

Math For Computer Graphics

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1 Linear Transformations

A point (or vertex) in three dimensional space can be represented by the following vector: $\mathbf{v} = [x_0, y_0, z_0, 1]^T$. The extra dimension is solely for the convenience of matrix manipulation.

1.1 Translation, Rotation

Translation of a vertex $\mathbf{v} = [x_0, y_0, z_0, 1]^T$ in x direction for t_x , y direction for t_y , and z direction for t_z is a linear transformation, whose corresponding matrix is T :

$$T\mathbf{v} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \\ z_0 \\ 1 \end{bmatrix} = [x_0 + t_x, y_0 + t_y, z_0 + t_z, 1]^T \quad (1)$$

The matrices R_x, R_y, R_z that correspond to the rotations of a vertex $\mathbf{v} = [x_0, y_0, z_0, 1]^T$ respect to x, y, z for θ degrees are:

$$R_z = \begin{bmatrix} \cos \theta & -\sin \theta & 0 & 0 \\ \sin \theta & \cos \theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, R_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta & 0 \\ 0 & \sin \theta & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, R_y = \begin{bmatrix} \cos \theta & 0 & \sin \theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin \theta & 0 & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

The Scaling matrix S is:

$$S = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3)$$

Recall matrix under multiplication is not an abelian group, i.e., matrix multiplication is associative, although not commutative. To transform a vertex \mathbf{v} with scaling, rotation, and translation, the order of the matrix is important:

$$TR_xR_yR_zS\mathbf{v} \quad (4)$$

OpenGL will normalize all the vertices before rendering to screen, which means if the setted screen is not a square, the final rendering will be distorted.

1.2 Projection

2 Colors in Fragment Shader

2.1 Interpolation

Assuming a triangle have three vertices A, B, C with arbitrarily assigned coordinates $(1, 0, 0), (0, 1, 0), (0, 0, 1)$. Barycentric Coordinate system can assign every points inside the triangle with coordinates (a, b, c) such that $a+b+c = 1$ and value of a, b, c are inversely proportional to its distance from vertices A, B, C .

Algorithm 2.1 (Barycentric Coordinates).

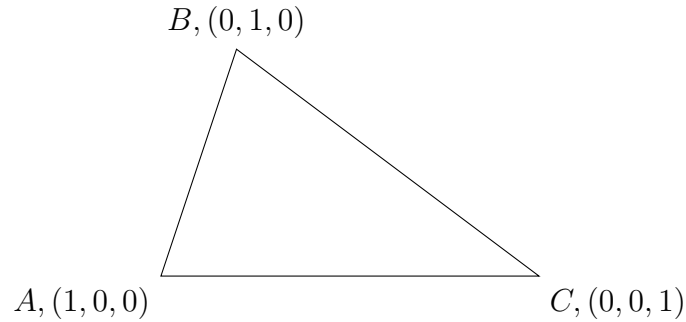


Figure 1: Barycentric Coordinate