Math For Computer Graphics

Harry Han

March 11, 2023

1 Linear Transformations

A point (or vertex) in three dimensional space can be represented by the following vector: $\boldsymbol{v} = [x_0, y_0, z_0, 1]^T$. The extra dimension is solely for the convenience of matrix manipulation.

1.1 Translation, Rotation

Translation of a vertex $\boldsymbol{v} = [x_0, y_0, z_0, 1]^T$ in x direction for t_x, y direction for t_y, and z direction for t_z is a linear transfomation, whose corresponding matrix is T:

$$T\boldsymbol{v} = \begin{bmatrix} 1 & 0 & 0 & x_t \\ 0 & 1 & 0 & y_t \\ 0 & 0 & 1 & z_t \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \\ z_0 \\ 1 \end{bmatrix} = [x_0 + x_t, y_0 + y_t, z_0 + z_t, 1]^T \tag{1}$$

The matrices R_x, R_y, R_z that correspond to the rotations of a vertex $\boldsymbol{v} = [x_0, y_0, z_0, 1]^T$ respect to x, y, z for θ degrees are:

$$R_z = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, R_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, R_y = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

1.2 Projection