1. Choose x_1, x_2, $a < x_1 < x_2 < b$

$h = \frac{b-a}{n} = \frac{b-a}{3}$

$f(x) = f(x_1)\frac{x-x_2}{x_1-x_2} + f(x_2)\frac{x-x_1}{x_2-x_1} + \frac{f''(\xi_x)}{2}(x-x_1)(x-x_2)$

$\Rightarrow \int_a^b f(x)\,dx = \int_a^b P_1(x)\,dx + \int_a^b E(x)\,dx$

$= \left[\frac{f(a)+f(x_1)}{2}(x_1-a) + \frac{f(x_1)+f(x_2)}{2}(x_2-x_1) + \frac{f(x_2)+f(b)}{2}(b-x_2)\right]$

$+ \frac{1}{2}\int_a^b f''(\xi_x)(x-x_1)(x-x_2)\,dx$

$= \frac{h}{2}[f(a)+f(b)+2f(x_1)+2f(x_2)] + \frac{1}{2}f''(\xi_x)\int_a^b (x-x_1)(x-x_2)\,dx$

$=$ Then $\frac{1}{2}f''(\xi_x)\int_a^b (x-x_1)(x-x_2)\,dx$

$= \frac{1}{2}f''(\xi)\left[\int_a^{x_1}(x-x_1)(x-x_2)\,dx + \int_{x_1}^{x_2}(x-x_1)(x-x_2)\,dx\right.$

$\left. + \int_{x_2}^{b}(x-x_1)(x-x_2)\,dx\right]$

$= -\frac{1}{2}f''(\xi)\left[\frac{1}{6}(x_1-a)^3 + \frac{1}{6}(x_2-x_1)^3 + \frac{1}{6}(b-x_2)^3\right]$

$= -\frac{1}{12}f''(\xi)\left[3\cdot\left(\frac{b-a}{3}\right)^3\right]$

$= -\frac{f''(\xi)}{108}(b-a)^3$

So $\int_a^b f(x)\,dx = \frac{h}{2}[f(a)+f(b)+2f(x_1)+2f(x_2)] - \frac{(b-a)^3}{108}f''(\xi)$

2. (a) Tropozoid Rule:

$f(x) = e^{3x}$ $\quad f'(x) = 3e^{3x}$ $\quad f''(x) = 9e^{3x}$

error: $\frac{h^2}{12}(b-a)f''(\xi) = \frac{(b-a)^3}{12n^2} f''(\xi)$

$$\frac{(b-a)}{12n^2} f''(\xi) \le 10^{-8}$$

$$12n^2 \ge (b-a)^3 f''(\xi) \times 10^8$$

$$|f''(\xi)| = |9e^{3x}| \le 9e^3$$

$$12n^2 \ge 9e^3 \times 10^8$$

$$n^2 \ge \frac{9e^3 \times 10^8}{12}$$

$$n \ge 38812.57$$

$$\approx 38813$$

Simpson's Rule:

$f'''(x) = 27e^{3x}$ $\quad f^{(4)}(x) = 81e^{3x}$ $\quad h = \frac{b-a}{2n}$

$$e = \frac{h^4}{180}(b-a)f^{(4)}(\xi) = \frac{(b-a)^5}{180n^4} f^{(4)}(\xi)$$

$$\frac{(b-a)^3}{180n^4} f^{(4)}(\xi) \le 10^{-8}$$

$$180n^4 \ge (b-a)^5 f^{(4)}(\xi) \times 10^8$$

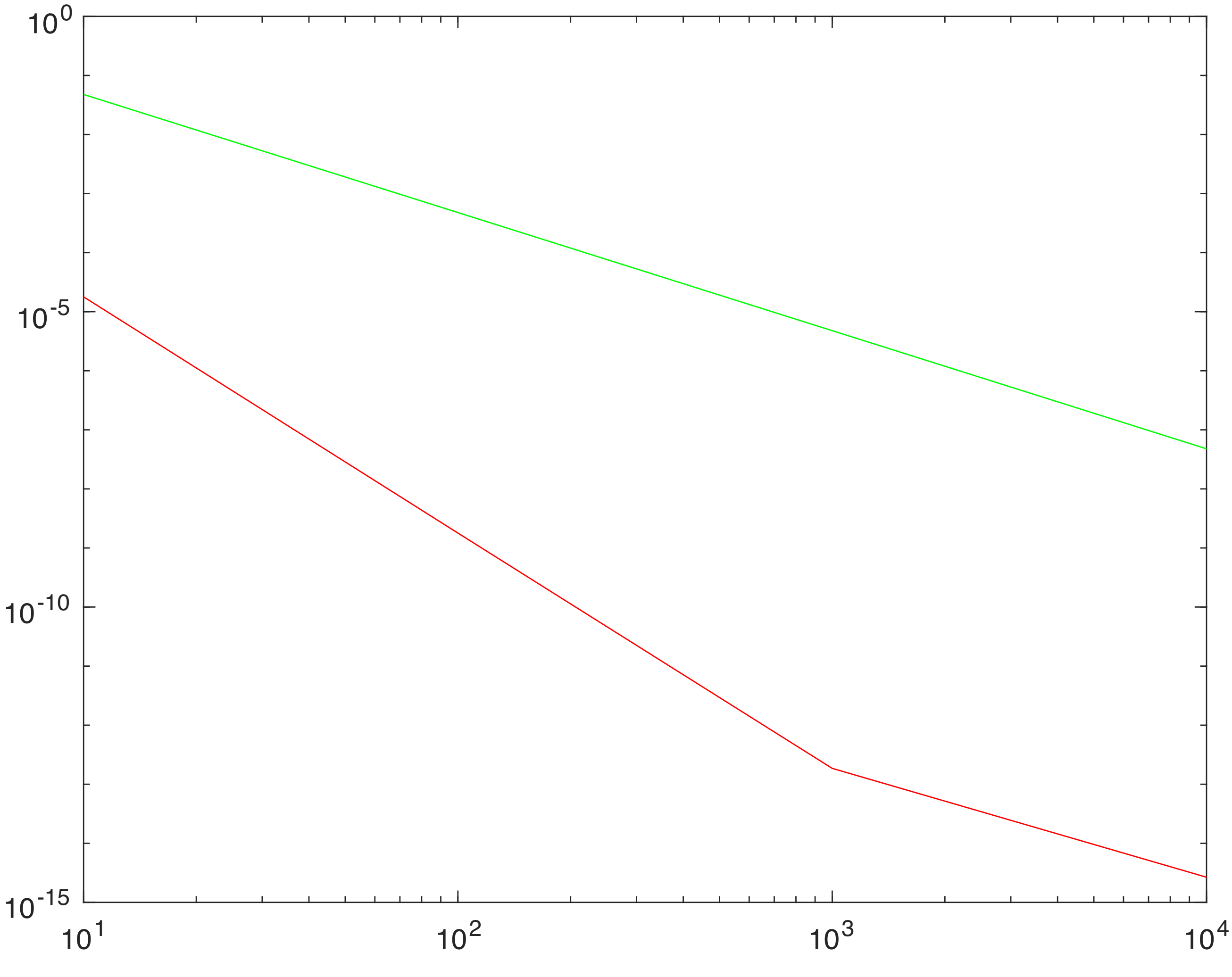
$$|f^{(4)}(\xi)| \le 81e^3$$

$$n^4 \ge \frac{81e^3 \times 10^8}{180}$$

$$n \ge 30064.08 \approx 173.39$$

(2) MATAB

It agrees with (a)



3. $S_0^1 1\,dx = 1 = A + B$

$S_0^1 x\,dx = \frac{1}{2} = \frac{1}{3}A + \frac{3}{4}B \Rightarrow \begin{cases} A = 3/5 \\ B = 2/5. \end{cases}$

$\int_0^1 f(x)\,dx \approx \frac{3}{5} f(\frac{1}{3}) + \frac{2}{5} f(\frac{3}{4})$

converts to $[a, b]$: $\Phi(t) = a + (b-a)t$

$\int_a^b f(x)\,dx = (b-a) \sum_{i=0}^{1} w_i f(\Phi(t_i))$

$= (b-a)[\frac{3}{5} f(\Phi(\frac{1}{3})) + \frac{2}{5} f(\Phi(\frac{3}{4}))]$

$= (b-a)[\frac{3}{5} f(a + (b-a)/3) + \frac{2}{5} f(a + (b-a) \cdot 3/4]$

$\int_0^\pi \sin x\,dx = (\pi - 0)[\frac{3}{5} f(\frac{\pi}{3}) + \frac{2}{5} f(\frac{3}{4}\pi)]$

$= (\frac{3}{5} \cdot \frac{\sqrt{3}}{2} + \frac{2}{5} \cdot \frac{\sqrt{2}}{2}) \cdot \pi = 2.521$

$\int_0^\pi \sin x\,dx = -\cos x \Big|_0^\pi = -\cos(\pi) + \cos(0)$

$= 2$

The exact valu is 0.52 smaller than approx

(4) $\int_1^2 f(x)\,dx = A f(0) + B f(1) + C f(3)$

$f(x) = 1 \quad f(x) = x \quad f(x) = x^2$

$\int_1^2 1\,dx = 1 = A + B + C$

$\int_1^2 x\,dx = \frac{1}{2} x^2 \Big|_1^2 = \frac{3}{2} = A \cdot 0 + B \cdot 1 + C \cdot 3 = B + 3C$

$\int_1^2 x^2\,dx = \frac{1}{3} x^3 \Big|_1^2 = \frac{8}{3} - \frac{1}{3} = \frac{7}{3} = A \cdot 0 + B \cdot 1 + C \cdot 9$

$= B + 9C$

$\Rightarrow A = -2/9 \quad B = 13/12 \quad C = 5/36$

$$\int_1^2 f(x)\,dx = -2/8\, f(0) + 13/12\, f(1) + 5/36\, f(3)$$