QF602 Derivatives

Lecture 1 - Introduction to Derivatives

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What are derivatives?

- Financial instruments that "derive" cash flows from price of some other asset.
- Typically, an agreement between two parties to exchange cash flows.
- Derivatives can be used to hedge against unfavorable price changes, or speculate on favorable price changes.
- Hedging means mitigate or reduce risks.

Types of Underlying Assets

- Physical assets such as agricultural products, precious metals, or fossil fuels.
- Financial assets such as stocks or bonds (or even other derivatives!).
- Intangible assets such as electricity or weather.
- Derivatives can also be written on an FX rate, or an interest rate.

Types of Derivatives

- Some Derivatives represent obligations. Both parties are required to follow through on transactions, irrespective of gain or loss.
- Other Derivatives represent options. One party has the right but not the obligation to buy or sell an asset. The other party receives a fee to sell that right.

Where can we trade them?

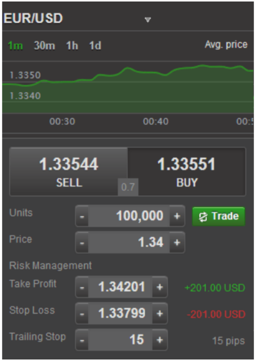
- Some Derivatives are traded on an exchange, such as Chicago Mercantile Exchange (CME).
- Exchange-traded Derivatives feature standardized terms and conditions, high liquidity, and price transparency.
- Clearing house operated by exchange serves as "central counter-party" to ensure that all transactions are completed.

Where can we trade them?

- Other Derivatives, known as "over-the-counter" (OTC) Derivatives, are direct bilateral agreements with no central counter-party.
- Generally, one party will be major financial institution acting as "market maker".
- OTC Derivatives have more flexibility for terms and conditions, but less price transparency and no protection from credit risk in general.

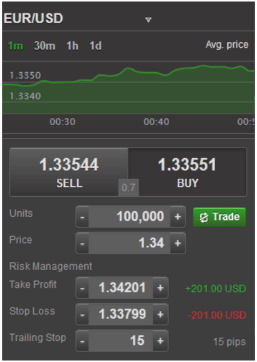
Market Marking

- If you want to long 100k EURUSD, what would you do?
- You find an FX broker, open and account, put some money into the account and then trade.
- Most of them will have a mobile app looks like below.



Market Making - Some FX lingos

- For EURUSD, the 2nd digit after the dot is called "the big figure"/handle (i.e. 3)
- The 4th digit after the dot is called one pip (i.e. 4)
- 100 pips = 1 big figure
- The last digit is 1/10 of a pip.



Market Making

- If you want to buy 100K EURUSD now, the price is 1.33551.
- But if you want to sell 100K EURUSD now, the price is 1.33544.
- Note that if you buy and sell simultaneously then you will guarantee to lose $100k \times (1.33544 - 1.33551) = \$7$.
- Who earns that ? Market makers.
- The \$7 is the bid-offer spread.
- Note that the natural PnL currency for EURUSD is USD. The first currency is called the "Foreign currency", the second currency is called the "Domestic currency".

Market Making

- Market makers are there to provide the liquidity to trade even in extreme market conditions, like the SNB unpegged CHF against EUR on 15 Jan 2015.
- On return, they charge a spread to provide that service.

Market Making

- EURCHF dropped to 0.85 from 1.2 in a matter of minutes.

Market Making

- A few brokers went bust because of that event.

City forex broker and West Ham sponsor goes bust as clients suffer huge losses from Swiss currency bombshell

- **Alpari UK applies for insolvency after clients rack up huge losses**
- **Swiss central bank shocked currency markets by removing euro peg**
- **Positions held by forex traders went haywire as Swiss franc soared 13-15% against euro, sterling and dollar**

By ADRIAN LOWERY FOR THISISMONEY.CO.UK
PUBLISHED: 11:11 GMT, 16 January 2015 | **UPDATED:** 12:23 GMT, 16 January 2015

Types of Derivatives

- Forward contract is an OTC derivative that represents obligation.
- Futures contract is an exchange-traded derivative that represents obligation.
- Option contract is an OTC or exchange-traded derivative that represents option.
- Forward rate agreement (FRA) is forward contract on (short-term) interest rate.
- Interest rate swap (IRS) is sequence of FRAs.
- And many more types....

Exchange-Traded vs OTC

- Central counterparty eliminates credit (default) risk
- Standardized products and pricing transparency
- Both parties subject to credit (default) risk
- Flexible products but pricing opacity

Credit Risk

- Credit risk is a risk that counterparty will default on terms of agreement.
- If a derivative represents obligations, then both parties will be exposed to credit risk (since either party could potentially incur loss).
- If a derivative is an option, in general, only the party with the right (long the option) will be exposed to credit risk.
- For OTC Derivatives, both parties are exposed to credit risk, although market maker is unlikely to default.
- In some cases, market maker might demand collateral or imposed by central counter-party (CPP).
- For exchange-traded derivatives, CPP bears all credit risk while customers receive almost complete protection.

Variation Margin

- In order to mitigate credit risk, banks usually sign a Credit Support Annex (CSA) between each other.
- Roughly speaking, CSA is a legal document that is part of an ISDA master agreement that describes the terms of how to post/receive collateral due to the mark-to-market (mtm) of the derivatives traded between the banks.
- The bank that has negative mtm needs to post the mtm amount to the other bank on a daily basis (depends on the terms of the CSA). This is the so-called variation margin.

Initial Margin

- Recently, for all global systematically important banks (G-SIBs), all non-CCP cleared derivatives are required to post initial margin regardless if a CSA is signed or not.
- This is to cover the "grace period risk". Roughly speaking, banks won't be asked to file bankrupcy if they miss a collateral posting by a short period of time. The grace period is usually 10 working days.
- Market can move a lot within the grace period.
- The industry is moving towards Standard Initial Margin Model (SIMM).

Credit Valuation Adjustment (CVA)

- CSA is usually signed between financial institutions because they have the infrastructure and the liquidity to support this.
- For small-medium sized corporate who use derivatives to hedge may be not feasible to ask them to post/receive collateral. This is a source of credit risk.
- In order to protect the bank in case the corporate defaults while the derivative has a positive mtm for the bank. The bank needs to charge an extra to buy "an insurance" for the transactions. The extra charge is CVA.

Time Value of Money

- \$100 to be paid 1 year from now (Future Value).
- 5% interest rate per annum, denote as r.
- Present value (PV) can be computed as PV = Future Value x Discount Factor.
- $DF(0, T) = exp(-rT) = 0.9512$, where $T = 1, r = 5\%$.
- We assume continuous compounding in this course unless otherwise stated.

Forward contract

- A forward contract is an agreement between 2 parties to buy or sell an asset at maturity.
- No cash flows until maturity.
- The price agreed to buy or sell the asset at maturity is call the strike price, K.
- The value of the forward contract V at t is computed as:

$$V(t) = (F(t, T) - K) \times DF(t, T)$$

.

- The forward contract that has 0 value at inception has strike price equals to the forward price at time 0, i.e. $V(0) = 0$, if $F(0, T) = K$.

Forward price

- $S(t)$ denotes the price of a stock at time t.
- $F(t, T)$ denotes the forward price at time t with maturity T, $t \leq T$. Note that $F(T, T) = S(T)$.
- Consider a stock that pays no dividends and is worth \$50, $S(0) = \$50$.
- 6 month interest rate is 6% per annum, $r = 6\%$.
- Forward price of a stock can be viewed as equal to the spot price plus the cost of carrying it.
- The cost of carry is equal to the interest that might be received if he had immediately sold the stock and invested in a risk-free investment.

Forward price

- Forward price can be computed as:

$$F(t, T) = S(t)exp(r(T - t)) = S(t)/DF(t, T)$$

To compute a 6 month forward price, we can set t=0, T=6/12 and we get

$$F(0, 6/12) = \$50 \times exp(6\% \times 6/12) = \$51.52.$$

Forward price

- So what if the stock provides an additional income to the stockholder?
- Stock pays dividend and one can also lend out the stock to earn an extra income via repurchasing agreement, aka repo.
- The cost of carry decreases.
- Dividend yield = 1% p.a., repo rate = 2% p.a. for 6 month period.
- The 6 month forward price now becomes

$$\$50 \times exp((6\% - 1\% - 2\%) \times 6/12) = \$50.76.$$

Futures contract

- A futures contract is an exchange-traded contract in which the holder has the obligation to buy/sell an asset on a future date at a market-determined price called the futures price.
- The contract specifications like quantity, time and place of delivery are determined by the exchange.
- The assets can be a commodity, a stock, an index, etc. . .
- Futures is considered safer than forward since the counterparty risk is almost totally eliminated.
- Futures contract are marked-to-market on a daily basis.
- Every buyer/seller must maintain a certain amount with an account at the exchange.
- If the price movement is against you, the balance of your account reduces. If it drops below the maintenance margin then you will need to top up to the initial margin.

Example: Mark-to-Market

- Futures price: \$100
- Initial margin: \$7 per contract
- Maintenance margin: \$4 per contract
- Investor A buys 500 contracts, while investor B sells (shorts) 500 contracts
- Initial margin: $\$500 \times \$7 = \$3,500$
- Maintenance margin: $\$500 \times \$4 = \$2,000$

Example: Mark-to-Market

Trading Day	Settlement Price
1	\$99
2	\$97
3	\$98
4	\$95

- Settlement price is the price used for mark-to-market.

Investor A's Margin Account.

- Day 1
  - PnL $= (\$99 - \$100) \times 500 = -\$500$
  - Balance $= \$3500 - \$500 = \$3000$
- Day 2
  - PnL $= (\$97 - \$99) \times 500 = -\$1000$
  - Balance $= \$3000 - \$1000 = \$2000$
- Day 3
  - PnL $= (\$98 - \$97) \times 500 = \$500$
  - Balance $= \$2000 + \$500 = \$2500$
- Day 4
  - PnL $= (\$95 - \$98) \times 500 = -\$1500$
  - Balance $= \$2500 - \$1500 = \$1000$
  - Receives a margin call to deposit \$2500 to bring the balance up to initial margin.
- Total PnL $= \$1000 - \$3500 = -\$2500$

Investor B's Margin Account.

- Day 1
  - PnL $= (\$100 - \$99) \times 500 = \$500$
  - Balance $= \$3500 + \$500 = \$4000$
- Day 2
  - PnL $= (\$99 - \$97) \times 500 = \$1000$
  - Balance $= \$4000 + \$1000 = \$5000$
- Day 3
  - PnL $= (\$97 - \$98) \times 500 = -\$500$
  - Balance $= \$5000 - \$500 = \$4500$
- Day 4
  - PnL $= (\$98 - \$95) \times 500 = \$1500$
  - Balance $= \$4500 + \$1500 = \$6000$
- Total PnL $= \$6000 - \$3500 = \$2500$

Futures Price vs Forward Price

- A Futures price may be different from a forward price when a margin account earns interest.
- If risk-free interest rate is constant, then the futures price will be the same as the forward price.
- If risk-free interest tends to move in the same direction as the futures price (i.e. positively correlated), then the long party tends to benefit at expense of the short party.
- To compensate, the futures price will be higher than the corresponding forward price.
- Conversely, if risk-free interest tends to move in the opposite direction as the futures price (i.e., negatively correlated), then the short party tends to benet at expense of the long party.
- To compensate, the futures price will be lower than the corresponding forward price.

"Proof" of Futures price equals to Forward price if interest rates are constant.

- Assume the margin is paid every Δ years, and that the maturity $T = n\Delta$. The interest rate is r. Consider the following strategy:
- At time 0, long $e^{-r(n-1)\Delta}$ futures contracts at futures price $\mathcal{F}(0, T)$.
- At time Δ, increase the position to $e^{-r(n-2)\Delta}$ contracts at futures price $\mathcal{F}(\Delta, T)$. Note that we can do this at no cost.
- At time $i\Delta$, increase the position to $e^{-r(n-i-1)\Delta}$ contracts at futures price $\mathcal{F}(i\Delta, T)$ for $i = 2, ..., n-2$.
- At time $(n-1)\Delta$, increase the position to 1.

"Proof"

- With this strategy, we receives:
  - At time Δ, we receive mtm gain of $(\mathcal{F}(\Delta, T) - \mathcal{F}(0, T))e^{-r(n-1)\Delta}$. We can invest this at rate r and thus it compounds up by $e^{r(n-1)\Delta}$ to time $n\Delta = T$.
  - At time $(i+1)\Delta$, we receive mtm gain of $(\mathcal{F}((i+1)\Delta, T) - \mathcal{F}(i\Delta, T))e^{-r(n-i-1)\Delta}$, which compounds up by $e^{r(n-i-1)\Delta}$ to time T.
- Therefore, the value at time $T = n\Delta$ of the cash flows received from mtm gains or losses is

$$
\begin{aligned}
& \sum_{i=0}^{n-1}(\mathcal{F}((i+1)\Delta, T) - \mathcal{F}(i\Delta, T))e^{-r(n-i-1)\Delta}e^{r(n-i-1)\Delta} \\
= \; & \mathcal{F}(n\Delta, T) - \mathcal{F}(0, T) \\
= \; & S(T) - \mathcal{F}(0, T).
\end{aligned}
$$

"Proof"

- So an initial holding of $e^{-rT}\mathcal{F}(0,T)$ of cash, plus the cost-less futures trading strategy given above, results in a portfolio with value at T:

$$\mathcal{F}(0,T) + (S(T) - \mathcal{F}(0,T)) = S(T).$$

- However, consider the portfolio consisting of $e^{-rT}F(0,T)$ of cash invested at r, plus one forward contract. This is also worth $S(T)$ at time T. Therefore

$$\mathcal{F}(0,T) = F(0,T)$$

.

Common futures contracts

- The most liquid futures contracts in the world are futures on US Treasuries (the bond future) and German government bonds (the bund future). Here the bond price and interest rate are almost always negative correlated. Therefore, the bond price and money market account are negatively correlated, and so in general $\mathcal{F}(t, T) < F(t, T)$.
- However, since most liquid bond futures have maturities of three months or less, the size of this convexity correction is often small.

Option Contracts

- Call option confers right (but not obligation) for holder (or long party) to buy underlying asset from writer (or short party)
- Put option confers right (but not obligation) for holder (or long party) to sell underlying asset to writer (or short party)
- Long party makes upfront payment of option premium (or price or value) to short party
- Transaction occurs when option is exercised,at fixed exercise price (or strike price)
- Option expires if not exercised before date of expiration (or maturity)

Option Contract

- European-style option may only exercised on date of expiration
- American-style option may be exercised at any time up to date of expiration
- Bermudan-style option may be exercised at xed times up to day of expiration (i.e., half way between Europe and America.)
- Option may be exchange-traded or OTC
- Exchange-traded options tend to be more standardized and liquid then OTC options
- Long party may be exposed to default risk for OTC options (but not short party)

Example: Option on IBM

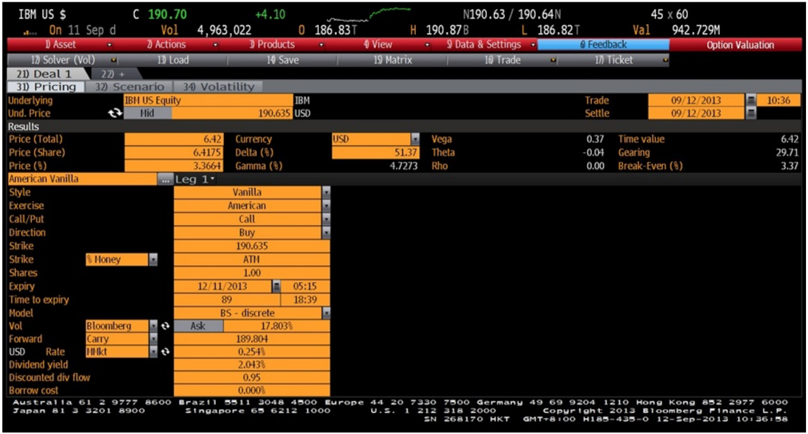


Figure: Bloomberg option pricer

IBM stock price

Figure: IBM stock price.

Moneyness

- Let $S(t)$ be the spot price of an underlying asset at time t.
- Let K be the exercise price.
- Option is in-the-money when exercise is profitable: $S(t) > K$ for call and $S(t) < K$ for put.
- Option is out-of-the-money when exercise is not profitable: $S(t) < K$ for call and $S(t) > K$ for put.
- Option is at-the-money when $S(t) = K$.
- Note that this is not the only definition of moneyness.
- Moneyness can be set with respective to forward in some market like interest rates option.
- For FX option, ATM strike is the strike such that a straddle has no delta. See Chapter 3.5 in Foreign Exchange Option Pricing: A Practitioner's Guide by Iain Clark.

Option Value

- Option Value = Intrinsic Value + Time Value.
- Intrinsic value is payoff from immediate exercise: $max(S(t) - K, p)$ for call option and $max(K - S(t), 0)$ for put option.
- Time value is value of not exercising since intrinsic value may increase if we don't exercise.

Put-Call Parity

- Let $c(t)$ and $p(t)$ be values of European call and put respectively, with the same underlying, exercise price K and maturity T.
- Combination of long call and short put at the same strike delivers the same payoff as long a forward contract.
- If no cash flows for the underlying asset before maturity T, then no-arbitrage relation for put-call parity holds:

$$c(t) - p(t) = (F(t, T) - K)DF(t, T)$$

.

Put-Call Parity

- Rearrange to get different interpretation:

$$c(t) + K \times DF(t, T) = p(t) + F(t, T) \times DF(t, T)$$

- Left side represents a portfolio A, where enough money is deposited into an interest-bearing account to cover exercise of the call option.
- Right side represents a portfolio B, where put provides protection against drop in value of the underlying asset.

Example: Put-Call Parity

- Call price: \$7.5.
- Put price: \$4.25.
- Exercise price: \$100.
- Underlying price: \$99.
- Time to maturity: 6 months.
- Risk-free interest rate: 10%, $DF(0, 0.5) = 0.9512$.
- Forward price $F(0, 0.5) = \$99/0.9512 = \$104.08$
- Dividend yield and repo rate $= 0$.

Example: Put-Call Parity

- Value of portfolio A:

$$7.5 + 100 \times 0.9512 = 102.62.$$

- Value of portfolio B:

$$4.25 + 104.08 \times 0.9512 = 103.25.$$

- B is worth more than A, so arbitrage opportunity exists.
- Buy low, sell high: short B and buy A for immediate profit of 63 cents.

American option

- European option can only be exercised at the maturity.
- American option can be exercised at any time before the maturity.
- $c(t), p(t)$ be an European call and put and $C(t), P(t)$ be an American call and put.
- It is obvious that $C(t) \geq c(t)$ and $P(t) \geq p(t)$ if all other terms are the same.
- Is there a situation that $C(t) = c(t)$ and $P(t) = p(t)$?

American call option

- Without dividends, never exercise an American call early.
- Exercise early requires paying the exercise price early, hence loses the time value of money because he doesn't receive interest on this cash amount.
- On the other hand, he would receive future dividends for holding the stock.
- If dividend yield is higher than the interest rate then it MAY BE optimal to exercise.

American put option

- Without dividends, it can be optimal to exercise an American put early.
- consider a put with $K = 100$ on a stock with S(t) very very close to 0.
- $S(t)$ cannot go any lower and the put option pays $max(K - S(t), 0) = 100$ is the maximum that one can earn for holding this put option.
- Exercise now gives 100 today.
- Exercise later gives 100 later.

Black Scholes

Key assumptions:

- Volatility is constant over time.
- Underlying is traded continuously and is log-normally distributed.
- One can always short sell.
- No transaction costs.
- One can sell any fraction of a share.
- One can borrow and lend cash at a constant risk free rate.
- Stock pays a constant dividend yield.

Risk neutral pricing

- The fundamental assumption behind risk-neutral pricing is to use a replicating portfolio of assets with known prices to remove any risk.
- In Black Scholes world, options are considered to be redundant in the sense that one can replicate the payoff of an European option on stock using the stock itself and risk-free bonds.
- Since options can be replicated and their theoretical values do not depend upon investors' risk preferences.
- The idea of replication is one of the most important contributions by Black and Scholes.

Black Scholes Formula

- Let $S(0)$ be the spot price at time 0.
- σ be the volatility of the log return of the underlying.
- r and q be the interest rate and dividend yield respectively.
- Forward price at time 0 with maturity T is

$$F(0, T) = S(0)e^{(r-q)T}$$

- The price of an European call option is given by

$$DF(0, T)(F(0, T)N(d_1) - KN(d_2))$$

$$d_1 = \frac{ln(F/K) + \sigma^2 T}{\sigma\sqrt{T}}, d_2 = d_1 - \sigma\sqrt{T}$$

$$N(a) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{a} e^{-u^2/2} du$$

Black Scholes Formula

- The price of an European put option is given by

$$DF(0,T)(KN(-d_2) - F(0,T)N(-d_1))$$

- Note that $N(-a) = 1 - N(a)$.
- It is easy to show that Black Scholes Call minus Black Scholes Put becomes

$$DF(0,T)(F(0,T) - K).$$

- Put Call parity works!!!!!
- In fact, this is a model-free result and must be satisfied by any non-arbitrage models.