QF602 Derivatives

Lecture 2 - Option Strategies and Exotics

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Option Strategies

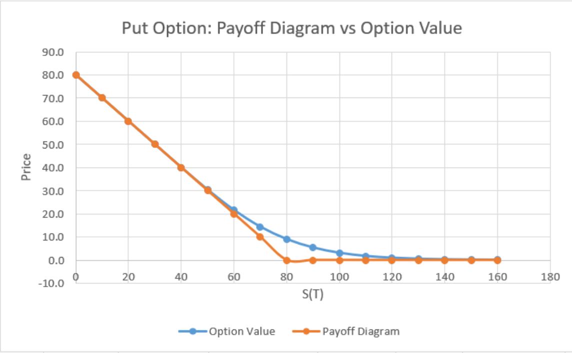
- We will look at the payoff of different portfolios of stocks and options in this lecture.
- The portfolio is also called option strategy.
- They are used to express trader's view of the future of the underlying (i.e. speculation) or for hedging

Option Strategies

- Protective Put
- Covered Call
- Bull spread
- Bear spread
- Butterfly spread
- Condor spread
- Ratio Spread
- Straddle
- Strangle

Notations

- Let $Put(t, T, K)$ be the value of an European put option with strike K at time t.
- The payoff of an European put option can be represented as $Put(T, T, K)$, i.e. the intrinsic value.

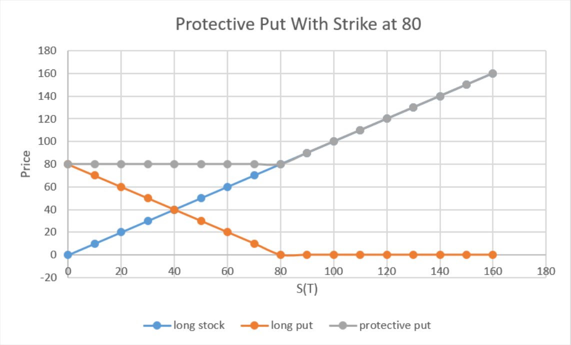


Protective Put

- A portfolio consists of a put option and the underlying stock.
- Long protective put is equivalent to holding a stock and a put option on the same stock.
- The main motivation for buying a protective put is mainly for hedging.
- A fund manager holds 10,000 stocks of IBM at 80.
- The stock is trading at 90.
- He is worried about the market going down during the next 3 months but doesn't want to sell his shares.
- The fund manager decides to buy 10,000 3-month European puts on IBM with strike at 80.
- Buying a protective put enables an investor to fix the maximum loss that he could potentially suffer.

Payoff Diagram for Protective Put

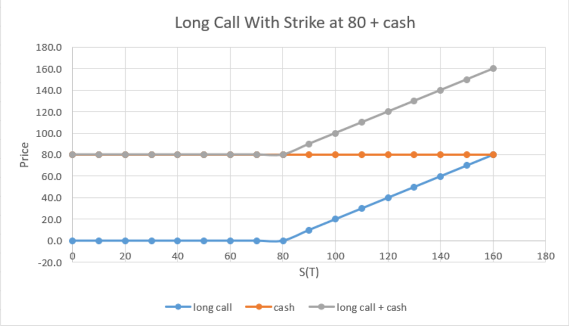
- Note that the premium of the option is not included in the payoff diagram.



Equivalent to Call + Cash

- Applying put-call parity, we can show that

$$Put(T, T, K) + S(T) = Call(T, T, K) + K$$



Covered Call

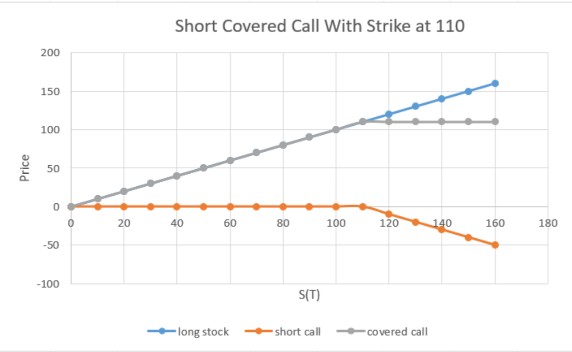
- A portfolio consists of a call option and short selling the underlying stock.
- The payoff of a covered call is

$$Covered\_Call(T, T, K) = Call(T, T, K) - S(T)$$

- Writing a covered call is a popular hedging strategy.
- A trader holds some shares and believes the price will increase in the long term but decline in the short term.
- The trader will receive an extra income in the form of call option premium.

Payoff Diagram for Short Covered Call

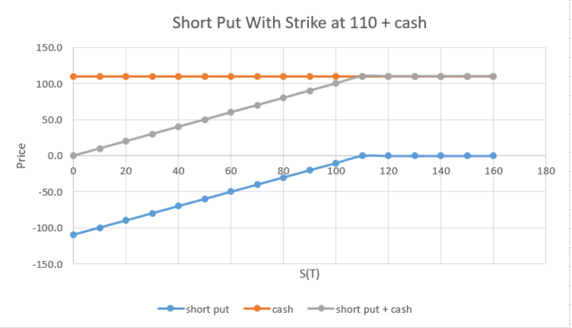
- The trader "gives up" the upside above 110 until the maturity of the option.
- To compensate the potential expected short term stock price decrease.



Equivalent to Short Put plus Cash

- We can show that writing a covered call is equivalent to short a put plus cash:

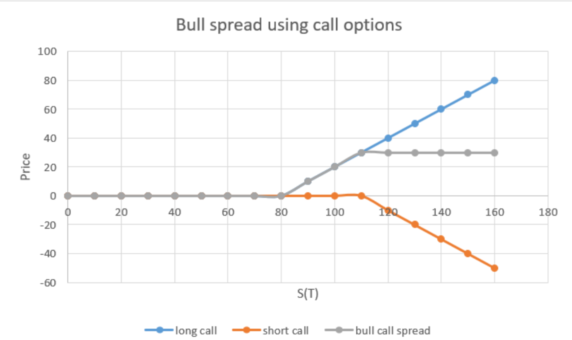
$$S(T) - Call(T, T, K) = Put(T, T, K) + K$$



Bull Spreads

- Bull spreads (or call spreads) are one of the most popular strategies and corresponding to a bullish view on the market.
- Trader believes an asset is going to increase above a specific strike K1 but will not be able to reach a level K2, K1<K2.

$$Bull\_Spreads(T, T, K1, K2) = Call(T, T, K1) - Call(T, T, K2)$$

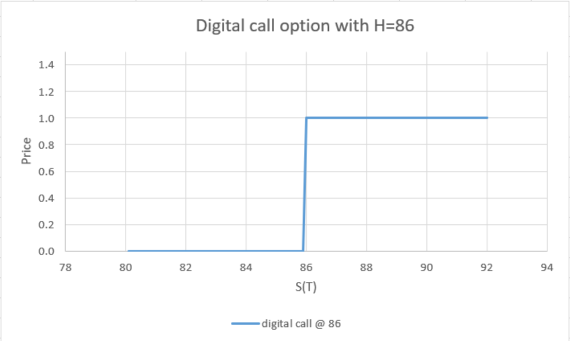


Digital Call Option

- Digital options, aka binary options, pay a specific coupon when a barrier or trigger event occurs.
- European digital call option has the payoff

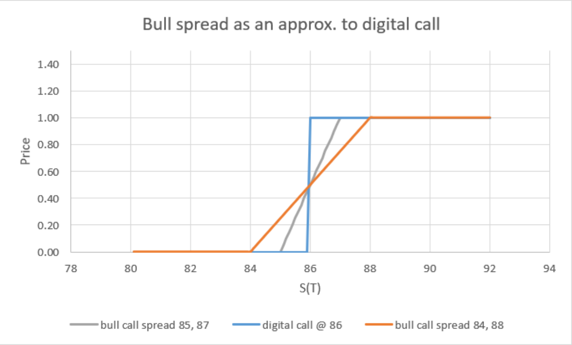
$$1_{S(T)>H}$$

- 1 is the indicator function and H is the barrier.
- The payoff reads: if $S(T) > H$ then it pays \$1, else \$0.



Bull Spread vs Digital Call

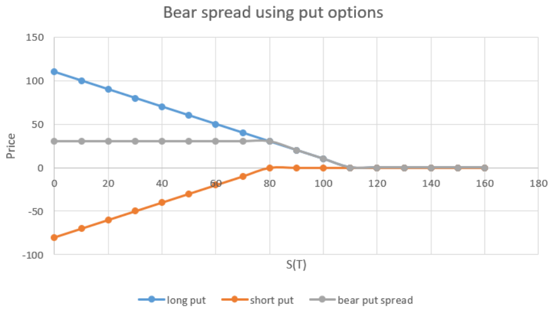
- We can see the similarity between bull spread and digital call.
- In fact, banks use bull spread to price digital call.



Bear Spreads

- Similar to bull spread but is for expressing a bearish view on the market.
- Trader believes an asset is going to decrease below a specific strike K2 but will not be able to reach below a level K1, K1<K2.

$$Bear\_Spreads(T,T,K1,K2) = Put(T,T,K2) - Put(T,T,K1)$$

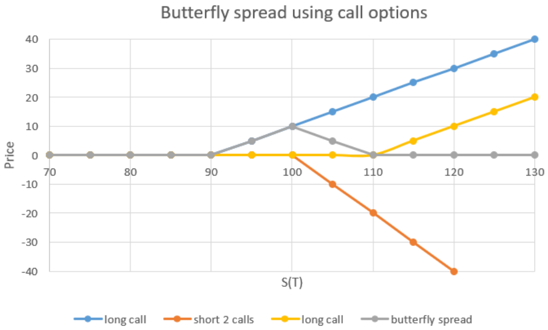


Butterfly Spreads

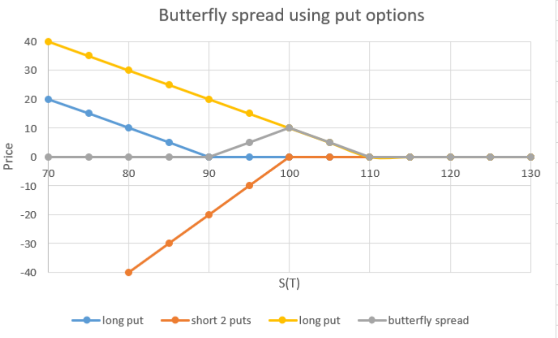
- It is considered to be a neutral strategy, neither bullish or bearish.
- One can regard it is a combination of a bull and a bear spread.
- Traders use it to express a view that the underlying will be traded within a range.
- There are 3 strikes to specify a butterfly and can be constructed using calls or puts.
- Using calls, the payoff of a butterfly spread, $K1 < K2 < K3$

$$Call(T, T, K1) - 2Call(T, T, K2) + Call(T, T, K3)$$

Payoff Diagram of Butterfly Spreads



Payoff Diagram of Butterfly Spreads

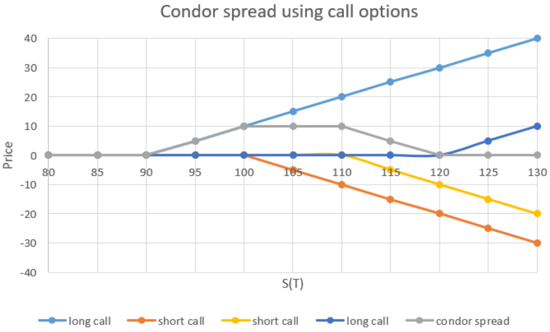


Condor Spreads

- Condor spreads is similar to the butterfly spreads except it involves 4 different strikes.
- Like butterfly spreads, condor spreads is a short volatility strategy.
- There are 4 strikes to specify a condor and can be constructed using calls or puts.
- Using calls, the payoff of a condor spread, $K1 < K2 < K3 < K4$

$$Call(T, T, K1) - Call(T, T, K2) - Call(T, T, K3) + Call(T, T, K4)$$

Payoff Diagram of Condor Spreads



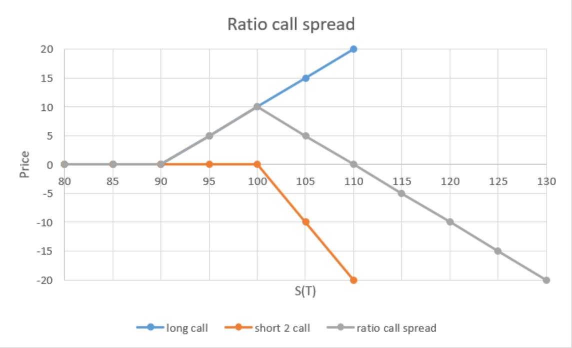
Ratio Spreads

- A generalization of bull or bear spreads.
- The payoff of a ratio call spread is:

$$N1Call(T, T, K1) - N2Call(T, T, K2)$$

- $K1 < K2$, $N1$ and $N2$ are the numbers of Calls at $K1$ and $K2$ respectively.
- Say $N1 = 1, N2 = 2$. The premium is lower than bull spreads for the same strikes because selling more calls.
- Upside risk is not protected.

Ratio Call Spreads



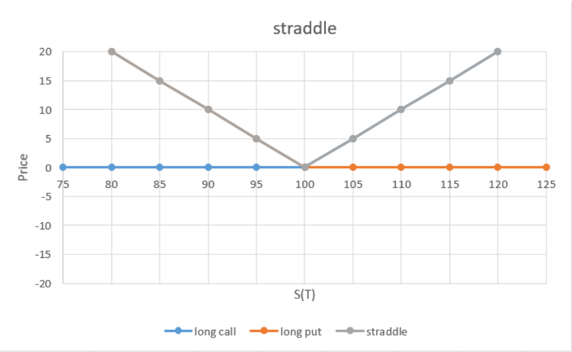
Straddles

- Straddles consist of long a call and a put on the same underlying at the same strike.
- The payoff of a straddle is:

$$Call(T, T, K) + Put(T, T, K)$$

- This is almost a pure volatility trade, i.e. no risk with respective to the underlying directional movement.
- The delta of straddle is close to 0 as the call and put delta cancels each other but the vega is doubled.

Straddles


straddle
20
15
10
5
0
-5
-10
-15
-20
Price
75
80
85
90
95
100
105
110
115
120
125
S(T)
long call
long put
straddle

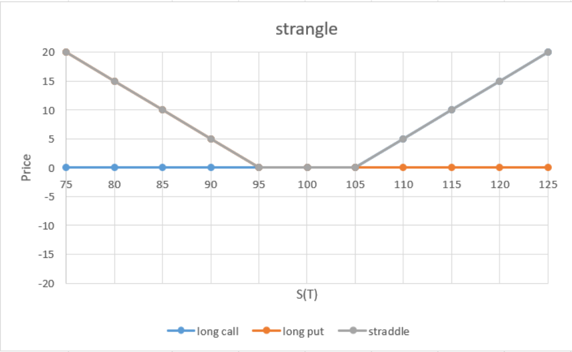
Strangle

- Strangle is a generalization of straddle with call and put are at different strikes.
- The premium is cheaper than straddle.
- The payoff of a strangle is, K1<K2

$$Call(T, T, K1) + Put(T, T, K2)$$

- The delta of straddle can be set to zero depends on the choice of strikes.
- Vega is lower than straddle.

Strangle


strangle
20
15
10
5
0
-5
-10
-15
-20
Price
75
80
85
90
95
100
105
110
115
120
125
S(T)
long call
long put
straddle

Breeden-Litzenberger Formula

- Let $C(K)$ and $P(K)$ be European call and put option prices at K.
- For any twice differentiable European payoff $h(S_T)$, the present value of it, i.e. $V_0 = e^{-rT}E[h(S_T)]$, it can be replicated by a collection of European options:

$$\begin{aligned} V_0 &= e^{-rT}h(x) + h'(x)(C(x) - P(x)) \\ &+ \int_0^x h''(K)P(K)dK + \int_x^\infty h''(K)C(K)dK \end{aligned}$$

- If we pick x to be the forward price F, the above equation can be reduced to

$$V_0 = e^{-rT}h(F) + \int_0^F h''(K)P(K)dK + \int_F^\infty h''(K)C(K)dK$$

Breeden-Litzenberger Formula

- All option strategies that we discussed are special cases of the B-L formula.
- The implication is that if we know the option prices for all strikes then we can price any European payoffs and the most importantly, the prices are model independent.
- Example: $h(S_T) = S_T^2$

$$V_0 = e^{-rT}F^2 + 2\int_0^F P(K)dK + 2\int_F^\infty (K)C(K)dK$$

- Assume $dS_t = \sigma S_t dW$, $r = 0$, $\sigma = 0.2$, we can compute $V_0 = E[h(S_T)]$ analytically:

$$\begin{aligned}
E[h(S_T)] &= E[S_T^2] \\
&= E[(S_0 e^{-0.5\sigma^2 T + \sigma\sqrt{T}x})^2] \\
&= S_0 e^{-\sigma^2 T} E[e^{2\sigma\sqrt{T}x}] \\
&= S_0 e^{-\sigma^2 T} e^{2\sigma^2 T} \\
&= S_0 e^{\sigma^2 T}
\end{aligned}$$

Breeden-Litzenberger Formula

- Assume $S_0,\ T = 1$ then

$$V_0 = S_0^2 e^{\sigma^2 T} = 100e^{0.2^2} = 104.08$$

- The B-L formula in the discrete settings:

$$V_0 = S_0^2 + 2\sum_{i=1}^{N} P(K_i)(K_{i+1} - K_i) + 2\sum_{i=N}^{M} C(K_i)(K_{i+1} - K_i)$$

- where N is the number of put options, M is the number of call options, $K_N = F$.
- The more granular the strikes are, the closer you get to the closed form solution.

Exotic Structures

We briefly discuss a few of the most popular exotic structures for FX, Equity and Interest Rates.

- Target Redemption
- Memory Autocallable
- Range Accruals

Target Redemption

- This is a popular structure for FX. A typical structure has maturity less than 1 year and with monthly fixing dates.
- At each fixing date, T_i, the holder of the structure has the payoff which is a function of the prevailing FX rate, $X(T_i)$. The simplest one is

$$f(X(T_i)) = X(T_i) - K.$$

- There is an additional feature called target $\bar{A}$ which defines the redemption event:

$$\bar{A} \le A_j := \sum_{i=1}^{j} \max(f(X(T_i)), 0)$$

where $T_j : 1 \le j \le N$ is the current time and N is the total number of fixings.

Target Redemption

- The target $\bar{A}$ is the maximum profit that the holder would get from this structure.
- Also note that if the individual $f(X(T_i))$ is less than 0 (i.e. a loss for the holder), it would not reduce the accumulated value A_j.
- There are many different flavours with different specifications of f.
- Some structure may also have a termination trigger on each fixing dates. For example, let H be a barrier level, if $X(T_i) < H$ then the whole structure is knocked out.

Memory Autocallable

- Autocallable is a popular structure for Equity. A typical structure has a few stocks as the underlying which pays a certain amount at the maturity if the structure is not autocalled (aka knocked out).
- Maturity is typically less than 1 year.
- The autocall event is usually defined as a barrier level and expressed as a percentage of the current stock price. If all the stocks are above the corresponding barrier level then the structure is considered as autocalled.
- Memory autocallable is a variation such that if a stock was above the barrier, it will be "remembered". So that all the stocks don't need to be above the barriers at the same time. This feature makes memory autocallable easier to be knocked out than the non-memory counterpart.

Range Accruals

- This is a popular structure for rates. The maturity of the trade is usually more than 10 years.
- The underlying can be a single rate (3m USD LIBOR) or a spread on two CMS rates (USD 10y - USD 2y).
- The exotic coupon receiver pays floating LIBOR rates and receives

$$5\% \times A$$

where $A = \frac{1}{N}\sum_{i=1}^{N} 1_{L<S(T_i)<U}$.
- In other words, if the rate/spread $S(T_i)$ is between L and U for every fixing date T_i, then the exotic coupon receiver receives the full 5%.
- Note that range accruals is nothing more than a series of digital options.