Countable and Non-Countable Sets

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Countability

Countability

A set S is countable if there exists a bijection $f : S \to \mathbb{N}$.

- $\mathbb{N}$ is countable.
- $\mathbb{Z}$ is countable.
- $\mathbb{Q}$ is countable.
- $\mathbb{R}$ is uncountable.

In particular, a countable set must contain infinitely many elements.

Countability

Let C_i be countable sets.

- $C_0 \cup C_1$ is countable.
- By induction, $\bigcup_{i=0}^{n} C_i$ is countable, for finite n.
- $C_0 \times C_1$ is countable.
- By induction, $X_{i \in 1, \cdots, n} C_i = C_0 \times C_1 \times \cdots \times C_n$ is countable, for finite n.
- $L = \bigcup_{i \in \mathbb{N}} C_i$ is countable: an surjection from $N \times N$ to L
- $G = X_{i \in \mathbb{N}} C_i$ is NOT countable!

Countability

Consider real number from 0 to 1, $[0, 1]$.

$0.000000000000 \cdots$

$\cdots$

$0.100200000000 \cdots$

$\cdots$

$0.123456789101 \cdots$

$\cdots$

$G = X_{i \in \mathbb{N}} C_i$ is NOT countable!

Injection from $[0, 1]$ to G: G is not countable.