Probability Note

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Contents

1 Basics Definitions

Definition 1.1 (Sample Spaces)**.** A sample space is a set of all possible outcomes of an experiment.

Definition 1.2 (Events)**.** An event is a subset of the sample space.

Definition 1.3 (Probability Mass Function)**.** Let $\mathbb{S} = \{x_1, x_2, \dots\}$ be a sample space and domain of function F. F is its probability mass function provided that: $0 < F(x_i) < 1$ and $\sum_{i=1} F(x_i) = 1$.

Definition 1.4 (Probability of an Event)**.** Let A be an event and F be its probability mass function. Then $P(A) = \sum_{x_i \in A} F(x_i)$.

Definition 1.5 (Independence of an Event)**.** Let A and B be two events. Then A and B are independent if $P(A \cap B) = P(A)P(B)$.

Definition 1.6 (Random Variable)**.** A random variable is a function on sample space.

Definition 1.7 (Idenpendence of Random Variable)**.** Two random variables X and Y are independent if and only if $P(X = x, Y = y) = P(X = x)P(Y = y)$ for all x and y.

Definition 1.8 (Expected Value of a Random Variable)**.** Let X be a random variable. Its expected value is $E(X) = \sum_i P(x_i)x_i$