4.5 Euler’s Equations

2 Project Work

2.1 Project requirements

Program 1 implemented 4 Stage Runge-Kutta methods to solve Euler's equations (5) numerically. To test the program, we chose $A = 1.5,\ B = 1, C = 0.5$ and $\omega_1(0) = \omega_2(0) = \omega_3(0) = 1$.

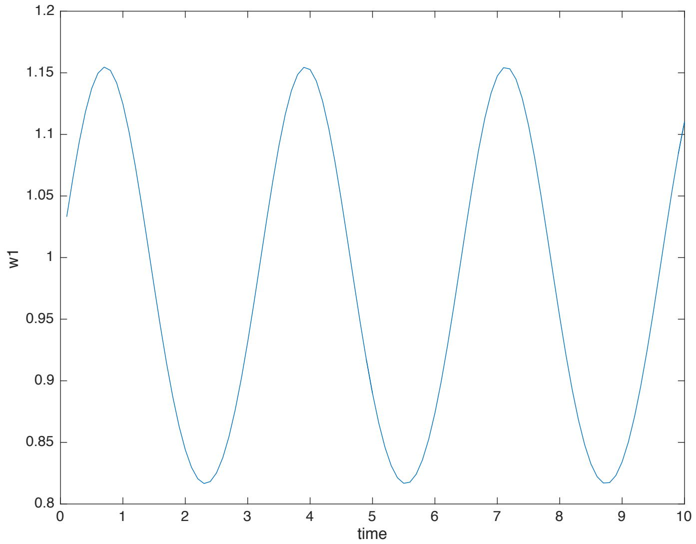


Figure 1: Plot Of $\omega_1(t)$ against with t

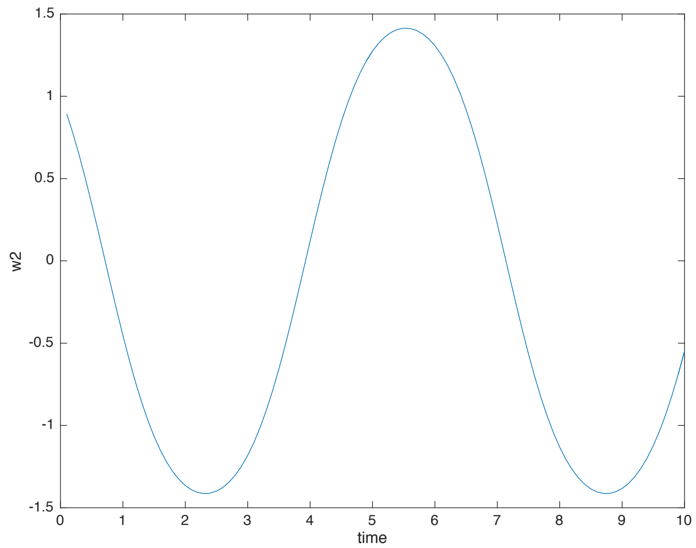


Figure 2: Plot Of $\omega_2(t)$against with t

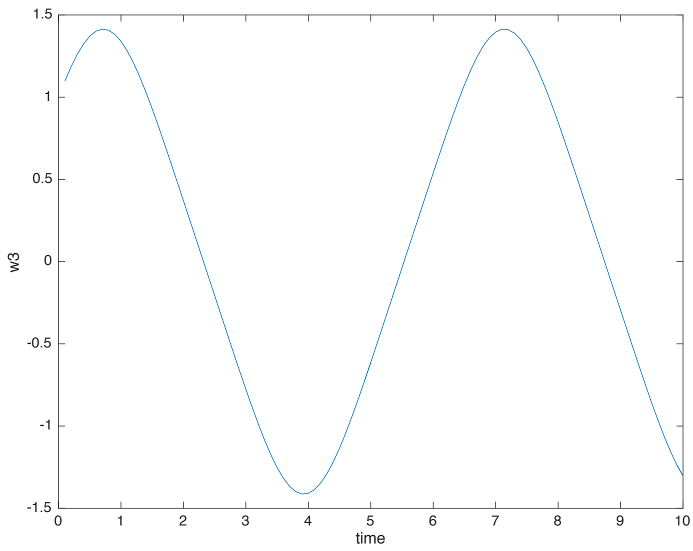


Figure 3: Plot Of $\omega_3(t)$ against with t

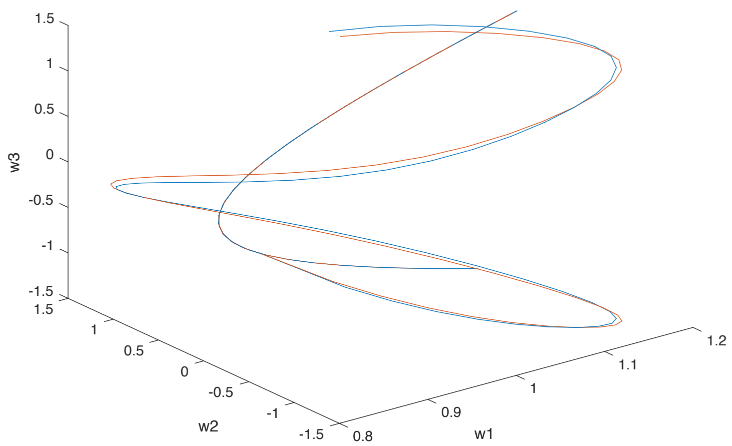


Figure 4: 3-D phase plots Of $\omega_1, \omega_2, \omega_3$

Equations (6) and (7) are used to check the accuracy of the numerical results. Table 1 displays the value of E and H^2 at the beginning and end of runs.

Table 1: Display of E and H^2

	E	H^2
Beginning of runs	1.5000	3.5000
End of the runs	1.5000	3.5000

Question 1 Let A, B and $C = a_1, a_2$ and a_3 respectively. Moreover, $a_i = \max(a_1, a_2, a_3)$, $a_k = \min(a_1, a_2, a_3)$ and a_j be the middle constant, i.e.

$$a_i < a_j < a_k$$

Now, we define $\omega_1'(0) = \omega_i(0)$, $\omega_2'(0) = \omega_j(0)$, and $\omega_3'(0) = \omega_k(0)$. Hence, without loss of generality, we have

$$a_i \frac{d\omega_1'}{dt} + (a_k - a_j)\omega'_2\omega'_3 = 0$$
$$a_j \frac{d\omega_2'}{dt} + (a_i - a_k)\omega'_3\omega'_1 = 0$$
$$a_k \frac{d\omega_3'}{dt} + (a_j - a_i)\omega'_1\omega'_2 = 0$$

Hence constants $A, B\ and\ C$ can always be taken in a way that $A > B > C$, without loss of generality.

In case $A = B \neq C$, Euler equation becomes

$$A\frac{d\omega_1}{dt} + (A - C)\omega_2\omega_3 = 0$$
$$A\frac{d\omega_2}{dt} + (A - C)\omega_1\omega_3 = 0$$
$$C\frac{d\omega_3}{dt} = 0$$

This can be solved analytically. We have

$$\omega_1 = \omega_1(0)\cos(kt)$$
$$\omega_2 = \omega_2(0)\cos(kt)$$
$$\omega_3 = \omega_3(0)$$

Where $k = \omega_3(0)(1 - \frac{C}{A})$. Hence, in the case, the body processes about the OZ axis with frequency k. Furthermore, if $A > C$, the direction of procession is clockwise, and if $A < C$, the direction of procession is anticlockwise.

In case $A = B = C$, $\omega_1, \omega_2\ and\ \omega_3$ are all constants. Hence the rigid body continue to spin with the same axis.

<u>Explanation of $B = 1$</u> Let $\frac{A}{B} = a\ and\ \frac{C}{B} = b$, then Euler's equations become

$$\frac{d\omega_1}{dt} = (\frac{b}{a} - \frac{1}{a})\omega_2\omega_3$$
$$\frac{d\omega_2}{dt} = (b - a)\omega_1\omega_3$$
$$\frac{d\omega_3}{dt} = \left(\frac{a}{b} - \frac{1}{b}\right)\omega_1\omega_2$$

Hence without loss of generality, we can chose take $B = 1$ and $A = a, C = b$.

<u>The scaling factor</u> Let $t' = Et$, where $E = \frac{A}{2}\ \omega_1^2(0) + \frac{B}{2}\omega_2^2(0) + \frac{C}{2}\omega_3^2(0) = E$

Consider

$$\int A\frac{d\omega_1}{dt}\omega_1 + B\frac{d\omega_2}{dt}\omega_2 + C\frac{d\omega_3}{dt}\omega_3 dt = E\int A\frac{d\omega'_1}{dt'}\omega_1 + B\frac{d\omega'_2}{dt'}\omega_2 + C\frac{d\omega'_2}{dt'}\omega_3 dt$$
$$E = \frac{A}{2}\ \omega_1^2 + \frac{B}{2}\omega_2^2 + \frac{C}{2}\omega_3^2 = E\frac{A}{2}\ \omega_1'^2 + \frac{B}{2}\omega'^2_2 + \frac{C}{2}\omega_3'^2$$
$$\frac{A}{2}\ \omega_1'^2 + \frac{B}{2}\omega'^2_2 + \frac{C}{2}\omega_3'^2 = 1$$

Hence with the re-scaling, $t' = Et$ is equivalent to choosing $E = 1$.

2.2 Results requirements

Question 2 Let us set the initial conditions as $\omega_1(0) = \sqrt{\frac{2}{A}}\ \omega_2(0) = 0.1\ and\ \omega_3(0) = 0.1$, and run the program. Figure 5 is a 3-D phase space plot of of ω_1, ω_2 and ω_3, which choose the OX axis for ω_1, without loss of generality. Thus, we obtain a stable solution that $\boldsymbol{\omega}(t)$ is rotating around the OX axis with small amplitude deviation from (1,0,0) before scaling.

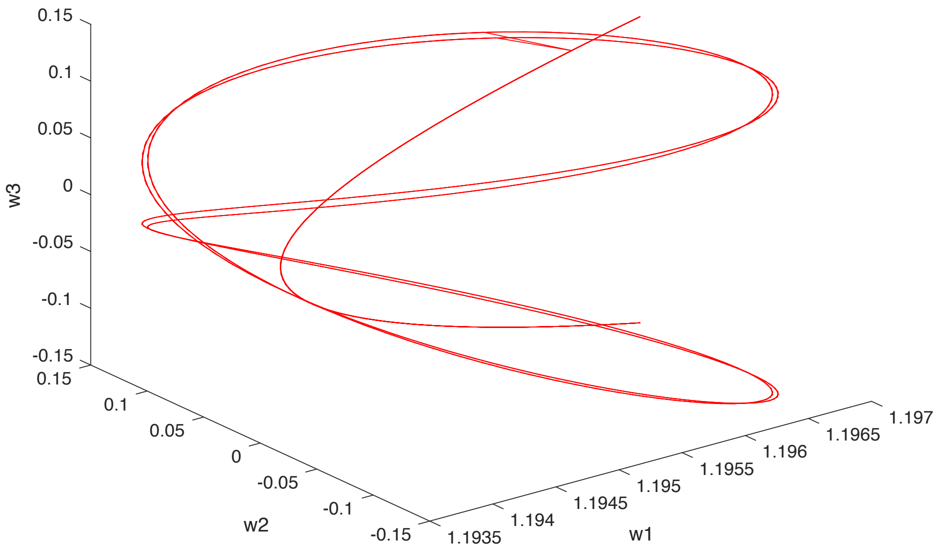


Figure 5: 3-D plots of ω_1, ω_2 and ω_3

Similarly, a small amplitude deviation from $(0,0,\sqrt{\frac{2}{C}})$ gives us stable solutions near the OZ axis. (see Figure 6)

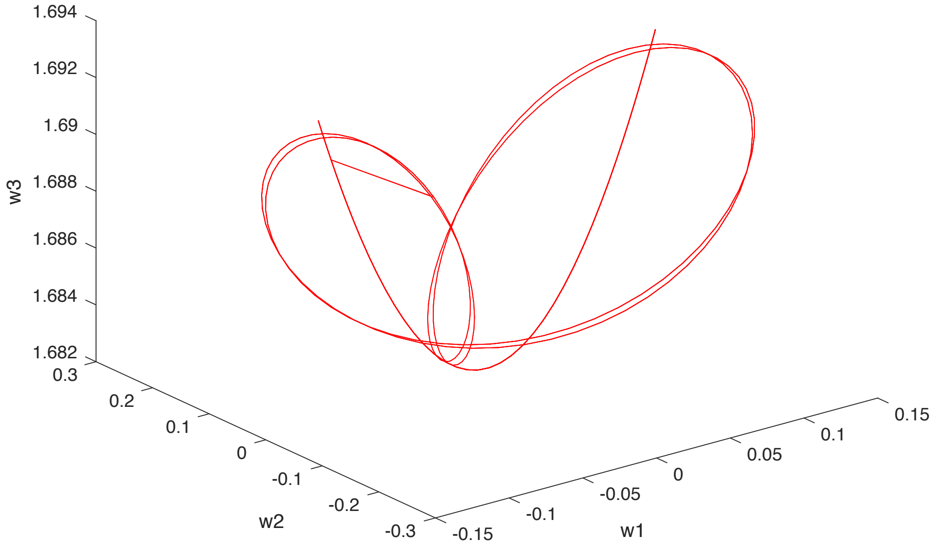


Figure 6

Question 3

(i) In case $\omega_1 \approx \sqrt{\frac{2}{A}}$, the other two Euler's equations become

$$\frac{d\omega_2}{dt} + \sqrt{\frac{2}{A}}(A - C)\omega_3 = 0 \qquad (1)$$

$$\frac{d\omega_3}{dt} - \sqrt{\frac{2}{A}}\frac{1-A}{C}\omega_2 = 0 \ \ (2)$$

By differentiating (1) and substitute it into (2), we can get a 2nd order differential equation,

$$\ddot{\omega}_3 + \frac{2(1-A)(C-A)}{AC}\omega_3 = 0$$

Let $f = \sqrt{\frac{2(A-1)(A-C)}{AC}}$, and we have

$$\omega_2 = \omega_2(0)cos(ft)$$

The period T satisfies $T = \frac{2\pi}{f}$, hence the analytic expression for the period is

$$T = \pi\sqrt{\frac{2AC}{(A-1)(A-C)}}$$

Figure 7 is a plot of ω_2 *and* ω_3 against t, with conditions $\omega_1 \approx \sqrt{\left(\frac{2}{A}\right)}$ *and* $\omega_2(0) = \omega_3(0) = 0.01$. We can deduce that the period is around 8.

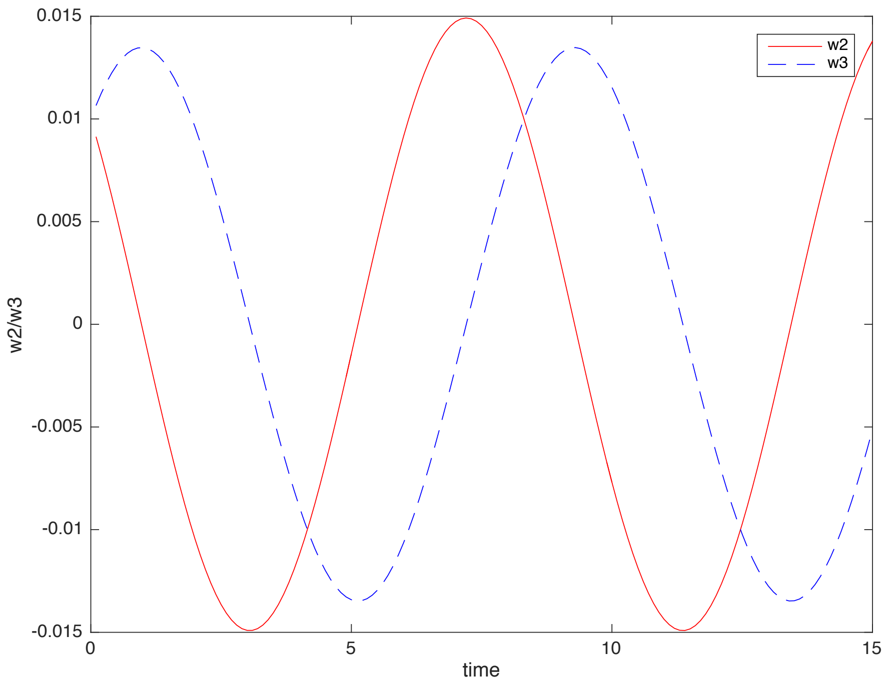


Figure 7: Plots of ω_2 *and* ω_3 against t

(ii) Similarly, in case $\omega_3(t) \approx \sqrt{2/C}$, we have

$$f = \sqrt{\frac{2(1-C)(A-C)}{AC}}$$

and the period satisfies

$$T = \pi\sqrt{\frac{2AC}{(1-C)(A-C)}}$$

From Figure 8, we can deduce that the period is also around 10.

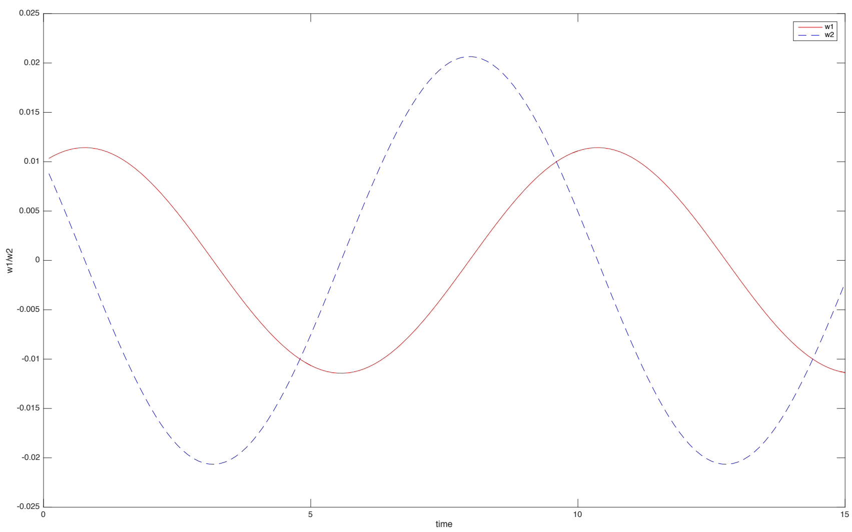


Figure 8: Plots of ω_1 *and* ω_2 against t

In comparison of (i) and (ii), the case $\omega_3(t) \approx \sqrt{2/C}$ has slightly longer period.

<u>Consistency with Question 2</u>

Question 4 In both case $\boldsymbol{\omega}(t) = \left(\sqrt{\frac{2}{A}}, 0,0\right)$ *and* $\boldsymbol{\omega}(t) = \left(0,0, \sqrt{\frac{2}{C}}\right)$, the numerical results are consistent with equation (6) and (7), as shown in Table 2 and 3

Table 2

$\boldsymbol{\omega}(t) = (\sqrt{2/A}, 0,0)$	E	H^2
Beginning of runs	1.0001	2.8001
End of the runs	1.0001	2.8001

Table 3

$\boldsymbol{\omega}(t) = (0,0, \sqrt{2/C})$	E	H^2
Beginning of runs	1.0001	1.4003
End of the runs	1.0001	1.4003

Question 5 When $\boldsymbol{\omega}(0)$ is very close to OY axis, we can set $\omega_1(0) = \omega_3(0) = 0.01, and\ \omega_2 = 1$ to investigate the solutions. Figure 9 shows 3D plots of $\omega_1, \omega_2\ and\ \omega_3$ against *t* in the same graph.

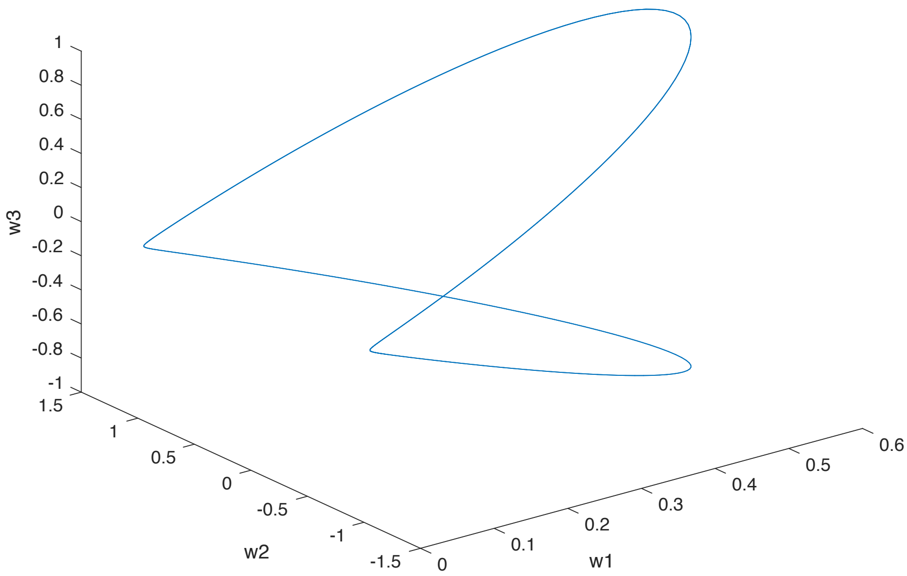


Figure 9: the 3D plot

Physical description The motion is stable and the largest amplitude oscillation is around ω_2.

Question 6

Inspired from Question 5, let us set $\omega_2(0) = 0$ and $\omega_1(0) = 1$ (w.o.l.g) and vary $\omega_3(0)$ to seek for such solution. From the computed results, we can discover that as $\omega_3(0) \to 1.632993162 \ldots (\sqrt{\frac{8}{3}})$; $\omega_1, \omega_3 \to 0$ and $|\omega_2| \to 1.8$. See Figure 10 and 11 for few iterations.

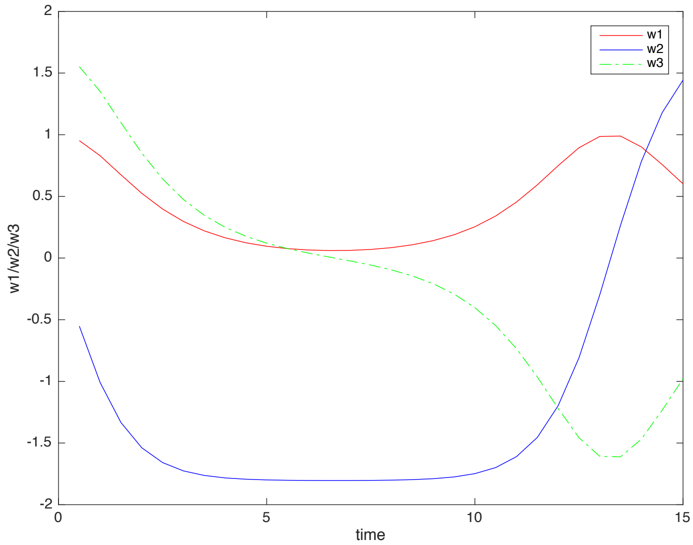


Figure 10: case $\omega_3(0) = 1.63$

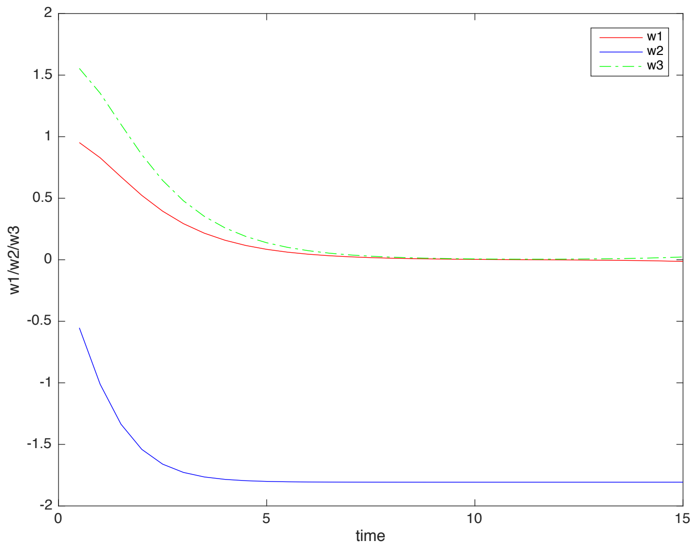


Figure 11: case $\omega_3(0) = 1.633$

Hence such solution exists.

Simulate the solution numerically   The numerical solution only works for finite time. When t is very large, $\boldsymbol{\omega}$ jumps periodically. (see Figure 12) This is plausible as this solution is not stable. Also from the numerical results, we discover that $H^2 \to 2E$, hence H must satisfies $H = \sqrt{2E}$.

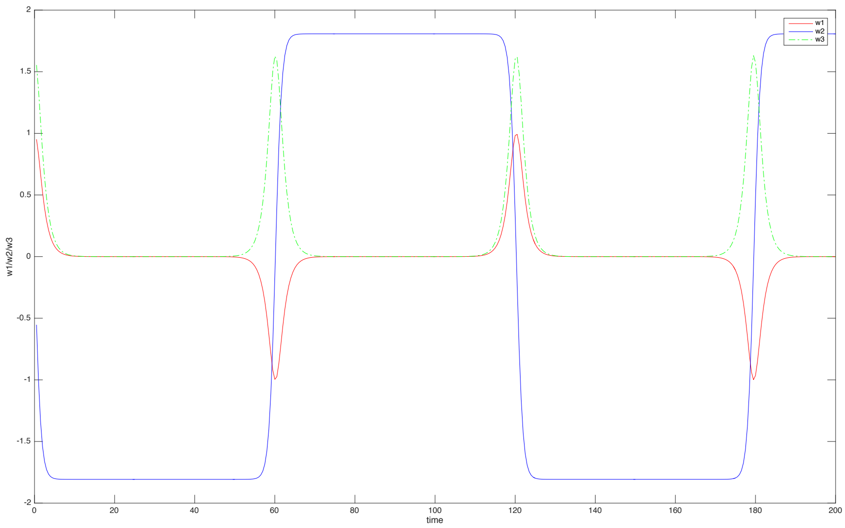


Figure 12: : case $\omega_3(0) = \sqrt{\frac{8}{3}}$

Question 7

We separation all possible qualitative types of motion in 3 cases:
Noted: for rang of behaviour of each type try different values of A, B and C, we can see the range of behaviour for each type.

i) $H^2 > 2E$
The body oscillates at a near constant rate around a particular axis.
Example: $\mathrm{A} = 1.4, \mathrm{B} = 1, \mathrm{C} = 0.7,\ \omega_1 = \omega_2 = \omega_3 = 1\ at\ t = 0$

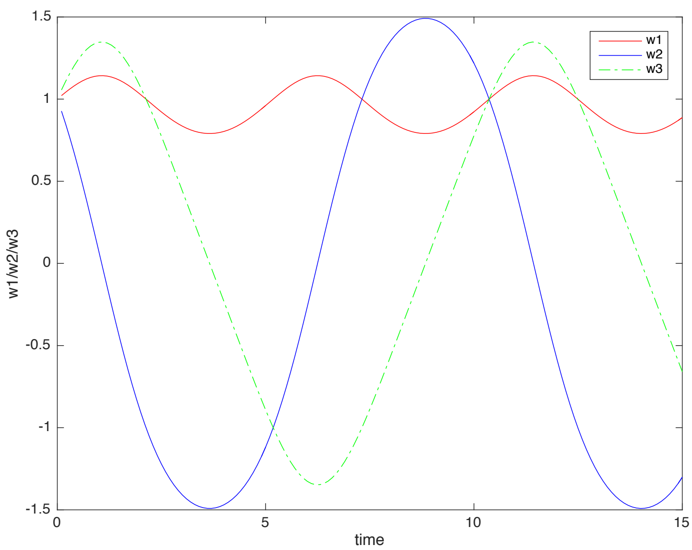


Figure 13

Range of behaviour For $H^2 \gg 2E$, the body oscillates at a very rapidly. For $H^2 \approx 2E$, the body oscillates at a slower rate.

ii) $H^2 = 2E$(See Question 6) which is the equation of boundaries of these regions. Hence as thee boundaries are approached, the solutions believe like:
$\boldsymbol{\omega}(t)$ begins away from OY axis, but tends towards the steady unstable solution, parallel to OY as $t \to \infty$. (as described in Question 6)

iii) $H^2 < 2E$
Example: $\mathrm{A} = 0.5, \mathrm{B} = 1, \mathrm{C} = 0.5,\ \omega_1 = \omega_2 = \omega_3 = 1\ at\ t = 0$

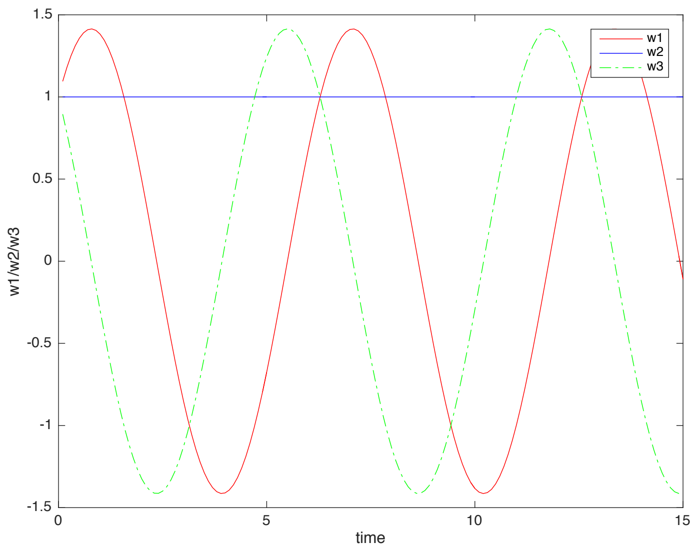


Figure 14

the body oscillates at an slower rate compared with case $H^2 > 2E$, and no components of $\boldsymbol{\omega}(t)$ oscillates at constant rate.

The choice of $\frac{A}{B}\ and\ \frac{C}{B}$ determines the which axis that the body oscillates (approximately) a constant rate. E.g. if $\frac{A}{B} > \frac{C}{B}$, then the oscillation around ω_1 at (approximately) a constant rate, vice versa (see Figure 13 and 14)

Question 8

Case $A = 1.4, B = 1\ and\ C = 0.7$

- Effect on Question 5 (obtained from Program 11)

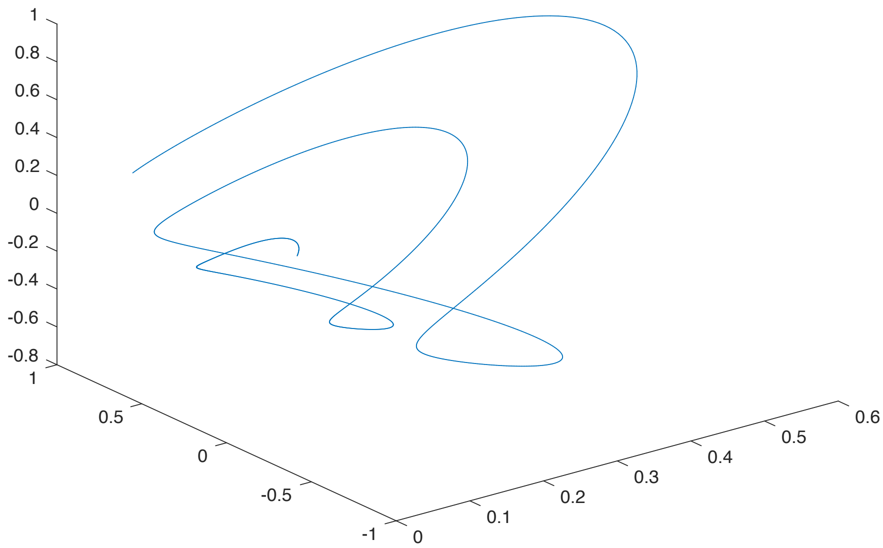
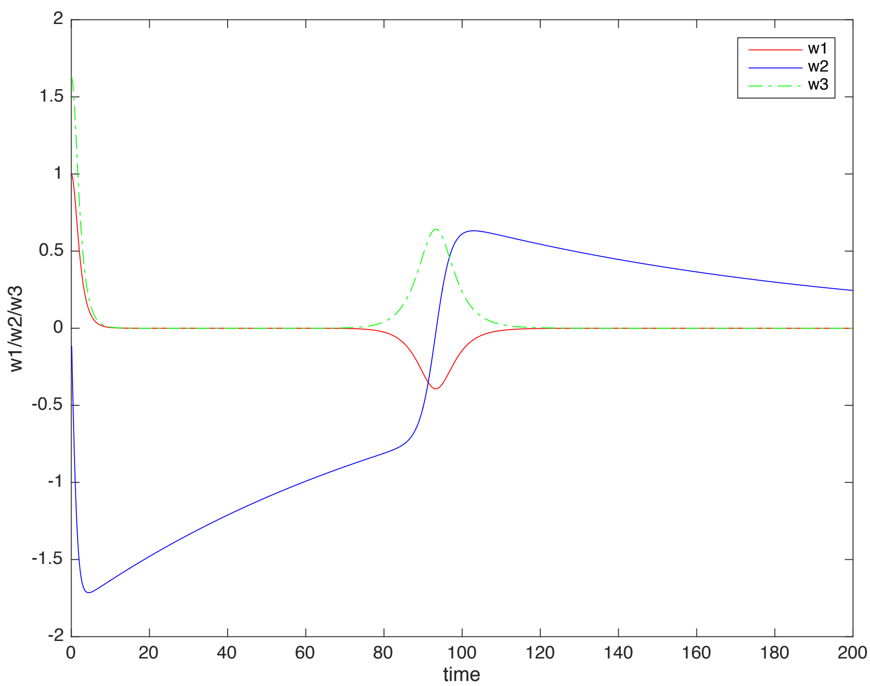


Figure 15

Affect on Question 6



Due to damping takes place, the condition $H^2 = 2E$ will not hold.

Affect on Question 7: follows from Question 6, the classifications would not be valid. In general, the affect acts like a damping to the motion of the body, hence the amplitudes of oscillations decrease against with time. (see figure 16)

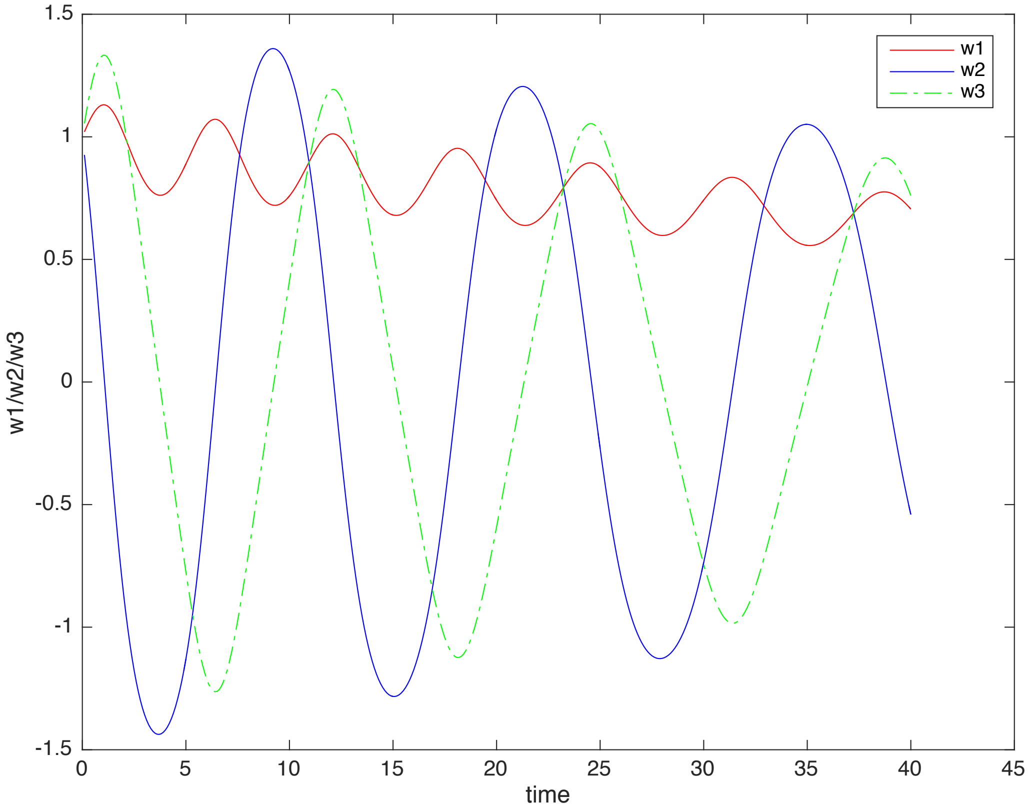


Figure 16

Question 9

The classification in Question 7 would still be useful. There is still a division of the solution space into regions. However, we need to change the boundaries from $H^2 = 2E$ to a non-linear boundaries with the damping factor taken into account.

Reference:

1. Tong, D. Lecture notes on Classical Dynamics (2015)
2. Cowley S.J. Lecture notes on Numerical Analysis (2014)

Appendix

Program 1 (to produce figure 1-3)

```
%Solve Euler's Equation numerically
n=1;
A=1.5;
B=1;
C=0.5;
t0=0;
tn=10;
%step size h
step=0.1;
U0=1;
V0=1;
W0=1;
m=(tn-t0)/step;
f=@(t,V,W)((B-C)*V*W/A);
g=@(t,U,W)((C-A)*W*U/B);
h=@(t,U,V)((A-B)*U*V/C);
w1=zeros(1,m);
t=zeros(1,m);
E=0.5*A*U0^2+0.5*B*V0^2+0.5*C*W0^2;
H=sqrt(A^2*U0^2+B^2*V0^2+C^2*W0^2);
display([E,H^2])

%4-stage explicit RK methods
while n<m+1
   t1=t0+step;
   a1=f(t0,V0,W0);
   b1=g(t0,U0,W0);
   c1=h(t0,U0,V0);

   a2=f(t0+0.5*step,V0+0.5*step*b1,W0+0.5*step*c1);
   b2=g(t0+0.5*step,U0+0.5*step*a1,W0+0.5*step*c1);
   c2=h(t0+0.5*step,U0+0.5*step*a1,V0+0.5*step*b1);

   a3=f(t0+0.5*step,V0+0.5*step*b2,W0+0.5*step*c2);
   b3=g(t0+0.5*step,U0+0.5*step*a2,W0+0.5*step*c2);
   c3=h(t0+0.5*step,U0+0.5*step*a2,V0+0.5*step*b2);

   a4=f(t0+step,V0+step*b3,W0+step*c3);
   b4=g(t0+step,U0+step*a3,W0+step*c3);
   c4=h(t0+step,U0+step*a3,V0+step*b3);

   U1=U0+step*(a1/6+a2/3+a3/3+a4/6);
   V1=V0+step*(b1/6+b2/3+b3/3+b4/6);
   W1=W0+step*(c1/6+c2/3+c3/3+c4/6);
   t(n)=t1;
   w1(n)=U1;
   plot(t,w1)
xlabel('time')
ylabel('w1')
%disp([t1,U1])

t0=t1;
```

```
U0=U1;
V0=V1;
W0=W1;
n=n+1;
end

E=0.5*A*U1^2+0.5*B*V1^2+0.5*C*W1^2;
H=sqrt(A^2*U1^2+B^2*V1^2+C^2*W1^2);
display([E,H^2])
```

Program 4 (to produce figure 4)

```
%Solve Euler's Equation numercailly
n=1;
A=1.5;
B=1;
C=0.5;
t0=0;
tn=10;
%step size h
step=0.1;
U0=1;
V0=1;
W0=1;
m=(tn-t0)/step;
f=@(t,V,W)((B-C)*V*W/A);
g=@(t,U,W)((C-A)*W*U/B);
h=@(t,U,V)((A-B)*U*V/C);
w1=zeros(1,m);
w2=zeros(2,m);
w3=zeros(1,m);
t=zeros(1,m);
E=0.5*A*U0^2+0.5*B*V0^2+0.5*C*W0^2;
H=sqrt(A^2*U0^2+B^2*V0^2+C^2*W0^2);
display([E,H^2])

%4-stage explicit RK methods
while n<m+1
    t1=t0+step;
    a1=f(t0,V0,W0);
    b1=g(t0,U0,W0);
    c1=h(t0,U0,V0);

    a2=f(t0+0.5*step,V0+0.5*step*b1,W0+0.5*step*c1);
    b2=g(t0+0.5*step,U0+0.5*step*a1,W0+0.5*step*c1);
    c2=h(t0+0.5*step,U0+0.5*step*a1,V0+0.5*step*b1);

    a3=f(t0+0.5*step,V0+0.5*step*b2,W0+0.5*step*c2);
    b3=g(t0+0.5*step,U0+0.5*step*a2,W0+0.5*step*c2);
    c3=h(t0+0.5*step,U0+0.5*step*a2,V0+0.5*step*b2);

    a4=f(t0+step,V0+step*b3,W0+step*c3);
    b4=g(t0+step,U0+step*a3,W0+step*c3);
    c4=h(t0+step,U0+step*a3,V0+step*b3);

    U1=U0+step*(a1/6+a2/3+a3/3+a4/6);
    V1=V0+step*(b1/6+b2/3+b3/3+b4/6);
    W1=W0+step*(c1/6+c2/3+c3/3+c4/6);
    t(n)=t1;
    w1(n)=U1;
    w2(n)=V1;
```

```
    w3(n)=W1;
    plot3(w1,w2,w3)
xlabel('w1')
ylabel('w2')
zlabel('w3')

t0=t1;
U0=U1;
V0=V1;
W0=W1;
n=n+1;
end

E=0.5*A*U1^2+0.5*B*V1^2+0.5*C*W1^2;
H=sqrt(A^2*U1^2+B^2*V1^2+C^2*W1^2);
display([E,H^2])
```

Program 6 (for figure 5,6)

```
%Answer Q2 with 3D plots
n=1;
A=input('please input your A=');
C=input('please input your C=');
B=1;
t0=0;
tn=500;
%step size h
step=0.01;
U0=0.01;
V0=1;
W0=0.01;
m=(tn-t0)/step;
w1=zeros(1,m);
w2=zeros(2,m);
w3=zeros(1,m);
t=zeros(1,m);
E=0.5*A*U0^2+0.5*B*V0^2+0.5*C*W0^2;
H=sqrt(A^2*U0^2+B^2*V0^2+C^2*W0^2);
display([E,H^2])
f=@(t,V,W)(E*(B-C)*V*W/A);
g=@(t,U,W)(E*(C-A)*W*U/B);
h=@(t,U,V)(E*(A-B)*U*V/C);
%4-stage explicit RK methods
while n<m+1
    t1=t0+step;
    a1=f(t0,V0,W0);
    b1=g(t0,U0,W0);
    c1=h(t0,U0,V0);

    a2=f(t0+0.5*step,V0+0.5*step*b1,W0+0.5*step*c1);
    b2=g(t0+0.5*step,U0+0.5*step*a1,W0+0.5*step*c1);
    c2=h(t0+0.5*step,U0+0.5*step*a1,V0+0.5*step*b1);

    a3=f(t0+0.5*step,V0+0.5*step*b2,W0+0.5*step*c2);
    b3=g(t0+0.5*step,U0+0.5*step*a2,W0+0.5*step*c2);
    c3=h(t0+0.5*step,U0+0.5*step*a2,V0+0.5*step*b2);

    a4=f(t0+step,V0+step*b3,W0+step*c3);
    b4=g(t0+step,U0+step*a3,W0+step*c3);
    c4=h(t0+step,U0+step*a3,V0+step*b3);

    U1=U0+step*(a1/6+a2/3+a3/3+a4/6);
```

```
    V1=V0+step*(b1/6+b2/3+b3/3+b4/6);
    W1=W0+step*(c1/6+c2/3+c3/3+c4/6);
    t(n)=t1;
    w1(n)=U1;
    w2(n)=V1;
    w3(n)=W1;
    %plot(t,w3)
plot3(w1,w2,w3,'r');
xlabel('w1');
ylabel('w2');
zlabel('w3');
t0=t1;
U0=U1;
V0=V1;
W0=W1;
n=n+1;
end

E=0.5*A*U1^2+0.5*B*V1^2+0.5*C*W1^2;
H=sqrt(A^2*U1^2+B^2*V1^2+C^2*W1^2);
display([E,H^2])
```

Program 9 (for figure 9)

```
%ans to Q5
%Solve Euler's Equation numercailly
n=1;
A=1.4;
B=1;
C=2.5;
t0=0;
tn=20;
%step size h
step=0.01;
U0=1;
V0=0;
W0=0.820;
m=(tn-t0)/step;
f=@(t,V,W)((B-C)*V*W/A);
g=@(t,U,W)((C-A)*W*U/B);
h=@(t,U,V)((A-B)*U*V/C);
w1=zeros(1,m);
w2=zeros(1,m);
w3=zeros(1,m);
t=zeros(1,m);
E=0.5*A*U0^2+0.5*B*V0^2+0.5*C*W0^2;
H=sqrt(A^2*U0^2+B^2*V0^2+C^2*W0^2);
display([E,H^2])

%4-stage explicit RK methods
while n<m+1
    t1=t0+step;
    a1=f(t0,V0,W0);
    b1=g(t0,U0,W0);
    c1=h(t0,U0,V0);

    a2=f(t0+0.5*step,V0+0.5*step*b1,W0+0.5*step*c1);
    b2=g(t0+0.5*step,U0+0.5*step*a1,W0+0.5*step*c1);
    c2=h(t0+0.5*step,U0+0.5*step*a1,V0+0.5*step*b1);

    a3=f(t0+0.5*step,V0+0.5*step*b2,W0+0.5*step*c2);
    b3=g(t0+0.5*step,U0+0.5*step*a2,W0+0.5*step*c2);
    c3=h(t0+0.5*step,U0+0.5*step*a2,V0+0.5*step*b2);
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    a4=f(t0+step,V0+step*b3,W0+step*c3);
    b4=g(t0+step,U0+step*a3,W0+step*c3);
    c4=h(t0+step,U0+step*a3,V0+step*b3);

    U1=U0+step*(a1/6+a2/3+a3/3+a4/6);
    V1=V0+step*(b1/6+b2/3+b3/3+b4/6);
    W1=W0+step*(c1/6+c2/3+c3/3+c4/6);
    t(n)=t1;
    w1(n)=U1;
    w2(n)=V1;
    w3(n)=W1;
    plot3(w1,w2,w3)
xlabel('w1')
ylabel('w2')
zlabel('w3')

t0=t1;
U0=U1;
V0=V1;
W0=W1;
n=n+1;
end

E=0.5*A*U1^2+0.5*B*V1^2+0.5*C*W1^2;
H=sqrt(A^2*U1^2+B^2*V1^2+C^2*W1^2);
display([E,H^2])
```

Program 10 (for Question 7)

```
% Sovling Q6
n=1;
A=0.25;
B=0.27;
C=0.30;
t0=0;
tn=50;
%step size h
step=0.2;
U0=1;
V0=1;
W0=1;
m=(tn-t0)/step;
f=@(t,V,W)((B-C)*V*W/A);
g=@(t,U,W)((C-A)*W*U/B);
h=@(t,U,V)((A-B)*U*V/C);
w1=zeros(1,m);
w2=zeros(1,m);
w3=zeros(1,m);
t=zeros(1,m);
E=0.5*A*U0^2+0.5*B*V0^2+0.5*C*W0^2;
H=sqrt(A^2*U0^2+B^2*V0^2+C^2*W0^2);
display([E,H^2])

%4-stage explicit RK methods
while n<m+1
    t1=t0+step;
    a1=f(t0,V0,W0);
    b1=g(t0,U0,W0);
    c1=h(t0,U0,V0);

    a2=f(t0+0.5*step,V0+0.5*step*b1,W0+0.5*step*c1);
```

```
    b2=g(t0+0.5*step,U0+0.5*step*a1,W0+0.5*step*c1);
    c2=h(t0+0.5*step,U0+0.5*step*a1,V0+0.5*step*b1);

    a3=f(t0+0.5*step,V0+0.5*step*b2,W0+0.5*step*c2);
    b3=g(t0+0.5*step,U0+0.5*step*a2,W0+0.5*step*c2);
    c3=h(t0+0.5*step,U0+0.5*step*a2,V0+0.5*step*b2);

    a4=f(t0+step,V0+step*b3,W0+step*c3);
    b4=g(t0+step,U0+step*a3,W0+step*c3);
    c4=h(t0+step,U0+step*a3,V0+step*b3);

    U1=U0+step*(a1/6+a2/3+a3/3+a4/6);
    V1=V0+step*(b1/6+b2/3+b3/3+b4/6);
    W1=W0+step*(c1/6+c2/3+c3/3+c4/6);
    t(n)=t1;
    w1(n)=U1;
    w2(n)=V1;
    w3(n)=W1;

    plot (t,w1,'r',t,w2,'b',t,w3,'g-.')

xlabel('time')
ylabel('w1/w2/w3')
legend('w1','w2','w3')

t0=t1;
U0=U1;
V0=V1;
W0=W1;

n=n+1;
end

E=0.5*A*U1^2+0.5*B*V1^2+0.5*C*W1^2;
H=sqrt(A^2*U1^2+B^2*V1^2+C^2*W1^2);
display([E,H^2])
```

Program 11 (for Question 8)

```
%Question 8
n=1;
A=1.4;
B=1;
C=0.7;
t0=0;
tn=40;
k=0.01;
%step size h
step=0.1;
U0=1;
V0=1;
W0=1;
m=(tn-t0)/step;
f=@(t,U,V,W)(((B-C)/A)*V*W -k*U);
g=@(t,U,V,W)(((C-A)/B)*W*U-k*V);
h=@(t,U,V,W)(((A-B)/C)*U*V-k*W);
w1=zeros(1,m);
w2=zeros(1,m);
w3=zeros(1,m);
t=zeros(1,m);
E=0.5*A*U0^2+0.5*B*V0^2+0.5*C*W0^2;
H=sqrt(A^2*U0^2+B^2*V0^2+C^2*W0^2);
```

```
display([E,H^2])

%4-stage explicit RK methods
while n<m+1
    t1=t0+step;
    a1=f(t0,U0,V0,W0);
    b1=g(t0,U0,V0,W0);
    c1=h(t0,U0,V0,W0);

    a2=f(t0+0.5*step,U0+0.5*step*a1,V0+0.5*step*b1,W0+0.5*step*c1);
    b2=g(t0+0.5*step,U0+0.5*step*a1,V0+0.5*step*b1,W0+0.5*step*c1);
    c2=h(t0+0.5*step,U0+0.5*step*a1,V0+0.5*step*b1,W0+0.5*step*c1);

    a3=f(t0+0.5*step,U0+0.5*step*a2,V0+0.5*step*b2,W0+0.5*step*c2);
    b3=g(t0+0.5*step,U0+0.5*step*a2,V0+0.5*step*b2,W0+0.5*step*c2);
    c3=h(t0+0.5*step,U0+0.5*step*a2,V0+0.5*step*b2,W0+0.5*step*c2);

    a4=f(t0+step,U0+step*a3,V0+step*b3,W0+step*c3);
    b4=g(t0+step,U0+step*a3,V0+step*b3,W0+step*c3);
    c4=h(t0+step,U0+step*a3,V0+step*b3,W0+step*c3);

    U1=U0+step*(a1/6+a2/3+a3/3+a4/6);
    V1=V0+step*(b1/6+b2/3+b3/3+b4/6);
    W1=W0+step*(c1/6+c2/3+c3/3+c4/6);
    t(n)=t1;
    w1(n)=U1;
    w2(n)=V1;
    w3(n)=W1;

    plot (t,w1,'r',t,w2,'b',t,w3,'g-.')

xlabel('time')
ylabel('w1/w2/w3')
legend('w1','w2','w3')
%plot3(w1,w2,w3)

t0=t1;
U0=U1;
V0=V1;
W0=W1;


n=n+1;
end

E=0.5*A*U1^2+0.5*B*V1^2+0.5*C*W1^2;
H=sqrt(A^2*U1^2+B^2*V1^2+C^2*W1^2);
display([E,H^2])
```