Master Theorem

- The Master Theorem applies to:

- $T(n) = a T(n / b) + f(n)$
- Where $a >= 1$ and $b > 1$

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- We first calculate $c = \log_b a$
- 3 cases

Case # 1

- $T(n) = a T(n / b) + f(n)$
- If $f(n)$ is $O(n^d)$ and $d < \log_b a$, then
- // $f(n) <= e n^d < e n^{\log_b a}$
- $T(n) = \Theta(n^{\log_b a})$

- Important note:
- $d < \log_b a$, not $d <= \log_b a$

Case # 2

- $T(n) = a\,T(n/b) + f(n)$
- If $f(n)$ is $\Theta(n^{c})$ and $c = \log_b a$, then
- $T(n) = \Theta(n^{\log_b a} \log n)$

Case # 2f (fancy version)

- $T(n) = a\,T(n/b) + f(n)$
- If $f(n)$ is $\Theta(n^{c} \log^{k} n)$ and $c = \log_b a$, then

- $T(n) = \Theta(n^{\log_b a} \log^{k+1} n)$

Case # 3

- If $f(n)$ is $\Omega(n^{d})$ and $d > \log_{b} a$, AND
- There is a $C < 1$ and $n0$ such that

  $a f(n / b) <= C f(n)$ for all $n >= n0$

  (regularity condition, true very often)
- Then, $T(n) = \Theta(f(n))$

- Important note:
- $d > \log_{b} a$, not $d >= \log_{b} a$