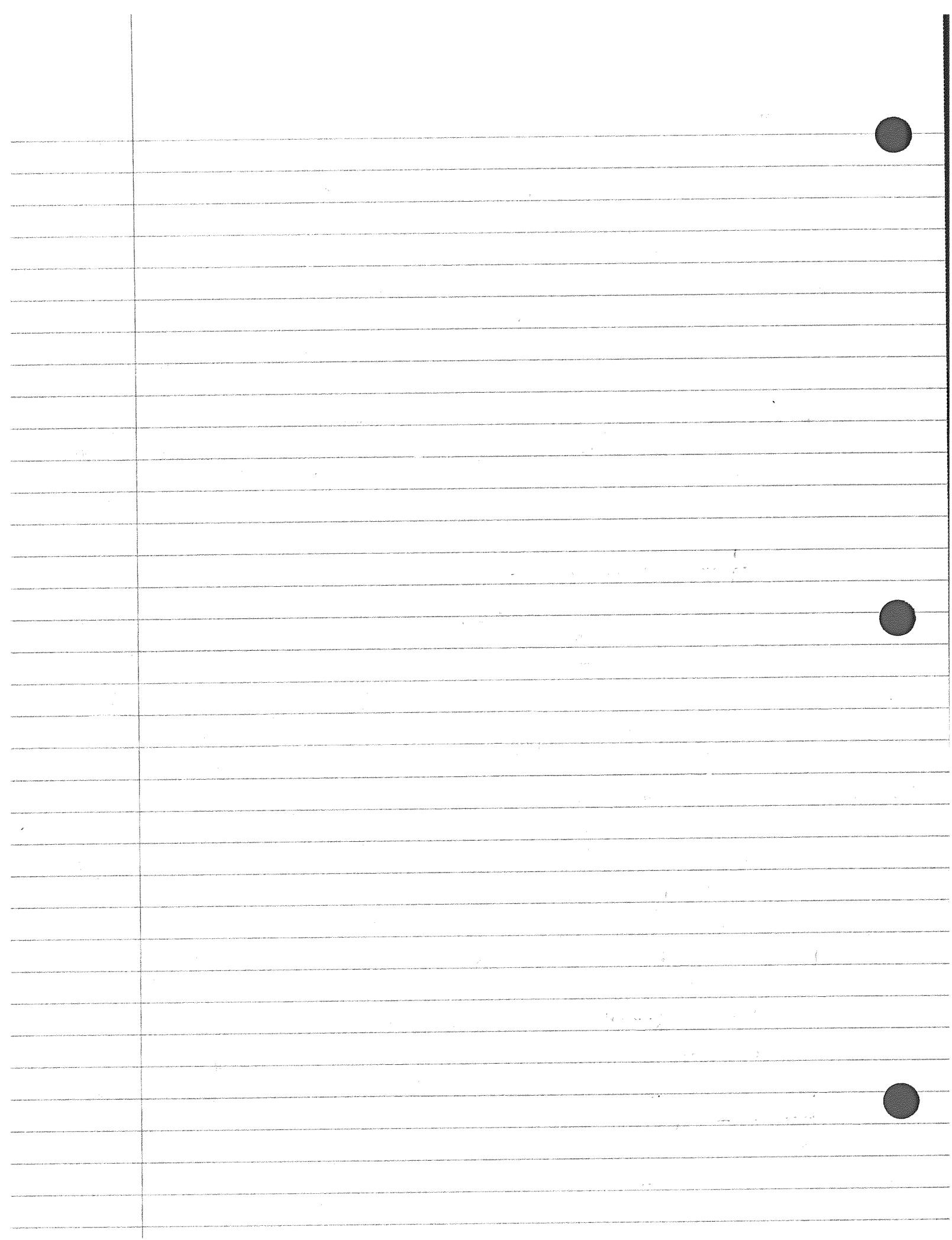
M_2 must be in sat, M_1 must be in linear.

$$\Rightarrow I_{d2} = I_d \Rightarrow \frac{1}{2} \mu C_o \left(\frac{w}{L} \right)_2 (V - V_B - V_T)^2 = \mu C_o \left(\frac{w}{L} \right)_1 (V - V_T - \frac{V_B}{2}) V_B$$

$$\text{let } k = \left(\frac{w}{L} \right)_2 / \left(\frac{w}{L} \right)_1 \Rightarrow V - V_T = V_{ov}$$

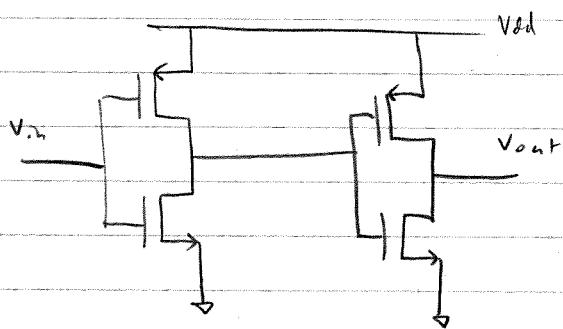
$$\Rightarrow k (V_{ov}^2 - 2V_B V_{ov} + V_B^2) = 2(V_B V_{ov}) - V_B^2$$

$$\Rightarrow k V_{ov}^2 - 2(k+1) V_B V_{ov} + (k+1) V_B^2 = 0$$

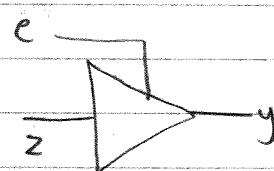


1999 - Wong

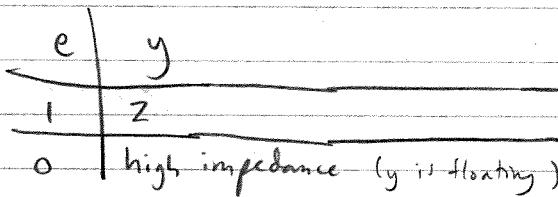
(1)



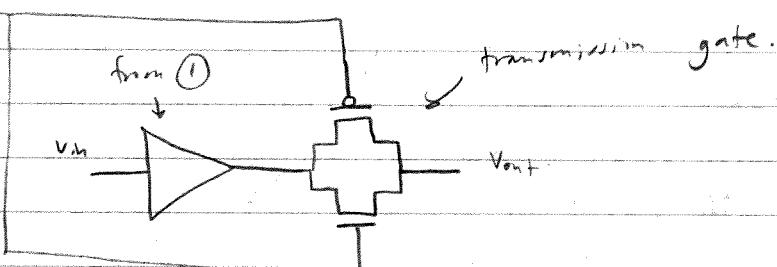
(2)



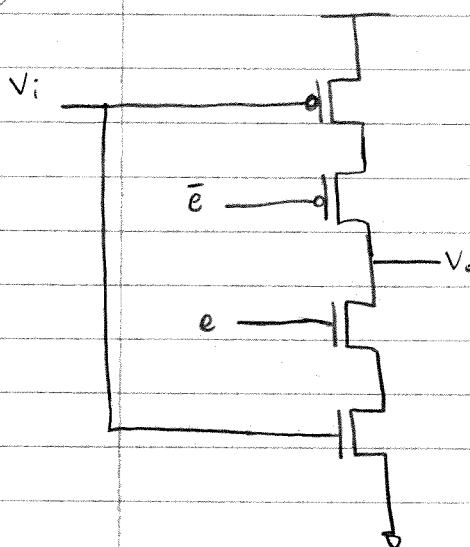
Digital design by Brown.



$e \rightarrow \Delta$



(3)



2006 - Shenoy

$$1. a) I_{DS} = \mu C_{ox} \frac{W}{L} (V_{GS} - V_t - \frac{V_{DS}}{2}) V_{DS}$$

$$g_m = \frac{\partial I_{DS}}{\partial V_{GS}} = \mu C_{ox} \frac{W}{L} V_{DS} \quad V_{DS} < V_{GS} - V_t$$

b) Gain is larger for Saturation mode:

$$\text{Saturation: } g_m = \underbrace{\mu C_{ox} \frac{W}{L} (V_{GS} - V_t)}_{> V_{DS}} \quad R_{DS_{sat}} \approx r_o \approx \frac{1}{\lambda I_{DS}} = \frac{V_A}{I_{DS}}$$

$$\text{Triode: } g_m = \mu C_{ox} \frac{W}{L} V_{DS} \quad R_{DS_{triode}} = \left(\frac{\partial I_{DS}}{\partial V_{DS}} \right)^{-1} \approx \frac{V_{DS}}{I_{DS}}$$

$$\frac{\partial I_{DS}}{\partial V_{DS}} = \mu C_{ox} \frac{W}{L} (V_{GS} - V_t - V_{DS}) \approx \mu C_{ox} \frac{W}{L} (V_{GS} - V_t) \quad \text{for } V_{DS} \ll V_{GS} - V_t$$

$$\Rightarrow R_{DS_{triode}} \approx \frac{1}{\mu C_{ox} \frac{W}{L} (V_{GS} - V_t)} \approx \frac{V_{DS}}{I_{DS}} \quad \text{for } V_{DS} \ll V_{GS} - V_t$$

$$\Rightarrow A_v = g_m R_{DS} \quad g_{m_{sat}} > g_{m_{triode}} \quad \text{and} \quad R_{DS_{sat}} > R_{DS_{triode}}$$

$$\Rightarrow A_{v_{sat}} > A_{v_{triode}}$$

$$2. a) ZVTC: \quad f_{3dB} \approx \left(\sum_{i=1}^3 R_i C_i \right)^{-1}$$

\Rightarrow dominant time constant is largest $R_i C_i$

\Rightarrow should decrease the capacitor that has the largest RC time constant.

b) No. ZVTC can only be used to estimate the dominant pole. The individual RC time constants do not correspond to individual poles in the system.
(but if $R_{dom} C_{dom} \gg R_i C_i \quad \frac{1}{R_i C_i} \approx f_{3dB}$)

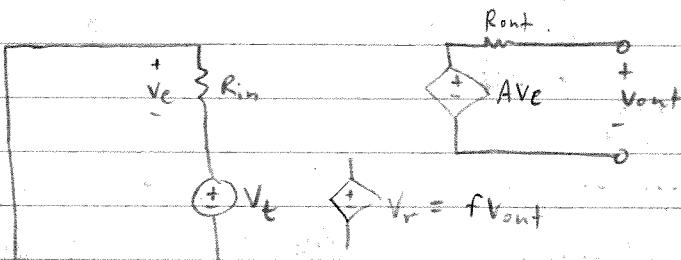
$$3. a) V_{out} = A v_e \quad V_e = V_{in} + f V_{out} \Rightarrow V_e = V_{in} - f V_{out}$$

$$V_{out} = A (V_{in} - f V_{out}) \Rightarrow V_{out} (1 + Af) = A V_{in} \Rightarrow \frac{V_{out}}{V_{in}} = \boxed{\frac{A}{1+Af} = G}$$

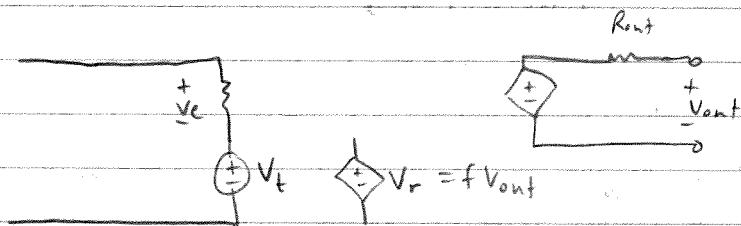
b) Blackman's Impedance Formula:

$$Z_{\text{port}} = Z_{\text{port}} \Big|_{(k=0)} \frac{1 + R \text{ (port shorted)}}{1 + R \text{ (port open)}}$$

$$\Rightarrow R_{\text{in}} \Big|_{f \neq 0} = R_{\text{in}} \Big|_{f=0} \frac{1 + R \text{ (port shorted)}}{1 + R \text{ (port open)}}$$



$$R \text{ (port shorted)} = - \frac{V_r}{V_t} = - \frac{-fAV_t}{V_t} = Af$$



$$R \text{ (port open)} = - \frac{V_r}{V_t} = - \frac{0}{V_t} = 0$$

$$\Rightarrow R_{\text{in}} \Big|_{f \neq 0} = R_{\text{in}} \Big|_{f=0} \frac{1+Af}{1+0}$$

$$\Rightarrow R_{\text{in}} \Big|_{f \neq 0} = (1+Af) R_{\text{in}} \Big|_{f=0}$$

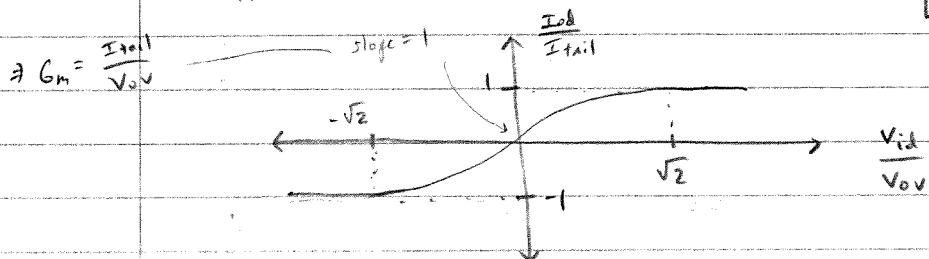
	Shunt	Series
OR	Input I	V
	V	I

⇒ This is Series-Shunt

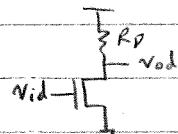
$$\Rightarrow Z_i' = Z_i (1 + T) \quad Z_o' = \frac{Z_o}{(1 + T)}$$

$$\Rightarrow R_{in, f \neq 0} = R_{in, f=0} (1 + Af)$$

$$4. a) \frac{I_{od}}{I_{tail}} = \frac{V_{id}}{V_{ov}} \sqrt{1 - \left(\frac{V_{id}}{2V_{ov}}\right)^2} \Rightarrow V_{ID} = \sqrt{2} (V_{GSO} - V_t)$$



$$b) A_{dm} = \frac{V_{od}}{V_{id}} \approx -g_m R_d \quad \text{half circuit:}$$

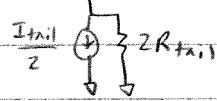


$$A_{cm} = \frac{V_{oc}}{V_{id}} \approx \frac{-g_m R_d}{1 + g_m (2R_{tail})} \quad \text{half circuit:}$$



$$\text{CMRR} = \left| \frac{A_{dm}}{A_{cm}} \right| \approx 1 + 2 g_m R_{tail}$$

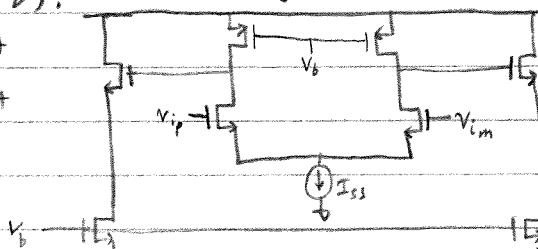
(= ∞ if $R_{tail} = \infty$, i.e. ideal tail current source)



5. A basic 2-stage op-amp consists of a voltage amplifier (C-S) and a voltage buffer (C-D).

can add cascodes to C-S stage to increase gain.

High BW, High Gain, High R_{in} High Swing, Low R_{out}



1998 - Dutton

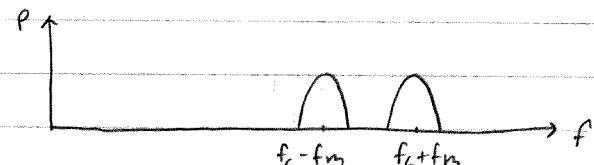
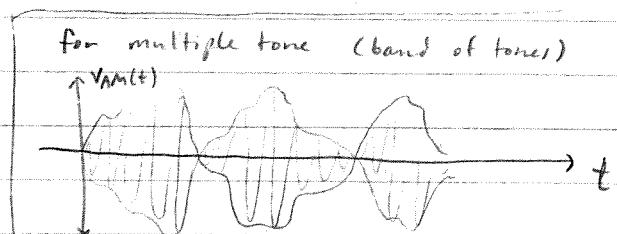
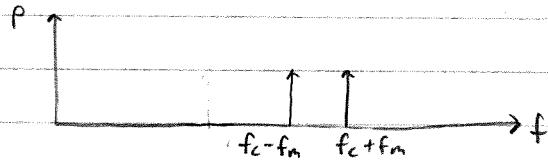
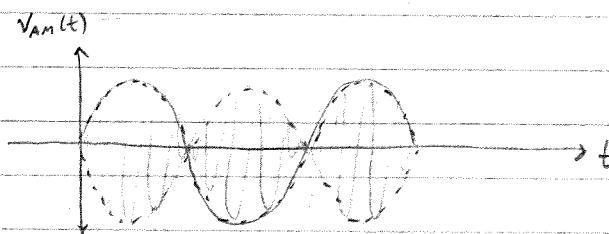
1. Ideal AM signal: for single tone

$$v_c(t) = V_c \sin(\omega_c t) \quad v_m(t) = V_m \sin(\omega_m t)$$

↑ carrier ↑ modulating signal

$$\Rightarrow v_{AM}(t) = V_c V_m \sin(\omega_c t) \sin(\omega_m t)$$

$$= \frac{V_c V_m}{2} [\cos((\omega_c - \omega_m)t) - \cos((\omega_c + \omega_m)t)]$$



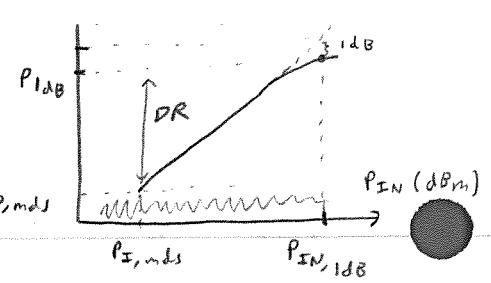
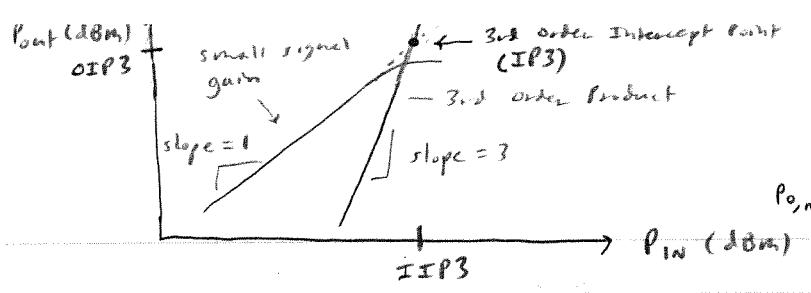
2. What are the limitations imposed by a real power amplifier on the ideal signal... how is the signal changed due to these real effects?

Give details of how a single transistor amplifier affects the results (for either MOS or BJT, what happens when the signals get too large?)

- MOS and BJT nonlinearity causes HD and IM.

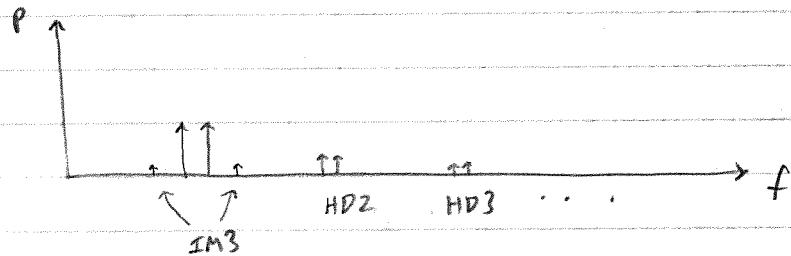
$$\text{MOS} \propto V_{in}^2 \quad \text{BJT} \propto \exp(V_{in}) = 1 + V_{in} + \frac{V_{in}^2}{2!} + \frac{V_{in}^3}{3!} + \dots$$

- Differential circuits help eliminate even harmonics, but we can't do this here since we have a single transistor amplifier.



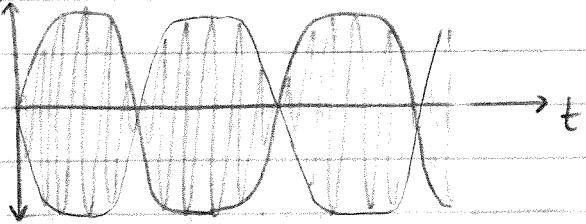
How does it look in frequency and time?

frequency:



If there are other signals being broadcast near your band, IM3 from your signal can interfere with theirs (if TX) or IM3 from their signals can interfere with your signal (if RX).

$V_{AM}(t)$



Gain compression
which causes
nonlinearity and
HD.

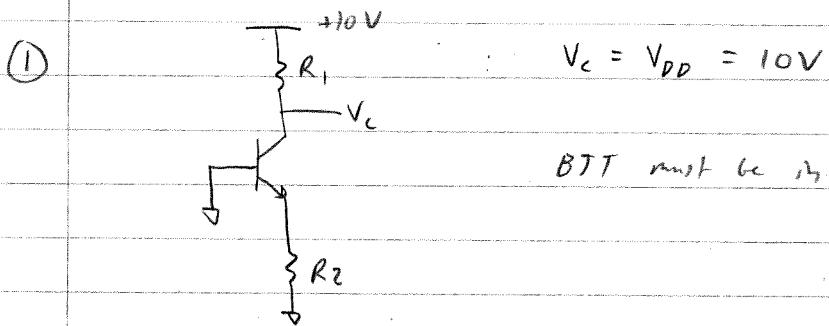
IM3 and HD3 will be worse for BJT than for MOS.

Give details of impact of the passive network
(i.e. only finite gain is possible and what happens
when the signals get too large) How does it look in frequency & time?

3.

AIK
Dutton

2004 - Shenoy



$$V_c = V_{DD} = 10V$$

BJT must be in cutoff.

(2) $g_m = \frac{I_c}{V_T} = \frac{g_I c}{kT}$ $I_c = I_s \exp\left(\frac{V}{V_T}\right) \Rightarrow \frac{dI_c}{dV} = \frac{1}{V_T} I_s \exp\left(\frac{V}{V_T}\right) = \frac{I_c}{V_T}$

(3) $g_m = \mu C_{ox} \frac{W}{L} (V_{gs} - V_t)$ $g_m + \text{noise} = \mu C_{ox} \frac{W}{L} V_{ds}$

(4) $\omega_t = \frac{g_m}{C_{gg}} = \frac{g_m}{C_{gs} + C_{gd}} \Rightarrow f_t = \frac{g_m}{2\pi(C_{gs} + C_{gd})} = \frac{3}{2} \frac{\mu V_{os}}{L^2}$

(5) $L = 32 \text{ nm}$ $f_t \approx 100 \text{ GHz}$

(6) Can use ZVTC if there is a dominant pole.

check
mit.
? $\Rightarrow f_{zvbc} = \left(2\pi \sum_{i=1}^n R_i C_i\right)^{-1}$

(7) Can increase 3dB freq. by reducing L, increasing g_m (burns more power). Want to increase g_m by increasing I_d (V_{os}) while keeping W constant, since normally W will increase C_s, C_{gd} , etc...

(8) Phase margin \Rightarrow Bode criterion $\phi|_{|T(s)|=1} = -180^\circ$
Gain margin

$T(s), RR$ at Loop gain. $GM = |T(s)| \Big|_{\phi(s) = -180^\circ}$

(9) All closed loop poles should be in left half plane.

To assure stability you must meet the Bode criteria.

\Rightarrow Phase margin $> 0^\circ$. In practical circuits, want Phase margin $> 30^\circ$ at least.

$|T(s)| < 1$ when $\phi(s) = -180^\circ$

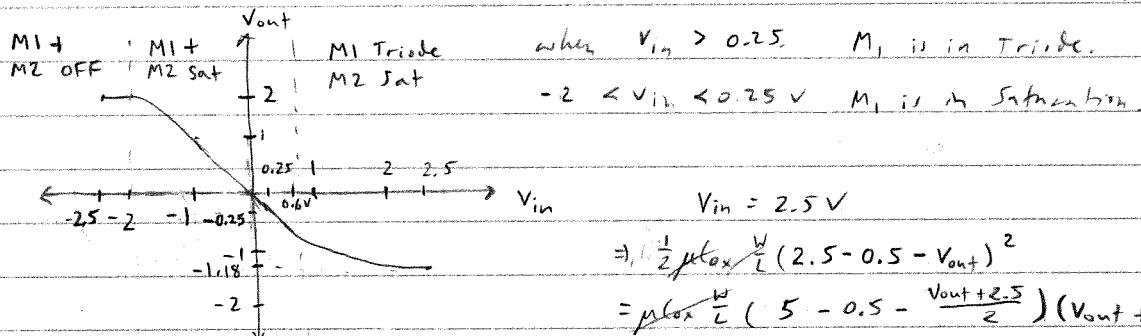
2005 Shenoy

$$1. a) (2.5 + V_{in} - V_T)^2 = (2.5 - V_{out} - V_T)^2$$

diode connected \Rightarrow M2 is always in SAT (or cutoff)

$$V_{GD} < V_T$$

- when $V_{in} < -2$, M_1 off $\Rightarrow V_{out} = V_{dd} - V_T$



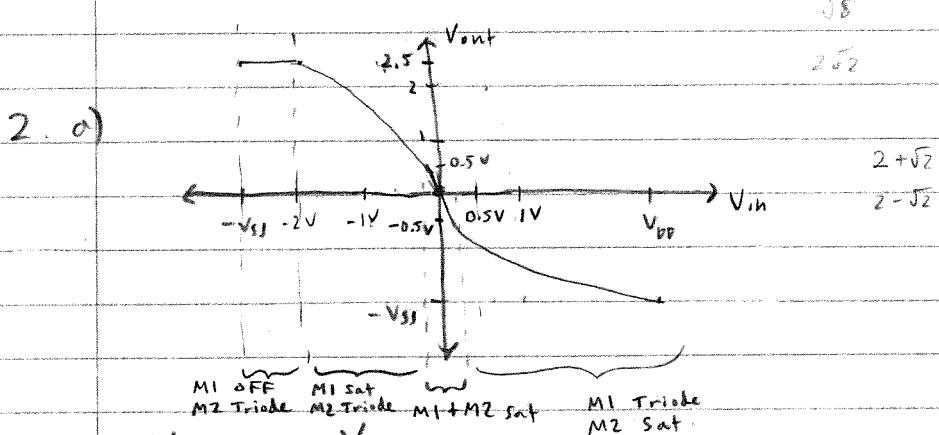
$$V_{in} = 2.5 \text{ V}$$

$$= \frac{1}{2} \mu_f \alpha \frac{W}{L} (2.5 - 0.5 - V_{out})^2$$

$$= \mu_f \alpha \frac{W}{L} (5 - 0.5 - \frac{V_{out} + 2.5}{2})(V_{out} + 2.5)$$

$$\Rightarrow V_{out} = -1.18 \text{ V}$$

$$b) V_{out_{max}} = V_{DD} - V_T = 2 \text{ V}$$



$$b) V_{out_{max}} = V_{DD}$$

$$c) \text{Gain is larger } -g_m (r_o || r_{o2}) \text{ vs. } -g_m (r_o || \frac{1}{g_m2}) \approx -\frac{g_m1}{g_m2} \approx -1$$

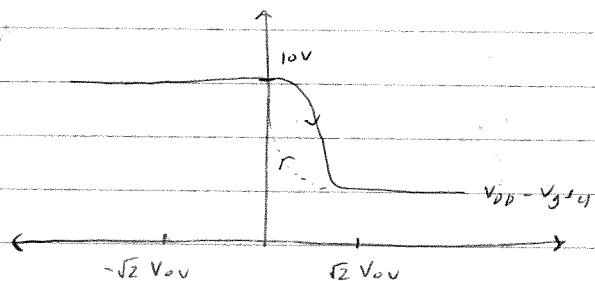
slope of V_{out} vs. V_{in} curve $\left(\frac{V_{out}}{V_{in}}\right)_{MAX}$ is clearly larger at $V_{in} = 0$, since M1 + M2 are in SAT, but in this case, M2 is not diode connected.

Check

Xhi!

?

3. a)



$$V_{DD} - V_g + \epsilon$$

b) Gilbert cell, used to cancel odd harmonics.

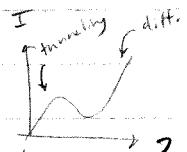
(say, oscillator)

1995 - Dutton

1.)

Lattice constants of semiconductors are on the order of 10 \AA , (5 \AA for Si) \Rightarrow which is comparable to the wavefunction of an electron.

Quantum Mechanics states that for an isolated atom there are a finite number of energy levels that an electron can occupy. Thus, electron energy is quantized. When you bring many atoms together, the levels split and form what look like continuous bands:



light spectrum of hydrogen gas. shows discrete lines corresponding to transitions between quantized states, Black body radiation, negative R of tunnel diode

3.

electrons can only move as waves if the lattice is perfect (no vibrations $\Rightarrow T=0$). Otherwise at $T > 0$, there is lattice scattering.

For example, for a resistor this will cause thermal noise: $i_n^2 = \frac{4kT}{R}$

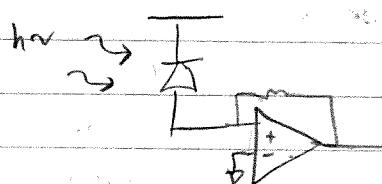
Wave properties would be observable in photonic devices

See next page.

Answers,

- 1) Semiconductors are composed of many atoms coming together. The existence of energy bands and band gaps can only be explained by quantum mechanics.

2)



Look up
photoelectric
effect.

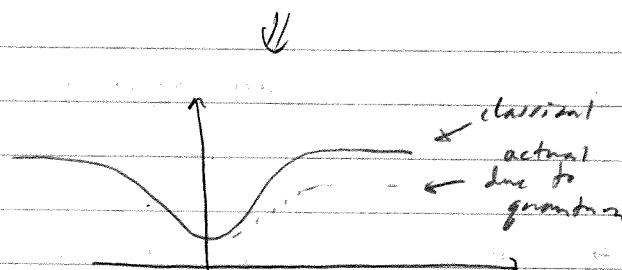
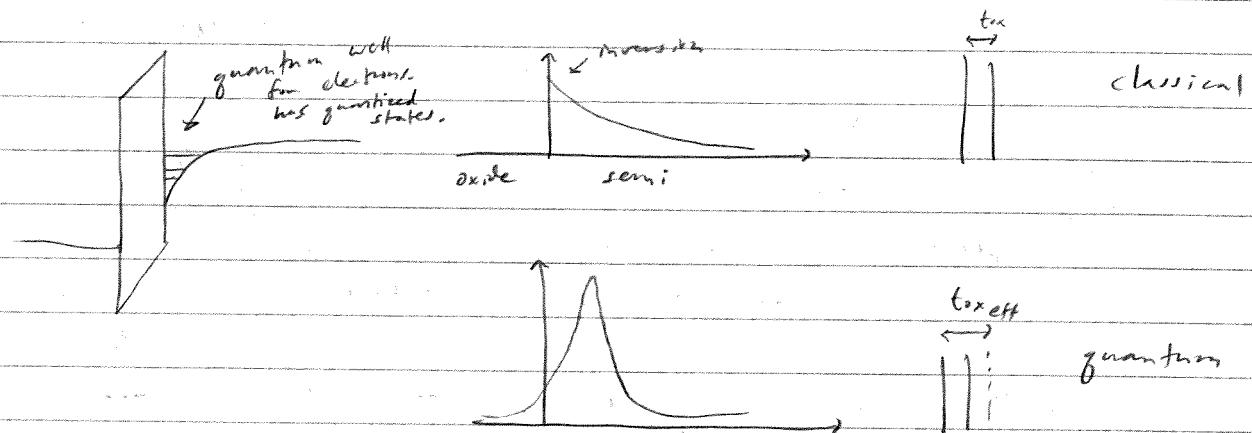
charge & until e, ht

parts are generated.

This shows that there is a bandgap.

OR photoelectric effect.

3. $I = q \cdot \frac{N}{t} \xrightarrow{\# \text{ of } e} \frac{I}{n}$



1996 - Dutton

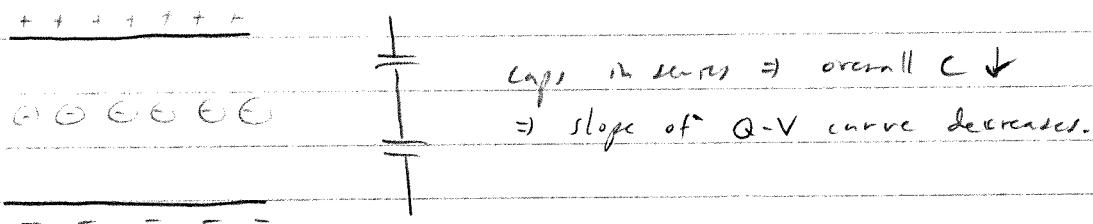
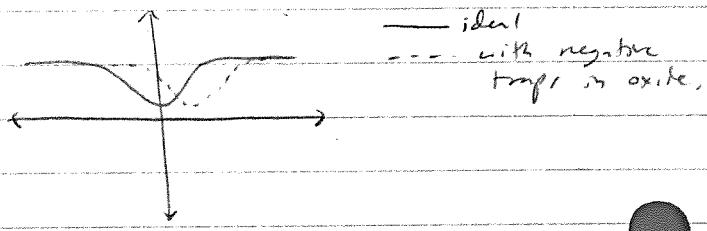
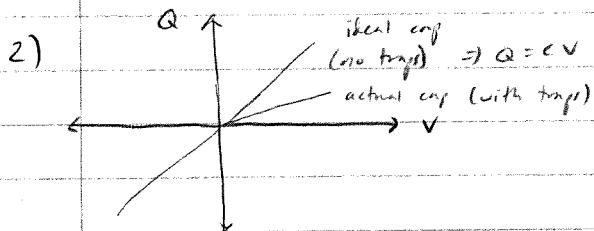
$$1) \quad 0.1\text{ }\mu\text{m} \times 0.1\text{ }\mu\text{m} \quad t_{ox} = 50\text{ \AA} = 5\text{ nm}$$

$$\rightarrow C_{ox} = \frac{\kappa \epsilon_0 A}{t_{ox}} \approx \frac{4 \cdot 8.85 \cdot 10^{-14} \text{ F/cm}^2 \cdot (0.1)^2 \cdot 10^{-8} \text{ cm}^2}{5 \cdot 10^{-7} \text{ cm}}$$

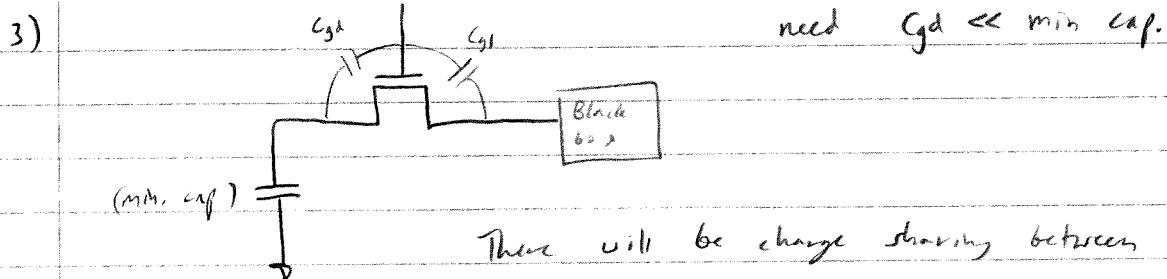
$$= 0.8 \cdot 8.85 \cdot 10^{-17} \text{ F} = 7 \cdot 10^{-17} \text{ F}$$

$$Q = CV \Rightarrow Q = 7 \cdot 10^{-17} \text{ C}$$

$$\frac{Q}{V} = \frac{7 \cdot 10^{-17} \text{ C}}{1.6 \cdot 10^{-19} \text{ C}} \approx 4 \cdot 10^2 = 400 \text{ e}^-$$

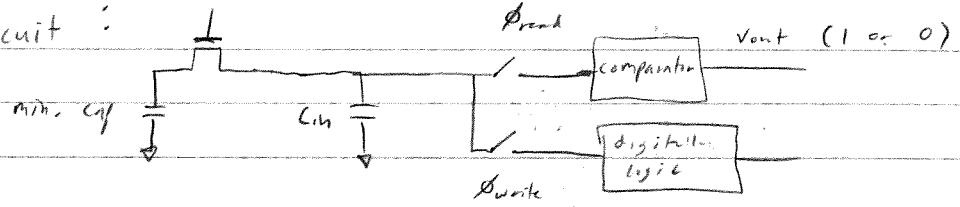


Charge moves onto traps through tunneling.



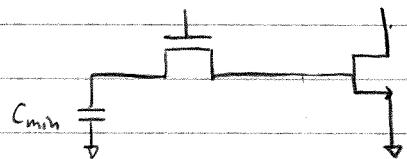
There will be charge sharing between min. cap and C_g . Also, I_{off} of pass transistor and I_m of Black box will continuously discharge min. cap \Rightarrow there is a limit to how long the memory can hold its state.

Black box circuit :



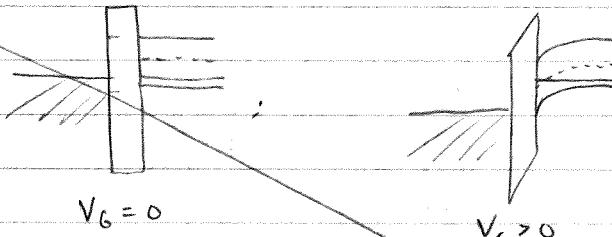
more detailed

for read:



$$C_{min} \frac{1}{f} \frac{1}{C_{gr.}}$$

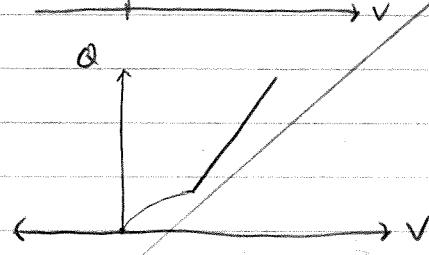
2)



$E_F > E_T$ at surface
 \Rightarrow traps fill with e^-
 \Rightarrow since charge doesn't
 recombine, this is effectively
 like inversion, $\Rightarrow C \rightarrow C_{ox}$

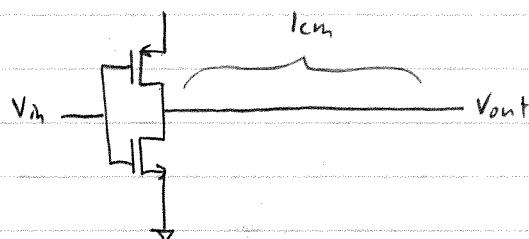
C
 with traps.

$$Q = VC$$



1994 Dutton

1.



$$\frac{1}{\sqrt{LC}} = \frac{c}{\sqrt{\epsilon_r \mu_r}}$$

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \quad v_{\text{prop}} = \frac{1}{\sqrt{\epsilon \mu}} = \frac{c}{\sqrt{\epsilon_r \mu_r}}$$

For lossless transmission line: velocity factor $VF = \frac{v_{\text{propagation}}}{c} = \frac{1}{c\sqrt{LC}}$
 $\Rightarrow VF \leq 1$

$$\text{where } c = 3 \cdot 10^8 \text{ m/s}$$

$$\frac{1}{\sqrt{\mu_0 \epsilon_0}} * v_{\text{propagation}} = \frac{1}{\sqrt{LC}} < c$$

L is distributed inductance (Henries per unit length)

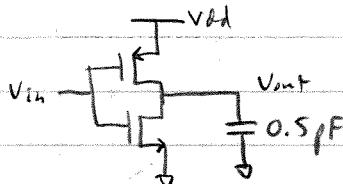
C is distributed capacitance (Farads per unit length)

\Rightarrow Absolute fastest it could be without inversion is:

$$\frac{1\text{cm}}{c \cdot VF} > \frac{1\text{cm}}{c} = \frac{10^{-2}\text{m}}{3 \cdot 10^8 \text{ m/s}} \approx 3.33 \cdot 10^{-11} \text{ s} = 33.3 \text{ ps}$$

with inversion

But if it will actually take longer than this since the inverter will have a propagation delay.



$$t_{p_{HL}} = \ln(2) R_{\text{on}} C_L$$

$$= 0.69 \left(\frac{3}{4}\right) \frac{C_L V_{DD}}{I_{D,\text{sat},n}} = 0.52 \frac{C_L V_{DD}}{(W/L)_n k_n V_{T,n} (V_{DD} - V_T - V_{D,\text{sat},n}/2)}$$

$$\text{for } V_{DD} \gg V_{D,\text{sat},n} \quad t_{p_{HL}} \approx 0.52 \frac{C_L}{(W/L)_n k_n V_{D,\text{sat},n}}$$

\Rightarrow we are limited by the time it takes to charge $C_L \approx 0.5 \text{ pF}$.

to decrease t_p , could increase $\frac{W}{L}$, V_{DD} (since $V_{D,\text{sat}} = V_{G1} - V_T = V_{DD} - V_T$), μ (or decrease V_T).

→ can only do this to an extent. After some point C_{gds} and C_{gd} (which get Miller multiplied at the output by a factor of 2) will dominate 0.5 pF and C_L .

(What is the dynamic power dissipation of the Inverter
 (i.e. in the transition from high to low, when both M_n and M_p
 are in saturation, there is power dissipated in the channel resistance)

2. Energy requirements: Must charge wire (0.5pF)

$$\Rightarrow \boxed{E_{\text{min}} = \frac{1}{2} C_w V_{dd}^2} = \frac{1}{2} (0.5\text{pF}) V_{dd}^2$$

↑
 fundamental limit.

It will actually take more energy due to dynamic power dissipation in the parasitic resistance of the wire during the changing transient,

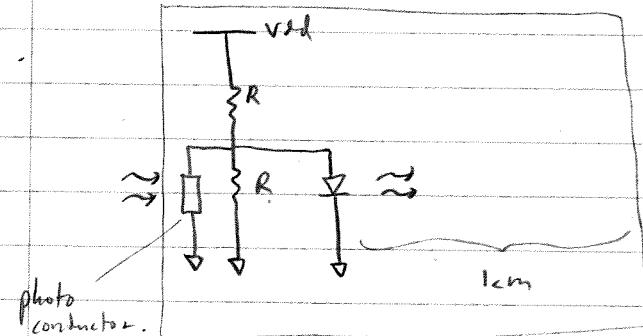
$P(t) = i(t) R_w$ can minimize $i(t)$ by transforming the voltage to high voltage. \rightarrow $V_{dd} \xrightarrow{\text{1cm}} V_{High} > V_{th} \xrightarrow{\text{1cm}} V_{dd}$ then transforming it back down.

After the inversion, $i_w = 0 \Rightarrow P_w = 0$

⇒ Zero static power in the wire.

But there is static power in the inverter due to finite gate leakage current and finite I_{off} (subthreshold current).

Could also do an optoelectronic based system:



Thus when V_{in} is low (no light incident), $R_{\text{photoconductor}}$ is high \Rightarrow LED does ON
 $\Rightarrow V_{out}$ is high (light emitted).

When V_{in} is high (light incident), $R_{\text{photoconductor}}$ is low \Rightarrow LED OFF
 $\Rightarrow V_{out}$ is low (no light emitted).

Must have $R_{\text{photoconductor, Light}} \ll R \ll R_{\text{photoconductor, Dark}}$

propagation time limited by speed of light $\Rightarrow 33.3 \text{ ps}$

plus it will take time for photoconductor to respond to incident light.

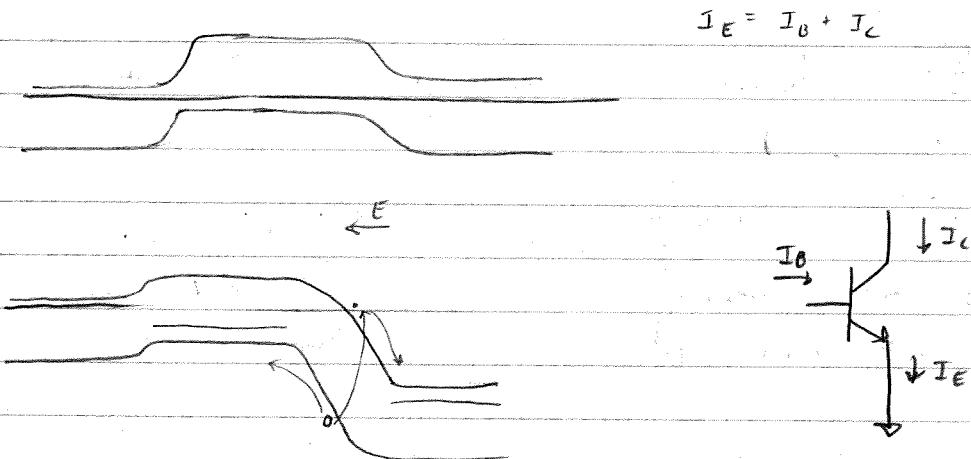
due to transit time of carriers. (can speed up by decreasing length of photoconductor, increasing V across it, or decreasing carrier lifetime).

2. Absolute minimum energy requirement would depend on how many photons are required to consider the output high.

In this implementation, there is high static power since current is always flowing through Rphotoconductor $\parallel \neq$.

1994 - Wong

1. For $V_{CB} = 2V$, why is the base current negative at low V_{BE} ($V_{BE} < 0.4V$)? Hint: $I_c = -I_B$ at low V_{BE}



For high V_{BE} , many e^- s diffuse into the depletion region due to the lowered barrier and some recombine with holes. These holes must be replaced and thus I_B is positive.

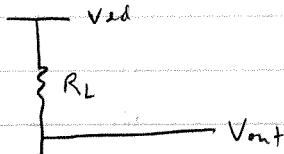
For low V_{BE} , fewer e^- s diffuse into the depletion region.
⇒ Current is actually dominated by generation in the B-C depletion region.

These holes flow into the base and must be removed
($I_E = I_B + I_C$, we are given that $I_C = -I_B \Rightarrow I_E = 0$)
⇒ I_B is negative and equal to I_C .

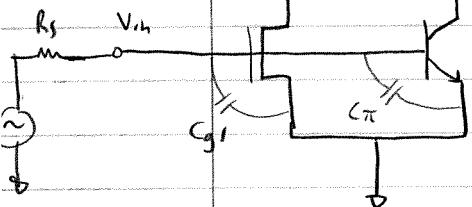
2. For $V_{CB} = 3V$, why is the base current negative even for $V_{BE} = 0.6$ to $0.8V$? Hint: The negative base current is not observed at lower V_{CB} (2V). The negative base current tracks the collector current.

Similar to part 1), negative base current means excess holes are generated in the reverse-biased base-collector junction. Since the negative base current tracks the collector current and only occurs at higher V_{CB} , it is related to the collector current (e^-) going through a larger potential drop (high \bar{E}). The highly accelerated electrons can cause impact ionization and generate excess electron-hole pairs.

2006 - Dutton



for max small signal voltage gain, at what operating point should we bias the new device?



$$I_{out} = \frac{1}{2} \mu C_{ox} \frac{W}{L} (V_{in} - V_{Th})^2 + I_o e^{V_{in}/V_T}$$

$$\Rightarrow g_m = \mu C_{ox} \frac{W}{L} (V_{in} - V_{Th}) + \frac{1}{V_T} I_o e^{V_{in}/V_T}$$

\Rightarrow want to maximize V_{in} such that MOS stays in sat and BJT stays in forward active.

$$\Rightarrow V_{dd} - I R_L > V_{in} \quad \text{Assume BJT current dominates.}$$

$$\Rightarrow V_{dd} - I_o e^{V_{in}/V_T} R_L = V_{in} \Rightarrow \text{solve for } V_{in}.$$

$$\frac{\partial I_o}{\partial V_{in}} e^{V_{in}/V_T} = \frac{I_c}{V_T}$$

$$\text{AC gain and bandwidth: } A_v = -\left(g_m^MOS + g_m^BJT\right) R_L$$

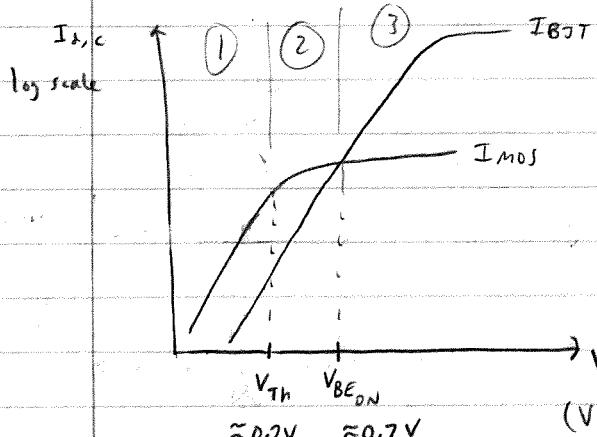
$$f_{3dB} \approx \frac{1}{2\pi \left[(1+A_v) [C_{gd} + C_{in}] + C_{gs} + C_{pi} \right] R_s}$$

$$G_a = \frac{\partial I}{\partial V_{in}} = \frac{\partial I_o(MOS)}{\partial V_{in}} + \frac{\partial I_c(BJT)}{\partial V_{in}} = g_m^MOS + g_m^BJT$$

$$= \mu C_{ox} \frac{W}{L} V_{ov} + \frac{I_c}{V_T}$$

To maximize G_a , we should maximize $(I_c + I_{ds})$

\Rightarrow we should bias in region 3.



$$\text{AC gain: } A_v = -G_a R_L \approx -\frac{I_c}{V_T \cdot R_L}$$

$$\text{AC bandwidth: } \frac{1}{R_s (C_{gs} + C_{pi} + (1/A_v) (C_{gd} + C_{in}))}$$

In other words, this device is not very useful. in ③, it looks like a BJT with $C_{\mu}' = C_{\mu} + C_{gs}$, $C_{\mu'}' = C_{\mu} + C_{gd}$.

In ① and ②, it looks like a MOSFET with $C_{g's}' = C_{gs} + C_{pi}$ and $C_{g'd}' = C_{gd} + C_{in}$. \Rightarrow lower BW \Rightarrow lower GBW product.

2004 - Dutton

$$C = \frac{\Delta Q}{\Delta V}$$

minimum Q is $g = 1.6 \cdot 10^{-19} C$

maximum V is $V_{BR} \approx 10 kV$

$$\Rightarrow \text{minimum } C \text{ is } \sim \frac{10^{-19}}{10^4} = 10^{-23} F$$

1998 - Wong

- What mechanisms are responsible for breakdown of MOSFET at large V_{DS} .

(2)

Punchthrough



depletion regions touch
⇒ current flows below
channel.

How does
punchthrough
work?
Impact ionization?

DIBL \Rightarrow tunneling

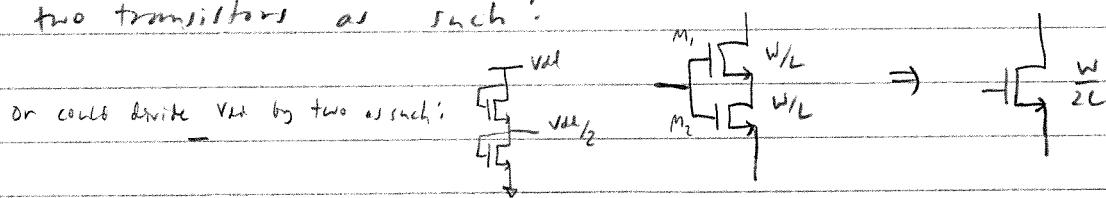


Reverse breakdown of D-substrate diode

- Given a MOSFET with $V_{BR} = 10V$, how can you modify the device or circuit to use it with $V_{DD} = 15V$

To reduce punchthrough, you can increase L ,
increase the doping of the bulk (to reduce size of S-D
depletion regions), Decrease the doping of the drain
(lightly doped drain) \Rightarrow lower E field \Rightarrow higher V_{BR} between D-substrate,
for given V

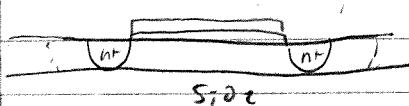
To increase L at a circuit level, you can combine
two transistors as such:



If M_1 is in SAT, M_2 must be in linear.

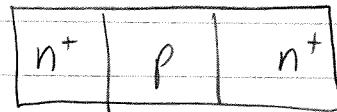
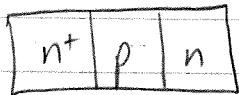
Voltage drop across each transistor will be less.

To prevent punchthrough, could use SOI.



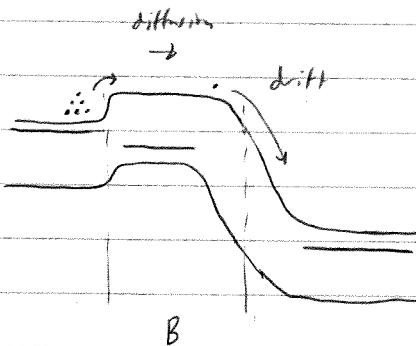
Now depletion regions can't touch
below the channel.

1997 - Wong



Compare the transit time of carriers inside the base of a BJT and that along the channel of a MOSFET.

Assume BJT in forward Active + Mos in saturation.

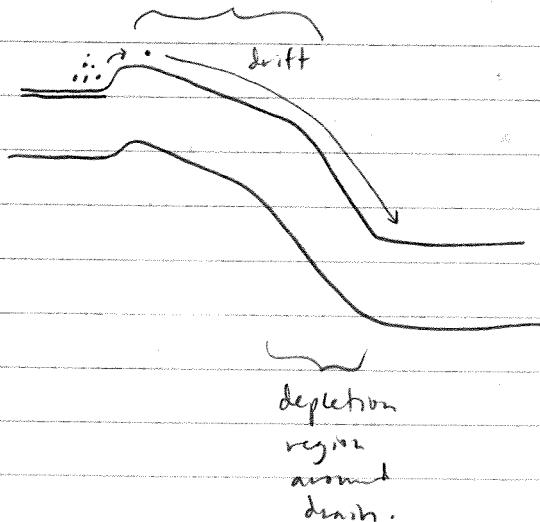


$$\text{BJT will have } \tau_{\text{BJT}} = \tau_{\text{diff}} + \tau_{\text{drift}}$$

$$V_d = \mu \bar{\epsilon}$$

$\downarrow \times \bar{\epsilon}$ of
mobility at depletion region (BC) $\frac{V_{CB}}{W_{\text{dep}}} \approx \frac{V_{CB}}{W_{\text{dep}}}$

MOSFET channel



$$\text{Mos will have } \tau_{\text{mos}} = \tau_{\text{drift}}$$

$$V_d = \mu \bar{\epsilon}$$

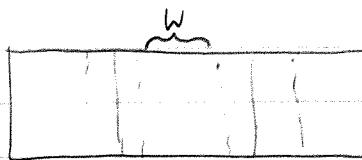
$\uparrow \text{channel mobility} \quad \approx \frac{V_{ds}}{L}$

Answer:

In base of BJT, carriers are minority carriers & move by diffusion.

$$\tau_{t_{BJT}} = \frac{8B}{I_c} \approx \frac{W^2}{2D_B} = \frac{W^2}{2 \frac{kT}{q} \mu}$$

from EE216 notes.



e) for uniform base doping, $\tau_B > \tau_{MOS}$

for graded base doping, τ_B will be less, because there will be an internal electric field in the base which will aid the transport across the base.

$$\tau_{MOS} = \frac{L^2}{\mu V_{DS}}$$

$$\tau = \frac{L}{v_d} = \frac{L}{\mu E} = \frac{L^2}{\mu V_{DS}}$$

$$\tau_{t_{BJT}} = \frac{W^2}{2(\frac{kT}{q}) \mu_B}$$

$$\tau_{MOS} = \frac{L^2}{V_{DS} \mu_H}$$

$\mu_H < \mu_B$ due to surface scattering.

But $V_{DS} \gg 2(\frac{kT}{q})$ usually.

\Rightarrow if $W_{base} = L_{channel}$, $\boxed{\tau_B > \tau_{MOS}}$

1993 - Dutton

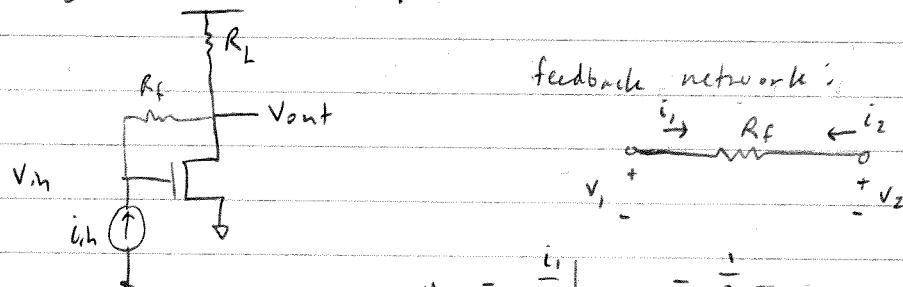
1. What is Miller Capacitance?

$$V_{in} \xrightarrow{C} k \cdot V_{in} \Rightarrow (1-k) C \xrightarrow{Z} \frac{1}{(1-k)} C$$

$$V_{in} \xrightarrow{Z} kV_{in}$$

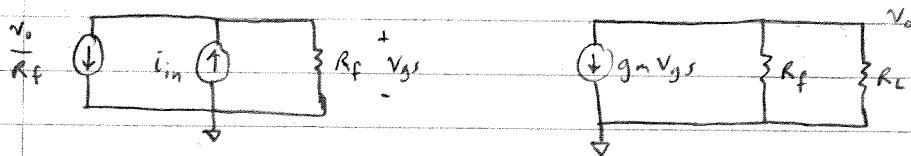
$$V_{in} \xrightarrow{Z} \begin{matrix} \text{+} \\ \text{-} \end{matrix} kV_{in} = V_{in} \xrightarrow{Z} \begin{matrix} \text{+} \\ \text{-} \end{matrix} -\frac{2}{k} \xrightarrow{Z} \begin{matrix} \text{+} \\ \text{-} \end{matrix} kV_{in}$$

2. How does shunt-shunt feedback resistance effect the input and output impedance of a single stage transistor amplifier.



$$g_{11} = \left. \frac{i_1}{v_1} \right|_{v_2=0} = \frac{1}{R_f} \quad y_{21} = \left. \frac{i_2}{v_2} \right|_{v_1=0} = \frac{1}{R_f}$$

$$f = \left. \frac{i_1}{v_2} \right|_{v_1=0} = -\frac{1}{R_f}$$



$$\Rightarrow a = \frac{v_o}{i_{in}} = -g_m R_f (R_f \parallel R_L) \quad f = -\frac{1}{R_f} \Rightarrow af = g_m (R_f \parallel R_L)$$

$$A_{CL} = \frac{a}{1+af} = -\frac{g_m R_f (R_f \parallel R_L)}{1 + g_m (R_f \parallel R_L)} \approx -R_f \text{ for } g_m (R_f \parallel R_L) \gg 1$$

$$Z_i = \frac{R_f}{1 + g_m (R_f \parallel R_L)} \quad Z_o = \frac{(R_f \parallel R_L)}{1 + g_m (R_f \parallel R_L)}$$

\Rightarrow input and output impedance are decreased by $(1 + T)$.

to compare to Miller, we must find $K = \frac{V_{out}}{V_{in}}|_{CL}$

$$A_{CL} = \frac{V_{out}}{V_{in}} \Rightarrow K = \frac{V_{out}}{V_{in} \cdot Z_{in}} = \frac{A_{CL}}{Z_i} = -g_m(R_f \parallel R_L) = -T$$

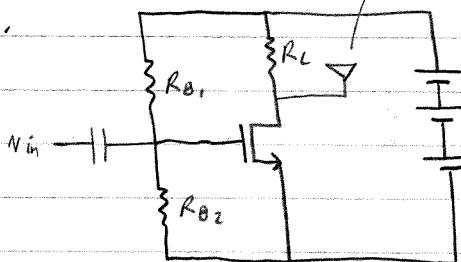
$$\Rightarrow V_i \xrightarrow[R_f]{\text{---}} V_o \Rightarrow \frac{R_f}{1-K} = \frac{R_f}{1+T} = \frac{R_f}{1+g_m(R_f \parallel R_L)} \quad \left\{ \begin{array}{l} R_f \left(\frac{K}{K-1} \right) = \frac{R_f (g_m(R_f \parallel R_L))}{1+g_m(R_f \parallel R_L)} \approx R_f \end{array} \right.$$

\Rightarrow This agrees with Miller for the input impedance, but not for the output impedance. $Z_o \approx (R_f \parallel R_L) \neq \frac{(R_f \parallel R_L)}{1+T}$

Not sure exactly why this is, but it may have to do with the fact that in using the 2-port model, we neglect feed forward through R_f .

2007 - Shenoy

1. a) C-S.



make antenna out of wire.

length should be $\sim \lambda/4$

for

$$\lambda f = c \quad \frac{3 \cdot 10^8}{3 \cdot 10^{10}} = 1 \text{ cm}$$

for 30 MHz, $\lambda = 1 \text{ cm}$

AM - 0.3-3 MHz $\Rightarrow 1 \text{ km} - 100 \text{ m}$

CB radio - 3-30 MHz $\Rightarrow 100 \text{ m} - 10 \text{ m}$

FM - 30-300 MHz $\Rightarrow 10 \text{ m} - 1 \text{ m}$

b) Goal with batteries is to maximize the output voltage swing
 \Rightarrow stack them in series to get highest possible rail.

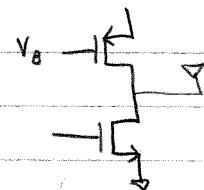
c) Goal with resistors is to DC bias the MOSFET and to provide a load R_L for the MOSFET to achieve gain $-g_m R_L$.

d) Yes. With just a resistive load, the maximum gain achievable is $\approx -\frac{2V_{dd}}{V_{ov}}$ since R_L sets both the DC biasing and output load resistance.

$$|-g_m R_L| = \left| -\frac{2I_d}{V_{ov}} \cdot R_L \right| \text{ but } I_d R_L < V_{dd} \Rightarrow \left| -g_m R_L \right| < \left| -\frac{2V_{dd}}{V_{ov}} \right|$$

With a PMOS, we can create an active load:

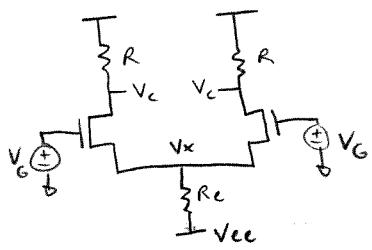
Thus we can get higher gain: $-g_m (r_{on} \parallel r_{op})$



e) To increase $3dB$ frequency reduce any series resistance R_s at the input and if possible, the G_S and G_D of the MOSFETs since the dominant pole of this high gain amplifier will likely be:

$$W_{3dB} \approx \frac{1}{R_s (C_{gd}) (1 + g_m (r_{on} \parallel r_{op}))}$$

due to Miller multiplication of G_D .



2. a) $\frac{V_X - V_{OE}}{R_E} = 2 k_n' \left(\frac{W}{L}\right) (V_G - V_X - V_T)^2$

$$\frac{V_X - V_{OE}}{R_E} = A (V_A - V_X)^2 = A (V_A^2 - 2V_A V_X + V_X^2) \quad \text{let } B = AR_E$$

$$\Rightarrow V_{OE} = V_X - B(V_A^2 - 2V_A V_X + V_X^2)$$

$$\Rightarrow BV_X^2 - (2BV_A + 1)V_X + (BV_A^2 + V_{OE}) = 0$$

$$\Rightarrow V_X \approx \frac{(2BV_A + 1) \pm \sqrt{4BV_A + 1 - 4BV_{OE}}}{2B}$$

$$\Rightarrow V_{OE} \uparrow \Rightarrow V_X \uparrow$$

OR qualitatively: $V_{OE} \uparrow \Rightarrow V_X = V_{OE} + 2IDR_E \uparrow \Rightarrow ID \uparrow$

Cutoff when $V_G - V_X < V_T$

Saturation when $V_G - V_X > V_T$ and $V_C - V_G > -V_T$
 $V_G - V_C < V_T$

Triode when $V_G - V_X > V_T$ and $V_G - V_C > V_T$

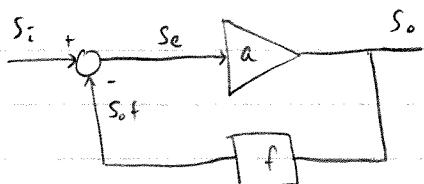
\Rightarrow as V_{OE} is increased

Triode \rightarrow Saturation \rightarrow Cutoff

b) as V_{OE} is increased, V_X increases also.

When $V_{OE} = V_{CC}$, $V_X = V_{CC}$ since MOSFETs are in cutoff.

3. a)



derive closed loop gain:

$$S_o = (S_i - S_o f)a = aS_i - afS_o$$

$$\Rightarrow (1+af)S_o = aS_i \Rightarrow \frac{S_o}{S_i} = \frac{a}{1+af} = A$$

b) derive closed loop 3dB frequency if $a(s) = \frac{a_0}{1 + \frac{s}{\omega_{3dB}}}$

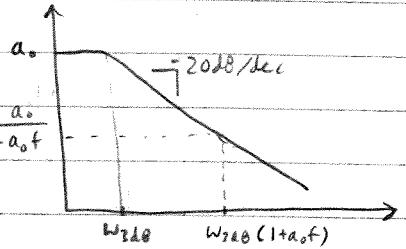
$$A = \frac{\left(\frac{a_0}{1 + \frac{s}{\omega_{3dB}}}\right)}{1 + \frac{a_0 f}{1 + \frac{s}{\omega_{3dB}}}} = \frac{a_0}{1 + \frac{s}{\omega_{3dB}} + a_0 f}$$

$$= \frac{\frac{a_0}{(1+a_0 f)(1 + \frac{s}{\omega_{3dB} \cdot (1+a_0 f)})}}{1 + \frac{s}{\omega_{3dB, CL}}} = \frac{A}{1 + \frac{s}{\omega_{3dB, CL}}}$$

$$\Rightarrow \boxed{\omega_{3dB, CL} = (1+a_0 f) \omega_{3dB}}$$

c) No, Gain bandwidth product is the same.

$$a_0 \cdot \omega_{3dB} = \frac{a_0}{(1+a_0 f)} \cdot \omega_{3dB} \cdot (1+a_0 f)$$



2005 - Murmann

a) $I_{out} = (1+k) I_{in}$. Feedback used to match source voltages of the two rightmost transistors.
 \Rightarrow current amplifier.

b). By inspection: $T(s) = -g_m \left(\frac{1}{g_m} \parallel \frac{1}{sC} \right) \left(g_m (r_o \parallel \frac{1}{sC}) \right)$

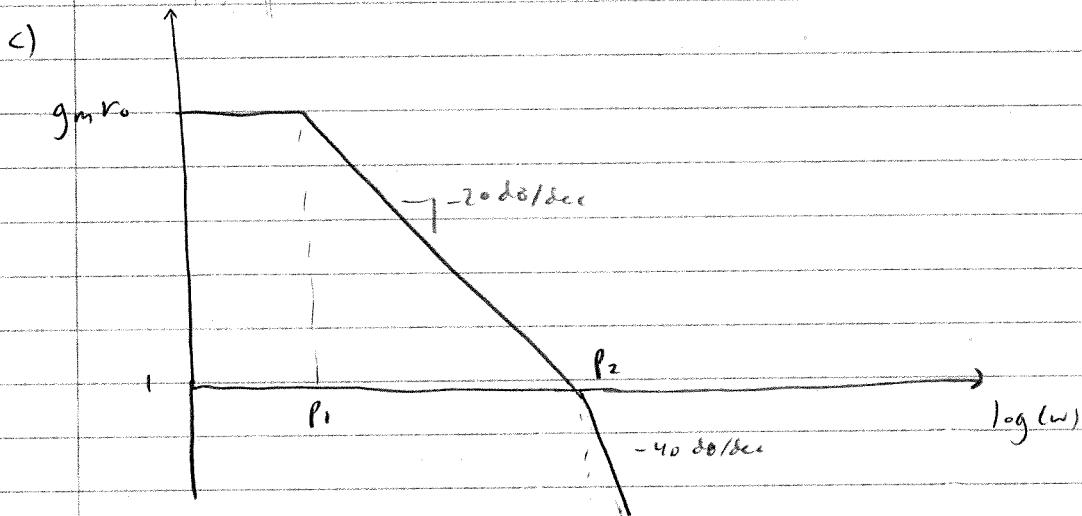
assuming nodes (1) and (2) are isolated

$$\Rightarrow T(s) = -g_m r_o \left(\frac{1}{(1+s r_o C)(1+s \frac{C}{g_m})} \right)$$

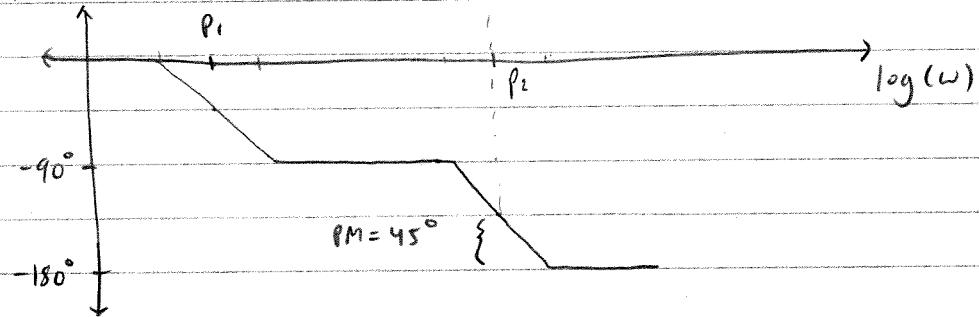
$$W_{P_1} = \frac{1}{r_o C}$$

$$W_{P_2} = \frac{1}{(\frac{1}{g_m}) C}$$

$20 \log |T(j\omega)|$



$\phi(T(j\omega))$



2006 - Murmann

A) a) $A_{vA} = -g_{m_2} \left(\frac{1}{g_{m_1}} \right)$ I_d is the same

$$g_m = \mu C_{ox} \frac{W}{L} V_{ov} = \frac{2I_d}{V_{ov}} = \sqrt{2\mu C_{ox} \frac{W}{L} I_d}$$

$$\Rightarrow g_{m_2} = 3g_{m_1} \quad \Rightarrow \quad A_{vA} = -3$$

b) $A_{vA} = - \frac{g_{m_2}}{1 + g_{m_2}(-\frac{1}{g_{m_1}})} g_{m_1} = -3 \left(\frac{1}{1 - 3/2} \right) = -\frac{3}{1/2} = 6$

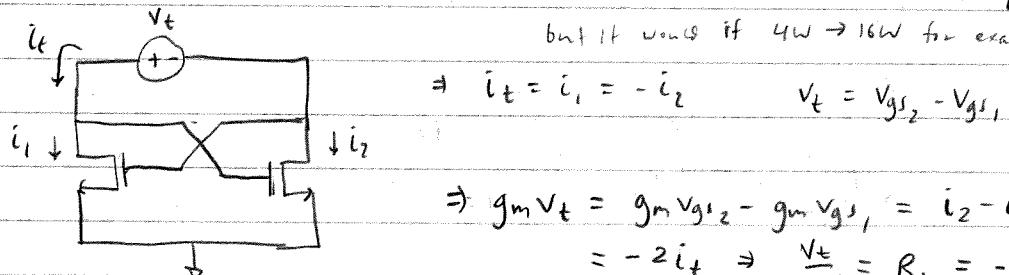
\Rightarrow higher gain, but polarity of output is reversed.

B) positive feedback \Rightarrow possible instability, latch up

must have $|T(s)| < 1$ for $s \neq 0$

$$T(s) = -\left(\frac{g_{m_2}}{g_{m_1}}\right)\left(-\frac{g_{m_2}}{g_{m_1}}\right) = \frac{4}{9} \quad \Rightarrow \text{in this case, it should be stable, not latch up}$$

but it would if $4W \rightarrow 16W$ for example.



$$\Rightarrow g_m v_t = g_m v_{gs2} - g_m v_{gs1} = i_2 - i_1$$

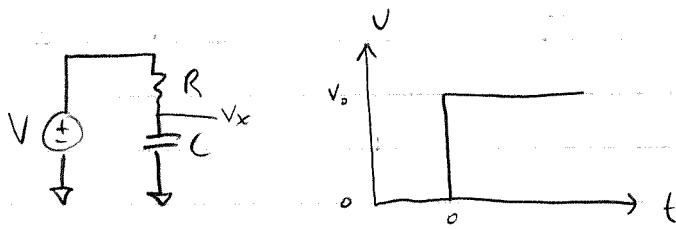
$$= -2i_t \Rightarrow \frac{v_t}{i_t} = R_{in} = -\frac{2}{g_m}$$

$$\frac{-\frac{2}{g_m}}{R} \Rightarrow \left. \begin{array}{c} \text{half circuit } R \\ -\frac{1}{g_m} \end{array} \right\} \quad \left. \begin{array}{c} \\ -\frac{1}{g_m} \end{array} \right\}$$

differential R

1993 - Meng

1. a)



Derive instantaneous power consumption of RC circuit as a function of time.

$$i(t) = C \frac{dV_x}{dt} = \frac{V - V_x}{R}$$

$$i(t) = \frac{V_0}{R} e^{-t/RC} \quad P = i(t) \cdot V_0 = \frac{V_0^2}{R} e^{-t/RC}$$

b) What is total energy needed to charge cap from 0 to V_0 .

$$\boxed{\frac{1}{2} CV_0^2} = \int_0^\infty P_c(t) dt$$

c) R effectively limits the amount of current that can flow into (charge) the cap. RC time constant determines the time it takes to charge C to V_0 .

since $P = \frac{E}{t}$, P is effected by t_{charge} and thus R .

We are changing the voltage instantaneously, but due to the resistor, the voltage across the capacitor cannot change instantaneously. The delay in the rise time of $V_x(t)$ is related to R .

Energy of the capacitor only depends on the charge required to develop a voltage of V_0 across it.

$$Q = CV \Rightarrow E = \frac{1}{2} QV$$

2. a) $E = \frac{1}{2} CV_{dd}^2 \Rightarrow$ if V_{dd} is reduced by factor of 2,

E is reduced by a factor of 4

b) Time would increase by a factor of ~ 2 .

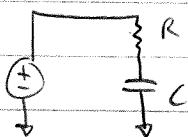
$$I \propto V_{dd}^2$$

assuming $V_{dd} \gg V_T$.

$$t = \frac{Q}{I} \quad Q = CV \quad I \propto V^2 \Rightarrow t_0 = \frac{Q_0}{I_0} \propto \frac{CV_0}{V_0^2}$$

$$\Rightarrow t_{new} \propto \frac{C \frac{V_0/2}{(V_0/2)^2}}{V_0} = 2 \frac{CV_0}{V_0^2} = 2t_0.$$

Answers.



$$V_c(t) = F.V. + (I.V. - F.V.) e^{-t/RC}$$

$$= V_0 + (0 - V_0) e^{-t/RC}$$

$$= V_0 (1 - e^{-t/RC}) \quad t \geq 0$$

$$i(t) = C \frac{dV_c}{dt} = C V_0 \frac{1}{RC} e^{-t/RC} = \frac{V_0}{R} e^{-t/RC} \quad t \geq 0.$$

$$P(t) = V_0 \cdot i(t) = \frac{V_0^2}{R} e^{-t/RC}$$

$$P_c(t) = i(t) V_c(t) = \frac{V_0^2}{R} (e^{-t/RC} - e^{-2t/RC})$$

$$P_R(t) = i(t)^2 R = \frac{V_0^2}{R} e^{-2t/RC}$$

$$\Rightarrow P(t) = P_c(t) + P_R(t)$$

???

redo.

2004 - Meng

$$\textcircled{1} \quad k' \left(\frac{W}{L}\right) (V_G - V_S - V_T)^2 = \frac{V_S + 1.5 \text{ V}}{10k\Omega} = \frac{3.5 - V_G}{10k\Omega}$$

Assuming $V_D = V_G$

$$\Rightarrow 3.5 - V_G = 10V_S + 15 \Rightarrow V_G = -11.5 - 10V_S$$

~~$$k' \left(\frac{W}{L}\right) (-11.5 - 10V_S - 0.5)^2 = 5(-12 - 10V_S)^2 = \frac{V_S + 1.5}{10k\Omega}$$~~

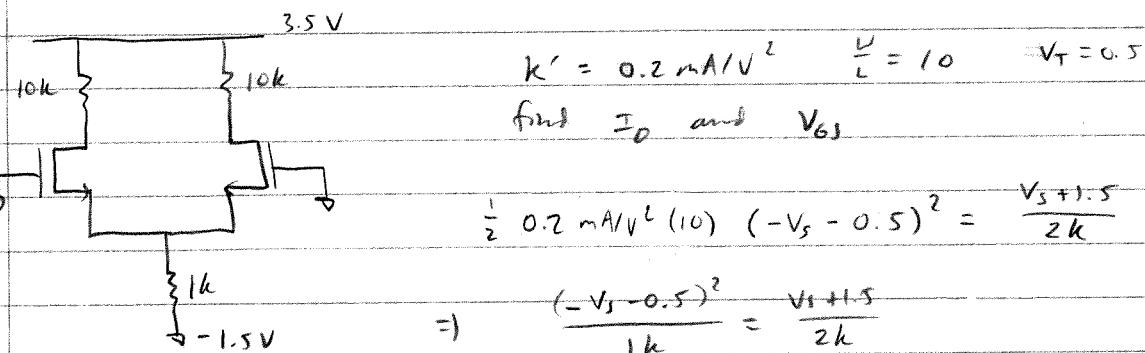
$$V_S = -1 \text{ V} \text{ or } 0.5 \text{ V} \quad I_{\text{drain}} = 0.5 \text{ mA}$$

$$I_D = 0.25 \text{ mA}$$

$$V_{GS} = 1 \text{ V}$$

$$V_{D\text{cm}} = 1 \text{ V}$$

$$\Rightarrow V_{DS} = 0.5 \text{ V} \quad V_{DS} = 2 \text{ V} \Rightarrow \text{saturation} \quad \checkmark$$



$$\Rightarrow 2V_S^2 + 2V_S + 0.5 = V_S + 1.5 \Rightarrow 2V_S^2 + V_S - 1 = 0$$

$$\Rightarrow V_S = \frac{-1 \pm \sqrt{1+8}}{4} = -1 \text{ V or } 0.5 \text{ V}$$

for 0.5 V , $I_D = \frac{2V}{2k} = 1 \text{ mA} \Rightarrow V_D = 3.5 \text{ V} - 10k \cdot 1 \text{ mA} = -6.5 \text{ V} \quad \times$

for -1 V , $I_D = \frac{0.5 \text{ V}}{2k} = 0.25 \text{ mA} \Rightarrow V_D = 3.5 \text{ V} - 10k \cdot 0.25 \text{ mA} = 1 \text{ V} \quad \checkmark$

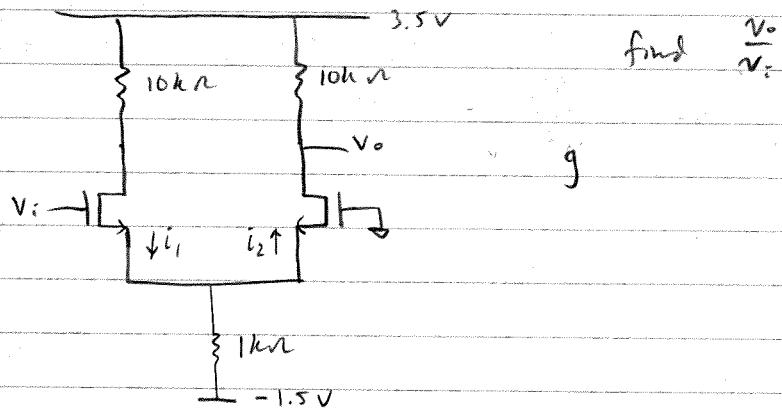
$$\Rightarrow \boxed{V_{GS} = 1 \text{ V}} \quad \boxed{I_D = 0.25 \text{ mA}}$$

?? redo.

$$(3) \quad A_{CD} = \frac{\left(1k\Omega \parallel \frac{1}{gm_2}\right)}{\left(1k\Omega \parallel \frac{1}{gm_2}\right) + \frac{1}{gm_1}} = \frac{1}{3}$$

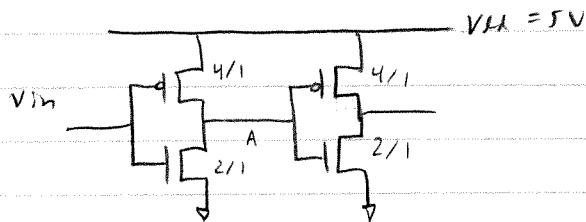
$$A_{CG} = gm_2 (10k\Omega) = 10$$

$$\Rightarrow \frac{V_o}{V_i} = A_{CD} \cdot A_{CG} = \boxed{\frac{10}{3}}$$



1994 - Meng

1.



$$k_n' = 20 \mu\text{A}/\text{V}^2 \quad k_p' = 10 \mu\text{A}/\text{V}^2$$

① What is gate cap in standard CMOS technology? (for now Assume 10fF)

$$C_{\text{ox}} \approx 8\text{fF}/\mu\text{m}^2 \quad \text{for } 32\text{nm technology.}$$

② Gate delay from input to node A?

Assume transistor is in saturation during charging.

$$I_{\text{sat}} = \frac{1}{2} k_n' \cdot \left(\frac{V_{\text{dd}}}{2}\right) (V_{\text{dd}} - V_T)^2 = \frac{1}{2} 40 \mu\text{A}/\text{V}^2 \cdot 25 \text{V}^2 = 0.5 \text{mA}$$

$$t_f \approx \frac{CV_{\text{dd}}}{I_{\text{sat}}} = \frac{5\text{V} \cdot 10\text{fF}}{0.5 \cdot 10^{-3}} = \frac{50 \cdot 10^{-15}}{5 \cdot 10^{-4}} = 0.1 \text{ns.}$$

$$t_f = \frac{C \Delta V}{I} = \frac{C_1 V_{\text{dd}}}{\frac{1}{2} k_n' \frac{W}{L} (V_{\text{dd}} - V_T)^2} \approx \frac{2 C_{\text{gs}}}{k_n' \left(\frac{W}{L}\right) V_{\text{dd}}}$$

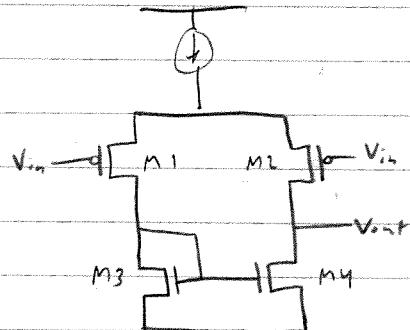
$$\text{(In reality, } t_f = \frac{3.4 C_{\text{gs}}}{k' \left(\frac{W}{L}\right) V_{\text{dd}}})$$

③ Design an output stage that aims at minimizing the delay of driving a capacitance 1000 times bigger than gate capacitance. Estimate the number of stages needed and the total delay.

???

Look this up.

2.



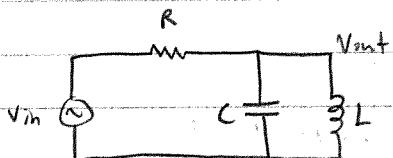
Assume the only offset contribution is from the mismatch in the W/L of the input transistors. Neglect transistor output resistance.

Params	\uparrow	\downarrow	Does not change
W_1, W_2	✓		
L_1, L_2		✓	
W_3, W_4			✓
L_3, L_4			✓
I		✓	
t_{ox}		✓	

1995 - Meng

1. a) Inductor: $V = L \frac{dI}{dt}$ Capacitor: $I = C \frac{dV}{dt}$

b)



$$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow f_0 = \frac{1}{2\pi\sqrt{LC}} = 1 \text{ GHz}$$

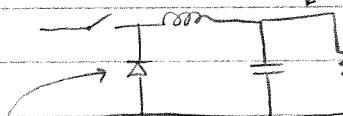
$$\Rightarrow LC = \left(\frac{1}{2\pi \cdot 10^9}\right)^2 \approx \frac{1}{40 \cdot 10^{18}} = 2.5 \cdot 10^{-20}$$

$$\Rightarrow \text{let } L = 2.5 \text{ nH} \quad C = 10 \text{ pF}$$

$$Z_{max} = j\omega_0 L \parallel \frac{1}{j\omega_0 C} = \frac{j\omega_0 L \cdot \frac{1}{j\omega_0 C}}{j\omega_0 L + \frac{1}{j\omega_0 C}} = \frac{j\omega_0 L}{1 - \omega_0^2 LC}$$

$$\Rightarrow |Z_{max}| = \frac{L/\sqrt{LC}}{|1 - 1|} = \infty \quad \leftarrow \text{only theoretically, in reality } L \text{ and } C \text{ will have finite Q.}$$

2. a) $V_{L_{ON}} = V_{in} - V_{D_{ON}} - V_{out} = L \frac{dI_{L_{ON}}}{dt}$



b) $V_{L_{OFF}} = -V_{D_{ON}} - V_{out} = L \frac{dI_{L_{OFF}}}{dt}$

"Free-wheeling" diode.
See MIT 6.334
L1, 58

assume pole of this RC lowpass
is much
lower than
switching
freq. Thus
 V_{out} remains
constant for
 T_{ON} and T_{OFF}

c) In P.S.S. $\langle \frac{dI_L}{dt} \rangle = 0$ otherwise I_L would change from cycle to cycle \Rightarrow violating P.S.S.

$$\Rightarrow \left(\frac{dI_{L_{ON}}}{dt} \right) T_{ON} + \left(\frac{dI_{L_{OFF}}}{dt} \right) T_{OFF} = 0 \quad D = \frac{T_{ON}}{T_{ON} + T_{OFF}}$$

$$\Rightarrow (V_{in} - V_{D_{ON}} - V_{out}) T_{ON} = -(-V_{D_{ON}} - V_{out}) T_{OFF}$$

$$\Rightarrow T_{ON} V_{in} = T_{ON} V_{D_{ON}} + T_{OFF} V_{D_{ON}} + (T_{ON} + T_{OFF}) V_{out}$$

if we further assume that $V_{D_{ON}} \approx V_{D_{OFF}} \approx 0V$

$$\frac{V_{out}}{V_{in}} = \frac{T_{ON}}{T_{ON} + T_{OFF}} \Rightarrow \boxed{\frac{V_{out}}{V_{in}} = \frac{T_{ON}}{T_{total}}}$$

1. Synchronous Buck Converter

* Inductor Volt-Second Balance :

$$V_L(t) = L \frac{di_L(t)}{dt} \Rightarrow \frac{1}{L} \int_0^{T_s} V_L(t) dt = i_L(T_s) - i_L(0)$$

Integrate over one complete switching period.

In P.S.S., the net change in inductor current is zero from cycle to cycle.

$$\Rightarrow 0 = \int_0^{T_s} V_L(t) dt$$

\Rightarrow The total area (or volt-seconds) under the inductor voltage waveform is zero whenever the converter operates in steady state.

$$\Rightarrow \langle V_L \rangle = \frac{1}{T_s} \overbrace{\int_0^{T_s} V_L(t) dt}^0 = 0 \Rightarrow \text{average inductor voltage is zero in P.S.S.}$$

* Capacitor Charge Balance :

integrate over one complete switching period.

$$i_C(t) = C \frac{dV_C(t)}{dt} \Rightarrow \frac{1}{C} \int_0^{T_s} i_C(t) dt = V_C(T_s) - V_C(0)$$

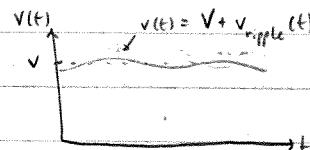
In P.S.S., the net change in capacitor voltage is zero from cycle to cycle.

$$\Rightarrow 0 = \int_0^{T_s} i_C(t) dt$$

\Rightarrow The total area (or charge) under the capacitor current waveform is zero whenever the converter operates in P.S.S.

$$\Rightarrow \langle i_C \rangle = \frac{1}{T_s} \int_0^{T_s} i_C(t) dt = 0 \Rightarrow \text{average capacitor current is zero in P.S.S.}$$

* Small Ripple Approximation

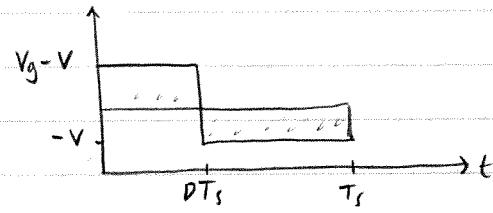


In a well designed converter, the output voltage ripple is small.

\Rightarrow the waveforms can be easily determined by ignoring the ripple.

$$\|v_{\text{ripple}}\| \ll V \Rightarrow v(t) \approx V$$

$$a) \phi_1: V_L = V_g - V = L \frac{d(i_L)}{dt}$$

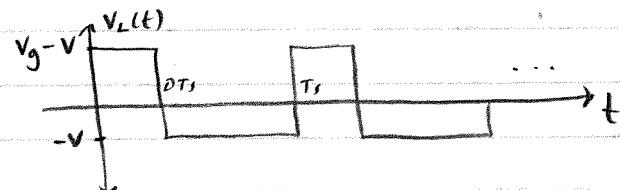


$$\phi_2: V_{L2} = -V = L \frac{d(i_L)}{dt}$$

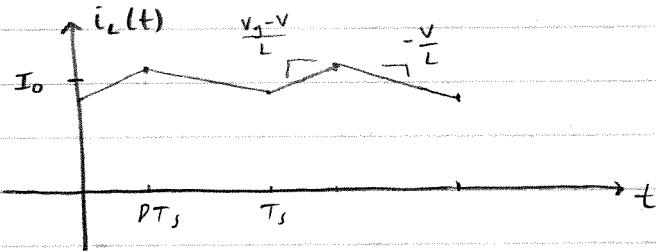
$$\langle V_L \rangle = 0 \Rightarrow (V_g - V) DT_s + (-V)(1-D)T_s = 0$$

$$\Rightarrow DT_s V_g - DT_s V - VT_s + VT_s = 0 \Rightarrow V = DV_g$$

$$b) \phi_1: \frac{d(i_L)}{dt} = \frac{V_g - V}{L}$$



$$\phi_2: \frac{d(i_L)}{dt} = -\frac{V}{L}$$

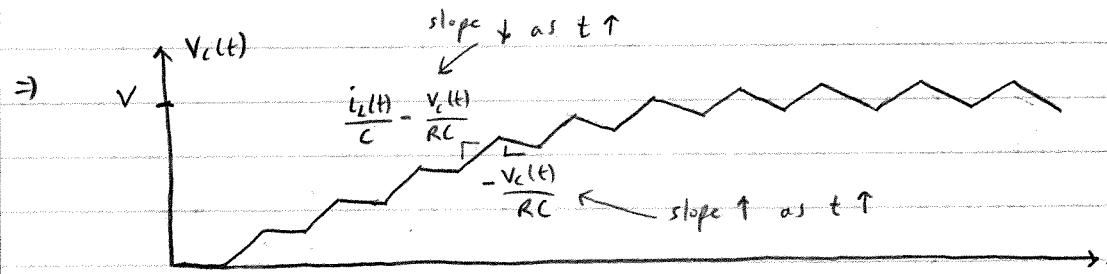


$$2. a) v(t) = v_c(t)$$

$$\phi_1: i_c(t) = -\frac{v_c(t)}{R} = C \frac{dv_c(t)}{dt} \Rightarrow \frac{dv_c(t)}{dt} = -\frac{v_c(t)}{RC}$$

$$\phi_2: i_c(t) = i_L(t) - \frac{v_c(t)}{R} = C \frac{dv_c(t)}{dt} \Rightarrow \frac{dv_c(t)}{dt} = \frac{i_L(t)}{C} - \frac{v_c(t)}{RC}$$

$$v_c(0) = 0V \quad v_c(\text{final}) = V$$



Assume $R_L = 0$

b) $\phi_1: i_{C_1} = C \frac{dV_C}{dt} = -\frac{V}{R}$ $V_{L_1} = V_g$

$\phi_2: i_{C_2} = C \frac{dV_C}{dt} = I_L - \frac{V}{R}$ $V_{L_2} = V_g - V$

$$\Rightarrow V_g D T + (V_g - V)(1-D)T = 0 \quad \text{let } D' = 1-D$$

$$\Rightarrow V_g (D+D') = D'V \Rightarrow V_g = D'V \Rightarrow V = \frac{1}{1-D} V_g$$

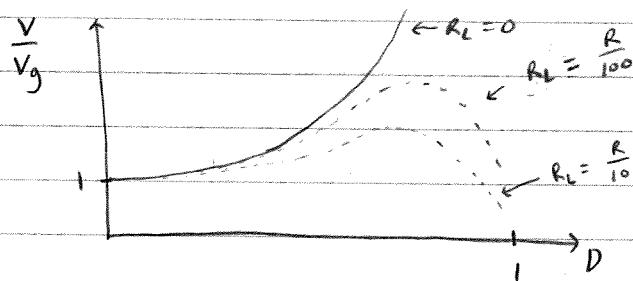
If $R_L \neq 0$,

Not sure how to derive this.

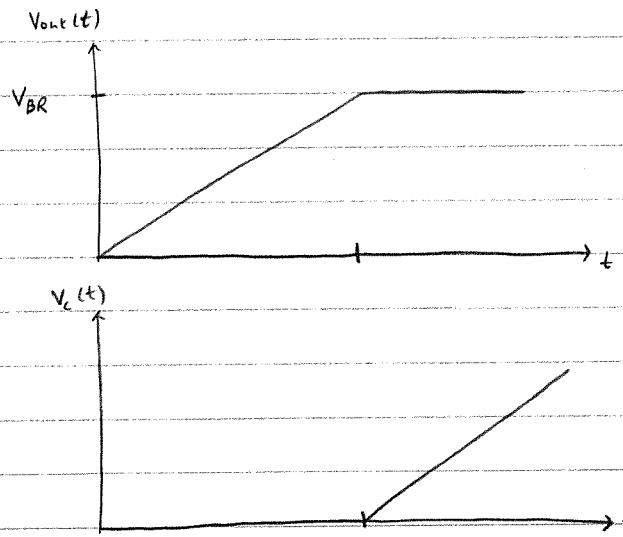
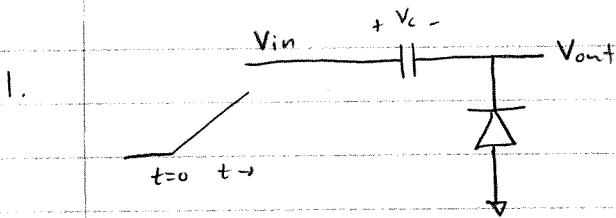
$$V = \frac{1}{\frac{R_L}{R(1-D)} + 1} V_g$$

How is the steady state average output voltage V influenced by R_L and D ?

$$D \uparrow \Rightarrow V \uparrow, \quad R_L \uparrow = V \downarrow$$



2010 - Murmann

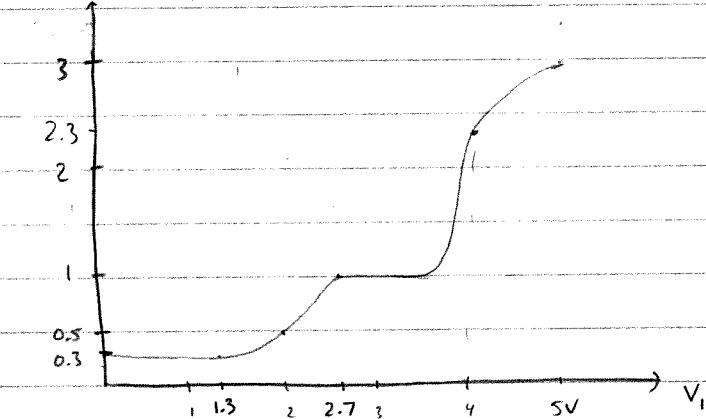


2. for $V_I = 2V$, $I_{d_1} = I_{d_2} = 0.5mA = \frac{1}{2} 4mA/V^2 (2V - V_x - 1V)^2$

$$\Rightarrow 1V - V_x = \sqrt{\frac{1}{4}} \Rightarrow V_x = 1 - \frac{1}{2} = 0.5V$$

for $V_I = 0V$, $1mA = \frac{1}{2} (4mA) (1 - V_x)^2 \Rightarrow V_x = 1 - \frac{1}{\sqrt{2}} \approx 0.3V \Rightarrow M_1 \text{ turns on at } V_I = 1.3V$

for $V_x = 1V$, M_2 is OFF $\Rightarrow 1mA = \frac{1}{2} (4mA) (V_I - 1 - 1)^2 \Rightarrow V_I = 2 + \frac{1}{\sqrt{2}} \approx 2.7$



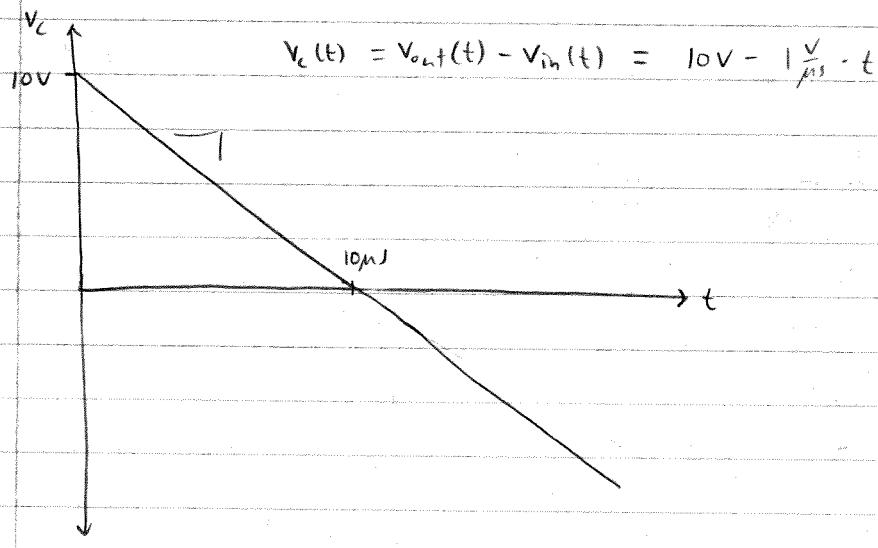
for $V_I = 4V$, $V_D = 5V - 1mA \cdot 2k\Omega = 3V \Rightarrow I_D = 1mA = \frac{1}{2} (4mA) ((3 - V_x) - \frac{1}{2} (3 - V_x)) (3 - V_x)$

$\Rightarrow V_x = 2.3V$. and we know that V_x cannot rise above 3V, since $V_D = 3V$

at $t=0$, M1 is off $\Rightarrow 1mA$ flows through $1k\Omega$

$$\Rightarrow V_{out}(0) = 11V \Rightarrow V_{in}(0) = 1V \text{ since } V_c(0) = 10V$$

$$i = C \frac{dV}{dt} \Rightarrow \frac{dV}{dt} = \frac{i}{C} = \frac{1mA}{1nF} = 1 \frac{V}{\mu s}$$



$$V_{in}(t) = 1V + 1 \frac{V}{\mu s} \cdot t \quad \text{for } 0 < t < 2\mu s$$

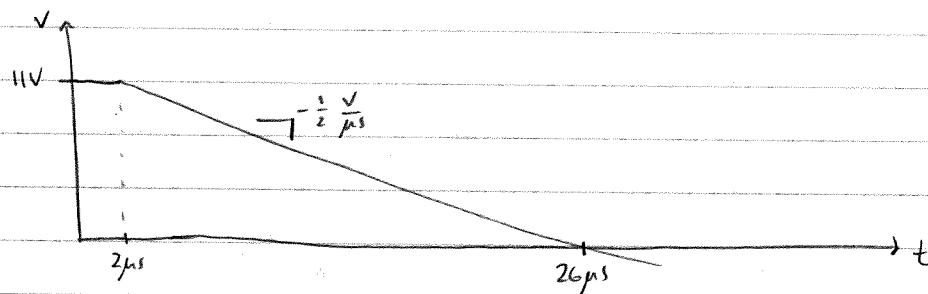
\Rightarrow at $t = 2\mu s$, M1 turns on.

$$V_{out}(t) = 11V \quad 0 < t < 2\mu s$$

$$V_{out}(t) = 11V - (1A \cdot 1k\Omega) = 11 - (V_{in} - 3) = 14V - V_{in}(t) \quad t > 2\mu s$$

$$\Rightarrow V_c(t) = V_{out}(t) - V_{in}(t) = V_{out}(t) - (14V - V_{out}(t)) = 2V_{out}(t) - 14V = 10V - 1 \frac{V}{\mu s} t$$

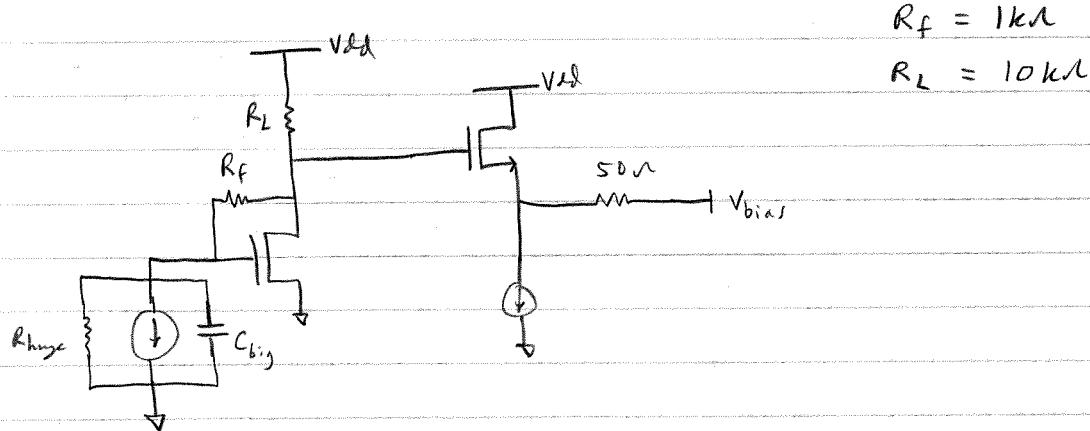
$$\Rightarrow V_{out}(t) = 12V - \frac{1}{2} \frac{V}{\mu s} t \quad \text{for } t > 2\mu s$$



2008 - Murmann

1. $A_v = \frac{V_o}{V_i} = -g_m (2k\text{n} || 2k\text{n} || (2k\text{n} + r_o)) = -g_m (1\text{k}\Omega) = \boxed{-10}$

2. TIA . high R_o . Low R_{in} . $\frac{100\text{mV}}{100\mu\text{A}} = 1\text{k}\Omega$ gain



first order bandwidth expression: $\omega_{3dB} = \frac{1}{R_f \cdot C_{bias}}$

1000 1000

1000 1000

1000

1000

1000

1000

1000

1000

1000

1000

1000

1000

1000

1000

1000

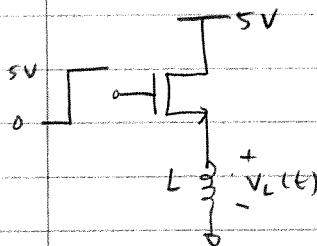
1000

1000

1000

1994 - Wooley

$$V_T = 1V$$



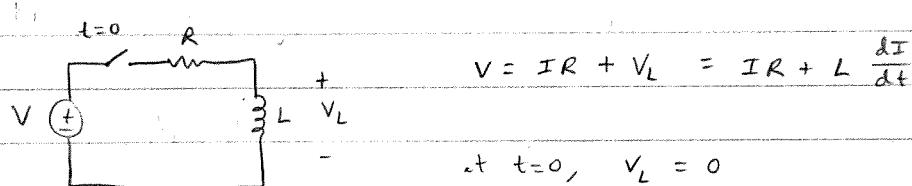
$$V_L(t) = 0 \text{ for } t < 0$$

What happens to $V_L(t)$ for $t > 0$?

$$V_L = L \frac{dI_L}{dt} \Rightarrow \text{current through inductor cannot change instantaneously.}$$

M1 must always be in saturation or cutoff for $t > 0$, since $V_g = V_d = 5V$

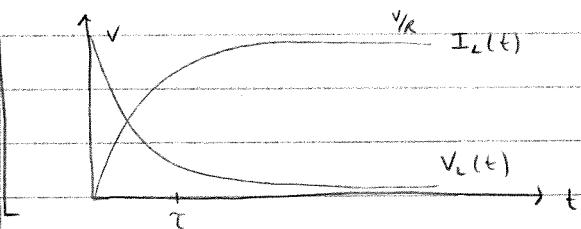
Consider the case of a basic LR circuit?



$$V = IR + V_L = IR + L \frac{dI}{dt}$$

$$\text{at } t=0, V_L = 0$$

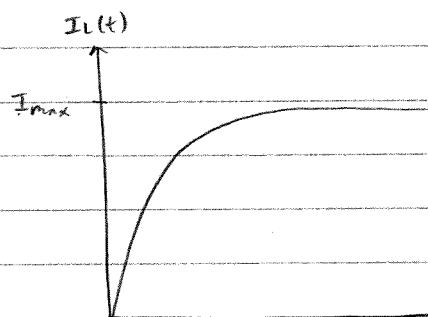
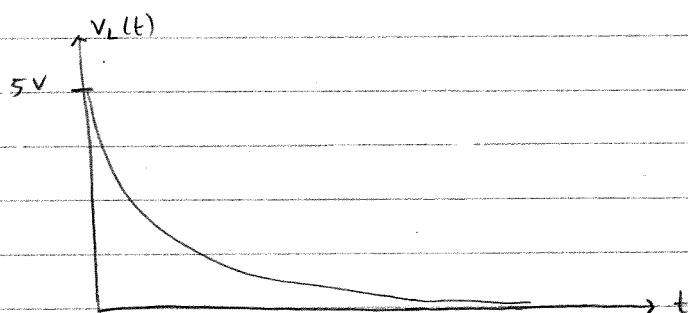
$$\Rightarrow I_L(t) = \frac{V}{R} \left(1 - e^{-t/\tau}\right) \quad \text{where } \tau = \frac{L}{R}$$



Initial conditions: $V_L(0^-) = 0V$ $V_L(0^+) = L \frac{dI}{dt} = 5V$ if we assume a perfect step input ($\frac{dI}{dt} \rightarrow \infty$)
 $I_L(0) = 0A$

$$\text{Steady State: } I_L(t) \Big|_{t \rightarrow \infty} = I_d = \frac{1}{2} \mu_0 k \frac{V}{L} (5V - 1V)^2$$

$$V_L(t) = L \frac{dI}{dt} = 0V$$



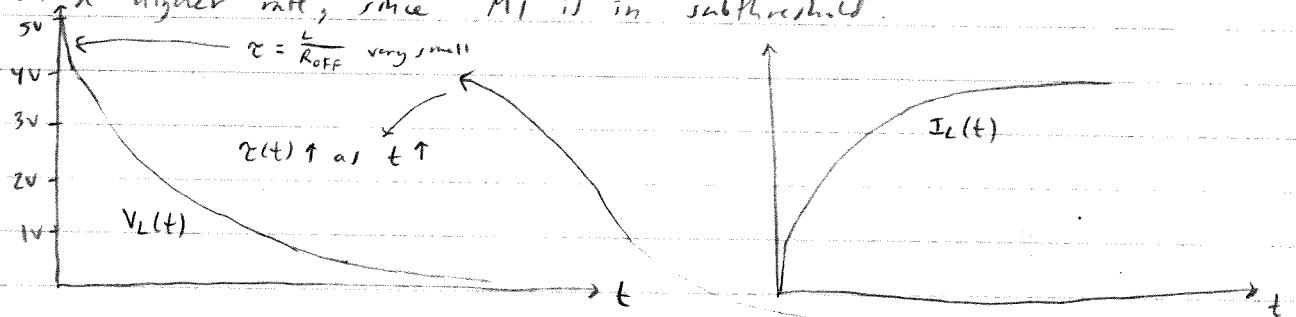
for $4V < V_L(t) < 5V$, M1 is in subthreshold

$$\Rightarrow I_d = I_0 e^{\frac{V_{gs} - 3V_T}{\gamma}} \quad \text{where } V_T = \frac{kT}{q} + \gamma > 1 \quad V_{gs} = 5 - V_L(t)$$

for $4 < V_L(t) < 5V$, M1 is in saturation

$$\Rightarrow I_d = \frac{1}{2} \mu_0 \times \frac{W}{L} (V_{gs} - V_T)^2 \quad V_{gs} - V_T = 5 - V_L(t) - 1 = 4 - V_L(t)$$

\Rightarrow initially ($4V < V_L(t) < 5V$), we might expect current to increase at a higher rate, and voltage $V_L(t)$ to decrease at a higher rate, since M1 is in subthreshold.



We expect L to decay from $5V$ to $0V$ with a time dependent $\frac{L}{R}$ time constant $\tau(t) = \frac{L}{R} = \frac{L}{(g_m R)} = g_m(t) L$

Initially: Subthreshold: $g_m = \frac{I_d}{3V_T}$ where $\gamma > 1$. This is very large g_m ??

then: Saturation: $g_m = \sqrt{\mu_0 \times \frac{W}{L} (2I_d)}$ $I_d \uparrow$ as $t \uparrow$

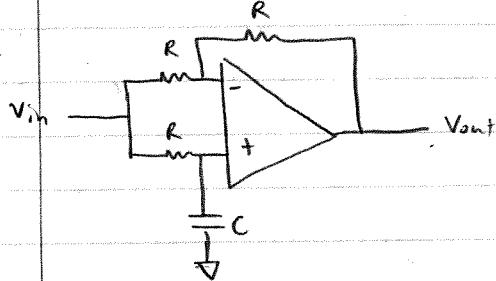
$$\Rightarrow g_m \uparrow \propto t \uparrow \Rightarrow \tau(t) = g_m L \uparrow \text{as } t \uparrow$$

However in our simulations, we see that τ is shortest in subthreshold, which disagrees with our $\frac{1}{g_m}$ assumption. Perhaps this just means that the $\frac{1}{g_m}$ assumption only applies for SAT, not in sub. R_{0FF} is very large. This makes intuitive sense. $\Rightarrow \frac{L}{R_{0FF}}$ should be very small.

In case this small signal model of $\kappa = \frac{1}{g_m}$ does not apply, we could make a physical argument that as $V_L \downarrow$, $V_{gs} \uparrow$ and thus channel inversion becomes stronger \Rightarrow more charge in the channel $\Rightarrow R_{channel} \downarrow$.

$$\frac{1}{1+j\omega RC} = \frac{1-j\omega RC}{1+\omega^2 R^2 C^2}$$

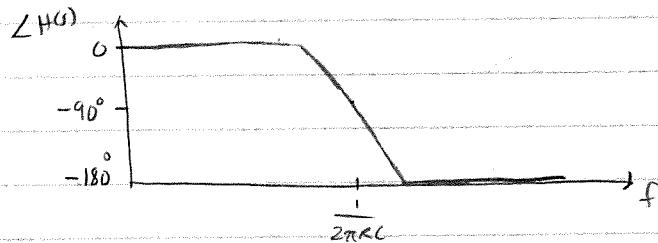
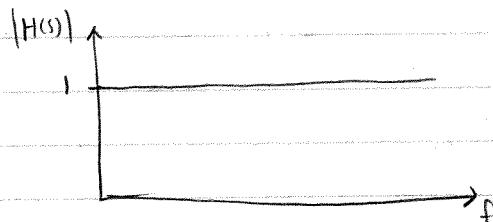
2001 - Lee



$$H(s) = \frac{1-sRC}{1+sRC}$$

$$|H(s)| = \frac{\sqrt{1^2 + \omega^2 R^2 C^2}}{\sqrt{1^2 + \omega^2 R^2 C^2}} = 1$$

$$\begin{aligned}\angle H(j) &= \angle(1-sRC) + \angle(1+sRC) \\ &= \angle(1-sRC) + \angle(1-sRC) \\ &= \tan^{-1}(-\frac{\omega RC}{1}) + \tan^{-1}(-\frac{\omega RC}{1}) = 2\tan^{-1}(-\omega RC)\end{aligned}$$



Response to step input: Laplace transform of $v(t)$ $\xrightarrow{\text{Laplace}}$ $H(s)$

can compute inverse Laplace transform of $\frac{1}{s} \left(\frac{1-sRC}{1+sRC} \right)$

OR, could recognize that all single pole step responses have the form:

$$s(t) = A + B e^{-t/\tau} \quad \text{where } \tau = \frac{1}{\omega p}$$

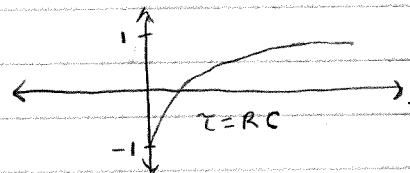
To find initial + final conditions, consider the circuit for $s=\infty$ and $s=0$:

at $t=0$, $s \rightarrow \infty \Rightarrow \frac{1}{sC} \rightarrow 0 \Rightarrow$ circuit looks like inverting amp $\Rightarrow \frac{V_{out}}{V_{in}} = -1 \Rightarrow V_{out}(0) = -1$

at $t \rightarrow \infty$, $s \rightarrow 0 \Rightarrow \frac{1}{sC} \rightarrow \infty \Rightarrow V_+ \text{ is open, } \Rightarrow \text{no current flow through my resistors.} \Rightarrow V_{out} = V_{in} \Rightarrow \frac{V_{out}}{V_{in}} = 1 \Rightarrow V_{out}(\infty) = 1$

$$\Rightarrow s(0) = A = 1 \quad s(\infty) = A + B = -1 \Rightarrow B = -2$$

$$\Rightarrow s(t) = 1 - 2e^{-t/\tau}$$



2005 - Woolley

1. Identical BJTs. Large β_0 & $r_o \Rightarrow I_c$ is independent of V_{ce} .
Depends only on V_{be} .

$$V_{be} \approx 0.7V \quad V_c = 0V \text{ at } t=0 \quad V_s = V_{be} + V_c$$

$$\Rightarrow \text{at } t=0, V_s = 0.7V$$

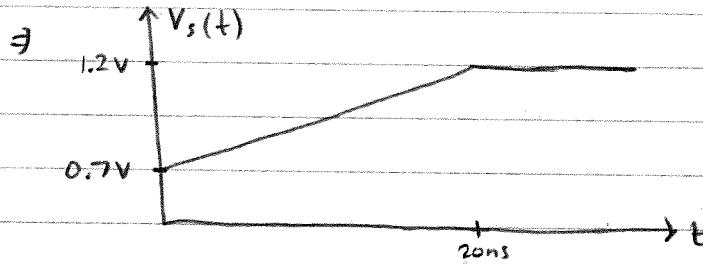
Since the bases of all four transistors are connected, they must each pull an equal current I_c .

$$\Rightarrow \text{at } t=0, I_{c1} = I_{c2} = I_{c3} = I_{c4} = 1mA,$$

$$\text{for } 0 < t < 20ns, I_{c1} = I_{c2} = I_{c3} = I_{c4} = 1.25mA$$

$$\Rightarrow i_c = I_{c1} - 1mA = 0.25mA \quad i_c = C \frac{dV}{dt} \Rightarrow \frac{dV}{dt} = \frac{i}{C}$$

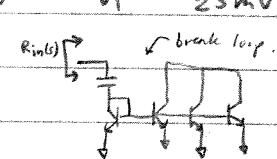
$$\Rightarrow V_c(t) = \frac{i}{C} \cdot t = \frac{0.25mA}{10\mu F} \cdot t \Rightarrow V_c(20ns) = \frac{0.25mA}{10\mu F} \cdot 20ns = 0.5V$$



$$g_m = \frac{I_c}{V_T} \approx \frac{1.25mA}{25mV}$$

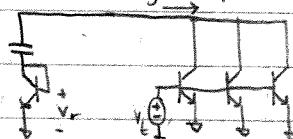
Note: There is negative shunt-shunt feedback

2. Small signal input impedance $|R_{in}(s)| = \left| \frac{1}{sC + \frac{1}{g_m}} \right|_{open\ loop}$



$$\Rightarrow |R_{in}(s)| \approx \frac{1}{sC} + \frac{1}{g_m} \approx \frac{1}{s(10\mu F)} + \frac{25mV}{1.25mA} \approx \frac{1}{s(10\mu F)} + 20\Omega$$

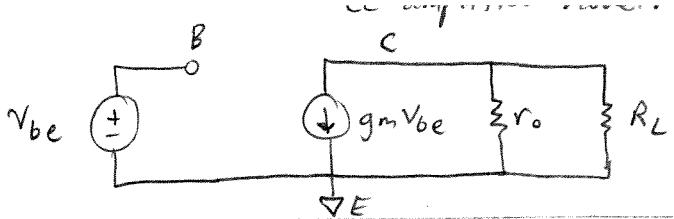
Find loop gain via RR analysis:



$$V_r = -3g_m V_{test} \left(\frac{1}{g_m} \right) = -3V_{test}$$

$$T = RR = -\frac{V_r}{V_e} = 3 \quad \text{Shunt-shunt feedback} \Rightarrow R_{in} \text{ is reduced by } 1+T = 4$$

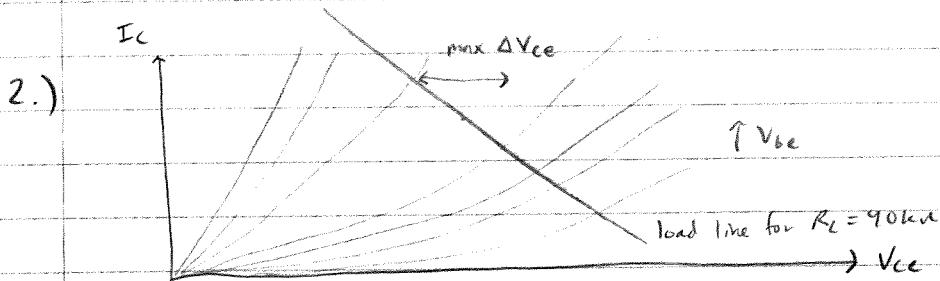
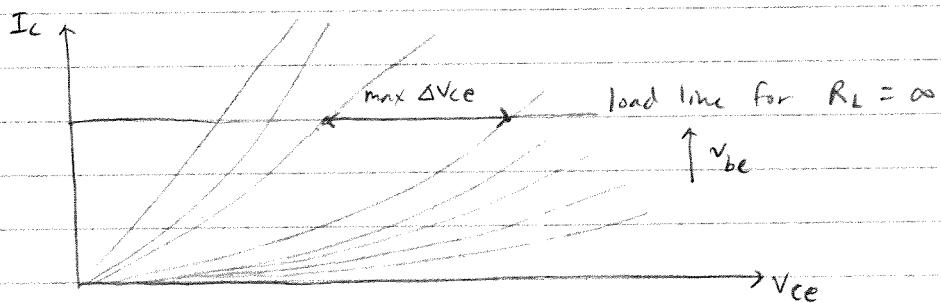
$$\Rightarrow |R_{in}(s)| = \frac{1}{4} \left(\frac{1}{sC} + \frac{1}{g_m} \right) = \frac{1}{s(40\mu F)} + 5\Omega$$



1995 - Lee

$$1.) \quad g_m = \frac{\partial i_c}{\partial v_{be}} ; \quad r_o = \frac{\partial v_{ce}}{\partial i_c}$$

$$\Rightarrow g_m r_o = \frac{\partial v_{ce}}{\partial v_{be}} \approx \frac{\Delta v_{ce}}{\Delta v_{be}} = \frac{6V}{0.2V} = 30$$



Load line is ~ perpendicular to IV curves at operating point $\Rightarrow r_o \approx R_L \Rightarrow g_m(r_o || R_L) \approx \frac{1}{2} g_m r_o = 15$

2002 - Lee

$$1. V_{out} = \frac{R_2}{R_1+R_2} V_{in}$$

2. Yes R_1 can have a negative value. But if R_1 's magnitude exceeds that of R_2 , the net resistance seen by the driving source (R_1+R_2) is negative. Depending on the nature of the driving impedance (R_s , parallel L and C) this negative resistance can result in instability + oscillation. If we ignore this possibility,

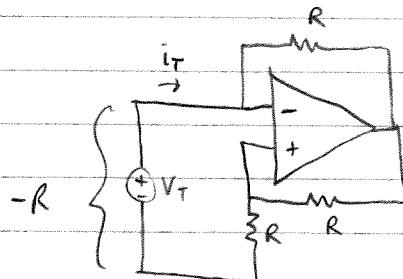
$$V_{out} = \frac{R_2}{R_1+R_2} V_{in} = -K V_{in} \text{ where } K \text{ is a constant.}$$

\Rightarrow the network produces an inversion in signal polarities.

3. Can a negative resistor be physically realized as an element with a total of no more than two terminals?
 Explain. [Clarification: There is a completely sealed black box, out of which only two wires emerge. Can there appear between those two terminals a negative resistance that functions forever?]

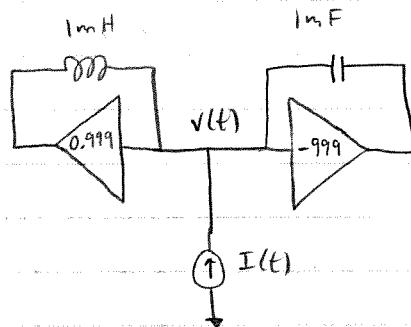
A negative resistor can supply energy ($P = I(-R) < 0$).
 \Rightarrow there needs to be an energy source somewhere.
 Since it's not allowed to be in the box, then more than two terminals are needed.

e.g. rail voltages needed for opamp implementation:

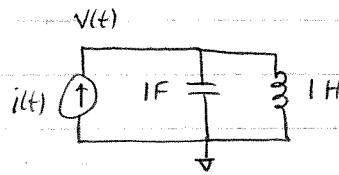


$$\begin{aligned} (V_T - i_T R) \frac{R}{R+R} &= V_T \\ \Rightarrow 2V_T &= V_T - i_T R \\ \Rightarrow V_T &= -i_T R \\ \Rightarrow \frac{V_T}{i_T} &= -R \end{aligned}$$

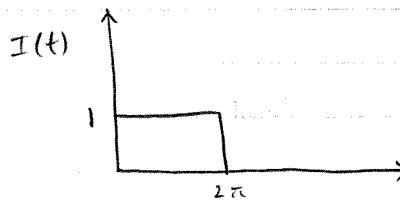
1997 - Lee



by Miller



$$Z(s) = \frac{V(s)}{I(s)} = \frac{\frac{1}{sc} \cdot sL}{\frac{1}{sc} + sL} = \frac{sL}{1 + s^2 LC} = \frac{s}{1 + s^2}$$



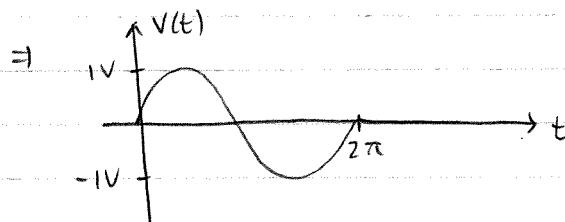
$$\Rightarrow I(t) = u(t) - u(t - 2\pi)$$

$$\Rightarrow I(s) = \frac{1}{s} - e^{-2\pi s} \frac{1}{s} = (1 - e^{-2\pi s}) \frac{1}{s}$$

$$\Rightarrow V(s) = I(s) Z(s) = \frac{1}{1+s^2} = \frac{e^{-2\pi s}}{1+s^2}$$

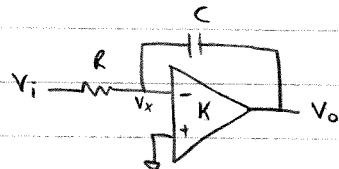
\downarrow
0 for $0 < t < 2\pi$

$$\Rightarrow V(t) = \sin(t) - \sin(t - 2\pi)$$



2000 - Lee

1.



Brute force Method:

$$\frac{V_i - V_x}{R} + \frac{V_o - V_x}{\frac{1}{sC}} = 0 \quad V_o = -K V_x$$

$$\frac{V_i - V_x}{R} = \frac{(K+1)V_x}{\frac{1}{sC}} \Rightarrow \frac{V_i}{R} = \left(\frac{(K+1)}{\frac{1}{sC}} + \frac{1}{R} \right) V_x = \left(\frac{K+1}{\frac{1}{sC}} + \frac{1}{R} \right) \left(-\frac{V_o}{K} \right)$$

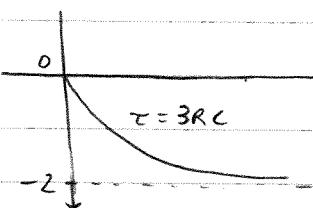
$$\Rightarrow \frac{V_o}{V_i} = -\frac{K}{R} \left(\frac{K+1}{\frac{1}{sC}} + \frac{1}{R} \right)^{-1} = \left(\frac{R(K+1) + \frac{1}{sC}}{\frac{1}{sC} \cdot R} \right)^{-1} \left(-\frac{K}{R} \right) = -\frac{K}{1 + sRC(K+1)}$$

$$\frac{V_o}{V_i} = -\frac{2}{1 + 3sRC} = H(s) \quad u(t) \Leftrightarrow \frac{1}{s}$$

$$\Rightarrow V_o(s) = H(s) \frac{1}{s} = -\frac{2}{s(1 + 3sRC)} = \frac{c_1}{s} + \frac{c_2}{(1 + 3sRC)}$$

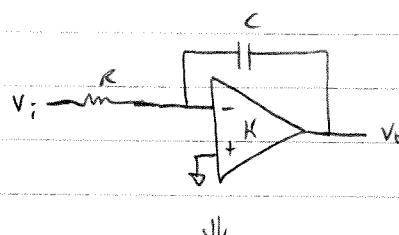
$$c_1 = -2 \quad c_2 = 6RC \quad \Rightarrow V_o(s) = -\frac{2}{s} + \frac{6RC}{1 + 3sRC} = -\frac{2}{s} + \frac{2}{3RC + s}$$

$$\Rightarrow V_o(t) = -2u(t) + 2e^{-t/3RC}$$



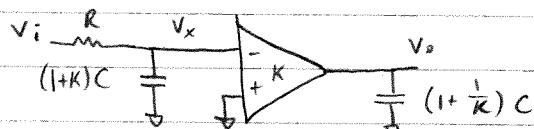
Intuitive Method: Using Miller:

$$\frac{V_x}{V_i} = \frac{\frac{1}{s(3C)}}{\frac{1}{s(3C)} + R} = \frac{1}{1 + 3sRC}$$



$$V_o = -KV_x = -2V_x \Rightarrow \frac{V_o}{V_i} = \frac{V_o}{V_x} \frac{V_x}{V_i} = \frac{V_o}{V_x} \frac{V_x}{V_i}$$

$$= -\frac{2}{1 + 3sRC} \quad \checkmark$$

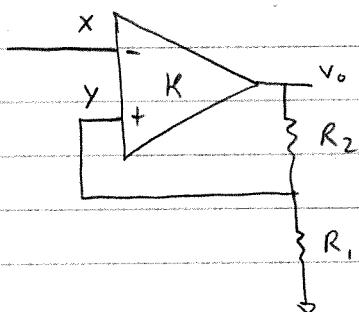


for step response, this is a one pole system,

The final value is DC = cap is open $\Rightarrow V_x = V_i \Rightarrow V_o = -2V_i = -2V$

The initial value is 0 since cap is shorted. Time constant is $3RC = \tau$.

2.



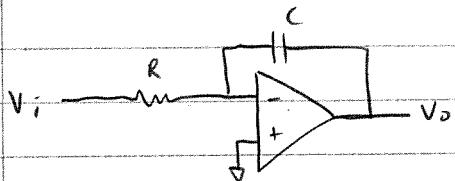
$$\frac{R_1}{R_1 + R_2} = \frac{1}{K}$$

$$\Rightarrow V_y = \frac{V_o}{K}$$

$$V_o = K(V_y - V_x)$$

$$\Rightarrow V_o(1 - \frac{1}{K}) = -K V_x$$

$$\Rightarrow \frac{V_o}{V_x} = -\frac{K}{(1-1)} = -\infty \Rightarrow \text{positive feedback boosts gain to } \infty.$$

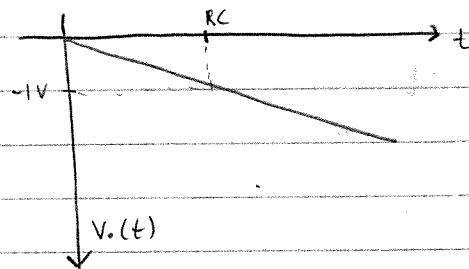


Brute force Method :

Integration with factor of $-\frac{1}{RC}$

$$\frac{V_i}{R} = -\frac{V_o}{sC} \Rightarrow \frac{V_o}{V_i} = -\frac{1}{SRC} = H(s)$$

$$V_o(s) = -\frac{1}{s^2 RC} \Rightarrow V_o(t) = -\frac{1}{RC} t = -\frac{t}{RC}$$



Another Method : use expression from Q1 and let K → -∞

$$\Rightarrow \left. \frac{V_o}{V_i} \right|_{K \rightarrow -\infty} = -\frac{K}{1 + (K+1)SRC} \Big|_{K \rightarrow -\infty} = -\frac{1}{SRC}$$

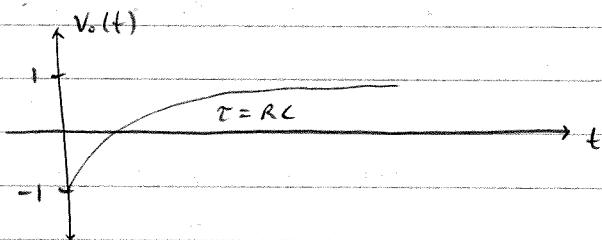
2001 - Lee (Continued)

$$V_o(s) = \frac{1}{s} \left(\frac{1-sRC}{1+sRC} \right) = \frac{1-sRC}{s(1+sRC)} = \frac{c_1}{s} + \frac{c_2}{1+sRC}$$

$$c_1 = 1 \quad c_2 = \frac{1+1}{-1/RC} = -2RC$$

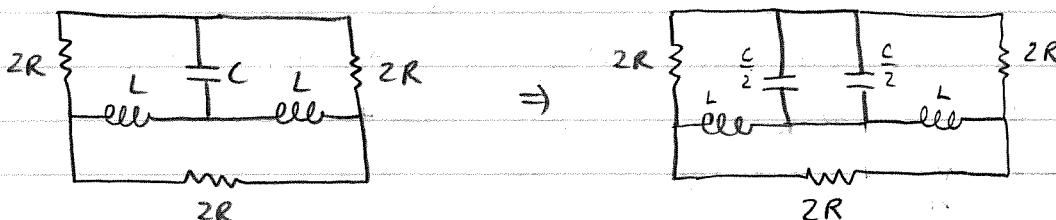
$$\Rightarrow V_o(s) = \frac{1}{s} - \frac{2RC}{1+sRC} = \frac{1}{s} - \frac{2}{\frac{1}{RC} + s}$$

$$\Rightarrow V(t) = u(t) - 2e^{-t/RC} = 1 - 2e^{-t/RC}$$



2004 - Lee

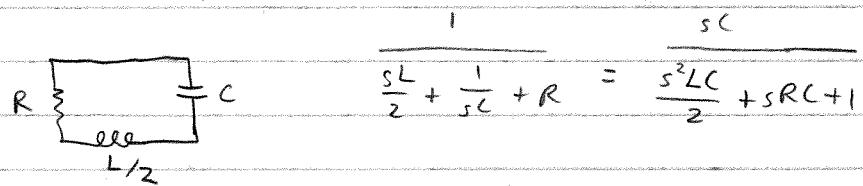
a)



Because we can specify the initial energies of the three energy storage elements independently (the initial inductor currents and capacitor voltage can all be chosen without conflict), there are three poles (three degrees of freedom).

- b) Symmetry enables simplified analysis. Suppose, for example we choose initial inductor currents of equal magnitude both flowing toward their common connection with the capacitor.

By symmetry, the left and right halves of the circuit behave the same way. The voltage across the bottom resistor thus remains zero, so we can take it out of the network and we may also fold the left half onto the right half. Then the network becomes:



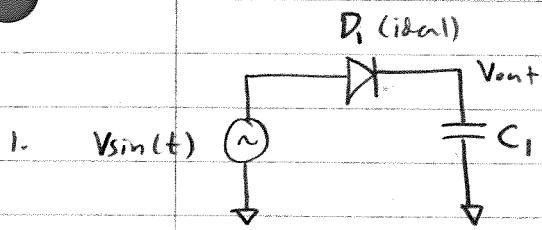
This second order network has poles at the roots of $\frac{s^2LC}{2} + sRC + 1$

$$\Rightarrow P_{1,2} = \frac{-RC \pm \sqrt{R^2C^2 - 2LC}}{LC} = -\frac{R}{L} \pm \sqrt{\frac{R^2}{L^2} - \frac{2}{LC}}$$

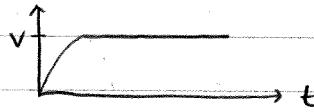
Now try a differential mode initial condition, using initial inductor currents that are equal in magnitude, but oriented so that one flows into the capacitor and the other out of it. This permits us to short across the capacitor (it never charges) together with the center of the bottom resistor. That decomposes the network into a simple LR network whose time constant is $\frac{3}{2} \frac{L}{R}$. The pole frequency is minus the reciprocal or

$$P_3 = -\frac{2}{3} \frac{R}{L}$$

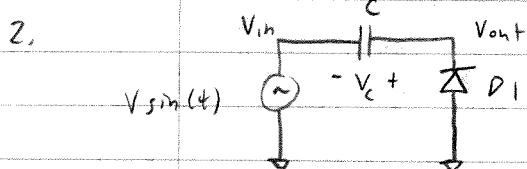
2005 - Lee



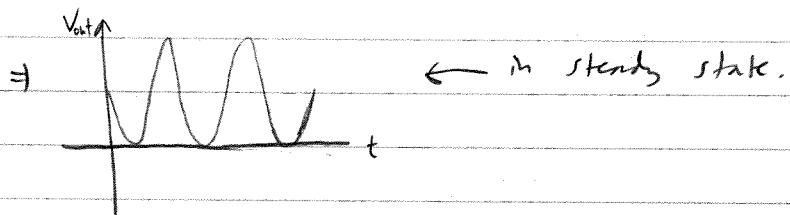
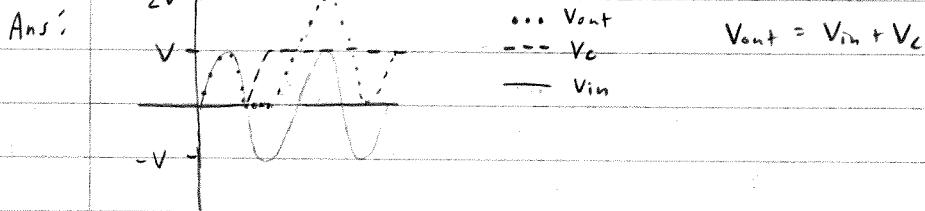
What is the steady state output voltage?



Ans: The diode charges C_1 up to V in the first quarter cycle. There is nothing to discharge the capacitor, so the output voltage stays at V forever. This circuit is a filtered halfwave rectifier.

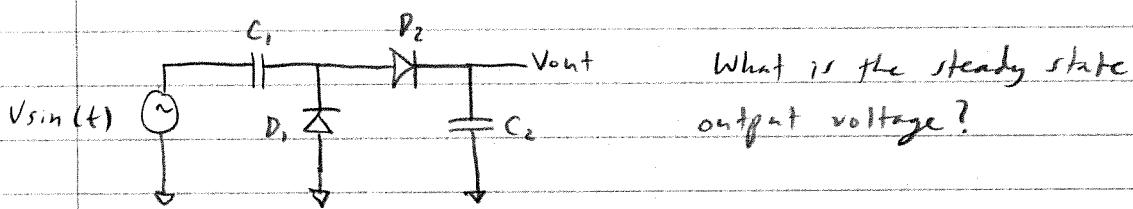


Sketch the output voltage in steady state.



The same three elements remain in series. The capacitor here charges up to V in the first negative quarter cycle, and stays charged forever. Note that the polarity of the capacitor's voltage is such that it adds V to the sinusoidal drive. The output is thus still a sinusoid, just shifted upward by V , swinging between ground and $2V$. This circuit is a level shifter (or clamp).

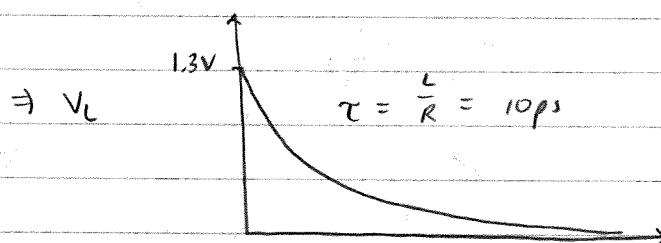
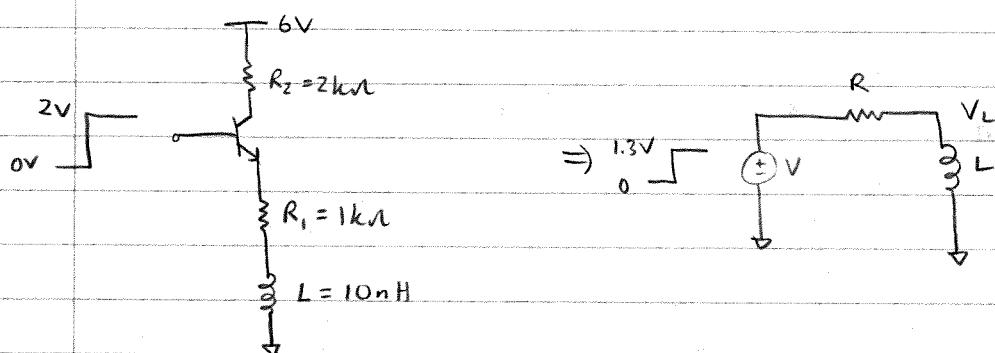
3.



Ans: 2V, On each negative going quarter cycle of the sinewave drive, the input capacitor charges up to V (left diode on, right diode off). On each positive going edge, some stored charge transfers to the output capacitor. Again, with nothing to discharge the output capacitor, this charge pumping continues until the output voltage is $2V$ (the peak output value of the previous circuit). This circuit is thus a voltage doubler.

If Woolley asks if you're done, say yes.
Otherwise he won't give you the second
part of the problem and you'll get a low score.

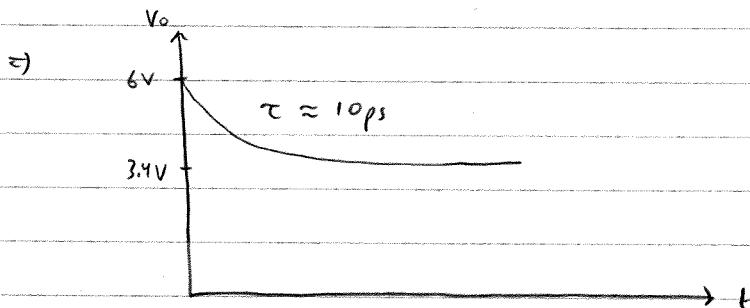
2002 - Woolley



initially, BJT is off $\Rightarrow V_o \text{ initial} = 6\text{V}$

at steady state, $V_o \sim 2 - 0.7 = 1.3\text{V}$ $V_L = 0$

$$\Rightarrow I = \frac{1.3\text{V}}{1\text{k}\Omega} = 1.3\text{mA} \Rightarrow V_o = 6\text{V} - 1.3 \cdot 2 = 3.4\text{V}$$

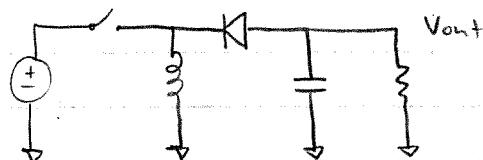


We expect something like this.

In reality when we simulated this circuit the τ was actually around 4ps.

2003 - Lee

1. $V_{in DC}$



Buck-Boost

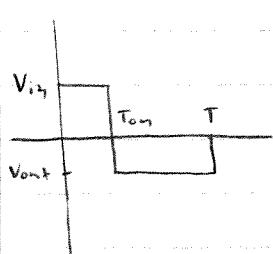
Calculate the steady state DC ratio $\frac{V_{out}}{V_{in}}$ if the switch

periodically closes for T_{on} then opens every T seconds.

All elements are ideal \Rightarrow zero diode drop, capacitor

filters perfectly and the loading by the resistor may be neglected. Assume inductor current is never zero

(\Rightarrow continuous mode). Hint $\langle I_L \rangle = 0$



$$V_{in} T_{on} + V_{out} T' = 0 \Rightarrow \frac{V_{out}}{V_{in}} = -\frac{T_{on}}{T'}$$

$$\text{let } D = \frac{T_{on}}{T} \Rightarrow \frac{V_{out}}{V_{in}} = -\frac{T_{on}}{T-T_{on}} \Rightarrow \frac{V_{out}}{V_{in}} = -\frac{D}{1-D}$$

range is 0 to $-\infty$.

2.

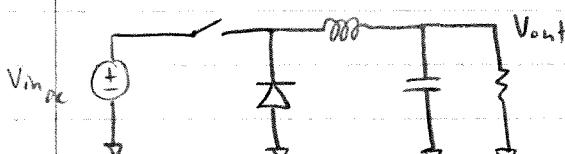


Boost

$$V_{in} T_{on} + (V_{in} - V_{out}) T' = 0$$

$$\Rightarrow V_{in} (T_{on} + T') = V_{out} T' \Rightarrow \frac{V_{out}}{V_{in}} = \frac{T_{on} + T'}{T'}$$

$$= \frac{T}{T-T_{on}} = \frac{1}{1-\frac{T_{on}}{T}} = \frac{1}{1-D} \Rightarrow \frac{V_{out}}{V_{in}} = \frac{1}{1-D} \quad \leftarrow \text{range is } 1 \text{ to } \infty$$



Buck

$$(V_{in} - V_{out}) T_{on} - V_{out} T' = 0$$

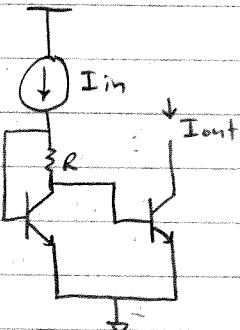
$$\Rightarrow V_{in} T_{on} = V_{out} (T' + T_{on}) = V_{out} (T)$$

$$\Rightarrow \frac{V_{out}}{V_{in}} = \frac{T_{on}}{T} \Rightarrow \frac{V_{out}}{V_{in}} = D \quad \leftarrow \text{range is } 0 \text{ to } 1$$

2007 - Lee

Plot the output current as a function of input current.

Label any features of relevance.



$$I_{in}R = V_{be2} - V_{be1}$$

$$I_{in} = I_o e^{V_{be2}/V_T} \quad I_{out} = I_o e^{V_{be1}/V_T}$$

$$\Rightarrow \frac{I_{in}}{I_{out}} = e^{(V_{be2} - V_{be1})/V_T} \Rightarrow V_{be2} - V_{be1} = V_T \ln \left(\frac{I_{in}}{I_{out}} \right)$$

$$= I_{in}R \Rightarrow \frac{I_{in}}{I_{out}} = e^{I_{in}R/V_T}$$

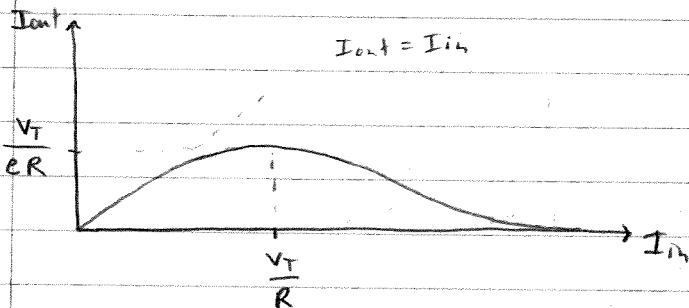
$$\Rightarrow I_{out} = I_{in} e^{-I_{in}R/V_T} \quad \Rightarrow I_{out} \approx I_{in} \text{ for small } I_{in}$$

$$I_{out} \approx 0 \text{ for large } I_{in}$$

$$\frac{\partial I_{out}}{\partial I_{in}} = I_{in} \left(-\frac{R}{V_T} e^{-I_{in}R/V_T} \right) + e^{-I_{in}R/V_T} = 0$$

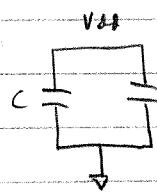
$$\Rightarrow -I_{in} \frac{R}{V_T} + 1 = 0 \Rightarrow I_{in} = \frac{V_T}{R} \quad \leftarrow \text{slope} = 0 \text{ at this point} \Rightarrow \max \text{ of } I_{out} \text{ vs. } I_{in}$$

$$\text{At this point, } I_{out} = \frac{V_T}{R} e^{-\left(\frac{V_T}{R}\right)\left(\frac{R}{V_T}\right)} = \frac{V_T}{eR}$$

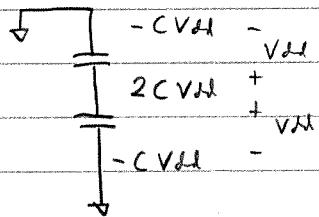


Wong 2010

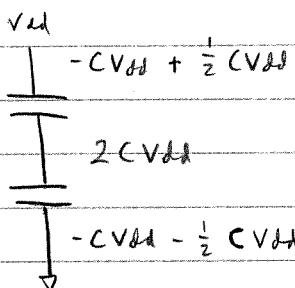
1.



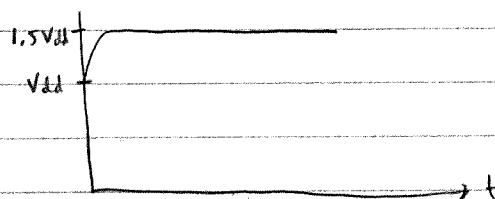
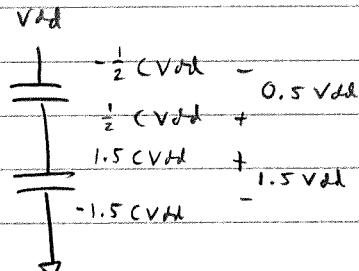
initial



final :



=



$$2. \quad E_i = \frac{1}{2} (2C) V_{dd}^2 = CV_{dd}^2 \quad E_f = \frac{1}{2} C \left(\frac{V_{dd}}{2}\right)^2 + \frac{1}{2} C \left(\frac{3V_{dd}}{2}\right)^2 = 1.25 CV_{dd}^2$$

$$3. \quad \text{if } C_{left} \ll C_{right}, \quad V_{out_{max}} = 2V_{dd}$$

4. ??

OR : 1) charge trapped on center node

$$2) \quad C = \frac{Q}{V_{pp}} \Rightarrow Q = CV_{pp}$$

$$3) \quad \text{total } Q \text{ on center node} = 2CV_{pp}$$

4) charge redistributes.

$$5), \quad Q_1 + Q_2 = 2CV_{dd}$$

$$C_1 = \frac{Q_1}{V_{out}' - V_{pp}} \quad C_2 = \frac{Q_2}{V_{out}' - 0}$$

$$Q_1 = C_1 (V_{out}' - V_{pp}) \quad Q_2 = C_2 V_{out}'$$

$$\Rightarrow C_1 (V_{out}' - V_{pp}) + C_2 V_{out}' = 2CV_{dd}$$

$$\Rightarrow V_{out}' (C_1 + C_2) = 3CV_{dd} \Rightarrow V_{out}' = \frac{3C}{2C} V_{dd} = 1.5V_{dd}$$

2005 - Dutton

1.
 - 1) Gate delay
 - 2) Gate pitch
 - 3) I_{ON}/I_{OFF}

2. In general, Scaling means $V_t \downarrow \Rightarrow V_{DD} \downarrow$ and also, there are increased short channel effects.

$$f_T = \frac{g_m}{C_{gg}} \approx \frac{3}{2} \frac{\mu V_{ov}}{L^2} \Rightarrow \text{as } L \downarrow, f_T \uparrow \text{ good.}$$

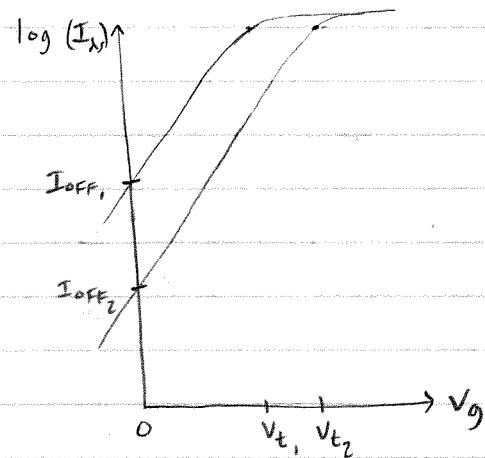
as $V_{DD} \downarrow$, voltage swing is reduced. Cascaded stages become less attractive.

Simple square law models cannot be used. Better to use a g_m/I_S approach.

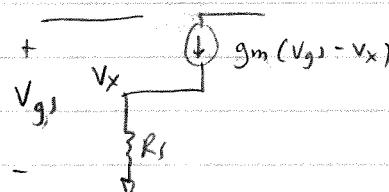
2009 - Dutton

1. Do lower V_t devices have lower "off currents" (for $V_{gs} = 0V$)?

No. Minimum S is 60mV/decade. \Rightarrow the lower the V_t , the higher the "off current" for a given on currents



2. MOS



$$V_x = g_m (V_{gs} - V_x) R_s$$

$$V_x (1 + g_m R_s) = g_m R_s V_{gs}$$

$$V_x = \frac{g_m R_s}{1 + g_m R_s} V_{gs}$$

$$\Rightarrow i_d = g_m V_{gs} \left(1 - \frac{g_m R_s}{1 + g_m R_s} \right) = \frac{g_m}{1 + g_m R_s} V_{gs}$$

$$\Rightarrow G_m = \frac{g_m}{1 + g_m R_s}$$



$$V_x = \left(1 + \frac{1}{\beta} \right) g_m (V_{be} - V_x) R_E$$

$$\Rightarrow V_x = \frac{\left(1 + \frac{1}{\beta} \right) g_m R_E V_{be}}{1 + \left(1 + \frac{1}{\beta} \right) g_m R_E}$$

$$g_m (V_{be} - V_x) = g_m \left(\frac{V_{be}}{1 + \left(1 + \frac{1}{\beta} \right) g_m R_E} \right)$$

$$\Rightarrow G_m = \frac{g_m}{1 + \left(1 + \frac{1}{\beta} \right) g_m R_E}$$



BJT gm is degraded slightly more since

there is base current (vs. no gate current for MOS).

3. No. Intrinsic gate delay is the RC product τ of a MOSFET inverter driving its own gate capacitance.

Gate delay depends on R as well as C , and $R \propto \mu$
⇒ mobility matters. $\Rightarrow \boxed{F}$

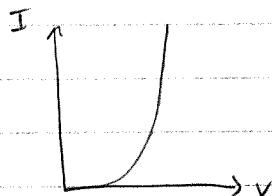
a) Yes.

b) Bias it in far right region by placing a resistive load \parallel current source at the output & choosing $V_{control}$ such that the operating point is at point O .

c) Limitations are finite output resistance ($\frac{\delta I_{out}}{\delta V_{out}} \neq 0$),
Limited output swing to keep the gain high (must stay in far right region). And high voltage operating point.

2010 - Dutton

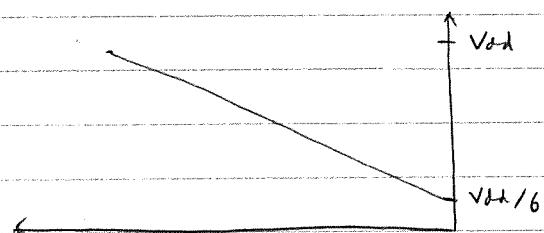
A)



pN diode, forward biased,

B)

JFET as $V_{in} \downarrow$, depletion region grows \Rightarrow channel narrows $\Rightarrow R_{dr} \uparrow$
 $\Rightarrow V_{out} \uparrow$



2000 - Wooley

$$V_T = 1V \quad \mu C_{ox} = 100 \mu A/V^2 \quad \frac{W}{L} = 5 \quad A = 20 \quad R_m = ?$$

$$\frac{1}{2} \mu C_{ox} \frac{W}{L} (V_{gs} - V_T)^2 = 1mA \Rightarrow 0.25mA (V_{gs} - 1V)^2 = 1mA$$

$$\Rightarrow V_{gs} - 1V = \sqrt{4} = 2V \Rightarrow V_{gs} = 3V, \quad V_{ov} = 2V$$

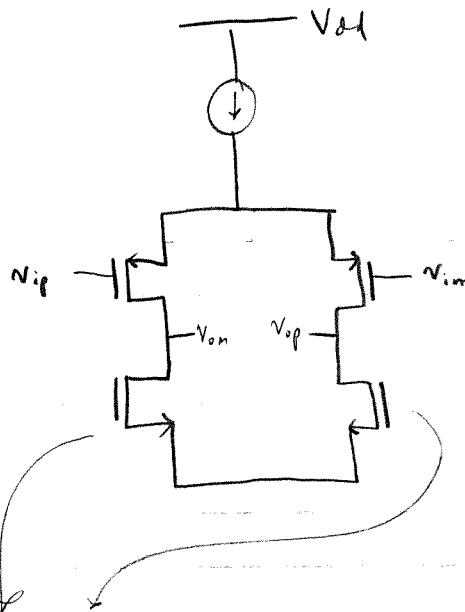
$$g_m = \frac{2Id}{V_{ov}} = 1mA$$

Using Blackman's Impedance Formula:

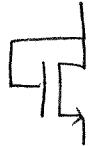
$$Z_{k=0} = \frac{1}{g_m} \quad RR_{(\text{short})} = - \left(\frac{-g_m R V_t A}{V_t} \right) = g_m R A$$

$$RR_{(\text{open})} = 0 \Rightarrow Z = R_{in} = \frac{1}{g_m} \left(\frac{1 + g_m R A}{1 + 0} \right) = \frac{1}{g_m} + RA = [21k\Omega]$$

2003 - Wong



①



diode connected :

High

Linearity (if load was p-type)
so not in this case.

Low

Gain

Low

Output swing

Linearity:

If load was p-type, $A_v = -\frac{g_{mp1}}{g_{mp2}} = \frac{\sqrt{2\mu_0 C_{ox} (W/L)_1} I_D}{\sqrt{2\mu_0 C_{ox} (W/L)_2} I_D} = \sqrt{\frac{(W/L)_1}{(W/L)_2}}$ \Rightarrow independent of process variations, (μ, C_{ox}, V_t)

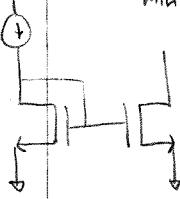
Gain:

$$A_v = -\frac{g_{mp}}{g_{mn}}$$

Output swing:

$$V_{out, min} = V_t + V_{ov}$$

②



Current source load :

High Gain

Medium Output swing.

Gain

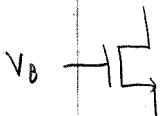
$$A_v = -g_{mp} (r_o^p \parallel r_o^n)$$

$$r_o = \frac{1}{2I_D} \quad 2 \times \frac{1}{2} = \frac{1}{2} \Rightarrow r_o \propto \frac{L}{I_D}$$

but for fixed current, $L \uparrow \Rightarrow \left(\frac{W}{L}\right) \downarrow \Rightarrow V_{ov} \uparrow$

\Rightarrow higher gain \Rightarrow lower voltage swing.

③



Triode Load:

Medium Gain (but susceptible to process variations)

High Output Swing

Gain:

$$A_v = -g_{mp} (r_{op} \parallel r_{on} \parallel R_{on})$$

$$\text{But } R_{on} = \frac{1}{\mu_n C_{ox} \left(\frac{W}{L}\right)_2 (V_b - V_t)}$$

\Rightarrow susceptible to process variations

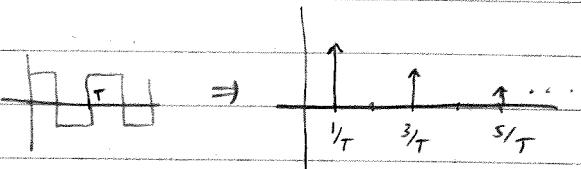
(μ, C_{ox}, V_t)

Output swing:

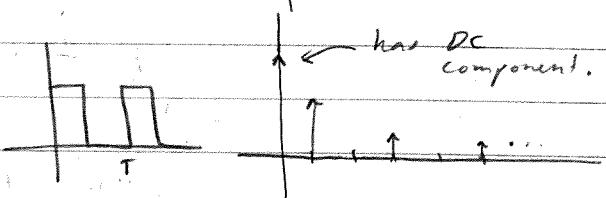
$$V_{out, min} = 0$$

2009 - Wong

- Power spectrum of square wave is:

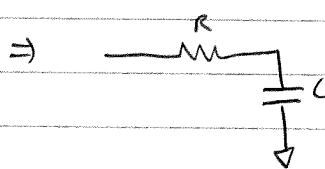


- Power spectrum of positive square wave is:



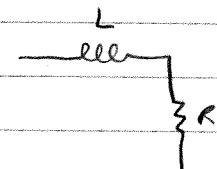
In this case, we want to extract only the DC component.

\Rightarrow use a LPF with $f_p \ll \frac{1}{T}$



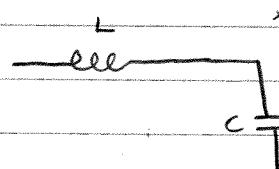
$$\frac{1}{2\pi RC} \ll \frac{1}{T}$$

$$2\pi RC \gg T$$



$$\frac{R}{2\pi L} \ll \frac{1}{T}$$

$$2\pi \frac{L}{R} \gg T$$



$$\frac{1}{2\pi \sqrt{LC}} \ll \frac{1}{T}$$

$$2\pi \sqrt{LC} \gg T$$

except for peaking
at resonant
frequency,
this filter
has a
low pass
behavior.

- If the output has to drive a heavy load, how would you modify the circuit?

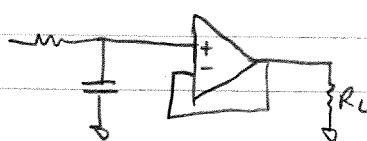
Heavy load \Rightarrow small R_L

R-C: Won't work. R_L will increase BW.

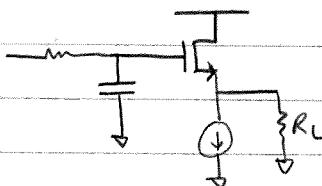
R-L: Will work. R_L will decrease BW.

LC: ω_0 stays the same. Peaking reduced ($Q = \frac{R}{\sqrt{L/C}} = \omega_0 R C$)

Also could add unity gain voltage buffer capable of driving the heavy load.



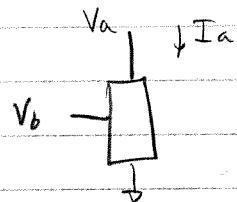
or



then there will be a level shift here of $V_{out} + V_t$.

2007 - Dutton

$$I_a = \left(V_b + \frac{V_a}{10} \right)^2$$

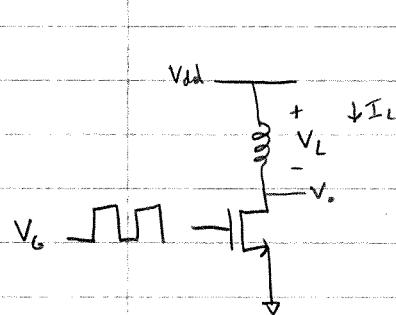


$$g_m = \frac{\partial I_a}{\partial V_b} = 2(V_b + \frac{V_a}{10})$$

$$g_{ds} = \frac{\partial I_a}{\partial V_a} = \frac{2}{10} (V_b + \frac{V_a}{10})$$

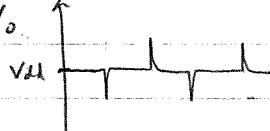
$$\Rightarrow g_m r_o = \frac{g_m}{g_{ds}} = 10 \leftarrow \text{intrinsic gain.}$$

2006 - Kovacs



$$V_L = L \frac{di}{dt} \quad i_L : \text{square wave} \Rightarrow V_L : \text{square wave}$$

$$V_o = V_{dd} - V_L \Rightarrow V_o :$$



Capacitor + Inductor Basics:

$$\text{Capacitor: } \frac{1}{C} \int_{t_0}^t \frac{+V_c + i_c}{-V_c} dt \quad i = C \frac{dV}{dt}$$

$$v = \frac{1}{C} \int_{t_0}^t i dt + v(t_0)$$

$$P = iv = Cv \frac{dv}{dt}$$

$$E = \int P dt = \int Cv dv = \frac{1}{2} Cv^2$$

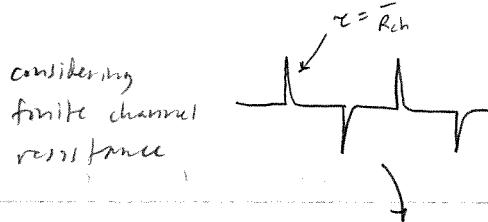
Inductor:

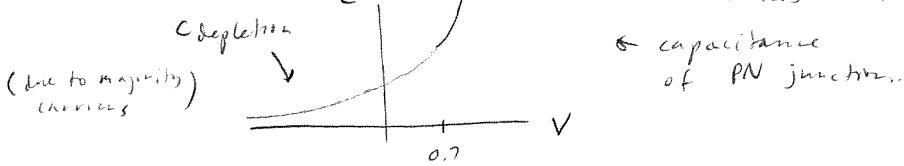
$$\frac{1}{L} \int_{t_0}^t \frac{+V_L + i_L}{-V_L} dt \quad V = L \frac{di}{dt}$$

$$i = \frac{1}{L} \int_{t_0}^t V dt + i(t_0)$$

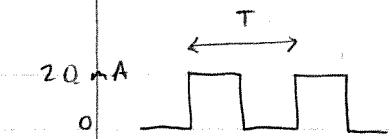
$$P = iv = Li \frac{di}{dt}$$

$$E = \int P dt = \int L i di = \frac{1}{2} Li^2$$

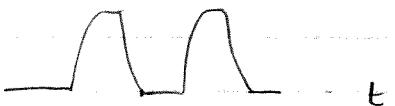
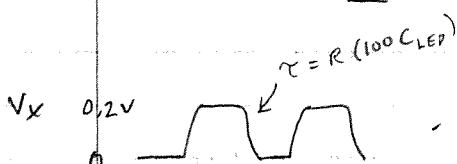




2005 - Kovacs

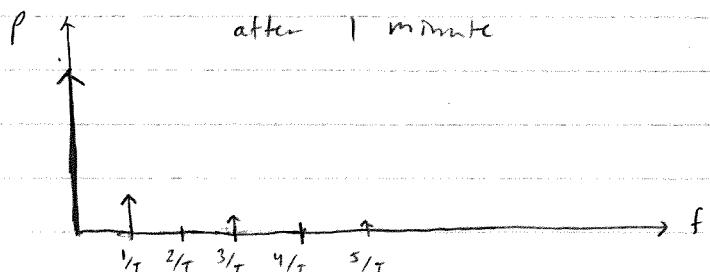
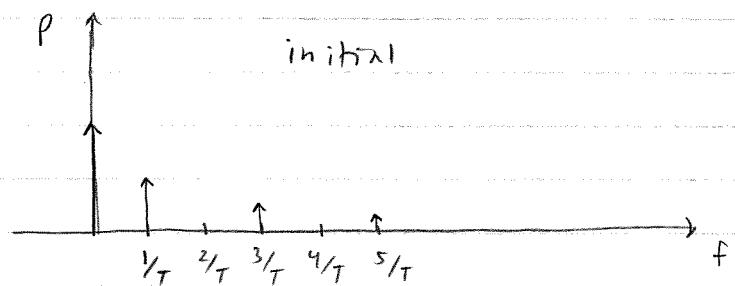


$$(20 \text{ mA} \cdot 100) \cdot 0.1 \Omega = 0.2 \text{ V}$$



as time passes, Resistor heats up ($P = I^2 R$)
and as $T \uparrow$, $R \uparrow$. As $R \uparrow$, V_x increases
and $\tau = R(100 C_{LED})$ increases. \Rightarrow output is smoother and
has increased power (especially in its DC component)

=> Power spectrum:

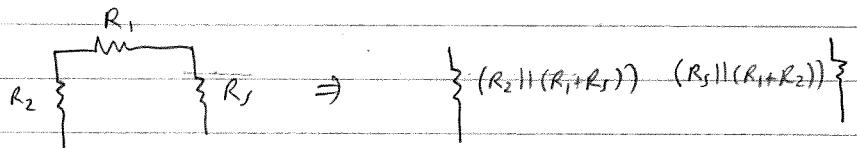


2009 - Poon

1. Series - Series

$$2. f = \frac{R_S R_2}{R_1 + R_2 + R_S}$$

3. loading:



$$\Rightarrow a = -g_{m_1} (r_{o1} || r_{o3}) (-g_{m_5} r_{o5}) \frac{g_{m_6}}{1 + g_{m_6} (R_S || (R_1 + R_2))}$$

$$\Rightarrow a = \frac{g_{m_1} g_{m_5} g_{m_6} (r_{o1} || r_{o3}) r_{o5}}{1 + g_{m_6} (R_S || (R_1 + R_2))}$$

$$4. R'_{out} = (1 + g_{m_6} (R_S || (R_1 + R_2))) (1 + af)$$

How to find f :

If feedback is shunt at input, short input feedback node and calculate feedback current.

If feedback is series at input, open input feedback node and calculate the feedback voltage.

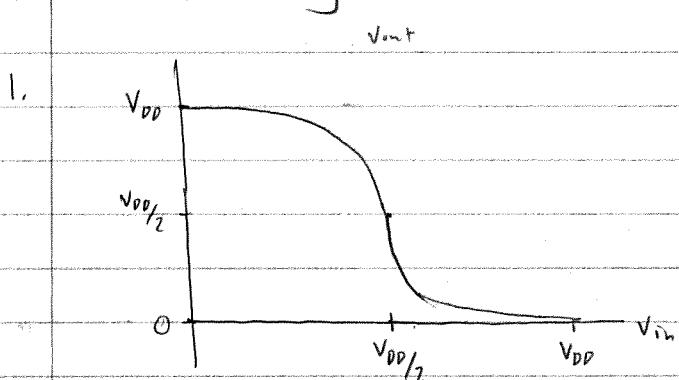
If feedback is shunt at output, drive feedback network with a voltage source.

If feedback is series at output, drive feedback network with a current source.

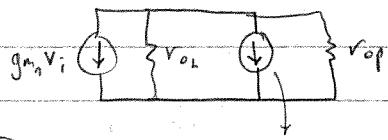
How to find loading effects:

If feedback is shunt at input (output), short input (output) feedback node to find feedback loading on output (input). If feedback is series at input (output), open the input (output) feedback node to calculate the output (input) feedback loading.

2005 - Wong



2. $N + P \text{ in sat} \Rightarrow$

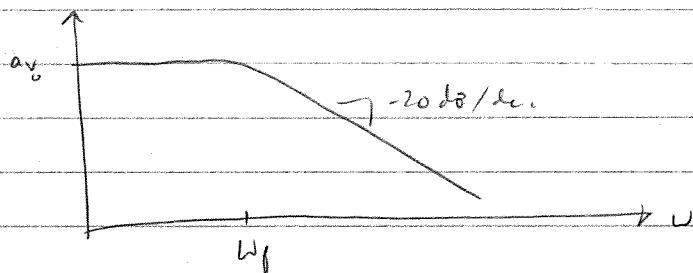


$$\Rightarrow a_v = -(g_{mn} + g_{mp})(r_{on} || r_{of})$$

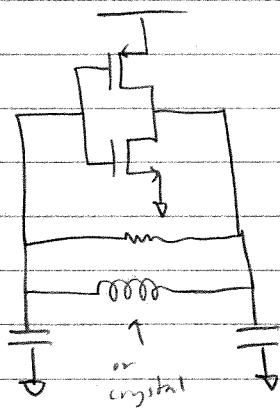
3. Dominant pole due to Miller multiplied $C_{gd} + C_s$ at input

Assume R_s at input.

$$\Rightarrow w_p = \frac{1}{R_s(2C_{gd}(1+(g_{mn}+g_{mp})(r_{on}||r_{if})) + 2C_s)}$$

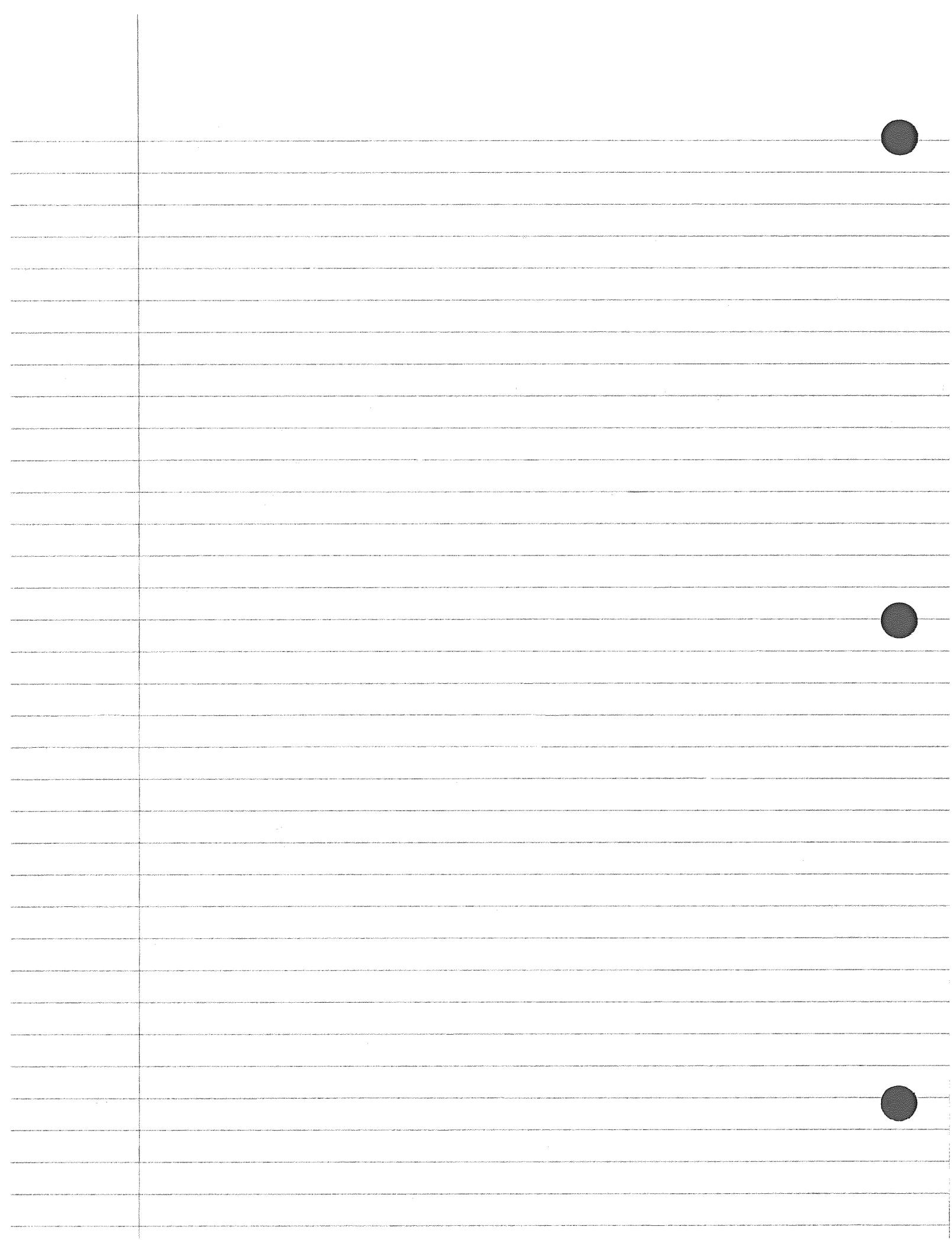


4.



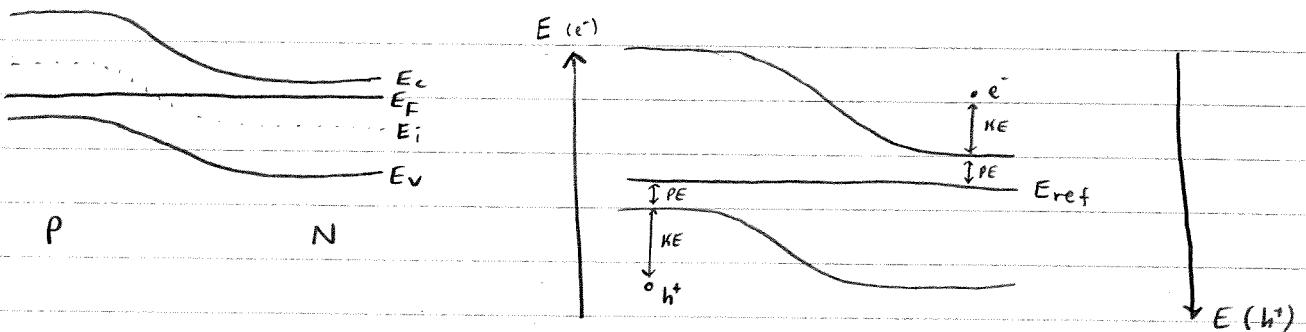
5. $\omega = \frac{1}{\sqrt{LC}}$ amplitude determined by R ? V_{ds} ?

Devices



1. Draw an energy band diagram for a p/n junction.

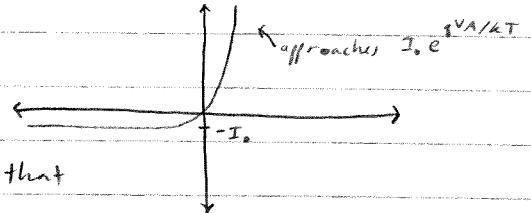
What does the vertical axis represent (Total E, KE, or PE)?



The vertical axis is total energy. E_c and E_v are lines of PE and zero KE for electrons and holes, respectively. $\leftarrow \times$
 KE is the energy above E_c for electrons and below E_v for holes.

2. Draw the I-V characteristic for the p/n junction and explain what physically happens in terms of the band diagram above when you apply a bias to the junction. What is the "turn on voltage"?

$$\text{Ideal PN Junction : } I = I_0 (e^{\frac{V_A - V_b}{kT}} - 1)$$



At zero bias there is an electrostatic barrier (V_{b0}) that provides a balance between drift and diffusion of carriers in the junction. Under forward bias the barrier is reduced, favoring carrier diffusion over drift. The carriers have a Boltzmann distribution in energy, thus the number of carriers which exceed the remaining barrier (are injected as minority carriers into the opposite conductivity region) is proportional to $\exp[(V_A - V_{b0})/kT]$, providing an exponential dependence of current on voltage. Under reverse bias, the barrier is increased, reducing the injection of carriers and favoring drift. However, the concentration of minority carriers can only go to zero and the number which reach the p/n junction is limited by diffusion from the bulk, which remains essentially constant with bias, producing a saturation current independent of bias. The threshold voltage V_{th} is just the extrapolation from the high forward current region to zero current and is usually about $\frac{2}{3} E_{A0}$ (at room temperature).

- implies forward bias.
3. What happens to the injected carriers? What energy do the photons have if the recombination is radiative?

The injected carriers are minority carriers, which are at a higher concentration than their thermal equilibrium concentrations and they are surrounded by a far larger number of majority carriers, thus they recombine (either radiatively (band to band) or nonradiatively) to move toward their equilibrium value. If the recombination is radiative, then the photon energy, $h\nu$, is equal to E_g .

4. How is it possible to get $h\nu = E_g$ photons emitted when we've only had to apply $V_{th} \sim \frac{2}{3} \frac{E_g}{q}$ volts bias to get forward current to flow? Is this a violation of Conservation of Energy and the Laws of Thermodynamics?

The minority carriers only require about $\frac{2}{3} \frac{E_g}{q}$ to be injected because these are the carriers farthest up (highest KE) in the Boltzmann distribution, so the "added" energy is KE from the lattice.

5. What would happen if the diode was placed in thermal isolation and forward biased but with a window where photons could escape? Could I make a refrigerator with such a diode?

Yes. If the diode were placed in a thermally isolated environment and forward biased with only radiative recombination occurring, the diode (initially) loses energy, thus its temperature drops and it serves as a refrigerator. \rightarrow (Later external source will provide energy to it)

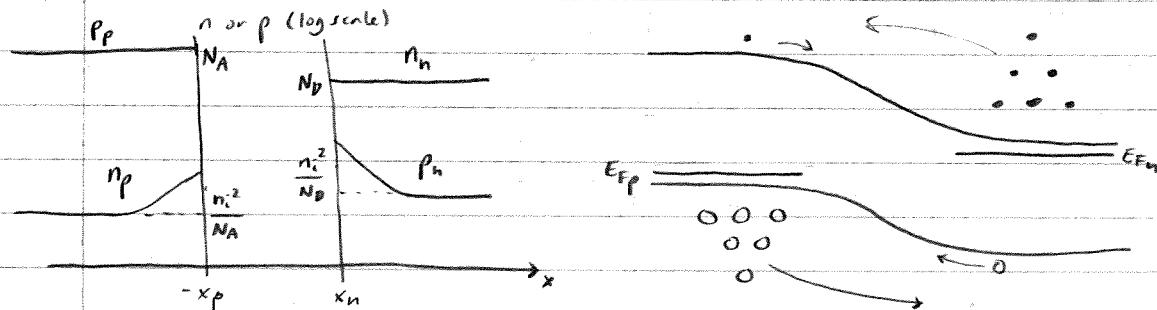
6. What would the ideal "asymptotic" IV characteristic look like and what would be the values of V_{th} and T ?

Ideally, the asymptotic IV characteristic would have zero current until reaching $V_A = \frac{E_g}{q}$ whereupon the current increases extremely rapidly, since I is proportional to $e^{(V_A - V_t)/kT}$ and in the limit, T is going to 0K.

7. Do you think the same thing will happen with a Schottky barrier (M-S diode)? Why or why not?

No. Because forward current is a result of majority carriers from the semiconductor being injected as hot ($KE = \Phi_b$) into the metal, which has no bandgap, thus all of the KE is dissipated by scattering and thermal energy is deposited into the metal. \Rightarrow we can't use this as a refrigerator.

Relevant Basics: PN Junction

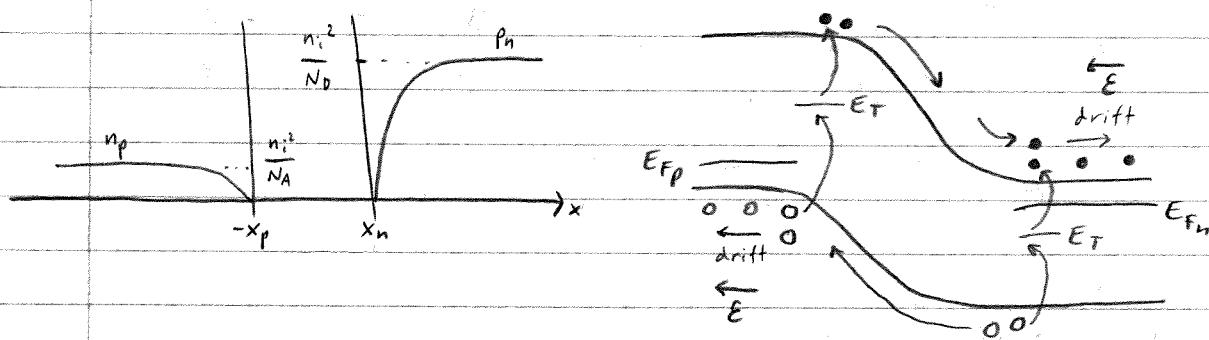


$$\Delta n_p (-x_p) = \frac{n_i^2}{N_A} (e^{qV_A/kT} - 1)$$

$$\Delta p_n (x_n) = \frac{n_i^2}{N_D} (e^{qV_A/kT} - 1)$$

excess minority holes recombine here with majority electrons. These electrons are replaced by electrons from the contact through drift,

n or p (linear scale)



minority electrons that diffuse into depletion region are replaced through generation.

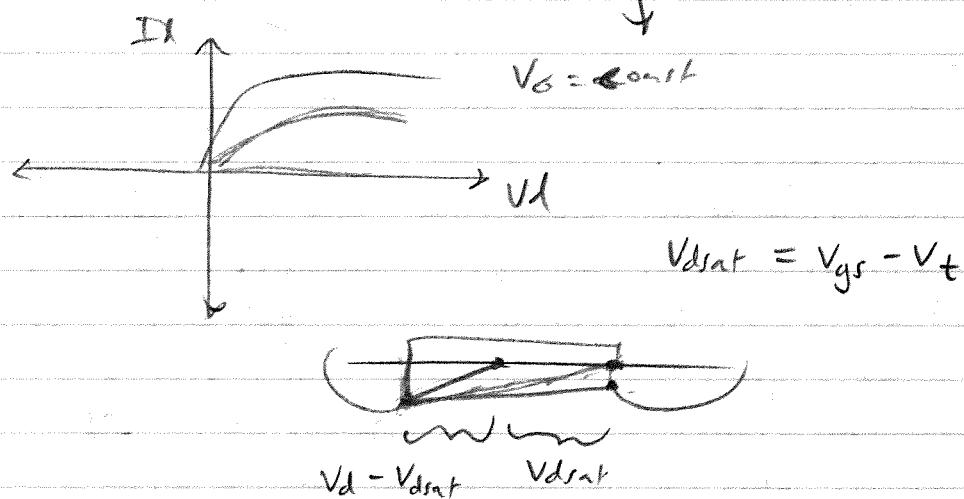
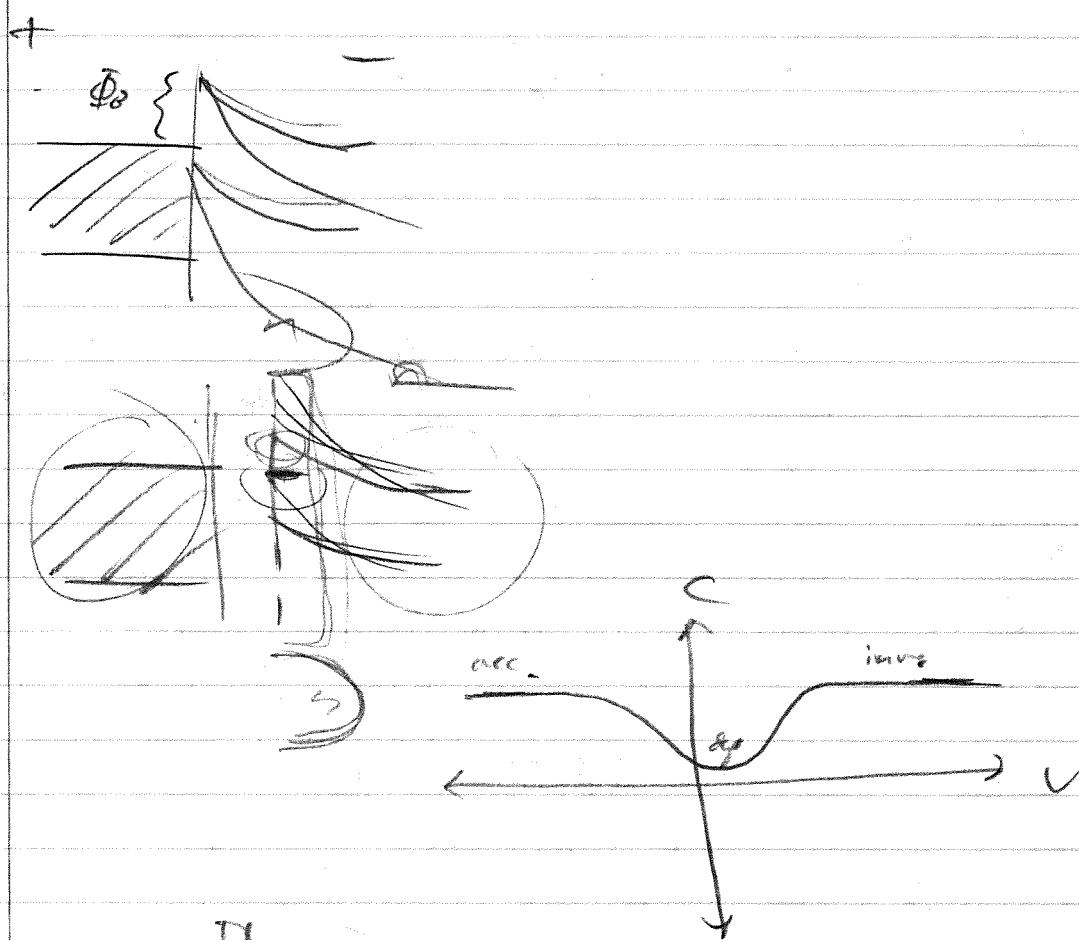
This creates a slight excess of majority carriers near the junction, which creates an E field that sweeps majority carriers towards the contacts.

M-S Diode:

Forward bias: $E_{FM} - \Phi_b$

\Rightarrow no recombination of injected carriers on metal side.

1996 Harris



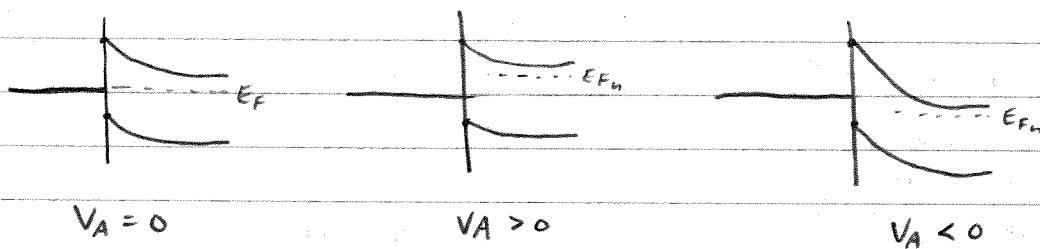
A; 1. If I deposit a metal layer onto a semiconductor to form an MS junction:

A. What occurs at the interface or junction?
carriers move & sheet of charge forms, at metal interface
and depletion region forms, \Rightarrow band bending.



B. Draw an energy band diagram and explain what happens when a voltage is applied to the junction.

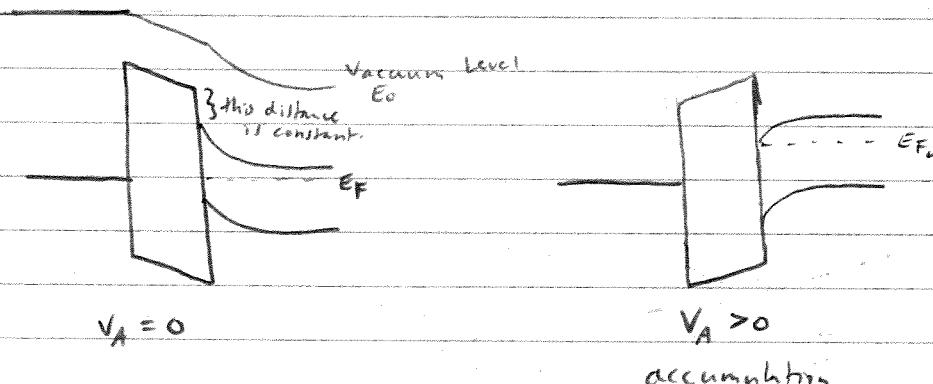
+ V_A - Forward Bias



2. If I now insert a thin insulator between the metal and semiconductor:

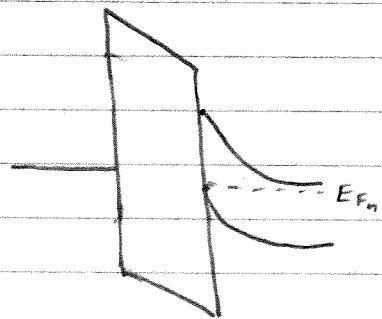
C. What does the insulator do? Prevents current flow

Draw an energy band diagram. Explain what happens with bias:



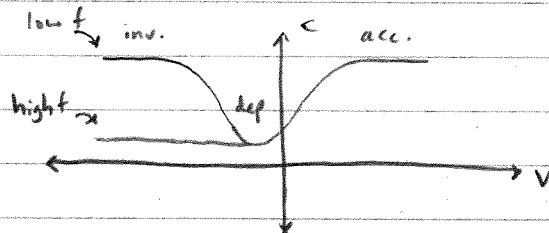
Pierret

pg. 575



D. Since there is no current flow, an IV curve is not meaningful. What measurement might be used to characterize an MIS structure and explain what you expect it to look like?

C-V measurement.



- E. What causes the inversion layer to form? minority carriers.

(which are formed by thermal generation).

3. I can utilize this structure to form a transistor:

F. What is the basis of forming such a transistor: like MOSFET

G. From where do carriers originate that form the inversion layer?
they mostly come from the source + drain.

H. Draw IV characteristics.

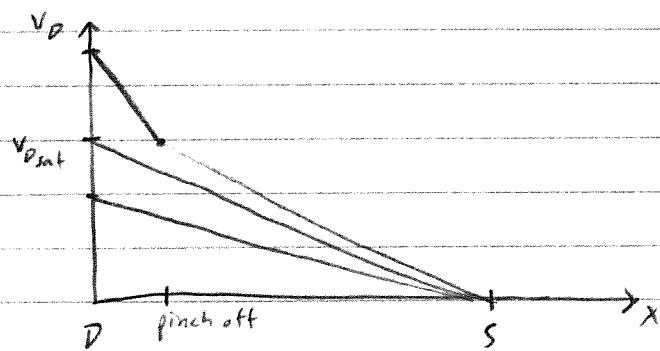


I. What are V_{th} and V_{Dsat} .

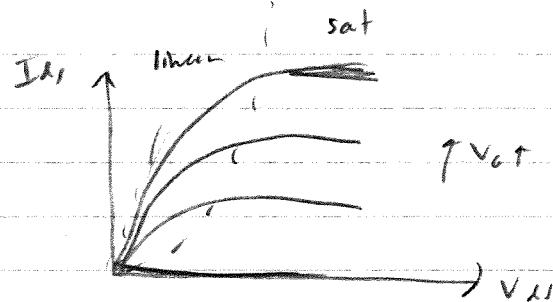
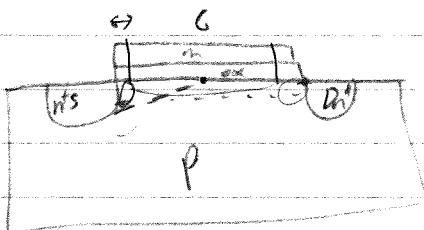
V_{th} : $P_n = N_0$ V_{Dsat} : voltage where channel pinches off at drain.
when inversion layer starts to form.

J. What does the potential look like along the channel for

$$V_D < V_{Dsat} \text{ and } V_D > V_{Dsat}$$



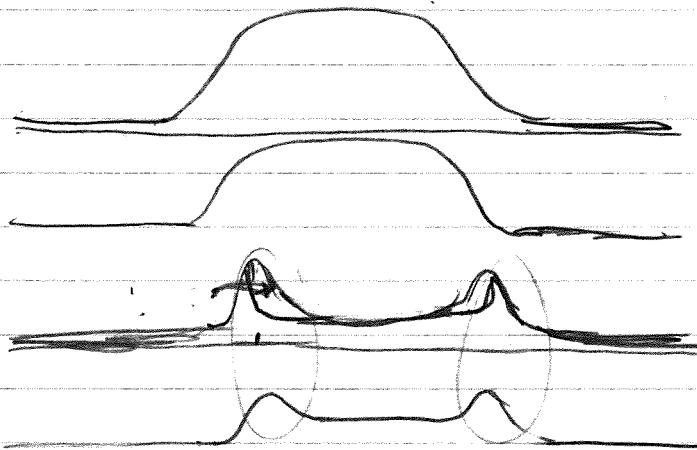
1997 - Harris



$$V_{ds} < V_{gs} - V_{th}$$

$$V_{ds} > V_{gs} - V_{th} \quad V_{gs} < V_R$$

$I_D \propto V_{ds}$



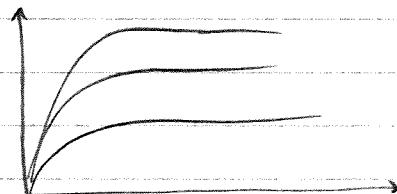
$$\partial E \quad Q = CV \quad \cancel{E = \frac{Q}{C}} \quad V = \frac{Q}{C}$$

~~100mV~~

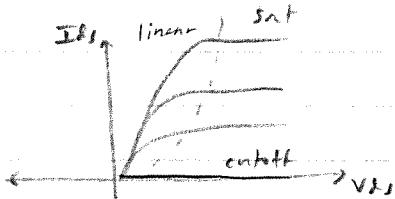
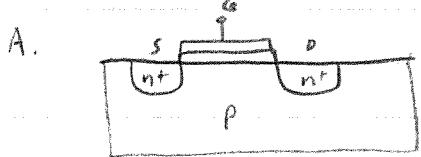
$$100\text{mV} = \frac{1\text{eV}}{C}$$

$$C = \frac{1}{100}$$

100 times less.



1. Sketch cross-section of MOSFET. Draw IV curves. Describe 2 or 3 most important regions and how these are related to the physics within the device.



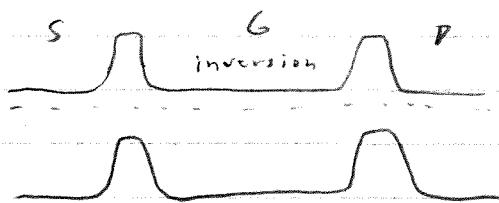
2. What happens if gate does not overlap Source and Drain regions?

A. Neglecting fringing fields, the transistor does not function.

3. Can an inversion region still form under gate with applied V_{GS} ?

A. Yes. This is a MOS cap. The device first goes into deep depletion and eventually forms an inversion layer under the gate from thermal generation of carriers.

4. Draw band diagram assuming a gate bias creates an inversion layer.



5. If I make the p regions narrow, I claim I can make a new type of transistor. What might be the important physical mechanisms operative in such a device?

A. Tunneling: If I apply a positive bias to D, an electron can tunnel from S to channel and then from channel to D. Thus get I_{DS} .

6. I would like to make a device that could operate at room temp. If I could make the gate (and hence inversion region) so small that the tunneling of a single electron into this small region increases the potential by 100 meV

how small would this have to be (for instance compared to the gate capacitance of a MOSFET)?

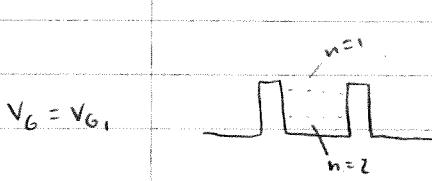
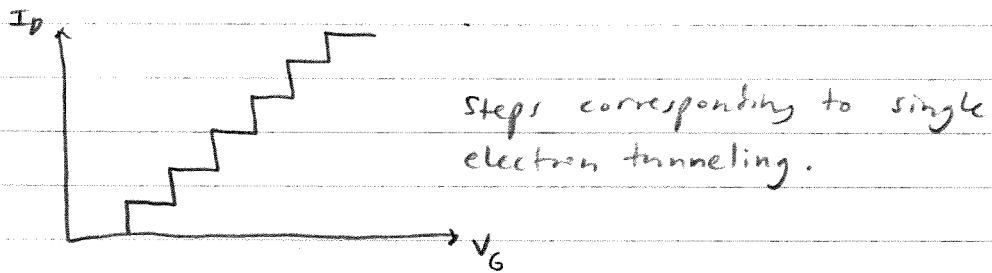
$$C = \frac{dQ}{dV} = \frac{1.6 \cdot 10^{-19}}{0.1} = 1.6 \cdot 10^{-18} \text{ F}$$

which is very small compared to a typical MOSFET gate $C_g = 10^{-15} \text{ F/m}^2$ or $1 \text{ aF}/\mu\text{m}^2$

$$Q = CV \Rightarrow C = \frac{Q}{V}$$

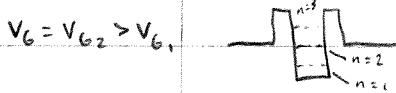
charge of e^- : 100 aV

7. What might you guess the I-V characteristic to look like if I had such a small structure and current transport limited by tunneling of single electrons.

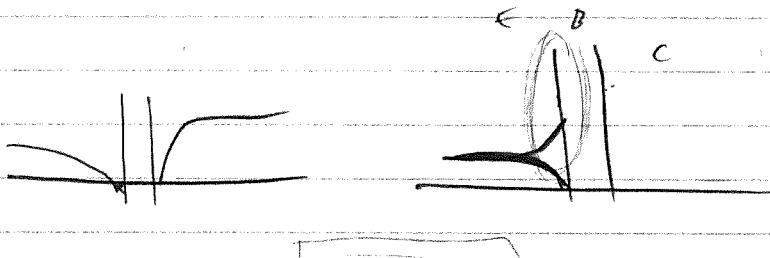


$V_G = V_{G1}$ when an e^- tunnels into the well, the energies of the allowable states increase \Rightarrow other e^- s can't tunnel into the well.

If you apply a larger gate voltage V_G , the well becomes deeper \Rightarrow there are more available states in the well. \Rightarrow can have more electrons in the well simultaneously.



1994 - Harris



1. What are the advantages of MOSFET + associated technology to BJTs.

Lower power, simpler fabrication technology,
natural device isolation, higher integration levels, high
input impedance and no gate leakage \Rightarrow charge
storage, complementary architecture.

2. What areas has the bipolar transistor remained important?

Analog and A/D circuits, microwave and high speed
digital circuits, BiCMOS circuits, high power devices.

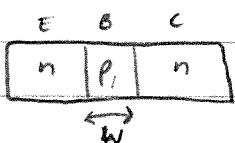
3. Why has bipolar remained faster.

It has been easier to control the base width to very
small dimensions by ion implantation and epitaxy than
the source drain spacing for MOSFETs by lithography.

As devices are scaled to very small dimensions, parasitic
capacitances no longer scale and the high current drive
capability and lower voltage operation of bipolar transistors
enables them to operate at higher speeds.

$$\tau = \frac{W^2}{2D_B}$$

$$= \frac{W^2}{2\mu V_T}$$



$$\tau = \frac{L}{V_{th}} \Rightarrow \tau = \frac{L}{\mu E} = \frac{L^2}{\mu V_{th}}$$



$$\tau = \frac{L^2}{\mu V_{th}}$$

but BJTs are
still faster since
MOSFET speed is
limited by parasitic
capacitance.

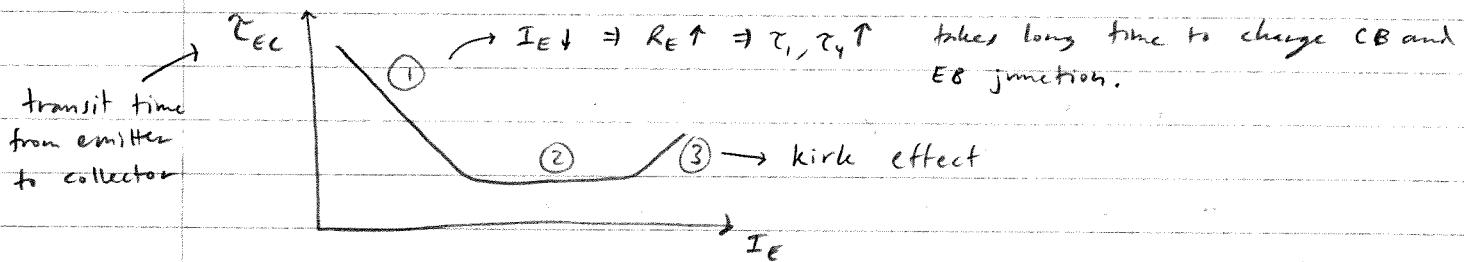
$$E = \frac{V}{L}$$

Since $\mu_B > \mu_{ch}$ \leftarrow due to lattice
scattering, lower carrier
concentration

$$\tau_{BJT} < \tau_{MOS}$$

but $V_I \ll V_{th} \Rightarrow \tau_{BJT} > \tau_{MOS}$

The high speed performance of a BJT varies with emitter current as follows; How is this related to the physics of the device operation?



τ_1 : charge tht junction τ_2 : time for charge to cross the base,

$\tau_3 = R_E C_{j_{BE}}$ $R_E = \frac{\delta V_{BE}}{\delta I_E}$, $I_E = I_s e^{\frac{qV_{BE}}{kT}}$

$\tau_4 = R_C C_{j_{CB}}$ $R_C = \frac{\delta V_{CB}}{\delta I_C} = \frac{V_T}{I_C}$

$\tau_2 = \frac{W^2}{2D_B}$

$\tau_3 = \frac{Wc}{2V_{sat}}$

$\tau_{EC} = \tau_1 + \tau_2 + \tau_3 + \tau_4$

τ_3 : time for charge to drift across CB depletion width;

τ_4 : time to charge CB junction

?

Kirk effect:

1993 - Harris

① What are the properties of the "ideal" semiconductor and why?

1. Bandgap - 1.2-1.5eV (direct for optical devices)

- Too Small ($< 1.2\text{eV}$) : large n_i , inability to control $n + p$, poor operating temperature range, large leakage currents, low breakdown voltage (for FETs and BJT)

- Too Large ($> 1.5\text{eV}$) : insulator (unable to ionize impurities and obtain free carriers), poor ohmic contacts, high power dissipation and high V_{TH} for FET, BJT, diodes and lasers.

2. Naturally Forming Oxide - Good insulator, passivates

(makes nonreactive to air) the surface, has no interface states (trap states), thermally stable, good high temperature diffusion + ion implantation barrier, easily etched, has good etch selectivity to semiconductor. $K = 3.9$
(for Si, $K = 11.7$)

3. Easily Grown - and prepared as large area wafers that are physically robust, thermally and chemically stable, compatible with both metals and insulators, able to be controllably doped n or p over a broad range and to high densities, able to be semi-insulating and have a lower dielectric constant (< 2)

4. High Mobility - for both e^- and h^+ (and equal for complementary circuits), low resistivity for diodes, lasers, BJT, etc., high v_{dsat} which is reached at lower electric fields for high speed and low power dissipation.

5. High Thermal Conductivity - and small thermal expansion coefficient and one that matches to its oxide and metals.

6. Heterojunction Possibilities - to be able to fabricate higher performance lasers, HBTs, MODFETs, OEICs, QW devices,
optoelectronic quantum well.

7. Temperature Independence - of properties.

8. High Breakdown Field - for high power devices

?
low effective mass
→ lower density
of states
→ less tunneling

9. Small and Equal Effective Mass - Better tunneling for
low resistance ohmic contacts, lower I_{th} for lasers,
and higher current in tunneling devices.

10. Short / Long Carrier Lifetime - Short for high speed devices,
Long for CCDs, solar cells, and high power devices.

11. Widely Abundant, Low Cost, Non-Toxic material.

(2) If you could choose one property to be temperature independent, which would you choose?

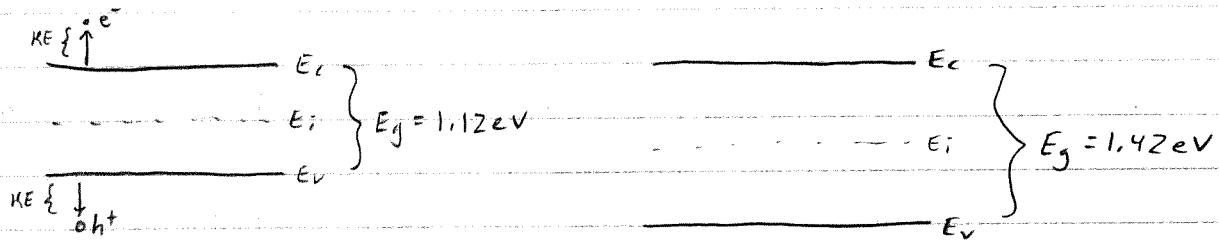
$$\text{Bandgap} - (E_g(T) = E_g(0) - \frac{\alpha T^2}{T + \beta}, E_g(0), \alpha, \beta \text{ are constants})$$

Temperature variation in bandgap causes threshold variations in
FETs and HBTs, wavelength of lasers, and leakage currents in
all devices.

Mobility - a high, temperature independent mobility for both e^- and h^+
makes complementary, high speed circuits much easier to design.

2005 - Harris

1.



$$\text{Si : } \frac{m_n}{m_0} = 0.26 \quad n_p/m_0 = 0.39$$

$\text{GaAs : } \frac{m_n}{m_0} = 0.068 \Rightarrow \text{much higher } \mu_n$
 $m_p/m_0 = 0.5 \Rightarrow \text{lower } \mu_p$
 $\Rightarrow \text{worse for CMOS (digital logic)}$
 $\text{but good for high frequency (MMIC)}$

Indirect bandgap.

a) Vertical axis is total Energy.

$$h\nu = E_g$$

band to band generation.
of $e^- h^+$ pair.

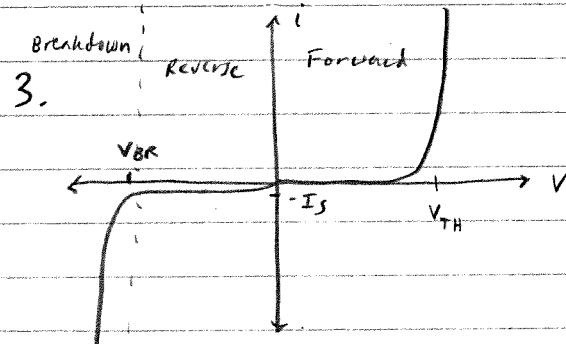
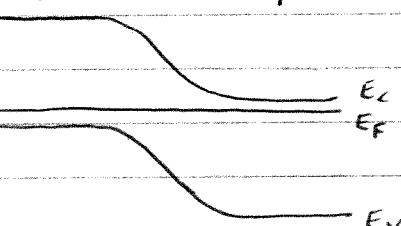
Direct bandgap.

Wider bandgap \Rightarrow less sensitive to heat, resistance to radiation damage (good for space electronics), highly resistive in pure state with high dielectric constant \Rightarrow ideal electrical substrate (provides natural isolation).

$$h\nu = 1.5E_g$$

d) Excess energy is transferred to lattice as heat (phonons).

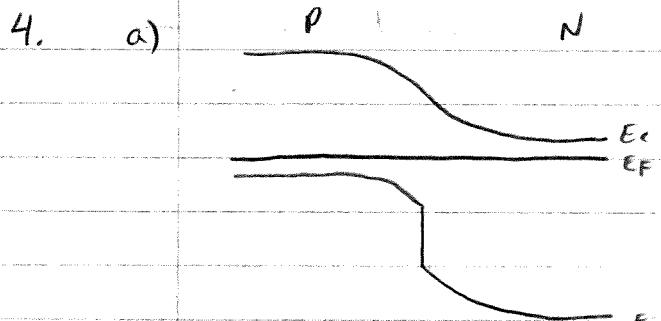
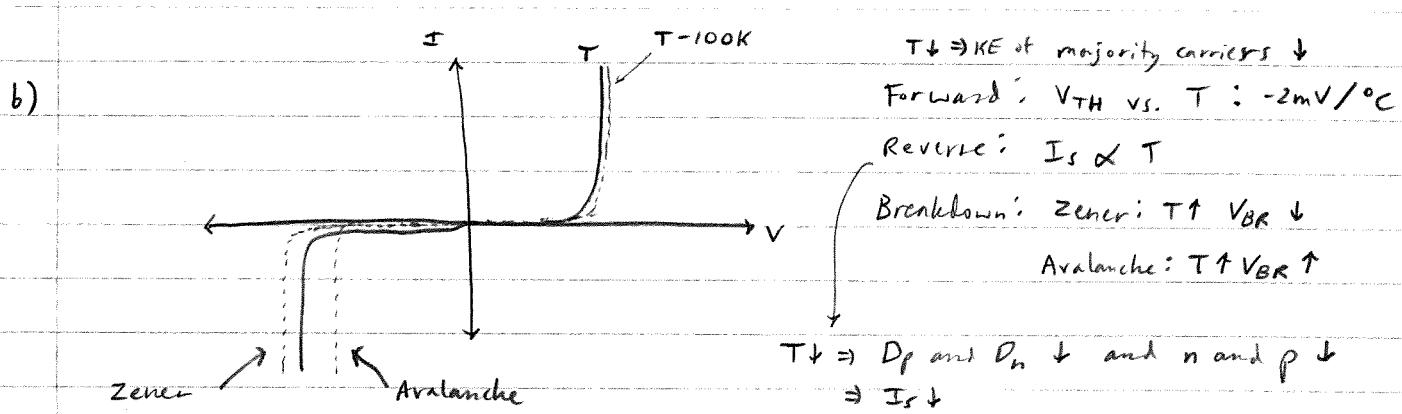
2.



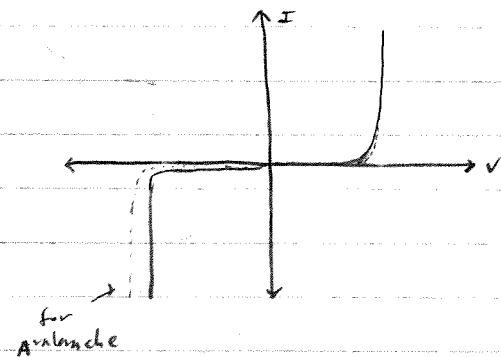
a) Forward: Barrier lowering \Rightarrow diffusion of majority carriers
 \Rightarrow exponential. $I = I_s (\exp(\frac{qV}{kT}) - 1)$

Reverse: Drift of minority carriers, Barrier increases but doesn't matter.
 \Rightarrow saturates. $I_s = qA \left(\frac{D_p}{L_p} \frac{n_i^2}{N_D} + \frac{P_N}{L_p} \frac{n_i^2}{N_A} \right)$

Breakdown: Due to Zener breakdown (if voltage is low) heavily doped or Avalanche breakdown (if voltage is high) lightly doped



- b)
- Reverse current I_s decreases, since N_i decreases (E_g larger) on n-side.
Forward current reduced by factor of 2 (e^- current dominates, h^+ see large barrier).
Possible increase in V_{BR} for avalanche since E_g is higher on n-side.



- c)
- Yes. This would be advantageous in a HBT. If you make the emitter of an npn HBT a wider bandgap material, you could heavily dope the base (which would reduce the negative effects of Base width modulation and r_b which leads to current crowding) without sacrificing emitter efficiency.

$$\gamma = \frac{I_{Eh}}{I_{Eh} + I_{Ep}}$$

Since holes from base would see a large barrier due to the hetero structure.

or for an LED
to prevent photons
from being
reabsorbed:

act as
window
 $\rightarrow h\nu$

or for a solar
cell. can't
allow more light
to reach the
depletion region

\Rightarrow increase
solar
efficiency,
less absorption in higher

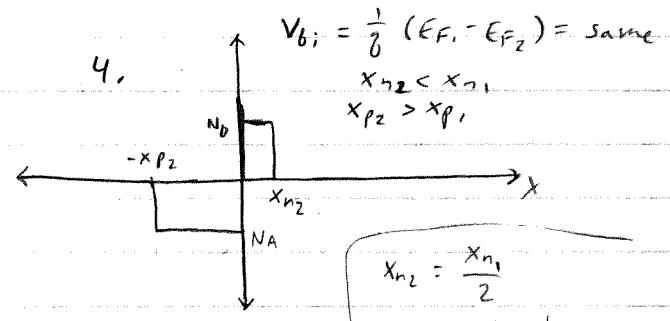
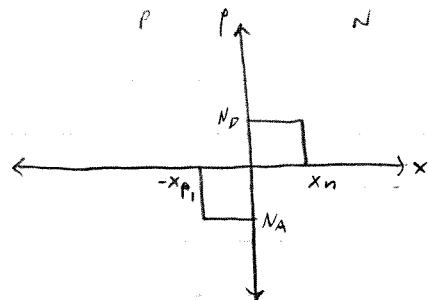
$\rightarrow h\nu$

2003 - Harris

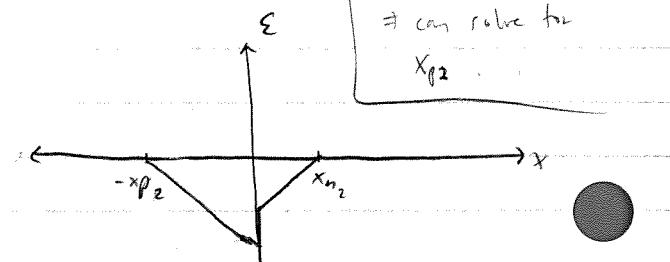
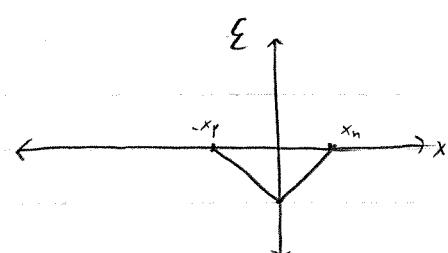
1. What is the depletion approximation for a pn junction?

- depletion region is fully depleted of mobile charge carriers.
- therefore there is no recombination in the depletion region.
- net charge density outside of depletion region is zero.

2.

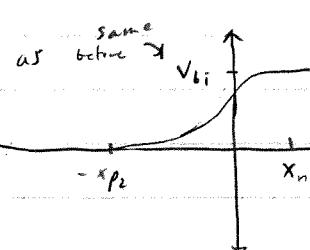
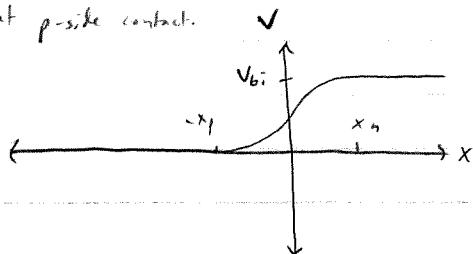


$$V_{bi} = \frac{kT}{q} \ln \left(\frac{N_D N_A}{n_i^2} \right)$$

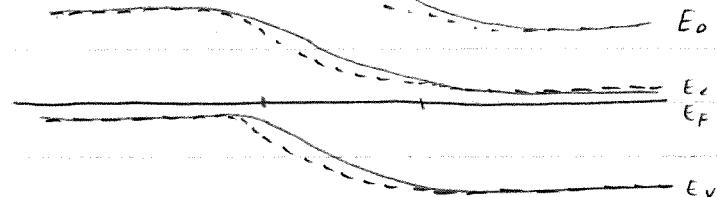


Let

$V=0$ at p-side contact.



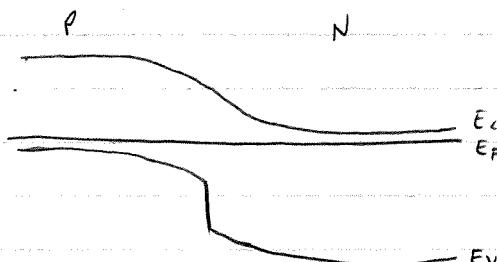
3.



5.

original
--- with sheet charge

6.



2004 - Harris

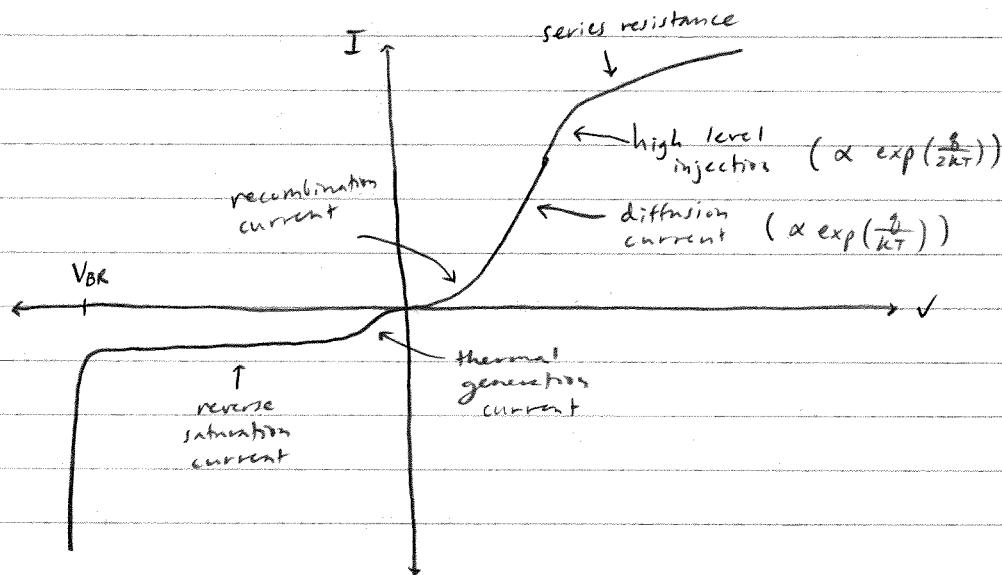
$$1. \quad a) \quad I = I_s (\exp(\frac{qV_A}{kT} - 1)) \quad I_s = gA \left[\frac{D_n n_p}{L_n} + \frac{8D_p p_n}{L_p} \right]$$

- underlying assumptions:

- no recombination or generation in depletion region.
- low level injection.
- $\epsilon = 0$ in quasineutral regions (Depletion approximation).
- assume drift current of minority carriers at the edge of the depletion region is zero. $\swarrow ?$
- (law of the junction holds: $np = n_i^2 (\exp(\frac{qV_A}{kT}))$)

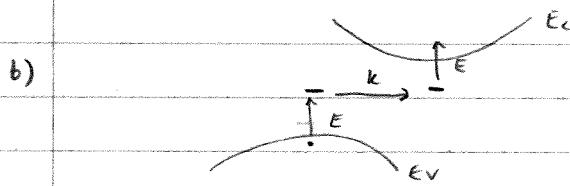
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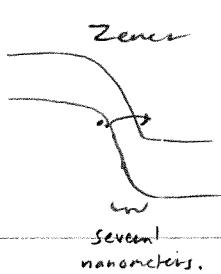
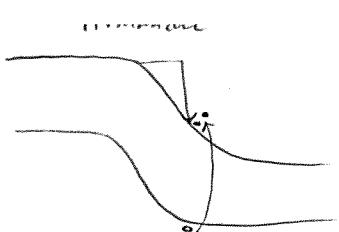
3.



4. Because their concentration is comparable to the minority carrier concentration \Rightarrow their current dominates for low voltage.

- a) NO. They are more effective when E_T is close to E_i . (close to center of the band). And more effective for indirect semiconductors. Thermal generation and recombination in indirect semiconductors requires energy and momentum simultaneously. With added trap states, you can get generation/recombination through various changes in only energy or only momentum.





5. Avalanche and Zener (tunneling) breakdown.

a) Eg.

6. "Zener" diodes for voltage references.

1995 - Harris

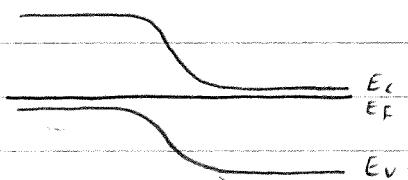
1. LED is a forward biased PN Junction. Injected minority carriers recombine radiatively with majority carriers.
2. Different Bandgaps. Doesn't have to be direct bandgap, but indirect is much less efficient. Indirect materials must incorporate an optically efficient trap.
3. The laser is also a p/n junction diode operating in the same basic manner as the LED except that it is inside a Fabry-Perot cavity which provides optical feedback and when the number of injected carriers is high enough (threshold current), a population inversion is reached and this combined with the feedback produces stimulated emission and an optical oscillator or laser. Below threshold, the laser is an LED - it emits only spontaneous, not stimulated photons.
population inversion: density of electrons in conduction band is larger than in the valence band.
Degenerately dope both sides and forward bias the PN junction. Then a region in the depletion region will have population inversion.
4. The laser output is controlled by optical feedback of the Fabry-Perot cavity, which only occurs over a very narrow angular range. A laser can only be made with a direct band gap material because the trap transitions saturates and cannot achieve a population inversion.
5. Yes. LED - changes color due to change in E_g . Laser changes only slightly due to thermal expansion of optical cavity.

5. The laser output is polarized along either the horizontal or vertical plane of the laser. Since the cavity controls the output either the cavity shape or reflectivity of the mirrors is different for TM vs. TE polarized light. And one mode is strongly favored.

6.

2001 - Harris

1. a)



e) forward - high level injection

reverse - breakdown.

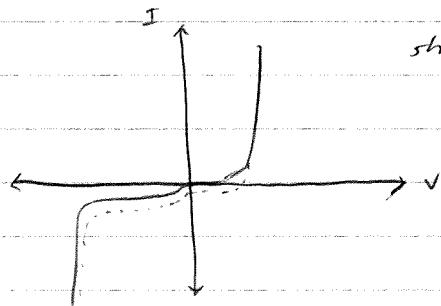
f) KE of electron or hole must be greater than E_g

2. b) $V_{TH} = -2\pi\sqrt{kT}/e$ $T \downarrow V_{TH} \uparrow$

$V_{BR} : T \downarrow V_{BR} \uparrow$ zero $T + V_{BR} \uparrow$ Avalanche

dI/dV : ? .

3. a)



shine light \Rightarrow shifts current law

$$I_t = I_{dark} - I_{ph}$$

b) producing energy. $I \cdot V < 0$

c) Solar cell.

d) use narrow base diode.

mainly

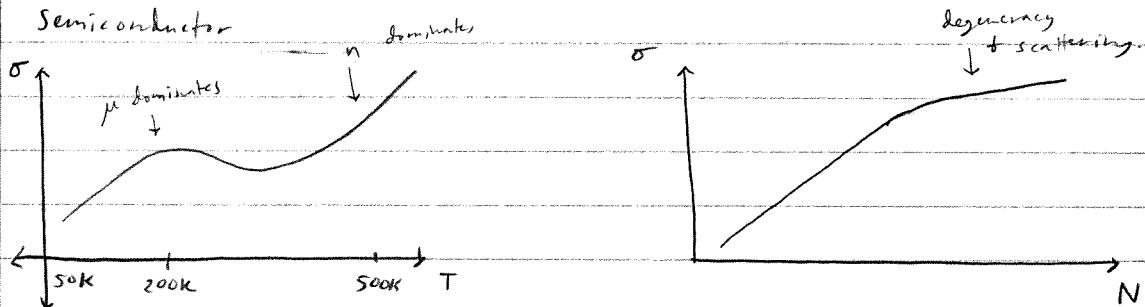
e) drift of minority carriers. Yes there is a contribution from the depletion region, which is the generation current from thermal generation inside the depletion region.

f) heterojunction.

2000 - Harris

1. Conductivity

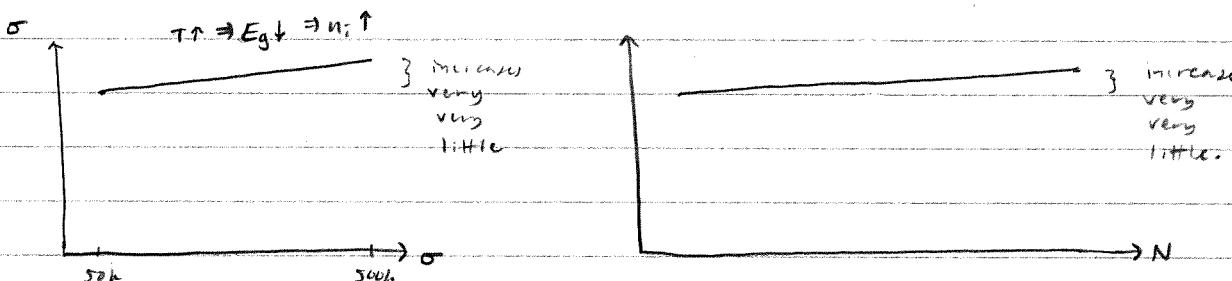
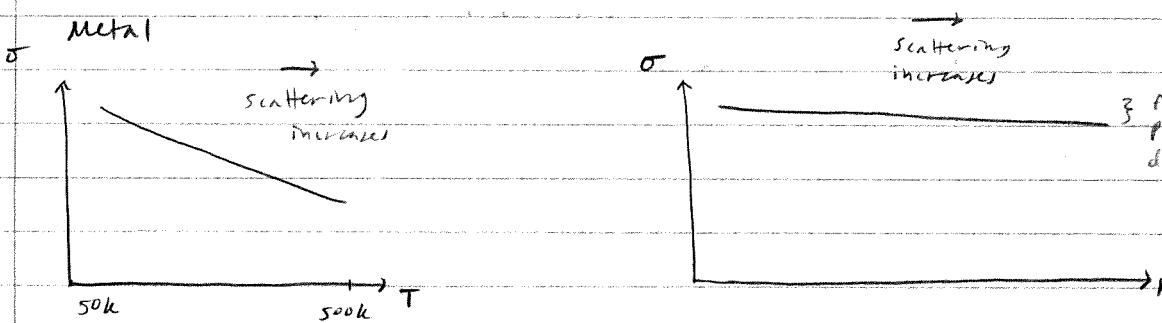
2. Semiconductor



$$\sigma = g \cdot n \cdot \mu$$

Annotations: 200k, 200k

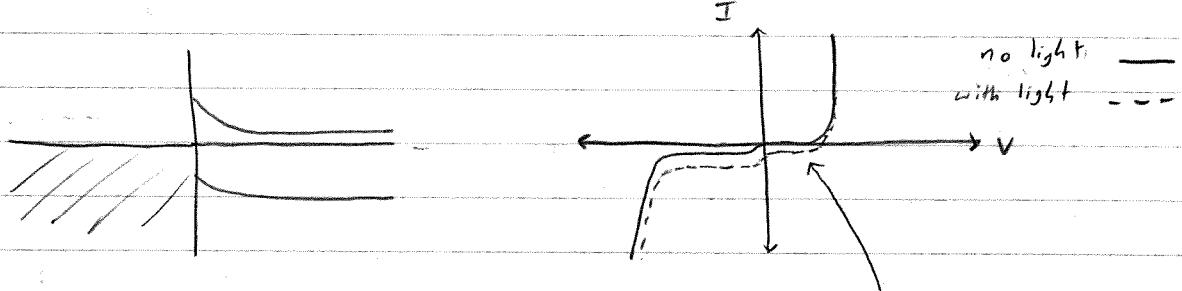
3. Metal



3. ideal properties of semiconductor

2002 - Harris

1. a)

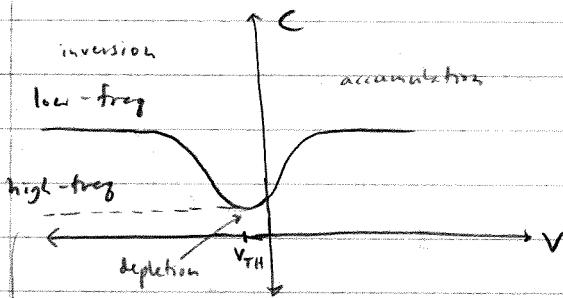


b) generation in depletion region \Rightarrow IV curve shifts down;

2. a) prevents current from flowing, and since there is a voltage drop across the insulator, you also reduce the band bending in the semiconductor.

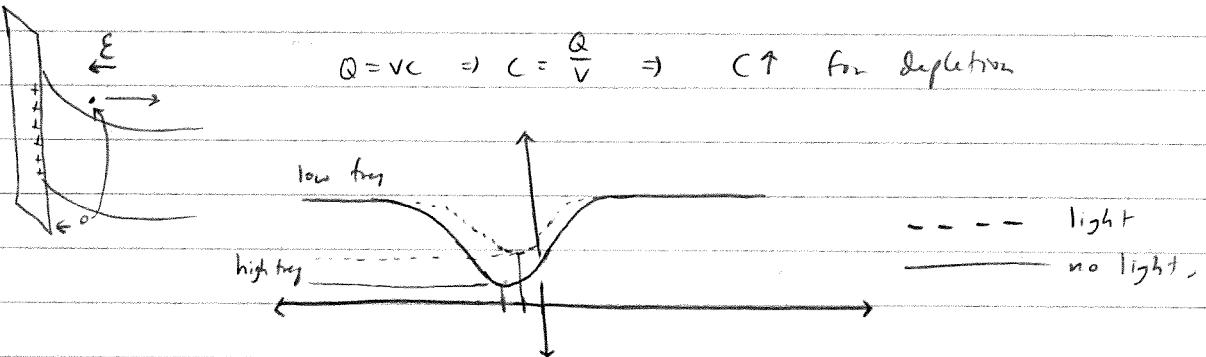
b) C-V measurement. Threshold voltage, doping, interface charge, thickness of oxide, V_b .

c)



minority carriers can't respond fast enough.

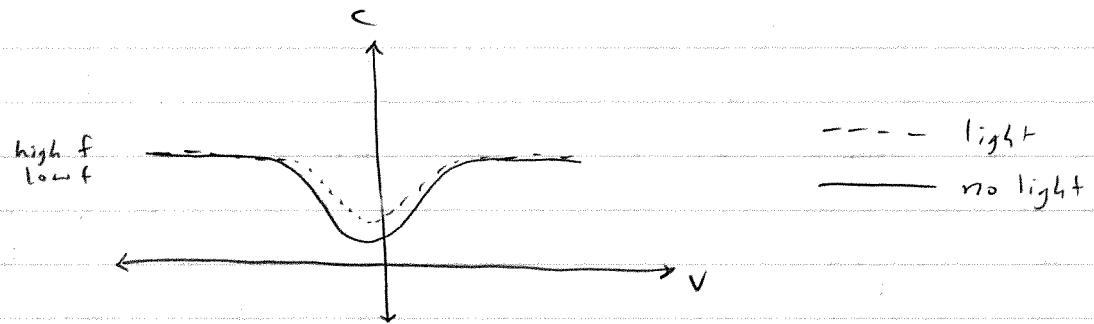
d)



When V is very negative or very positive, there is no depletion region, so the generated carriers just recombine and have no effect.

$|V_{TH}|$ decreases, $C_{\text{depletion}}$ increases.

3. Yes.



with light: $|V_{TH}|$ decreases, C_{DG} increases.

No difference between low + high freq, because now have source + drain to provide minority carriers.

IV - above V_{TH} - roughly the same.

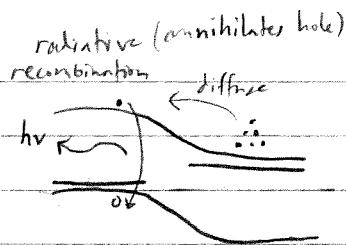
below V_{TH} - subthreshold current increases due to weak channel formed by h^+

by gate from photogeneration.

(can't turn MOSFET off very well).

2008 - Harris

1. Forward biased PN junction.



2. Different bandgap energy E_g . $\lambda = \frac{1.24}{E}$ $E_g \leq E_{\text{photon}} \leq E_g + kT$

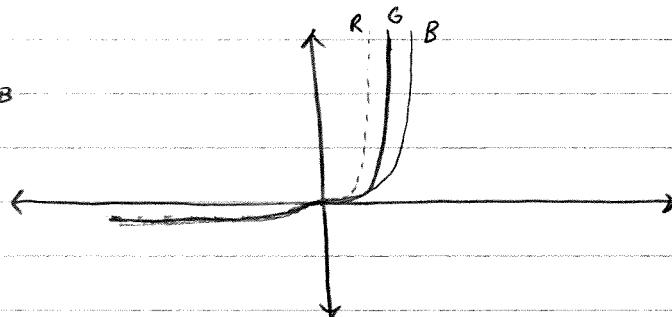
No. Doesn't have to be direct bandgap semiconductor (though these are much more efficient). If it's an indirect bandgap semiconductor, it must incorporate an optically efficient trap.

3. Can mix Red Green and Blue LEDs to get white light.

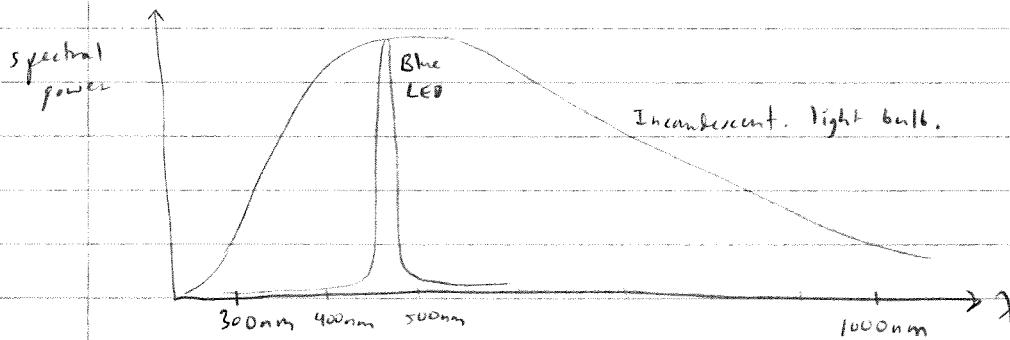
(Or, you can use a phosphor material to convert monochromatic light from a blue or UV LED to broad spectrum white light (much like how a fluorescent light bulb works).)

$$4. V_{TH} \approx \frac{2}{3} E_g \quad \lambda \propto \frac{1}{E_g} \Rightarrow f \propto E_g \Rightarrow E_{GR} < E_{GG} < E_{GB}$$

$$\Rightarrow V_{THR} < V_{THG} < V_{THB}$$



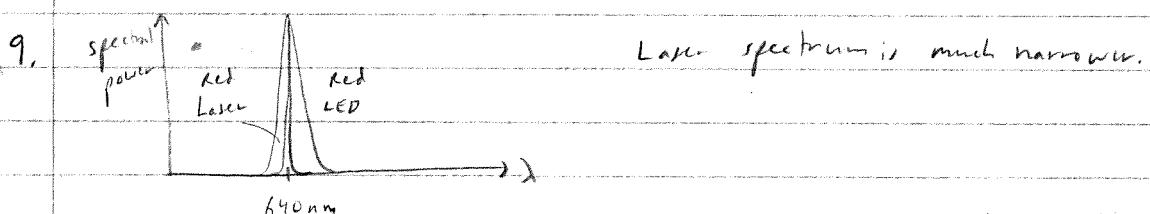
$$5. \lambda_R \approx 640\text{nm} \quad \lambda_G \approx 550\text{nm} \quad \lambda_B \approx 470\text{nm}$$



6. Kinetic energy of carriers (kT)

7. Temperature decreases $\Rightarrow E_g \uparrow \Rightarrow V_{TH} \uparrow$ and $\lambda \downarrow$ ($f \uparrow$)

also, the total integrated spectral power may decrease since there will be fewer energetic carriers available as $t \rightarrow \infty$.



8. The laser is also a p-n junction diode operating in the same basic manner as the LED except that it is inside a Fabry-Perot cavity which provides optical feedback and when the number of injected carriers is high enough (threshold current), a population inversion is reached and this, combined with the feedback produces stimulated emission and an optical oscillator or laser.

Below threshold, the laser is an LED - it emits only spontaneous, not stimulated photons.

The laser output is strongly directional because the output is largely controlled by the optical feedback from the Fabry Perot cavity, which only occurs over a very narrow angular range. Photons not emitted directly down the optical axis experience no feedback and gain.

10. Yes. The LED output will "change color" ($\lambda \downarrow$) since $E_g \uparrow$ as $T \downarrow$. The laser output is controlled by the optical feedback, hence the fabry-perot cavity resonant wavelength. Since the linear thermal expansion coefficient is very small compared to the bandgap shift, the laser output remains at the same wavelength until there is insufficient gain at this wavelength and then the laser either stops oscillating or it "jumps" to a different wavelength where there is another Fabry-Perot resonance and sufficient gain to oscillate at this new wavelength.

2001 - Please

1. - Scaling \Rightarrow increased precision and speed.
 - Digital circuits are less sensitive to noise.
 - Analog design is more specialized \Rightarrow doing a digital design can be more readily achievable.
 - Digital designs can sometimes be flexible (e.g. FPGA)
 - \Rightarrow can change the design to implement a fix or improve performance through software rather than hardware.
 - for storage digital can be exactly reproduced, and doesn't degrade over time.

2. - Sampling rate

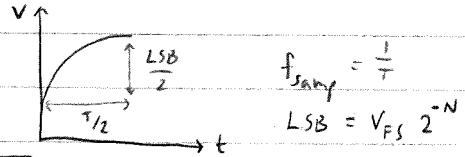
- Power dissipation
- Resolution (number of bits)
- Accuracy (LSB)
- Quantization Error
- Linearity (DNL, INL)
- Dynamic Range

3. Compares continuous input to a number of discrete levels (with a comparator). Based on these comparisons, a digital representation of the input is generated as the output.

Ideally, if the two inputs to a comparator are the same, the output will equal $\frac{V_{cc}}{2}$ which is an undetermined state (neither high nor low). (But in most practical comparators, finite mismatch will cause the output to either go high or low.)

4. Yes, the Heisenberg uncertainty principle does set a limit on the combination of high speed and high resolution.

Specifically, $\Delta p \Delta x \geq \hbar$ can alternatively be stated as $\Delta E \Delta t \geq \hbar$
in the context of an ADC, $\Delta E = \frac{(\text{LSB})^2}{R} \cdot \frac{T}{2}$ $\Delta t = \frac{T}{2} \Rightarrow \Delta E \Delta T = \frac{1}{R} \left(\frac{\text{LSB} \cdot T}{4} \right)^2 \geq \hbar$



$$f_{\text{samp}} = \frac{1}{T}$$

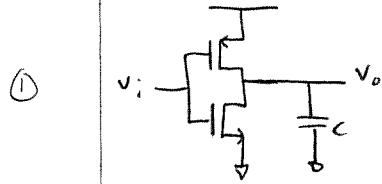
$$\text{LSB} = V_{FS} 2^{-N}$$

$$2^N \cdot f_{\text{samp}} \leq \frac{V_{FS}}{4\sqrt{\hbar R}}$$

Thermal noise limits resolution, Jitter noise limits conversion speed.

↑ Reduces noise & increases resolution as more transitions occur to increase resolution

2002 - Please



② $\frac{1}{2} C V_{dd}^2 + \frac{1}{2} C (-V_{dd})^2 = CV_{dd}^2$ is dissipated in the mosfets during the clock transitions.

Since $V_{gs} \downarrow$ for smaller devices, since $V_{th} \downarrow$ for smaller devices.

③ per cycle: $C + V_{dd} \downarrow \Rightarrow P \downarrow$

but f will increase. \Rightarrow power may actually increase since there will be more cycles.

also leakage current will increase as we scale down. So off current will increase and more static power will be dissipated.

④ 2nd law of thermodynamics:

in order to do work, you must dissipate power (increase entropy).

But to make a battery, we must reduce entropy. So we must find materials that have low entropy (high initial potential energy). And find ways to efficiently extract energy.

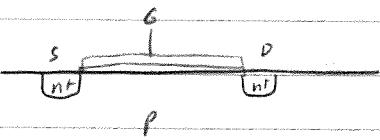
Doesn't benefit from scaling like IC's because you fundamentally need a certain amount of volume to store a certain amount of energy.

Could also try to use supercapacitors. But these tend to degrade after ~ 100 cycles.

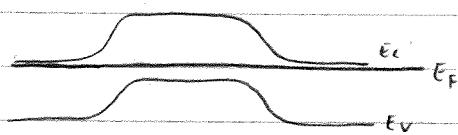
↓
some people are trying to
use graphene for this.

2007 - Harris

I. A.



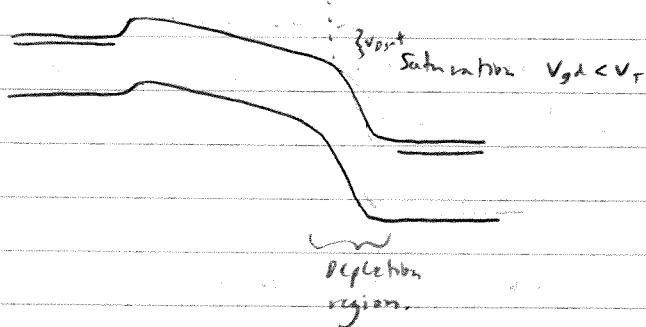
B.



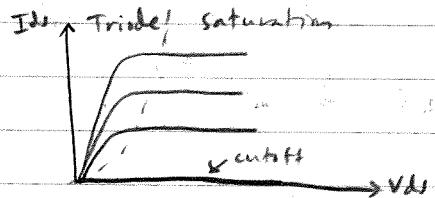
C.



D.



E.



cutoff: see (B)

Triode: see (D), current is due to drift of carriers in channel.
thus $Id = A \cdot nE \propto Vds$
since channel does not pinch off.

Saturation: see (D). current saturates since after pinch off, there is simply a reverse biased p-n junction. The drop across the channel from S to pinch off point is V_{dsat} . This voltage remains constant and determines the current Id_s .

2000 - Please

1. Make more money (smaller gate pitch \Rightarrow more chips per wafer)

Increase speed, reduce supply voltage, reduce power consumption, reduce size, Moore's Law.

2. Speed $\propto \frac{1}{L}$ since $I_{DS} \propto \frac{1}{L}$ ($\tau = \frac{CV}{I} \Rightarrow f_{max} = \frac{I}{CV}$)

$f = \frac{I}{CV}$
 $C = \frac{1}{k^2}$
 $I = \frac{k}{V}$
 $\Rightarrow f \propto V$

energy per clock cycle = $C V_{DD}^2$ \Rightarrow (only changes if interconnect V_{DD} reduces since V_{TH} + with scaling))

3. R only affects the charging time and not the total energy stored (dissipated) - since $E = CV^2$ per cycle.

4. No. This is equivalent to increasing the R. $E = CV^2$ regardless.

5. 1) atomic size (for thickness of oxide, length of channel, etc.)
2) Gate leakage (due to quantum tunneling)
3) Subthreshold current.

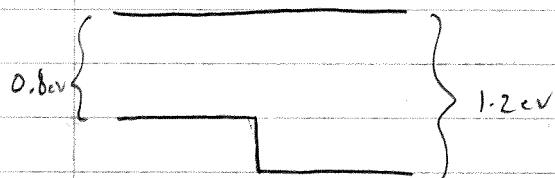
Thermodynamic limit: $\Delta Q = E = kT \ln(2)$

Heisenberg uncertainty: $\Delta p \cdot \Delta x > h$

$$\Delta E \cdot \Delta t > h \Rightarrow f_{max} \approx 25 \text{ THz}$$

$$\Delta x_{min} \sim 1.5 \text{ nm}$$

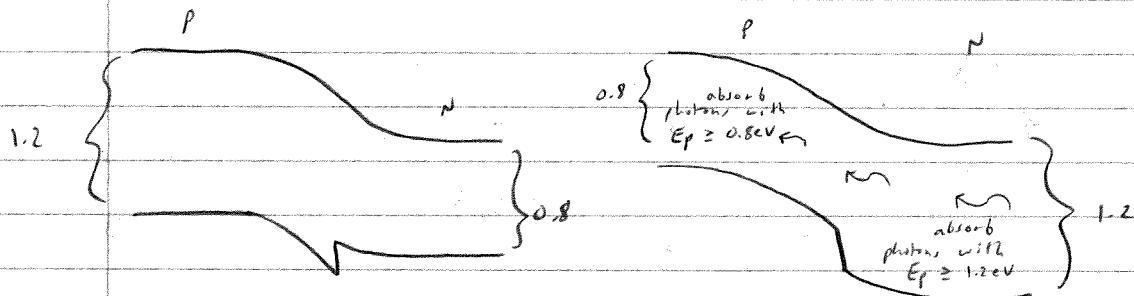
Pre 1993 - Harris



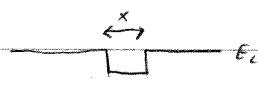
Heterojunction is junction of two materials with different bandgaps (but typically with matching lattice constants)

Could be used for HBT: increases emitter efficiency.

Could be used for LED or solar cell: wide band gap material acts as a transparent window to allow photons in/out without causing generation.



assume $\chi_1 = \chi_2$ in these sketches.



what happens as x gets small? it will behave

as a quantum well \Rightarrow energy states will become discrete rather than continuous.

can use these structures to make a laser. trap charges in quantum well to get population inversion.

Boundary conditions for Poisson's Eq:

$$\nabla \cdot E = \frac{\rho}{K_s \epsilon_0}$$

$$\frac{\partial E}{\partial x} = \frac{\rho}{K_s \epsilon_0}$$

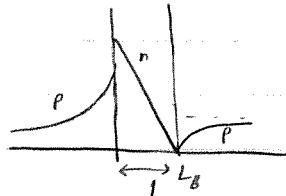
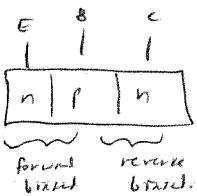
for 1-D.

- for MOSFET what's the limit to band bending in depletion?

$$(E_c - E_F) > (E_F - E_V)_{V_G=0}$$



- How does NPN BJT work?



$$\text{Emitter efficiency: } \gamma = \frac{I_{E\bar{p}}}{I_{E\bar{p}} + I_{E\bar{n}}}$$

determined by doping of base relative to doping of collector.

Why not metal-p-n



$$\gamma = \frac{I_{E\bar{n}}}{I_{E\bar{n}} + I_{E\bar{p}}} \Rightarrow \text{terrible emitter efficiency for metal-p-n}$$

1996 - Please

1. MOSFET : Digital + Analog CMOS,

and MMIC / RF

BJT : High performance analog! Not often used as switch due to base current.

JFET : Voltage controlled resistor.

2. Zero off current. Zero gate leakage. High gain.

High switch speed (fast turn on + off transient).

Small
gate pitch

⇒ more
parallel

3. Gate leakage : tunneling due to thickness of gate. but there will always be some chance of tunneling.
(quantum). set by physics.

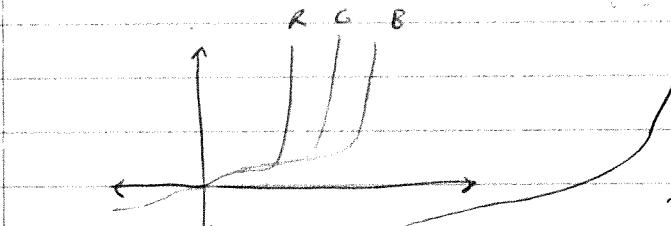
Switching speed : determined by technology (gate length, mobility, parasitic capacitance, inter-connect parasitics, isolation)

4. Yes. Much better on/off ratio. OFF current of a relay is very close to zero. Roger Howe makes these. Can be lower power than MOSFET for switching. Can tolerate larger temperature variations than MOSFET.

problem is mechanical reliability / endurance.
has finite number of switching cycles.

2010 - Harris

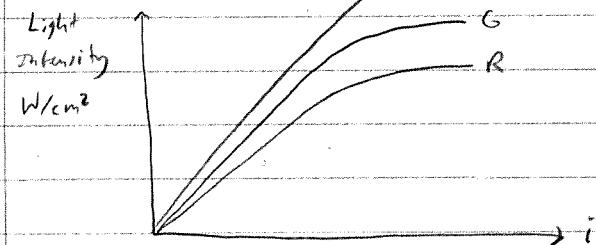
1.



L saturates at high i ,
since as $i \uparrow \Rightarrow T \uparrow \Rightarrow \tau_r \downarrow$
 \Rightarrow nonradiative recombination (through traps) increases since $\tau_r \downarrow$

$$\tau_r = \frac{1}{v_{th} g \cdot N_T} \xrightarrow{\frac{2}{3} kT} \Rightarrow \text{proportion of radiative recombination decreases.}$$

2.

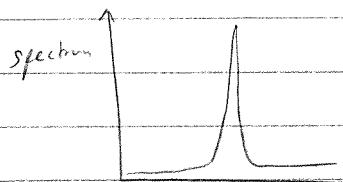


$$\lambda = \frac{1.24}{E}$$

$$\Rightarrow E = \frac{1.24}{\lambda}$$

\Rightarrow Blue light has higher energy photons than green + red light.

3.



$$h\nu > E_6 \quad \text{so} \quad h\nu = E_g$$

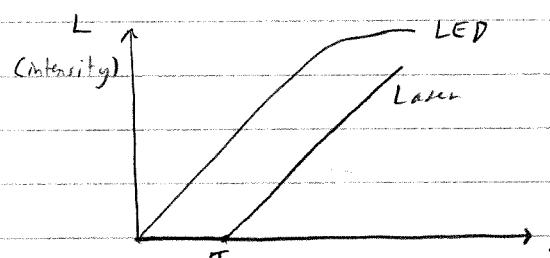
$$\text{why is } \lambda_{max} = \frac{1.24}{E_6}$$

since you can't have carriers in Bandgap \Rightarrow minimum energy difference from recombination is E_6 .

4.

5. For laser: must heavily dope to achieve population inversion, must use direct bandgap material.
Need a Fabry-Pérot resonance cavity.

6.



7.

$I_{TH} \rightarrow$ needed to achieve population inversion.

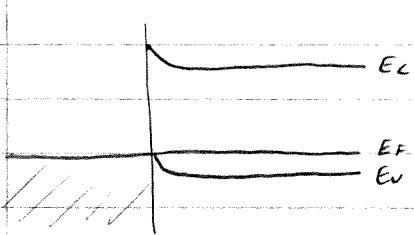
8. Below I_{TH} , the laser acts as a simple LED (only spontaneous emission, not stimulated emission). Thus, you can use it as a solar cell.

For solar cell, want to maximize diffusion length to collect carriers.

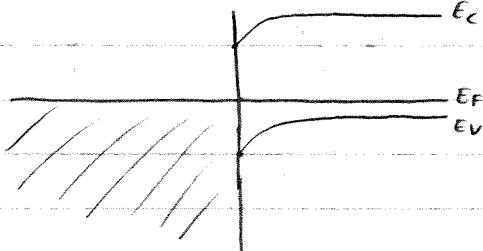
(for LED + laser you want recombination. For a solar cell, you don't).

2003 - Vuckovic

1. - $\Phi_m > \Phi_s \Rightarrow$ ohmic (contact)
- $\Phi_m < \Phi_s \Rightarrow$ rectifying (diode)



$\Phi_m > \Phi_s$ Ohmic



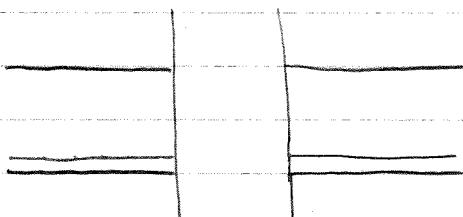
$\Phi_m < \Phi_s$ Rectifying.

2. Inversion : Diagram 2 , Bias point d

Accumulation : Diagram 4 , Bias point a

Depletion : Diagram 3

3.



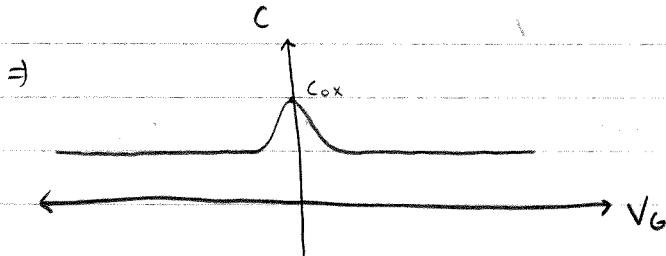
$V_G = 0$

depletion, inversion

accumulation.

$V_G < 0V$

(mirror image for $V_G > 0V$) due to symmetry.



4. Intuitively, this should behave like a PN diode.

$V_C = V_E \Rightarrow V_{BE} = V_{CE} \Rightarrow$ BJT can only operate in

E-B

C-B

Saturation ($V > 0$) \Rightarrow Forward Biased

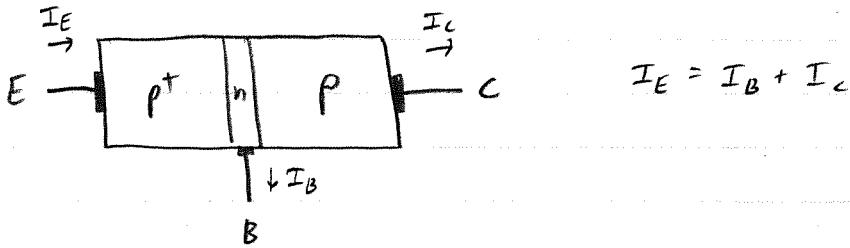
Forward Biased

OR

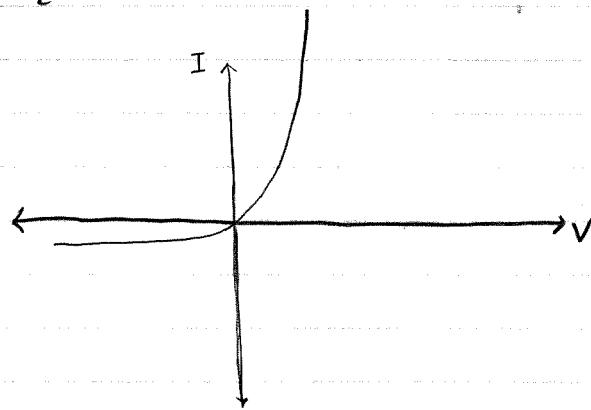
Cutoff ($V < 0$) \Rightarrow Reverse Biased

Reverse Biased.

$$\beta = \frac{I_c}{I_B} \quad \alpha = \frac{I_c}{I_e}$$



$$I = I_B = I_e - I_c$$



Ebers-Moll: $I_E = I_{F0} (e^{V_{EB}/V_T} - 1) - \alpha_R I_{R0} (e^{V_{CB}/V_T} - 1)$
 $I_C = \alpha_F I_{F0} (e^{V_{EB}/V_T} - 1) - I_{R0} (e^{V_{CB}/V_T} - 1)$

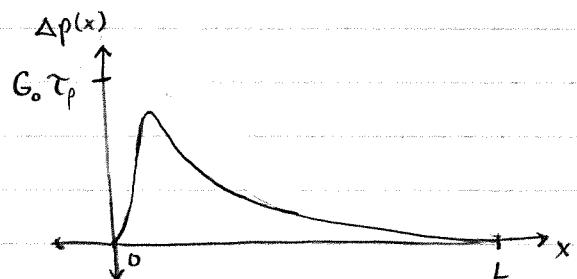
\Rightarrow in this problem, $V_{EB} = V_{CB} = V$

$$= I_E - I_C = [I_{F0} (1 - \alpha_F) + I_{R0} (1 - \alpha_R)] (e^{V/V_T} - 1)$$

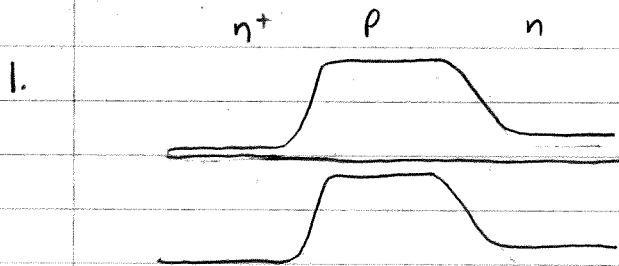
$$\frac{\partial \Delta p_n}{\partial t} = D_p \frac{\partial^2 \Delta p_n}{\partial x^2} - \frac{\Delta p_n}{\tau_p} + G_L(x) \Rightarrow 0 = D_p \frac{\partial^2 \Delta p_n(x)}{\partial x^2} - \frac{\Delta p_n(x)}{\tau_p} + G_0 e^{-kx}$$

Boundary Conditions: $\Delta p_n(0) = \Delta p_n(L) = 0$

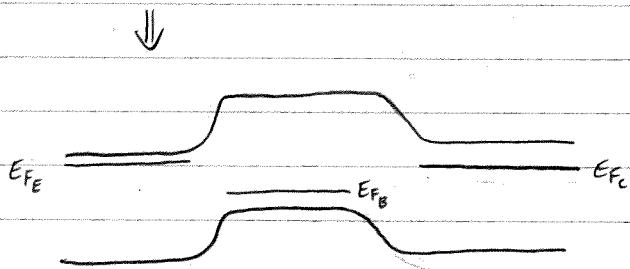
Rough Sketch:



2006 - Penmans



Linear region \Rightarrow Saturation \Rightarrow BE and BC both forward biased.



$$\text{Current gain: } \beta = \frac{I_c}{I_b} = \frac{\alpha_{dc}}{1 - \alpha_{dc}}$$

$$\alpha_{dc} = \gamma \alpha_T \quad \gamma = \frac{IE_n}{I_E} \quad \text{emitter efficiency}$$

$$\alpha_T = \frac{I_{Cn}}{I_{E_n}} \quad \text{base transport factor.}$$

Can improve current gain β by:

$$\text{decrease base doping} \Rightarrow N_E \gg N_B \Rightarrow \gamma = \frac{1}{1 + \frac{D_E}{D_B} \frac{N_B}{N_E} \frac{W}{L_E}} \uparrow$$

$$\text{decrease base width} \Rightarrow W \ll L_B \Rightarrow \alpha_T = \frac{1}{1 + \frac{1}{2} \left(\frac{W}{L_B} \right)^2} \uparrow$$

$$\Rightarrow \beta_{dc} = \frac{1}{\frac{D_E}{D_B} \frac{N_B}{N_E} \frac{W}{L_E} + \frac{1}{2} \left(\frac{W}{L_B} \right)^2} \uparrow$$

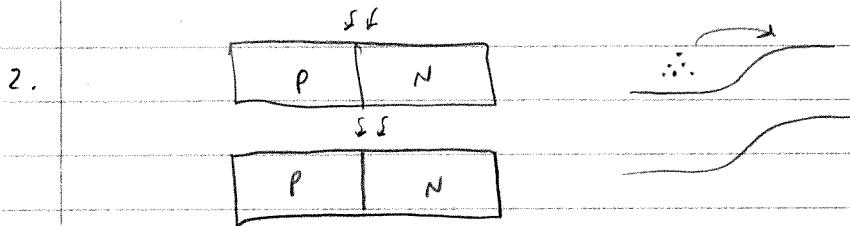
2. Strange because it seems to violate conservation of energy. But it actually doesn't. The extra energy comes from the KE (due to kT) of the carriers.

? electron gain
3.

$$I = \sqrt{\frac{2Vg}{m_e^*}} \times n \times g = \sqrt{\frac{2Vg}{m_{e_0}}} \times n \times g$$

2007 - Peumans

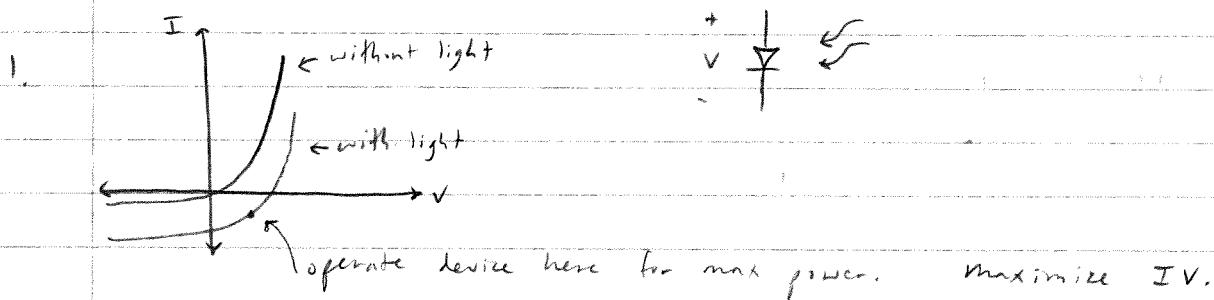
1. Yes, due to KE of carriers.



sunlight \rightarrow heat \rightarrow $T \uparrow \Rightarrow kT \uparrow \Rightarrow KE \uparrow$

No

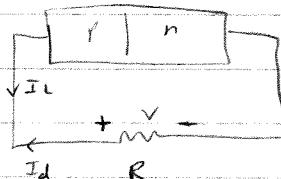
2008 - Penmans



What are the physical constraints between light in and e^- out.
 (physical effects that must be considered)

- absorption coefficient α (and spectrum)
- diffusion length
- length of p and n regions.
- width of depletion region.

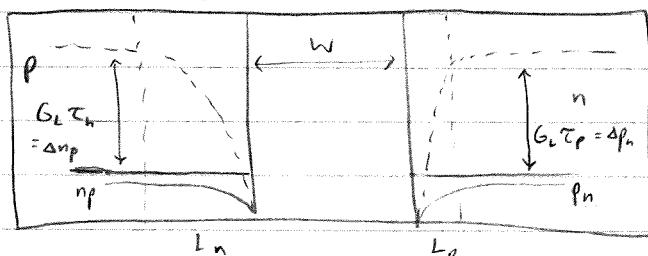
$$I_{\text{total}} = I_L - I_d$$



$$V_{oc} = \frac{kT}{q} \ln \left(\frac{I_L}{I_s} \right) \quad I_{sc} = I_s e^{\frac{V_{oc}}{N_v}}$$

— without light
-- with light.

reverse biased:



$$L_n > L_p \text{ for Si}$$

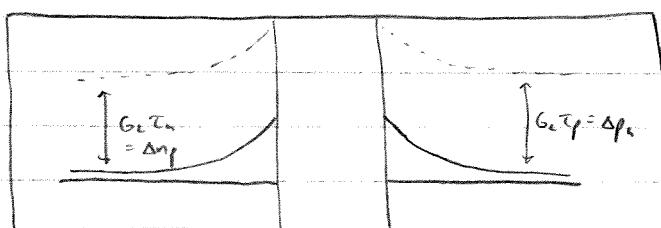
since $\mu_n > \mu_p$

$$L_n = \sqrt{D_n \tau_n}$$

$$= \sqrt{(\mu_n \frac{kT}{q}) \tau_n}$$

"optimum bias":

slight forward bias



What determines V_{max} ? $V_{oc} = \frac{kT}{q} \ln \left(\frac{I_L}{I_s} \right) \Rightarrow T, I_L \text{ and } I_s$.

How can you get more voltage? Increase T, light intensity, or decrease I_s .

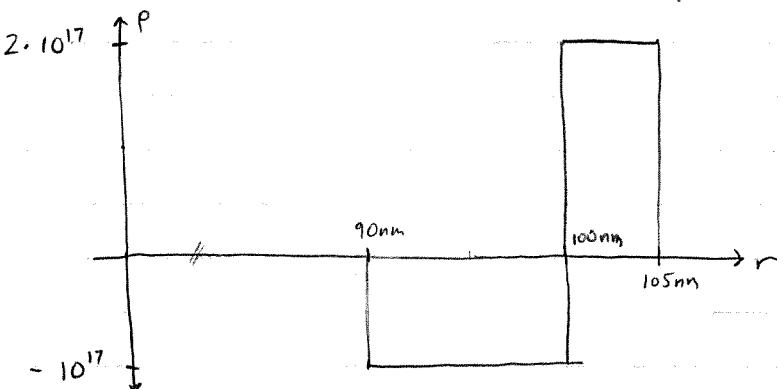
2006 - Howe

a) Assume: $x_p \ll R \Rightarrow$ can approximate with a 1-D model.

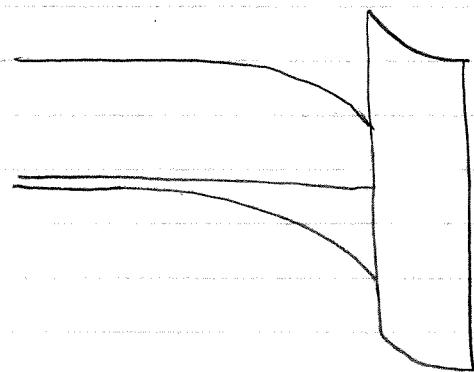
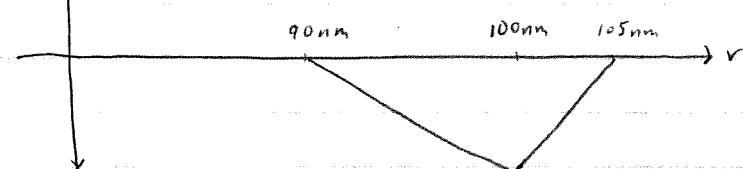
If you assume
the holes are
somewhat able
to leave
the wire.

assume
 E field outside
wire = 0.

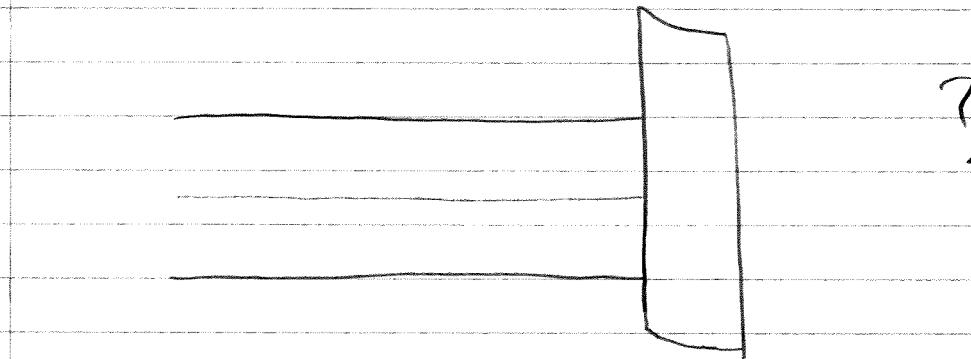
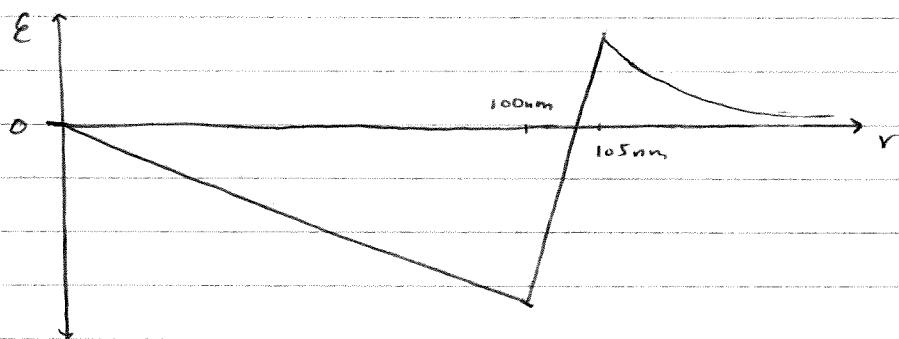
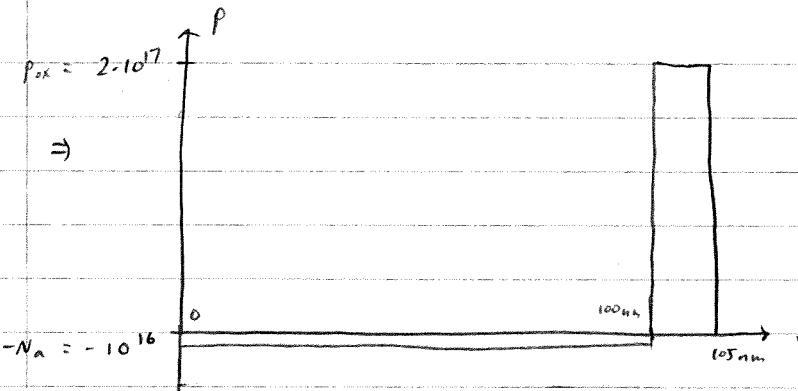
$$p_{ox} \approx \frac{10^{11} \text{ cm}^{-2}}{t_{ox}} \quad \text{since } t_{ox} \ll R \Rightarrow p_{ox} = \frac{10^{11} \text{ cm}^{-2}}{5 \cdot 10^7 \text{ cm}} = 2 \cdot 10^{17} \text{ cm}^{-3}$$



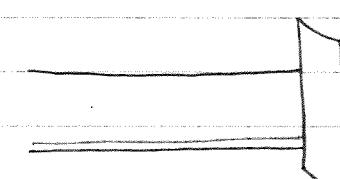
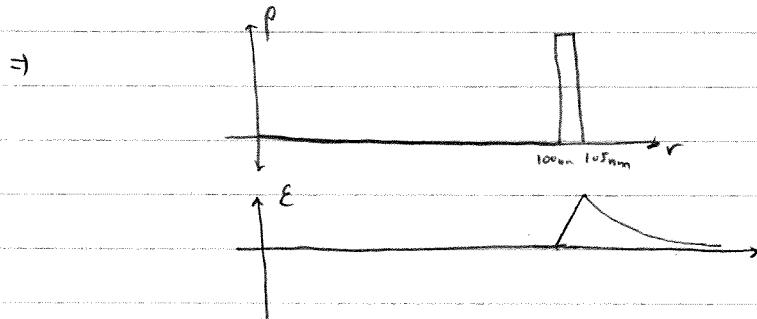
$$x_p = 10 \text{ nm} \ll R = 100 \text{ nm} \checkmark$$

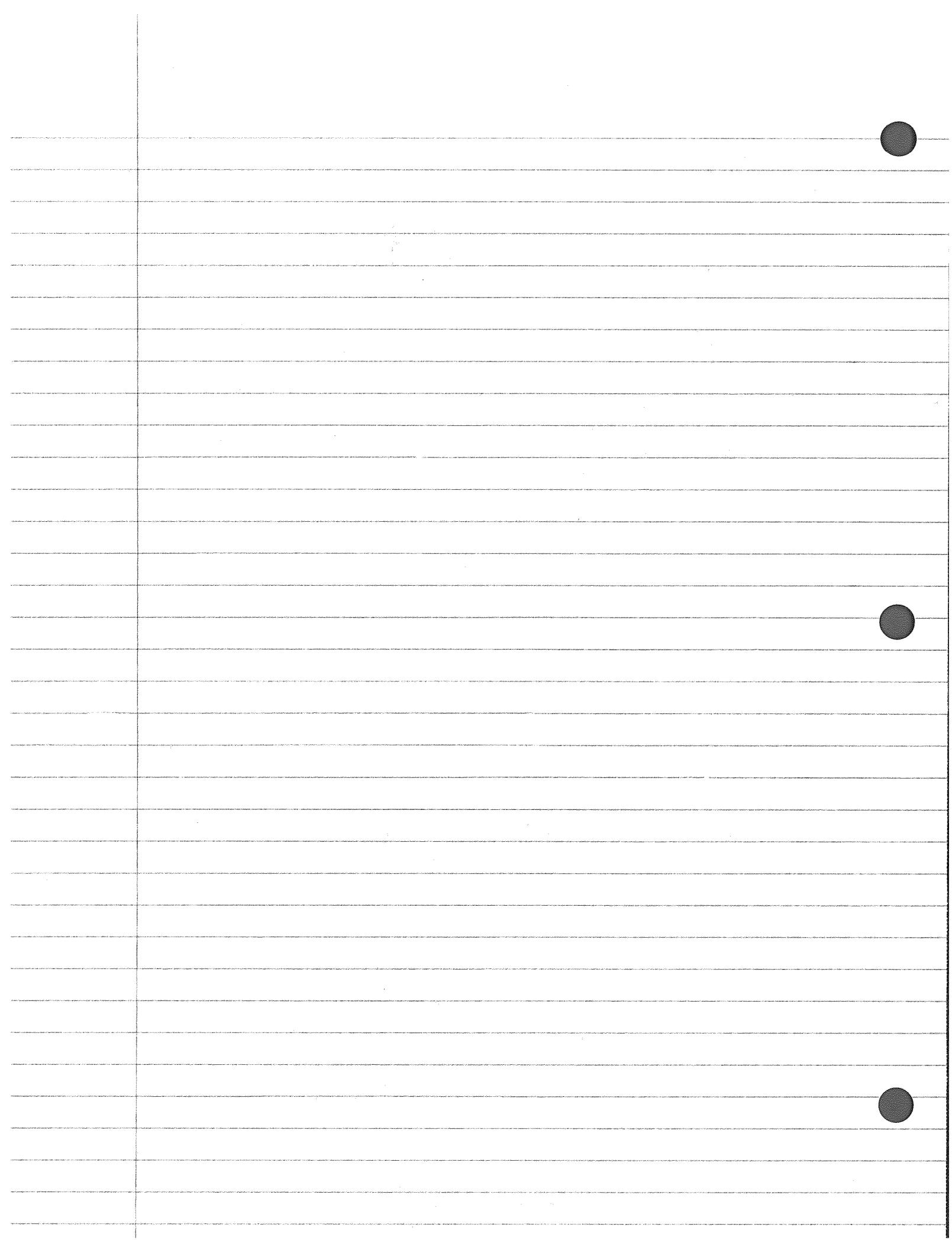


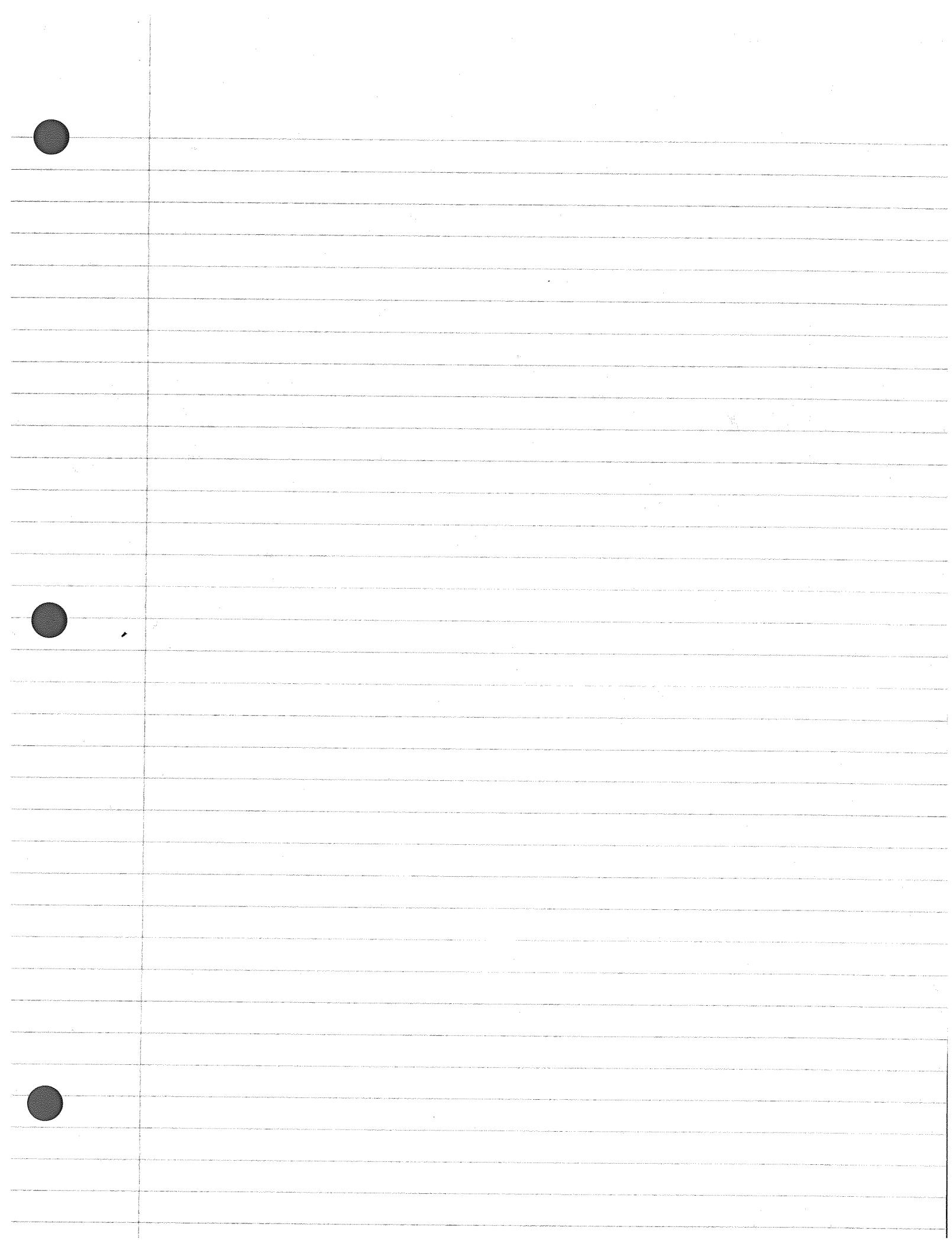
If $N_a = 10^{16} \text{ cm}^{-3}$, depletion region will extend all the way to
the center of the cylinder, since $R \cdot N_a = 10 \cdot t_{ox} \cdot N_a < t_{ox} p_{ox}$
 $= 20 t_{ox} N_a$.

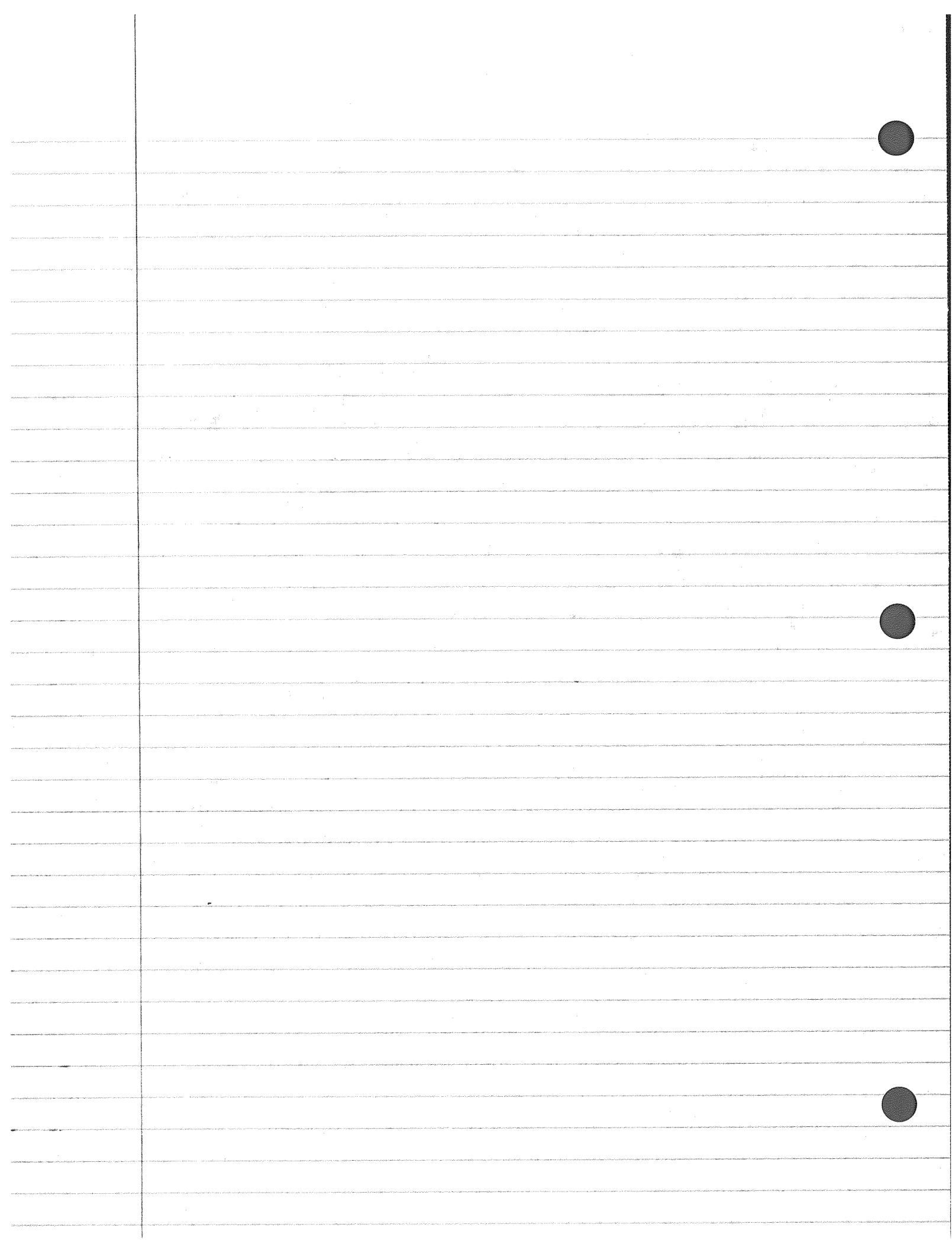


OR If you assume that holes cannot be removed, E outside the wire is positive + E field at R is 0, due to Gauss's Law.









2006 Vuckovic

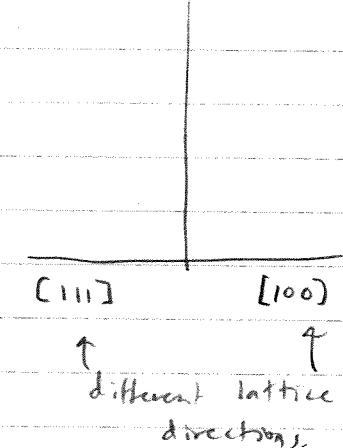
2. Calculated with quantum mechanics. Assume wave functions of electrons in a crystal lattice have a Bloch form

$$\Psi_k(\vec{r}) = e^{i\vec{k}\cdot\vec{r}} u_k(\vec{r}) \quad \leftarrow \text{periodic since crystal lattice is periodic.}$$

Apply periodic boundary conditions.

Along the curves, we have the allowable energy states for various momentum values. Probability of electrons is zero in band gaps and higher in valleys than in peaks. Probability distributions of electrons in space in such materials is periodic.

\vec{k} - wave vector ($\vec{k} = h \cdot \vec{p}$)



These graphs basically show the allowed states for electrons and holes at different momentum for different lattice directions.

a) 1.42 eV

b) $L \quad m^* = \frac{1}{\hbar^2 \cdot \frac{d^2 E}{dk^2}} \Rightarrow$ sharper curve \Rightarrow lower m^*

c) 10^{18} cm^{-3} dopants will be fully ionized.

(but actually $N_c = 4 \cdot 10^{17}$ so this doping is actually degenerate. So the concentration of electrons in the conduction band should be less than 10^{18} cm^{-3})

d) $n = N_c \exp\left(\frac{E_F - E_c}{kT}\right) \approx 10^{-5}$

assume $E_F = E_D \Rightarrow n_L = N_c \cdot \exp\left(\frac{1.4 - 1.71}{kT}\right) \approx 10^{13}$

if we assume $n_F \approx 10^{18}$, $\Rightarrow \frac{n_c}{n_F} \approx 10^{-5} \Rightarrow 0.001\%$

dopant momentum is uniform in \vec{k} space, since the location is fixed (Heisenberg uncertainty principle).

e) GaAs has a direct bandgap
Si has an indirect bandgap.

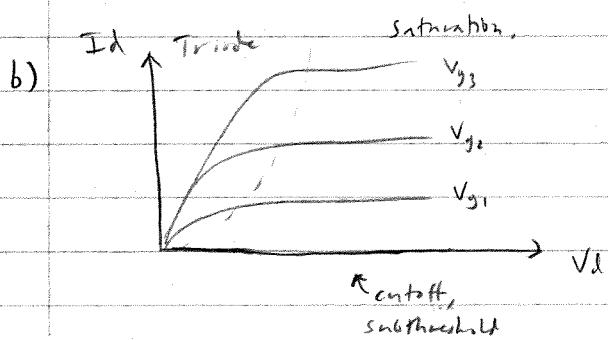
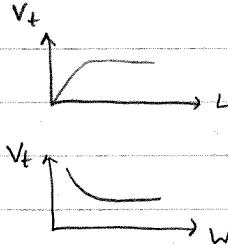
2004 Vuckovic

1. a) $N_A \uparrow, V_T \downarrow$

$t_{ox} \uparrow, V_T \uparrow$

$L \downarrow, V_T \downarrow$

$W \downarrow, V_T \uparrow$



c) Subthreshold: diffusion. Some energetic electrons diffuse into "channel" region above subthreshold: drift.

+ surmount barrier +

above subthreshold: drift.

$$\text{Subthreshold: } I_D = I_0 e^{g\left(\frac{V_G - V_T}{kT}\right)} (1 - e^{-\delta V_D/kT})$$

	Sub	above sub
$V_G \uparrow$	\uparrow	\uparrow
$V_T \uparrow$	\uparrow	\uparrow

d) Yes. $V_G \uparrow \Rightarrow I_D \uparrow$ because you have more inversion

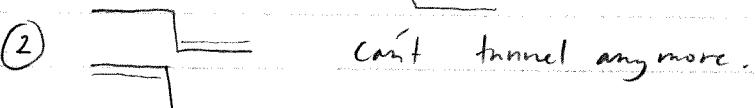
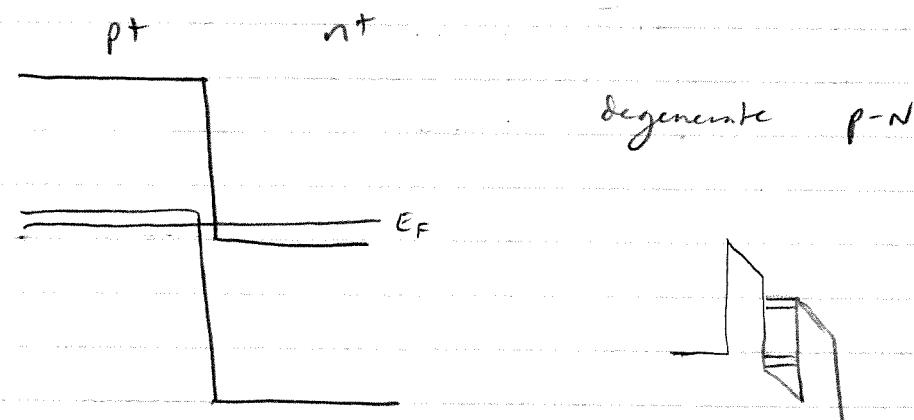
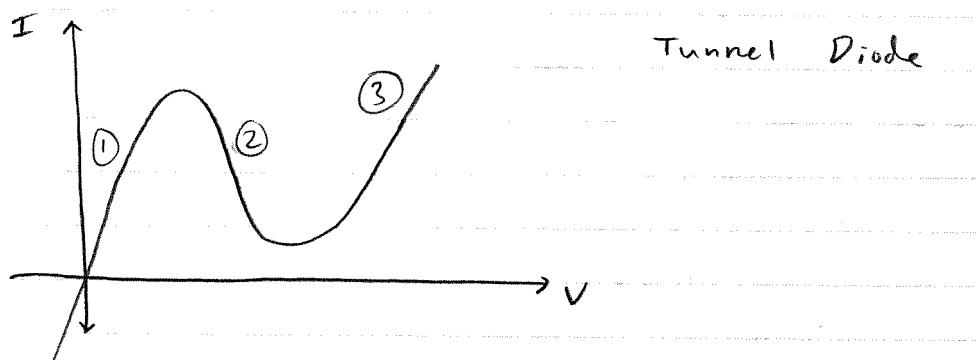
layer charge in the channel. + the more charge you have in the channel, the greater your drift current.

e) Yes. $\leftarrow ?$. Look into this.

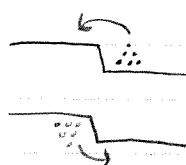
f) $I_D \downarrow$ because barrier to diffusion is increased. ($V_{BS} \uparrow$)

g) $I_D \uparrow$ because barrier to diffusion is increased ($V_{BS} \downarrow$)

2.

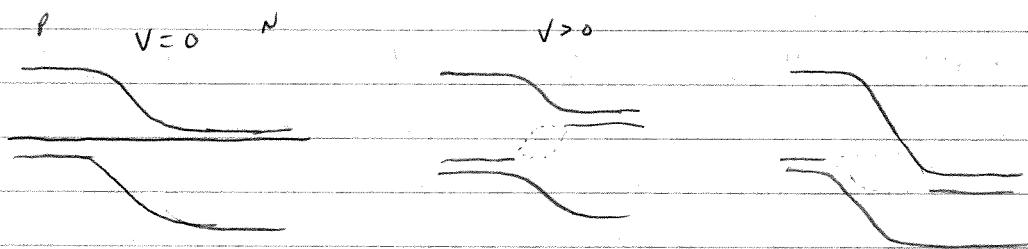


③ Diffusion current. (Forward biased PN)



2003 - Leumann's

(1)



(2)

$$I = I_0 (e^{\frac{qV}{kT}} - 1)$$

$$I_0 = gA n_i^2 \left(\frac{D_p}{L_p} \frac{1}{N_d} + \frac{D_n}{L_n} \frac{1}{N_A} \right)$$

(3)

$$I_{\text{total}} = I_{\text{diff}} - I_{\text{ph}} \quad \text{when you shine a light on it.}$$

(4)

$$\eta = \frac{P_{\text{out,full}}}{P_{\text{input,full}}} = \frac{I_{\text{max}} V_{\text{max}}}{(\text{light intensity}) A} \quad \text{solar cell efficiency.}$$

(5)

$\sim 10\%$. (actually typically 20-40%)

1993 - Pianetta

① n-type GaAs

① N_D , shallow

② N_D , 0.25 eV below E_C and shallow.

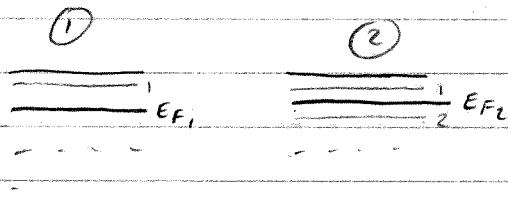
Just look at conductivity at 300K.

\Rightarrow for ②, doping is $10kT$ below $E_C \Rightarrow$ very little ionization
but shallow dopants ionize \Rightarrow will have higher conductivity.

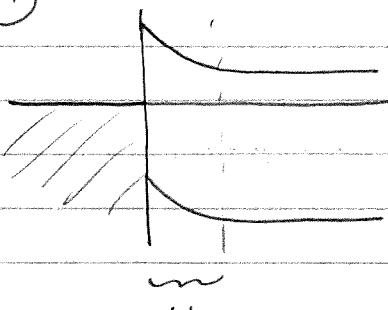
① will have higher conductivity since its dopants will ionize.
can't tell the difference between ① + ② in terms of just conductivity.

②

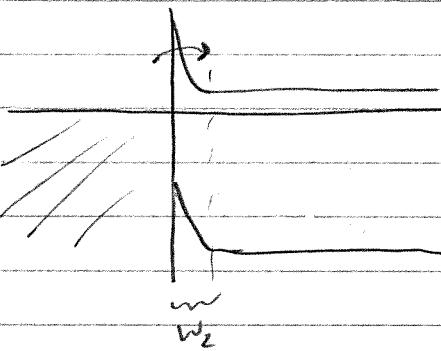
$$E_{F_2} > E_{F_1}$$



①



②



$W_2 < W_1$ because there
are more dopants in ②
 \Rightarrow shorter depletion region.

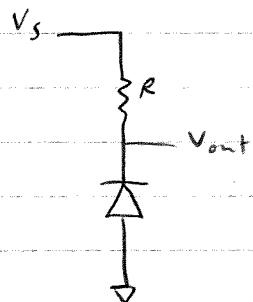
More tunneling

$$\Rightarrow I_2 > I_1$$

\Rightarrow You can distinguish ① from ② by
doing a contact resistance measurement. ($I_2 > I_1$
for a given V_{Applied})

- For low doping CV measurement would measure depletion width
- For high doping, tunneling occurs \Rightarrow measure current.

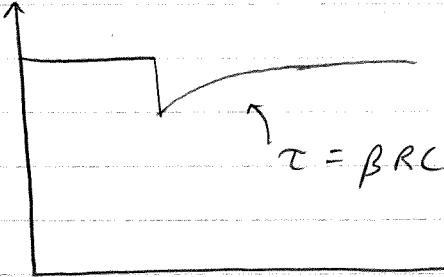
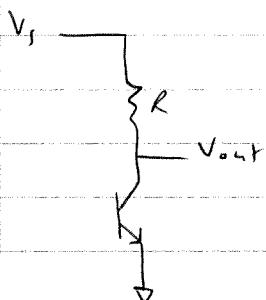
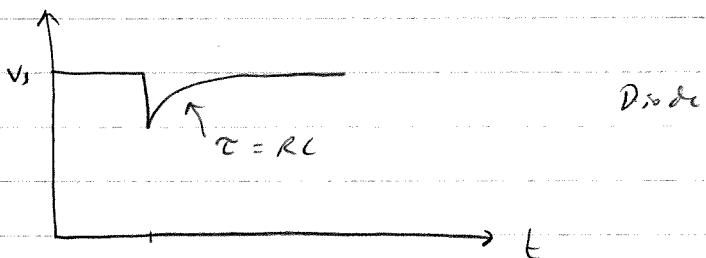
2002 - Miller



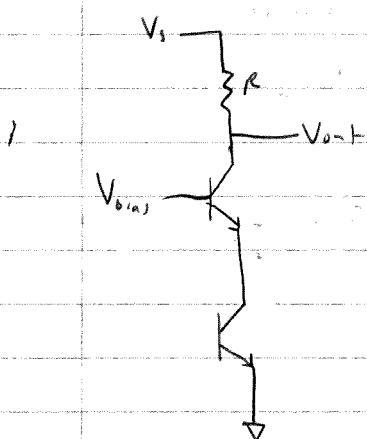
$$\text{dark: } V_{\text{out}} = V_s - I_o R$$

$$I_o = 0 \Rightarrow V_{\text{out}} = V_s$$

$$\text{Light pulse: } V_{\text{out}} = V_s - I_L R$$



possible solution: use cascade.

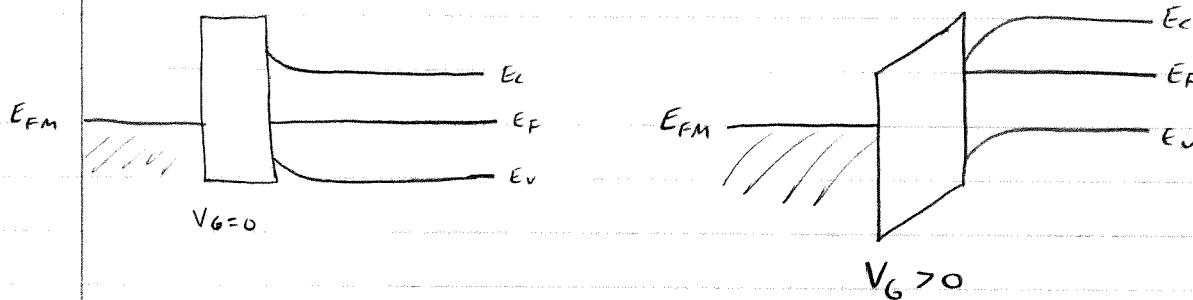


1995 - Pianetta

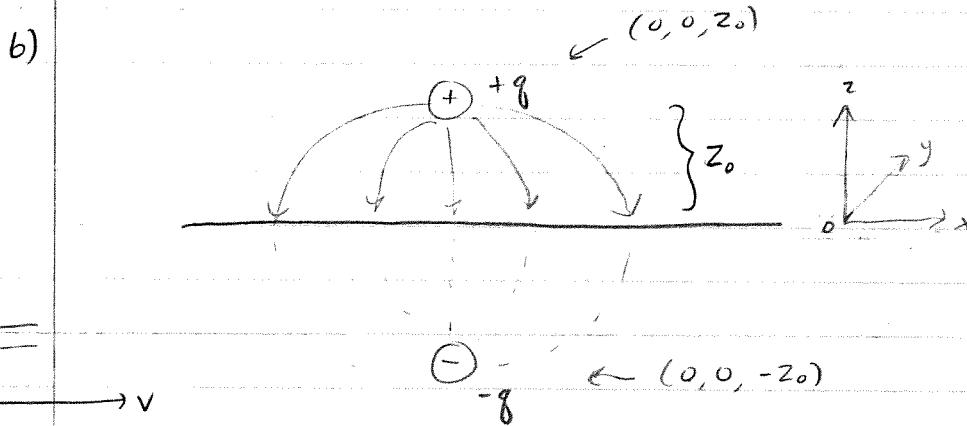
1. a) $1\text{\AA} = 10^{-10}\text{m} = 0.1\text{nm} \Rightarrow 100\text{\AA} = 10\text{nm}$

Doesn't mention doping \Rightarrow assume intrinsic:

Similar to MoS₂. Assume $\Phi_M > \Phi_{Si}$



b) Assuming a conductivity inversion layer is formed



potential in cartesian coordinates: (let $V=0$ for $z=0$)

$$V(x, y, z) = \frac{(q/4\pi\epsilon_0)}{\sqrt{x^2+y^2+(z-z_0)^2}} - \frac{(q/4\pi\epsilon_0)}{\sqrt{x^2+y^2+(z+z_0)^2}}$$

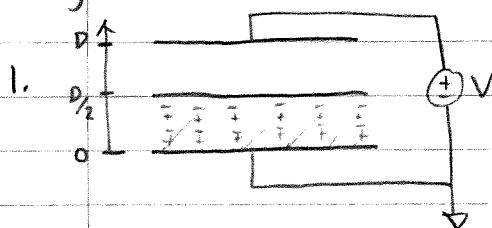
$$E_z(x, y, 0) = -\frac{\partial}{\partial z}(V(x, y, z))|_{z=0} = -\frac{2(q/4\pi\epsilon_0)z_0}{(x^2+y^2+z_0^2)^{3/2}}$$

$$\sigma_{induced}(x, y) = -\frac{(qz_0/2\pi)}{(x^2+y^2+z_0^2)^{3/2}}$$

$$\Rightarrow \sigma_{induced}(r) = \frac{-qz_0 r}{(r^2+z_0^2)^{3/2}} \Rightarrow \text{falls off as } \frac{1}{r^2}$$

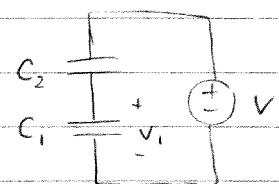
electrons collect mostly right below probe tip with circular symmetry.

1996 - Pianetta



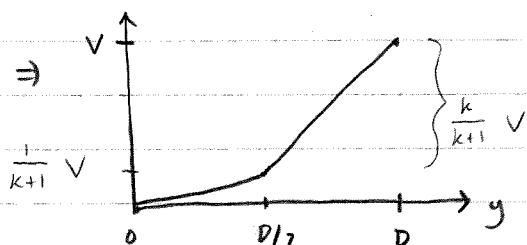
$$E = -\nabla V$$

$$\Rightarrow V(y) = - \int E(y) dy$$



$$E(y) = \begin{cases} E_1, & 0 < y < D/2 \\ E_2, & D/2 < y < D \end{cases}$$

$E_1 < E_2$ due to dielectric
(dielectric constant $k > 1$)



$$C_1 = \frac{k \epsilon_0 A}{(D/2)}$$

$$C_2 = \frac{\epsilon_0 A}{(D/2)}$$

$$V_1 = \frac{C_2}{C_1 + C_2} V = \frac{1}{k+1} V$$

$$V_2 = \frac{C_1}{C_1 + C_2} V = \frac{k}{k+1} V$$

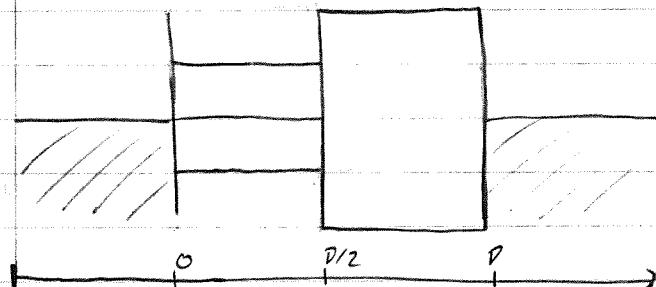
2.

Metal Si Air

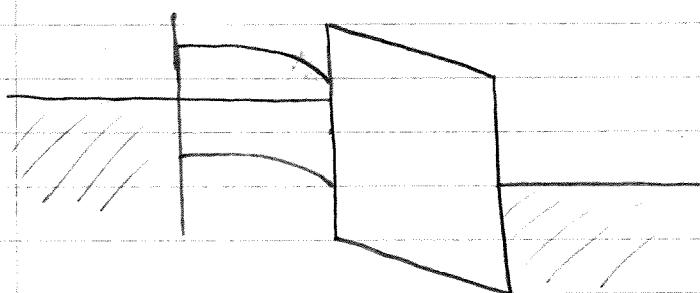
Metal

Assume $\Phi_s = \Phi_a$

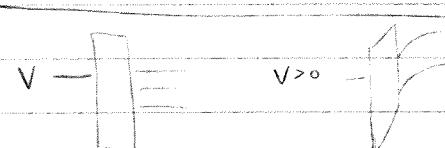
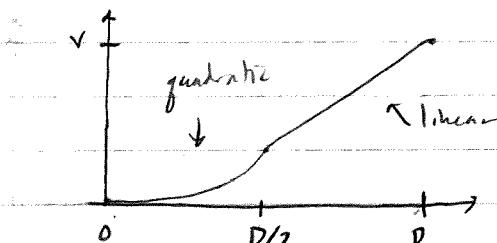
for simplicity



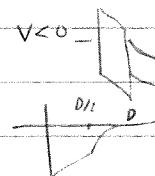
$$V = 0$$



$$V > 0$$

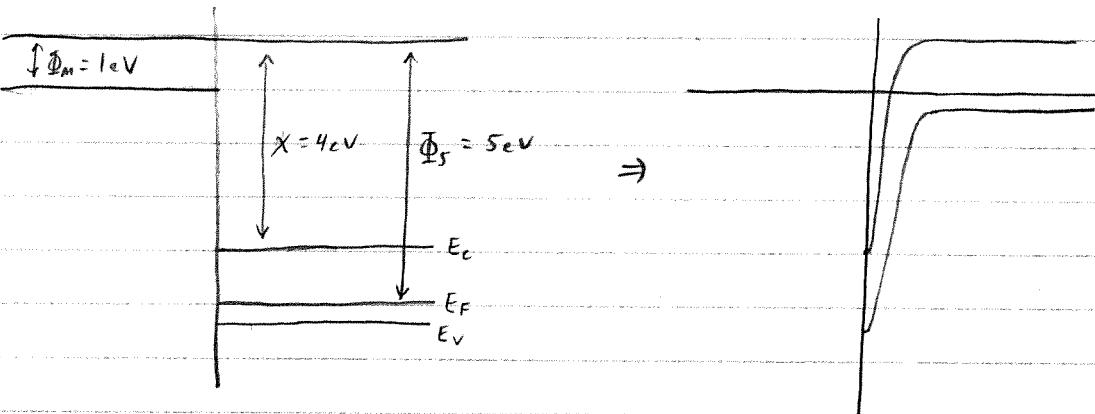


$$V > 0$$



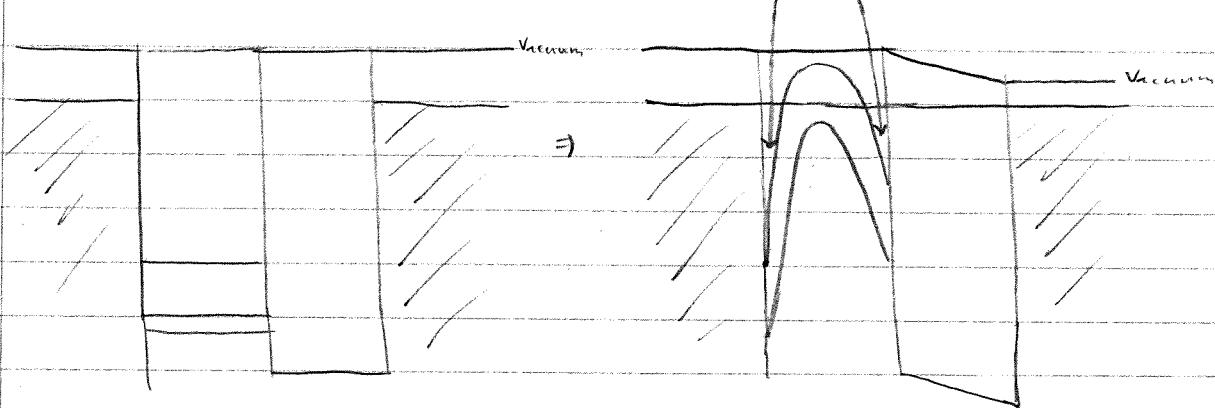
probably will have degenerate semiconductor
at the interface and possibly a quantum well.

3.



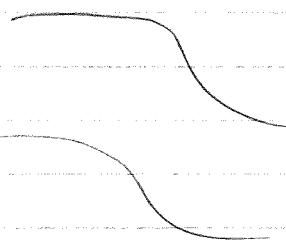
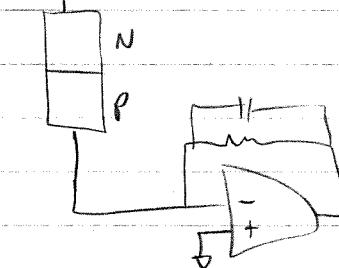
The band diagram shows very strong bending and even possible accumulation depending on the type of semiconductor. There were no other expected answers to this part, primarily this part was to allow students to give me their ideas on what might happen in such a structure which was similar to a familiar device, but with very different physical constants.

M s Am M Accumulation / Inversion here.



2000 - Pianetta

V_{dd}



BJT,

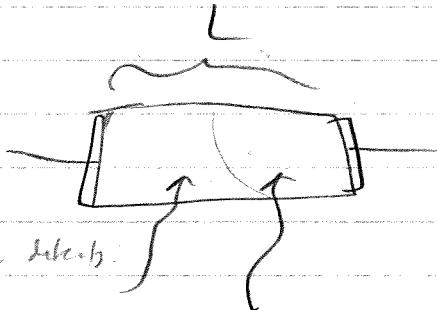
$$10^{10} \text{ s}^{-1}$$

$$e^i = 1.6 \cdot 10^{-19} C \cdot 10^{10} = 1.6 \cdot 10^{-7} C/s$$

$$1.6 \text{ nA}$$

$h\nu$

$$\mathcal{P} \propto D_n \gg L$$



pure, no traps, few surface states, impurities.

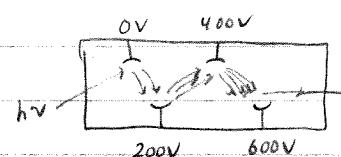
$\alpha \downarrow$

reverse biased

1. PN Junction, Silicon film between two ohmic contacts,

PIN Diode, BJT, MOSFET, Photomultiplication \rightarrow avalanche diode.

$$2. \approx 10^{10} \cdot 10^{-19} = 10^{-9} A = 1 \text{nA}$$



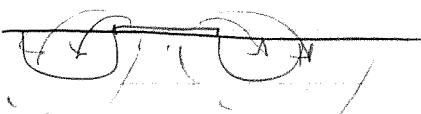
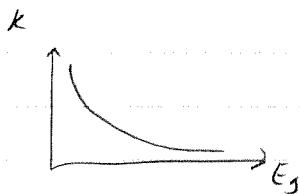
3. 1nA is small \Rightarrow should have dark current no more than 10% of the photocurrent. \Rightarrow if 1V is applied

$$R_{light} \approx \frac{1V}{1 \text{nA}} = 10^9 \Omega \quad R_{dark} \approx \frac{1V}{0.1 \text{nA}} = 10^{10} \Omega$$

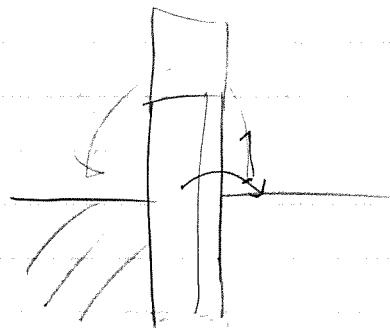
Long carrier lifetime, want them to be collected by electrodes, want $L \ll D_n$, antireflection coating at surface, reflecting coating at back, ohmic contact, heat sink, low impurities.

1999 - Pianetta

$$\text{high } k \Rightarrow \text{Low } E_g \Rightarrow \text{low } V_f$$



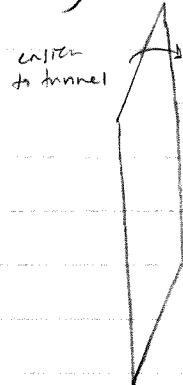
E field constant
to avoid breakdowns.



Barrier decreased

- A. $E_g \downarrow \Rightarrow 1 \Rightarrow$ Increased Tunneling \Rightarrow Increased Gate Leakage Current,
+ Increased thermionic emission
+ hot carrier injection

$E_g \uparrow \Rightarrow$ Tunneling reduced. \Rightarrow Decreased Gate Leakage Current.



$V \uparrow \Rightarrow$ more tunneling \Rightarrow leakage Current \uparrow
 $T \uparrow \Rightarrow$ leakage Current \uparrow since carriers
are more energetic \Rightarrow more of
them see the thin barrier
as shown in the drawing on
the left.

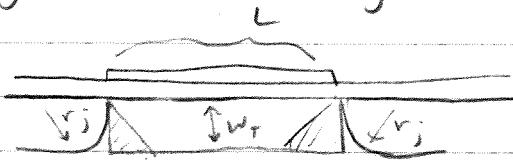
Tunneling is a function of barrier height and
thickness. height \downarrow , Tunneling \uparrow
thickness \downarrow , Tunneling \uparrow

Need to know operating voltage (FN tunneling) and
operating temperature (increased thermionic emission and $T \uparrow \Rightarrow E_g \downarrow$)

$$B. V_T = V_{(D_s=2\Phi_F)} = 2\Phi_F + \frac{K_s X_0}{K_o} \sqrt{\frac{q_N A}{K_s \epsilon_0}} \Phi_F$$

For scaling, we generally want to keep the \vec{E} field in the oxide constant (breakdown limitations).

When lateral dimensions are made smaller ($L \downarrow$), the source and drain assist in depleting the region under the gate:



$$\Delta V_T = - \frac{q N A W_T}{C_o} \frac{r_j}{L} \left(\sqrt{1 + \frac{2 W_T}{r_j}} - 1 \right)$$

(short channel)

\Rightarrow get unwanted short channel effects.

$$L_{min} = 0.4 [r_j X_0 (W_s + W_d)^2]^{1/3} \quad X_0 \text{ in } \text{\AA}$$

\Rightarrow To achieve smaller L_{min} , we must decrease X_0 . (thinner dielectric).

$$C_{ox} = \frac{K_o \epsilon_0 A}{X_0} \quad \text{Inversely proportional to oxide thickness.}$$

$$V_T \propto X_0 \quad \text{roughly proportional to oxide thickness.}$$

Problem with thinner oxides is more tunneling. Can solve this by either going to infinite bandgap or by increasing the thickness of the dielectric and then solving the resulting problems with lowered capacitance and increased threshold voltage by using a high K dielectric ($K_o \uparrow$).

2002 - Pianetta

- What determines the mobility of a semiconductor and explain the temperature dependence.

$$\mu_n = \frac{qT_e}{m_n^*} \quad \mu_p = \frac{qT_p}{m_p^*} \quad \text{where } \tau \text{ is mean time between collisions}$$

and m^* is effective mass.

for low E , $v_d = \mu E$ where v_d is drift velocity.

Mobility depends on effective mass, lattice scattering,

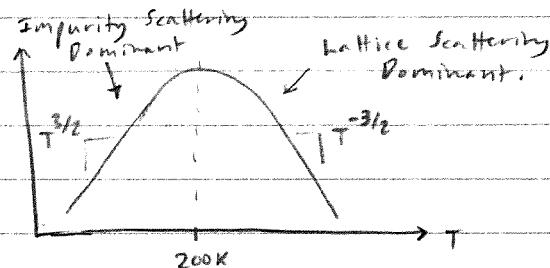
impurity scattering (defect scattering) and Temperature

↓
vacancies, interstitials, dislocations, grain boundaries.

$$\mu_I \propto \frac{T^{3/2}}{N_I} \quad N_I = N_d^+ + N_a^- \quad (\text{Total doping (not effective doping)})$$

$$\mu_L \propto T^{-3/2}$$

$$\frac{1}{\mu} = \frac{1}{\mu_I} + \frac{1}{\mu_L} \quad \text{Mathiessen's Rule.} \quad \left(\frac{1}{\mu} = \frac{1}{\mu_S} + \frac{1}{\mu_L} + \frac{1}{\mu_D} \leftarrow \text{defects} \right)$$



- What determines the mobility of polycrystalline silicon vs. single crystalline silicon?

Si Poly-Si

Scattering at grain boundaries.



Poly-Si can be deposited at lower temperatures than Si.

polycrystalline has lower μ than crystalline silicon.

2. Given three different materials: Si, Ge, GaAs.

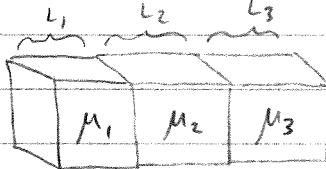
Can you explain why their mobilities are different?

$m^* = \frac{1}{\hbar^2 \left(\frac{\partial^2 E}{\partial k^2} \right)}$ \Rightarrow since Si, Ge, and GaAs have different (E-k) energy bands, they will all have different effective masses.

Also, Si, Ge, and GaAs will have different lattice properties such as atomic size and lattice constant which will lead to different amounts of lattice scattering, and thus different values of τ_c .

$\mu = \frac{q\tau_c}{m^*}$ \Rightarrow Si, Ge, and GaAs will have different mobilities.

4.



$$\text{and } p_1 = p_2 = p_3 = p$$
$$\text{and } n_1 = n_2 = n_3 = n \gg p$$

If we assume $L_1 = L_2 = L_3 = L$ and $A_1 = A_2 = A_3 = A$

$$\text{then } p = \frac{1}{\sigma} = \frac{1}{g(\mu_n n + \mu_p p)} \approx \frac{1}{g(\mu_n n)} \quad R = \frac{pL}{A}$$

$$\Rightarrow R_t = R_1 + R_2 + R_3 = \left(\frac{L}{gA} \left(\frac{1}{\mu_{n1}} + \frac{1}{\mu_{n2}} + \frac{1}{\mu_{n3}} \right) \right) = \frac{L}{gA} \left(\frac{1}{\mu_t} \right)$$

$$\Rightarrow \boxed{\frac{1}{\mu_t} = \frac{1}{\mu_1} + \frac{1}{\mu_2} + \frac{1}{\mu_3}}$$

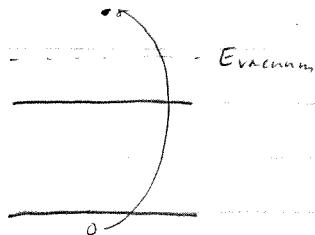
2001 - Pianetta

1.

X-ray detector using Si. How does it work?

- Photoelectric Effect. Since X-rays have such high energy, the generated carriers will have large kinetic energy beyond vacuum level. And carriers can also cause impact ionization.

In order to conserve energy,
of e^+ pairs
must be less than
X-ray energy
divided by bandgap
of Si.



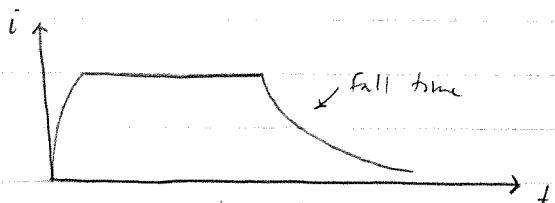
$$E_{x-ray} = \frac{1.24}{\lambda}$$

2.

Can use PN, PIN, Photoconductor, or Phototransistor (BJT)

3.

Expected signal:



For PIN

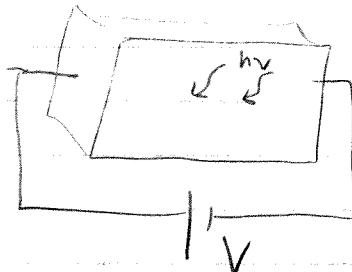
to reduce fall time, decrease C. $\Rightarrow Q \downarrow$

increase width of intrinsic region for PIN

For Photoconductor, to reduce fall time, increase bias V.

$Nd = \mu E \propto V \Rightarrow$ carriers removed faster. OR decrease

carrier lifetime so excess carriers are more quickly eliminated via recombination.



$$t_{rise} = \frac{1}{(\text{carrier transit})} = \frac{1}{(W_I / V_{sat})} \quad \text{but you can't make } V_I$$

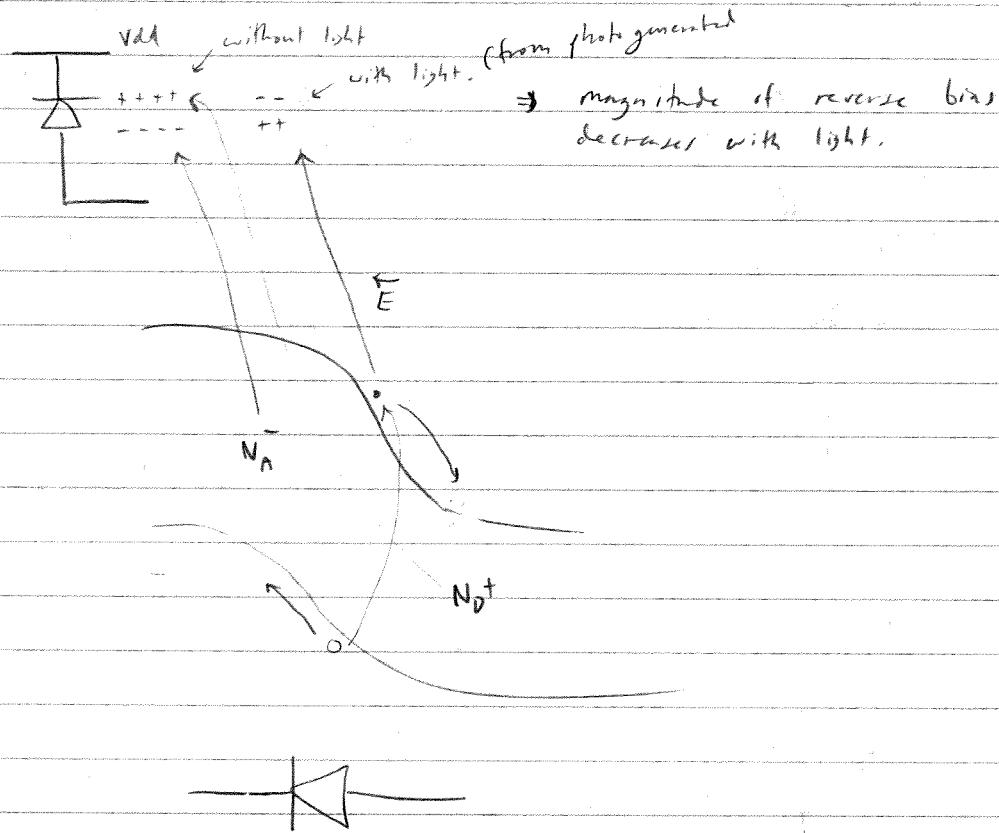
arbitrarily small. Eventually, the RC time constant associated

with the internal series R and junction capacitance

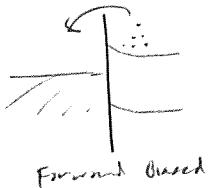
$$(C_J = \frac{k_T e A}{W_I}) \propto \frac{1}{W_I}$$

will limit the response of the diode.

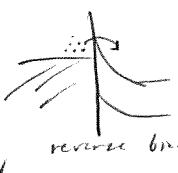
For PN and BJT to decrease fall time,
 you should also decrease $C_{ap} \Rightarrow Q \downarrow$
 \Rightarrow decrease doping to increase $W_{ap} \Rightarrow C_{ap} \uparrow$



Pre - 1993



Forward biased



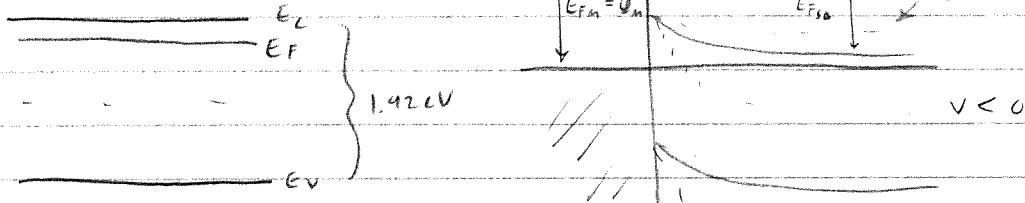
reverse biased.

Called
majority
carrier
device.

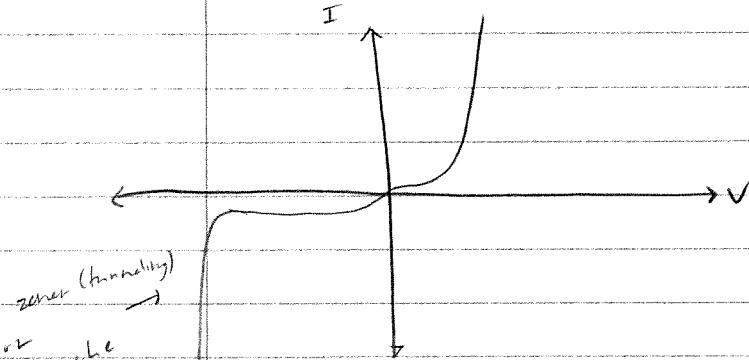
(recombination +
diffusion current
from minority
carrier is negligible)

Extrinsic

1)

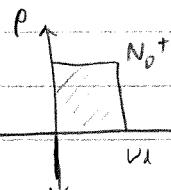


$$\Phi_B = \Phi_m - X \quad \Phi_s = \Phi$$



or
anomalous
breakdown

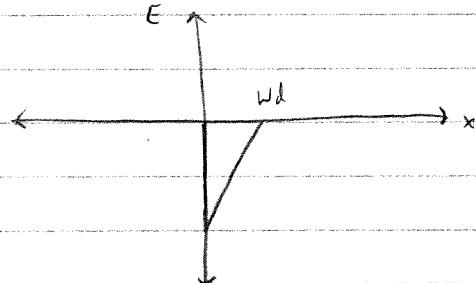
2) derive width of depletion region:



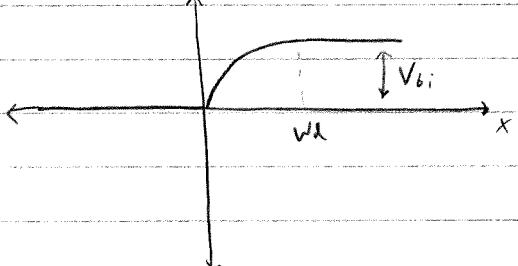
$$\frac{P}{k_b T} = \frac{\partial E}{\partial x} \quad \leftarrow \text{Poisson's Eq.}$$

$$\frac{dE}{dx} = \frac{P}{\epsilon}$$

$$E = -\frac{dV}{dx}$$



$$V = - \int E dx$$



$$V_{bi} = \frac{1}{\epsilon} (E_{Fm} - E_{Fs}) = \frac{1}{\epsilon} (\Phi_m - \Phi_s)$$

if you know doping N_D, you can solve for W_D.

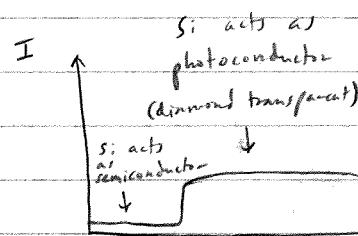
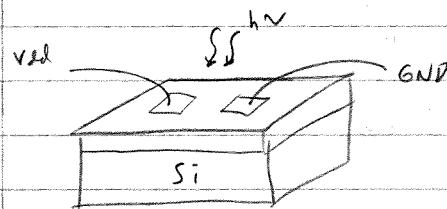
$$\text{OR: } W_D = \sqrt{\frac{2\epsilon_s (V_i - V_o)}{q N_D}}$$

$$\epsilon_s = \epsilon_r \epsilon_0 \text{ for GaAs}$$

Debye length describes how abrupt a junction is.



Pre-1993 - Pianetta



$$\rho = \frac{h}{\lambda} \quad \lambda = \frac{1.24}{E_g}$$

? $I_L = \rho \frac{8 \pi^2 G_0}{Z_t} (1 + \frac{W}{\mu_n}) A \cdot L$

diamond is an insulator.

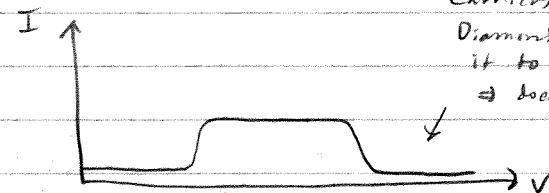
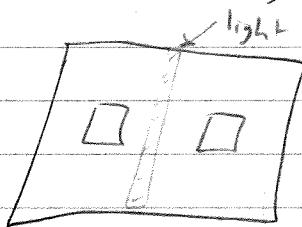
but diamond can become a semiconductor when sufficient # of carriers are excited.

diamond absorbs the photon
⇒ carriers aren't generated in Si. (Diamond is opaque)

But only the photo carriers generated right next to the contacts can contribute to current.



Now if light is only shown over this area:



metal - n-GaAs. When you apply negative bias, the Fermi level will decrease

at the surface and the donors ionize ⇒ ionized impurities contribute positive charge at interface ⇒ depletion width decreases ⇒ tunneling

← at lower applied voltage. When you apply positive voltage, nothing really changes

- Electron tunneling - direct tunneling $t_{ox} < 3nm$ FN tunneling $t_{ox} > 5nm$



1998 - Wang

$$KE \propto kT \quad T$$

$$KE + U$$

$$KE \uparrow \quad U \downarrow$$

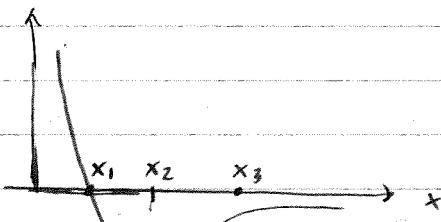
2 atoms

When $T = 0K$

$x = x_2$ since the system will go to the minimum energy state. (point where $F=0 \Rightarrow$ stable point)

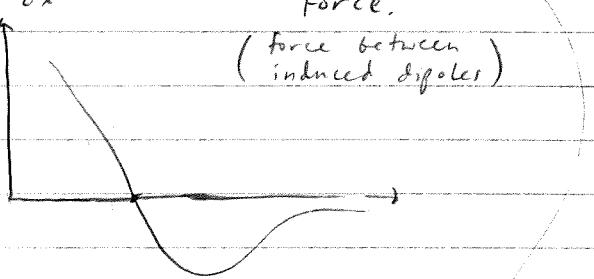
$$KE = \frac{3}{2} kT$$

$U(x) \leftarrow$ potential energy between 2 atoms,



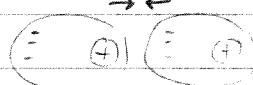
$$F = -\frac{\partial U}{\partial x} \leftarrow \text{Van der Waals Force.}$$

(force between induced dipoles)



When $T \uparrow$, what happens to x_{avg} ?

average distance increases with increasing Temperature.



$$F = -\frac{\partial U}{\partial x}$$

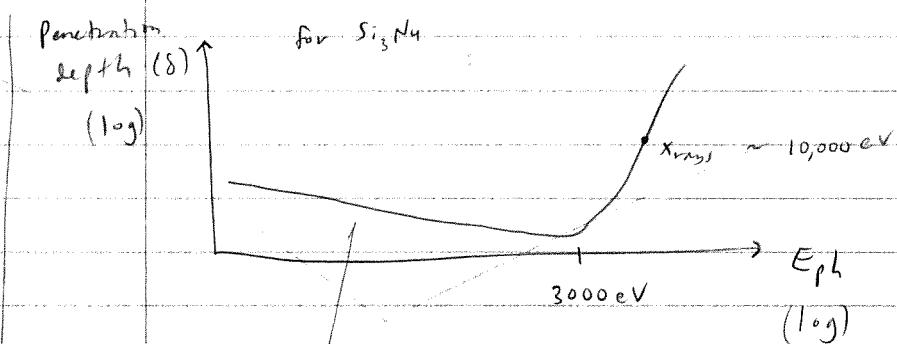
$$\Rightarrow F_{left} > 0, \quad x < x_2$$

$$F_{right} < 0, \quad x > x_2$$

$|F_{right}| < |F_{left}|$ since the slope is lower on the right side, x_{avg} will be greater than x_2 when $T \uparrow$

This is the mechanism for thermal expansion.

2008 - Pianetta



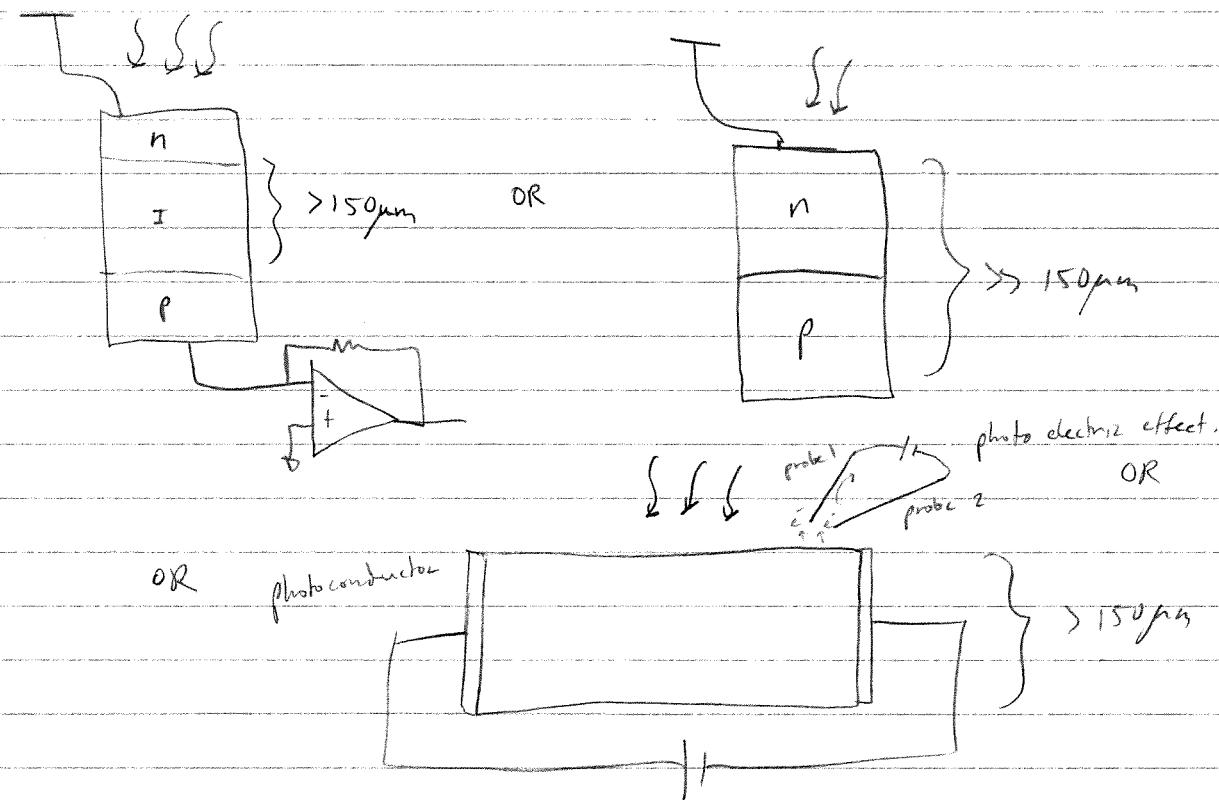
$$\delta = \sqrt{\frac{1}{W\mu\sigma}} \quad \text{only applies to good conductors.}$$

↑ frequency when $\sigma \gg W\epsilon_r E_0$.

we actually care about α , not δ .

How would you use a Si device to efficiently detect photons with an energy as high as 10,000 eV? Note that the penetration depth of such a photon is about 150 μm versus a few μm for visible light photons.

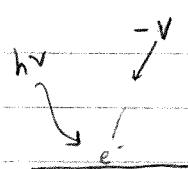
Ville



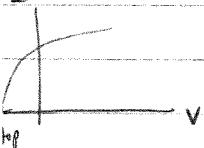
Could you use such a device to measure the energy of an individual photon?

OR use
photoelectric
effect to
measure $V_{stopping}$

Yes, using the fact that the electron created by the high energy photon has a very high kinetic energy which will be converted into a cascade of secondary electrons (due to ionization) and can be swept out of the device as a current pulse.



What determines the response time of the detector?



Concepts that should be understood include the time required to sweep the electrons out of the depletion region and the RC time constant of the device.

$$-8V_{stopping} = h\nu - \Phi_s$$

How could you make the detector faster?

Builds on answer to previous question including reducing sweep out time with higher voltage, reducing R and C . Ideas for reducing C include shaping the electrodes to have a small back electrode that can still collect the charge.

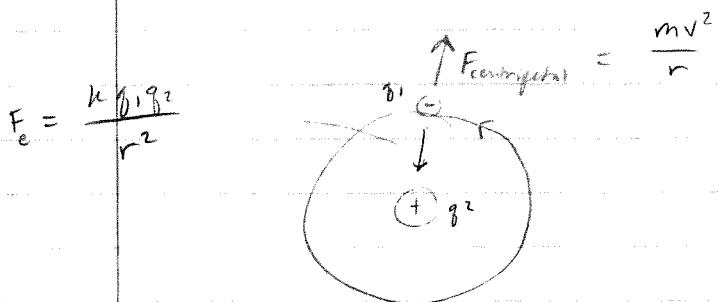
Can reduce R by reducing contact resistance, n and p wells. OR can use a semiconductor with higher μ such as GaAs.

2005 - Pianetta

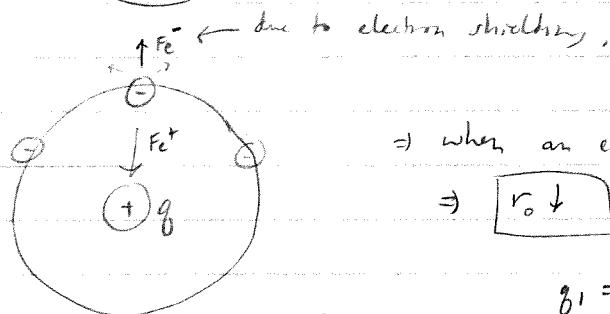
Consider a simplified model of an atom with a positive core and with the outer or valence electrons modeled as a spherical shell of charge. Also consider one of the core electrons with a binding energy E_b . If I were to remove charge from the outer valence shell, for example, by oxidizing of the atom, how does the binding energy of the core electron change?

$$F = \frac{-kq_1q_2}{r^2} \quad W = \int_{\infty}^{r_0} F(r) dr = -kq_1q_2 \int_{\infty}^{r_0} \frac{1}{r^2} dr$$

$$= -kq_1q_2 \left(-\frac{1}{r} \right)_{-\infty}^{r_0} = \frac{kq_1q_2}{r_0}$$



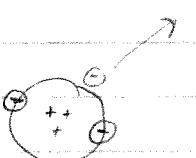
$$F_e = F_c$$



\Rightarrow when an electron is removed, $F_c \downarrow$

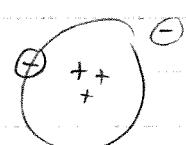
$$\Rightarrow r_0 \downarrow$$

$$q_1 = e^- \quad q_2 = e^-$$



$$\text{for removing the first } e^- : E_b = W = q_1 \Delta V = q_1 \int_{r_0}^{\infty} \vec{E}(r) dr = q_1 \int_{r_0}^{\infty} \frac{q_2}{4\pi\epsilon_0 r^2} dr = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r_0}$$

$$\text{for removing the 2nd } e^- : q_1 = e^- \quad q_2 = 2e^- \Rightarrow E_b \uparrow$$



1999 - Wang

- 1) How do you express or describe the momentum of an electron in a crystalline solid?

Crystal momentum is a momentum-like vector associated with electrons in a crystal lattice. It is defined by the associated wave-vectors k of the lattice:

$$p_{\text{crystal}} = \hbar k$$

$$p = \frac{\hbar}{\lambda}$$

DeBroglie

True momentum, or average momentum, is given by:

$$\langle p_x \rangle = \int_{-\infty}^{\infty} \Psi^* \frac{\hbar}{i} \frac{\delta \Psi}{\delta x} dx \quad \leftarrow \text{for momentum in } x\text{-direction}$$

for the 1-D case.

- 2) Is it true that the property of such an electron will not change if its wave vector is increased by a reciprocal lattice vector? If yes, what is the underlying physical law or theorem?

Yes. Bloch's theorem:

Consider a Bravais Lattice with translational symmetry vector \mathbf{a}
⇒ $U(\mathbf{r})$ is periodic. ($U(\mathbf{r}) = U(\mathbf{r+a})$).

$$\Rightarrow \Psi(\mathbf{r+a}) = e^{i\mathbf{k}\cdot\mathbf{a}} \Psi(\mathbf{r}) \Leftrightarrow \Psi(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}} u(\mathbf{r})$$

where $u(\mathbf{r}) = u(\mathbf{r+a})$
unit cell wavefunction with same periodicity as potential.

Reciprocal lattice is the set of all vectors \mathbf{K} that satisfy $e^{i\mathbf{K}\cdot\mathbf{a}} = 1$

\Rightarrow if $\psi(r+a) = e^{ika} \psi(r)$, then increasing wave vector

by a reciprocal lattice vector yields:

\downarrow equals 1 by definition

$$\psi(r+a) = e^{i(k+K)a} \psi(r) = e^{iKa} e^{ika} \psi(r) = e^{ika} \psi(r)$$

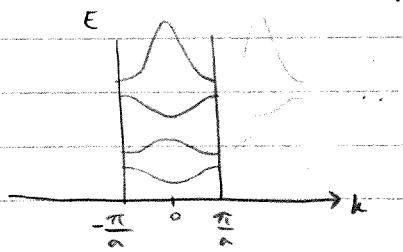
\Rightarrow wavefunction stays the same.

- 3) Show that the real (or average) momentum of the electron indeed does not change if its wavevector is increased by a reciprocal lattice vector.

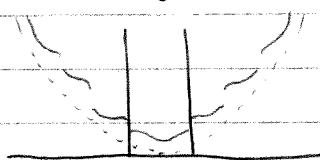
Although one may expect momentum to change, since crystal momentum ($\hbar k \rightarrow \hbar(k+K)$) changes. The real (average) momentum must not change, since the wavefunction stays the same.

$$\langle p_x \rangle = \int_{-\infty}^{\infty} \psi^* \frac{\hbar}{i} \frac{\partial \psi}{\partial x} dx \quad \leftarrow \text{this is for the 1-D case for momentum in the } x\text{-direction, but the 3-D case also depends only on wavefunction.}$$

- \Rightarrow This is also why E-k diagram repeats with period $\frac{2\pi}{a}$.



\Updownarrow identical representations



2003 - Wang

$$C_1 = \frac{2\epsilon_0 A}{d} \quad C_2 = \frac{2\epsilon_r \epsilon_0 A}{d} = \epsilon_r C_1$$

$$C_{\text{eff}} = \frac{\epsilon_r}{1+\epsilon_r} C_1 = \frac{\epsilon_r \epsilon_0 A}{d(\epsilon_r+1)}$$

$$F = QE = Q \frac{0}{\epsilon_r \epsilon_0}$$

$$= Q \cdot \frac{Q}{A \epsilon_r \epsilon_0} = \frac{Q^2}{A \epsilon_r \epsilon_0}$$

$$Q = CV = \frac{\epsilon_r \epsilon_0 A}{\frac{d}{2}(\epsilon_r+1)} \cdot V \Rightarrow F = \frac{\epsilon_r \epsilon_0 A V^2}{\left(\frac{d(\epsilon_r+1)}{2}\right)^2}$$

and think
why is it right?

$$\epsilon_r(E - E_i) = E$$

$$V = \frac{d}{2}(E - E_i) + \frac{d}{2}E$$

$$= \frac{d}{2} \cdot \frac{E}{\epsilon_r} + \frac{d}{2}E = \frac{d}{2} \left(1 + \frac{1}{\epsilon_r}\right) E$$

$$Q = CV = V \cdot \left(\frac{1}{1 + \frac{1}{\epsilon_r}}\right) \frac{2\epsilon_0 A}{d}$$

$$F = \frac{QE}{2} = \frac{1}{2} V \left(\frac{1}{1 + \frac{1}{\epsilon_r}}\right) \frac{2\epsilon_0 A}{d} \cdot V \cdot \frac{2}{d} \left(\frac{1}{1 + \frac{1}{\epsilon_r}}\right) = \boxed{\frac{2V^2 \epsilon_0 A}{(1 + \frac{1}{\epsilon_r})^2 d^2} = F}$$

$$(\text{for } \epsilon_r = 1, F = \frac{QE}{2} = \frac{CV E}{2} = \frac{\epsilon_0 A V^2}{2d^2})$$

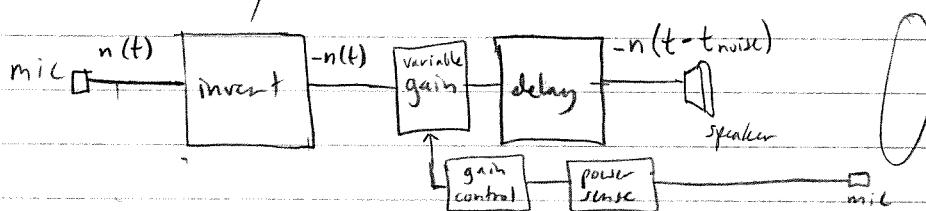


2006 - Please

$$t_{\text{antiv}} = \frac{d_2}{v_{\text{sound}}} < t_{\text{noise}}$$

180° phase modulator.

ear



$$t_{\text{noise}} = \frac{d_1}{v_{\text{sound}}} = \frac{d_1}{343 \text{ m/s}} \approx \frac{5 \text{ cm}}{343 \text{ m/s}} \approx \frac{15 \text{ cm}}{1000 \text{ m/s}} = \frac{15 \cdot 10^{-2} \text{ m}}{1000 \text{ m/s}} = 150 \mu\text{s}$$

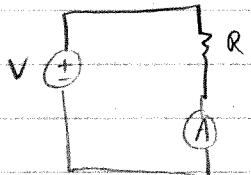
inversely

If we assume that the Λ block response is much faster than t_{noise} ($\sim 150 \mu\text{s}$), we can create a wave that will destructively interfere with the noise waveform before it hits the ear. In order for this to work, we must ensure that the volume gain is 1 for this system. So we should have some means of adjusting gain perhaps with feedback. Adjust gain until the volume sensed near the ear is minimized.

2005 - Please

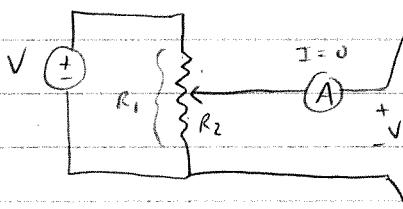
$$1. \quad IV = \frac{1}{TC}$$

2. Voltmeter:

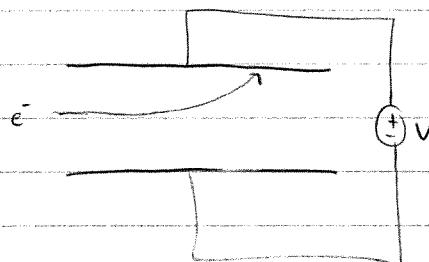


$$V = IR$$

want R to be large.



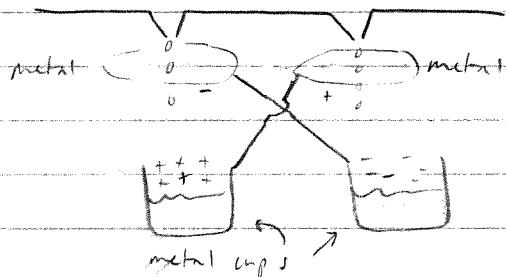
$$\frac{V}{R_1} = \frac{V_x}{R_2}$$



measure distance of deflection.
This will be related to V .

3. No.

4. water



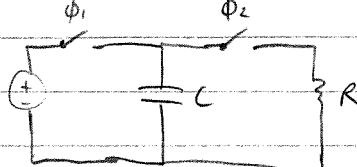
positive feedback.

Limitations: may have ESD sparks between caps.

5. Because the caps only form a capacitor which discharges.

6. Devise a voltmeter with infinite resistance. ($> 10^{14} \Omega$)

use C with
thick dielectric
(large tox)



measure discharge time of C .

This should be related to voltage.

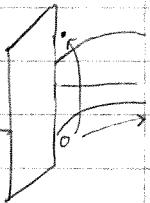
Input resistance of a cap should be similar to that at the gate of a MOSFET which is typically $> 10^{12} \Omega$ (due to leakage through dielectric, tunneling, etc.)

Pre 1993 - Please

- How does a TV Camera Work? Discuss problems of getting good collection efficiency, interconnecting and addressing of cells, diffusion lengths, and capacitor storage times.

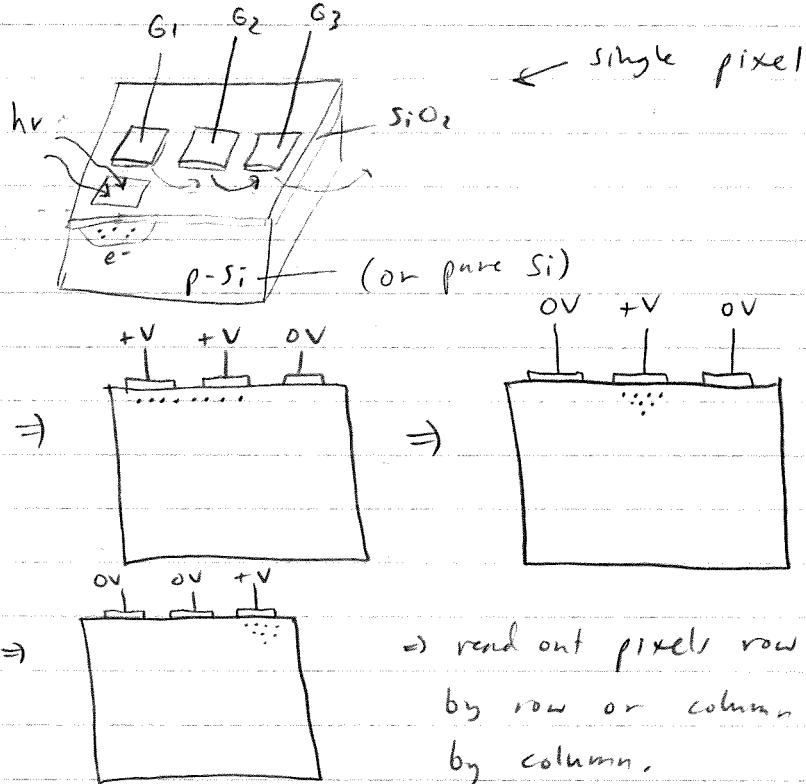
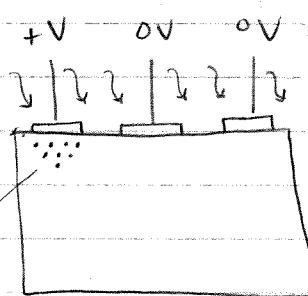
MOS II

biased in
deep depletion.



accumulated
charge is
proportional
to light
intensity.

CCD.



Collection efficiency:

- Quantum efficiency: absorption length $\propto \frac{1}{\lambda}$
 ⇒ short wavelength photons absorbed very near surface.
 long wavelength photons absorbed deeper in bulk where the pixel gate potential well is relatively shallow, rather than in the depletion region. ⇒ these resulting photoelectrons may migrate to a neighboring pixel (charge diffusion ⇒ image blurring).
- pure silicon is shiny (reflective), especially for short wavelength (bluer) photons. This can be reduced by using an antireflection coating like MgF_2 .

Diffusion Lengths: Doesn't really matter except for low wavelength photons.

But some surface recombination can degrade the charge transfer and thus the readout efficiency.

Capacitor Storage times: Purer si ⇒ higher R ⇒ higher storage time. But more expensive. There should also always be finite leakage current due to trap assisted tunneling through the oxide.

2. a) Explain how a microwave heats a chicken.

Microwaves use EM waves (2.45 GHz) to cause polarized molecules (H_2O) to oscillate, generating heat. Generates heat within the material.

b) Compare with infrared heating.

Infrared heating heats the material's surface first and then heat moves inward. This is slower and less energy efficient.

c) Do microwaves have higher or lower frequency than infrared?

Lower. Microwaves $\sim 2.45\text{ GHz}$. Infrared $\sim 1-400\text{ THz}$.

d) Can you devise a method for quick cooling the chicken, using the method that microwaves use to heat it? Not sure...

3. Why is a very long wave used for sonar?

skin depth $\delta = \left(\frac{2}{\omega \mu_0}\right)^{1/2} \propto \frac{1}{\sqrt{\omega}}$ \Rightarrow longer wave \Rightarrow lower frequency
 \Rightarrow greater skin depth \Rightarrow wave can travel farther.

4. Define an active device:

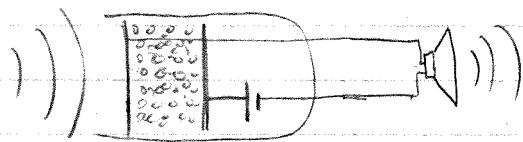
Any type of circuit component with the ability to electrically control electron flow. Also, active devices are capable of delivering power to the circuit (power gain greater than one).

Active: Vacuum Tubes, Transistors, Silicon Controlled Rectifiers, TRIACs.

Passive: Resistors, capacitors, inductors, transformers, diodes.

5. Can you use telephone speakers and microphones for power amplification?

Yes. Telephone microphones are carbon microphones which consist of 2 metal plates separated by granules of carbon. When pressure (i.e. from a sound wave) is applied to the mic, the granules of carbon are pressed together, decreasing resistance.



Thus, since the speaker power comes from the battery, it is possible to achieve power amplification.

6. a) If there is a current of $100\text{ A}/\text{cm}^2$ in a copper wire, what is v_d ?

$$I = q n v_d A \Rightarrow v_d = \left(\frac{I}{A}\right) / (nq) \quad \text{for copper, } n = 8.983 \cdot 10^{22} \text{ cm}^{-3} \quad q = 1.6 \cdot 10^{-19} \text{ C}$$

$$\Rightarrow v_d = 7.37 \cdot 10^5 \text{ m/s}$$

b) how long must you wait for the current pulse to traverse the wire?

Much less than that implied by the drift velocity. The wire acts as a transmission line. So $v_{\text{propagation}} = \frac{1}{\sqrt{LC}} \approx 0.75c$, typically.
 \uparrow speed of light
 \downarrow unit inductance unit capacitance

Since wire is full of carriers, you don't need to wait for a carrier on the far left side of the wire to drift to the far right before the signal arrives.

7. What are good properties of SiO_2 ?

Very few interactions, no damping bands.

Naturally forming oxide on Si. A chemically stable in air

(\Rightarrow passivates Si surface), Good insulator ($E_g \approx 9\text{ eV}$).

Good high temperature diffusion and ion implantation barrier.

Easily etched. Has good etch sensitivity to semiconductor (Si).

Reasonably high dielectric constant ($k=3.9$)

8. Why greater conductance for shorter channels?

$$G = \frac{1}{R} \Rightarrow G = \frac{A}{\rho L} \propto \frac{1}{L}$$

9. a) What is the mechanism that limits speed in today's VLSI chips?
propagation delay / transit frequency \Rightarrow minimum channel length.

Also thermal dissipation. Power dissipation / Area is increasing.

\Rightarrow increased operating temperature. Acceptable range: $-55^\circ\text{C} - 125^\circ\text{C}$ ($220\text{ K} - 400\text{ K}$)

b) Estimate transit time in a MOS transistor:

$$\tau = \frac{L}{v_d} = \frac{L}{\mu E} = \frac{L}{\mu \frac{V_{DS}}{L}} = \frac{L^2}{\mu V_{DS}} \Rightarrow \boxed{\tau = \frac{L^2}{\mu V_{DS}}}$$

c) What benefits on speed can be expected when you go to liquid N_2 temperature (77 K) in MOS and Bipolar circuits? What problems could you expect?

- Benefits - reduced lattice scattering, reduced thermal noise, higher μ than $T > 300\text{ K}$

- Problems - increased impurity scattering, process variations, \Rightarrow higher V_{TH} , V_T

$$V_{TH} (-1.2\text{ mV}/^\circ\text{C}) \uparrow, R (1\%/\text{ }^\circ\text{C}) \downarrow, V_T = \frac{kT}{q} (-2\text{ mV}/^\circ\text{C}) \uparrow$$

10. Is a transformer an active device? If not, why?

No. No power gain. $\frac{V_S}{V_P} = \frac{N_S}{N_P} = \frac{I_L}{I_S} \Rightarrow P_{in} = P_{out}$ for ideal transformer.

11. MOSFET: $g_m = \mu C_{ox} \frac{W}{L} V_{DS} = \sqrt{2\mu C_{ox} \frac{W}{L} I_D}$ BJT: $g_m = \frac{I_C}{V_T}$

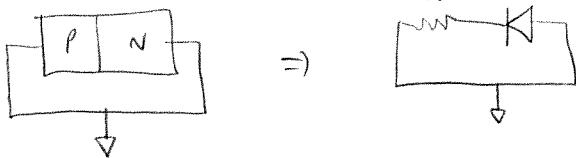
12. Do you increase g_m by increasing E_{ox} ?

Yes. $g_m = \mu C_{ox} \frac{W}{L} V_{DS} = \mu \frac{C_{ox} A}{L} \frac{W}{L} V_{DS} \propto E_{ox}$.

Is there a limit? not sure...

Yes. $k \propto \frac{1}{E_g} \Rightarrow$ at some point for high k , E_g will

be too low and leakage current will become too large.



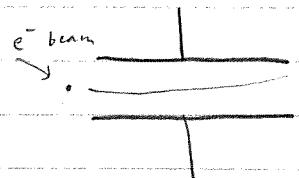
13. Will an electron beam directed right over a PN junction (short circuited) deflect the beam?

I think yes, due to the local electric field set up by V_{bi} .

14. a) A 10eV x-ray is incident upon the bottom plate of a $0.001\mu F$ open circuited capacitor. The photon excites an electron from the bottom plate to the top plate. What is the voltage across this cap?

$$Q = V/C \Rightarrow V = \frac{Q}{C} = \frac{1.6 \cdot 10^{-19}}{1 \cdot 10^{-9}} = 1.6 \cdot 10^{-10} \Rightarrow V = 0.16 \text{ nV}$$

b) How would you measure this voltage? describe the experiment in detail and find the fundamental physical limiting factor.



detect e^- , measure deflection, \Rightarrow calculate E
 \Rightarrow calculate voltage.

↑ thermal noise on the capacitor plates? Could possibly cool the capacitor to low temp (liquid $N_2 = 77K$)

2007 - Please

- A cathode ray oscilloscope operates by focusing and deflecting a beam of free electrons onto a screen.

Outline the limitations to the sensitivity, resolution and speed.

You may assume the electron lens is free of aberrations.

Speed: Time for beam to hit screen - almost instantaneous.

Fluorescent screen off \rightarrow on - almost instantaneous.

per pixel { response time CRT : < μ s
LCD : 2-5 ms

refresh rate CRT : limited by screen area and speed of lens circuitry.

ideally want 60Hz refresh rate per pixel (detectable by eye).

Resolution: Size of pixel is determined by beam width which is determined by beam aperture, and it is also determined by packing density of fluorescent molecules.
And by the resolution of the voltage you apply to the deflection lens.

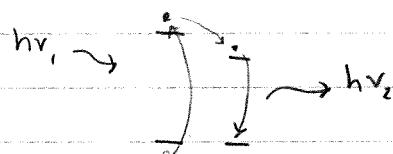
Eyes also have a limitation: Rayleigh criterion. Human eye can distinguish $1\text{ }\mu\text{m}$ distance. $\theta_R = 1.22 \frac{\lambda}{d}$ $\lambda \leftarrow$ wavelength of light.
 d = diameter of eye's lens diameter
 \Rightarrow resolution is limited by the eye, not the CRT.

Sensitivity: Limited by quantum efficiency (η) of the fluorescent screen.
of electrons you need to focus on the screen to emit a photon from the fluorescent molecule. Some energy will be dissipated as heat.

Contrast of CRT $>$ LCD

\rightarrow doesn't have true black.

Fluorescent Molecule:



e.g. Alamar Blue.

green \rightarrow \rightarrow red/orange

2008 - Please

1. Why do heat sinks have fins? To maximize surface area.

2. Express the area advantage of the above finned structure over an unfinned structure in terms of h, L_c, L_w .
assuming n fins.

$$A_{\text{unfinned}} = L \cdot z \quad A_{\text{finned}} = n(2h + L_c)z = \left(\frac{L}{L_w + L_c}\right)(2h + L_c)z$$

3. Choose h, L_c, L_w . Want L_c and L_w small so you have more fins.
Want h large so you have longer fins.

i.e. to maximize $A_{\text{finned}} = \left(\frac{L}{L_w + L_c}\right)(2h + L_c)z$

from Wikipedia: efficiency $\eta = \tanh\left(\sqrt{\frac{2h_f \cdot h}{k \cdot L_w}}\right)$

where h_f is a convection coefficient of the fins. (depends on the material of fins and fluid)

without fins: $A = n(L_c + L_w)z$

with fins: $A = n(L_c + 2h)z$

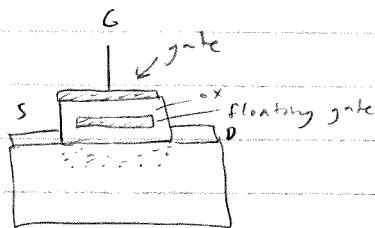
\Rightarrow must have $h > \frac{L_w}{2}$

FLASH How does flash memory work?

FLASH

write: if you apply $V_G > 0$

inversion layer forms + electrons tunnel onto floating gate. $\Rightarrow V_{TH} \uparrow$.



2010 - Please

1. 2009 Nobel Prize in Physics awarded for

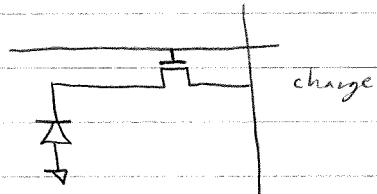
"the invention of an imaging semiconductor circuit - The CCD"

a) CCD or CMOS sensor (Active Pixel Sensor or Passive Pixel Sensor)

PPS: - 1 transistor per pixel

- small pixel, large fill factor
(% of pixel area that is sensitive to light)

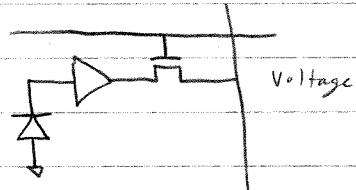
- But, slow, low SNR
- readout speed limited by row transfer time



APS: - 3-4 transistors per pixel

- fast, high SNR
- But, larger pixel, lower fill factor

- column readout time limits readout speed.



- As technology scaled below 0.5μm, APS pixel size and fill factor were no longer a problem \Rightarrow current technology of choice.

elaborate...

b) The CCD was invented at Bell Labs in 1969 as a new type of memory storage device. Though it did not prove to have a future in memory storage, the CCD gave rise to an explosion in digital imaging, with the first CCD based video cameras appearing in the early 1970s.

Although CCDs are to some extent now supplemented by competing technologies, their use in applications ranging from digital cameras to the Hubble space telescope has completely transformed image processing.

- c)
1. Low power. CCDs consume as much as 100 times more power than an equivalent CMOS sensor.

For CCD, entire array is switching all the time.

(high C, high V, and high f result in high $P = CV^2f$)

2. Low cost, CMOS chips can be fabricated on just about any standard silicon production line, so they are much cheaper than CCDs which require special processing.

⇒ don't have to worry about transfer efficiency.

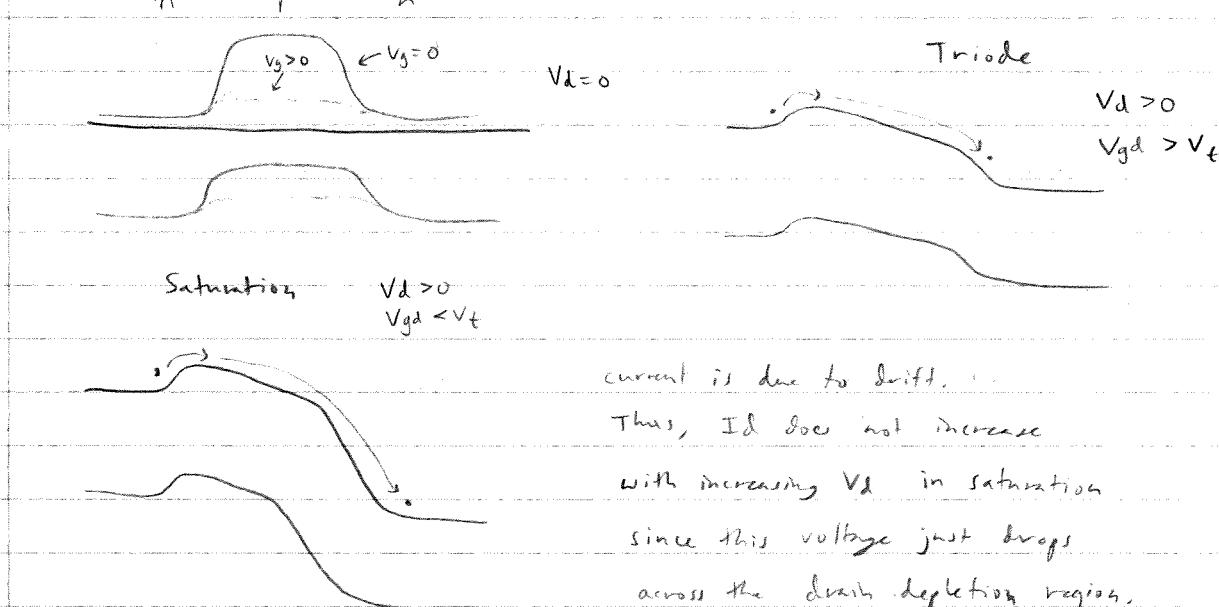
3. Faster. For CCDs, there is a tradeoff between transfer speed and transfer efficiency. This is not the case for CMOS sensors, since each pixel is read individually, (local Q→V conversion for APS)

4. Circuit Integration. Digital logic circuits, clock drivers, counters, and analog to digital converters can be placed on the same Si chip and at the same time as the photodiode array for CMOS sensors.

(You can't do this for CCDs). This enables CMOS sensors to participate in the process shrinks that move to smaller linewidths with a minimum of redesign, in a manner similar to ICs.

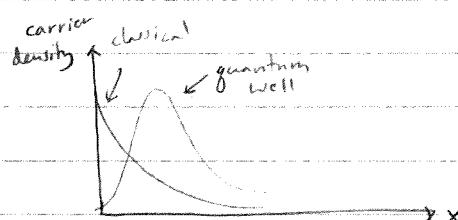
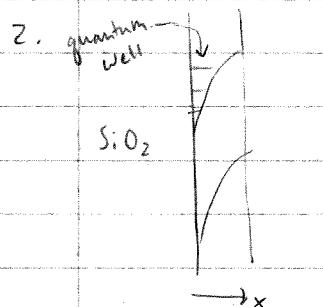
2007 - Nishi

- (100) is standard surface orientation for Si (highest μ)
sometimes people also use (110).



Thus, I_d does not increase with increasing V_d in saturation since this voltage just drops across the drain depletion region.
 $\Rightarrow \bar{E}$ in channel is \approx the same.

G Si Insulator

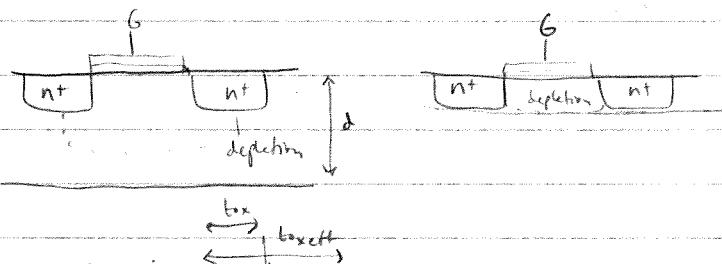


\Rightarrow carriers are mostly away from the surface
 \Rightarrow less surface scattering
 \Rightarrow increased mobility.

$$t_{ox\text{eff}} \uparrow \Rightarrow C_{ox} \downarrow \text{ but } \mu \uparrow$$

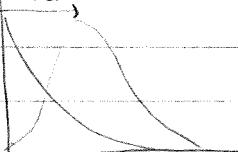
$\Rightarrow I_d \uparrow$ for given $(V_g - V_t)$ since $\mu \uparrow$ (but $V_t \uparrow$ so you need higher V_g)
 (see below)

classically:



\Rightarrow as $d \uparrow$, $V_t \downarrow$
 since less charge is needed to create the depletion region,
 since the depletion region is smaller.

But: For quantum:



so for very small d , $|V_t \uparrow|$ since $t_{ox\text{eff}} \uparrow$

$I_{D_{\text{sub}}} \uparrow$ since $C_{\text{ox}} \downarrow$ ($\frac{1}{I} \frac{C_{\text{ox}}}{C_{\text{sub}}}$) and $\mu \uparrow$

$$I_{D_{\text{sub}}} \approx \frac{W}{L} \alpha_e C_{\text{ox}} \left(\frac{kT}{\theta} \right)^2 (n-1) e^{(\nu_{GS} - \nu_T)/nkT} (1 - e^{-\theta \nu_{DS}/kT})$$

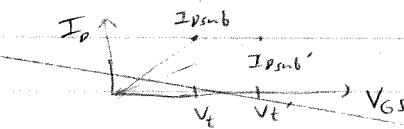
Simplified: from Razavi $I_{D_{\text{sub}}} = I_0 e^{(\nu_{GS}/\theta \nu_T)}$ where $\nu_T = \frac{kT}{\theta}$ $\theta > 1$

$I_{D_{\text{sub}}} \uparrow$ since $C_{\text{ox}} \downarrow$ since $t_{\text{ox}} \uparrow$

$$V_{ch} = \frac{\frac{V_G}{C_{\text{ox}}}}{\frac{C_{\text{sub}}}{C_{\text{sub}} + C_{\text{ox}}}} V_G \downarrow \Rightarrow \theta \uparrow$$

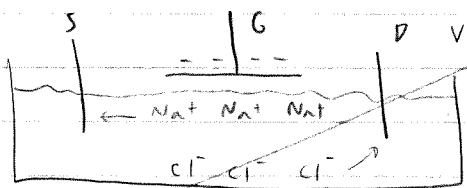
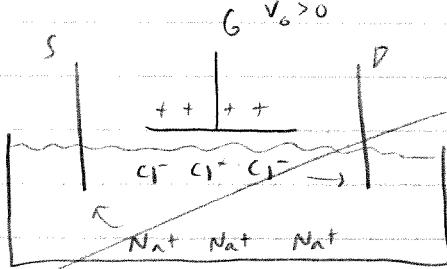
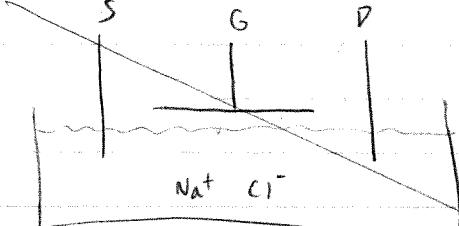
possibly due to capacitive divider

Can also think about this as $V_t \uparrow \Rightarrow$ for a given V_{GS} , $I_{D_{\text{sub}}} \downarrow$ should decrease.



$I_G \downarrow$ since $t_{\text{ox}} \uparrow \Rightarrow$ less tunneling.

3.



\Rightarrow conductive in both cases.
can't make a transistor.

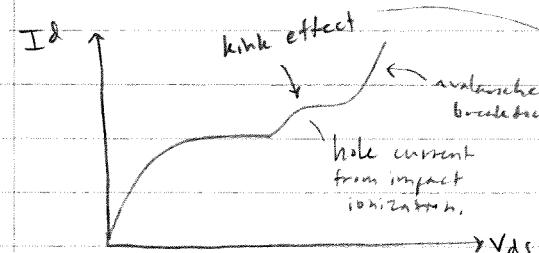
Should behave like a conductor with low mobility and zero leakage current.

Should still assume solid state. \Rightarrow ions = dopants

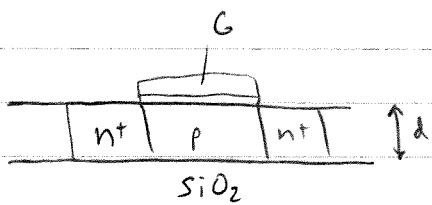
carriers have higher mass, higher volume \Rightarrow low mobility.

2003 - Nishi

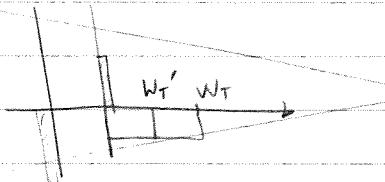
SOI
prevents
(CMOS latchup
and punchthrough)



only happens in partially depleted enhancement mode SOI MOSFET operated in strong inversion.
(also called floating substrate effect).



p substrate is floating. Not connected to GND.

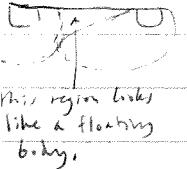


$W_T' = d$ since d is so small.

\Rightarrow we call the substrate partially depleted.

for SOI

kink effect
Can also happen
in a regular mos
if there's punchthrough



This region looks
like a floating
body.



positive feedback



turns on parasitic BJT.

Since substrate is floating, holes generated from impact ionization are collected by the source rather than the bulk GND connection.
This causes the kink.

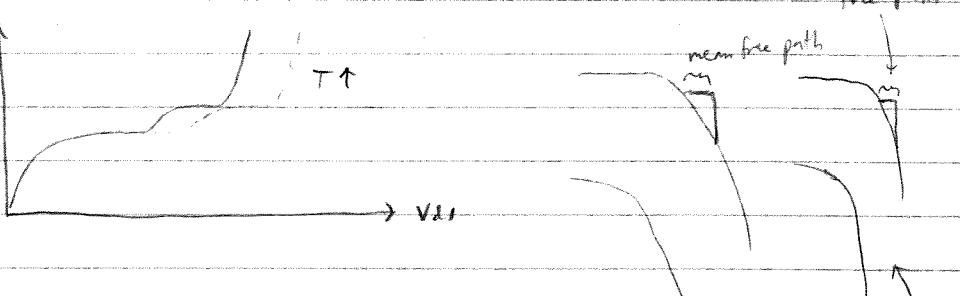
smaller mean free path

2. T dependence. I_D

$T \uparrow \Rightarrow$ lattice scattering \uparrow

\Rightarrow mean free path \downarrow

\Rightarrow need higher V_d for
impact ionization.

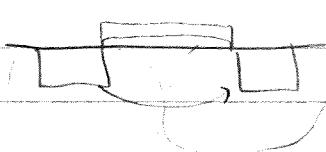


3. Parasitic BJT :

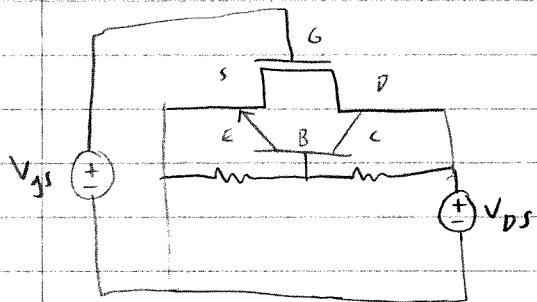
$$I_D \propto V_D^2$$

\Rightarrow for $V_g = 0$, $I_D \neq 0$

for punchthrough, not
parasitic BJT.



punch through
~~nonideal parasitic BJT~~
effective base width is 0.



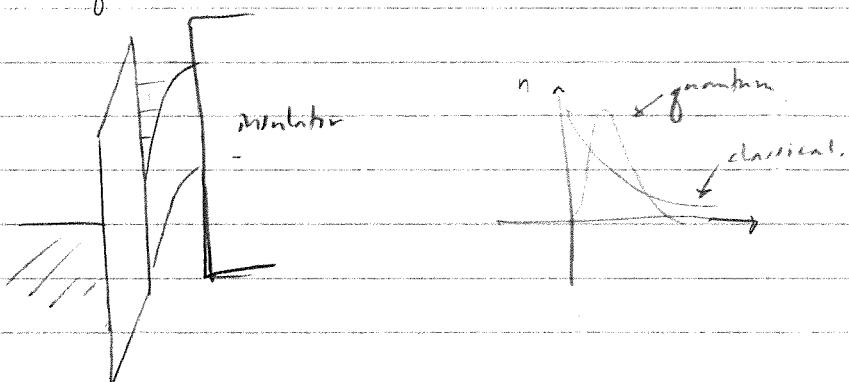
4. How do you prevent punch through?

use long channel, heavily doped substrate, use SOI,

5. Size effects: V_{ds} at which hump occurs is smaller.

also as you get very small, there can be tunneling from source to drain.

6. Last question



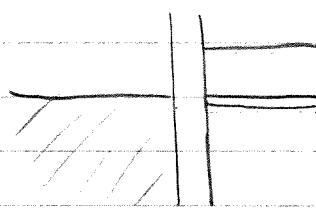
surface quantization / Volume inversion

2008 - Nishi

$$V_G = 0$$

M or p-type

1.



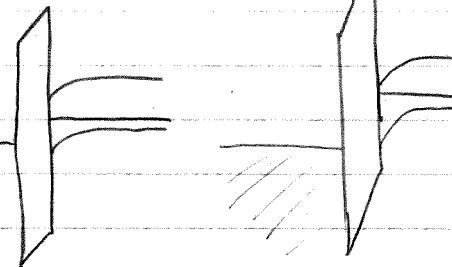
=

Depletion

$$V_G > 0$$

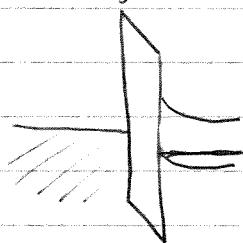
Inversion

$$V_G \gg 0$$



Accumulation

$$V_G < 0$$



Ac.

0V

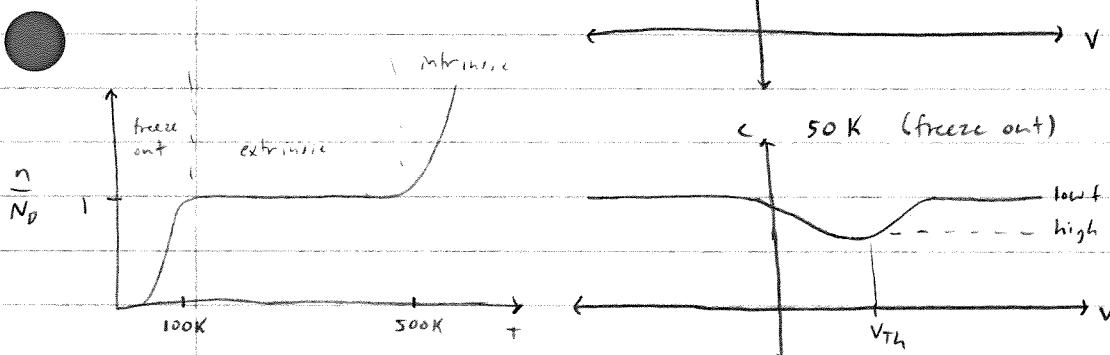
2nd

Low f

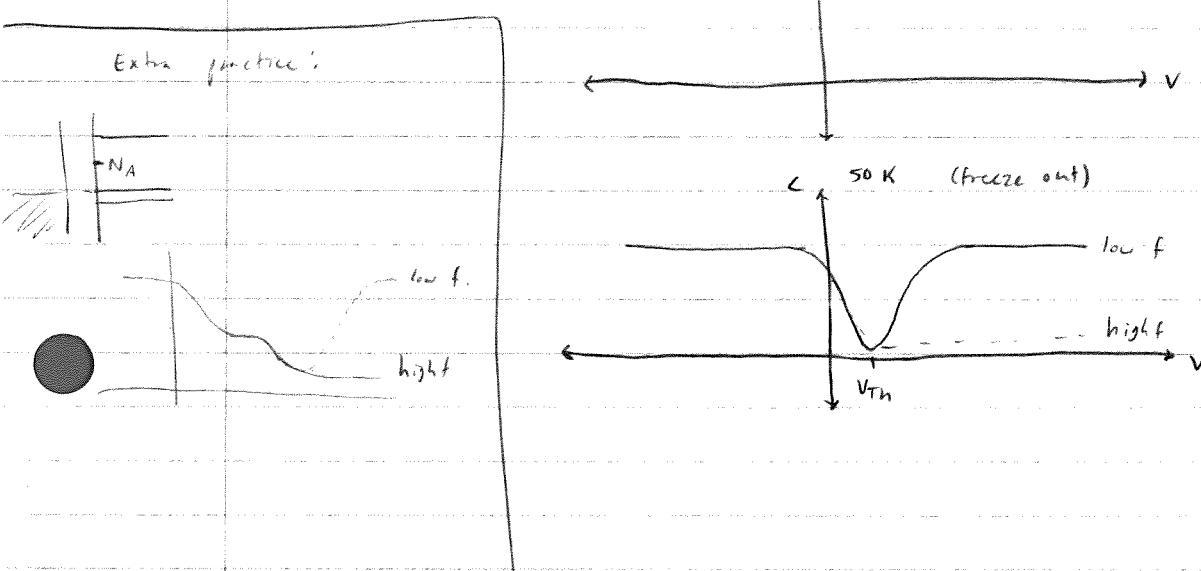
high f



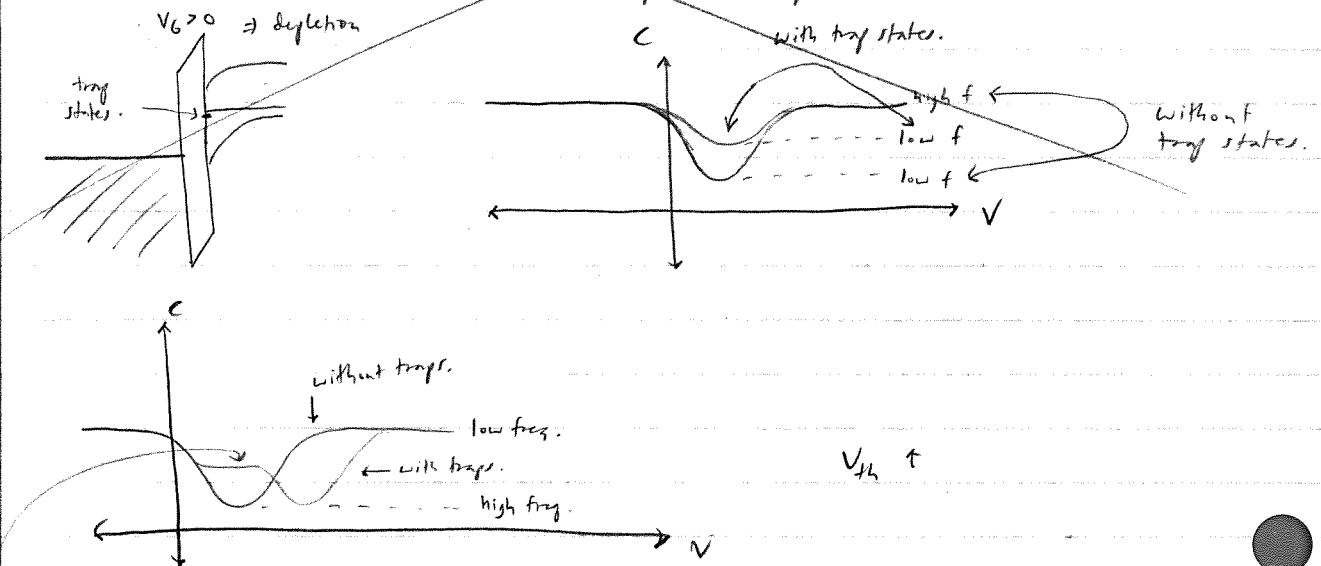
2. a) Highly Doped :



b) Lightly Doped :



~~3. Depletion capacitance will increase, since in depletion, these states will fall below the fermi level \Rightarrow they will fill with charge and contribute to the depletion charge. \Rightarrow the depletion width will decrease and the depletion capacitance will increase.~~



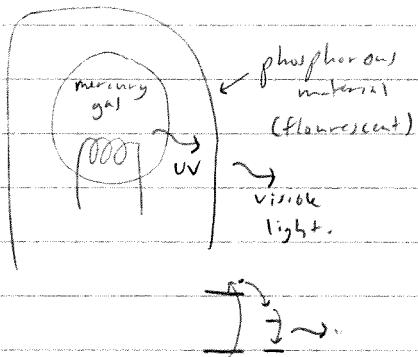
$\phi_s = 2\phi_f$. \leftarrow this is only accomplished by band bending in the silicon. \Rightarrow the max depletion W will be the same with or without traps, so $C_{depl, \min}$ will be the same with or without traps.

\bar{e} 's that would form inversion layer instead fill the traps. $\Rightarrow V_{th} \uparrow$.

1995 - Saraswat

Sensor in the box is a photodiode.

How does a fluorescent light work?

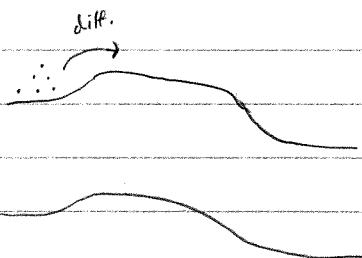


2006 - Nishi

1. MOSFET, low V_g + low V_d , I_{drift} dominates.

$$I_{drift} = n g(E) \mu \ll I_{CH}$$

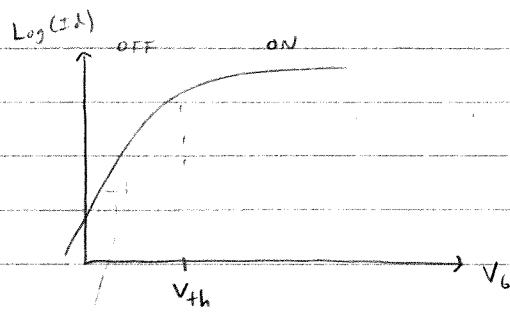
small, since V_g small small, since V_d small



$$I_{sub} = \mu_0 \frac{W}{L} C_{ox} (V_T)^2 (m-1) \exp\left(\frac{V_g - V_{th}}{m V_T}\right) \left(1 - \exp\left(-\frac{V_d}{V_T}\right)\right)$$

$$\text{where } V_T = \frac{kT}{q} \quad m = 1 + \frac{C_{dep}}{C_{ox}}$$

$$\frac{\frac{1}{V_g} C_{ox}}{\frac{1}{V_g/m} C_{dep}}$$



$$\frac{1}{\log(e)}$$

$$S\text{-factor} = \text{slope}_{\text{subthreshold}} = \frac{dV_g}{d(\log(I_d))} = S_t = 2.3 \frac{kT}{q} m = 2.3 \frac{kT}{q} \left(1 + \frac{C_{dep}}{C_{ox}}\right) \frac{V}{\text{decade-Amp}}$$

$C_{ox} \gg C_{dep}$

$$S_{t_{min}} = 2.3 \frac{kT}{q} (1+0) = \frac{60 \text{ mV}}{\text{decade}} \text{ at } T=300K$$

↑ holding V_d constant.

to decrease S -factor, you should decrease t_{ox} , decrease substrate doping, use a high K dielectric.

Typical S -factor is $80 - 120 \frac{\text{mV}}{\text{decade}}$

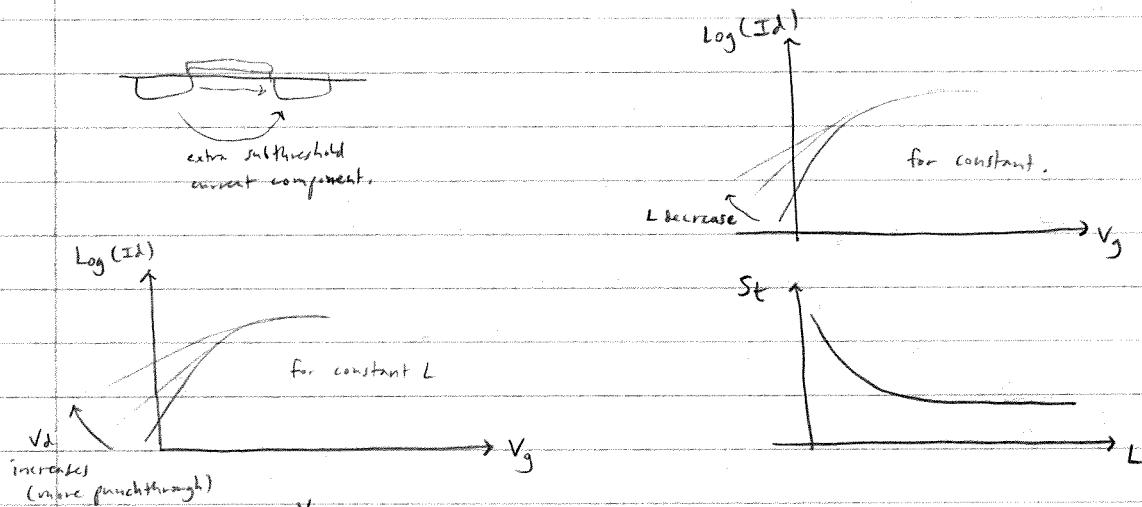
2. Long channel : $T \uparrow S_t \uparrow \quad \epsilon_s \uparrow S_t \uparrow$

$t_{ox} \downarrow S_t \downarrow$

$\epsilon_{ox} \uparrow S_t \downarrow$

$N_{bulk} \downarrow S_t \uparrow$

Short channel : S-factor will degrade (increase) if you have punch through



3. $S_t < 60 \frac{mV}{decade}$, what would you guess about that?

- 1) structure of device. (e.g. could be an electro-nano-mechanical relay or IMOS)
- 2) mob carrier transport mechanism (we assume diffusion for our theory)
- 3) temperature of operation. (ionization or tunneling might allow a better S_t.)
(if you decrease temperature, S decreases)

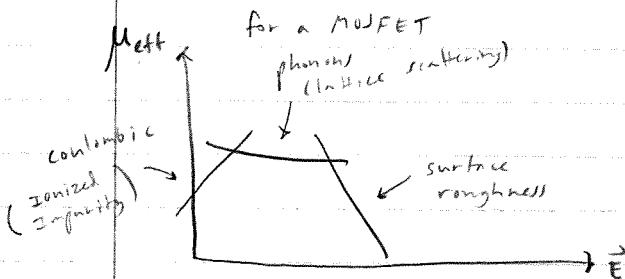
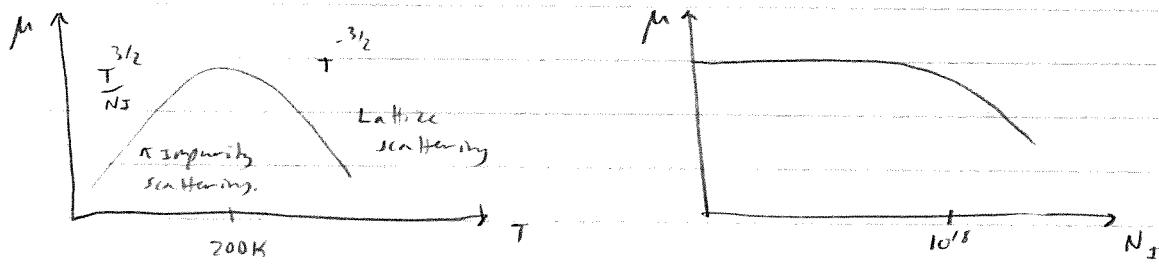
2009 - Nishi

$$1. \mu = \frac{g\tau}{m^*} \quad \tau \text{ is mean free time} \quad \tau = \frac{1}{p} \leftarrow \text{probability of scattering.}$$

$$2. \frac{1}{\mu} = \frac{1}{\mu_1} + \frac{1}{\mu_2} + \dots$$

$\rho = \rho_1 + \rho_2 + \dots$ if the mechanisms are independent.

$$\Rightarrow \frac{1}{\mu} = \frac{1}{\mu_1} + \frac{1}{\mu_2} + \dots \text{ if the mechanisms are independent.}$$

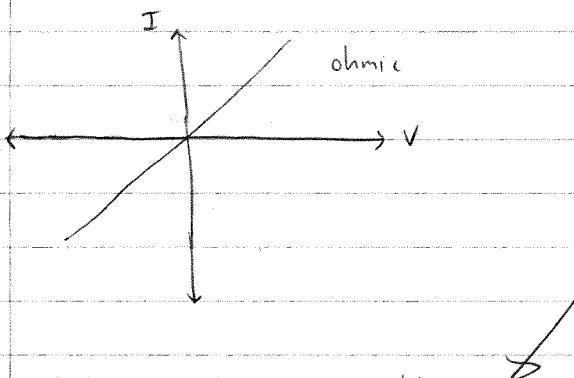
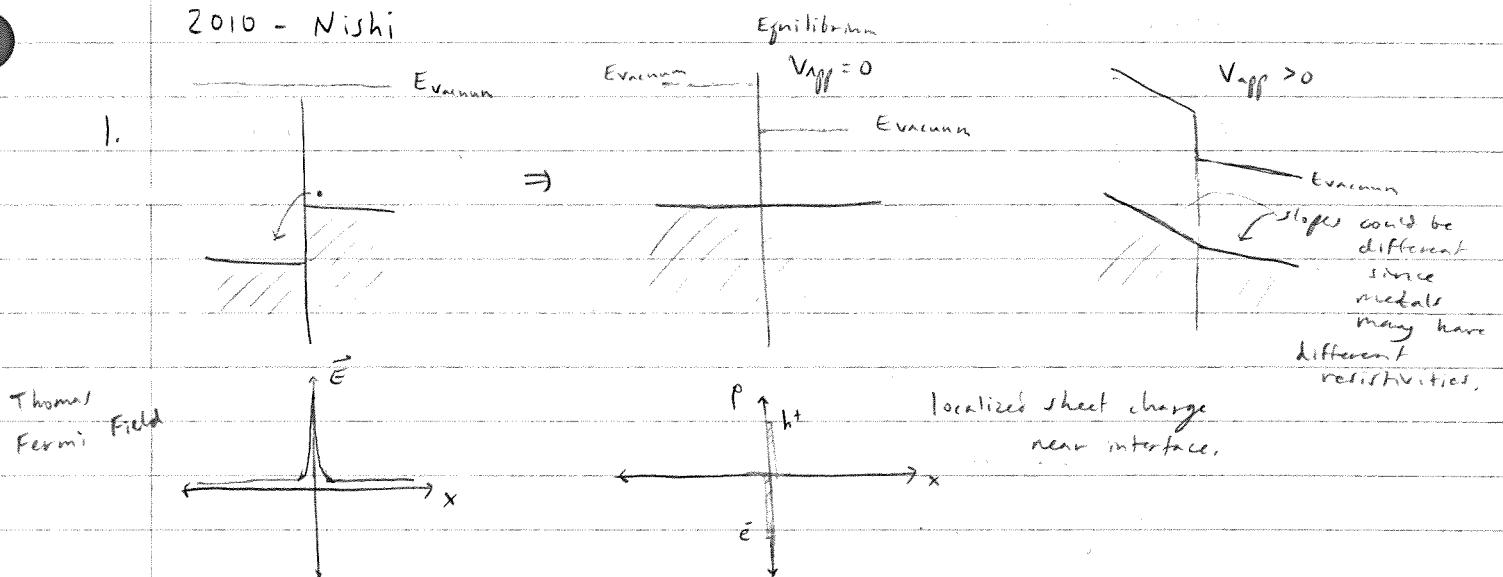


3. If size of the semiconductor is not large compared to the mean free path. Then you have ballistic transport so $\mu \neq \frac{g\tau}{m^*}$, since you can't use this statistical model which depends on mean free path. There is a high probability that many electrons will travel through the semiconductor without scattering at all.

If \bar{E} is very high, $v \neq \bar{v}\bar{E}$, $v = v_{sat} = v_{th}$

Also, if the different scattering mechanisms are not independent

2010 - Nishi



Note: For p-type, only deep trap states that are acceptor-like will have an effect ($W_{dep} \Rightarrow C_{dep} \uparrow, I_{tunnel} \uparrow$)
 For n-type, only deep trap states that are donor-like will have an effect ($W_{dep} \downarrow \Rightarrow C_{dep} \downarrow, I_{tunnel} \uparrow$)

2. MS contact, with trap states without trap states.

(Actually, for MoS-C, both types have an effect. ---)



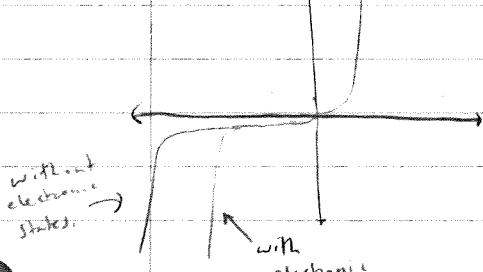
3. p-type. Assuming acceptor-like states at midgap:

states are ionized in depletion,

\Rightarrow decreases depletion width \Rightarrow more tunneling

$\Rightarrow I \uparrow$

Assuming donor-like states, it doesn't matter.



Assuming donor-like states at midgap:

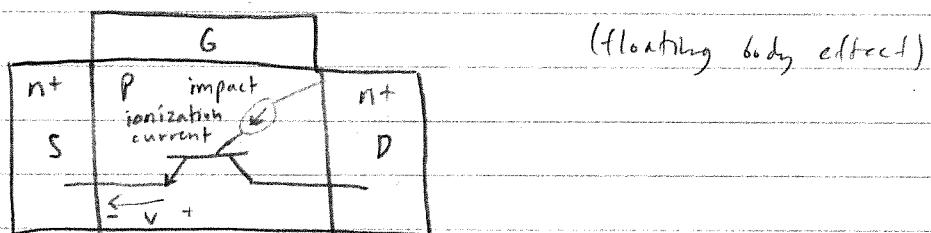
states are ionized in depletion,

\Rightarrow decreasing depletion width \Rightarrow more tunneling

$\Rightarrow I \uparrow$.

1996 - Saraswat

- Body of B is floating \Rightarrow the source junction can become forward biased, turning on a parasitic npn bipolar transistor between source and drain, in addition to the normal MOS transistor. This parasitic BJT can amplify impact ionization current (base current).



- Will this phenomenon take place in a PMOS transistor?

Yes, but the extent will be smaller, because β of a pnp is smaller than β of an npn since $\mu_p < \mu_n$

$$\beta_{DC} = \frac{D_E}{D_B} \frac{N_B}{N_E} \frac{W}{L_E} + \frac{1}{2} \left(\frac{W}{L_B} \right)^2 \quad \beta_{pnp} < \beta_{npn}$$

for npn: $D_E = D_P = \mu_p \frac{kT}{q}$ $D_B = D_N = \mu_n \frac{kT}{q} \Rightarrow \frac{D_E}{D_B} = \frac{\mu_p}{\mu_n} < 1 \Rightarrow \beta$ higher

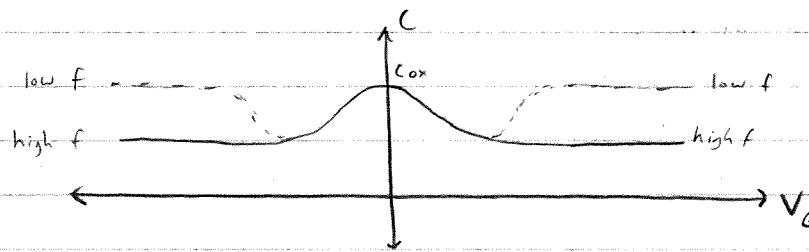
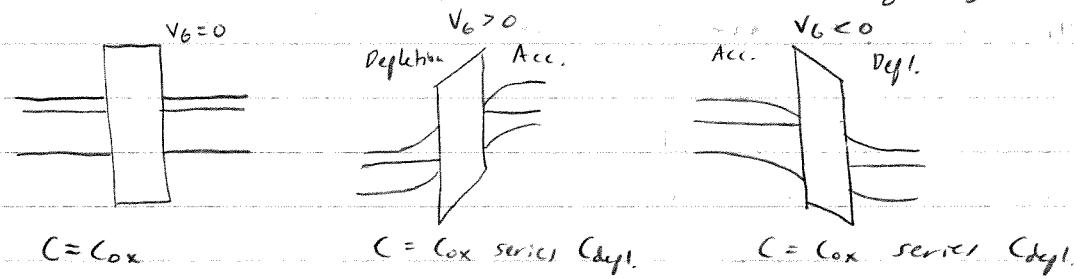
for pnp: $D_E = D_N = \mu_n \frac{kT}{q}$ $D_B = D_P = \mu_p \frac{kT}{q} \Rightarrow \frac{D_E}{D_B} = \frac{\mu_n}{\mu_p} > 1 \Rightarrow \beta$ lower

- The point defects act as generation-recombination centers, thus reducing the lifetime of minority carriers. This reduces the β of the parasitic BJT and hence the excess current disappears.

i.e. $L_B \downarrow \Rightarrow \beta_{DC} \downarrow$

1998 - Saraswat

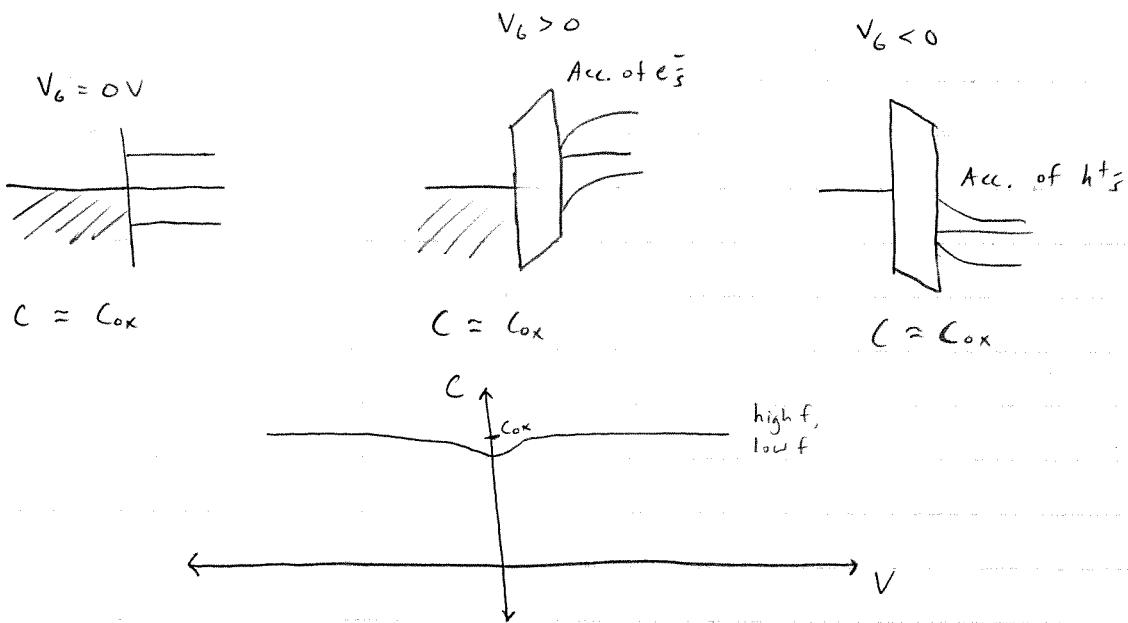
- At zero bias, the MOS capacitor will be in the flatband condition with gate oxide capacitance C_{ox} as the total capacitance. As the potential across the device is changed, the gate electrode will also be subjected to band bending similar to the substrate. When the substrate is depleted, the gate will be in accumulation and vice versa. The total capacitance will be C_{ox} in series with the Si depletion capacitance. As a result the CV characteristics for positive bias will be a mirror image of the characteristics for negative bias and will look like a bell shaped curve for high frequency measurement.



- At zero bias, the MOS cap will be in flatband condition with gate oxide capacitance, C_{ox} as the total capacitance and Si will be intrinsic with E_F at midgap. As the bias is made $+V$ or $-V$, the Fermi level at the surface will shift towards the conduction or valence band respectively. In each case, the surface will be accumulated with electrons or holes. Carrying this argument further, the C-V characteristic will look similar to the case of Q1.

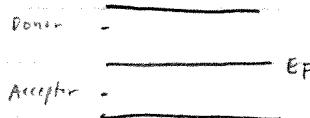
(I think this just means that it will also be symmetric. In this case, the bell should be upside down.)

(see next page)

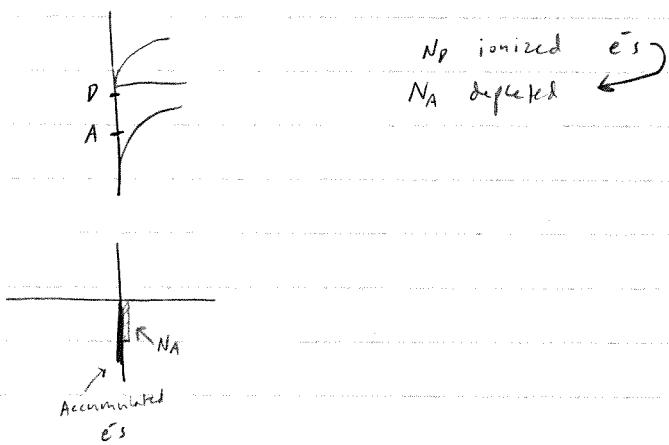


$$+ N_D = N_A$$

if E_F is at midgap 1 donor levels should be symmetric about E_F



in this case you can have simultaneous accumulation and depletion.



2000 - Saraswat

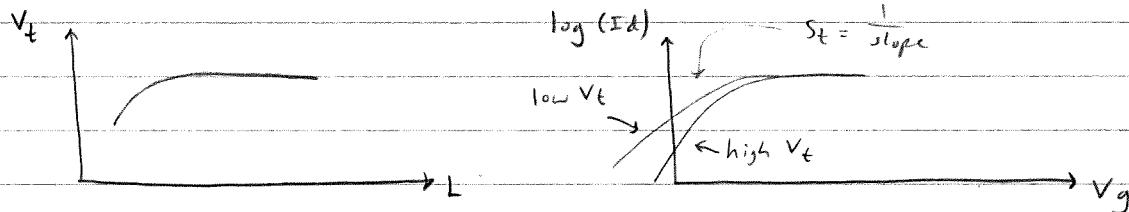
As $L \downarrow$: $500\text{nm} \rightarrow 5\text{nm}$

1. DIBL



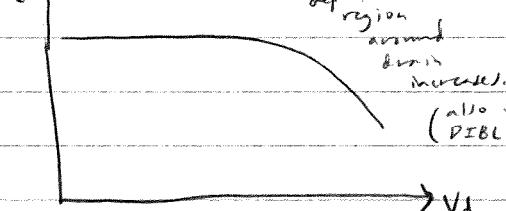
$\Rightarrow I_{\text{sub}} \uparrow, V_T \downarrow$

2.



for low V_T S factor increases for $V_T \downarrow$

3.

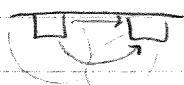


$\Rightarrow I_{\text{sub}} \uparrow$

S_f

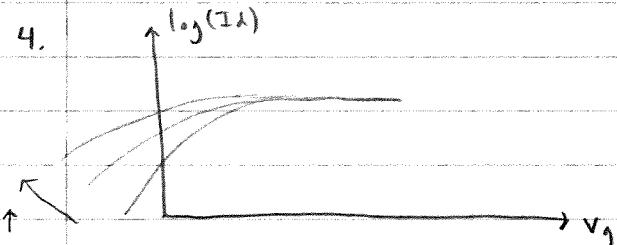
60mV
decade

3. punchthrough



$S_f \uparrow, I_{\text{sub}} \uparrow, Id \propto V_d^2$ at low V_g .

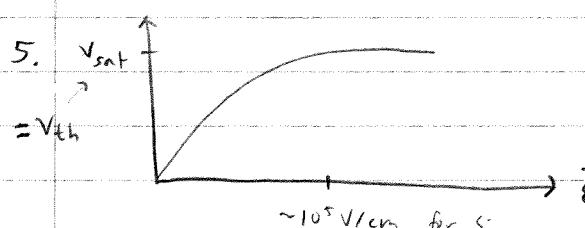
4.



as $S_f \uparrow$, can't control I_d with V_g .

V_g becomes less important. $I_{\text{sub}} \uparrow$

5.



V_{drift} saturates

$I_d \propto (V_g - V_t)^2$ is no longer valid.

now $I_d \propto (V_g - V_t)$

\Rightarrow linear rather than square dependence on $(V_g - V_t)$

6. Rchannel ↑

$$R = \frac{\rho l}{A} \times \frac{\rho l}{l^2} \times \frac{\rho}{l}$$

7. $R_{\text{contact}} \uparrow$ since area of contact is smaller. $R \propto p$

ρ is limited by limit of solid solubility which has already been reached.

mean free path,

+

8. But as $L \downarrow$, can have ballistic transport if $L < l$
⇒ almost no scattering. ⇒ V_d can exceed $V_{sat} = V_{th}$
⇒ higher g_m , I_d .

9. as $L \downarrow$, $E \uparrow$ ⇒ hot carrier effects + breakdown.

↓
because in practice, people don't
scale voltage down as $L \downarrow$ (in order
to keep making performance gains).

elaborate
here

	K	E_g
SiO_2	4	9
HfO_2	25	6

EOT is $\sim 1.2\text{nm}$ in modern processes

but t_{ox} is $\sim 10\text{nm}$ since it uses high k ,

2000 - Saraswat

As $t_{ox} \downarrow$: $100\text{nm} \rightarrow 1\text{nm}$



1. harder to deposit \rightarrow uniformity issue + reliability issue, \Rightarrow easier to breakdown.

2. dopant penetration from gate
into oxide or channel.

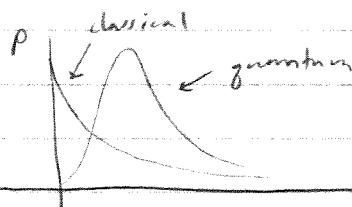


oxide $\Rightarrow I_{gate} \uparrow$ due to trap assisted tunneling + V_t will shift.
channel \Rightarrow decrease channel mobility due to increased surface scattering.

3. Quantization Effect

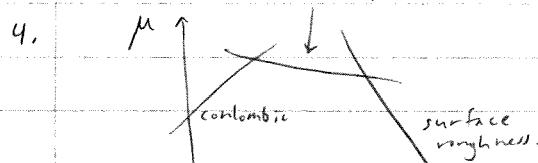
+ depletion in Poly-Si gate

(quantum well)



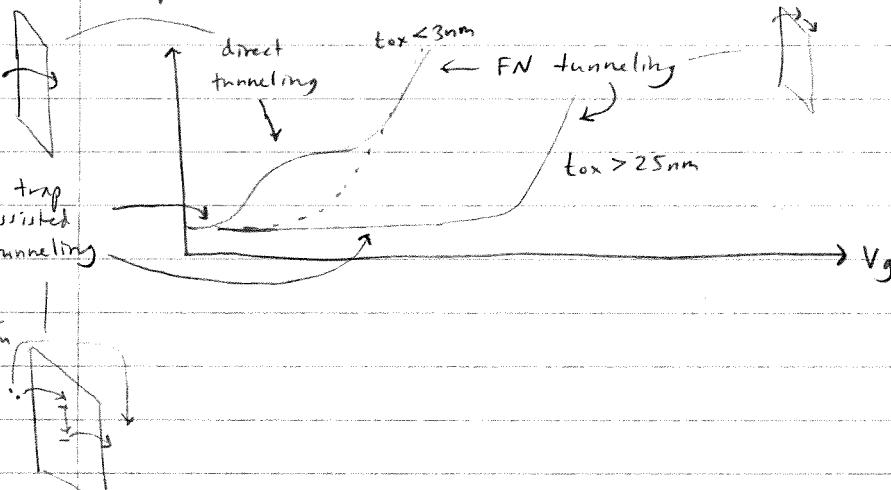
$$\Rightarrow EOT \uparrow \Rightarrow C_{ox} \downarrow \Rightarrow I_d \uparrow \Rightarrow g_m \downarrow$$

4. μ vs E (lattice/phonon)

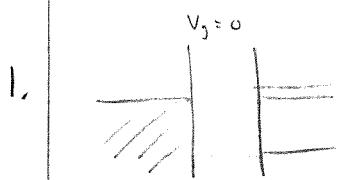


\vec{E}

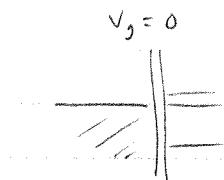
5. direct tunneling, even at small \vec{E} $J_{DT} \propto e^{-t_{ox}}$
especially for $t_{ox} < 3\text{nm}$



2003 - Saraswat



$$C_{ox_1} = \frac{\epsilon_0 A}{t_{ox_1}}$$



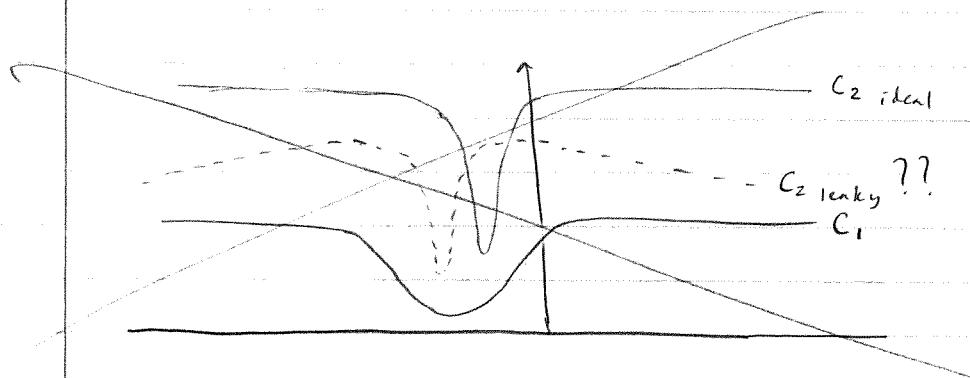
$$C_{ox_2} = \frac{\epsilon_0 A}{t_{ox_2}} > C_{ox_1}$$

$$C = \frac{Q}{V}$$

\Rightarrow leakage will take some Q away

\Rightarrow decrease C.

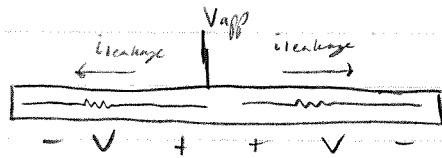
can model as



see next page.

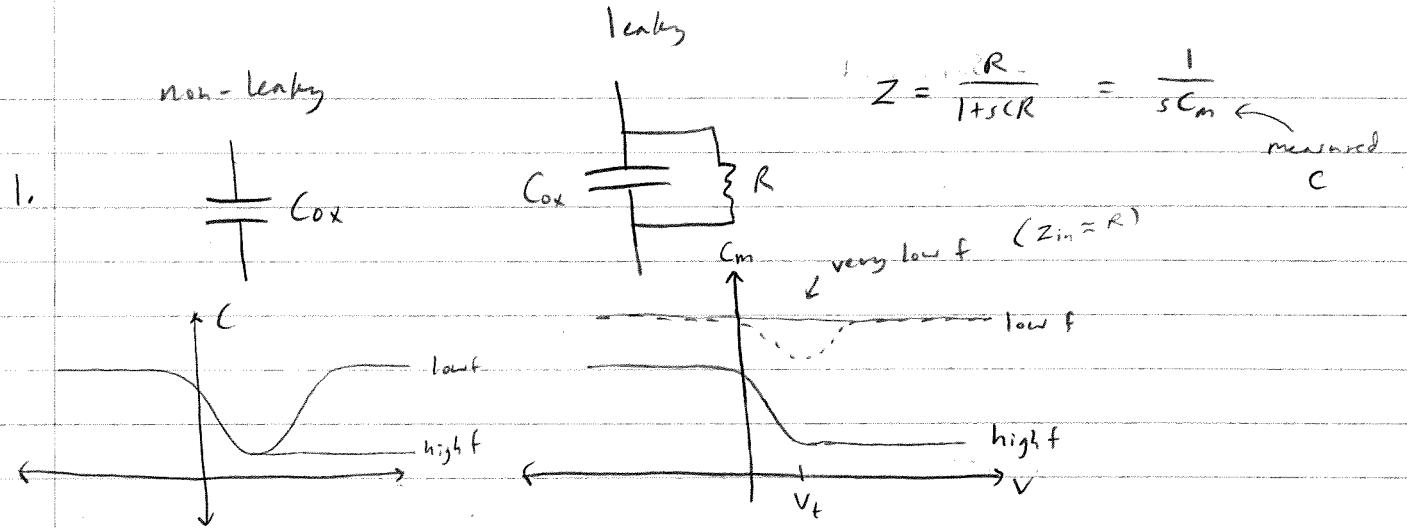
if $|V| \uparrow$, $J_{\text{tunnel}} \uparrow \Rightarrow C \uparrow$

2.



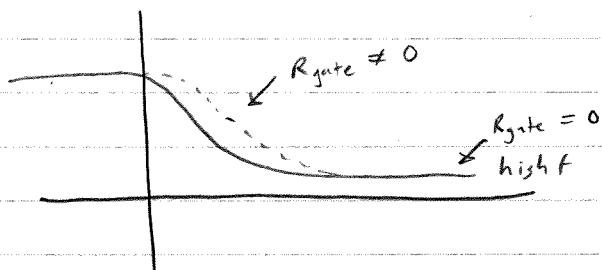
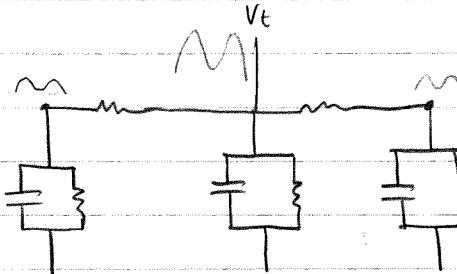
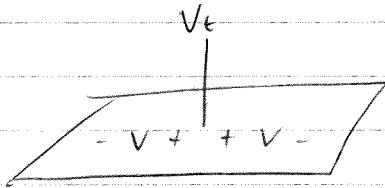
\Rightarrow Voltage at the edges will be less than Vapp

\Rightarrow in inversion + accumulation, the Capacitance per unit area will increase. compared to it there were no parasitic resistance in the contact.



2. Consider high f measurement,

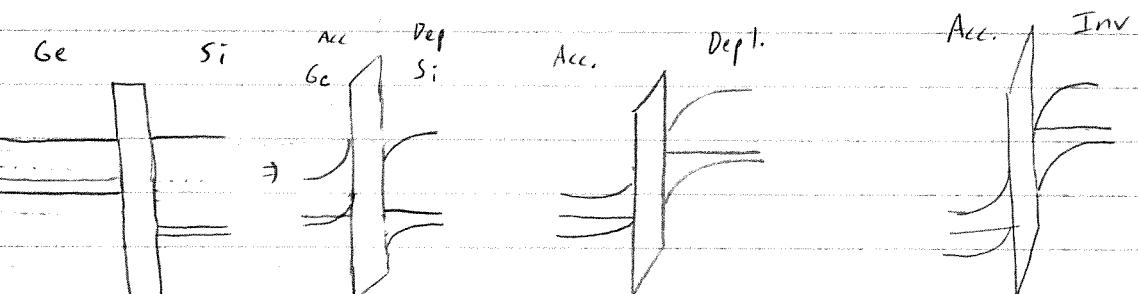
\Rightarrow from $0V$ to V_t , C_m should increase



Φ_s
SiO₂

$$\begin{array}{lll} \chi = 4 \text{ eV} & \chi = 4 \text{ eV} & \chi = 4 \text{ eV} \\ - K = 12 & - K = 16 & - K = 12 \\ \text{Si} - 1.12 \text{ eV} & \text{Ge} - 0.66 \text{ eV} & \text{GaAs} - 1.42 \text{ eV} \\ - n_i = 10^{10} \text{ cm}^{-3} & - n_i = 2 \cdot 10^{13} \text{ cm}^{-3} & - n_i = 2 \cdot 10^6 \text{ cm}^{-3} \end{array}$$

Assume $\Phi_{s_{Ge}} = \Phi_{s_{Si}}$



$$V_G = 0 \Rightarrow C = C_{ox}$$

$$V_G > 0 \quad C = \left(\frac{1}{C_{ox}} + \frac{1}{C_{dep}} \right)^{-1}$$

$$V_G >> 0 \quad C = C_{ox}$$

$$\text{Depl.} \quad \text{Acc.}$$

$$\text{Inv} \quad \text{Acc.}$$

$$V_G < 0 \Rightarrow C = \left(\frac{1}{C_{ox}} + \frac{1}{C_{dep}} \right)^{-1}$$

$$V_G \ll 0 \Rightarrow C = C_{ox}$$

$$E_g_{Ge} < E_g_{Si} \Rightarrow Ge \text{ needs less band bending to reach depletion.}$$

$$\Rightarrow V_T_{Ge} < V_T_{Si} \quad (\text{however the Workfunction difference shifts our CV curve to the left})$$

$$W_{MAX} = \sqrt{\frac{2K\epsilon_0}{gN_A}} (2\phi_F) \quad \text{where } 2\phi_F = \Phi_s \Rightarrow W_{MAX_{Si}} > W_{MAX_{Ge}}$$

total band bending

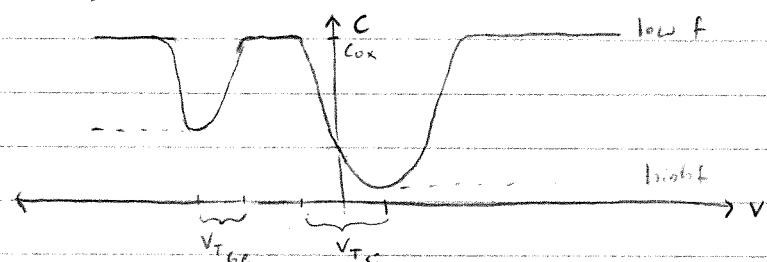
$$E_F - E_i = kT \ln \left(\frac{N_D}{n_i} \right) \Rightarrow \text{for } \frac{(E_F - E_i)_{Si}}{(E_F - E_i)_{Ge}} < 1$$

Assuming the difference in Φ_s is greater than the difference in K ,

$$\Phi_F = E_i - E_F \Rightarrow C_{dep_{Ge}} > C_{dep_{Si}} \Rightarrow \text{without light, our C-V is:}$$

or $E_F - E_i$

$$W_{MAX} \propto \sqrt{2\phi_F}$$

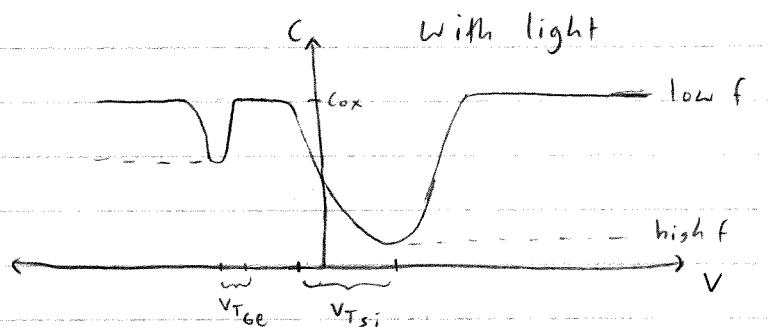


When light shines: $E = \frac{1.24}{\lambda} = \frac{1.24}{1.3} = 0.95 \text{ eV}$

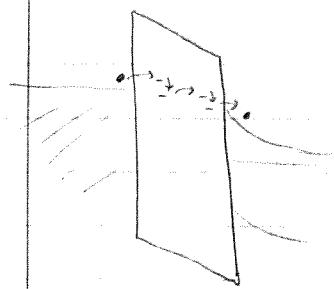
$$\Rightarrow E_{\text{Ge}} < E_{\text{ph}} < E_{\text{Ge}_{\text{Si}}} \Rightarrow \text{electron-hole pairs will be generated}$$

In Ge, but not Si. This will increase the depletion capacitance of Ge since the generated electrons in and around the depletion region will drift towards the SiO_2 interface and create a sheet of charge. This sheet of charge $\Rightarrow w_{\text{an}}$ will also lower the threshold voltage $V_{T_{\text{Ge}}}$. The Si CV won't change.

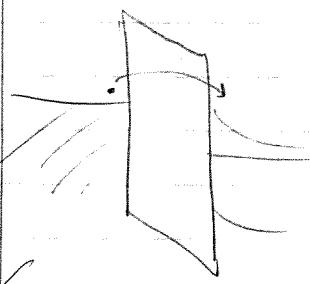
\Rightarrow The CV will look like:



2007 - Saraswat

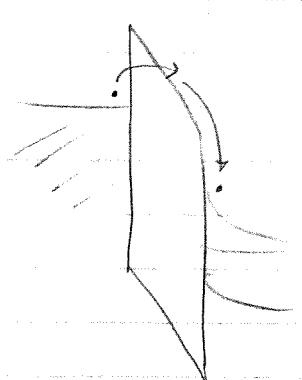


Small V_g . Trap assisted tunneling,
relatively constant & independent of V_g .



small V_g , for $t_{ox} < 3\text{nm}$
also have direct tunneling current

$$J_{DT} \propto e^{-t_{ox}}$$

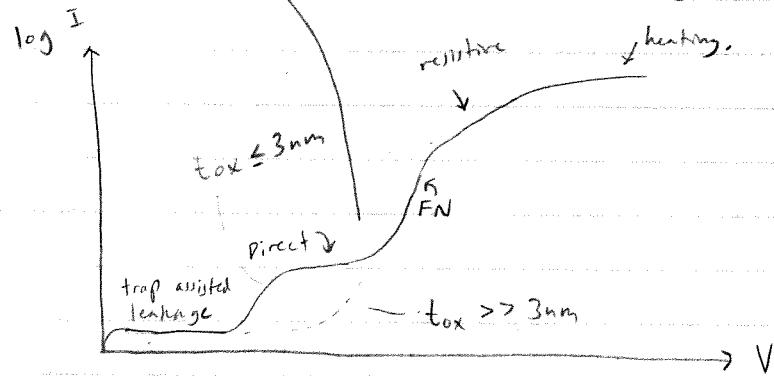


medium V_g , FN tunneling.

$$J_{FN} \propto e^{V_g}$$

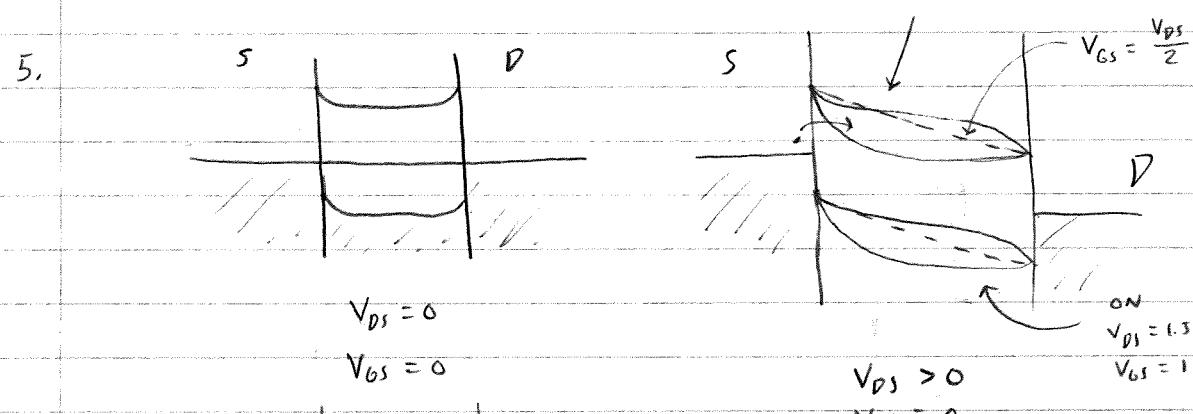
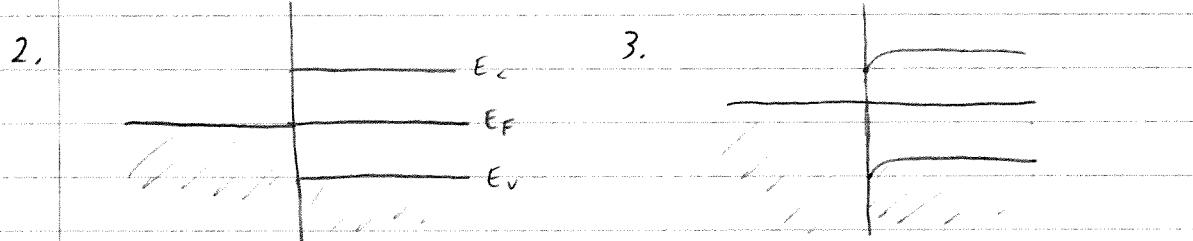
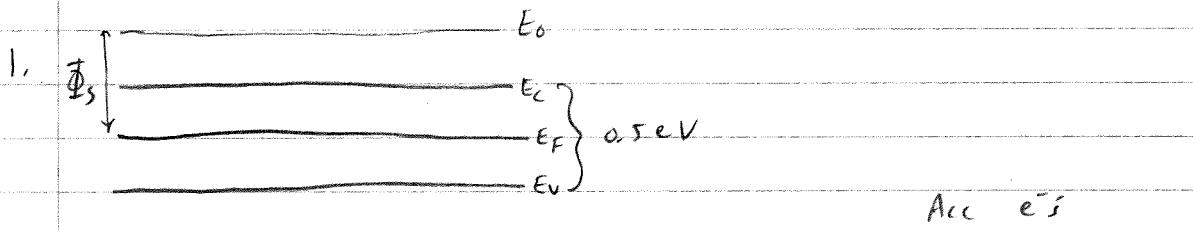
for direct tunneling,
both the height
and shape of the barrier.
 J_{DT}

At very high current, there may be
destructive breakdown. Current may
become limited by the resistance of the
Si bulk. At very very high current,
the Si bulk may heat up & resistance will increase.

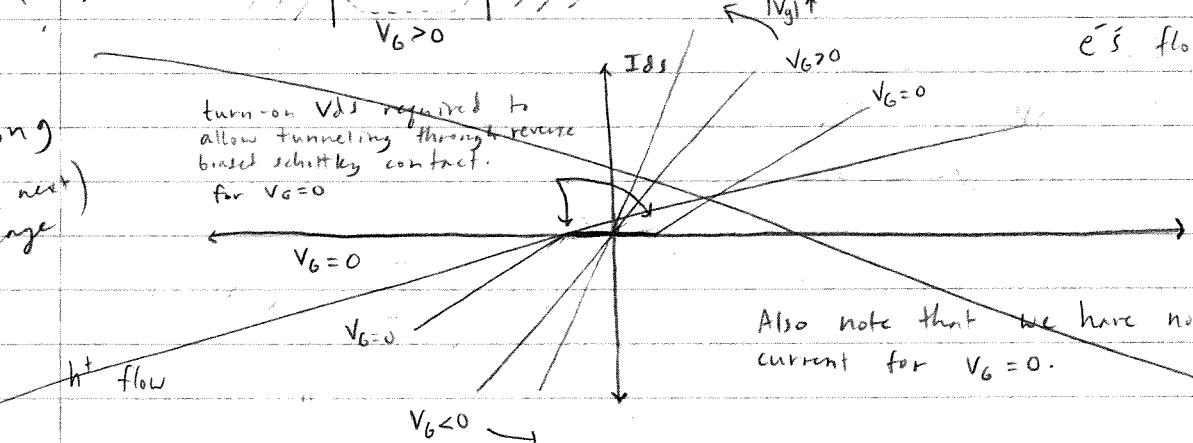
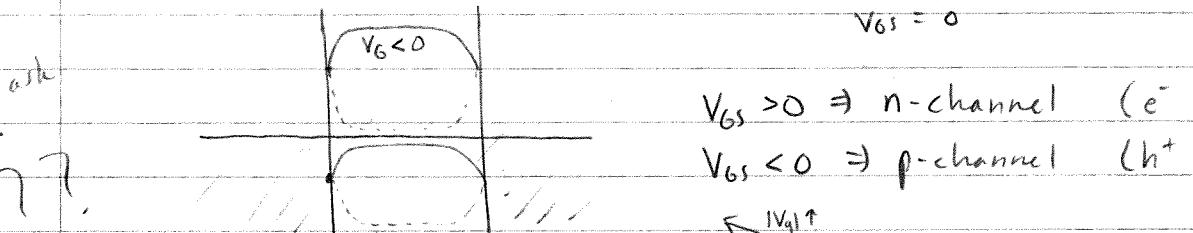


for very high V ,
there may also
be destructive
breakdown of the oxide.

2006 - Wong



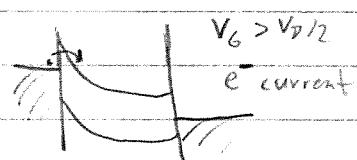
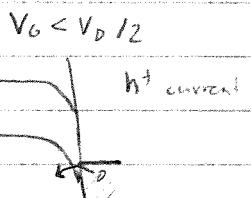
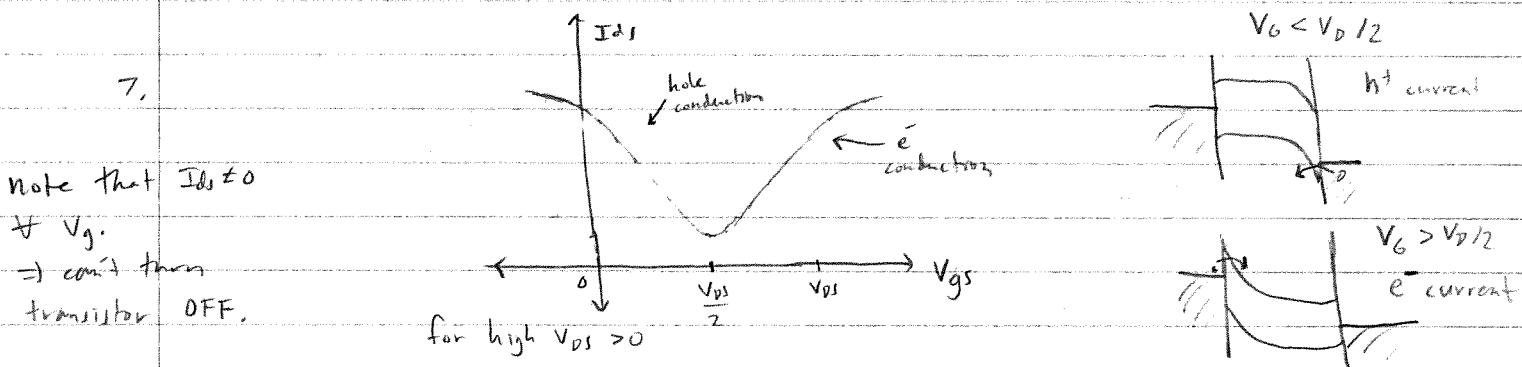
Maybe ask
Wong.



For ambipolar, S-D contacts must be metal. So that the source can supply both e^- s and h^+ s

(nt source can only supply e^- s. pt source can only supply h^+ s)

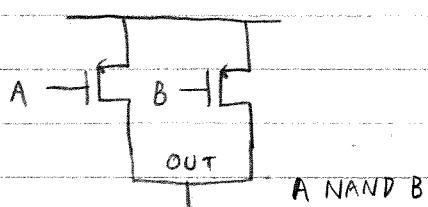
6. Could improve contacts by using a heavily doped (degenerate) n⁺ well at the S and D. \Rightarrow the depletion region will be very narrow, and direct tunneling can occur even at $V_{D1} = 0$. Thus the contacts become ohmic.



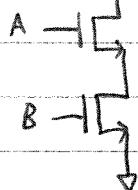
8. No. These ambipolar transistors will not work well in digital logic, where we'd like to be able to use MOSFETs as switches and thus be able to turn them OFF completely.

Wrong, it's
just since
you can't off
turn it off
with ambipolar

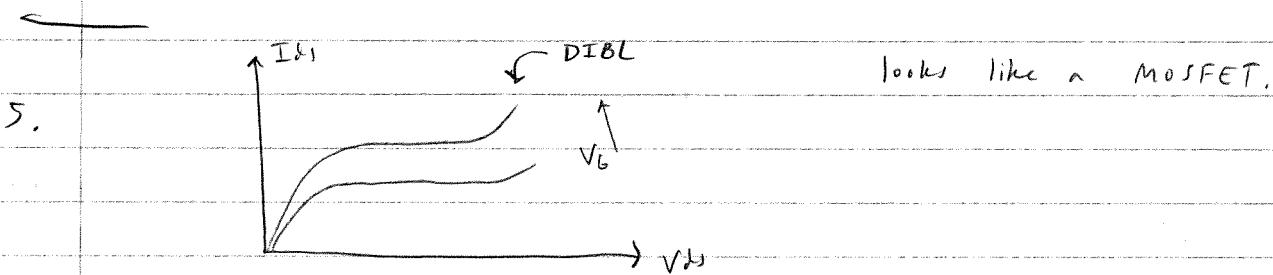
9.



A	B	OUT
1	1	0
1	0	1
0	1	1
0	0	1



No, this will not work if the transistor is made of the ambipolar transistor above.



right answer:
see last page of
devices.



see EE311 L10-1
slide 50.

2004 - Saraswat

Lindauer's principle.

- Ultimate limit to device scaling?

atomic size, because each device element needs at least one atom to exist. E.g. channel, source, drain, gate.
Unit cell length of Si is $\sim 5\text{ \AA} \approx 0.5\text{ nm}$

- What is the minimum time a device will need to switch its state.

Assume OFF \rightarrow ON is limited by transit time.

$$\Rightarrow \begin{array}{c} G \\ \square \\ \swarrow \quad \searrow \\ S \quad C \quad D \end{array} \quad KE = U \quad \frac{1}{2}mv^2 = qV$$
$$\Rightarrow v = \sqrt{\frac{2q}{m}V} \quad \text{assume max } v \text{ is } c = 3 \cdot 10^8 \text{ m/s}$$

$$\Rightarrow t_{\text{trans}} \approx \frac{0.5 \cdot 10^{-9}}{3 \cdot 10^8} = 1.6 \cdot 10^{-18} \text{ s}$$

- What will be the corresponding energy requirement.

if we model it as a parallel plate:

$$W = \int_s^D F \cdot ds$$



$$W = qe \int_s^D \vec{E} \cdot ds = qeV$$

entropy: S units of J/K joules per kelvin.
boltzmann constant

$$S_i = k \log (\# \text{ of states}) \Rightarrow \text{assume we have 10 bits.}$$

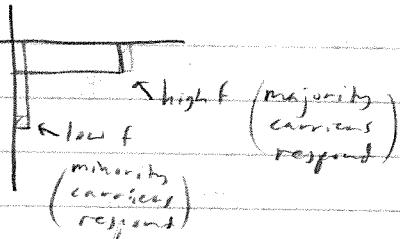
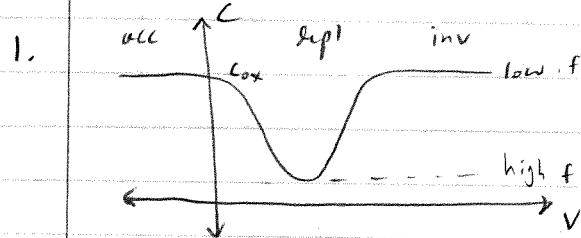
$\Rightarrow S_i = k \log (2^{10})$ if we switch to a specific desired combo of bits.

$$\Rightarrow S_f = k \log (1) = 0 \Rightarrow \Delta S = S_i - S_f = k \log (2^{10}) = \frac{\Delta Q}{T} \leftarrow \text{charge in Energy.}$$
$$\leftarrow \text{Temperature}$$

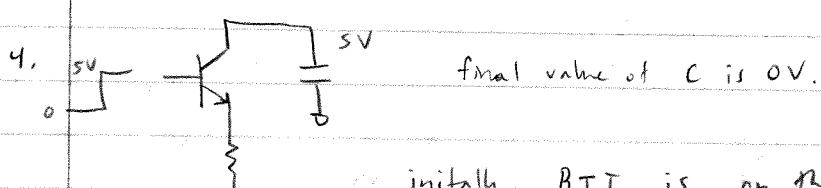
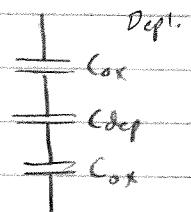
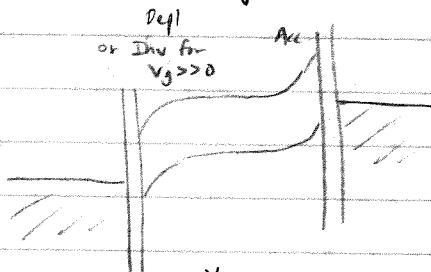
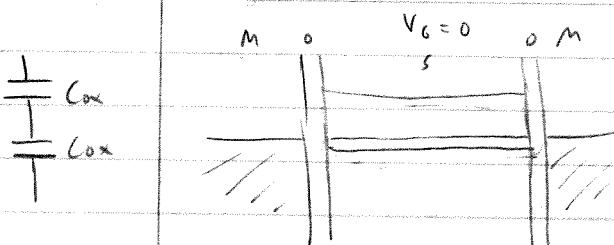
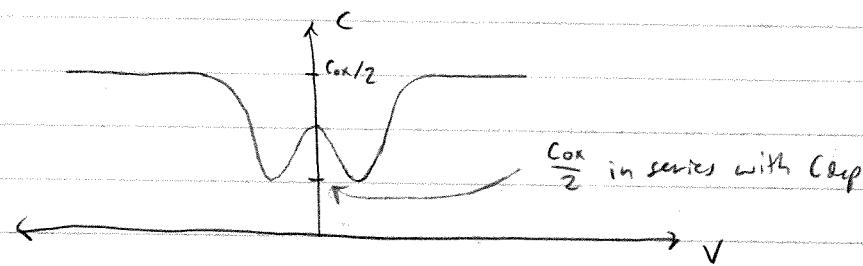
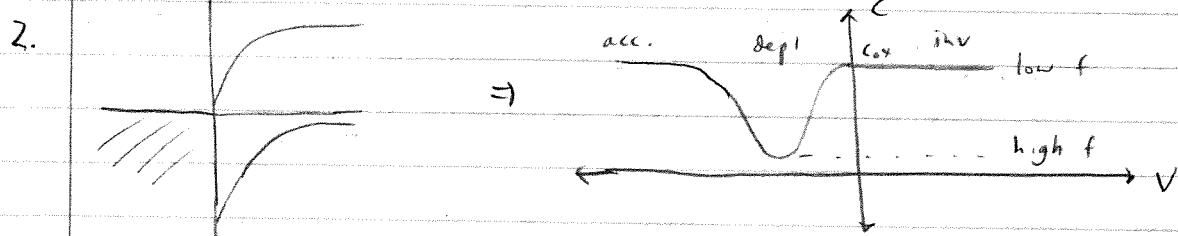
$$\Rightarrow \text{for } 300K \quad \Delta Q = 3000k \log(2)$$

$$\Rightarrow \text{for a single bit, the fundamental limit is } \boxed{\Delta Q = kT \ln(2)}$$

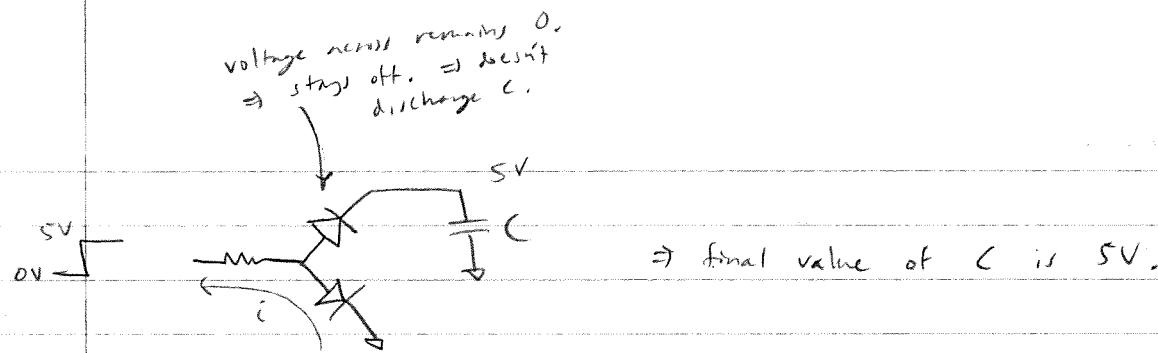
Simon
Pre-1993 - Wong



Minority carriers come from recombination/generation which are slow processes \Rightarrow can't respond at high freq.

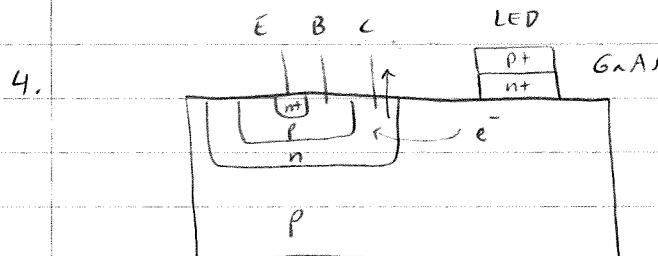


initially, BJT is on the brink of forward active
 $\Rightarrow V_{be} > 0 \quad V_{ce} = 0 \Rightarrow$ since $V_{be} > 0.7$, BJT turns on, starts drawing current + discharging C. $\Rightarrow V_{ce} > 0 \Rightarrow$ BJT enters saturation \Rightarrow looks like a resistor. So C will completely discharge.



Explain why the two diodes don't make a transistor.

In a BJT, the base is very narrow, so e's entering the base from the forward biased E-B diode get swept across and into the collector even though the V_{BC} diode is reverse biased. In the above case, the diodes are



completely independent, so the electrons simply leave through the resistor.

- e's simply contribute to collector current (leave through C), since $V_{BC} < 0 \Rightarrow$ b-C diode is reverse biased.

- Assume LED light is incident on the base. What happens in the base? How are the emitter, base and collector currents affected?

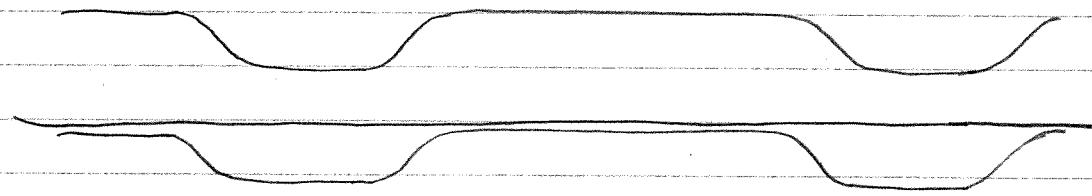
$e^- h^+$ pairs are generated in the base. e^- 's flow into the collector, and holes flow out of the base.
 $\Rightarrow i_b' \Rightarrow i_c'$

$$i_b' = i_b - \Delta i_b \quad i_c' = i_c - \beta \Delta i_b$$

2010 - Wong

1. Do you know how a CCD works?
2. Draw the energy band diagram along the SiO_2/Si interface for the following device structure and applied bias.

0V 5V 0V 0V 5V



3. Assume some e^- s are collected under G_2 , now we want to move the e^- s to the right under G_3 , what biases would you apply to the gates G_1 to G_5 ?

	G_1	G_2	G_3	G_4	G_5
ϕ_1	0	5	0	0	5
ϕ_2	0	5	5	0	5
ϕ_3	0	0	5	0	5

4. What are the forces acting on the electrons that move the charges from G_2 to G_3 ?

ϕ_2 : Diffusion

ϕ_3 : Drift

5. If we want to move the electrons faster, what would you do? You can change anything you want, including (but not limited to) applied biases, device structure, doping, and the materials of the device.

$$v_d = \mu_n E \Rightarrow \text{increase } E \text{ and } \mu_n.$$

$$D_n = \mu_n \frac{kT}{q} \Rightarrow \text{increase } \mu_n.$$

To Increase \bar{E} : Increase applied bias, decrease separation of gates.

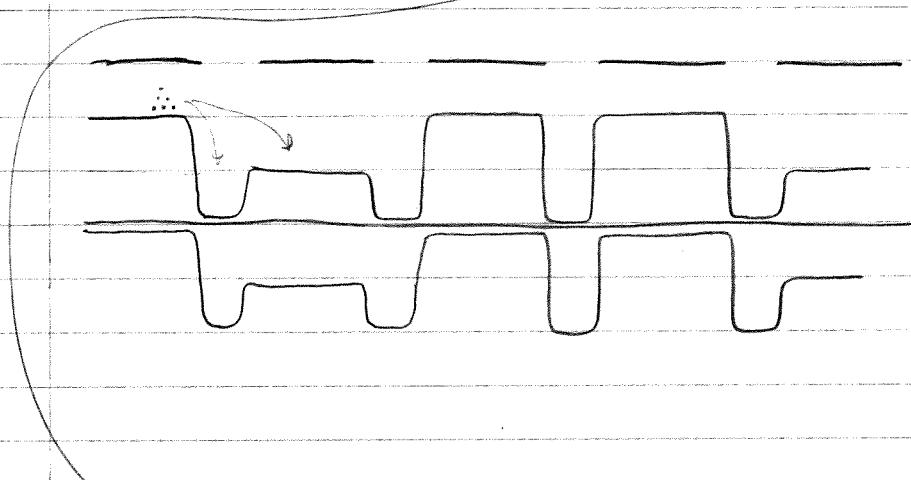
To Increase μ_n : Decrease p-type doping ($\propto \frac{1}{N_D^{3/2}}$ for low T).

Use higher mobility material like GaAs.

however, if we do this then $V_T \downarrow \Rightarrow$ we cannot increase applied bias, else we would get inversion.

? 6. If there is nt doping in the p-Si in between the gates, how does the band diagram look like and how does it effect the charge transfer?

Don't
Ans.

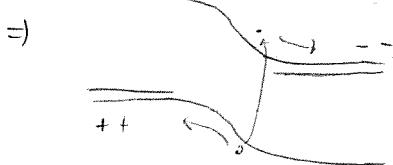
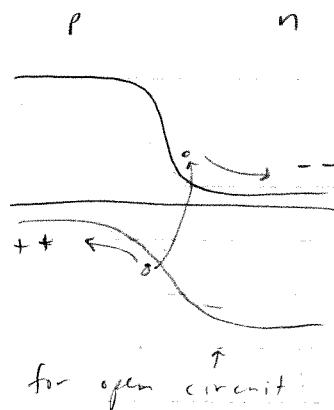
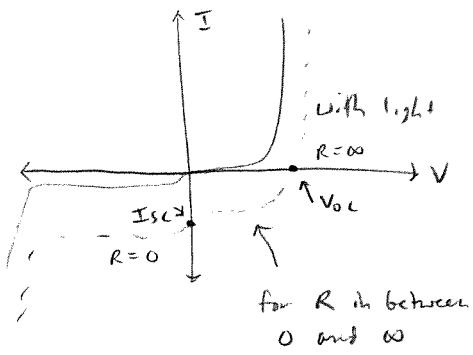


as the charges transfer, some are trapped in the nt regions.
 \Rightarrow this reduces the transfer efficiency + introduce some noise in the signal.

2008 - Wong

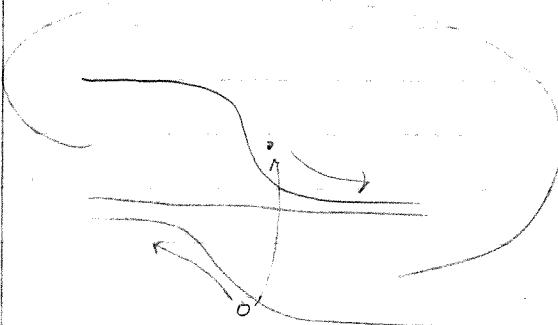
- P-N junction has V_{bi} generated $e^- h^+$ pairs in depletion region or around depletion region can be separated by V_{bi} .

2.



\Rightarrow PN junction becomes slightly forward biased, fermi levels split into quasi-fermi levels.

for short circuit



$$I_{sc} = I_c$$

Fermi level is at normal level
(from before any light is shone on it)

3.

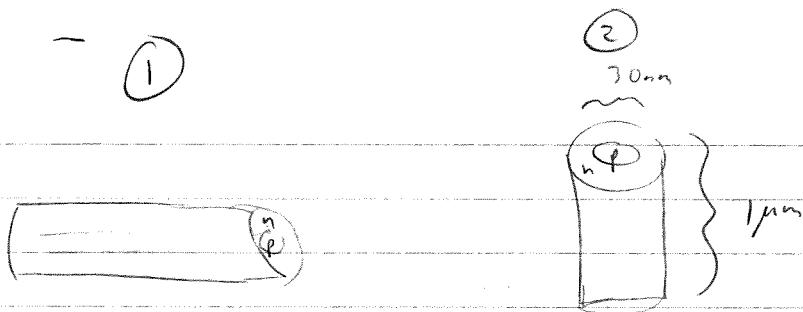
$$(E_F - E_i) = kT \ln\left(\frac{N_p}{n_i}\right) \text{ for n side}$$

$$(E_i - E_F) = kT \ln\left(\frac{N_A}{n_i}\right) \text{ for p side}$$

- Area of junction \rightarrow determines area of depletion region.
Want it to be large \Rightarrow don't want 3 and 4.

Also ③ + ④ will have more surface recombination as carriers diffuse into junction.

- ①



if α is large, would want to use ①, otherwise, should use ②.

(Note: optimal bandgap for solar is $\sim 1.4\text{eV}$ of GaAs is good, but one problem is it has higher absorptive coefficient, and has higher surface recombination than Si, since Si grows SiO_2 which passivates the surface very well, whereas GaAs does not).

5. Use PIN structure. OR



Antireflection coating on top.
Reflection coating on bottom.

also decreases contact resistance, & increases depletion width on other sides.
Make outer shell heavily doped and thin to allow light to go through without being absorbed. Make inner shell lightly doped to increase size of depletion region & increase diffusion length.

$$r = \frac{1}{C N T} \quad \begin{matrix} \uparrow \\ \text{trap coefficient} \end{matrix}$$
$$L_n = \sqrt{D_n \tau} \quad \begin{matrix} \leftarrow \\ \text{carrier lifetime} \end{matrix} = \sqrt{\mu_n \frac{kT}{q}} \tau \quad \begin{matrix} \rightarrow \\ n \end{matrix} \quad \begin{matrix} \leftarrow \\ L_n \end{matrix} \quad \begin{matrix} \rightarrow \\ p \end{matrix}$$

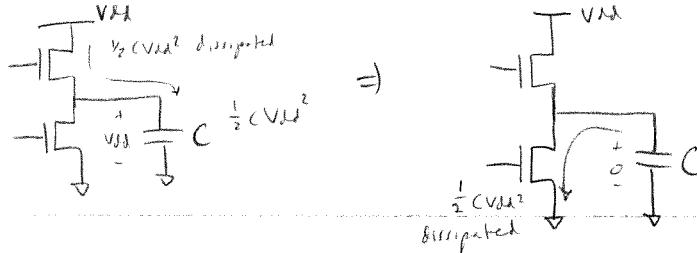
$\sigma = g(\mu_n + \mu_p)$
Make inner shell p and outer shell n^+ , since n^+ is more conductive and $L_n > L_p$ since $\mu_n > \mu_p$ so we want the large region to be p .

Make the semiconductor very pure (no traps, etc), that way the L_n .

Use a good heat sink since at high T , the semiconductor looks more intrinsic, and also $\mu \downarrow \Rightarrow L_n \downarrow \Rightarrow$ efficiency degrades.

\uparrow
doesn't look like a p-n junction anymore. just looks like a photoresistor.

$CVDD^2 / \text{cycle}$



2005 - Saraswat

scaling

\Rightarrow Power density $\propto 1.$

a) Static Power Dissipation ($I_{S\text{ub}}$)

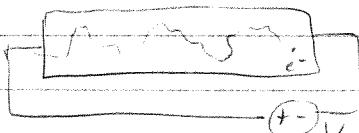
b) Dynamic Power Dissipation (Capacitor on/off) $P = CV_{DD}^2 f$

c) Oscillator + relay

d) Dynamic switching when both M1 + M2 are on

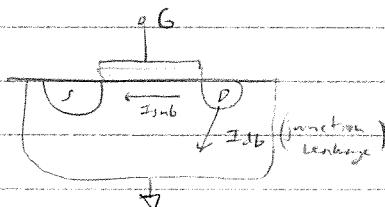


3. Resistive elements will convert electrical energy to thermal kinetic energy.



i 's accelerated by E ,
+ collide with lattice.
 \Rightarrow phonon scattering. \Rightarrow heat.

4. Transistors (channel resistance, junction leakage (to bulk), gate leakage, $I_{S\text{ub}}$), Resistors, Interconnects (Resistors).



dynamic { In the channel resistance, (resistance + contact)
during SAT + Triode,
static { From $I_{D\text{b}}$, $I_{S\text{ub}}$, and I_{gate} leakage
for static case.

(90% of I_{off} for 65nm)
(direct, trap assisted, FN)

Ignite can be from tunneling or hot carrier injection.

5. 1) Increase t_{ox} while using high K, high bandgap dielectric.

\Rightarrow t_{ox} is the same, but K_{ox} is higher \Rightarrow less gate leakage.
(but same active performance.)

2) $\mu \uparrow$, $L \downarrow$ (decrease channel resistance)

3) grow clean oxide with no traps (decreases I_{gate} from trap assisted tunneling)

4) use SOI to reduce junction leakage (I_{db}) and punch through.

\downarrow
improves isolation.

5) use a good heat sink.

6) decrease contact resistance

7) ~~use~~ trigate or finfet. !!

for constant
 E field scaling,

$CVDD^2 f = P$

\downarrow

$\frac{1}{\alpha} \downarrow$

$\alpha^2 \downarrow$

$I \rightarrow \frac{1}{\alpha^2}$

$\Rightarrow \frac{I}{CV} \rightarrow \alpha$

$P \rightarrow \frac{1}{\alpha^2}$

$A \rightarrow \frac{1}{\alpha^2}$

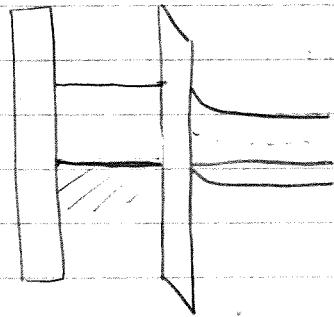
$\frac{P}{A}$ remains

A constant

\uparrow
power density.

2005 - Wong

1.



$$V_6 = 0 \text{ V}$$

$$\frac{E_F}{2} - 0.36 \text{ eV} = 0.2 \text{ eV} \Rightarrow V_{FB} = 0.2 \text{ V}$$



$$E_i - E_F = kT \ln\left(\frac{N_a}{n_i}\right) = 0.026 \ln\left(\frac{10^{16}}{10^{10}}\right) = 0.36 \text{ eV}$$

$$\phi_F = g(E_i - E_F) = 0.36 \text{ V}$$

$$V_{th} = 2\phi_F + \frac{k_s}{k_b T_{ox}} \sqrt{\frac{2qN_A}{k_s \epsilon_0} 2\phi_F}$$

$$= 2 \cdot 0.36 + \frac{3.9}{11.8} \cdot 10^{-6} \text{ cm} \sqrt{\frac{2 \cdot 1.6 \cdot 10^{-19} \cdot 10^{16} (0.72)}{3.9 \cdot 8.85 \cdot 10^{-14}}} = 0.92 \text{ V}$$

$$V_{inv} = V_{FB} + V_{th} = 0.2 + 0.92 = 1.02 \text{ V}$$

rule of thumb: $V_{inv} \approx g \cdot E_b \approx 1 \text{ V}$ for Si.

Wrong, consider just making a really thick oxide.

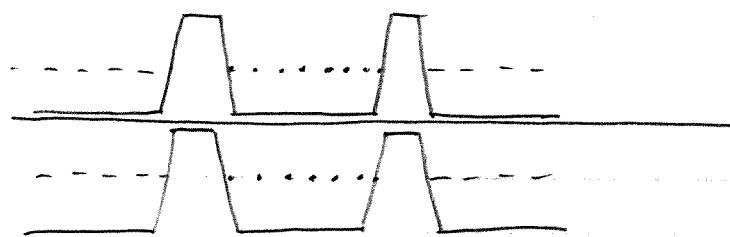
(But ϕ_s should be less than gE_b for inversion.)

Case A: —

Case B: ---

Case C:

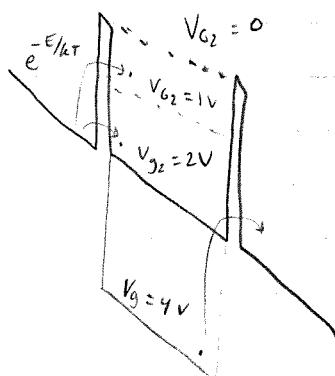
2.



3.

- Reduce gate spacing \Rightarrow fringing field will reduce bump. (Fringing fields)
- reduce substrate doping
- use n or n⁺ regions between gates. \times not a good idea.
- use heterostructure with lower bandgap material between gates.
- use lower bandgap material (e.g. Ge ~ 0.6 eV)

4.



Wong: There are no barriers as we've drawn.
The fringing field from the gates
removes the bump.
 \Rightarrow it should just look like a MOS.
(always ask if you should consider
(fringing fields))

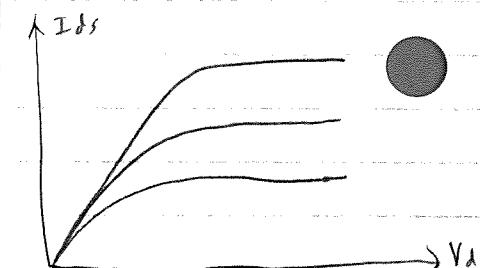
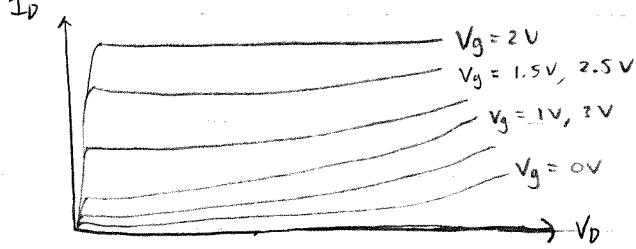
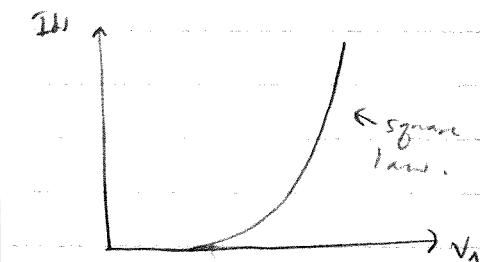
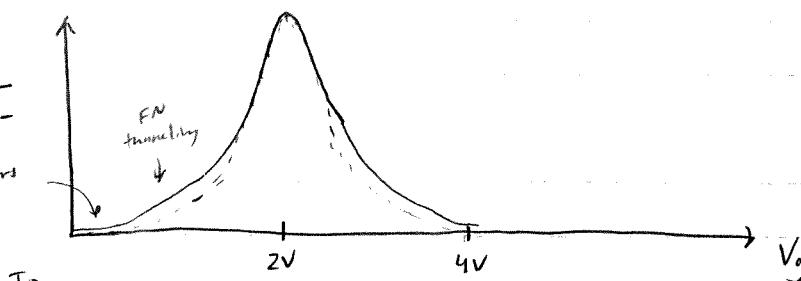
for $V_{G2} = 0V$ - no tunnelling, almost no diffusion.

$V_{G2} = 2V$ - tunnelling. (direct)

in between, we should see an exponential increase in current since only the carriers with sufficient energy can tunnel + carriers have a boltzmann energy distribution.

CASE A:

$V_D = 2V$ —
 $V_D = 50mV$ ---
hot carriers

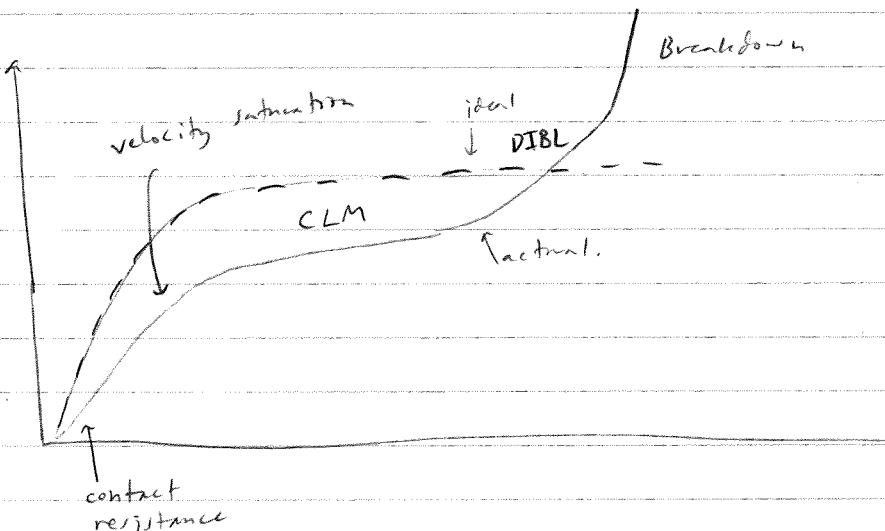
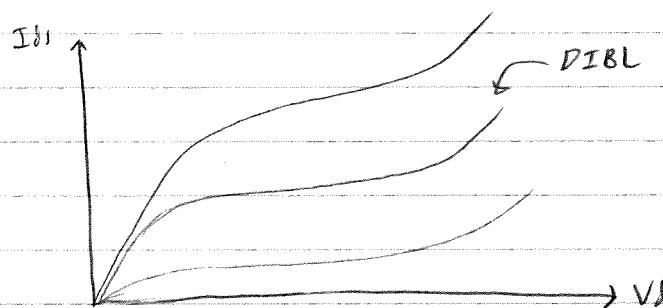
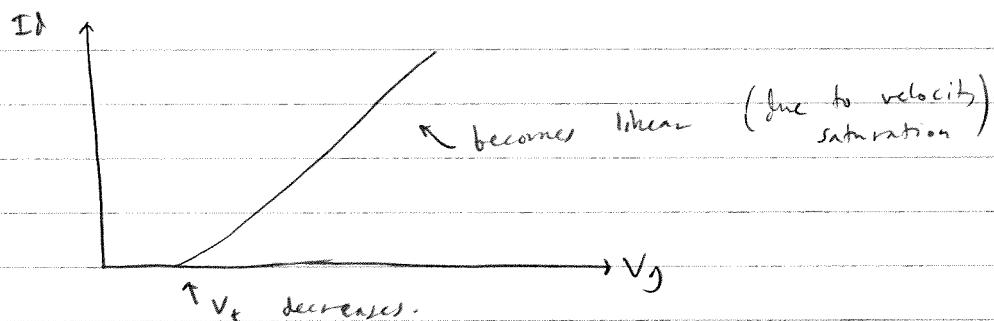


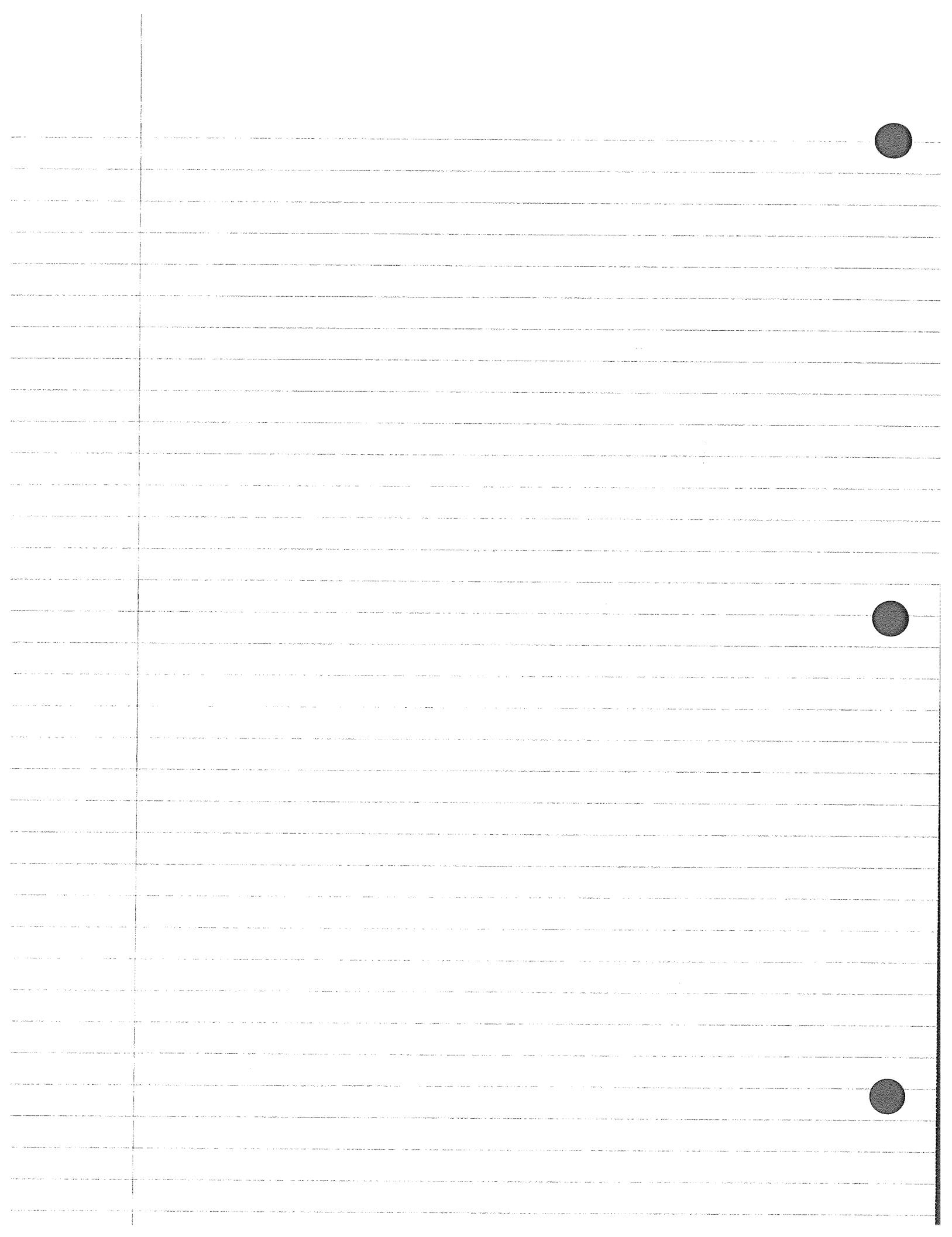
~~77~~ 5. $L \downarrow \Rightarrow$ steeper bending for a given $V_B \rightarrow$ more FN tunneling.

ask

Wong.

5. Now you should just consider short channel effects.

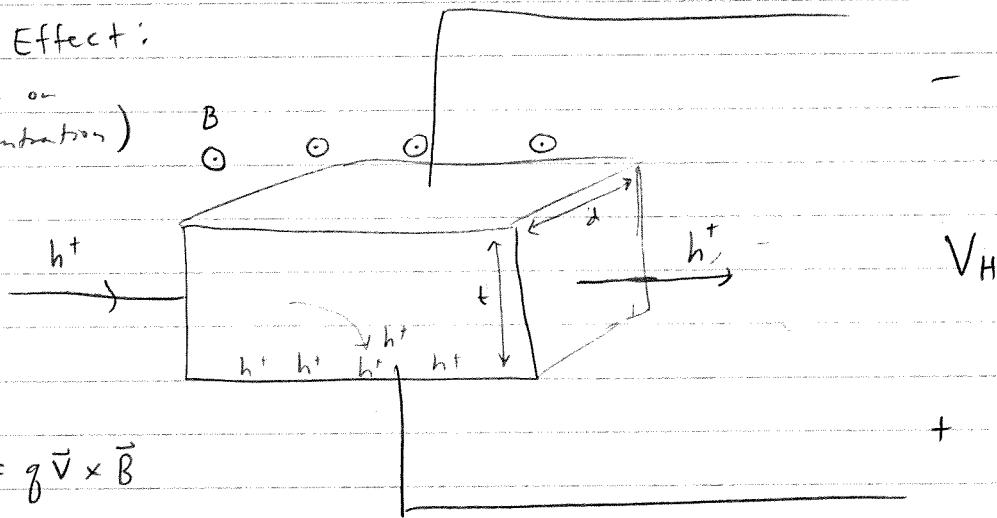




1995 - Wang

Hall Effect:

(can measure μ or carrier concentration)



$$F = q \vec{E} = q \vec{V} \times \vec{B}$$

$$E_y = \frac{V_H}{t} = |V| \cdot |B| = \frac{B \cdot I}{q g d \cdot t}$$

$$\Rightarrow \rho = \frac{d}{V_H} \cdot \frac{B \cdot I}{q \cdot d \cdot t} = \frac{IB}{qV_H d}$$

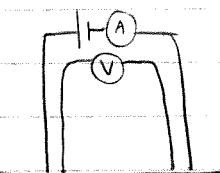
$$\Rightarrow \rho = \frac{IB}{qV_H d} \quad V_H = \frac{IB}{q\rho d}$$

$$n = -\frac{IB}{qV_H d} \quad V_H = -\frac{IB}{qnd}$$

Four Point Probe Measurement: Measures sheet resistance

Can determine σ or ρ

$$(\sigma = q\mu_n n + q\mu_p p)$$

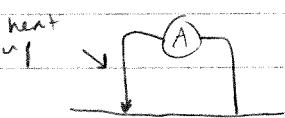


$$V = IR$$

$$R = \frac{V}{I}$$

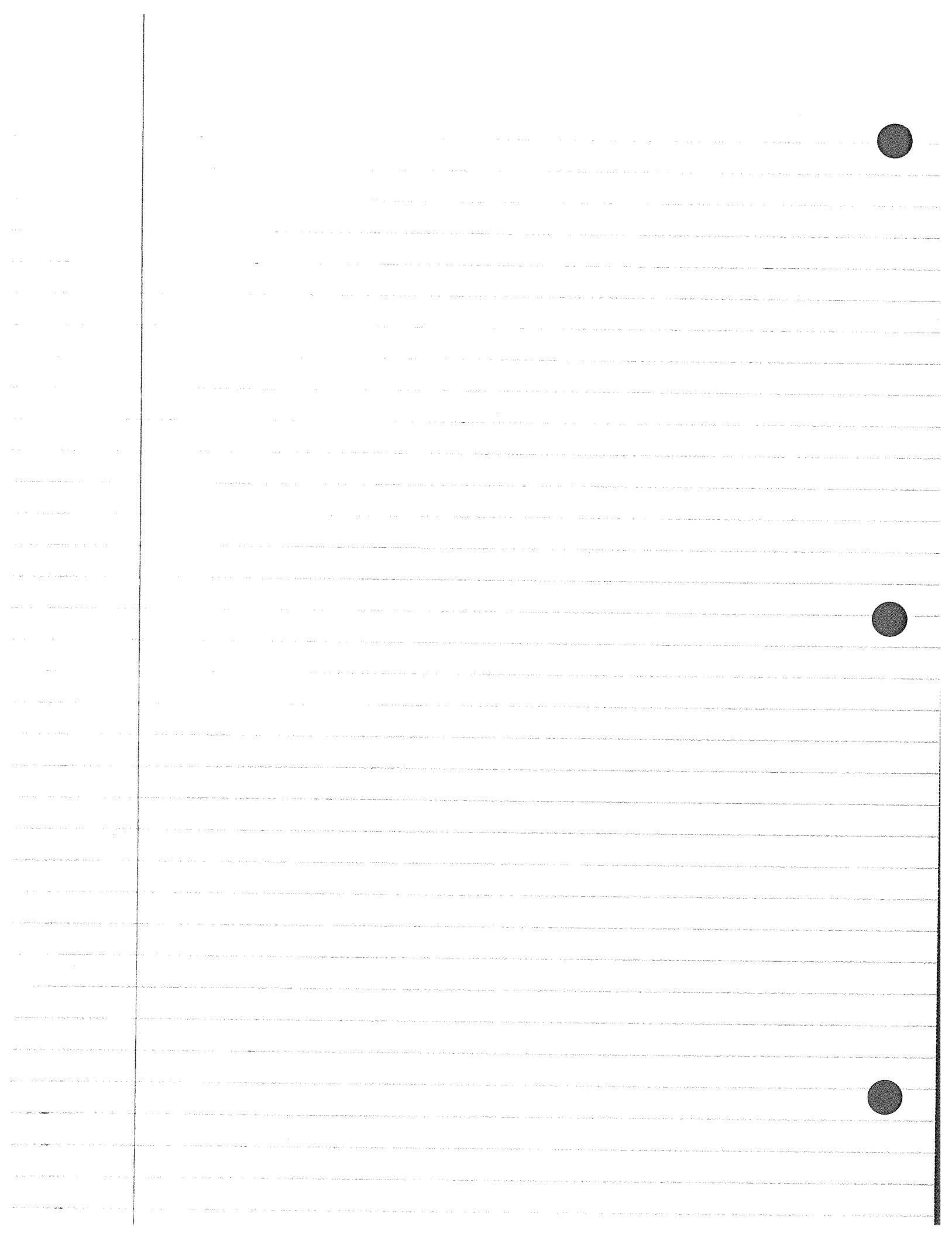
No current flows through voltmeter.

Hot Point Probe: Measures n-type or p-type.



if p-type, current flows away from hot probe.
if n-type, current flows into hot probe.

carriers always diffuse away from hot probe.



2004 - Saraswat

- Thermodynamic Limit:

$$E_b \geq kT \ln 2$$

- Quantum Mechanics

$$\Delta x \Delta p \geq h \Rightarrow x_{\min} \text{ (integration density)}$$

$$\Delta E \Delta t \geq h \Rightarrow \tau \text{ (switching speed)}$$

- Ultimate Limit:

after this \downarrow

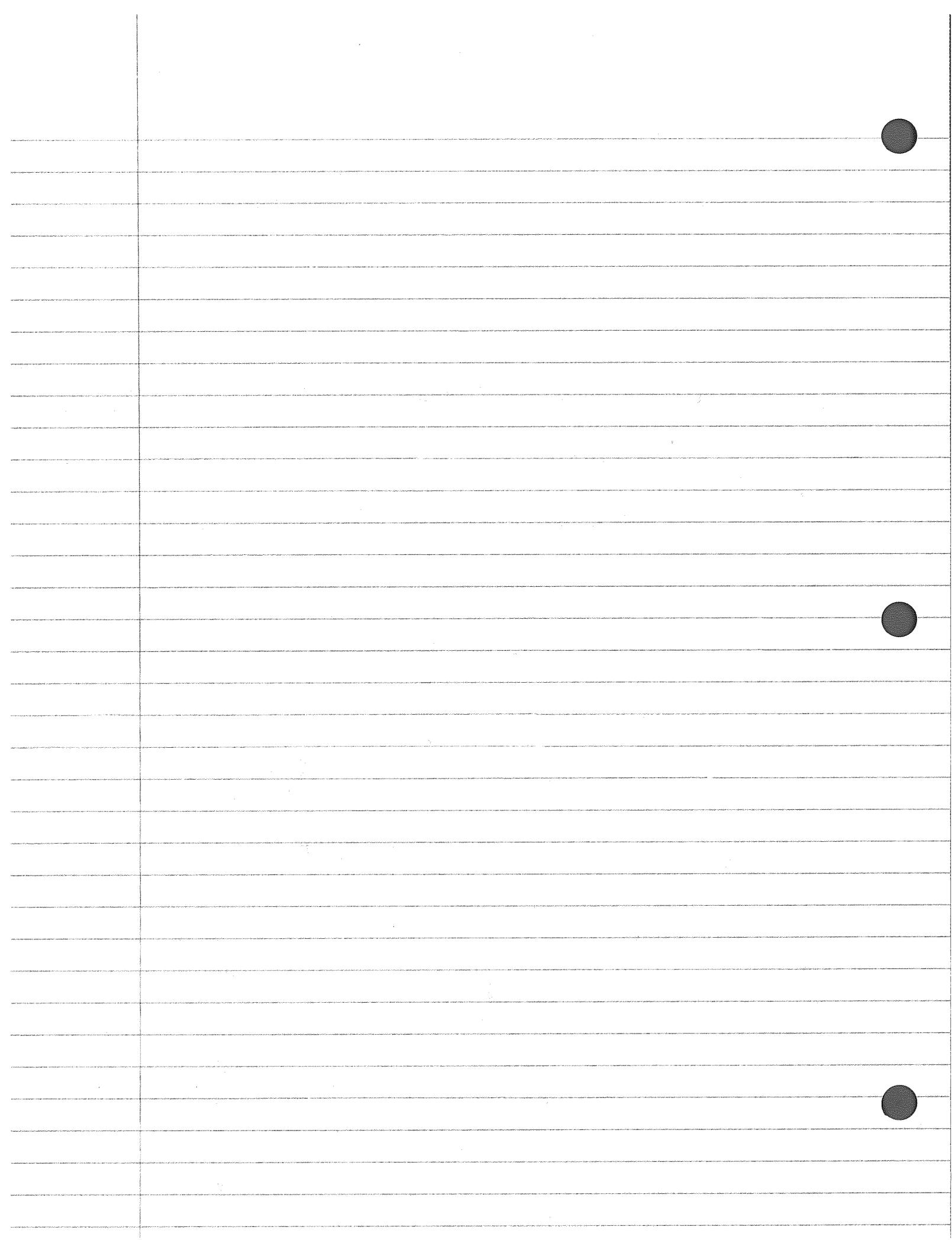
$$x_{\min} \approx 0.5 \text{ nm} \Rightarrow 5 \cdot 10^3 \text{ transistors/cm}^3$$

$$\text{Switching } t_{\min} \approx 0.04 \text{ ps} \Rightarrow 25 \text{ THz}$$

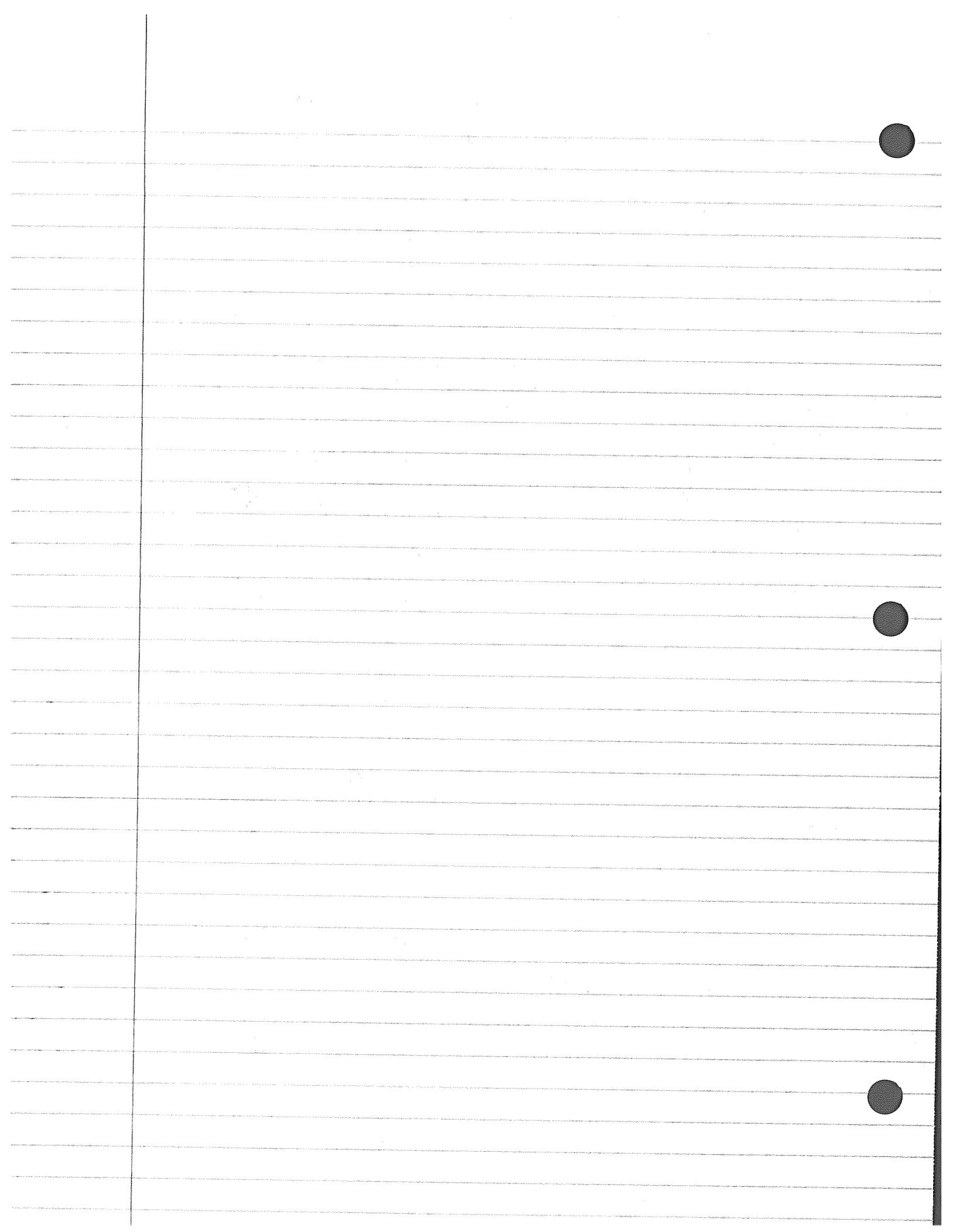
$$\text{Switching Energy } E_{\text{bit}} = 17 \text{ meV}$$

$$\text{Power} = 55 \text{ nW/bit}$$

- Total Power Density $\approx 370 \text{ W/cm}^2$

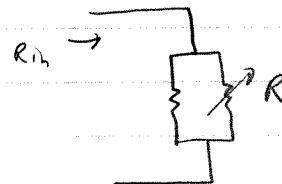
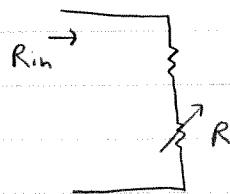


Systems



1995 - Boyd

①

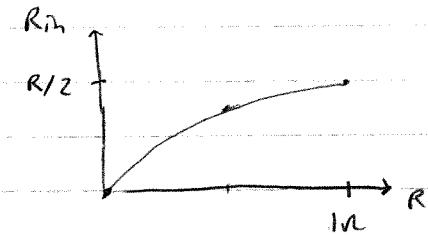
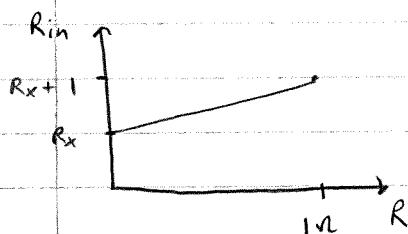


Sketch R_{in} vs R

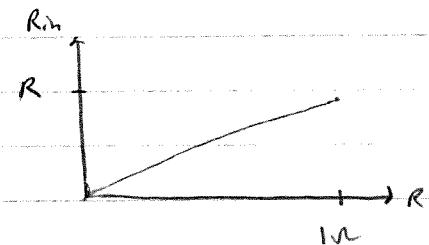
$$0 \leq R \leq 1\Omega$$

$$R_{in} = R_x + R$$

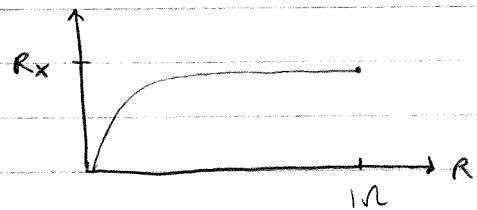
$$R_{in} = \frac{R_x - R}{R_x + R}$$



if $R_x = R$

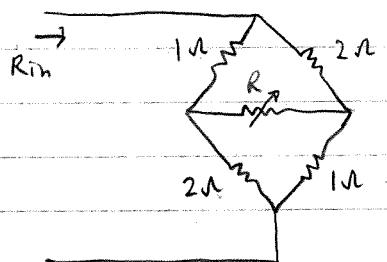


if $R_x > R$

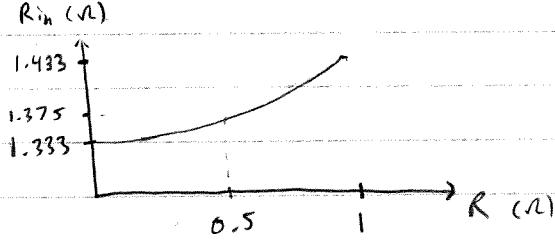


if $R_x \ll R$

②



$$0 < R < 1\Omega$$



$$\text{For } R=0 : R_{in} = \frac{2+1}{2+1} + \frac{2+1}{2+1} = \frac{4}{3} = 1.333$$

$$\text{For } R=1 : \quad \begin{array}{c} 1 \\ \diagup \quad \diagdown \\ 1 \quad 2 \end{array} \Rightarrow \begin{array}{c} 1-2 \\ \diagup \quad \diagdown \\ 1+2+1 \end{array} = 0.5$$

$$\frac{1-1}{1+2+1} = 0.25 \quad \frac{1-2}{1+2+1} = 0.5$$

$$\Rightarrow \begin{array}{c} 0.5\Omega \\ \diagup \quad \diagdown \\ 0.25\Omega \quad 0.5\Omega \\ | \quad | \\ 2\Omega \quad 1\Omega \end{array} \Rightarrow R_{in} = 0.5 + \left(\frac{\frac{9}{4} \cdot \frac{3}{2}}{\frac{7}{4}, \frac{3}{2}} \right) = 0.5 + \frac{27}{8} \cdot \frac{4}{15} = \frac{93}{30} = 1.433\Omega$$

$$\text{Similarly for } R=0.5 : R_{in} = 1.375\Omega$$

$$\text{since } \underbrace{(R_{in}|_{R=0} + R_{in}|_{R=1})}_2 = 1.383\Omega > 1.375\Omega$$

The R_{in} vs. R graph has positive second derivative.



? 11. related to positive definiteness?

1999 - Boyd

$$\text{Composite ranking } (A) = \mathbf{1}^T \mathbf{M}_A \mathbf{w}$$

$$\text{where } M_{A,j} = \begin{cases} 1 & \text{if ranked } j\text{th by evaluator } i \\ 0 & \text{if not ranked } j\text{th by evaluator } i \end{cases}$$

[Yes] This could happen if A, B, C, and D all receive the same score from every evaluator.

i.e. A receives 10 #1 rankings.

$$M_A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{etc. for } B, C, D.$$

$$\begin{aligned} &\text{This assures weight(1) < weight(2)} \\ &\quad < \text{weight(3)} < \text{weight(4)} \end{aligned}$$

$$\Rightarrow CR_A < CR_B < CR_C < CR_D \text{ No matter what.}$$

OR, this could also happen if 2 (e.g. A + B) 6, 12 receive 5 1sts and 5 2nd and the other 2 (e.g. C + D) both receive 5 3rds and 5 4ths.

$$\Rightarrow \text{No matter what } CR_A = CR_B < CR_C = CR_D$$

Now solve more mathematically.

Hint:

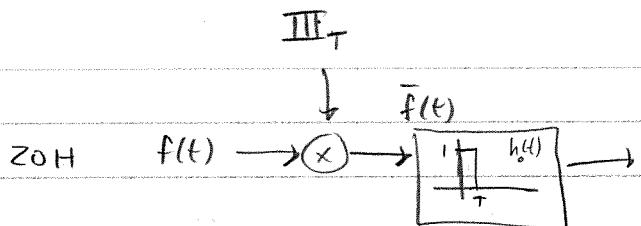
$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} & R_{13} & R_{14} \\ R_{21} & R_{22} & R_{23} & R_{24} \\ R_{31} & R_{32} & R_{33} & R_{34} \\ R_{41} & R_{42} & R_{43} & R_{44} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix} \quad 0 < w_1 < w_2 < w_3 < w_4$$

when $R_{11} = \# \text{ of times } A \text{ ranked 1st}$

$R_{12} = \# \text{ of times } A \text{ ranked 2nd}$

$R_{21} = \# \text{ of times } B \text{ ranked 1st.}$

$$\frac{1}{T} = 2B$$



$$\text{III}_p(x) = \sum_{k=-\infty}^{\infty} \delta(x - kp)$$

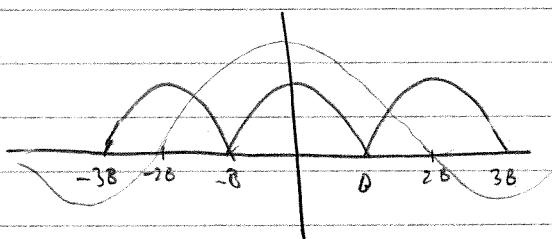
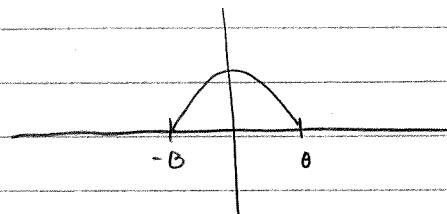
$$f(x) \text{III}_p(x) = \sum_{k=-\infty}^{\infty} f(x) \delta(x - kp) = \sum_{k=-\infty}^{\infty} f(kp) \delta(x - kp)$$

$\Rightarrow p$ is spacing of samples.

$$Fg = \Pi_p (Fg * \text{III}_p) \quad p = 2B$$

$$g(t) = \sum_{k=-\infty}^{\infty} g\left(\frac{k}{p}\right) \text{sinc}\left(p\left(t - \frac{k}{p}\right)\right)$$

$$h_s(t) = \Pi_T(t - T/2) \Leftrightarrow H(s) = e^{-2\pi i s T/2} T \text{sinc}(Ts)$$

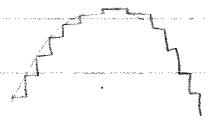


1993 - Nishimura

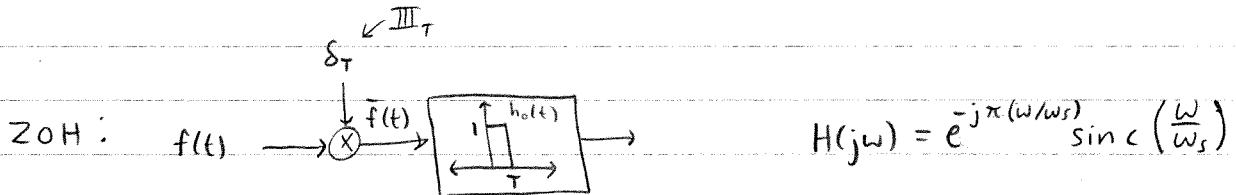
1. Increase the frequency of the sinewave from the function generator. You should hear the pitch increasing. Eventually, the pitch will suddenly drop due to aliasing. This should be roughly the Nyquist frequency. $\Rightarrow f_s = 2f_w$ where f_w is the frequency of the waveform generator at this point.

2. 10Hz is very low. But if this tone is sampled, we should also get a replica around $f_s = \frac{1}{T}$ we probably hear this higher tone.

* Dominant tone is at sampling frequency.



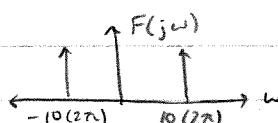
$$\frac{1}{T} = 28$$



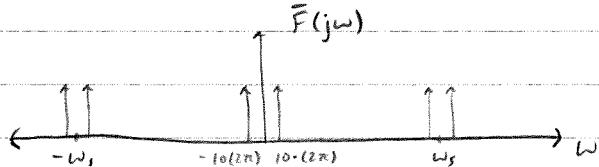
where $\omega_s = \frac{2\pi}{T}$ sampling frequency.

$$\Rightarrow \tilde{F}(j\omega) = \frac{1}{2\pi} F(j\omega) * \frac{2\pi}{T} \delta_{\frac{2\pi}{T}}(\omega) = \frac{\omega_1}{2\pi} F(j\omega) * \delta_{\omega_1}(\omega)$$

$$\Rightarrow \text{let } f(t) = \cos(2\pi(10)t)$$

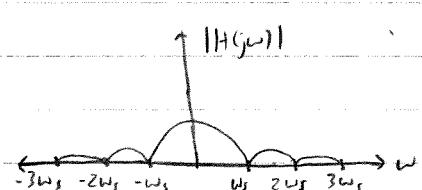


\Rightarrow

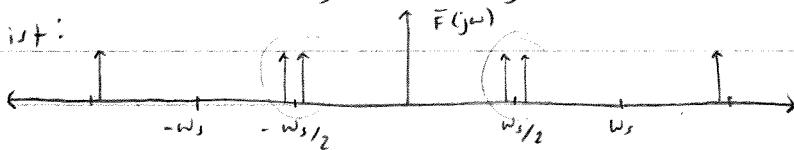


$$|H(j\omega)|$$

$$3. |H(j\omega)| = |e^{-j\pi(\omega/\omega_s)}| |\operatorname{sinc}\left(\frac{\omega}{\omega_s}\right)| = |\operatorname{sinc}\left(\frac{\omega}{\omega_s}\right)| \Rightarrow$$



4. The wavering nature of the output is due to the beating of two frequencies. Frequency domain sketch clearly shows why the two frequency components exist:



1996 - Nishimura

~~Since the frame rate of the Video camera or TV is below Nyquist for the fan rotation frequency, we will see aliasing.~~

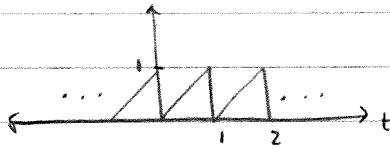
1. Given a TV frame rate of 30Hz + the fact that you see 3 markers appearing stationary, the fan is rotating at 10 Hz, one third of the sampling rate.

2. Yes, there are other rotational speeds that would give a video identical to what we've just seen.

$(10 + 30n)$ Hz rotations would give the same video, frame by frame, $n \in \mathbb{Z}$. Note too that a 20 Hz rotation would appear to be the same, but not on a frame by frame basis (i.e. we assume that you can tell the individual fan blades apart).

3. The fan appears to be nearly stationary since there are 3 fan blades and the rotation rate is one-third the sampling rate. If there were 2 fan blades and the rotation rate was one half the sampling rate, it would also appear stationary.

1999 - Osgood



$$\text{F.S. } D_n = \frac{1}{T} \int_{-\tau}^{t+\tau} f(t) e^{-j\omega_0 n t} dt = \int_0^1 t e^{-j2\pi n t} dt$$

$$\text{Integrate by parts: } u = t \quad dv = e^{-j2\pi n t} dt \quad v = \frac{j}{2\pi n} e^{-j2\pi n t}$$

$$= \left[\frac{j}{2\pi n} t e^{-j2\pi n t} \right]_0^1 - \frac{j}{2\pi n} \int_0^1 e^{-j2\pi n t} dt = \frac{j}{2\pi n} e^{-j2\pi n} + \frac{1}{4\pi^2 n^2} \left(e^{-j2\pi n t} \right]_0^1$$

$$= \frac{j}{2\pi n} + \frac{1}{4\pi^2 n^2} (e^{-j2\pi n} - 1) = \frac{j}{2\pi n}$$

$$\text{Parsevals Theorem: } \lim_{N \rightarrow \infty} \sum_{n=-N}^N |D_n|^2 = \frac{1}{T} \int_{t_0}^{t_0+T} |f(t)|^2 dt$$

$$D_0 = \int_0^1 f(t) dt = \frac{1}{2} \Rightarrow D_n = \begin{cases} \frac{1}{2}, & n=0 \\ \frac{j}{2\pi n}, & \text{otherwise} \end{cases}$$

$$\Rightarrow \sum_{n=-\infty}^{\infty} |D_n|^2 = D_0^2 + 2 \sum_{n=1}^{\infty} |D_n|^2 = \int_0^1 t^2 dt = \left[\frac{1}{3} t^3 \right]_0^1 = \frac{1}{3}$$

$$\Rightarrow 2 \sum_{n=1}^{\infty} \left(\frac{j}{2\pi n} \right)^2 = \frac{1}{3} - \frac{1}{9} = \frac{1}{12} \Rightarrow \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{4\pi^2}{24}$$

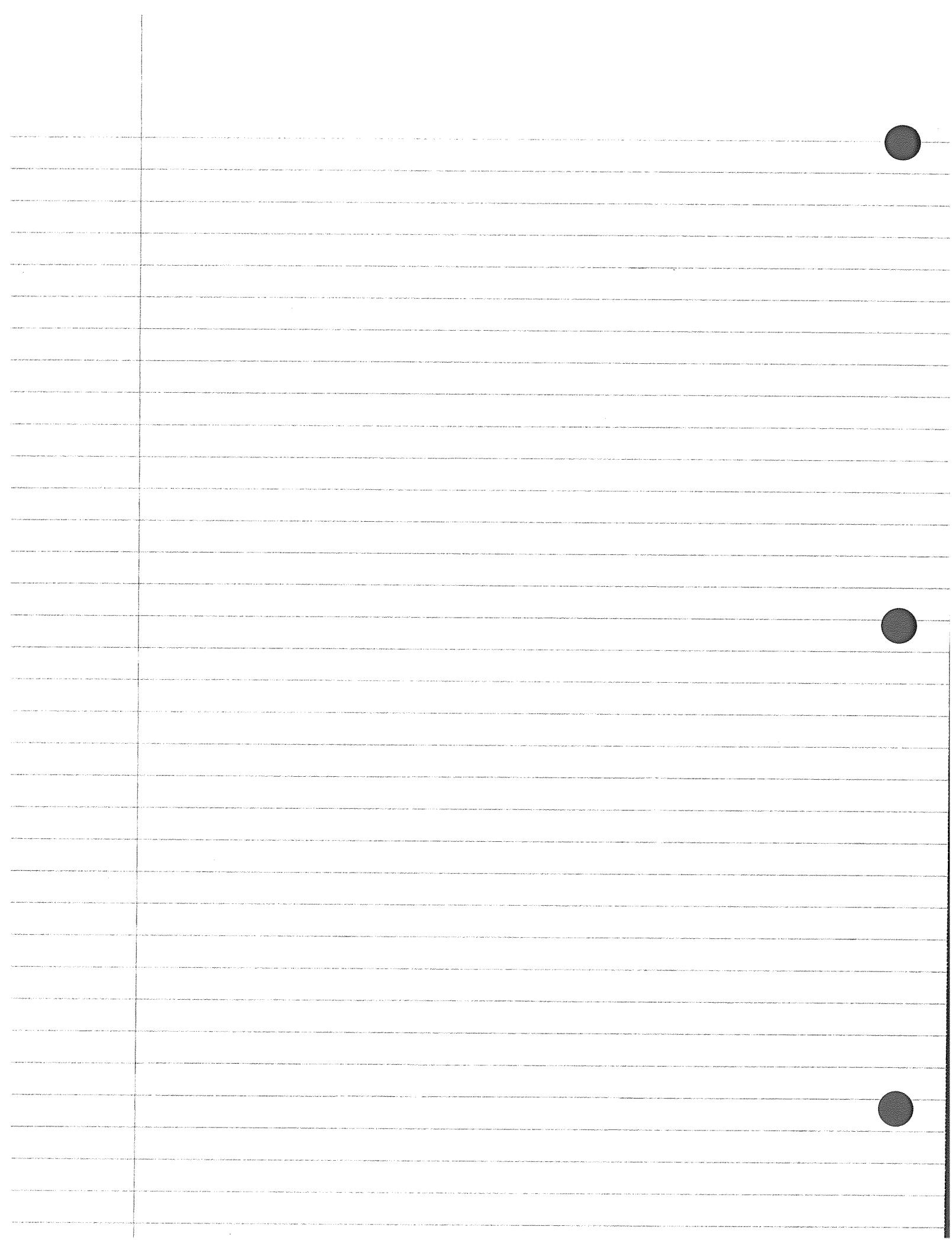
$$\Rightarrow \boxed{\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}} \quad \checkmark$$

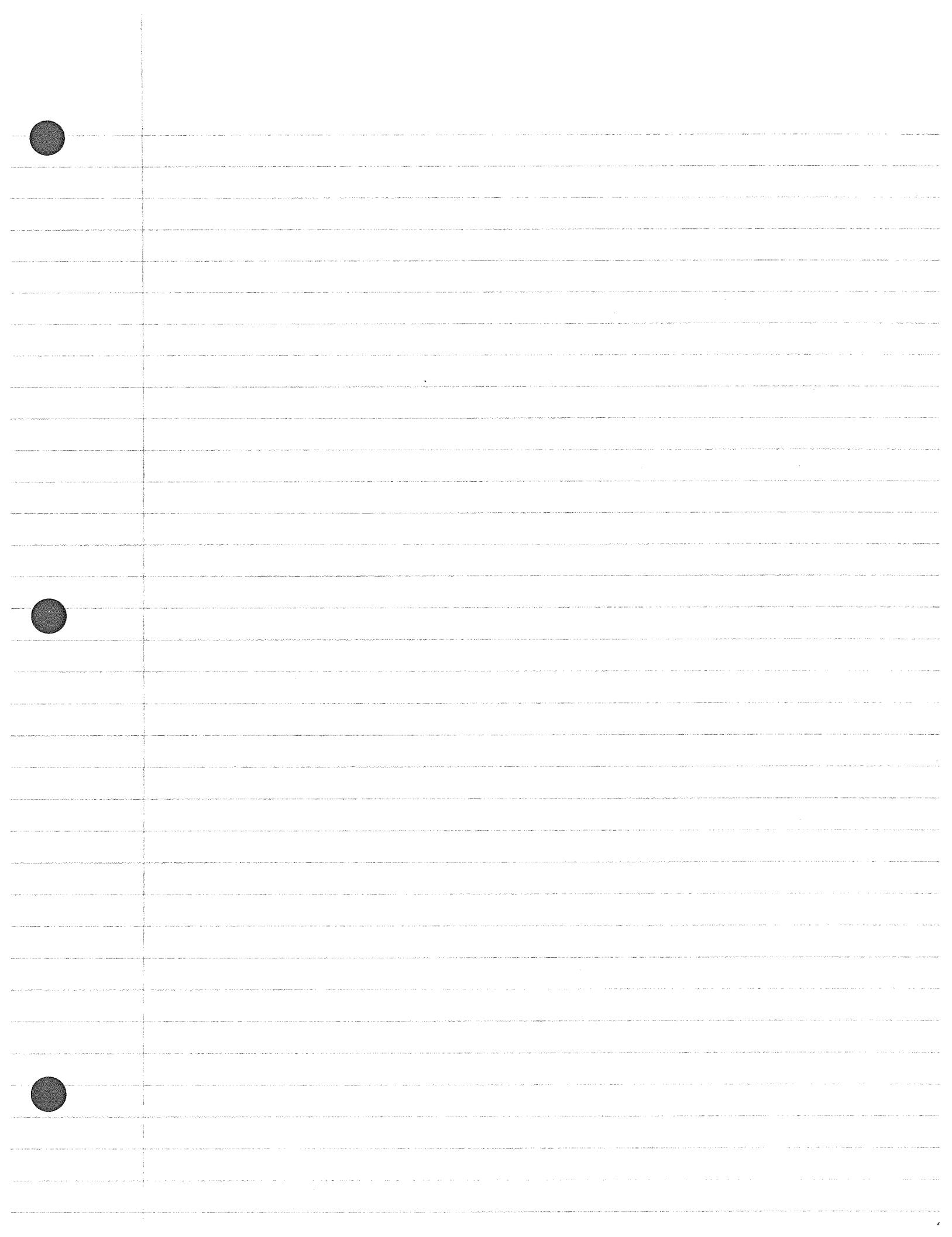
Fourier Transform exists for any energy signal and many power signals.

$$E_x = \lim_{t \rightarrow \infty} \int_{-T}^T |x(t)|^2 dt \quad \text{Energy Signal: } 0 < E_x < \infty, P_x = 0$$

$$P_x = \lim_{t \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |x(t)|^2 dt \quad \text{Power Signal: } 0 < P_x < \infty, E_x = \infty$$

Sawtooth is a power signal. Doesn't have a F.T. But it is periodic, so it definitely has a F.S.





1994 - Nishimura

1. $\cos(\omega t) \Leftrightarrow \pi(\delta(\omega + \omega_0) + \delta(\omega - \omega_0))$ $\angle F(j\omega) = 0$

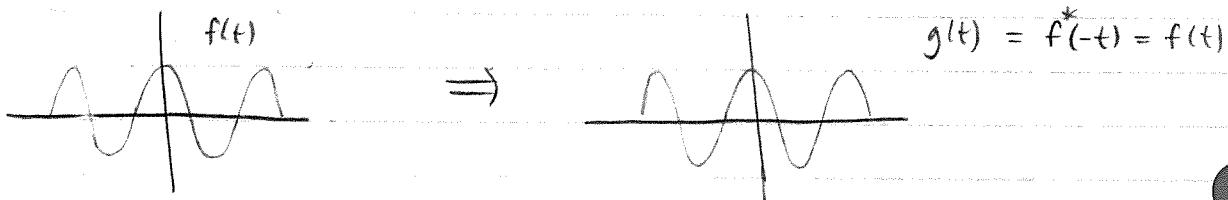
$\Rightarrow g(t) = f(t)$

$$\angle G(j\omega) = -\angle F(j\omega) \Rightarrow \tan\left(\frac{\text{Im}(G)}{\text{Re}(G)}\right) = -\tan\left(\frac{\text{Im}(F)}{\text{Re}(F)}\right)$$

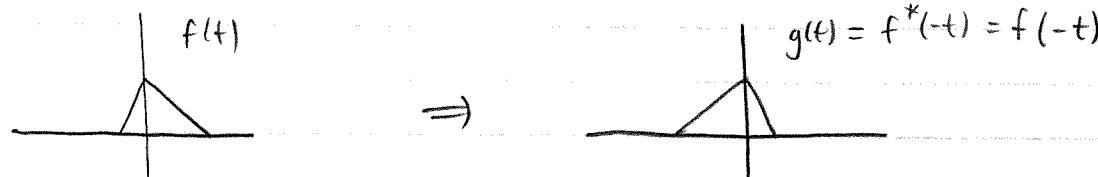
$$\Rightarrow \text{Im}(G) = -\text{Im}(F) \Rightarrow G(j\omega) = F^*(j\omega) \Rightarrow g(t) = f^*(-t)$$

If the input signal is real, its Fourier transform has hermitian symmetry.

a)



b)



A system is causal if the output depends only on past or present inputs. \Rightarrow non-causal

$$f^*(-(t-\tau)) = f^*(-t+\tau) = g(t+\tau) \Rightarrow$$
 time variant

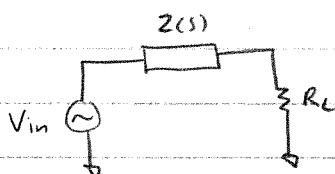
$$(\alpha f)^*(-t) \neq \alpha g(t) \text{ if } \alpha \in C \Rightarrow$$
 nonlinear

If the input is constrained to be real valued, the system behaves linearly.

2. a) Yes, one can throw away every other sample and still exceed the Nyquist frequency.
- b) Yes, model the "defective" sampling by $f(t)[\delta_T(t) - \delta(t)]$. The transform of this quantity is the original replicated spectrum shifted vertically based on the value of $f(0)$. Therefore, $f(0)$ can be determined by evaluating the spectrum in the gaps between replication islands. Since the values should be zero,
- $$f(t)[\delta_T(t) - \delta(t)] = f(t)\delta_T(t) - f(0)\delta(t) \Leftrightarrow \frac{1}{T} F(j\omega) * \delta_{\omega_0}(\omega) - f(0)$$
- ident sampled spectrum
- c) No, not possible because of aliasing.

1994 - Boyd

1. Assume:

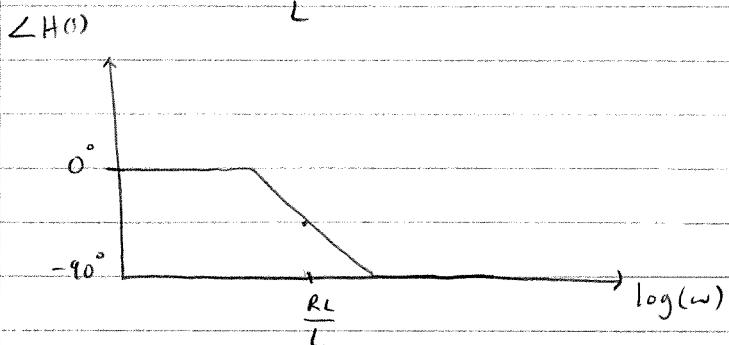
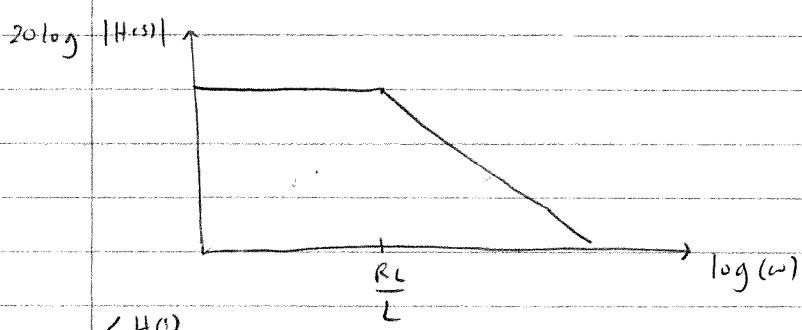


$$V_L(s) = \frac{R_L}{R_L + Z(s)} V_{in}(s) \quad \text{model } Z(s) \text{ as } -\frac{L}{s}$$

$$\Rightarrow Z(s) = sL \Rightarrow H(s) = \frac{R_L}{R_L + sL} = \frac{1}{1 + s \frac{L}{R_L}}$$

$$\text{pole at } \omega_p = -\frac{R_L}{L}$$

Ignore this part. It is wrong.



$\Rightarrow V_L(s)$ will always lag $V_{in}(s)$ by between 0° and 90° .

V_{in} looks like it's composed of 2 sinusoidal components: $\cos(\omega t)$ and $\cos(2\omega t)$. Changing R_L from 10Ω to 20Ω effectively doubles the bandwidth of the LPF. \Rightarrow we expect

$|V_L(s)|$ to increase and $\angle V_L(s)$ to increase as well.

~~this matches the waveform given.~~ \Rightarrow for $R_L = 5\Omega$, we would expect the bandwidth to shrink. $\Rightarrow |V_L(s)|$ should decrease and $\angle V_L(s)$ should decrease. Furthermore, since

$\omega_{p_{20\Omega}} = 2 \cdot \omega_{p_{10\Omega}}$ and $\omega_{p_{5\Omega}} = 2 \cdot \omega_{p_{10\Omega}}$ we can expect the

phase shift and magnitude decrease factor from 10Ω to 5Ω to be roughly the same as the decrease from 20Ω to 10Ω

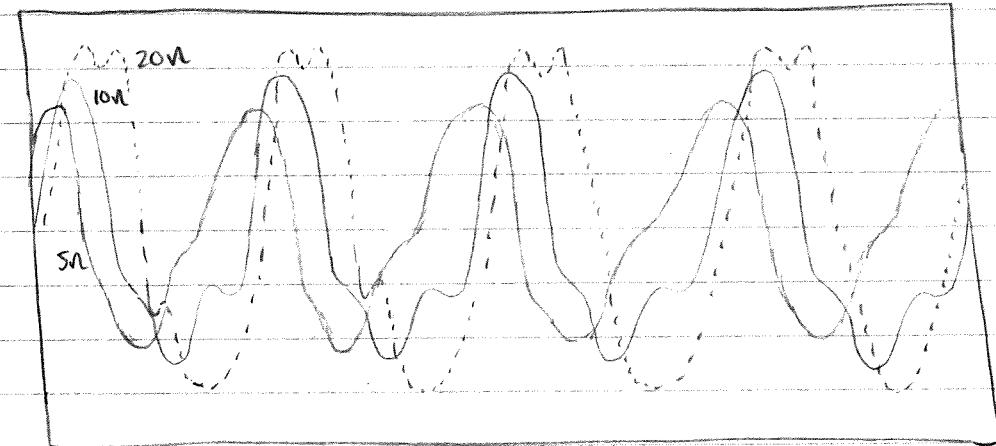
assuming the input frequencies are within ~ 2 decades of $\omega_{p_{10\Omega}}$.

We might also expect the amplitude of the high frequency component to be further reduced. So the waveform becomes smoother.

No. Wrong.

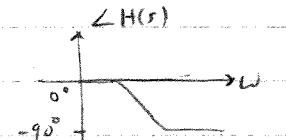
Actually,
10Ω waveform
isnt 20Ω waveform.

\Rightarrow We expect something like:



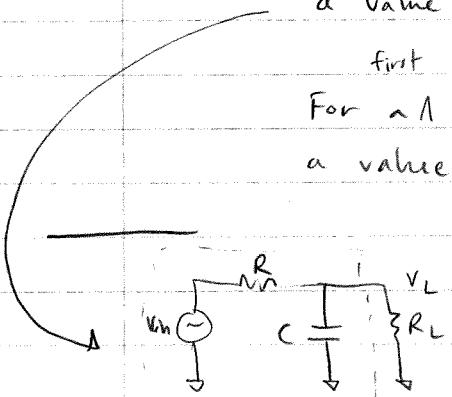
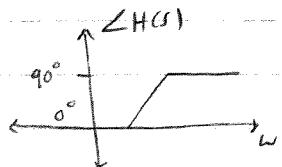
first order

Notes: For a 1 LPF, V_{out} will always lag V_{in} by a value between 0° and 90°



first order

For a 1 HPF, V_{out} will always lead V_{in} by a value between 0° and 90°



$$\begin{aligned} \frac{V_L(s)}{V_{in}(s)} &= \frac{\frac{R_L}{1+sR_L C}}{R + \frac{R_L}{1+sR_L C}} = \frac{R_L}{R + R_L(1+sR_L C)} \\ &= \frac{R_L}{R+R_L} \left(\frac{1}{1+s(R/(R_L)C)} \right) \end{aligned}$$

\Rightarrow Assume we have a signal around ω_s

$$\left| H(j\omega_s) \right|_{R_L=10\Omega} < \left| H(j\omega_s) \right|_{R_L=20\Omega} \text{ since } \frac{R_L}{R+R_L} \text{ increases for } R_L=20\Omega$$

$$\angle H(j\omega_s)_{R_L=10\Omega} > \angle H(j\omega_s)_{R_L=20\Omega} \text{ since pole decreases for } R_L=20\Omega$$

This is consistent with the observed waveforms. \Rightarrow for 5Ω , we expect the waveform to have smaller amplitude & greater phase as pictured above.

1997 - Boyd

$$1. \int_{-\infty}^{\infty} |u(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |U(j\omega)|^2 d\omega \quad y(j\omega) = H(j\omega) U(j\omega)$$

$$\int_{-\infty}^{\infty} |y(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |Y(j\omega)|^2 d\omega = \frac{1}{2\pi} \int_{-\infty}^{\infty} |H(j\omega) U(j\omega)|^2 d\omega$$

\Rightarrow No False. For example. Consider the case

when the system is a LPF with $H(j\omega) = \text{rect}\left(\frac{\omega}{4\pi}\right)$

$$\text{let } u(t) = 2 \text{rect}(t) \cos(10\pi t) \quad v(t) = \text{rect}(t)$$

$$\int_{-\infty}^{\infty} |u(t)|^2 dt \approx 2 \cdot \frac{1}{2\pi} \int_{-\infty}^{\infty} \text{sinc}\left(\frac{\omega}{2\pi}\right) d\omega \geq \int_{-\infty}^{\infty} |v(t)|^2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} \text{sinc}\left(\frac{\omega}{2\pi}\right) d\omega$$

$$\text{but } \int_{-\infty}^{\infty} |y(t)|^2 dt \neq \int_{-\infty}^{\infty} |z(t)|^2 dt$$

$$\text{since } U(j\omega) = \text{sinc}\left(\frac{\omega - 10\pi}{2\pi}\right) + \text{sinc}\left(\frac{\omega + 10\pi}{2\pi}\right) \quad V(j\omega) = \text{sinc}\left(\frac{\omega}{2\pi}\right)$$

$$\text{and } H(j\omega) = \text{rect}\left(\frac{\omega}{4\pi}\right)$$

$$\Rightarrow \frac{1}{2\pi} \int_{-\infty}^{\infty} (H(j\omega) U(j\omega))^2 d\omega \leq \frac{1}{2\pi} \int_{-\infty}^{\infty} (H(j\omega) V(j\omega))^2 d\omega$$

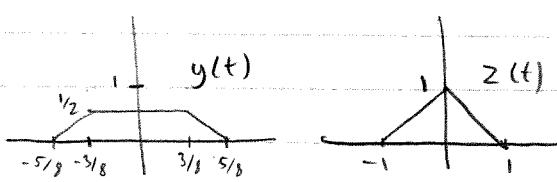
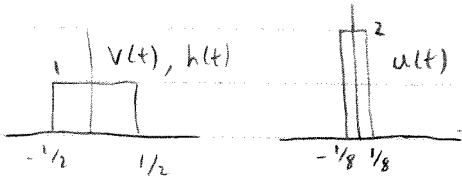
since most of $U(j\omega)$ will be filtered out,

$$2. \quad \boxed{\text{No}}. \quad y(t) = (u * h)(t) \quad z(t) = (v * h)(t)$$

$$\text{let } h(t) = \text{rect}(t) \quad u(t) = 2 \text{rect}(4t) \quad v(t) = \text{rect}(t)$$

$$\Rightarrow \max_t |u(t)| = 2 \geq \max_t |v(t)| = 1$$

$$\text{but } \max_t |y(t)| = \frac{1}{2} \neq \max_t |z(t)| = 1$$



- $$1. \quad \alpha f(x) \Leftrightarrow \alpha F(s) \quad |\alpha F(s)|^2 = \alpha^2 |F(s)| \neq \alpha G(s) \Rightarrow \text{non-linear} \\ f(x-\tau) \Leftrightarrow e^{j\omega \tau} F(s) \quad |e^{j\omega \tau} F(s)|^2 = |F(s)|^2 \Rightarrow \text{time variant} \\ \frac{G(s)}{F(s)} = \frac{|F(s)|^2}{F(s)} \quad g(x) = \text{autocorrelation of } f(x)$$

- $$f(-(x-\tau)) = f(-x+\tau) \neq g(x-\tau) \Rightarrow \text{time variant}.$$

- $$3. \quad H(s) = e^{-j\pi s} \quad 2\pi js \quad \text{Linear, time invariant.}$$

$$\frac{d}{dx} (\alpha f_1(x-1) + \beta f_2(x-1)) = \alpha \frac{df_1(x-1)}{dx} + \beta \frac{df_2(x-1)}{dx} = \alpha g_1(x) + \beta g_2(x) \quad \checkmark$$

$$\frac{d}{dx} f((x-\tau)-1) = \frac{d}{dx} f(x-\tau-1) = g(x-\tau) \quad \checkmark$$

$$y = \sum_{n=-\infty}^{\infty} \alpha f_1(x-n) + \beta f_2(x-n) = \alpha \sum_{n=-\infty}^{\infty} f_1(x-n) + \beta \sum_{n=-\infty}^{\infty} f_2(x-n) = \alpha g_1(x) + \beta g_2(x) \quad \checkmark$$

$$\sum_{k=-\infty}^{\infty} f((x-\tau)-n) = \sum_{n=-\infty}^{\infty} f(x-\tau-n) = g(x-\tau) \quad \checkmark$$

$$G(s) = F(s) \cdot S_1(s) \Rightarrow H(s) = S_1(s) \leftarrow \text{comb}(s)$$

$$5. \quad H(s) = \text{rect}(s) \delta_1(s) \sin^2(s) \quad \text{Linear (all elements linear)}$$

Time variant (If input shifted, linear phase in S-domain that should result does not because of replication islands that are not filtered out.)

Also, interpolation filter does linear interpolation. So consider a slightly shifted version of the input. The piecewise linear output cannot be a shifted version of the original piecewise linear input. Note that if the interpolation filter is a sinc interpolator of appropriate bandwidth, the system is LTI.

$$6. \quad \int_{-\infty}^{\infty} \alpha f_1(u) + \beta f_2(u) du = \alpha \int_{-\infty}^{\infty} f_1(u) du + \beta \int_{-\infty}^{\infty} f_2(u) du = \alpha g_1(x) + \beta g_2(x) \quad \checkmark$$

$$\int_{-\infty}^x f(u-\tau) du = \int_{-\infty}^{x-\tau} f(m) dm = g(x-\tau) \Rightarrow \text{Linear Time-invariant}$$

$$H(s) = \frac{1}{2} S(s) + \frac{1}{j2\pi s}$$

$$f(t) \rightarrow \boxed{\quad} \rightarrow g(t)$$

- 1) Yes
- 2) No.
- 3) No. Output must go negative at some points
- 4) No.
- 5) Yes $\frac{1}{1+R}$ $R+1 = 1/(1+R)$
- 6) No. Time-Variant
- 7) Yes. Narrow Bandpass filter.

2) let T be the period of $f(t)$. Clearly $g(t)$ does not have period T .

$$\Rightarrow f(t) \rightarrow g(t) \text{ and } f(t+T) = f(t) \rightarrow g(t) \neq g(t+T)$$

$$4) f(t) \rightarrow g(t) \Rightarrow g(t) = w(t) f(t) \text{ where } w(t) \text{ is a windowing function.}$$

$$\Rightarrow f(t-\tau) \rightarrow w(t) f(t-\tau) \neq g(t-\tau) = w(t-\tau) f(t-\tau)$$

$$x_{in} \quad x_{out} = z^{-100} (x_{in} + \alpha x_{out})$$

$$x_{out} (1-\alpha) = z^{-100} x_{in}$$

$$\frac{x_{out}}{x_{in}} = \frac{z^{-100}}{1-\alpha z^{-100}}$$

$$1-\alpha z^{-100} = 0 \Rightarrow z^{-100} = \frac{1}{\alpha}$$

$$z^{100} - \alpha = 0 \quad z^{100} = \alpha \quad \alpha^{1/100}$$

2004 - Boyd

a) $u \rightarrow [z^{-1}] \rightarrow \dots \rightarrow [z^{-1}] \rightarrow y$

$$x(t+1) = Ax(t) + Bu(t) \quad y(t) = Cx(t) + Du(t)$$

$$\Rightarrow x(t) = \begin{bmatrix} u(t-1) \\ \vdots \\ u(t-100) \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & \dots & 0 \\ 1 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & 0 & \dots & 0 \\ \vdots & & & & \ddots & 1 \\ 0 & \dots & 0 & 1 \end{bmatrix}_{100 \times 100}$$

$$B = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{100 \times 1}$$

$$C = [0 \ 0 \ \dots \ 0 \ 1]_{1 \times 100} \quad D = 0$$

b) Eigenvalues of A are all 0.

The diagonal entries of a triangular matrix are its eigenvalues.

- The determinant of a triangular matrix equals the product of its diagonal entries.
- Since for any triangular matrix A, the matrix $SI - A$ (whose determinant is the characteristic polynomial of A) is also triangular.

c) $x(t)$, B, C and D remain the same.

$$\alpha = 10^{-5}$$

$$A = \begin{bmatrix} 0 & \dots & 0 & \alpha \\ 1 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ \vdots & & \ddots & & 1 \\ 0 & \dots & 0 & 1 & 0 \end{bmatrix}$$

d) Find the eigenvalues of A.

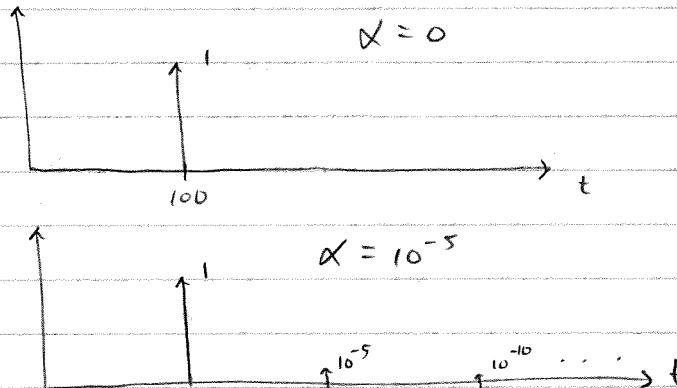
Eigenvalues of A are the poles of the transfer function:

$$Y = z^{-100} (U + \alpha Y) \Rightarrow Y(1 - \alpha z^{-100}) = z^{-100} U$$
$$\Rightarrow \frac{Y}{U} = \frac{z^{-100}}{1 - \alpha z^{-100}} = \frac{1}{z^{100} - \alpha}$$

$$\Rightarrow z^{100} - \alpha = 0 \Rightarrow z^{100} - 10^{-5} = 0 \Rightarrow z^{100} = 10^{-5}$$
$$\Rightarrow z = 10^{-5/100} e^{2\pi k i / 100} \quad k = 0, \dots, 99$$

These are points spaced 3.6° apart on a circle of radius $10^{-5/100} = 0.8913$. These eigenvalues are distinct, so A is diagonalizable.

e) How different is the impulse response of the system with feedback ($\alpha = 10^{-5}$) and without feedback ($\alpha = 0$)

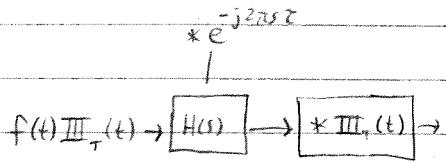


\Rightarrow There is little noticeable difference in the impulse response. One might expect a more dramatic change, since the transfer function with $\alpha = 10^{-5}$ has many poles in the right half plane.

2005 - Pauly

$$1) f(t) \text{III}_{\tau}(t) \quad \frac{1}{T} = 2B \Rightarrow T = \frac{1}{2B}$$

$$f(t) \text{III}_{\tau}(t) \Leftrightarrow F(s) * 2B \text{III}_{2B}(s)$$



$$\cdot f(t-\tau) \text{III}_{\tau}(t) \Leftrightarrow e^{-j2\pi s \tau} F(s) * 2B \text{III}_{2B}(s)$$

\Rightarrow can implement a phase shift $e^{-j2\pi s \tau}$ before resampling.

$$w = 2\pi s$$

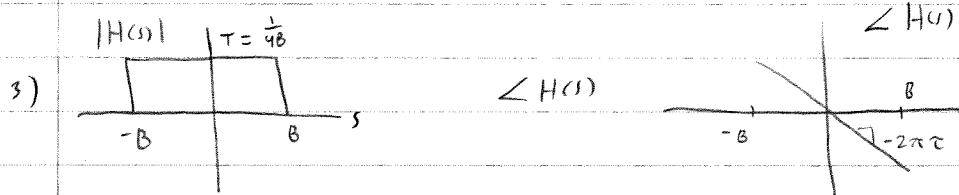
WRONG.
This would
be for
IIR



$$H(s) = \frac{1}{2B} \text{TT}_{2B} e^{-j2\pi s \tau}$$

$$e^{-j2\pi s \tau} = \cos(2\pi s \tau) - j \sin(2\pi s \tau) \Rightarrow \angle e^{-j2\pi s \tau} = \tan^{-1}\left(\frac{-\sin(2\pi s \tau)}{\cos(2\pi s \tau)}\right)$$

$$= \tan^{-1}(-\tan(2\pi s \tau)) = -2\pi s \tau$$



$$H(s) = \frac{1}{B} \text{TT}_{2B} e^{-j2\pi s \tau}$$

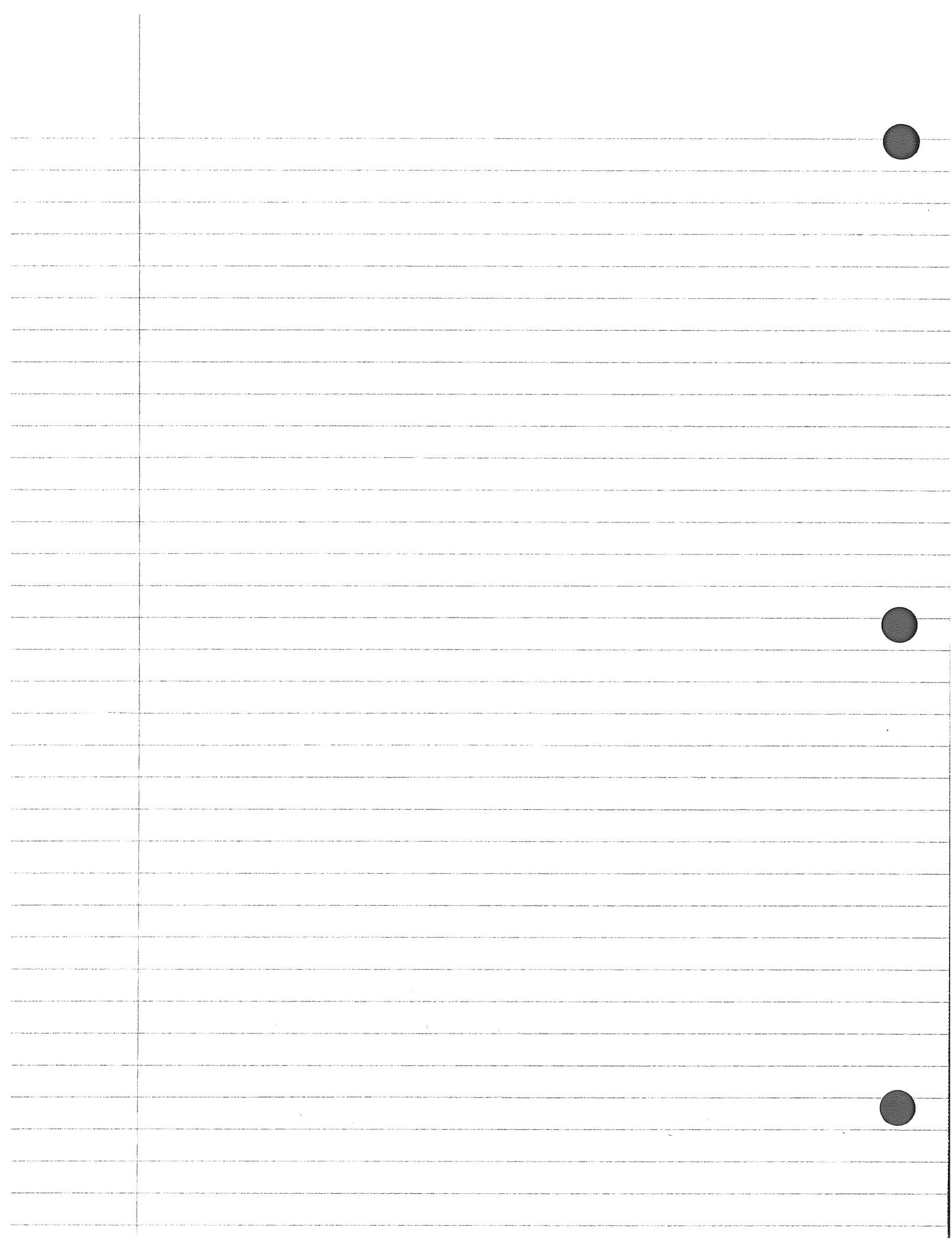
just decrease the amplitude of the FIR filter by a factor of 2.

2) for FIR, we must approximate this IIR filter.
The IIR is a shifted sinc. Thus, the FIR should be a truncated shifted sinc.

$$\Rightarrow h(t) = \text{rect}\left(\frac{t-\tau}{W}\right) \text{sinc}(2B(t-\tau)) \Rightarrow H(s) = \left(\frac{1}{2B} \text{TT}_{2B} * W \text{sinc}\left(\frac{s}{W}\right)\right) \cdot e^{-j2\pi s \tau}$$

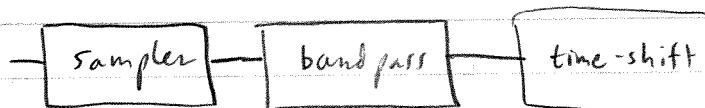
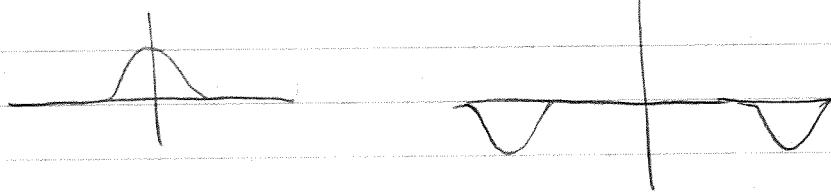
for $W \gg T$, this approximates a shifted rect (with some smoothed corners).

But the phase remains linear in the passband.



2010 - Nishimura

1.



by $\mathcal{H}(ax)$

$$\mathcal{H}(ax) \Leftrightarrow \frac{1}{|a|} \mathcal{H}\left(\frac{s}{a}\right)$$

$$\delta(t-\tau) \Leftrightarrow e^{-2\pi i s\tau}$$

we want phase shift of $-\pi$
for all s. $\Rightarrow e^{-\pi i}$

To approximate this, we could choose a τ s.t. $e^{-2\pi i a\tau} = e^{-\pi i}$

$$\Rightarrow \tau = \frac{1}{2a}$$

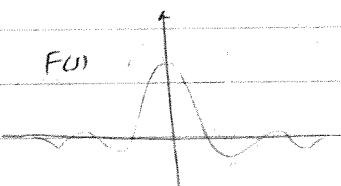
2. Yes. We know $f(x) \Leftrightarrow F(s)$ we want to find $\int_{-\infty}^{\infty} x f(x) dx$

and we know $\int_{-\infty}^{\infty} x^n f(x) dx = \left(\frac{j}{2\pi}\right)^n F(0)$

$$\Rightarrow \int_{-\infty}^{\infty} x f(x) dx = \boxed{\frac{j}{2\pi} F'(0)}$$

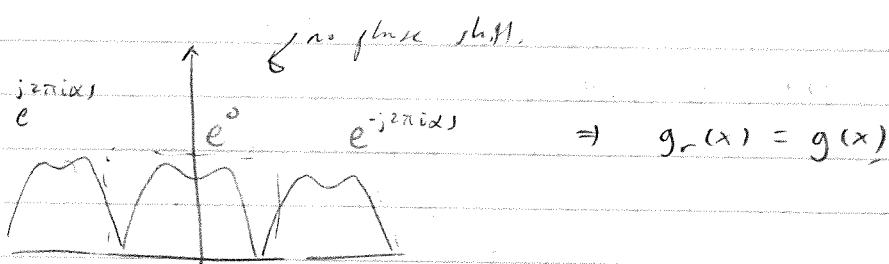
3. It would be best to agree and disagree with Dr. T.

Dr. T is partially correct, but additional constraints on s_0 and $F(s_0)$ are needed to avoid potential ambiguities in determining the center of mass.

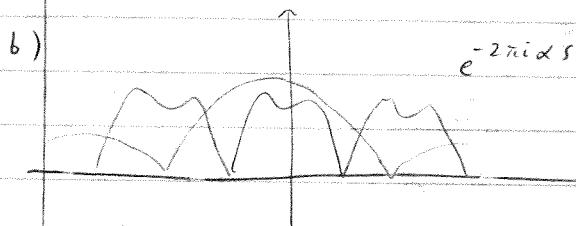


can't readily tell what center of mass is from a single point $F(s_0)$
(unless, of course, we somehow know that $F(s_0) = F'(0)$)

1998 - Nishimura



1. a) no

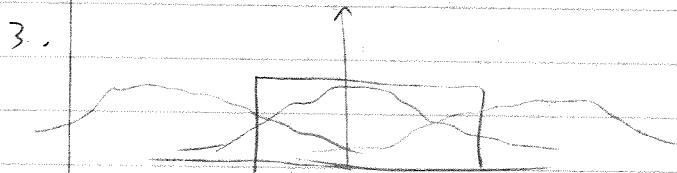


$$\epsilon = \int_{-\infty}^{\infty} |g_r(x) - g(x)|^2 dx = \int_{-\infty}^{\infty} |G_r(s) - G(s)|^2 ds$$

$$= \int_{-1/2}^{1/2} |\sin^2 s (G(s) - g(s))|^2 ds + \int_{-1/2}^{1/2} |e^s \sin^2 G(s)|^2 ds$$

$$+ \int_{1/2}^{\infty} |e^s \sin^2 G(s)|^2 ds$$

2. \Rightarrow doesn't depend on α .

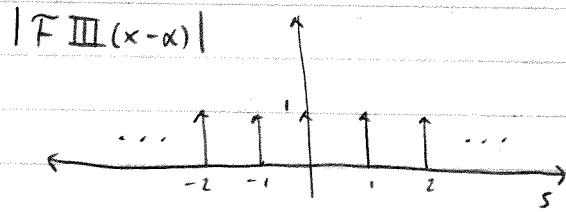


\Rightarrow yes. since phase shifted copies alias into the part that's filtered.

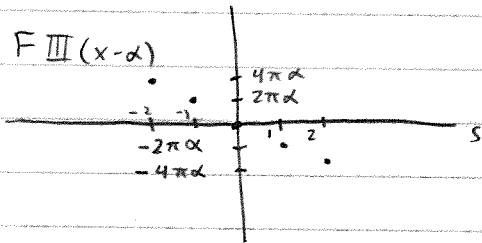
1.

$$\text{III}(x-\alpha) \Leftrightarrow e^{-2\pi i s \alpha} \text{III}(s) = \sum_{n=-\infty}^{\infty} e^{-2\pi i n \alpha} \delta(s-n)$$

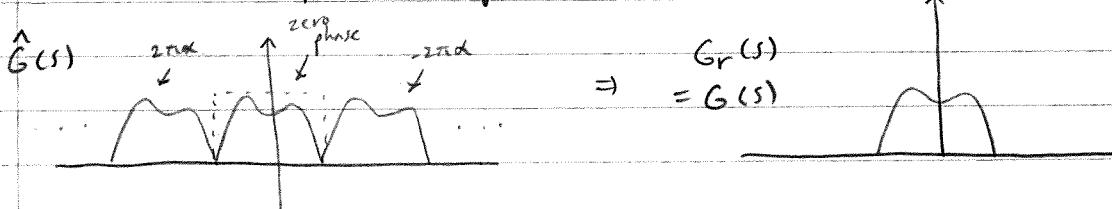
shifted shah?



$\angle F \text{III}(x-\alpha)$

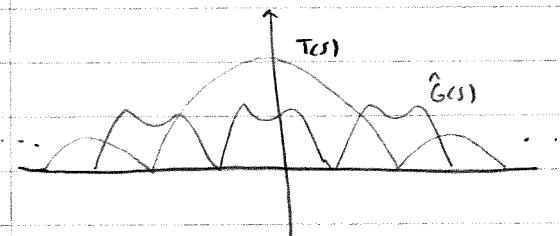


a) Assume we sample at Nyquist



$$\text{by Parseval's: } \epsilon = \int_{-\infty}^{\infty} |g_r(x) - g(x)|^2 dx = \int_{-\infty}^{\infty} |G_r(s) - G(s)|^2 ds = \int_{-\infty}^{\infty} 0 ds = 0$$

b) $t(x) = \Delta(x) \Rightarrow T(s) = \operatorname{sinc}^2(s)$



zero phase \Rightarrow
doesn't depend on α
 \checkmark

$$\epsilon = \int_{-\infty}^{\infty} |g_r(x) - g(x)|^2 dx = \int_{-\infty}^{\infty} |G_r(s) - G(s)|^2 ds = \int_{-\frac{1}{2}}^{\frac{1}{2}} |G_r(s) - G(s)|^2 ds$$

$$+ \int_{-\infty}^{-\frac{1}{2}} |G_r(s)|^2 ds + \int_{\frac{1}{2}}^{\infty} |G_r(s)|^2 ds \Rightarrow \boxed{\text{No}}, \epsilon \text{ won't depend on } \alpha$$

magnitude doesn't depend
on phase, and thus α .

2. No for $t(x) = \operatorname{sinc}(x)$

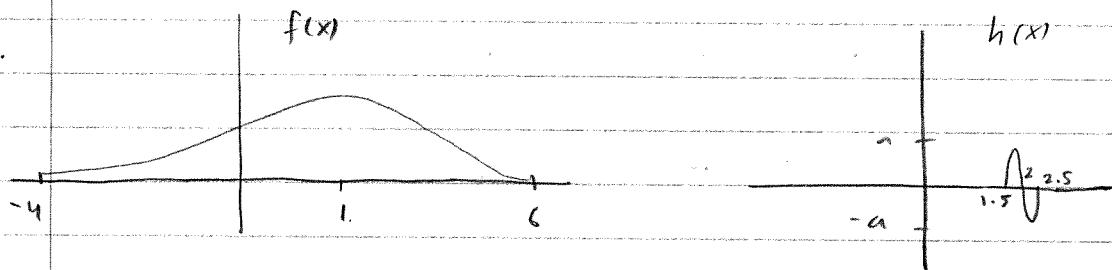
Yes for $t(x) = \Delta(x)$

3. Yes. Now islands with nonzero phase will alias into the passband (for both $t(x) = \operatorname{sinc}(x)$ and $t(x) = \Delta(x)$)

$\Rightarrow G_r(s) - G(s)$ will depend on $\alpha \Rightarrow \epsilon$ will depend on α .

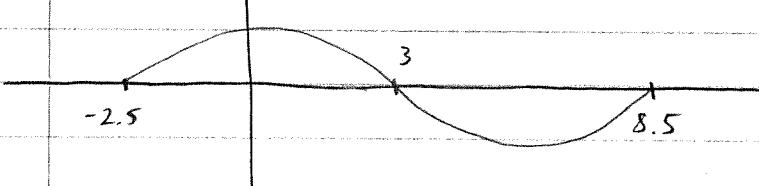
2009 - Nishimura

1.



$$f(x) * h(x)$$

approximate
takes the derivative
of $f(x)$.



2. a) Complex valued
 $f(x)$



$$|f(x)| \rightarrow$$

$$\angle f(x) \rightarrow$$

$$|g(x)|$$

$$+\angle g(x)$$

(more?)

$$\text{Sys: } |f(x)| = |g(x)|$$

phase of $f(x)$ passes through LTI system.

$$f(x) = r e^{i\theta} \quad g(x) = r e^{i\phi}$$

$$f_1(x) = r_1 e^{i\theta_1}$$

\Rightarrow [nonlinear].

sum of magnitude

$$f_2(x) = r_2 e^{i\theta_2}$$

\neq magnitude of sum.

e.g. consider $f_1(x) = -f_2(x) \Rightarrow |f_1(x) + f_2(x)| = 0 \neq |f_1(x)| + |f_2(x)| = 2|f_1(x)|$

b) time invariant? Yes, magnitude operation is time invariant,
and phase operation is time invariant since it passes
through an LTI system.

$$f(x) = r e^{i\theta}$$

through an LTI system.

$$f(x-\alpha) = r(x-\alpha) e^{i\theta(x-\alpha)}$$

$$g(x-\alpha) = r(x-\alpha) e^{i\phi(x-\alpha)}$$

\Rightarrow [time invariant]

c) magnitude operation is causal

but causality of overall system depends on causality of
LTI system that phase passes through.

2007 - Nishimura

1. Yes. This looks like simple time delay. $\Rightarrow H(s) = e^{-2\pi i s \alpha}$

2. BIBO stable: $\|h\|_1 = \int_{-\infty}^{\infty} |h(t)| dt < \infty \Rightarrow h(t) = \cos(t)$ is not BIBO stable.
for continuous case.

L^1 norm $\rightarrow \|h\|_1 = \sum_{n=-\infty}^{\infty} |h[n]| < \infty$ for discrete case.

$h(t)$ must fall off faster than $\frac{1}{x}$ \Rightarrow No

(aside: $\sum_{x=-\infty}^{\infty} \frac{1}{x^p}$ only converges for $p > 1$)

3. No this is time reversal which is not LTI (if time variant)

also, for an LTI system, you can't add new frequencies.

(can only remove frequencies) since we multiply by $H(s)$, from the graph, we can see a point on the right where $|F(s)|=0$ and $G(s) \neq 0$, which violates this principle.

for time reversal: $F(f(-t)) = F(-s) = R(-s) e^{j\theta(-s)}$

4. Yes

5. No $f(x) = \int_{-\infty}^{\infty} F(s) e^{j2\pi s x} ds$

$$\Rightarrow f(0) = \int_{-\infty}^{\infty} F(s) ds$$

from the graph we see that $\int_{-\infty}^{\infty} G(s) ds \neq 0$

and we are given that $g(0)=0 \Rightarrow$ False

6. No Fourier transform of autocorrelation always has zero phase.

$$F(\bar{g} * g) = |G(s)|^2$$

(aside: if $f(x)$ is real, $|F(s)|$ is even)

due to Hermitian symmetry. $F(-s) = F^*(s) \Rightarrow |F(-s)| = |F(s)|$

2005 - Osgood

Find the Fourier transform of $f(t) = |t|$

Existence of the Fourier Transform:

Conditions for the existence of the Fourier transform are complicated to state in general, but it is sufficient for $x(t)$ to be absolutely integrable (i.e. $x \in L_1$, meaning x has finite L_1 norm, $\|x\|_1 < \infty$)

$$\|x\|_1 = \int_{-\infty}^{\infty} |x(t)| dt < \infty$$

It is also sufficient for $x(t)$ to be square integrable ($x \in L_2$)

$$\|x\|_2 = \int_{-\infty}^{\infty} |x(t)|^2 dt < \infty$$

$f(t) = |t| \notin L_1 \text{ or } L_2$

but $f'(t) = \text{sgn}(t) \Rightarrow Ff'(t) = \frac{1}{\pi i s} = 2\pi i s Ff(s)$

$$\Rightarrow Ff(s) = -\frac{1}{2\pi^2 s^2}$$

2003 - Osgood

1. Express $f(t)$ as a finite sum of weighted sinusoids:

$$f(t) = \sum_{k=-N}^N c_k e^{2\pi i k t / T} \quad c_k = \bar{c}_k$$

$f(t)$ is periodic of period T .

There are $2N+1$ terms in the sum, so it takes $2N+1$ samples per period to determine $f(t)$ completely.

$$\mathcal{F}f(s) = \sum_{k=-N}^N c_k \delta(s - \frac{k}{T})$$

Spectrum goes from $-\frac{N}{T}$ to $\frac{N}{T}$.

Applying the sampling formula:

$$f(t) = \sum_{k=-\infty}^{\infty} f(t_k) \operatorname{sinc}(t-t_k) \quad \text{where } t_k = k \frac{T}{M}$$

where $M=2N+1$ defines the needed sample spacing (samples per period)

and $\rho = \frac{2N+1}{T} = \frac{M}{T}$ such that all δ 's in freq. domain are evenly spaced.

$$t_k = \frac{k}{\rho} = \frac{kT}{M}$$

\Rightarrow the sample points are spaced a fraction of a period apart, $\frac{T}{M}$,
and after $f(t_0), f(t_1), \dots, f(t_{M-1})$ the sample values
repeat (i.e. $f(t_n) = f(t_0), f(t_{n+1}) = f(t_1)$)

$$\Rightarrow f(t_{k+k'm}) = f(t_k) \quad \text{for } k, k' \in \mathbb{Z}$$

Using this periodicity of coefficients in the sampling formula, the single sampling sum splits into M sums w/

$$\sum_{k=-\infty}^{\infty} f(t_k) \operatorname{sinc}(t-t_k) = f(t_0) \sum_{m=-\infty}^{\infty} \operatorname{sinc}(pt-mM) + \dots + f(t_{M-1}) \sum_{m=-\infty}^{\infty} \operatorname{sinc}(pt-(M-1+mM))$$

The sums on the right have a simple closed form expression:

$$\sum_{m=-\infty}^{\infty} \text{sinc}(pt - k - m\Delta) = \frac{\text{sinc}(p(t - t_k))}{\text{sinc}(\frac{1}{\Delta}(t - t_k))}$$

$$\Rightarrow f(t) = \sum_{k=-N}^{N} c_k e^{2\pi i k t / T} = \sum_{k=0}^{2N} f(t_k) \frac{\text{sinc}(p(t - t_k))}{\text{sinc}(\frac{1}{\Delta}(t - t_k))}$$

$$= \sum_{k=0}^{2N} f(t_k) \frac{\text{sinc}((2v_{\max} + v_{\min})(t - t_k))}{\text{sinc}(v_{\min}(t - t_k))}$$

$$\text{where } p = \frac{2N+1}{T} \quad t_k = \frac{k}{p} = \frac{kT}{2N+1}$$

e.g. for $f(t) = \sin(2\pi t)$ $T=1$ $N=1$

\Rightarrow applying the sampling theorem:

$$f(t) = \sum_{k=-\infty}^{\infty} f(t_k) \text{sinc}(p(t - t_k)) \quad t_k = \frac{kT}{2N+1} = \frac{k}{3} \quad p = \frac{2N+1}{T} = 3$$

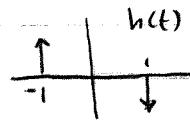
applying the finite sampling theorem:

$$\begin{aligned} f(t) &= \sum_{k=0}^{2N} f(t_k) \frac{\text{sinc}(p(t - t_k))}{\text{sinc}(\frac{1}{\Delta}(t - t_k))} \\ &= \sum_{k=0}^{2} f\left(\frac{k}{3}\right) \frac{\text{sinc}(3(t - t_k))}{\text{sinc}(t - t_k)} \\ &= 0 + \sin\left(2\pi \frac{1}{3}\right) \frac{\text{sinc}(3(t - \frac{1}{3}))}{\text{sinc}(t - \frac{1}{3})} + \sin\left(2\pi \frac{2}{3}\right) \frac{\text{sinc}(3(t - \frac{2}{3}))}{\text{sinc}(t - \frac{2}{3})} \\ &= \sin\left(\frac{2\pi}{3}\right) \frac{\text{sinc}(3t-1)}{\text{sinc}(t-\frac{1}{3})} + \sin\left(\frac{4\pi}{3}\right) \frac{\text{sinc}(3t-2)}{\text{sinc}(t-\frac{2}{3})} \end{aligned}$$

2002 - Nishimura

1.

Yes.



$$y(t) = x(t) * h(t)$$

2.

No.

LSI but not BIBO stable.

$$h(t) = u(t) \Rightarrow \int_{-\infty}^{\infty} |h(t)| dt < \infty$$

(Laplace)

Also, the system is an integrator with transfer function $\frac{1}{s}$

\Rightarrow pole is on jw axis \Rightarrow not BIBO stable.

BIBO stability requires all poles to be in left half plane.

3.

No.

Can't compress in time. This is the same as adding new frequencies. $a > 1$, $f(at) \Leftrightarrow \frac{1}{|at|} F(\frac{s}{a})$

Since $f * h \Leftrightarrow F(s) H(s)$, multiplication by $H(s)$ can only remove frequency content from $F(s)$.

LTI means you put a sine wave in, you get the same sine wave out, possibly with different amplitude & phase, but with the same frequency.

(constant at zero)

4.

regular gaussian

has zero phase. \Rightarrow Yes $F(e^{-\pi t^2}) = e^{-\pi s^2}$

$\xrightarrow{\text{Gaussian}} \text{Gaussian}$

5.

No.

$V_{in}(0) = 0$ but $V_{out}(0) \neq 0$

$\Rightarrow \alpha V_{in}(0) = 0 \neq \alpha V_{out}(0) = V_{out}(0) \Rightarrow$ nonlinear

2001 - Osgood

- Existence of Fourier Transforms:

if $\int_{-\infty}^{\infty} |f(t)|^2 dt < \infty$ or $\int_{-\infty}^{\infty} |f(t)| dt < \infty$,
then \hat{F} exists.

This condition is not satisfied for polynomials.

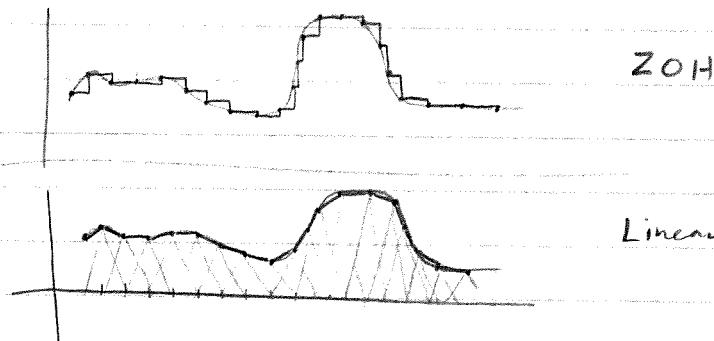
$$tf(t) \Leftrightarrow \left(\frac{j}{2\pi}\right) F'(s)$$

$$t^n f(t) \Leftrightarrow \left(\frac{j}{2\pi}\right)^n F^{(n)}(s)$$

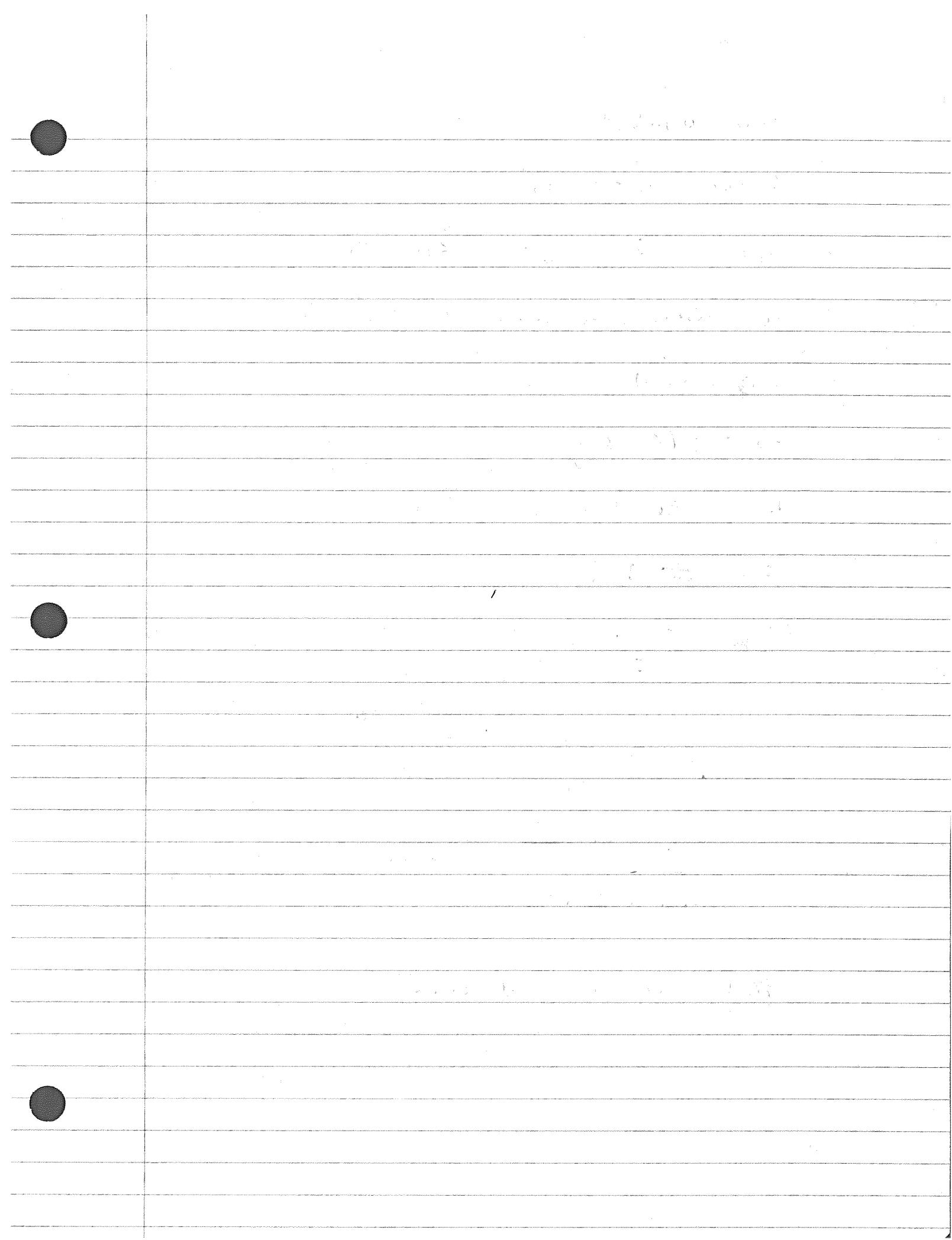
$$\text{let } f(t) = 1$$

$$\Rightarrow t^n \Leftrightarrow \left(\frac{j}{2\pi}\right)^n \delta^{(n)}(s) \Rightarrow \text{polynomials don't yield meaningful Fourier transforms.}$$

A better strategy would be interpolation using either zero-order hold or linear interpolation to approximate $f(t)$



expand on this: Ch 7 of Oppenheim



2000 - Nishimura

$$1. f_s = 2 \quad R(s) = \pi \left(\frac{s}{2} \right)$$

$$2. H(s) = 2\pi i s G(s) \quad f_s = 2 \quad R(s) = \pi \left(\frac{s}{2} \right)$$

$$3. H(s) = (G * G * G)(s) \quad f_s = 6 \quad R(s) = \pi \left(\frac{s}{6} \right)$$

$$4. H(s) = G(s+1) \quad f_s = 2 \quad R(s) = \pi \left(\frac{s+1}{2} \right)$$

$$5. H(s) = \frac{1}{2} (G(s-3) + G(s+3))$$

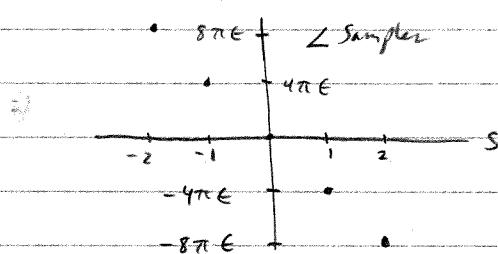
$$f_s = 2 \quad R(s) = \frac{1}{2} \left(\pi \left(\frac{s-3}{2} \right) + \pi \left(\frac{s+3}{2} \right) \right)$$

$$2. R(s) = \frac{1}{2\pi i s} \pi \left(\frac{s}{2} \right)$$

$$5. \boxed{\text{No}}. \quad \frac{1}{2} \text{III}(t-\epsilon) \Leftrightarrow e^{-2\pi i s \epsilon} \text{III}_2(s)$$

$\frac{1}{2} \leftarrow T = \frac{1}{f_s}$

$$= e^{-2\pi i s \epsilon} \sum_{k=-\infty}^{\infty} \delta(s-2k) = \sum_{k=-\infty}^{\infty} e^{-2\pi i s 2k} \delta(s-2k)$$



\Rightarrow at 3, $\hat{H}(s)$ has a $G(s) (1 + e^{-i12\pi\epsilon})$

at -3, $\hat{H}(s)$ has a $G(s) (1 + e^{i12\pi\epsilon})$

\Rightarrow unless ϵ satisfies a special property (i.e. $6\epsilon \in \mathbb{Z}$)

in which case we could reconstruct as in Q5.) we cannot recover $h(t)$.

2006 - Osgood

$$(f * g)[k] = \sum_{n=-\frac{N}{2}+1}^{\frac{N}{2}} f[n] g[n+k] = \sum_{n=0}^{N-1} f[n] g[n+k]$$

a) Show that $F(f * g) = \overline{Ff Fg}$

$$Ff[n] = \sum_{n=0}^{N-1} f[n] e^{-2\pi i nm/N} \Rightarrow \overline{Ff} = \sum_{n=0}^{N-1} f[n] e^{2\pi i nm/N}$$

$$Fg[n] = \sum_{n=0}^{N-1} g[n] e^{-2\pi i nm/N}$$

$$\overline{Ff Fg}[n] = \left(\sum_{n=0}^{N-1} f[n] e^{-2\pi i nm/N} \right) \left(\sum_{k=0}^{N-1} g[k] e^{-2\pi i km/N} \right)$$

$$F(f * g)[m] = \sum_{k=0}^{N-1} \left(\sum_{n=0}^{N-1} f[n] g[n+k] \right) e^{-2\pi i km/N}$$

$$= \sum_{k=0}^{N-1} \sum_{n=0}^{N-1} g[k] f[n] e^{-2\pi i (k-n)m/N}$$

$$e^{2\pi i nm/N} e^{-2\pi i (k+n)m/N}$$

$$F(f * g)[m] = \sum_{k=0}^{N-1} \left(\sum_{n=0}^{N-1} f[n] g[n+k] \right) e^{-2\pi i km/N}$$

$$= \sum_{k=0}^{N-1} \left(\sum_{n=0}^{N-1} f[n] e^{2\pi i nm/N} \right) g[n+k] e^{-2\pi i (k+n)m/N}$$

$$= \sum_{k=0}^{N-1}$$

2006 - Osgood

1.

$$(f \star g)[k] = \sum_{n=-\frac{N}{2}+1}^{\frac{N}{2}} f[n]g[n+k] = \sum_{n=0}^{N-1} f[n]g[n+k]$$

a) show that $\mathcal{F}(f \star g) = \overline{\mathcal{F}f} \mathcal{F}g$

$$\begin{aligned}\overline{\mathcal{F}f} \mathcal{F}g &= \sum_{p=0}^{N-1} \overline{f[p]} w^p \cdot \sum_{g=0}^{N-1} g[g] w^{-g} \\ &= \sum_{p=0}^{N-1} \sum_{g=0}^{N-1} f[p] g[g] w^{-(g-p)} \\ &= N \sum_{p=0}^{N-1} f[p] g[-p]\end{aligned}$$

$$\mathcal{F}[n] = \sum_{n=0}^{N-1} f[n] e^{2\pi i n m / N} \quad G[m] = \sum_{n=0}^{N-1} g[n] e^{-2\pi i n m / N}$$

$$\Rightarrow \overline{\mathcal{F}[n]} G[m] = \sum_{p=0}^{N-1} f[p] e^{2\pi i p m / N} \sum_{n=0}^{N-1} g[n] e^{-2\pi i n m / N}$$

$$= \sum_{p=0}^{N-1} \sum_{n=0}^{N-1} f[p] g[n] e^{-2\pi i m(n-p) / N} \quad k = n-p$$

$$= \sum_{k=0}^{N-1} \sum_{p=0}^{N-1} f[p] g[p+k] e^{-2\pi i m k / N}$$

b) $(f \star g)(0)$ max value is $\sum_{n=0}^{N-1} f[n]^2$, if $f[n] = g[n]$

2007 - Osgood

$$f(t) = \int_{-\infty}^{\infty} F(s) e^{j2\pi st} ds$$

$$y(t) = \int_{-\infty}^{\infty} F(s) H(s) e^{j2\pi st} ds$$

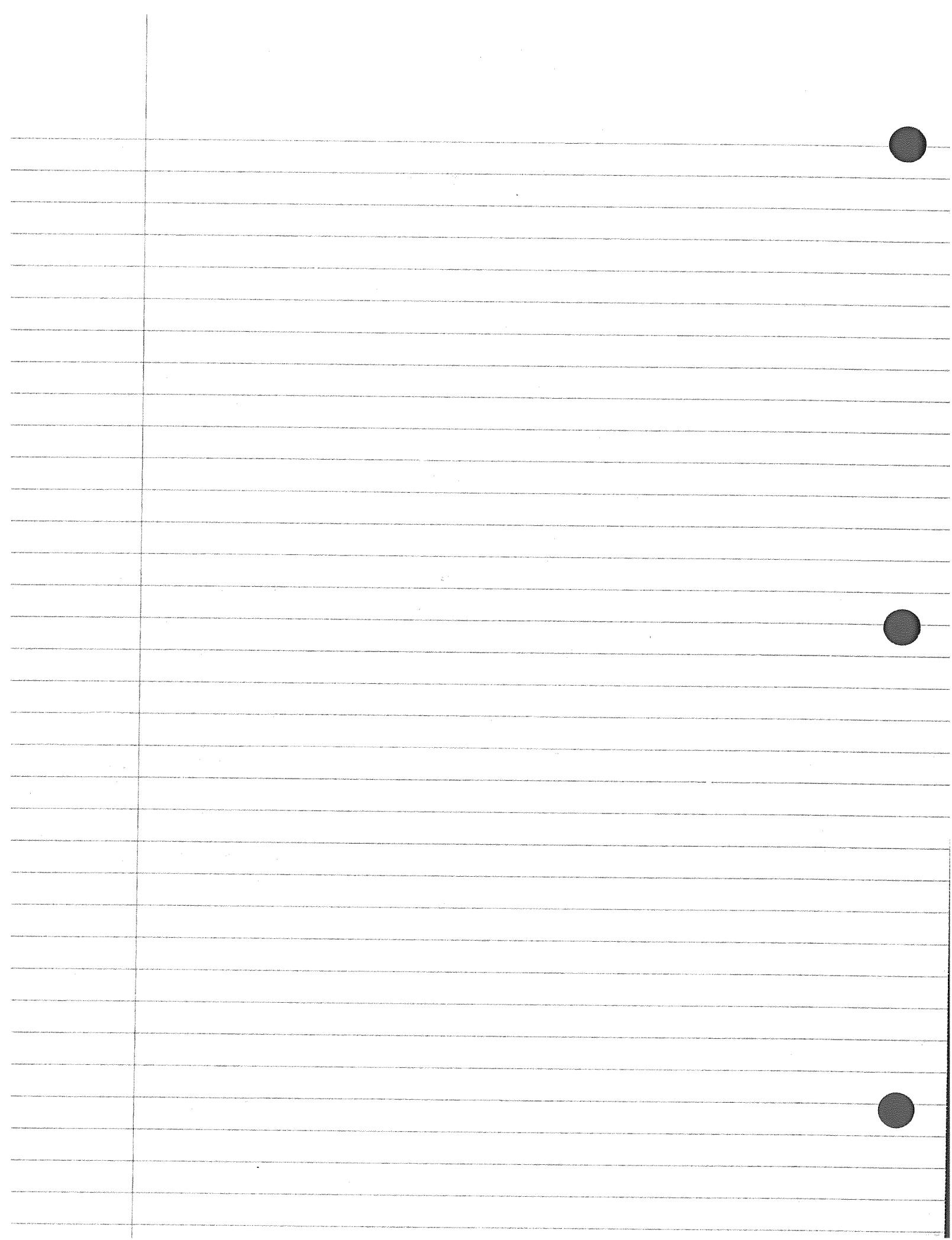
$$= \int_{-\infty}^{\infty} F(s) \int_{-\infty}^{\infty} h(\tau) e^{-j2\pi s\tau} d\tau e^{j2\pi st} ds$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(s) h(\tau) e^{j2\pi s(t-\tau)} ds d\tau$$

$$= \int_{-\infty}^{\infty} h(\tau) \int_{-\infty}^{\infty} F(s) e^{j2\pi s(t-\tau)} ds d\tau$$

$$= \int_{-\infty}^{\infty} h(\tau) f(t-\tau) d\tau$$

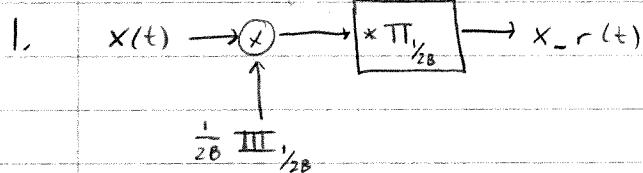
$$= (h * f)(t) \Rightarrow LTI$$



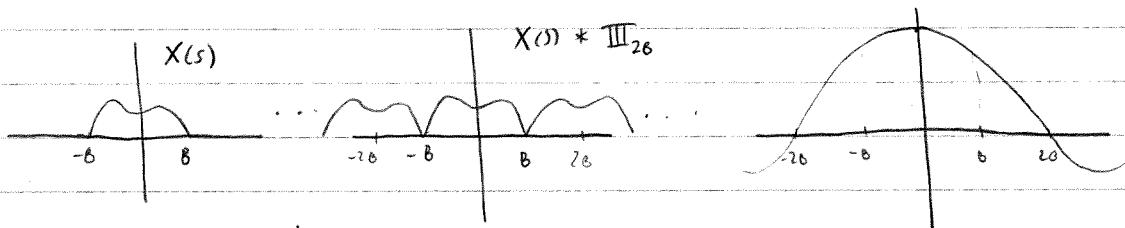
2009 - Pauly

11
12

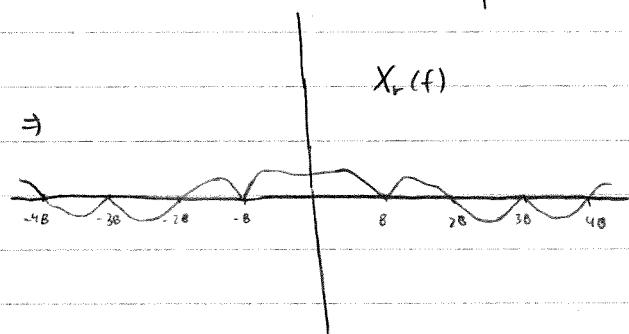
$Z(s)$



$$Z(s) = \frac{1}{2B} \sin(\frac{s}{2B})$$



$X_r(f)$



2. LPF $x_r(t)$. $H(s) = \frac{2B}{\sin(\frac{s}{2B})} \pi_{\frac{1}{2B}}$

(or you could predict the signal)

1995 - Nishimura

1. No.

2. Yes.

$$3. \operatorname{sgn}(t) \Leftrightarrow \frac{1}{\pi i s} \Rightarrow \frac{1}{2}(1 + \operatorname{sgn}(t)) \Rightarrow \frac{1}{2}(\delta(s) + \frac{1}{\pi i s})$$

$$\Rightarrow \frac{1}{2}(\delta(-t) - \frac{1}{\pi i t}) \Leftrightarrow \frac{1}{2}(1 + \operatorname{sgn}(t)) = H(t)$$

$$\Rightarrow h(t) = \frac{1}{2}\delta(t) - \frac{1}{2\pi i t}$$

$$\Rightarrow x(t) * h(t) = \frac{1}{2}x(t) - \frac{1}{2\pi i t} * x(t)$$

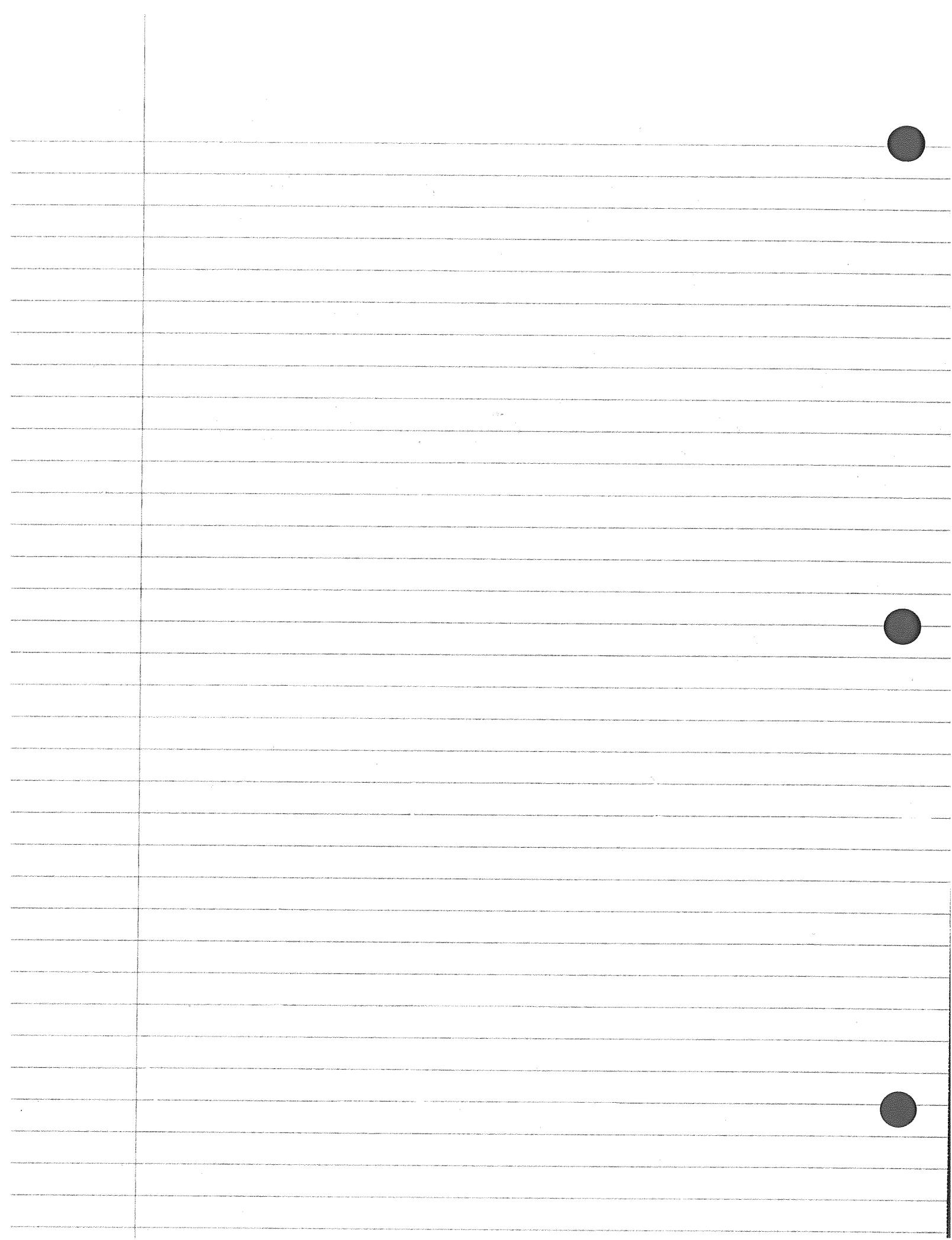
$$= \frac{1}{2}x(t) + \underbrace{\frac{i}{2\pi} (t * x(t))}_{\text{imaginary}}$$

$$\Rightarrow g(t) = \frac{1}{2}x(t)$$

64 samples.

2. $T = 1s \Rightarrow$ freq. resolution is $\frac{1}{64} \text{ Hz}$





2007 - Panly

$x(t)$

Bandlimited signal A , Sample at seemingly random but known times. Can you reconstruct $x(t)$? What conditions do you need?

Maybe if average sampling rate is above Nyquist.

??

To the first order, you could do linear interpolation.

1998 - Osgood

Let $R(t)$ denote the population of robins
 $W(t)$ denote the population of worms

If left completely alone, each population would ideally grow exponentially.

$$P(t) = P_0 e^{\alpha t} \quad \alpha \in \mathbb{R}, \alpha > 0$$

Can write this as $\frac{dP}{dt} = \alpha P(t)$

Interaction between the populations (Robins eating worms)
should have a \sim linear effect on each population.

$$\Rightarrow \frac{dR}{dt} = A \cdot R(t) + B \cdot W(t)$$

$$\frac{dW}{dt} = C \cdot W(t) - D \cdot R(t)$$

??

2004 - Osgood

if $f(t)$ is real, $|Ff|$ is even + $\angle Ff$ is odd
due to hermitian symmetry properties.

Since $f[n]$ is real + even, we expect $F[m]$ to be
real and even (zero imaginary component).

Student entered $[11100\dots01111]$ when he should
have entered $[111100\dots01111]$. Thus, the $f[n]$ he
entered was not even. It was a shifted version of $f[n]$.
Accordingly, $F[m]$ ended up with an imaginary component (nonzero phase).

, 2006 - Nishinura

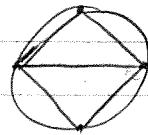
$$\frac{f_m}{f_s} = \frac{16}{128} = \frac{1}{8} \Rightarrow f_s = 8 \cdot f_m$$

max point the FFT can represent is $\frac{f_s}{2}$

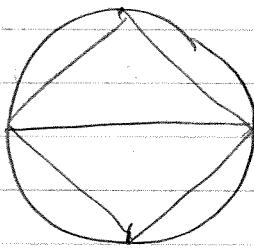
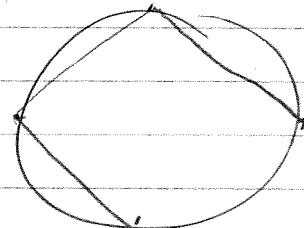
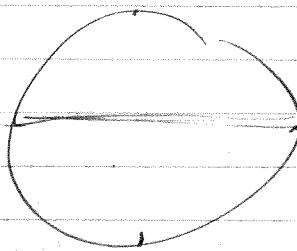
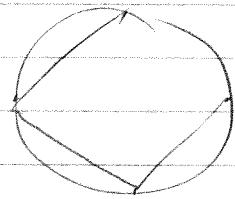
1. Check power connections.
2. f_m was reduced or f_{sample} was increased.
3. Signal being clipped \Rightarrow nonlinearity from ADC.
4. Spike noise in time signal (white noise)
5. Spectral leakage $\Rightarrow \frac{f_m}{f_s}$ is not rational.
6. time signal rectified

2000 - Osgood

$$F[m] = \sum_{k=0}^3 f[k] e^{-2\pi i k m / 4}$$



$$F[m] = f[0] e^0 + f[1] e^{-\pi i m / 2} + f[2] e^{-\pi i m} + f[3] e^{-3\pi i m / 2}$$



2000 - Spielman

1.

* Question of sampling rate. Is the function band limited?

treat rainy day on 1/3 as an impulse.

$$\Rightarrow \text{impulse response decays as } e^{-t/\tau} \quad 5(1-\frac{1}{e}) \approx (0.63)(5) = 3.15$$

$$\Rightarrow \tau \approx 1 \text{ day} \quad \text{BW} = \frac{1}{2\pi\tau} = f_{3dB}$$

$$\approx \frac{1}{6 \text{ days}} = \frac{1}{6} \text{ cycles/day}$$

\Rightarrow Nyquist is $\frac{1}{3}$ cycles/day \Rightarrow 2 measurements per day
is above Nyquist.

Is this an LTI system? To first order, it should be.

Linear since more rain \Rightarrow more water \Rightarrow faster flow rate.

Linear if we assume river width is relatively constant

Time invariant since delaying rain causes a delay in
water level rising.

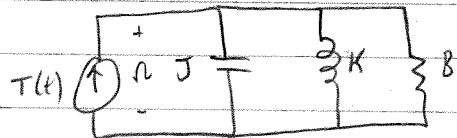
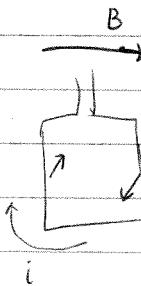
2. 2 cycles/day by Nyquist.

3. $r'(t) \Leftrightarrow 2\pi i R(s)$

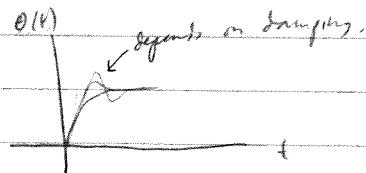
I would say no. We can see that based on 1/3,
this strategy would cause a significant error.

2001 - Spielman

$$1. \quad F = g\vec{v} \times \vec{B} = \vec{i} \times \vec{B}$$



2. a)



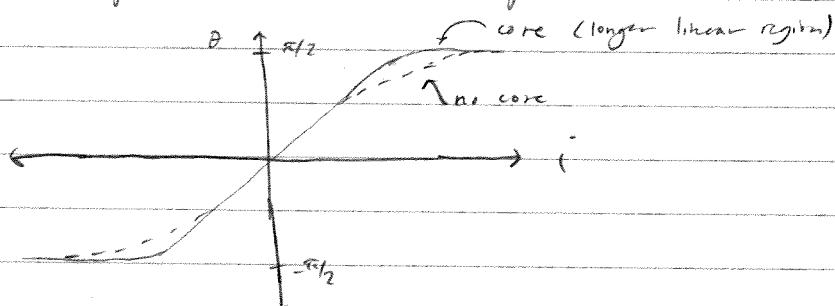
$\theta(t)$ depends on damping.

will also have noise,
which will be strongest
near resonant frequency.

b) critical damping. Increase air resistance around needle.

3. nonlinear, but still time invariant

- Resonant frequency unchanged.
- For small θ , system is LTI as before.
- Can compute steady state response

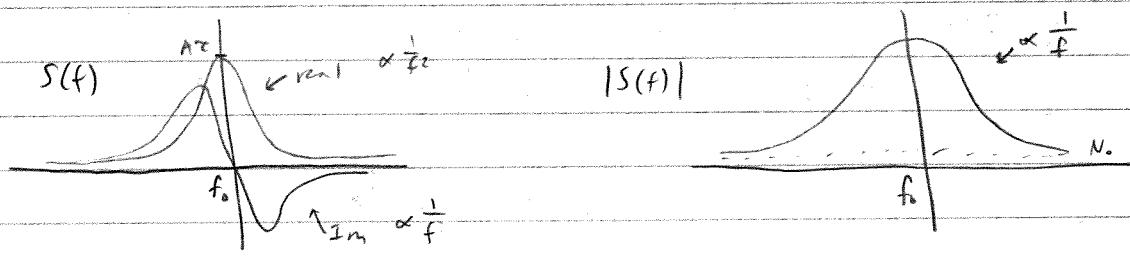


2003 - Spicelman

$$1. s(t) = A e^{-t/\tau} e^{j 2\pi f_0 t} + n(t)$$

$$e^{-at} \Leftrightarrow \frac{1}{a+2\pi i s}$$

$$2. S(f) = A \tau \left(\frac{1}{1 + (2\pi \tau (f-f_0))^2} - j \frac{2\pi \tau (f-f_0)}{1 + (2\pi \tau (f-f_0))^2} \right) + N_0$$



comprised by τ

$$\text{FWHM} \propto 1/\tau$$

$$\text{FWHM} \propto 1/\tau$$

2. Additive noise term in $S(f)$ is zero mean, complex WGN.

White noise in $|S(f)|$ is white, it is in general not Gaussian.

The noise is well approximated zero mean WGN where the signal component is large + Rayleigh distributed where the signal is small, and obeys Rician statistics for intermediate values.

2009 - Spielman

1. No

Yes

$$h(t) = 3\delta(t) - 5$$

2. Yes

No

$$h(t, t_0) = e^{-i2\pi t t_0} \quad \delta(t - t_0) \rightarrow e^{i2\pi t t_0}$$

3. Yes

Yes

$$\delta(t)$$

4. Yes

Yes

$$\pi_{2w} \Leftrightarrow 2w \operatorname{sinc}(2wt) = h(t)$$

5. Yes, No.

$$h(t) = \begin{cases} \delta(t - t_0), & -T < t_0 < T \\ 0, & |t| > T \end{cases}$$

2010 - Spielman

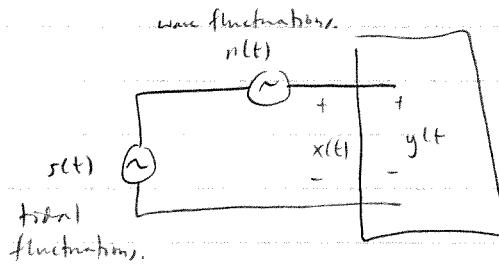
1. Yes. $y(t) = x(t)$

$$\frac{Y(f)}{X(f)} = H(f)$$

2. Advantages: inexpensive, easy to build, durable.

Disadvantages: need a boat to make the measurement.

More importantly, data will be noisy. The measurements are subject to unwanted variations from surface wave fluctuations. An improved device would filter out this unwanted high frequency noise.



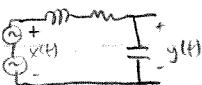
3. There are lots of choices for better measurement systems which can incorporate the desired low-pass filtering.

damping factor
 $\zeta = \frac{1}{2Q}$

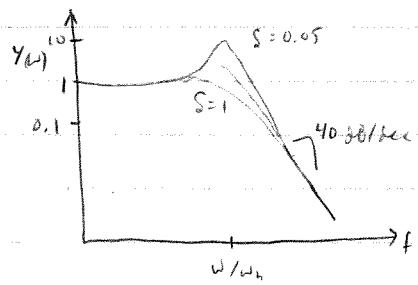
$$S = \frac{1}{2Q}$$

critical damping when
 $S = 1$

$$(Q = \frac{1}{2})$$



4. Device 2 is a low pass filter (equivalent to an LRC circuit). The frequency response would look something like (depending on the choice of A , L , + D):



In choosing A , L , and D , we want to avoid an underdamped system with a natural frequency in the range of ocean surface waves, typically on the order of 0.1 Hz.

$$\omega_n = \frac{1}{\sqrt{LC}} \text{ where } L_n \text{ is the inductance}$$

(fluid analog of electrical inductance) and C_f is fluid capacitance. The damping ratio equals $\frac{1}{2Q} = S = \frac{R}{2\sqrt{LC}}$ where R_f is fluid resistance. We want surface wave fluctuations $\gg \omega_n$ and tidal variations $\ll \omega_n$.

Damped Harmonic Oscillators in General:

$$\frac{d^2x}{dt^2} + 2\zeta\omega_0 \frac{dx}{dt} + \omega_0^2 x = 0 \quad \omega_0 = \sqrt{\frac{k}{m}} = \frac{1}{\sqrt{LC}}$$

$$\zeta = \frac{c}{2m\omega_0} = \frac{1}{2Q}$$

Overdamped $\zeta > 1$

$$Q = \frac{1}{2\zeta}$$

Critically damped $\zeta = 1$

Underdamped $\zeta < 1$

5. a) Increasing A increases the fluid capacitance, thereby decreasing the natural frequency ω_n .

b) Increasing L increases the fluid inductance, thereby decreasing the natural frequency ω_n , and increases the fluid resistance (thereby increasing the damping).

c) Increasing D decreases the fluid inductance, thereby increasing the natural frequency ω_n) and decreases the fluid resistance (thereby decreasing damping).

