1) $T(n) = c + \underline{T(n-1)}$ A.

$T(x) = c + T(x-1)$

① $x = n-1 \Rightarrow \underline{T(n-1)} = c + T(n-1-1)$

$= c + T(n-2)$.

$T(n) = c + c + T(n-2)$

$= 2c + \underline{T(n-2)}$

② $x = n-2 \Rightarrow \underline{T(n-2)} = c + T(n-2-1)$

$= c + T(n-3)$.

$\therefore T(n) = 2c + c + T(n-3)$

$= 3c + T(n-3)$.

$\therefore T(n) = k \cdot c + T(n-k)$.

Base Case : $T(0) \Rightarrow n-k = 0$

$\therefore n = k$.

$\therefore T(n) = \underline{n \cdot c} + \underbrace{T(0)}_{\Theta(1)} = cn + \Theta(1)$

$\therefore \Theta(n)$.

note : $\Theta(n) \Leftrightarrow c \cdot n$.

2) MCS :

$T(n) = \underline{c_1 + c_2} + 2T(\frac{n}{2}) + \underline{\Theta(n)} + \underline{c_3}$.

$= \cancel{c} + 2T(\frac{n}{2}) + c \cdot n$

$\therefore T(n) = 2T(\frac{n}{2}) + cn$.

$T(x) = 2T(\frac{x}{2}) + c \cdot x$.

① $x = \frac{n}{2}$, $\Rightarrow T(\frac{n}{2}) = 2 \cdot T((\frac{n}{2}) \div 2) + c \cdot \frac{n}{2}$.

$= 2 \cdot T(\frac{n}{2^2}) + c \cdot \frac{n}{2}$

$$T\left(\frac{n}{2}\right) = 2T\left(\frac{n}{2^2}\right) + c \cdot \frac{n}{2}.$$

$$T(n) = 2\left[2 \cdot T\left(\frac{n}{2^2}\right) + c \cdot \frac{n}{2}\right] + cn.$$

$$= 2^2 T\left(\frac{n}{2^2}\right) + cn + cn.$$

$$= 2^2 \underline{T\left(\frac{n}{2^2}\right)} + 2 \cdot cn.$$

② $x = \frac{n}{2^2}$. $\Rightarrow$ $\underline{T\left(\frac{n}{2^2}\right)} = 2T\left(\frac{n}{2^3}\right) + c \cdot \frac{n}{2^2}.$

$$\therefore T(n) = 2^2\left[2 T\left(\frac{n}{2^3}\right) + c \cdot \frac{n}{\cancel{2^2}}\right] + \underline{2cn}.$$

$$= 2^3 T\left(\frac{n}{2^3}\right) + 3cn$$

pattern ? $T(n) = 2^k T\left(\frac{n}{2^k}\right) + k \cdot cn.$

Base Case ? $T(1) \rightarrow \frac{n}{2^k} = 1$

$\therefore n = 2^k.$

$\therefore k = \lg n.$

$$\therefore T(n) = \underline{2^{\lg n}} \cdot \underline{T(1)} + \lg n \cdot cn.$$

$$= n \underbrace{T(1)}_{\Theta(1)} + c n \lg n.$$

$\therefore \Theta(n \lg n)$

3) B S:

$T(n) = c + T(\frac{n}{2})$ 1.

$T(x) = c + T(\frac{x}{2})$ A.

① $x = \frac{n}{2} \Rightarrow T(\frac{n}{2}) = c + T(\frac{n}{2^2})$

$\therefore T(n) = c + c + T(\frac{n}{2^2}) = 2c + T(\frac{n}{2^2})$

② $x = \frac{n}{2^2} \Rightarrow T(\frac{n}{2^2}) = c + T(\frac{n}{2^3})$

$\therefore T(n) = 2c + c + T(\frac{n}{2^3})$

$= 3c + T(\frac{n}{2^3})$

pattern? $T(n) = k \cdot c + T(\frac{n}{2^k}) \rightarrow$ true

Base case: $T(1)$ $\therefore \frac{n}{2^k} = 1$ $\therefore n = 2^k$

$k = \lg n$.

$\therefore T(n) = \lg n \cdot c + T(1)$

$= c \lg n$

$\therefore \Theta(\lg n)$

4). $T(n) = 3T(\frac{n}{2}) + 1$ w/ $T(1) = 1$.

$T(x) = 3 \cdot T(\frac{x}{2}) + 1$ A.

① $x = \frac{n}{2}$, $T(\frac{n}{2}) = 3 \cdot T(\frac{n}{2^2}) + 1$.

$\therefore T(n) = 3[3 \cdot T(\frac{n}{2^2}) + 1] + 1$.

$= 3^2 \cdot T(\frac{n}{2^2}) + 3 + 1$.

② $x = \frac{n}{2^2}$, $T(\frac{n}{2^2}) = 3 \cdot T(\frac{n}{2^3}) + 1$.

$$T(n) = 3^2[3 \cdot T(\frac{n}{2^3}) + 1] + 3 + 1$$

$$= 3^3 T(\frac{n}{2^3}) + 3^2 + 3 + \underline{\underline{1}} \cdot 3^0$$

pattern: $T(n) = 3^k \cdot T(\frac{n}{2^k}) + 3^{k-1} + 3^{k-2} + \cdots + 3 + 3^0$

$$= 3^k \cdot T(\frac{n}{2^k}) + \sum_{i=0}^{k-1} 3^i$$

$$= 3^k \cdot T(\frac{n}{2^k}) + \frac{3^k - 1}{3 - 1}$$

$$\frac{3^k}{2} - \frac{1}{2}$$

Base Case: $\frac{n}{2^k} = 1$

$\therefore n = 2^k$, $\underline{k = \lg n}$

$\therefore T(n) = 3^{\lg n} \cdot T(1) + \frac{1}{2}(3^{\lg n} - 1)$

$$= 3^{\lg n} + \frac{1}{2} 3^{\lg n} - \frac{1}{2}$$

$\therefore 3^{\lg n} = 3^{\frac{\log_3 n}{\log_3 2}} = (3^{\log_3 n})^{\frac{1}{\log_3 2}} = n^{\log_2 3}$

$\therefore T(n) = \frac{3}{2} n^{\lg 3} - \frac{1}{2}$

$$\log_2 3 = \frac{\log_3 3}{\log_3 2} = \frac{1}{\log_3 2}$$