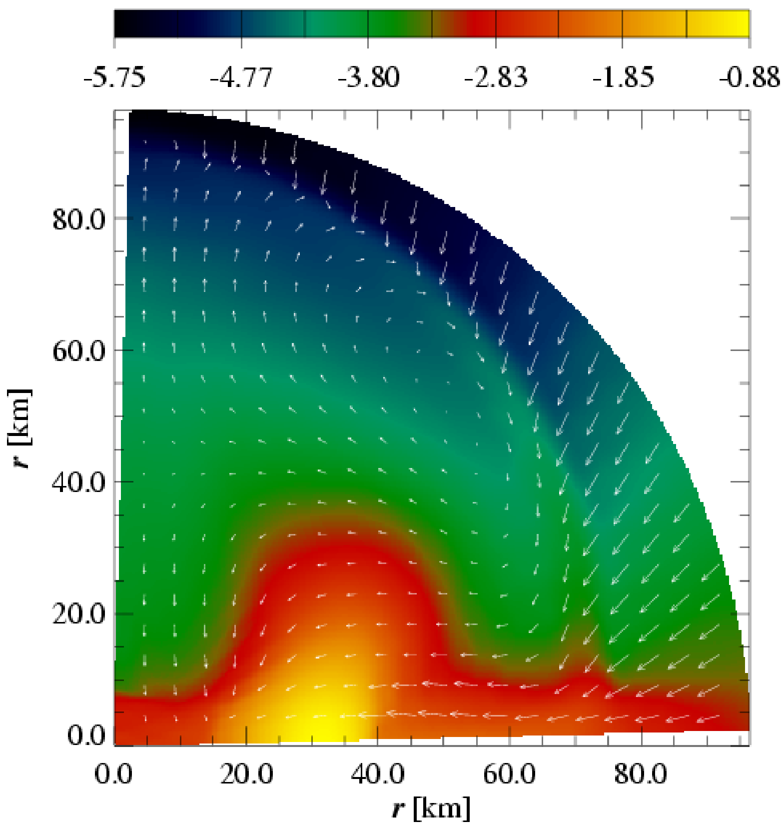

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Gravitational Waves from Supernova Core Collapse:

What could the Signal tell us?

Work done at the MPA in Garching

Dimmelmeier, Font, Müller, *Astron. Astrophys.*, 388, 917–935 (2002), astro-ph/0204288
Dimmelmeier, Font, Müller, *Astron. Astrophys.*, 393, 523–542 (2002), astro-ph/0204289

Physics of Core Collapse Supernovæ

Physical model of core collapse supernova:

- Massive progenitor star ($M_{\text{progenitor}} \approx 10 - 30 M_\odot$) develops an iron core ($M_{\text{core}} \approx 1.5 M_\odot$).
- This approximate 4/3-polytrope becomes unstable and collapses ($T_{\text{collapse}} \approx 100$ ms).
- During collapse, neutrinos are practically trapped and core contracts adiabatically.
- At supernuclear density, hot proto-neutron star forms (EoS of matter stiffens $\Rightarrow$ bounce).
- During bounce, gravitational waves are emitted; they are unimportant for collapse dynamics.
- Hydrodynamic shock propagates from sonic sphere outward, but stalls at $R_{\text{stall}} \approx 300$ km.
- Collapse energy is released by emission of neutrinos ($T_\nu \approx 1$ s).
- Proto-neutron subsequently cools, possibly accretes matter, and shrinks to final neutron star.
- Neutrinos deposit energy behind stalled shock and revive it (delayed explosion mechanism).
- Shock wave propagates through stellar envelope and disrupts rest of star (visible explosion).
- Neutron star might develop triaxial instabilities due to gravitational wave backreaction.

Gravitational Waves from Core Collapse Supernovæ

Difficulties with observing core collapse supernova:

So far we only see optical light emission (light curve) of the explosion (hours after collapse – envelope optically thick).

Gravitational waves are direct means of observation of stellar core collapse!

Challenge: Such burst signals are very complex!
$\Rightarrow$ We need realistic prediction of signal from relativistic numerical simulations!

Our contribution:

Relativistic simulations of rotational core collapse to a neutron star in axisymmetry, and publicly available gravitational wave signal catalogue from a parameter study.

Gravitational Waves from Core Collapse Supernovæ

During the various evolution stages, simulations of core collapse face many challenges:

- Physical complexity:

  Many and complicated aspects of physics involved.

  Some of the physics like supernuclear EoS uncertain.

  Initial rotation state of iron core not well known.

- Numerical difficulties:

  Many different time and length scales (comoving coordinates, FMR, AMR).

  Multidimensional treatment might be crucial
  (convection in proto-neutron star and neutrino heating region, triaxial instabilities, Rayleigh–Taylor instabilities in envelope, rotation, magnetic fields, . . .).

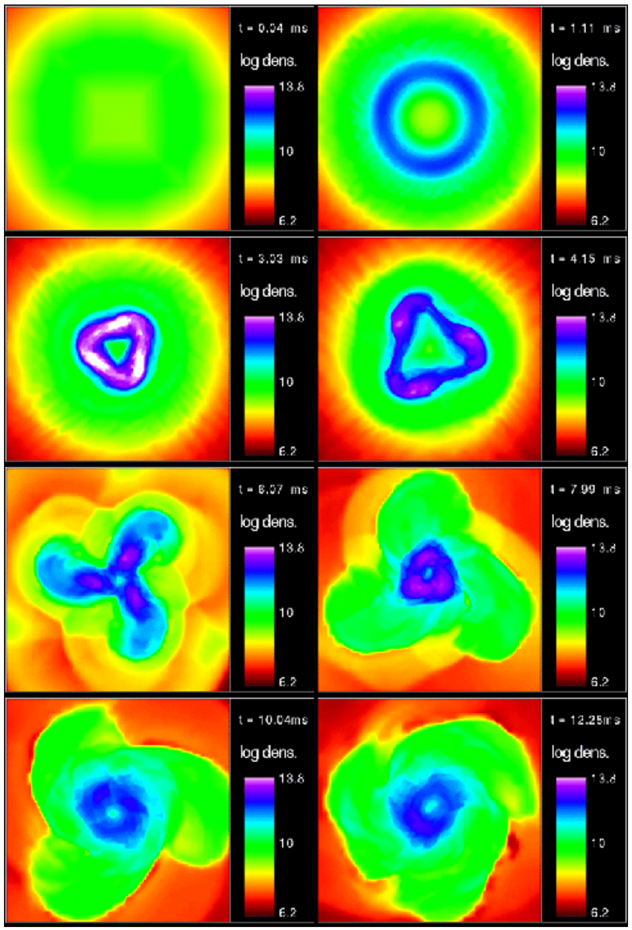
  Solution of Boltzmann transport equations for consistent treatment of neutrinos.

$\Rightarrow$ Numerical simulations are very complicated, many approximations necessary.

$\boxed{\text{But: Measurement of signal waveform will reveal new physics!}}$

Gravitational waves will put constraints on rotation states of iron core and neutron star, supernuclear EoS, degree of convection, . . .

Gravitational Waves from Core Collapse Supernovæ



Example:

Development of triaxial instabilities on dynamic or secular timescales can be an important source of gravitational waves.

Signal amplitude is comparable to core collapse signal.

This process will yield particular waveform structure.
Waveform reveals information about supernuclear EoS.

Such simulations can only be done in 3d codes.

Gravitational Waves from Core Collapse Supernovæ

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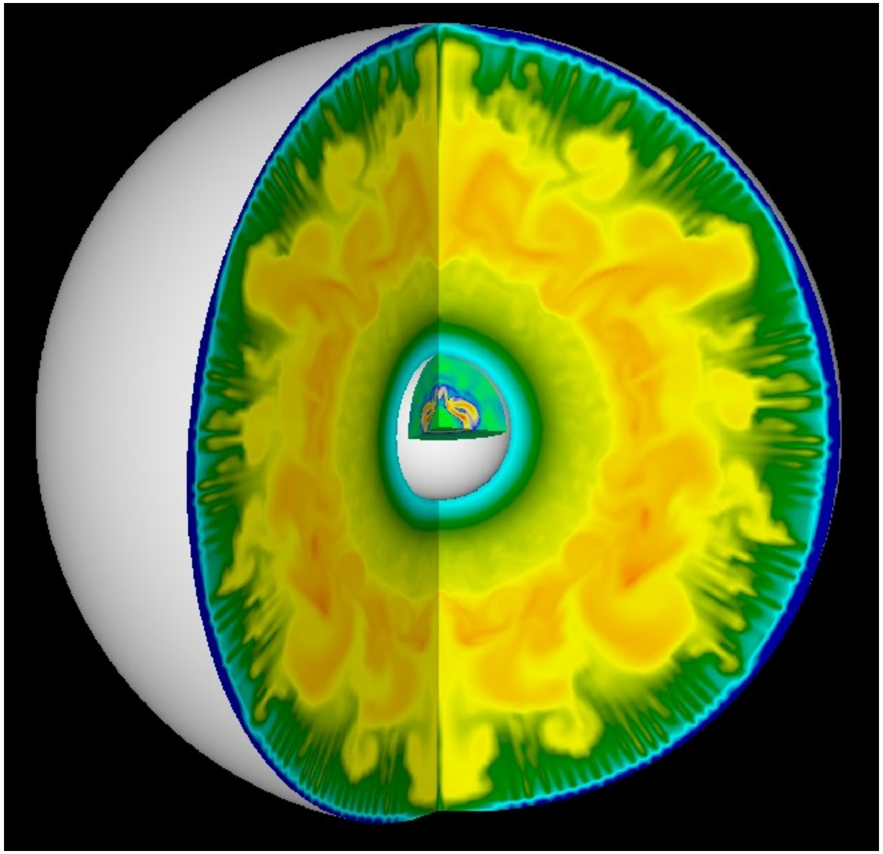
Gravitational Waves from Core Collapse Supernovæ



Another example:

Convection in both

- the proto-neutron star,
- the remainder of the iron core, and
- the stellar envelope

can have important consequences on dynamics of supernova explosion.

In inner, dense regions, convection can produce considerable amount of gravitational radiation.

Now concentrate on gravitational waves from core bounce.

Model Assumptions

To reduce complexity of problem, we have assumed

- axisymmetry and equatorial symmetry,
- rotating $\gamma = 4/3$ polytropes in equilibrium as initial models,
  with $\rho_{\mathrm{c\,ini}} = 10^{10}\ \mathrm{gm\ cm^{-3}}$, $R_{\mathrm{core}} \approx 1\,500\ \mathrm{km}$, and various rotation profiles and rates,
- simplified ideal fluid equation of state, $P(\rho, \epsilon) = P_{\mathrm{poly}} + P_{\mathrm{th}}$ (neglect complicated microphysics),
- constrained system of Einstein equations (assume conformal flatness for 3-metric).

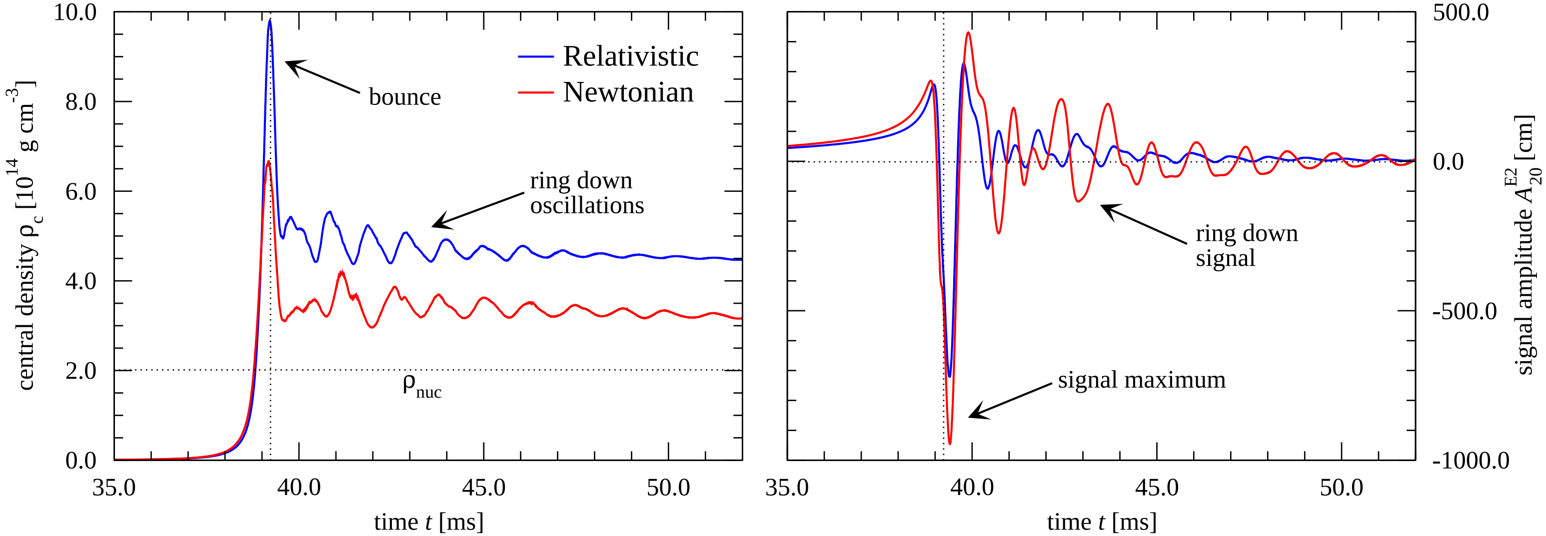
Goals

We have built an axisymmetric GR hydro code and performed parameter study of 26 models to

- extend research on Newtonian rotational core collapse by Zwerger and Müller to GR,
- obtain more realistic waveforms as "wave templates" for interferometer data analysis,
  (wave templates are important and actually being used in data analysis:
  VIRGO data analysis group has used Zwerger's catalogue (Pradier et al., 2000),
  and already uses our results (Chassande-Mottin, 2002)),
- have 2d GR hydro code for comparison with future simulations and as basis for extension.

Regular Collapse

Model A: Slow, almost uniform rotation, fast collapse ($\approx$ 40 ms), soft supernuclear EoS.



- Deep dive into potential, high supernuclear densities, single bounce, subsequent ring down.
- GR simulation: Higher central density and signal frequency, but *lower* signal amplitude.

  Explanation: GW signal is determined by accelation of *extended* mass distribution:

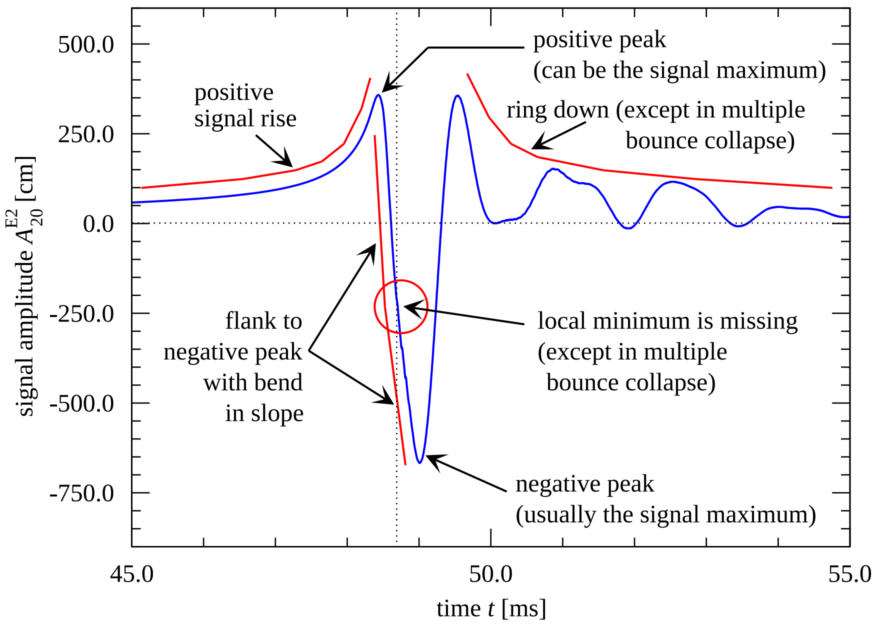
$$A^{E2}_{20} = \ddot{Q} \propto \frac{d^2}{dt^2} \int dV \rho \boxed{r^2}. \longleftarrow \text{ weight factor!}$$

In relativistic gravity core is more compact. $\Rightarrow$ Gravitational waves can have smaller amplitude!

Max Planck Institute for Astrophysics, Garching, Germany

Typical Features of Gravitational Wave Signals

Every waveform shares some or all of several more or less clearly identifiable features.



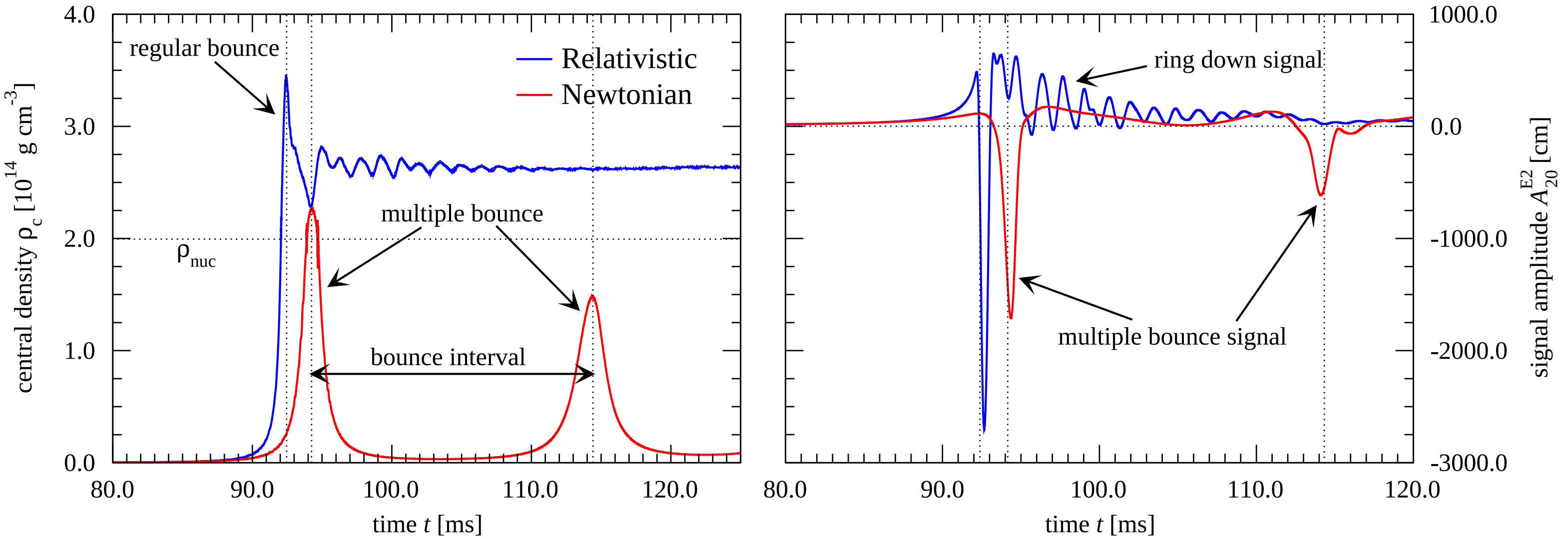
Our waveform catalogue shows dependence of these features on the parameters.

These features can be used to train filters and search algorithms (together with information about signal amplitude and frequency).

Conversely, in detected signal, these features allow for conclusions about physics of core collapse.

Change of Collapse Dynamics

Model B: Slow, almost uniform rotation, slow collapse ($\approx$ 90 ms).

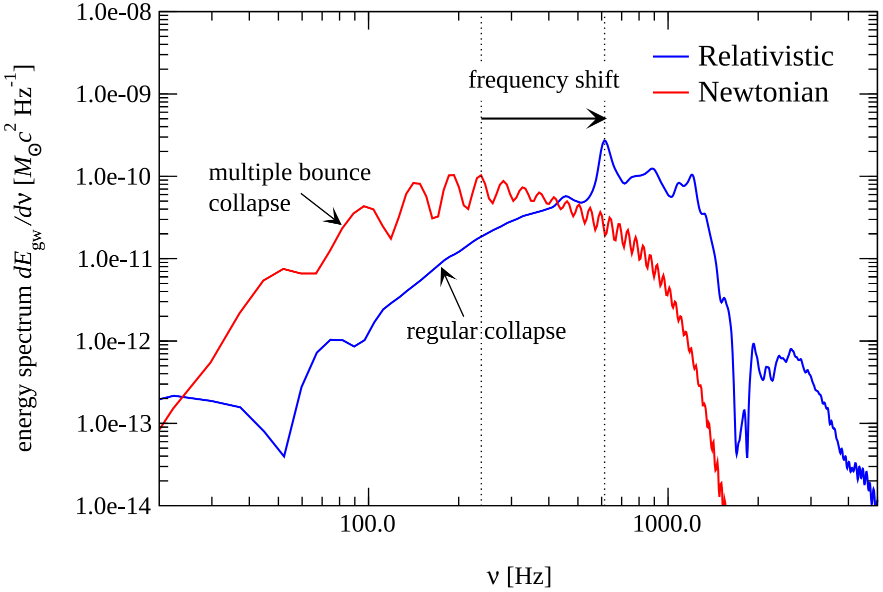


- Rotation increases strongly during collapse (angular momentum conservation!).
- Newtonian: Nuclear density hardly reached, multiple centrifugal bounce with re-expansion.
- GR: Nuclear density easily reached, regular single bounce.
- Relativistic simulations show multiple bounces only for few extreme models.

Strong *qualitative* difference in collapse dynamics and thus in signal form.

Change of Collapse Dynamics

A change in collapse dynamics is clearly visible in energy spectrum.



- **Multiple bounce collapse has broad round spectrum which peaks at relatively low frequency.**
- **Regular collapse has steeper spectrum with pronounced peak at higher frequency.**

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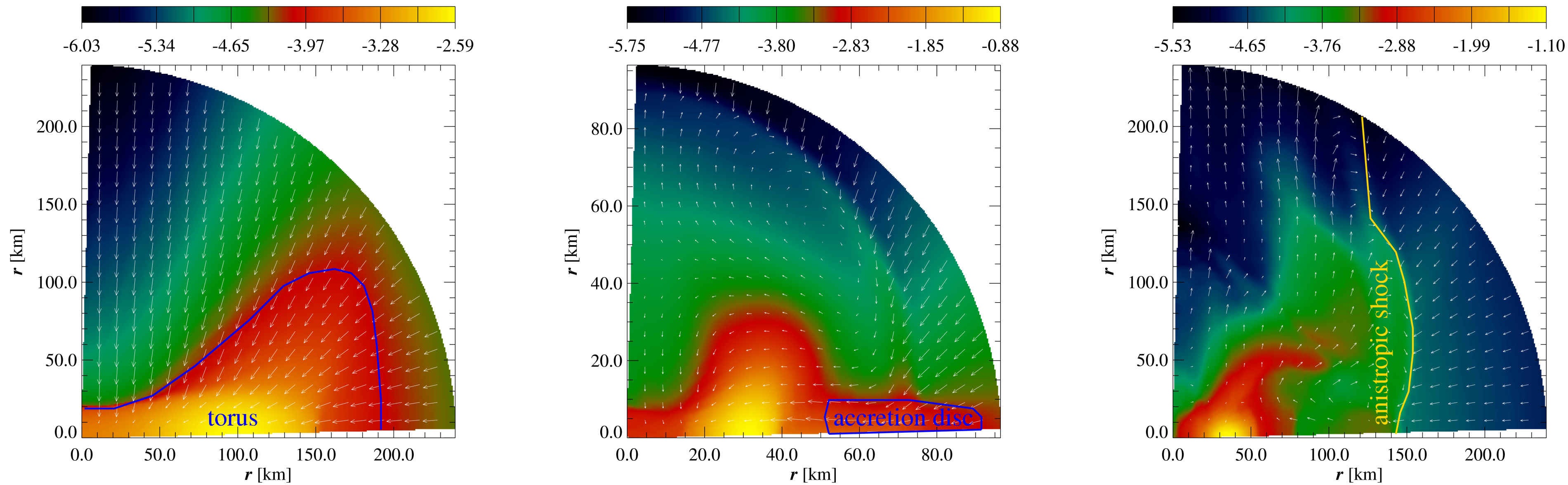
Rapidly Rotating Models

Model C: Fast and extremely differential rotation, rapid collapse (≈ 30 ms).

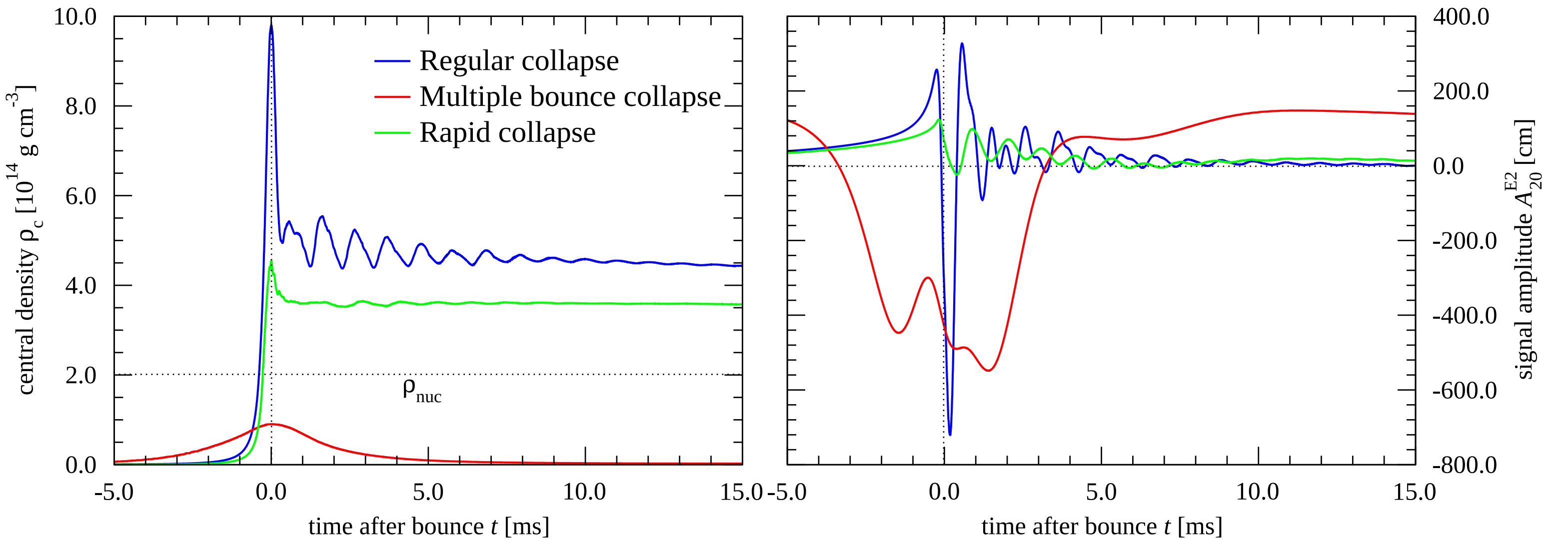


- Initial model has toroidal density shape; torus becomes more pronounced during contraction.
- Proto-neutron star is surrounded by disc-like structure, which is accreted.
- After bounce, strongly anisotropic shock front forms.
- Bar instabilities are likely to develop on dynamical timescale (particularly in GR and with differential rotation (cf. Shibata et al., 2002)); they will produce characteristic signal!

Gravitational Wave Signals

Just as in Newtonian gravity, in our relativistic simulations we observe

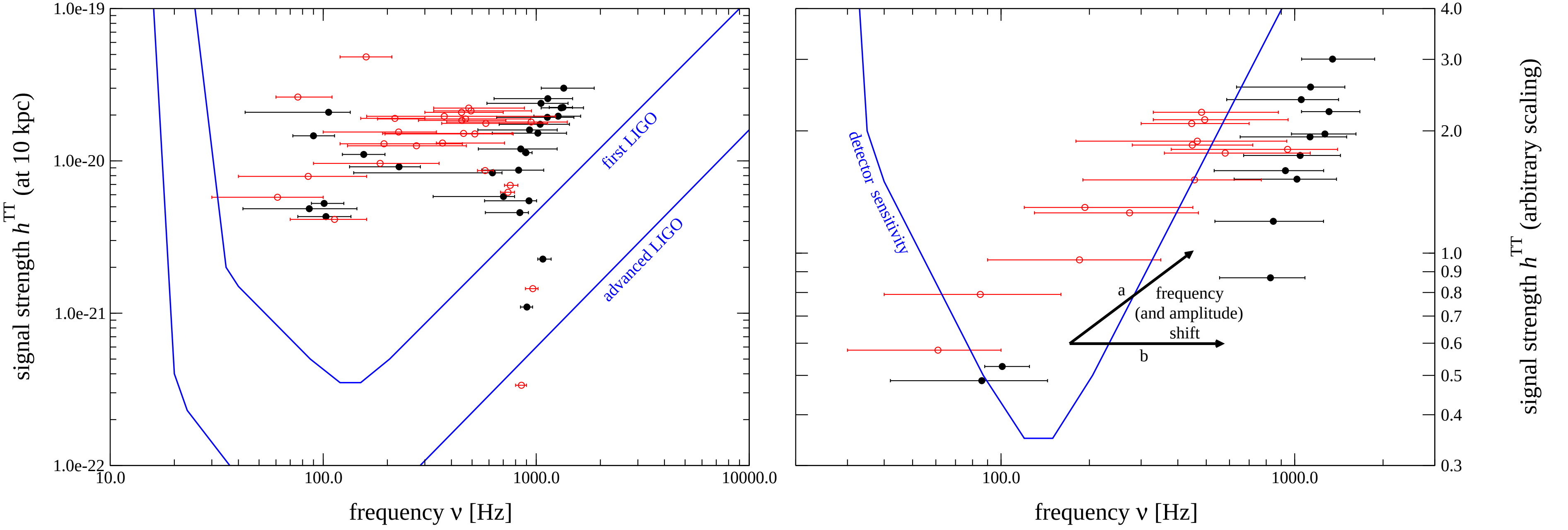
- three normal collapse types,
  regular collapse (signal shows one large negative peak, and clear ring down),
  multiple bounce collapse (signal shows distinct multiple large negative peaks, and no ring down),
  rapid collapse (signal shows one small positive maximum peak, and low-amplitude ring down),
- a separate class of rapidly and differentially rotating models (which form torus).



These collapse types can be identified both in density evolution and in signal waveform.

Gravitational Wave Signals

Influence of relativistic effects on signals: Investigate amplitude–frequency diagram.



- **Spread of the 26 models does not change much. ⇒ Signal of galactic supernova detectable.**
- **On average: Amplitude remains at $h^{\mathrm{TT}} \approx 10^{-23} \cdot 10\ \mathrm{Mpc}/R$, frequency increases to $\nu \approx 1000$ Hz.**

If close to detection threshold: Signal could leave sensitivity window in relativistic gravity!

Future Directions

Further developments of our axisymmetric CFC code:

- Include more accurate approximation of spacetime metric (CFC Plus) (get closer to formation of rotating black hole).
- Extend code to 3d, and investigate dynamic development of triaxial instabilities.
- Add more realistic microphysics and check robustness of wave signal (simple extension of supernuclear EoS, tabulated EoS (Lattimer–Swesty), neutrino effects).
- Use better extraction method for gravitational waves (currently quadrupole formula is used).

Ongoing or planned other projects:

- Assess quality of CFC approximation by comparison to fully relativistic simulations (Cartoon method used by Shibata – Problems with wave extraction!).
- Use AEI Cactus Code to simulate axisymmetric and fully 3-dimensional core collapse.
- Make AMR work with multidimensional hydrodynamic core collapse codes.
- Extract gravitational waves from existing multidimensional Newtonian simulations.
- Add (full or approximate) relativistic gravity to existing Newtonian codes.
- Extend existing spherical Newtonian codes with sophisticated microphysics to multidimensions.

Summary

We have obtained the first gravitational wave templates from simulations of rotational supernova core collapse in *general relativity.*

Our simulations show:

- We can identify same three collapse types as in Newtonian simulations.
- Remnants are more compact with higher central densities compared to Newtonian gravity.
- Many previous multiple bounce models collapse to supernuclear densities in relativity.
- On average, signal amplitude does not change
(we still find $h^{\mathrm{TT}} \approx 10^{-23} \cdot 10\ \mathrm{Mpc}/R$ for axisymmetric supernova core collapse).
- However, frequency increases to $\nu \approx 1000$ Hz
(particularly due to suppression of multiple bounce collapse).
- Relativistic effects increase rotation rate; many models could develop triaxial instabilities.

Our wave templates replace Zwerger catalogue; we have made them publicly available:
`http://www.mpa-garching.mpg.de/Hydro/RGRAV/`.

This area of research is currently very active and exciting.
The emerging interaction between numerical relativity and hydrodynamics, and data analysis is stimulating on both sides.