Dark-matter admixed white dwarfs

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We study the equilibrium structures of white dwarfs with dark matter cores formed by non-self-annihilating dark matter (DM) particles with mass ranging from 1 GeV to 100 GeV, which are assumed to form an ideal degenerate Fermi gas inside the stars. For DM particles of mass 10 GeV and 100 GeV, we find that stable stellar models exist only if the mass of the DM core inside the star is less than $O(10^{-3})$ $M_\odot$ and $O(10^{-6})$ $M_\odot$, respectively. The global properties of these stars, and in particular the corresponding Chandrasekhar mass limits, are essentially the same as those of traditional white dwarf models without DM. Nevertheless, in the 10 GeV case, the gravitational attraction of the DM core is strong enough to squeeze the normal matter in the core region to densities above neutron drip, far above those in traditional white dwarfs. For DM with particle mass 1 GeV, the DM core inside the star can be as massive as $\sim 0.1 M_\odot$ and affects the global structure of the star significantly. In this case, the radius of a stellar model with DM can be about two times smaller than that of a traditional white dwarf. Furthermore, the Chandrasekhar mass limit can also be decreased by as much as 40%. Our results may have implications on to what extent type Ia supernovae can be regarded as standard candles - a key assumption in the discovery of dark energy.

I. INTRODUCTION

It has been widely accepted that more than 80% of matter in the universe is dark matter (DM), most of which believed to be non-baryonic, possibly weakly interacting massive particles (WIMP). Compelling evidences for DM include observations of the flatness of galactic rotation curves, measurements of the Cosmic microwave background and baryon-acoustic oscillations (see, e.g., [1–3] for reviews). Nevertheless, the properties of DM particles, including their spin, mass and interactions, are still largely unknown.

The effects of various DM candidates on stellar evolution and structure have been discussed in the literature. It is hoped that constraints on the properties of DM particles may be obtained from observations of stellar objects such as the sun or compact stars. Such studies can be divided into two classes: non-self-annihilating DM and self-annihilating DM.

Self-annihilating DM affects a star by supplying energy through their annihilations into photons. The role of DM in the first generation stars as stellar seeds, together with the possibility of DM annihilation as the energy source in the first phase of stellar evolution are discussed in [4–8]. The DM annihilation energy might provide sufficient pressure to delay the star from entering the main-sequence stage. Also, the main-sequence lifetime can be extended [9]. It was pointed out in [10, 11] that the DM self-annihilation energy may broaden the path in the H-R diagram. By considering DM self-annihilation in the sun, authors of [12, 13] suggest that solar neutrino flux and helioseismology data can be used to constrain DM particle properties. In compact stars, DM annihilation becomes the only heat-generation mechanism, and their cooling curves or luminosity may be altered [14–20], which may in turn provide information on the DM properties, scattering cross-sections, as well as local DM distributions.

Non-self-annihilating DM affects a star through its gravity by accumulating in the stellar core, or through its cooling of stellar materials by scattering with ordinary particles. In [21, 22] it is suggested that the cooling by DM scattering may increase the minimum mass for hydrogen-burning stars and a longer main-sequence lifetime. It has also been suggested [23] that the solar composition problem may be solved by admixing non-self-annihilating DM. The changes in the orbits of stellar objects due to the increase of stellar mass from accreted DM are studied in [24–26]. Convection zone profile may also be changed, bringing impact on the helium flash [27]. Similar studies are applied to compact stars. The first study is pioneered in 1989 by Goldman and Nussinov [28], which sets a limit on DM particle mass and scattering cross-section by considering very old compact stars. The idea of using compact stars to probe the DM particle properties is further elaborated in [14, 17, 20, 29]. Limits on the mass and scattering cross-sections for different classes of DM, such as bosonic and fermionic DM with different scattering channels, are derived from compact star observations [30–33]. In [34–36], models of non-self-annihilating DM, including asymmetric dark matter and mirror matter, are examined. It is found that a more compact neutron star is resulted when a DM core is included. The gravity from the accumulated DM might even be strong enough to trigger a phase transition from nuclear matter to quark matter inside the star and produce a gamma-ray burst [37].

In our previous work [38, 39], we considered non-self-annihilating DM particles of mass ~ 1 GeV to study the equilibrium structure and radial oscillations of DM

admixed neutron stars using a general relativistic two-fluid formulation. In particular, we found a new class of compact stars which consists of a small normal matter (NM) core with radius of a few kilometers embedded in a ten-kilometer sized DM halo. Here NM refers to ordinary particles in the Standard Model.

In this paper, we extend our investigation by employing the two-fluid formulation to study white dwarfs (WD) with DM cores. We shall in general refer to these stellar models as hybrid white dwarfs (HWD) in the following. The impact of annihilating DM on the cooling of traditional WD is well studied [14, 16, 18–20, 29, 40]. However, the study of non-self-annihilating DM on the structure of WD is a relatively unexplored area. A recent study using bosonic condensate DM can be found in [41]. If the mass and/or radius of traditional WD near the Chandrasekhar limit are altered significantly due to the presence of DM cores, the initial conditions of Type Ia supernova explosions would be affected, making it doubtful whether they can still be regarded as standard candles - a key assumption in the discovery of dark energy.

In this work, we shall study the equilibrium structure of HWD by assuming that the DM particles are non-self-annihilating fermionic particles with particle mass m_{DM} ranging from 1 to 100 GeV. The mass range is chosen based on two reasons. First, the most popular DM candidate, WIMP, is expected to have a mass of the order 100 GeV. *Second, data from the DAMA, CoGeNT, and CRESST experiments [42–44] are consistent with detecting light DM particles with a few GeV, though the results are in conflict with the null results reported by CDMS and XENON [45, 46]. More recently, the CDMS-II collaboration has also reported signals that are consistent with DM particles with mass ~ 9 GeV [47].*

The major difference between our current work and our previous work [38, 39] lies in the NM equation of state (EOS). The pressure inside a HWD is due mainly to a degenerate electron gas instead of nuclear matter. Apart from the EOS, the typical length scales of NM and DM in a HWD differ by many orders of magnitude. As we shall see below, for DM particles with mass 100 GeV, the typical size of a DM core can be smaller than the radius of the whole star by a factor of 10^6. The outline of the paper is as follows: In Sec. II, we briefly outline the formulation for constructing HWD. In Sec. III, we study the structure of HWD for different DM particle masses in detail. Sec. IV summarizes our results and discusses possible future investigation. Finally, Appendix A discusses briefly the radial oscillation modes of HWD. Appendix B discusses how our proposed HWD might be formed. We use units where $G = c = 1$ unless otherwise noted.

II. FORMULATION

In our previous work [38, 39], we study the structure and oscillations of DM admixed neutron stars using a general relativistic two-fluid formalism. Here we adopt the formulation to study HWD with DM cores. The essential structure equations and numerical technique for constructing a two-fluid star can be found in [39] (see also [48] for the full derivations). Here we only outline the essential equations.

For a static and spherically symmetric spacetime $ds^2 = -e^{\nu(r)}dt^2 + e^{\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$, the structure equations for a two-fluid compact star are given by [48]

$$A_0^0 p' + B_0^0 n' + \frac{1}{2}(Bn + Ap)\nu' = 0,$$

$$C_0^0 p' + A_0^0 n' + \frac{1}{2}(An + Cp)\nu' = 0,$$

$$\lambda' = \frac{1 - e^\lambda}{r} - 8\pi r e^\lambda \Lambda,$$

$$\nu' = -\frac{1 - e^\lambda}{r} + 8\pi r e^\lambda \Psi, \qquad (1)$$

where n and p are the number densities of NM and DM, respectively. The primes refer to derivatives with respect to r, and the coefficients A, B, C, A_0^0, B_0^0, and C_0^0 are functions of the master function Λ, which is the negative of the thermodynamics energy density. The generalized pressure Ψ is calculated from the master function Λ. We refer the reader to [39, 48] for the explicit expressions.

In the two-fluid formalism, the master function Λ plays the role of the EOS information needed in the structure calculation. In this work, we assume that DM couples with NM only through gravity. Hence, the master function is separable in the sense that

$$\Lambda(n, p) = \Lambda_{\mathrm{NM}}(n) + \Lambda_{\mathrm{DM}}(p), \qquad (2)$$

$\Lambda_{\mathrm{NM}}(n)$ and $\Lambda_{\mathrm{DM}}(p)$ being the negative of energy densities of NM and DM, respectively.

To model the NM, we choose the APR EOS [49] to describe the high-density nuclear matter. As we shall see in Sec. III, we need to model matter in the nuclear density range ($\sim 10^{14}$ g cm^{-3}) because the NM in the core of a HWD can indeed reach this density range. At lower densities we use the SLY4 [50] and BPS EOS [51]. On the other hand, we consider degenerate ideal Fermi gas EOS for DM. We shall consider DM in the mass range from 1 GeV to 100 GeV.

III. RESULTS

In this section, we study the stellar properties of HWD with different DM core masses M_{DM} and particle mass m_{DM}. Before presenting our results in detail, let us first give a brief summary of our finding.

Since the DM core is described by an ideal Fermi gas, and it is well known that the maximum stable mass of a self-gravitating Fermi gas depends on the particle mass m_{DM}, thus the maximum amount of DM that can exist inside a stable HWD is determined by m_{DM}. As we shall see below, a stable HWD can only have a tiny DM

core ($M_{DM} \sim 10^{-6} M_\odot$) if the core is composed of massive DM particles with $m_{DM} = 100$ GeV. If the mass of the DM core increases beyond the maximum stable limit, the whole star would collapse promptly to a black hole. On the other hand, for low-mass DM particle $m_{DM} = 1$ GeV, the DM core can reach the level $M_{DM} \sim 0.1 M_\odot$ and affect the structure of the star significantly. The relation between m_{DM} and the maximum DM core mass can be obtained from the energy argument of Landau (see, e.g. [52]), from which we have the scaling relation $M_{DM(max)} \sim m_{DM}^{-2}$.

For $m_{DM} > 100$ GeV, the DM core becomes so small in spatial size that it is difficult to be resolved. However, extrapolating our results to $m_{DM} > 100$ GeV, we expect almost no change to the mass-radius relation.

A. Chandrasekhar mass limit and Moon-sized HWD

Here we study the mass-radius relation of HWD composed of DM with different particle mass m_{DM} ranging from 1 GeV to 100 GeV. In Fig. 1, we plot the mass-radius relations of HWD with DM core formed by $m_{DM} = 100$ GeV DM particles. Results for three different DM core mass M_{DM} are plotted together with the case without DM. For these massive DM particles, we find that HWD models cannot be constructed with $M_{DM} \gtrsim 5 \times 10^{-6} M_\odot$, the value of which is set by the maximum mass limit of a self-gravitating degenerate Fermi gas. It should be noted that the mass of the NM fluid inside the DM core also decreases the stability of the DM core. The total masses of these HWD models are dominated by the NM fluid, and hence the DM cores have negligible effects on the stellar structures as shown in Fig. 1.

Decreasing the DM particle mass can increase the maximum stable mass limit of the degenerate DM cores inside HWD. One may thus expect to see a significant difference between HWD and traditional WD models in the low m_{DM} regime.

In Fig. 2, we show the mass-radius relation of HWD for $m_{DM} = 10$ GeV. We see that the maximum stable mass of the DM core increases to about $M_{DM} = 2 \times 10^{-3} M_\odot$, beyond which no HWD model can be constructed. Although the DM core can now be more massive than that in the case of $m_{DM} = 100$ GeV, it is still not large enough to change the mass-radius relation significantly. In particular, the DM core has little effect on the Chandrasekhar mass limit of WD. Hence, for non-self-annihilating massive DM particles, we conclude that stable HWD can only have tiny DM cores ($M_{DM} << M$). As a result, the global properties of these stellar models are very similar to traditional WD.

Next we study the effects of low-mass DM with $m_{DM} = 1$ GeV. We plot the corresponding mass-radius relation in Fig. 3. The maximum stable mass limit of the DM core is $M_{DM} = 0.075 M_\odot$ in this case. In con-
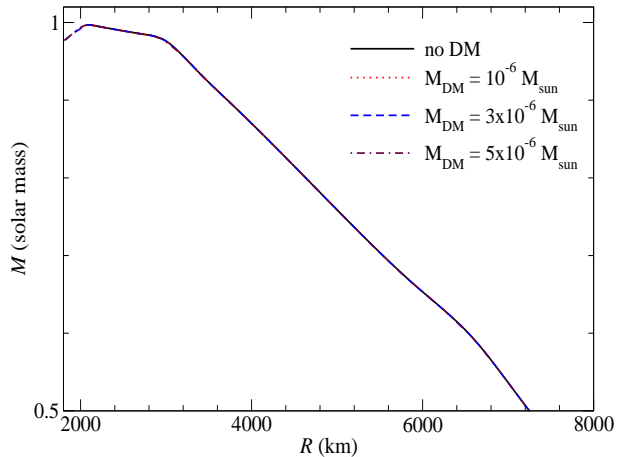

FIG. 1: Mass-radius relations of HWD for different DM core mass M_{DM}. The DM particle mass is $m_{DM} = 100$ GeV. We choose M_{DM} up to the value $5 \times 10^{-6} M_\odot$, beyond which no stable DM core can exist.
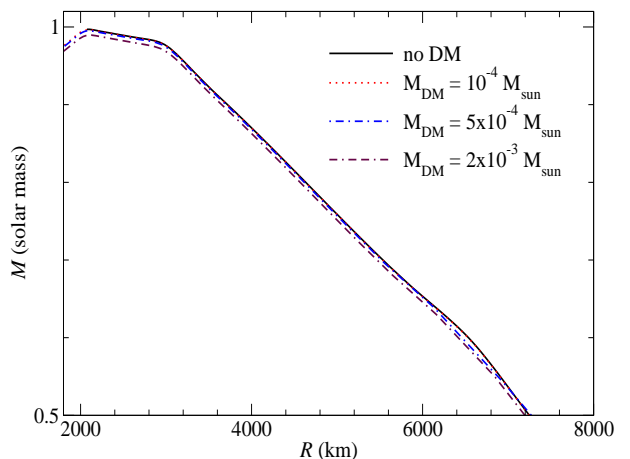

FIG. 2: Same as Fig. 1, but for $m_{DM} = 10$ GeV.

trast to the previous cases $m_{DM} = 10$ and 100 GeV, we now see that the mass-radius relation of HWD depends sensitively on M_{DM}. In particular, the Chandrasekhar mass limit decreases from about $1 M_\odot$ to $0.6 M_\odot$ as M_{DM} increases from 0 to $0.075 M_\odot$. However, the radius of the stellar model corresponding to the Chandrasekhar mass limit does not depend sensitively on M_{DM}. It should be noted that while both fluids contribute to the gravitational potential, the pressure of each fluid does not support the other one. Therefore, extra pressure gradient is needed for one fluid to balance the extra gravitational force exerted by the other fluid. However, because the DM core is much more compact than the NM, a slight increase in M_{DM} results in a large gravitational attraction in the core and induces the collapse of the Chandrasekhar-mass model. In order to achieve a new stable equilibrium model, the HWD should have a much lower M_{NM}. Note that the total mass of a HWD is dominated by the NM, thus the Chandrasekhar
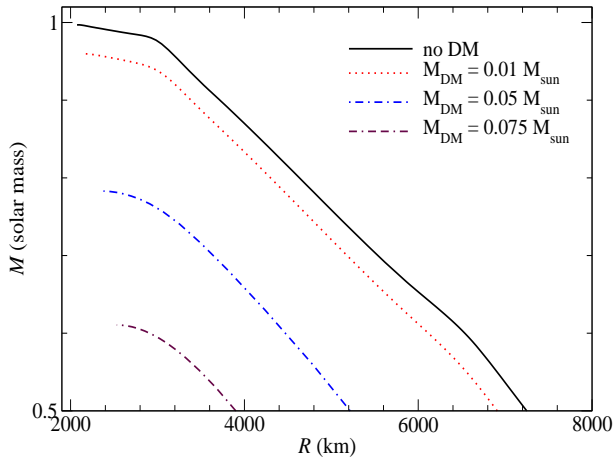
FIG. 3: Same as Fig. 1, but for $m_{DM} = 1$ GeV.

FIG. 4: Total mass M is plotted against the NM central density ρ_c for $m_{DM} = 10$ GeV. The region between the two vertical dashed lines is the "forbidden" region in which no stable traditional white dwarf can exist. The arrow in the figure indicates the neutron drip density.

mass drops significantly with a slight increment in $M_{\rm DM}$. On the contrary, the radius of the Chandrasekhar-mass model displays a mild change. Along the sequence of Chandrasekhar-mass models, the increase in $M_{\rm DM}$ leads to a smaller radius because of a stronger gravitational attraction. On the other hand, the decrease in $M_{\rm NM}$ makes a HWD larger. As these factors partially cancel each other, the radius of the Chandrasekhar-mass model is thus insensitive to $M_{\rm DM}$.

One may also notice that, for a given total mass M, the radius of the star decreases significantly as the amount of DM increases. Fig. 3 shows that the radius of a 0.6 $M_\odot$ traditional WD without DM is about 6600 km. Due to the strong gravity of the DM core, a HWD with the same total mass and $M_{DM} = 0.075 M_\odot$ has a radius of 2800 km only. For comparison, the radius of the moon is about 1700 km, whereas the largest satellite in the solar system, Ganymede, has a radius of about 2600 km.

B. HWD with central densities above neutron drip

Since DM and NM are assumed to couple with each other only through gravity in our calculations, the NM is supported by its own pressure against gravitational attraction, which is sourced by both fluids. Therefore, comparing to traditional WD models without DM, the presence of a DM core in a HWD can provide extra gravitational attraction to squeeze the NM in the core region to a higher density.

It is known that the global structure of a traditional WD is determined by the EOS below neutron drip ($\rho_{\rm drip} \approx 4 \times 10^{11}$ g cm^{-3}), while that of a neutron star is determined mainly by the EOS near or above nuclear density ($\rho_{\rm nuc} \approx 2.8 \times 10^{14}$ g cm^{-3}). In the domain between $\rho_{\rm drip}$ and $\rho_{\rm nuc}$, the nuclei become more neutron rich and the electron gas degeneracy pressure drops significantly as the density increases. As a result, no stable traditional WD can have central densities in this range.
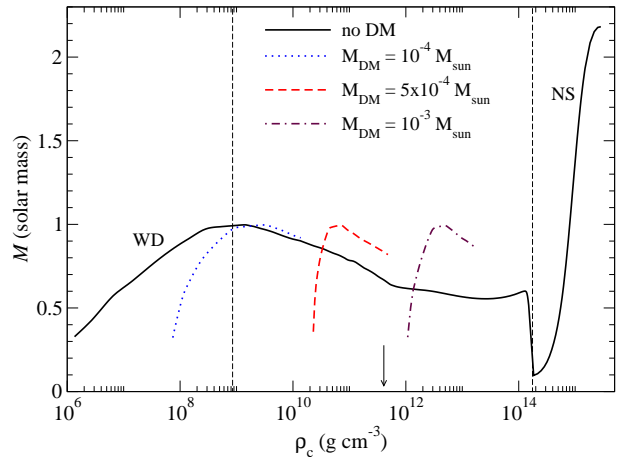
Here we show that, in the presence of a DM core, it is indeed possible for the NM inside stable HWD to have a central density in this domain.

In Fig. 4 we plot the total mass M against the NM central density ρ_c for $m_{DM} = 10$ GeV. Three sequences of HWD with different DM core masses M_{DM} are shown. For comparison, the case without DM (solid line) is also plotted. The vertical line at a lower density divides the stable (labeled as WD) and unstable branches of traditional WD models, while the other vertical line at a higher density marks the onset of the stable branch of neutron stars (labeled as NS). The region between the two vertical dashed lines is thus the "forbidden" region in which no stable traditional WD can exist for our chosen NM EOS model. It is seen clearly from Fig. 4 that stable HWD can exist in this traditional forbidden region. For a given M, the NM central density of a HWD increases by a few orders of magnitude as M_{DM} increases from $10^{-4} M_\odot$ to $10^{-3} M_\odot$. In particular, the HWD models with $M_{DM} = 10^{-3} M_\odot$ have central densities above neutron drip. Note, however, that the maximum stable mass of HWD is insensitive to M_{DM} as we have shown in Fig. 2. To further illustrate how the NM central density changes with M_{DM}, we plot in Fig. 5 ρ_c against M_{DM} for three different sequences of fixed total mass $M = 0.6$, 0.8 and 0.95 $M_\odot$. We see that the values of ρ_c for the three sequences get closer as M_{DM} increases. They can even reach the range of nuclear-matter density ($\sim 10^{14}$ g cm^{-3}) when $M_{DM} = 2 \times 10^{-3} M_\odot$, beyond which no stable HWD can be constructed as we have discussed above.

In Fig. 6, we plot the density profiles for three different stellar models with the same total mass $M = 0.83 M_\odot$ and $m_{\rm DM} = 10$ GeV. The upper panel is the density profile for a traditional WD without DM. The middle panel shows the NM (solid line) and DM (dashed line) den-
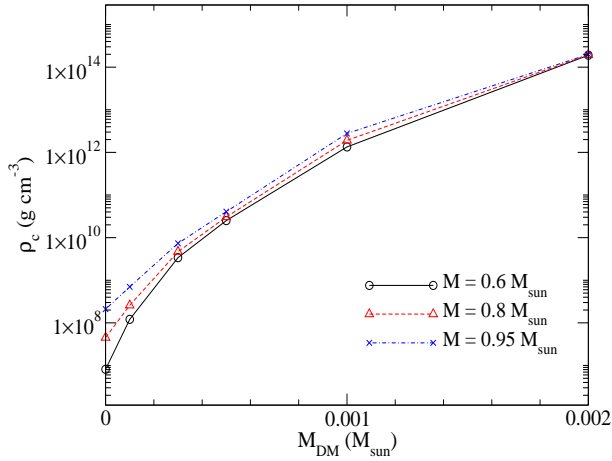
FIG. 5: NM central density ρ_c is plotted against DM core mass M_{DM} for three sequences of HWD with fixed total mass M. The DM particle mass is fixed at $m_{DM} = 10$ GeV.
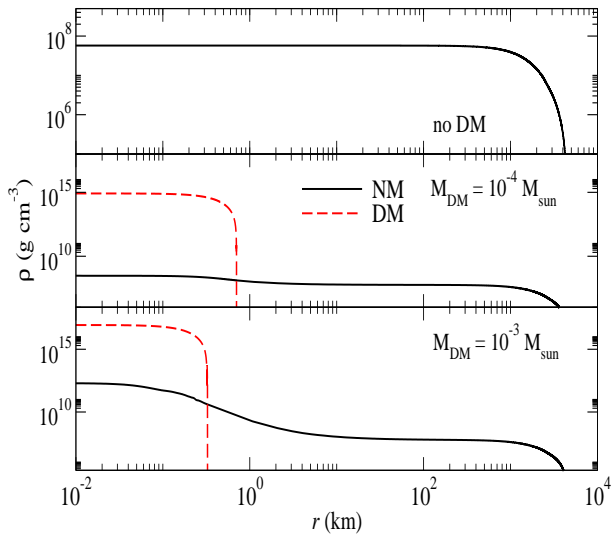

FIG. 6: Density profiles for three stellar models with the same total mass $M = 0.83 M_\odot$. The upper panel shows the profile for a traditional WD without DM. The middle and lower panels show the profiles of two HWD models with different DM core mass M_{DM}. The DM particle mass is fixed at $m_{DM} = 10$ GeV. The solid and dashed lines represent the profiles of NM and DM, respectively.

sity profiles for a HWD with $M_{DM} = 10^{-4} M_\odot$, while the bottom panel shows the profiles for a HWD with $M_{DM} = 10^{-3} M_\odot$. Fig. 6 shows that the presence of a DM core affects the distribution of NM near the core significantly. As M_{DM} increases and the DM core becomes more compact, the NM in the core region with a size ~ 1 km can be squeezed to density above neutron drip. However, slightly outside the DM core, the NM density drops quickly to the level commonly found in the core of a traditional WD. It should be noted that despite the difference in the density profiles in the core regions of the HWD and traditional WD models in Fig. 6, their global
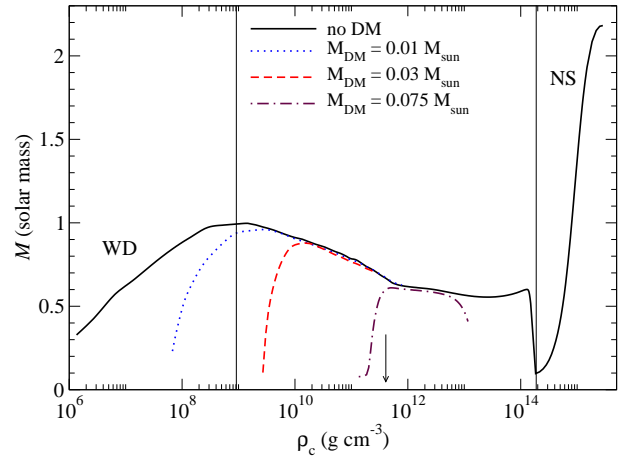

FIG. 7: Same as Fig. 4, but for $m_{DM} = 1$ GeV.

properties such as their masses and radii, are essentially the same. Our results thus suggest that HWD models with a tiny DM core are consistent with many observed WD candidates. However, it also means that it would be challenging to distinguish these HWD and traditional WD models observationally.

Finally, we plot M against ρ_c for the case $m_{DM} = 1$ GeV in Fig. 7 for comparison. Similar to the case $m_{DM} = 10$ GeV, we see that the stable branch (with $dM/d\rho_c > 0$) of HWD migrates to the traditional forbidden region as M_{DM} increases. The central density can reach above neutron drip for the case $M_{DM} = 0.075 M_\odot$. In contrast to the previous case with $m_{DM} = 10$ GeV, the maximum stable mass of HWD now depends more sensitively on M_{DM}. In particular, it is interesting to note that the curves for HWD models are all bound above by the traditional-WD curve. We have checked the stability of HWD by analyzing the radial oscillation modes of these stars as we have done previously for DM admixed neutron stars [39]. In particular, we show in Appendix A that the DM oscillation modes found in [39] also exist in HWD.

IV. DISCUSSION

We have used a general relativistic two-fluid formulation to study the equilibrium structure of HWD with DM cores, with the DM particle mass m_{DM} and the DM core mass M_{DM} as parameters. The DM particles are assumed to be non-self-annihilating and form an ideal degenerate Fermi gas.

For massive DM particles with $m_{DM} = 100$ GeV, we find that stable HWD can only sustain a tiny DM core with $M_{DM} \sim 10^{-6} M_\odot$. The masses of these HWD are dominated by the NM fluid, and hence the global structures of these stars are essentially the same as traditional WD without DM. If the DM core mass increases beyond $\sim 10^{-6} M_\odot$, the HWD would become unstable and col-

lapse promptly to a black hole. For less massive DM particles with $m_{DM} = 10$ GeV, the DM core inside a HWD can be as large as $M_{DM} \sim 10^{-3} M_{\odot}$, but still it is not massive enough to affect the global structures, such as the mass and radius, of the star significantly. Nevertheless, the gravitational attraction of the kilometer-sized DM core can now squeeze the NM in the core region to density above neutron drip. In some cases, the NM central density of the star can even reach the range of nuclear-matter density. The properties of NM in the inner cores of these HWD are very different from those of traditional WD which have typical central densities below neutron drip. However, from the observational point of view, these HWD and traditional WD models could be indistinguishable because their global properties are essentially the same.

In our view, the more interesting result in our study is obtained from the case of low-mass DM particles with $m_{DM} \sim 1$ GeV. The DM core of a HWD in this case can reach the level $M_{DM} \sim 0.1 M_{\odot}$ and affect the structure of the star significantly. In particular, the Chandrasekhar mass limit of these HWD depends sensitively on M_{DM} and can decrease by 40% as we increase M_{DM} from 0 to $0.075 M_{\odot}$. Moreover, the radii of these HWD can be as small as ~ 3000 km. Comparing to HWD formed by massive DM particles, these moon-sized HWD could be more easily distinguished from traditional WD, which have typical radii ranging from about 5000 km to 10000 km. Note that while our results presented here are based on one particular NM EOS model, we have in fact tried different EOS models and found that our results still hold qualitatively.

Our study focuses only on non-self-annihilating DM. In general, other DM particle models may be considered, such as self-annihilating DM. Also, DM accretion could be included. However, we remark that even if we consider DM accretion, from the analysis in [15], a compact star cannot accumulate DM in the mass range of the DM core described here in cosmological timescale, except when the HWD is embedded in a region of ultra-high DM density. For the non-self-annihilating DM we considered in this paper, one possible way for a WD to acquire a relatively massive DM core is that the DM is already trapped inside the star during its early stage of proto-star formation. In Appendix B, we provide an order-of-magnitude analysis on how much DM can be trapped in that scenario.

In recent years, with the advancement in WD observation using double-lined eclipsing binaries, the mass and radius of the WD in WD-main-sequence binary systems can now be accurately measured [53, 54]. The properties of a number of WD have been precisely measured [55–58]. It is thus not inconceivable that moon-sized HWD (if exist) could be detected in the near future. The detection of such kind of small HWD will be a hint that DM particles are low-mass ($m_{DM} \sim 1$ GeV) and non-self-annihilating. However, we cannot exclude the possibility that compact WDs are formed by other mechanisms, such as with the help of a strange matter core [59–61].

Finally, let us remark that as the Chandrasekhar mass limit of HWD formed by 1-GeV scale DM particles depends sensitively on M_{DM}, the initial conditions of Type Ia supernovae might not be as universal as generally assumed, making it doubtful to what extent Type Ia supernovae can be regarded as standard candles - a key assumption in the discovery of dark energy. It will be interesting to extend our work to investigate how the presence of a DM core would affect the results of a Type Ia supernova such as its luminosity, light curve, and nucleosynthesis yields etc. This will be our future investigation.

Appendix A: Radial oscillation Modes of HWD

In [39] we studied the radial oscillation modes of DM admixed neutron stars and found a new class of modes which are characterized mainly by the oscillations of DM fluid. We have also employed the formulation of [39] to compute the oscillation modes of HWD. Similar to the study of DM admixed neutron stars, the oscillation modes of HWD can be divided into two classes, namely the NM fluid modes and the DM fluid modes. The former class of modes is driven mainly by NM and depends weakly on the properties of DM fluid such as the DM core mass M_{DM}. These modes reduce properly to the fluid modes of a traditional WD model, with the same total mass, as M_{DM} tends to zero. On the other hand, the second class of modes depend sensitively on the DM fluid.

To illustrate the difference between the two classes of modes, we plot in Fig. 8 the mode frequency squared ω^2 as a function of the central DM density ρ_{DM}. The total mass of the HWD is fixed at $M = 0.6 M_{\odot}$ and the DM particle mass is $m_{DM} = 1$ GeV. In the lower panel of Fig. 8, we show the first three NM modes (solid lines labeled by $n = 1$, 2 and 3) and notice that their frequencies are of the order of Hz. In the upper panel, we plot the fundamental DM fluid (dashed line) and the $n = 30$ and $n = 50$ NM modes (solid lines) for comparison. It is noted that the frequency of the fundamental DM mode is much higher than that of the NM mode ($n = 1$) because the DM core is much more compact than the whole star. It can also be seen that the DM mode depends much more sensitively on ρ_{DM} as we found previously in the case of DM admixed neutron stars [39].

Appendix B: Possible Formation Mechanisms of HWD

We notice that there is yet no related study on this issue. Here, we make an order-of-magnitude analysis on possible formation mechanisms of HWD. A stellar object may acquire DM by accretion [15, 62–64]. Alternatively, DM could be trapped in the star formation stage. We now estimate the amount of trapped DM particles in a
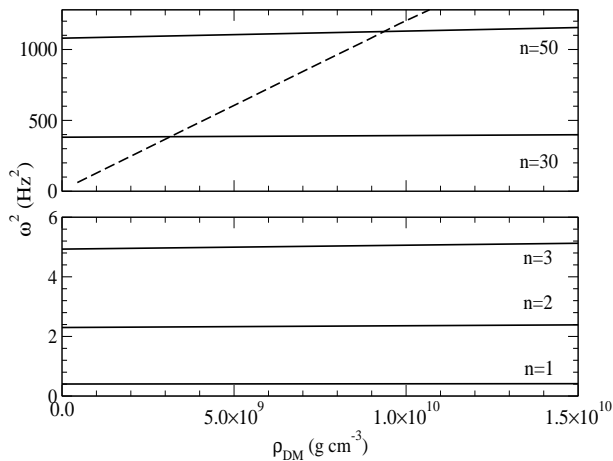
FIG. 8: The frequency squared ω^2 for different oscillation modes are plotted against the central DM density ρ_{DM}. The first three NM modes are plotted in the lower panel. In the upper panel, the fundamental DM mode (dashed line) and the $n = 30$ and $n = 50$ NM modes (solid lines) are plotted. The total mass of the star is fixed at $M = 0.6 M_\odot$ and the DM particle mass $m_{DM} = 1$ GeV.

collapsing molecular cloud of mass M_{NM0}, with density ρ_{NM}, temperature T and radius R. We assume that both the densities of NM and DM are constant, and the effects of rotation and magnetic field are neglected. Within the same volume, there is also a mass of DM given by $M_{DM0} = \frac{4}{3}\pi R^3 \rho_{DM}$. The total energy of this system is given by

$$E_{\text{tot}} \sim -\frac{2}{5}\frac{G(M_{\text{NM0}} + M_{\text{DM0}})^2}{R} + \frac{3}{2}\frac{M_{\text{NM0}}}{m_{\text{H}}}kT. \quad \text{(B1)}$$

Here, m_{H} is the atomic mass of a hydrogen atom. Note that we include only the internal energy of NM but not of DM.

Solving Eq. (B1) by requiring $E_{\text{tot}} = 0$, we obtain the Jean's radius and then the Jean's mass for both NM and DM. *The progenitor of a compact HWD with the mentioned M_{NM} and M_{DM} is a protostar of mass $\sim 1-10 M_\odot$, with trapped DM of mass $\sim 10^{-2} M_\odot$, and a mass ratio $M_{\text{DM0}}/M_{\text{NM0}} \geq 10^{-2}$. This condition can be satisfied for $\rho_{\text{NM}} \sim 100~GeV/cm^{-3}$ and $\rho_{\text{DM}} > 1~GeV/cm^{-3}$, which can exist around halo center according to observational and N-body simulation results [65, 66].*

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