QF602: Derivatives

Lecture 3:
Options Strategies

Option Strategies

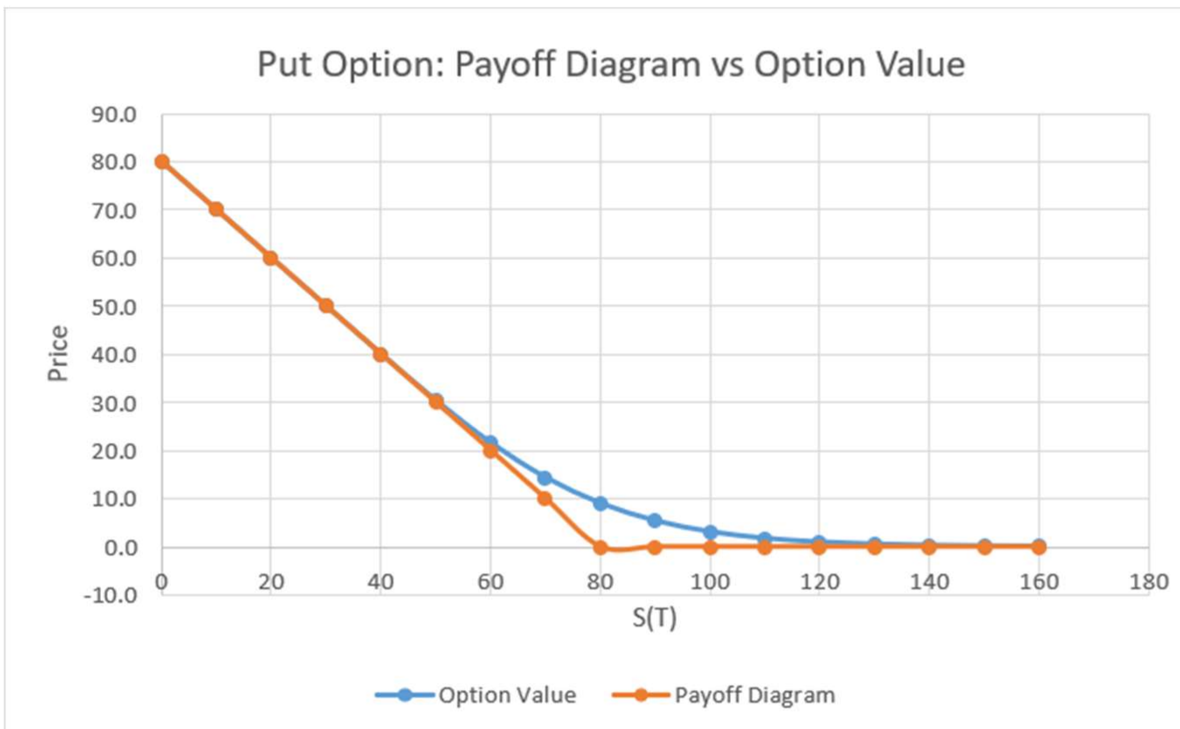
- We will look at the payoff of different portfolios of stocks and options in this lecture.
- The portfolios is also called option strategies.
- They are used to express trader's view of the future of the underlying (i.e. speculation) or for hedging.

Option Strategies

- Protective Put
- Covered Call
- Bull spread
- Bear spread
- Butterfly spread
- Condor spread
- Ratio Spread
- Straddle
- Strangle

Payoff Diagram vs Option value

- Let Put(t,T,K) be the value of an European put option with strike K at time t.
- The payoff of an European put option can be represented as Put(T,T,K), i.e. the intrinsic value.



Protective Put

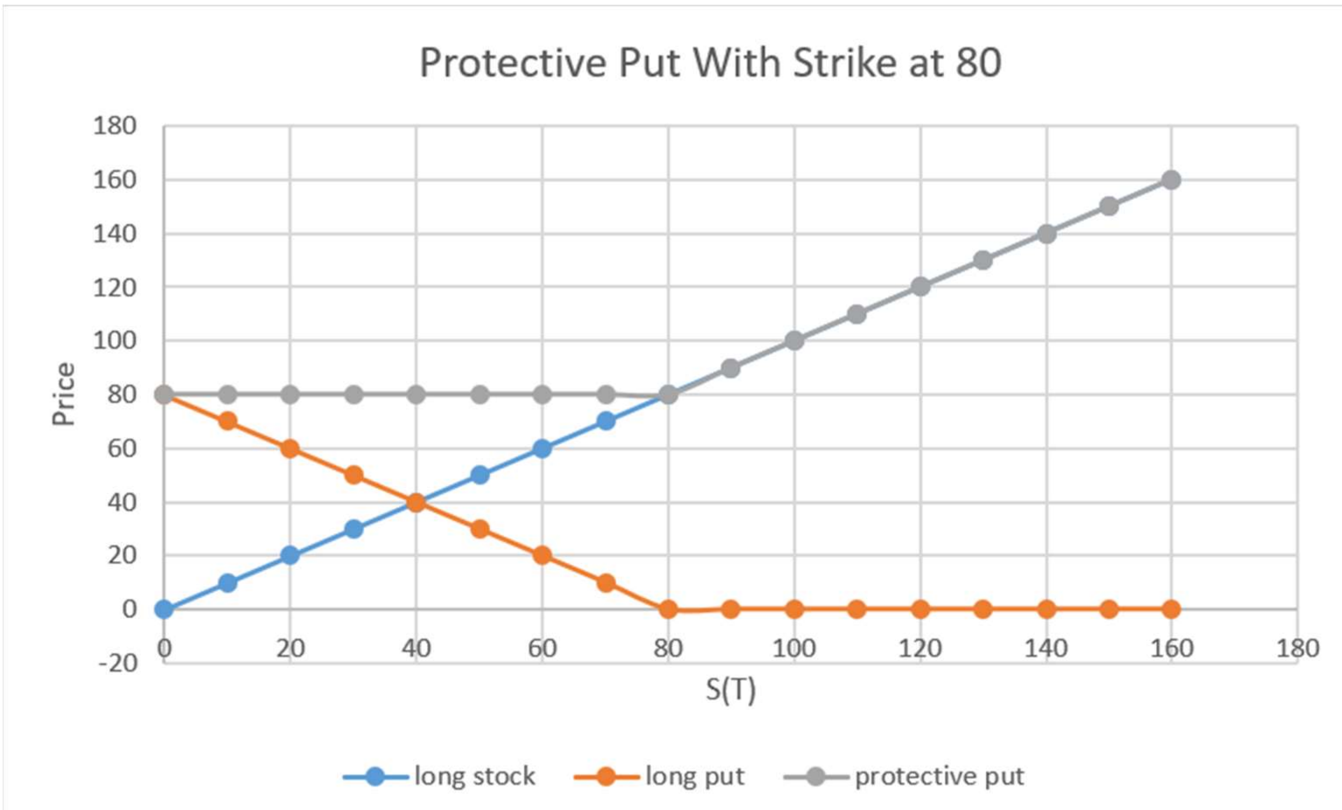
- A portfolio consists of a put option and the underlying stock.
- Long protective put is equivalent to holding a stock and a put option on the same stock.
- The main motivation for buying a protective put is mainly for hedging.

Protective Put

- A fund manager holds 10,000 stocks of IBM at $80.
- The stock is trading at $90.
- He is worried about the market going down during the next 3 months but doesn't want to sell his shares.
- The fund manager decides to buy 10,000 3-month European puts on IBM with strike at $80.
- Buying a protective put enables an investor to fix the maximum loss that he could potentially suffer.

Payoff Diagram for Protective Put

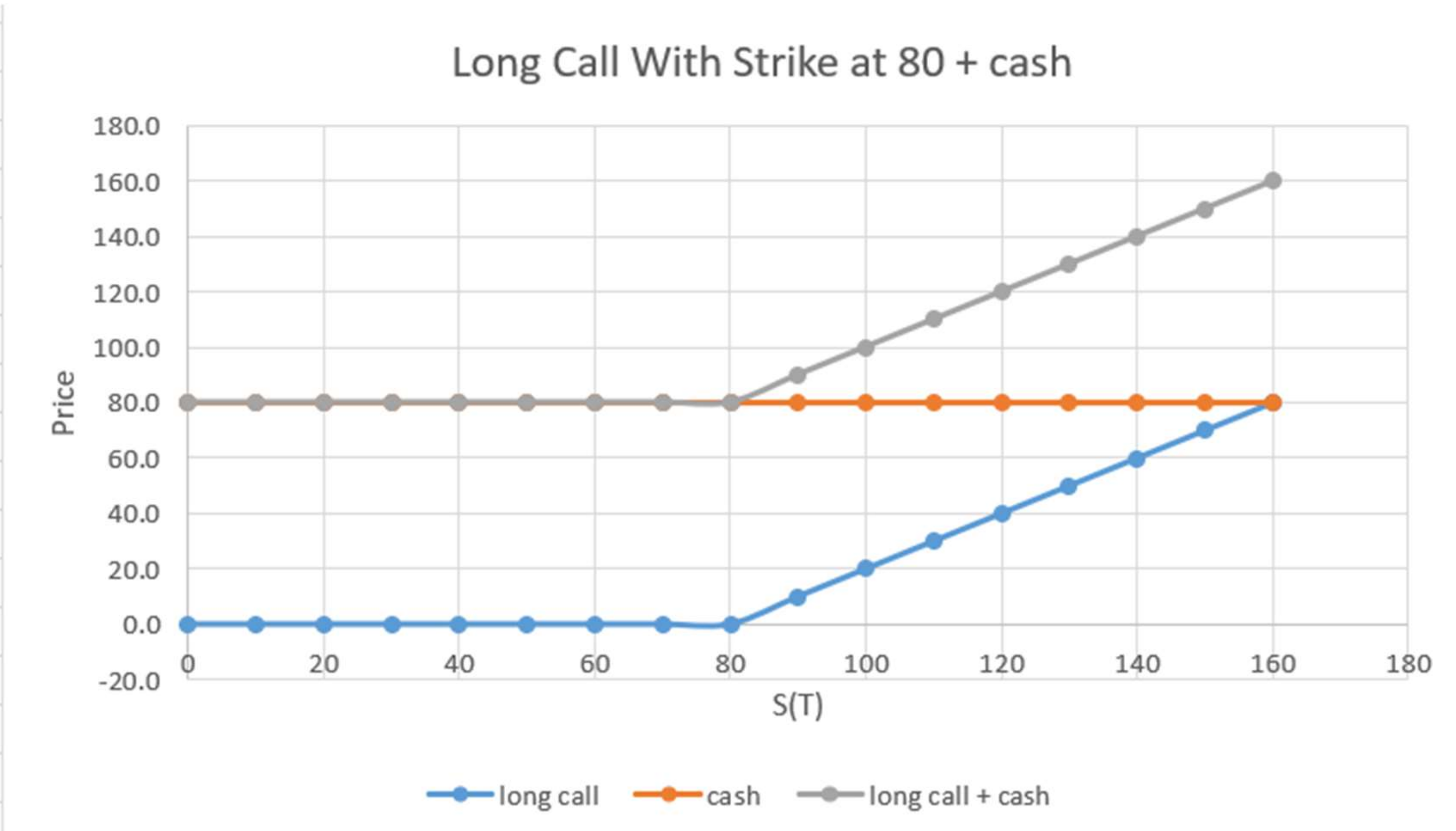
- Note that the premium of the option is not included in the payoff diagram.



Equivalent to Call + Cash

- Applying put-call parity, we can show that

$$Put(T,T,K) + S(T) = Call(T,T,K) + K$$



Covered Call

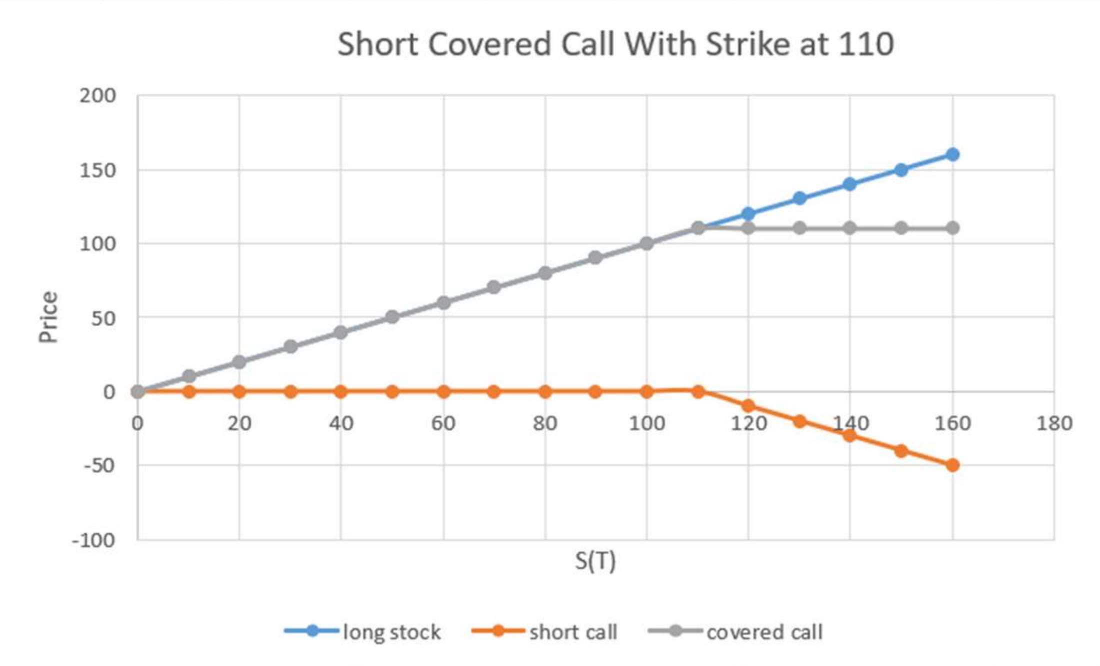
- A portfolio consists of a call option and short selling the underlying stock.
- The payoff of a covered call is

$$\text{Covered Call}(T,T,K) = \text{Call}(T,T,K) - S(T)$$

- **Writing** a covered call is a popular hedging strategy.
- A trader holds some shares and believes the price will increase in the long term but decline in the short term.
- The trader will receive an extra income in the form of call option premium.

Payoff Diagram for Short Covered Call

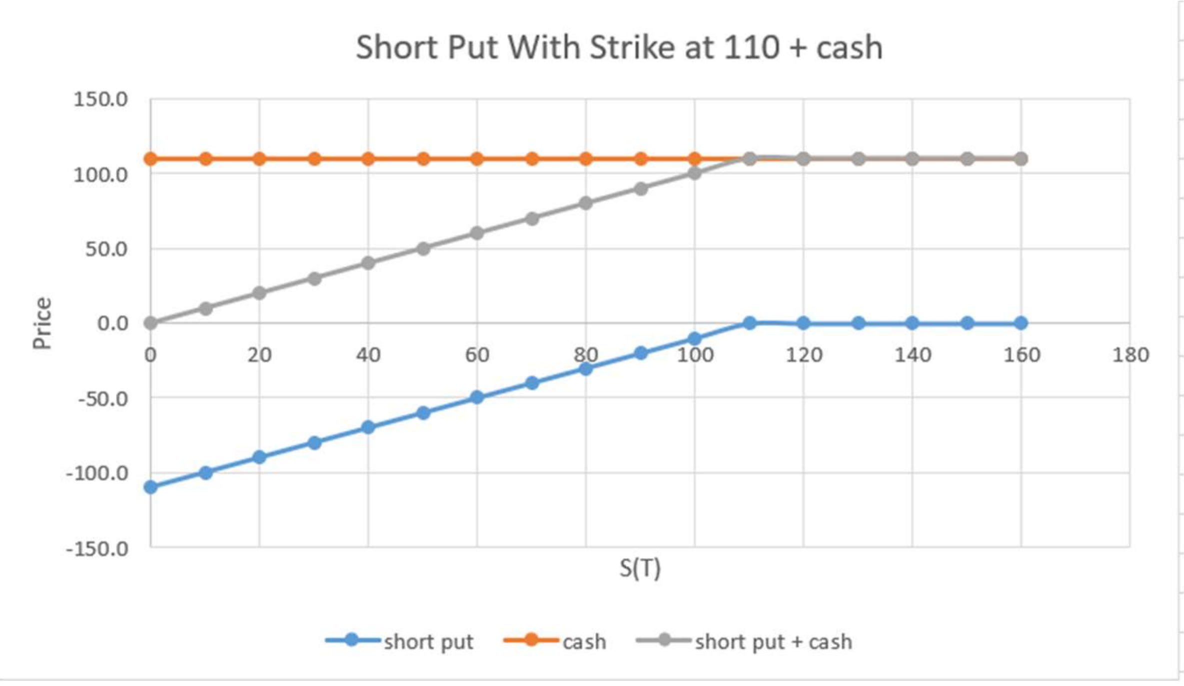
- The trader "gives up" the upside above 110 until the maturity of the option.
- To compensate the potential expected short term stock price decrease.



Equivalent to Short Put plus Cash

- We can show that writing a covered call is equivalent to short a put plus cash:

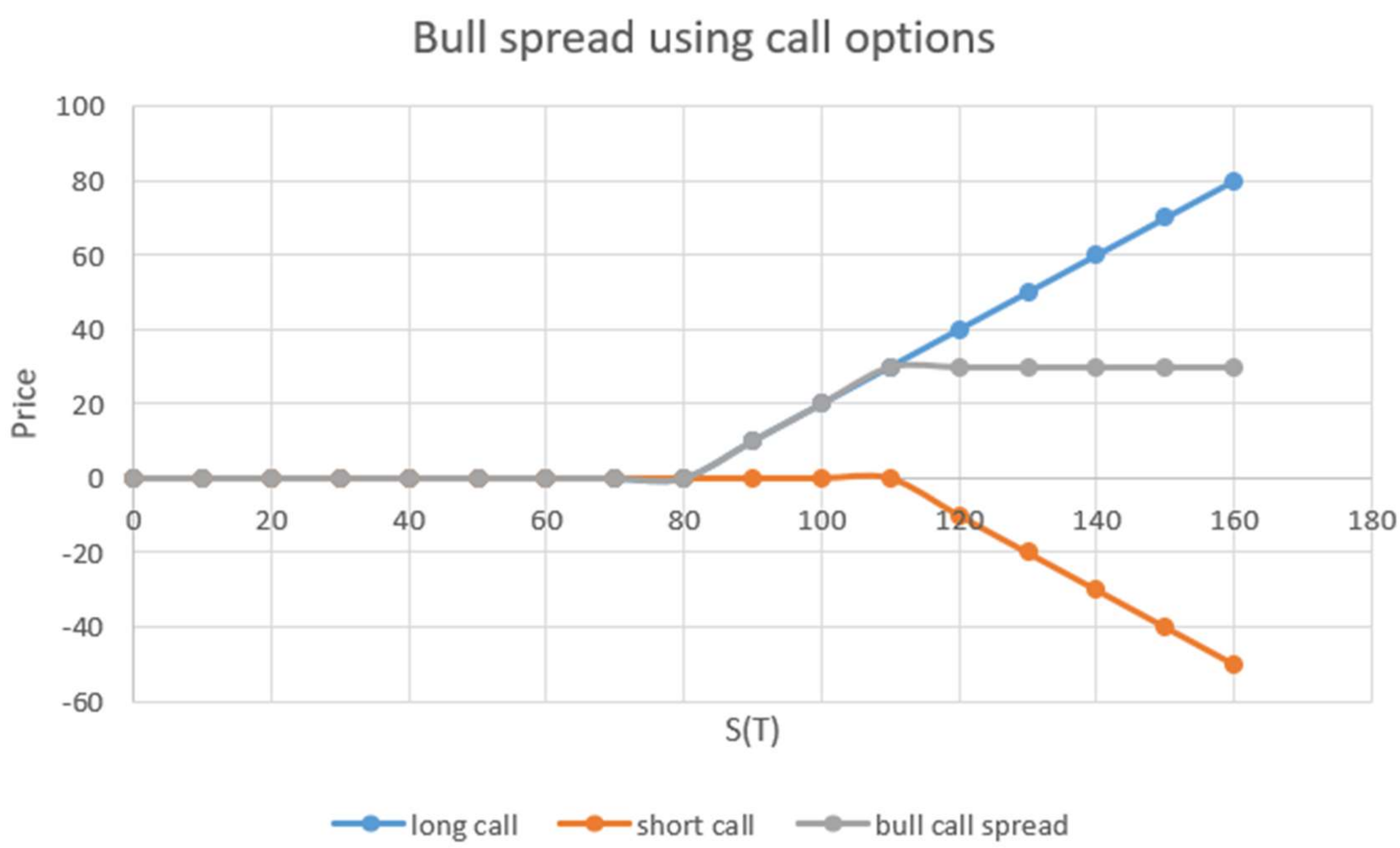
$$S(T) - Call(T,T,K) = Put(T,T,K) + K$$



Bull Spreads

- Bull spreads (or call spreads) are one of the most popular strategies and corresponding to a bullish view on the market.
- Trader believes an asset is going to increase above a specific strike K1 but will not be able to reach a level K2, K1<K2.
- Bull Spreads(T,T,K1,K2) = Call(T,T,K1) – Call(T,T,K2)

Payoff Diagram of Bull Spreads

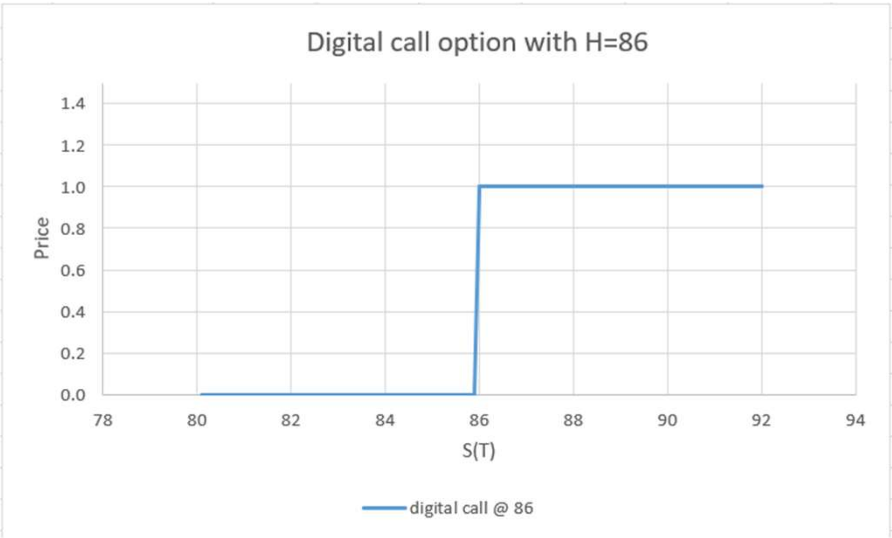


Digital Call Option

- Digital options, aka binary options, pay a specific coupon when a barrier or trigger event occurs.
- European digital call option has the payoff

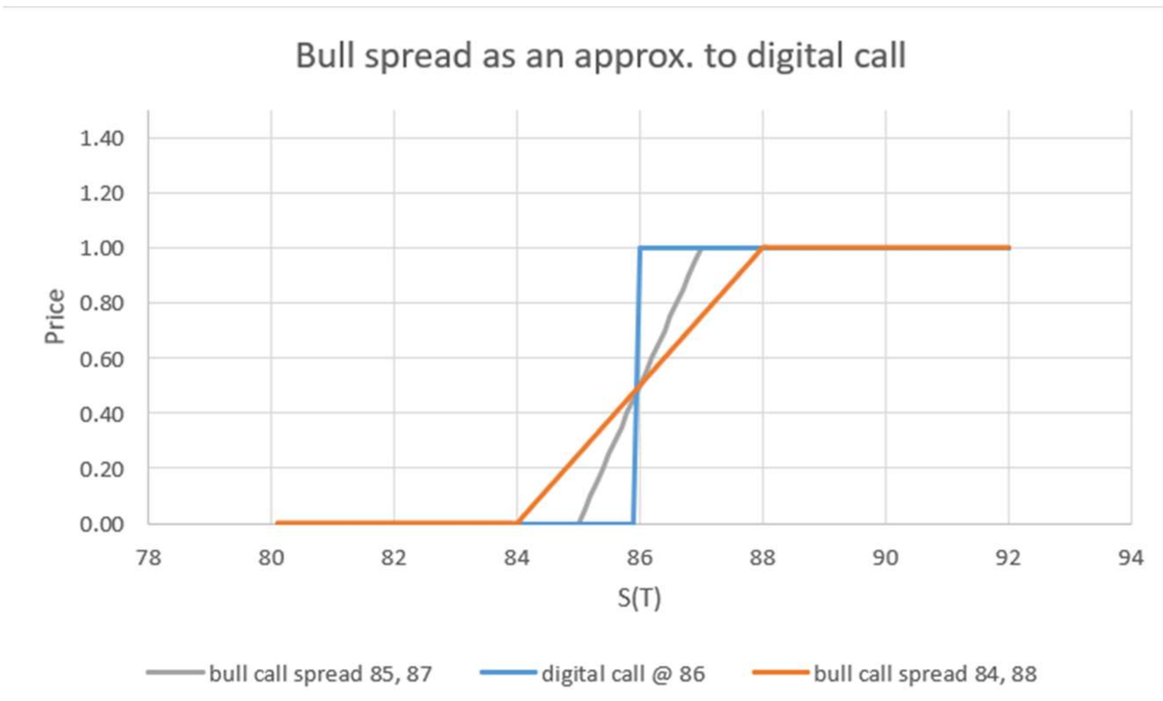
$$1_{S(T)>H}$$

- 1 is the indicator function and H is the barrier.
- The payoff reads: if S(T)>H then it pays \$1, else \$0.



Bull Spread vs Digital Call

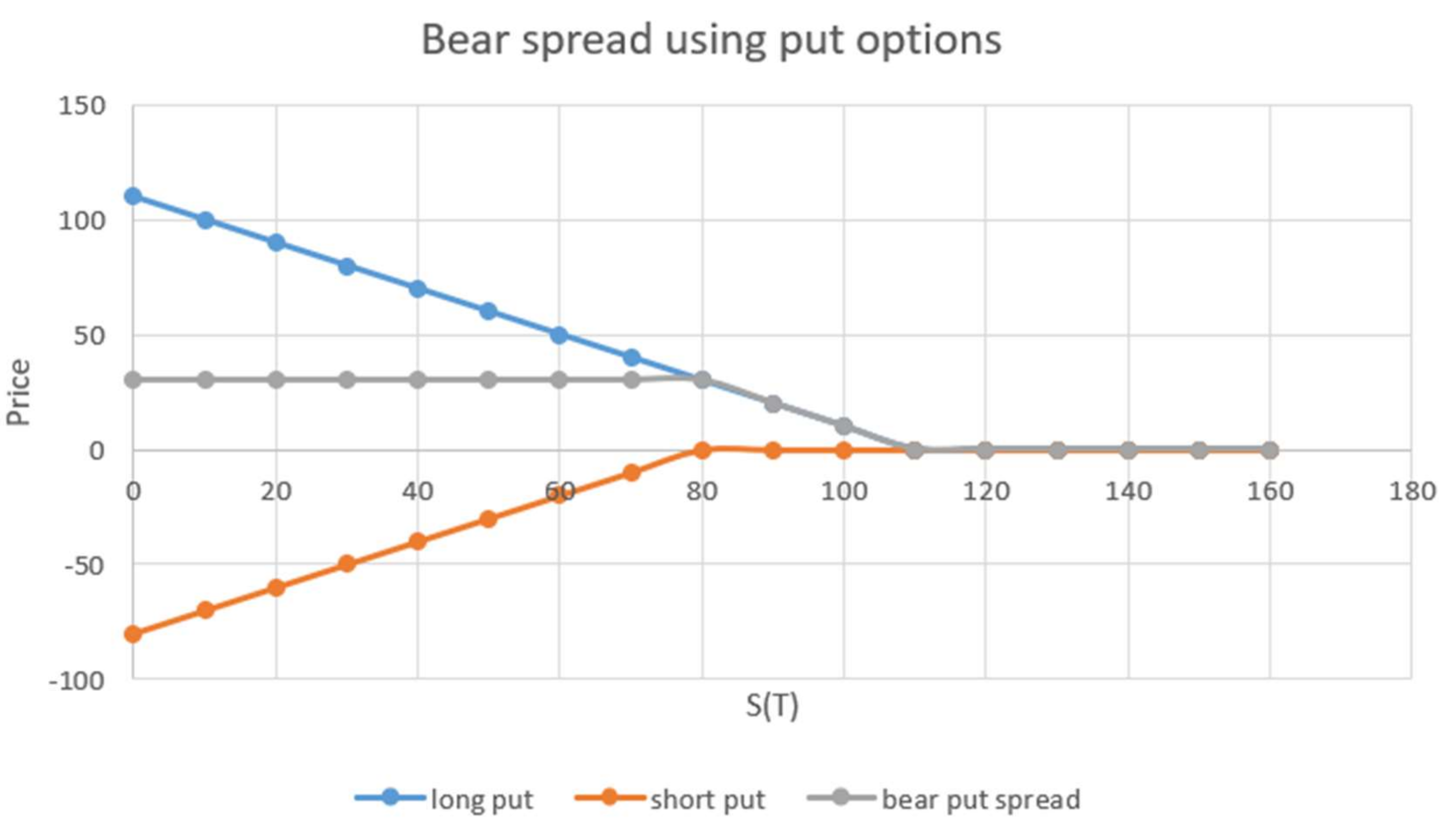
- We can see the similarity between bull spread and digital call.
- In fact, some banks use bull spread to price digital call.



Bear Spreads

- Similar to bull spread but is for expressing a bearish view on the market.
- Trader believes an asset is going to decrease below a specific strike K2 but will not be able to reach below a level K1, K1<K2.
- Bear Spreads(T,T,K1,K2) = Put(T,T,K2) – Put(T,T,K1)

Payoff Diagram of Bear Spreads



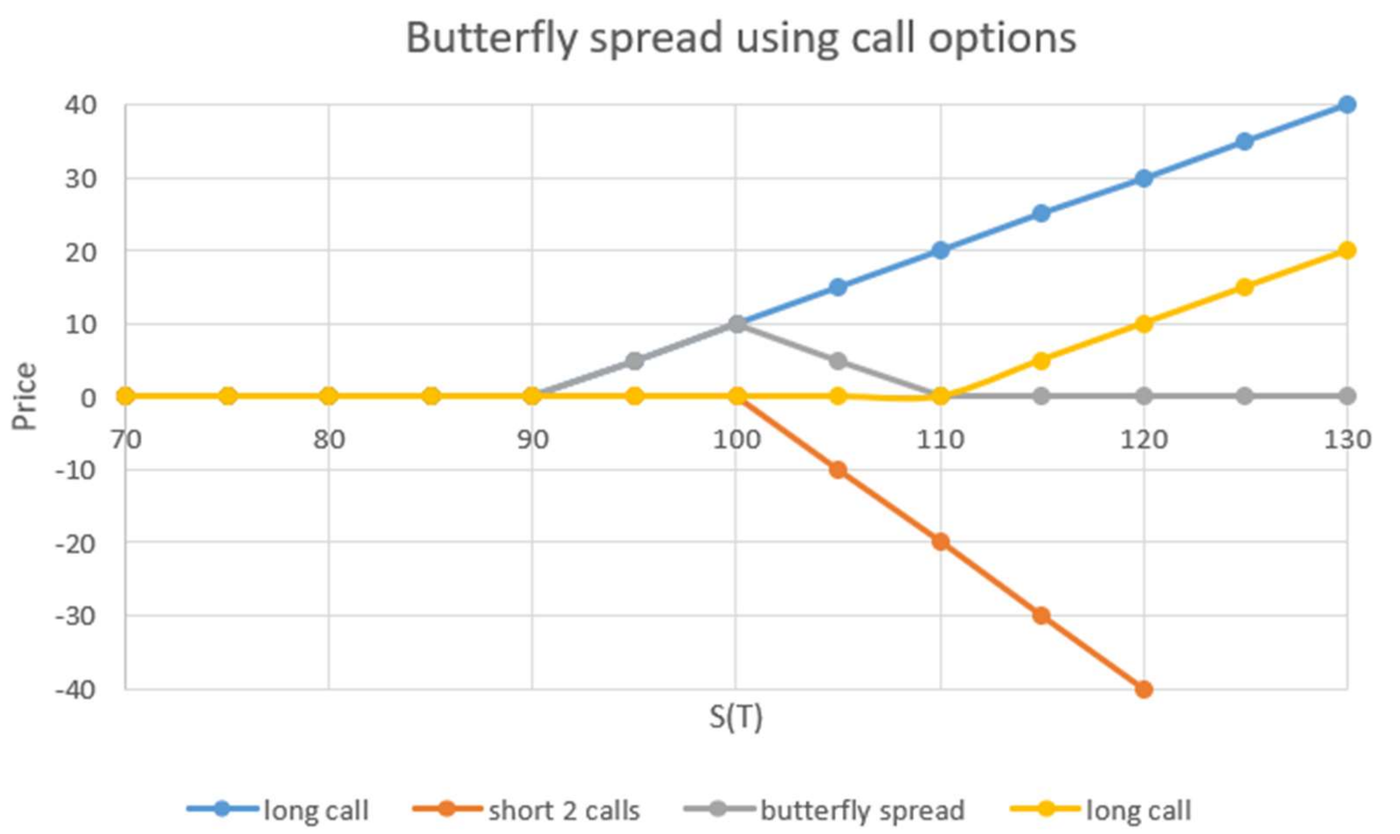
Butterfly Spreads

- It is considered to be a neutral strategy, neither bullish or bearish.
- One can regard it is a combination of a bull and a bear spread.
- Traders use it to express a view that the underlying will be traded within a range.
- There are 3 strikes to specify a butterfly and can be constructed using calls or puts.
- Using calls, the payoff of a butterfly spread:

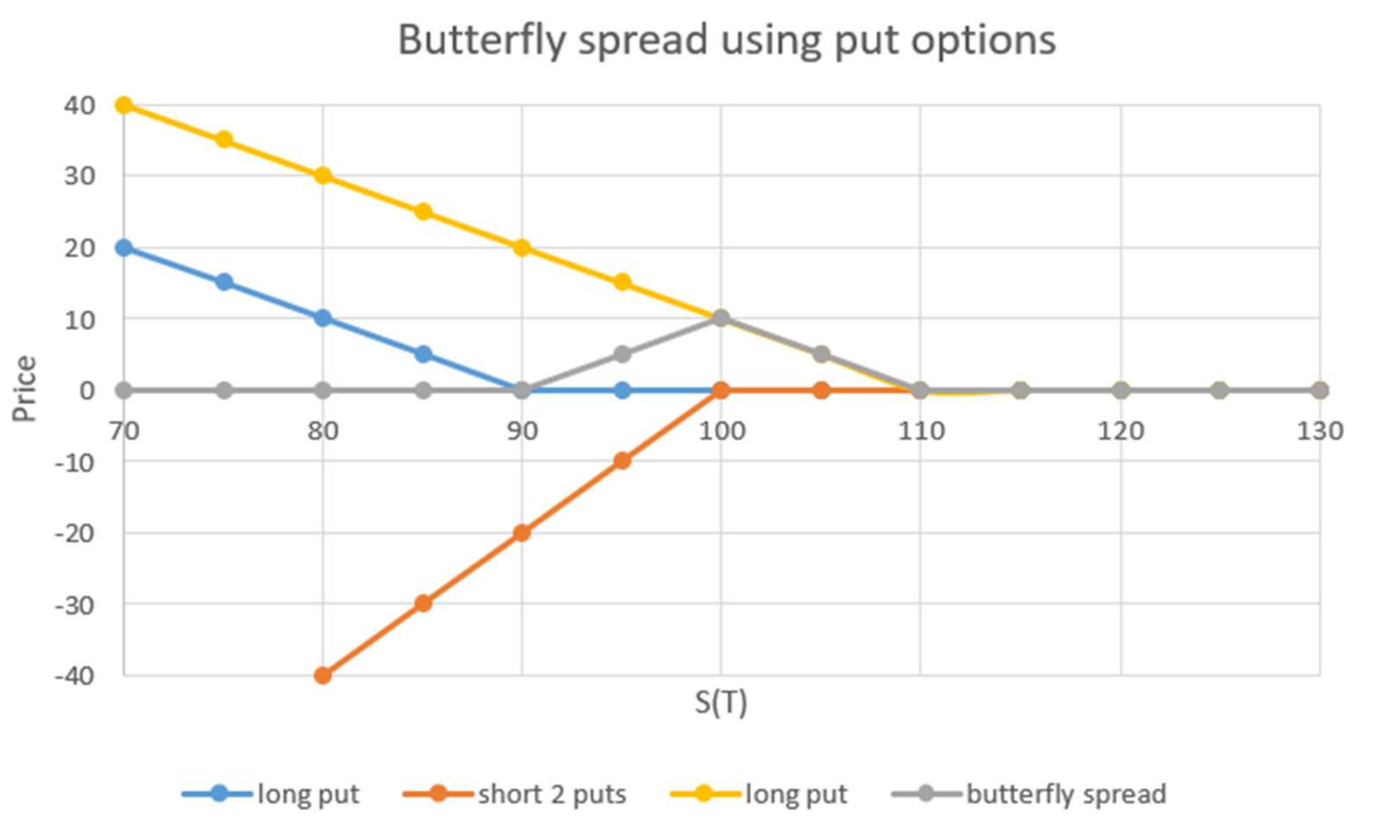
$$Call(T,T,K1) – 2*Call(T,T,K2) + Call(T,T,K3)$$

- $K1<K2<K3$

Payoff Diagram of Butterfly Spreads



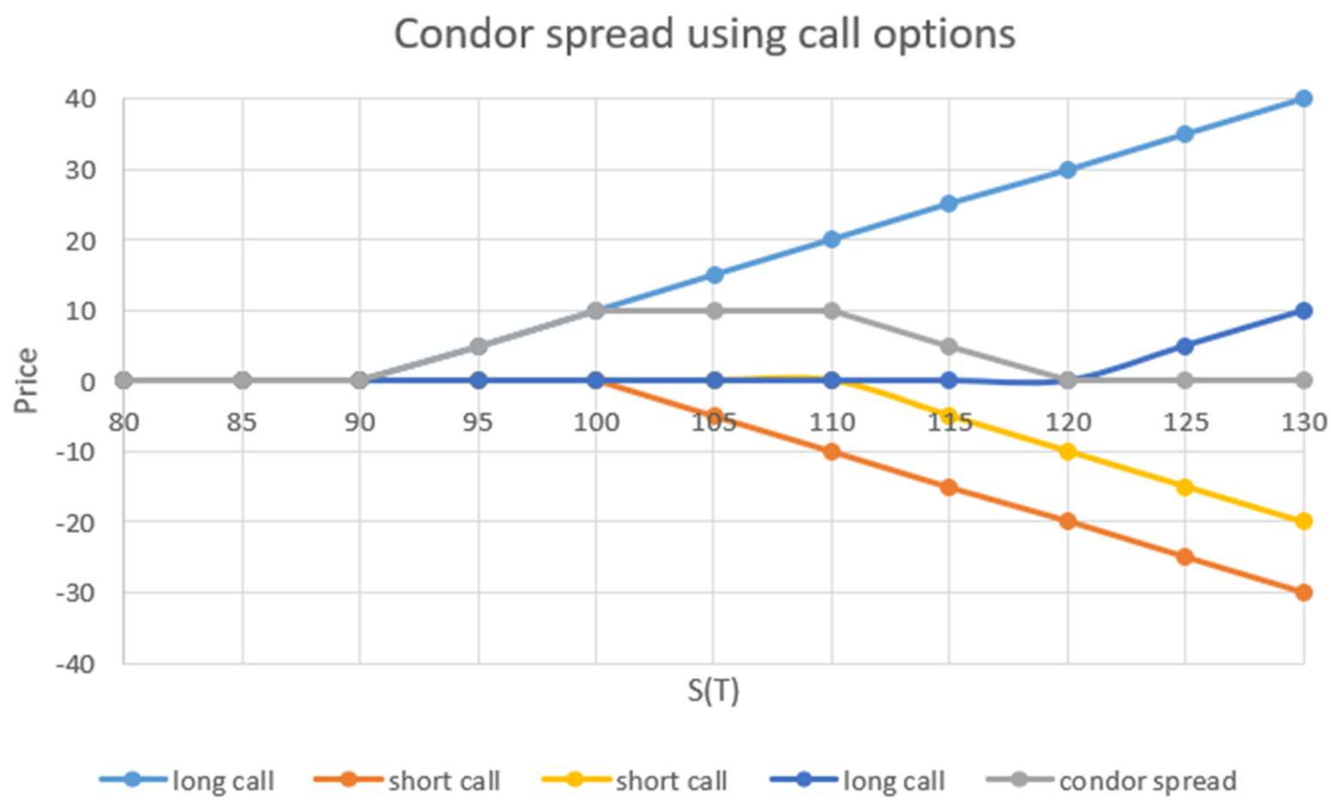
Payoff Diagram of Butterfly Spreads



Condor Spreads

- Condor spreads is similar to the butterfly spreads except it involves 4 different strikes.
- Like butterfly spreads, condor spreads is a short volatility strategy.
- There are 4 strikes to specify a condor and can be constructed using calls or puts.
- Using calls, the payoff of a condor spread:
  Call(T,T,K1) – Call(T,T,K2) – Call(T,T,K3) + Call(T,T,K4)
- K1<K2<K3<K4

Payoff Diagram of Condor Spreads



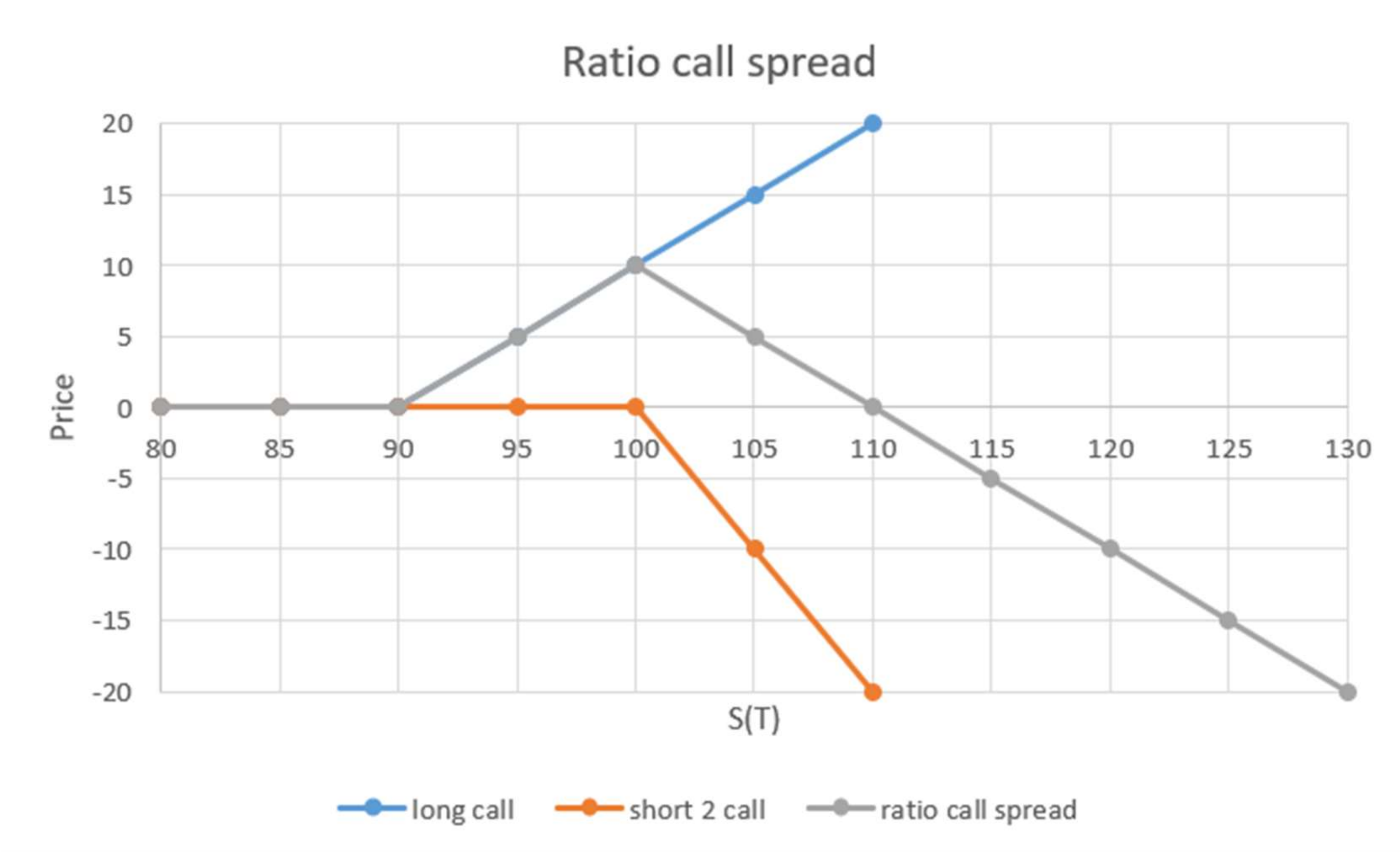
Ratio Spreads

- A generalization of bull or bear spreads.
- The payoff of a ratio call spread is:

$$N1 * Call(T,T,K1) - N2 * Call(T,T,K2)$$

- $K1<K2$, N1 and N2 are the numbers of Calls at K1 and K2 respectively.
- Say $N1=1$, $N2=2$. The premium is lower than bull spreads for the same strikes because selling more calls.
- Upside risk is not protected.

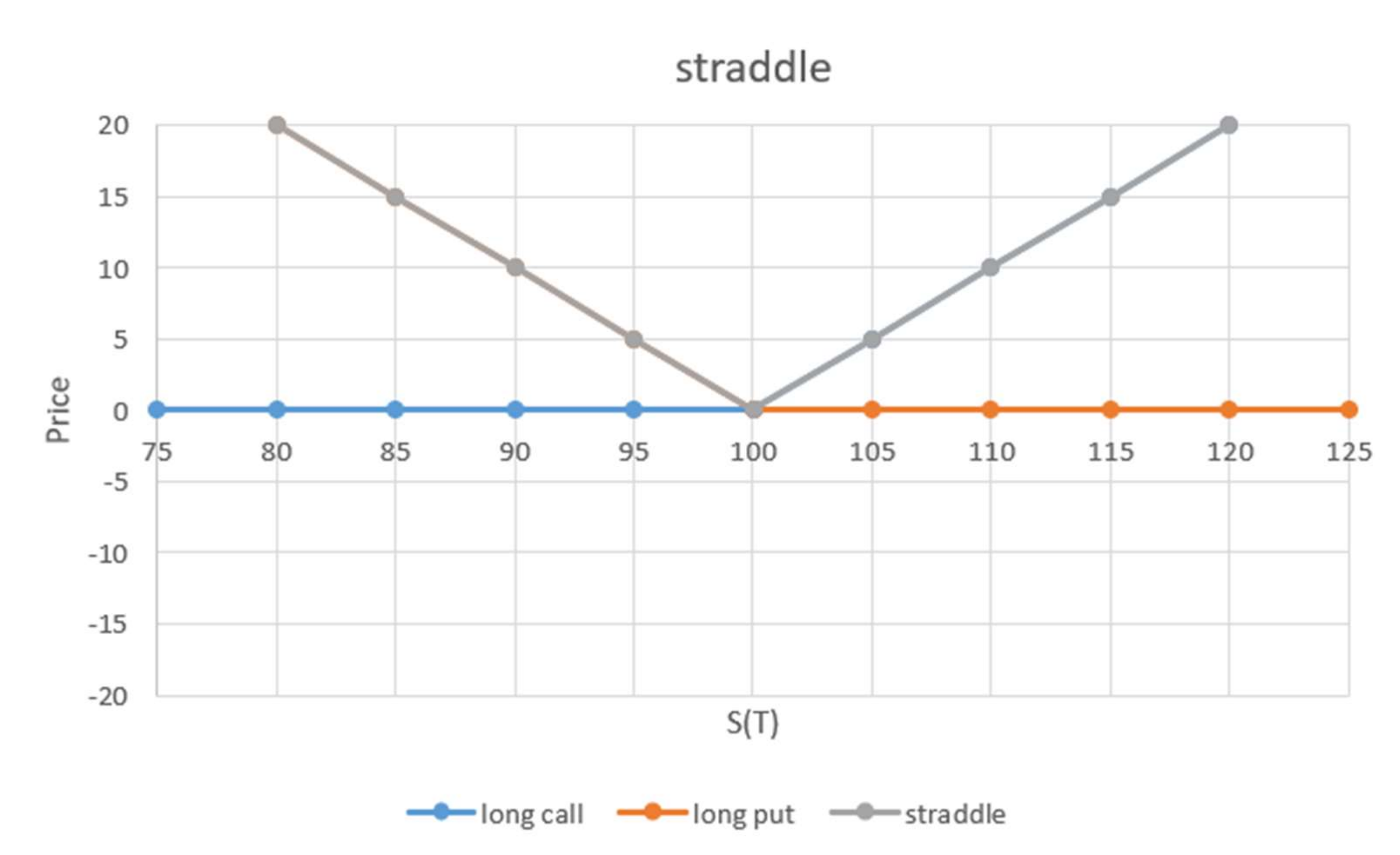
Ratio Call Spreads



Straddles

- Straddles consist of long a call and a put on the same underlying at the same strike.
- The payoff of a straddle is:
    - Call(T,T,K) + Put(T,T,K)
- This is almost a pure volatility trade, i.e. no risk with respective to the underlying directional movement.
- The delta of straddle is close to 0 as the call and put delta cancels each other but the vega is doubled.

Straddles

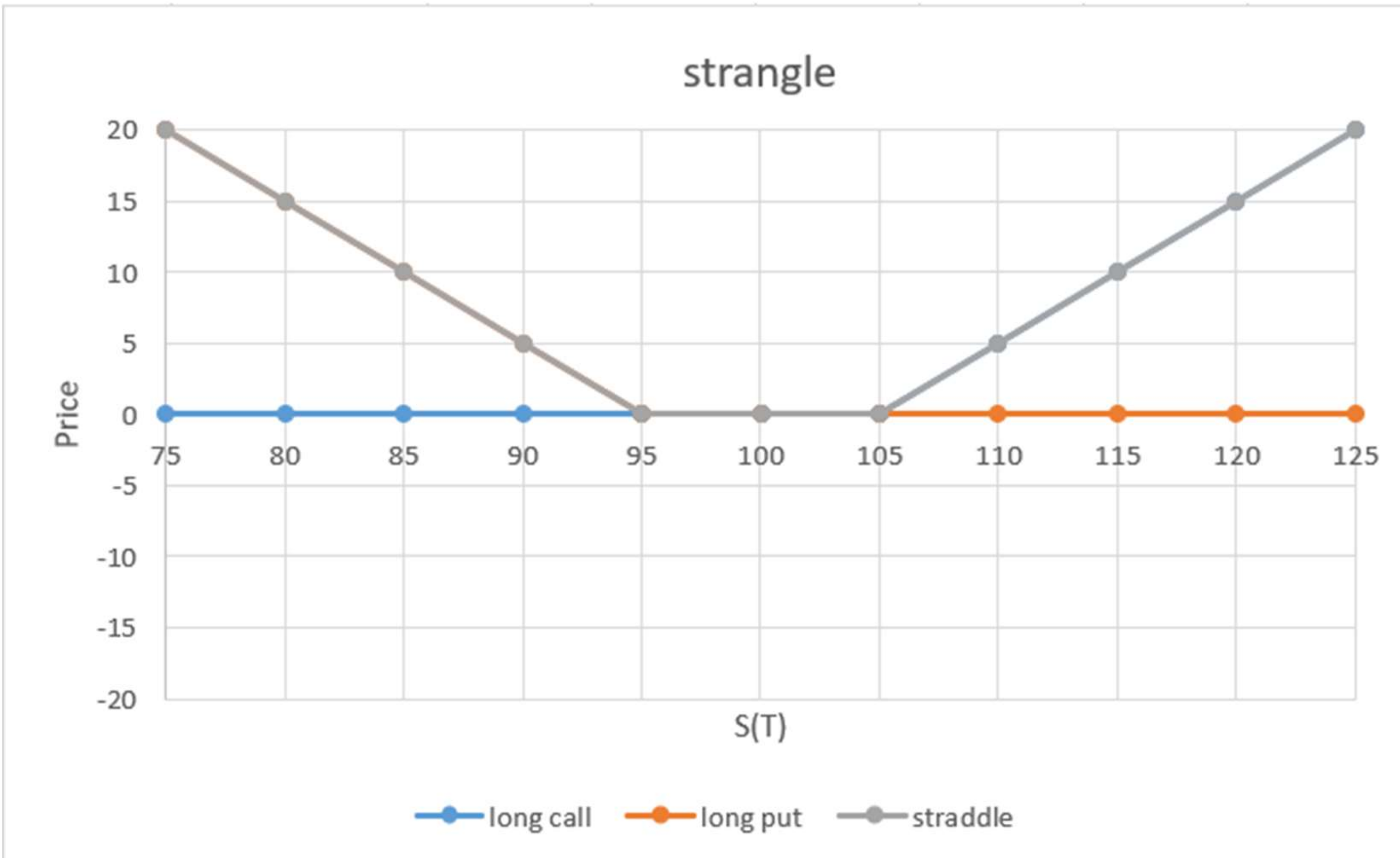


straddle
20
15
10
5
0
-5
-10
-15
-20
Price
75
80
85
90
95
100
105
110
115
120
125
S(T)
long call
long put
straddle

Strangle

- Strangle is a generalization of straddle with call and put are at different strikes.
- The premium is cheaper than straddle.
- The payoff of a strangle is:
    - Call(T,T,K1) + Put(T,T,K2)
- K1<K2
- The delta of straddle can be set to zero depends on the choice of strikes.
- Vega is lower than straddle.

Strangle



strangle
20
15
10
5
0
-5
-10
-15
-20
Price
75
80
85
90
95
100
105
110
115
120
125
S(T)
long call
long put
straddle

Breeden-Litzenberger Formula

- Let C(K) and P(K) be European call and put option prices at K.
- For any twice differentiable European payoff $h(S_T)$, the present value of it, i.e. $V_0 = e^{-rT}E[h(S_T)]$, it can be replicated by a collection of European options:

$$V_0 = e^{-rT}h(x) + h'(x)\big(C(x) - P(x)\big) + \int_0^x h''(K)P(K)dK + \int_x^\infty h''(K)C(K)dK$$

- If we pick x to be the forward price F, the above equation can be reduced to

$$V_0 = e^{-rT}h(F) + \int_0^F h''(K)P(K)dK + \int_F^\infty h''(K)C(K)dK$$

Breeden-Litzenberger Formula

- All option strategies that we discussed are special cases of the B-L formula.
- The implication is that if we know the option prices for all strikes then we can price any European payoffs and the most importantly, the prices are model independent.
- Example: $h(S_T) = S_T^2$

$$V_0 = e^{-rT}F^2 + 2\int_0^F P(K)dK + 2\int_F^\infty C(K)dK$$

- Assume $dS_t = \sigma S_t dW, r = 0, \sigma = 0.2$, we can compute $V_0 = E[h(S_T)]$ analytically:

$$E[h(S_T)] = E[S_T^2] = E\left[\left(S_0 e^{-0.5\sigma^2 T + \sigma\sqrt{T}x}\right)^2\right] = S_0 e^{-\sigma^2 T} E[e^{2\sigma\sqrt{T}x}]$$

$$= S_0 e^{-\sigma^2 T} e^{2\sigma^2 T} = S_0 e^{\sigma^2 T}$$

Breeden-Litzenberger Formula

- Assume S_0, T=1, then

$$V_0 = S_0^2 e^{\sigma^2 T} = 100e^{0.2^2} = 104.08$$

- The B-L formula in the discrete settings:

$$V_0 = S_0^2 + 2\sum_{i=1}^{N} P(K_i)(K_{i+1} - K_i)$$

$$+2\sum_{i=N}^{M} C(K_i)(K_{i+1} - K_i)$$

  - Where N is the number of put options, M is number of call options, $K_N = F$.
  - The more granular the strikes are, the closer you get to the closed form solution.