$b > 1$

$T(n) = aT(\frac{n}{b}) + f(n)$, $a \geq 1$, $b \geq 2$.

case 1: if $f(n) = O(n^d)$ and $d \leq \log_b^a$, then
$T(n) = \Theta(n^{\log_b a})$

$< \Rightarrow \times$

case 2: if $f(n) = \Theta(n^c)$ and $c = \log_b^a$, then.
$T(n) = \Theta(n^{\log_b a} \lg n)$

case 3: if $f(n) = \Omega(n^d)$ and $d > \log_b^a$, then
$T(n) = \Theta(f(n))$

case 2f: if $f(n) = \Theta(n^c \lg^k n)$ and $c = \log_b^a$, then
$T(n) = \Theta(n^{\log_b a} \lg^{k+1} n)$

e.g. 1. $T(n) = 9T(\frac{n}{3}) + n$.
step 1: $a = 9$, $b = 3$, $f(n) = n$, $\log_b^a = \log_3^9 = 2$.
if case #1: $f(n) = O(n^d) \Rightarrow$ we need to find the d. $d < 2$.
$\therefore d : 1 \sim 2$
$\therefore d = 1.5$
$\therefore$ Case #1 $\therefore T(n) = \Theta(n^2)$.

e.g. 2. $T(n) = T(\frac{2n}{3}) + 1$.
$a = 1$, $b = \frac{3}{2} = 1.5$, $f(n) = \boxed{1}$, $\log_b^a = \log_{1.5}^1 = 0$
if case #1? $\rightarrow d < 0$ $d \geq 0$
if case #2? $\rightarrow f(n) = \Theta(n^c) = \Theta(n^0) = \Theta(1)$
$\therefore$ Case #2
$\therefore T(n) = \Theta(n^0 \lg n) = \Theta(\lg n)$.

$b > 1$

$T(n) = aT(\frac{n}{b}) + f(n)$, $a \geq 1$, $b \geq 2$.

case 1: if $f(n) = O(n^d)$ and $d \leq \log_b a$, then
$T(n) = \Theta(n^{\log_b a})$

$<\Rightarrow \times$

case 2: if $f(n) = \Theta(n^c)$ and $c = \log_b a$, then.
$T(n) = \Theta(n^{\log_b a} \lg n)$

case 3: if $f(n) = \Omega(n^d)$ and $d > \log_b a$, then
$T(n) = \Theta(f(n))$

case 2f: if $f(n) = \Theta(n^c \lg^k n)$ and $c = \log_b a$, then
$T(n) = \Theta(n^{\log_b a} \lg^{k+1} n)$

e.g. 1. $T(n) = 9T(\frac{n}{3}) + n$.
step 1: $a = 9$, $b = 3$, $f(n) = n$, $\log_b a = \log_3 9 = 2$.
if case #1: $f(n) = O(n^d) \Rightarrow$ we need to find the d.
$\therefore d : 1 \sim 2$ $d < 2$.
$\therefore d = 1.5$
$\therefore$ Case #1 $\therefore T(n) = \Theta(n^2)$.

e.g. 2. $T(n) = T(\frac{2n}{3}) + 1$.
$a = 1$, $b = \frac{3}{2} = 1.5$, $f(n) = \boxed{1}$, $\log_b a = \log_{1.5} 1 = 0$
if case #1? $\rightarrow d < 0$ $d \geq 0$
if case #2? $\rightarrow f(n) = \Theta(n^c) = \Theta(n^0) = \Theta(\boxed{1})$
$\therefore$ Case #2
$\therefore T(n) = \Theta(n^0 \lg n) = \Theta(\lg n)$.

e.g.3. $T(n) = 3T(\frac{n}{4}) + n\lg n$.

$a=3$, $b=4$, $f(n) = n\lg n$, $\log_b^a = \log_4^3 = 0.793$. <1

$\therefore$ if case #1 ? $\Rightarrow f(n) = O(n^d)$, $d < 0.793$. ✗

if case #2 : $\Rightarrow f(n) = \Theta(n^c) = O(n^{0.793})$. ✗

Try case #3 : $\Rightarrow f(n) = \Omega(n^d)$ $d > 0.793$. ✓

$d = 0.8, 0.9 \ldots$

$\therefore T(n) = \Theta(f(n)) = \Theta(n\lg n)$, $f(n)$ is DT.

e.g.4. $T(n) = 2T(\frac{n}{2}) + n\lg n$.

$a=2$, $b=2$, $f(n) = n\lg n$, $\log_b^a = \log_2^2 = 1$

if case #1 ? $f(n) = O(n^d)$, $d<1$ ✗

if case #2 ? $f(n) = \Theta(n^c) = \Theta(n)$ ✗

if case #3 ? $f(n) = \Omega(n^d)$, $d>1$.

$\Omega(n^d) = \Omega(n \cdot n^{d-1})$ $\quad n\lg n$ ✗

if case #2f : $f(n) = \Theta(n^1 \lg^k n)$ $\quad k=1$ ✓

$\therefore$ 2f $\Rightarrow T(n) = \Theta(n\lg^2 n)$

e.g.6. $T(n) = 9T(\frac{n}{3}) + n\lg n + n^2 + 5$.

$\therefore a=9$, $b=3$, $f(n) = n\lg n + n^2 + 5$, $c = \log_b^a = 2$.

$\therefore f(n) = \Theta(n^c) = \Theta(n^2)$ $\Rightarrow$ case 2.

$\therefore T(n) = \Theta(n^c \lg n) = \Theta(n^2 \lg n)$.

× apply MS:

a. $T(n) = 1.5\,T(\frac{n}{3}) + n^2 + \lg n$.

b. $T(n) = 2T(n-1) + \lg n$. b ×

c. $T(n) = 2T(\frac{n}{3}) + 3T(\frac{n}{4}) + n^2$.

d. $T(n) = 4T(\frac{n}{2}) + f(n)$, w/ $f(n) = O(n^2)$

case 1

$\log_b a = \log_2 4 = 2 > d = 2$

$2 > \cancel{d}\, 2$ ×

$\therefore$ ×