# 23.6 Accretion Discs

1. **Fluids Equation**

**Question 1**

1. By using equation (3), the RHS of equation (5) can be simplified as

and hence by product rule

From equation (2), we can deduce

As required.

1. Consider

Since , we have

Also since , the RHS of equation (5) can be written

Hence we can obtain equation (6) from equation (5)

**Question 2**

1. Substitute equation (2) into equation (6), we obtain

hence

therefore, we have

1. In a steady state, we have ,

Substitute equation (8) into equation (7), we obtain

integrate with respect to R,

Where A= and B are arbitrary constants.

Now apply inner boundary condition

hence

Hence our original equation becomes

Figure 1 is a plot of in units of against



Figure 1

By substituting we obtain

Hence, we obtain the condition

such that equation (6) remains the same for these dimensionless variables.

**Question 3**

1. Substitute and and into equation (6), (with dimensionless variables)

As is independent of , we have

Hence .

1. By definition of derivatives,

And applied the definition again for the second derivative, we obtain

Then equation (9) becomes

which simplifies to

**Question 4**

Choose which is smaller than we obtain time evolution of surface density.



Figure 2

**Question 5**

1. Program from Question 4 is adapted to produce table 1. It contains height and

position (r) of the peak in surface density at the times used in Question 4.

Table 1

|  |  |  |
| --- | --- | --- |
| **Time** | **Height** | **Position** |
| 0 | 1.0000 | 1.0000 |
| 0.002 | 0.6231 | 1.0000 |
| 0.008 | 0.2933 | 1.0000 |
| 0.032 | 0.0946 | 1.0000 |
| 0.128 | 0 | 0.0004 |
| 0.512 | 0 | 0.0004 |

Furthermore, the program is also adapted to produce the time evolution of the total angular momentum. (Notes it is in units of )



Figure 3

Figure 4 is the position of the peak angular momentum surface density as a function of time. Note that the position should be at for all time. However, at later stage, the maximum height becomes computationally zero, which causes the position to drops to 0.0004.



Figure 4

1. Figure 5 is a (zoomed in) plot of angular momentum and surface density



Figure 5

Hence the angular momentum is slightly shifted to the right in the x-axis compared with the surface density.

**Question 6**

Substitute into equation (8),

The condition for to satisfies, such that equation (8) remains the same for these dimensionless variables is still

Hence we have

By inverse chain rule, we obtain

Let , we obtain

Where as in Question 3.

Using the similar set up as in Question 4, we have

The radial velocity (in term of dimensionless variables) may be discretised as

Hence, by adopting the program in Question 4, we obtain a plot of radial velocity as a function of radius in the time steps.



Figure 6

**Question 7**

By using the radial velocities in Question 6, we obtain figure 7, a plot of the evolution of particles’ orbits for



Figure 7

Figure 8 is a plot of maximum radius attained by the particles as a function of its initial radius.



Figure 8

Figure 9 is a plot of time took to reach the maximum radius as a function of its initial radius.



Figure 9

Figure 10 is a pot of the time it takes to reach a boundary (at the lower bound) as a function of its initial radius.



Figure 10

**Question 8**

From figure 10 in Question 7, all the initial radii have reached the inner boundary by

Hence, by using equation (12), we deduce that

where is the initial mass that has reached the inner boundary by and is the total initial mass. Numerical results are obtained from program ‘Question 8’, by using composite Simpson’s rule.

Hence, the fraction of the initial mass that reached the inner boundary by is approximately 97%.

From figure 8, we can see that the maximum radius attained by the particle is below the outer boundary. Hence the fraction of the initial mass that reached the outer boundary by is 0.

From the results in Question 4, we obtain table 2

Table 2

|  |  |  |
| --- | --- | --- |
| Initial radius  (reached inner boundary by ) | Surface density | Mass at |
| 0.9 | 0 | 0 |
| 0.95 | 0 | 0 |
| 1.0 | 0 | 0 |
| 1.05 | 0 | 0 |
| 1.1 | 0 | 0 |

From table 2, we conclude that the fraction of the remain value of the initial mass that has reached the inner boundary is 0 at . And similarly, the fraction of the initial mass that has reached the outer boundary is also 0 at .

Hence, we can obtain table 3 to compare these values

Table 3

|  |  |  |  |
| --- | --- | --- | --- |
| in terms of fraction of the initial mass | | | |
| reached the inner boundary by | 97% | the remain value that has reached the inner boundary | 0% |
| reached the outer boundary by | 0% | the remain value that has reached the outer boundary | 0% |

Hence, we conclude that the result for the inner boundary is fair accuracy. The results for the outer boundary are both zero, due to the effect of diffusion (see equation 9).

**Appendix**

**Program for Question 2**

x = linspace(1,100,1000);

y=(1-x.^(-1/2))/(6\*pi);

plot(x,y)

xlabel('R/Rin(in units m dot)')

ylabel('nu\*sigma(in units m dot)')

**Program for Question 4**

X1=0.02;

Xn=2;

dX=2/100;

m=100;

X=zeros(m,1);

X(1)=dX;

for i=2:m

X(i)=X(i-1)+dX;

end

dtau=0.0001;

n=(0.52/dtau);

sigma=zeros(m,n+1);

f=zeros(m,n+1);

g=zeros(m,n+1);

for i=1:m

sigma(i,1)=exp(-((X(i)-1)^2)/0.001);

end

for i=1:m

g(i,1)=3\*sigma(i,1)\*X(i);

f(i,1)=4\*(X(i)^3)\*sigma(i,1);

end

for j=1:n+1

for i=j+1:m-j

f(i,j+1)=f(i,j)+(dtau/(dX^2))\*(g(i+1,j)-2\*g(i,j)+g(i-1,j));

sigma(i,j+1)=f(i,j+1)/(4\*X(i)^3);

g(i,j+1)=3\*sigma(i,j+1)\*X(i);

end

end

plot(X.^2,sigma(:,1))

hold on

plot(X.^2,sigma(:,3))

plot(X.^2,sigma(:,9))

plot(X.^2,sigma(:,33))

plot(X.^2,sigma(:,129))

plot(X.^2,sigma(:,513))

xlabel('r')

ylabel('sigma')

title('tau=0,0.002,0.008,0.032,0.128,0.512')

hold off

**Program for Question 5**

X1=0.02;

Xn=2;

dX=2/100;

m=100;

X=zeros(m,1);

X(1)=dX;

for i=2:m

X(i)=X(i-1)+dX;

end

dtau=0.0001;

n=(0.52/dtau);

sigma=zeros(m,n+1);

f=zeros(m,n+1);

g=zeros(m,n+1);

for i=1:m

sigma(i,1)=exp(-((X(i)-1)^2)/0.001);

end

for i=1:m

g(i,1)=3\*sigma(i,1)\*X(i);

f(i,1)=4\*(X(i)^3)\*sigma(i,1);

end

for j=1:n+1

for i=j+1:m-j

f(i,j+1)=f(i,j)+(dtau/(dX^2))\*(g(i+1,j)-2\*g(i,j)+g(i-1,j));

sigma(i,j+1)=f(i,j+1)/(4\*X(i)^3);

g(i,j+1)=3\*sigma(i,j+1)\*X(i);

end

end

[M0,I0]=max(sigma(:,1))

[M1,I1]=max(sigma(:,3));

hold on

[M2,I2]=max(sigma(:,9));

[M3,I3]=max(sigma(:,33));

[M4,I4]=max(sigma(:,129));

[M5,I5]=max(sigma(:,513));

hold off

a=[M0^2,I0\*dX;M1^2,I1\*dX;M2^2,I2\*dX;M3^2,I3\*dX;M4^2,I4\*dX;M5^2,I5\*dX]

**Program for Question 6**

dX=2/100;

m=100;

dtau=0.0001;

n=(0.52/dtau);

X=zeros(m,n+1);

X(1,1)=dX;

for i=2:m

X(i,1)=X(i-1,1)+dX;

end

f=zeros(m,n+1);

g=zeros(m,n+1);

for i=1:m

g(i,1)=log(3\*sigma(i,1)\*X(i,1));

f(i,1)=-3/(4\*X(i,1).^2);

end

for j=1:n+1

for i=1:m-j

if sigma(i,j)<(10^-40)

X(i,j+1)=0;

else

X(i,j+1)=X(i,j)+(dtau/dX)\*f(i,j)\*(g(i+1,j)-g(i,j));

f(i,j+1)= -3/(4\*(X(i,j+1).^2));

g(i,j+1)=log(3\*sigma(i,j+1)\*X(i,j+1));

end

end

end

v=zeros(m,n+1);

for j=1:n

for i=1:m

v(i,j)=(X(i,j+1)^2-X(i,j)^2)/dtau;

end

end

xlabel('r')

ylabel('surface density/radial volcity')

title('tau=0,0.002,0.008,0.032,0.128,0.512')

hold on

plot(X(:,3).^2,v(:,3),'g')

plot(X(:,9).^2,v(:,9),'r')

plot(X(:,33).^2,v(:,33),'g')

plot(X(:,129).^2,v(:,129),'r')

plot(X(:,513).^2,v(:,513),'g')

hold off

**Program for Question 7**

dX=2/100;

m=100;

X=zeros(m,1);

X(1)=dX;

for i=2:m

X(i)=X(i-1)+dX;

end

dtau=0.0001;

n=(1/dtau);

sigma=zeros(m,n+1);

f=zeros(m,n+1);

g=zeros(m,n+1);

for i=1:m

sigma(i,1)=exp(-((X(i)-1)^2)/0.001);

end

for i=1:m

g(i,1)=3\*sigma(i,1)\*X(i);

f(i,1)=4\*(X(i)^3)\*sigma(i,1);

end

for j=1:n+1

for i=j+1:m-j

f(i,j+1)=f(i,j)+(dtau/(dX^2))\*(g(i+1,j)-2\*g(i,j)+g(i-1,j));

sigma(i,j+1)=f(i,j+1)/(4\*X(i)^3);

g(i,j+1)=3\*sigma(i,j+1)\*X(i);

end

end

clear m n X f g dX dtau

dX=2/100;

m=100;

dtau=0.0001;

n=(1/dtau);

X=zeros(m,n+1);

X(1,1)=dX;

for i=2:m

X(i,1)=X(i-1,1)+dX;

end

f=zeros(m,n+1);

g=zeros(m,n+1);

for i=1:m

g(i,1)=3\*sigma(i,1)\*X(i,1);

f(i,1)=4\*(X(i,1)^3)\*sigma(i,1);

end

for j=1:n+1

for i=1:m-j

if f(i,j)<(10^-4)

X(i,j+1)=0;

else

X(i,j+1)=X(i,j)-(dtau/dX)\*(1/f(i,j))\*(g(i+1,j)-g(i,j));

f(i,j+1)=4\*(X(i,j+1)^3)\*sigma(i,j+1);

g(i,j+1)=3\*sigma(i,j+1)\*X(i,j+1);

end

end

end

tau=zeros(1,n+1);

tau(1,1)=0;

for i=2:n+1

tau(1,i)=tau(1,i-1)+dtau;

end

plot(tau(1,1:80),X(round(sqrt(0.9)/dX),1:80).^2)

hold on

plot(tau(1,1:60),X(round(sqrt(0.95)/dX),1:60).^2)

plot(tau(1,1:60),X(round(sqrt(1.0)/dX),1:60).^2)

plot(tau(1,1:60),X(round(sqrt(1.05)/dX),1:60).^2)

plot(tau(1,1:60),X(round(sqrt(1.1)/dX),1:60).^2)

hold off

xlabel('r')

ylabel('radial volcity')

title('tau=0,0.002,0.008,0.032,0.128,0.512')

hold off

M=zeros(5,1);

I=zeros(5,1);

N=zeros(5,1);

for i=1:5

N(i)=0.85+i\*0.05

end

[M(1) I(1)]=max(X(round(sqrt(0.9)/dX),:).^2);

[M(2) I(2)]=max(X(round(sqrt(0.95)/dX),:).^2);

[M(3) I(3)]=max(X(round(sqrt(1.0)/dX),1:60).^2);

[M(4) I(4)]=max(X(round(sqrt(1.05)/dX),1:60).^2);

[M(5) I(5)]=max(X(round(sqrt(1.1)/dX),1:60).^2);

%plot(N,M)

%plot(N,I,'k')

**Program for Question 8**

x0=0.0004;

x1=4;

m=10000;

h=(x1-x0)/m;

x=x0:h:x1;

f=pi\*(x.^2).\*exp(-((x.^(1/2)-1).^2)/0.001);

I1=f(2);

for i=4:2:m-1

I1=I1+f(i);

end

I2=f(3);

for i=5:2:m-2

I2=I2+f(i);

end

I=(h/3)\*(f(1)+4\*I1+2\*I2+f(m))