QF602: Derivatives

Lecture 2:
Options

Option Contract

- Call option confers right (but not obligation) for holder (or long party) to buy underlying asset from writer (or short party)
- Put option confers right (but not obligation) for holder (or long party) to sell underlying asset to writer (or short party)
- Long party makes upfront payment of option premium (or price or value) to short party

Option Contract

- Transaction occurs when option is exercised, at fixed exercise price (or strike price)
- Option expires if not exercised before date of expiration (or maturity)

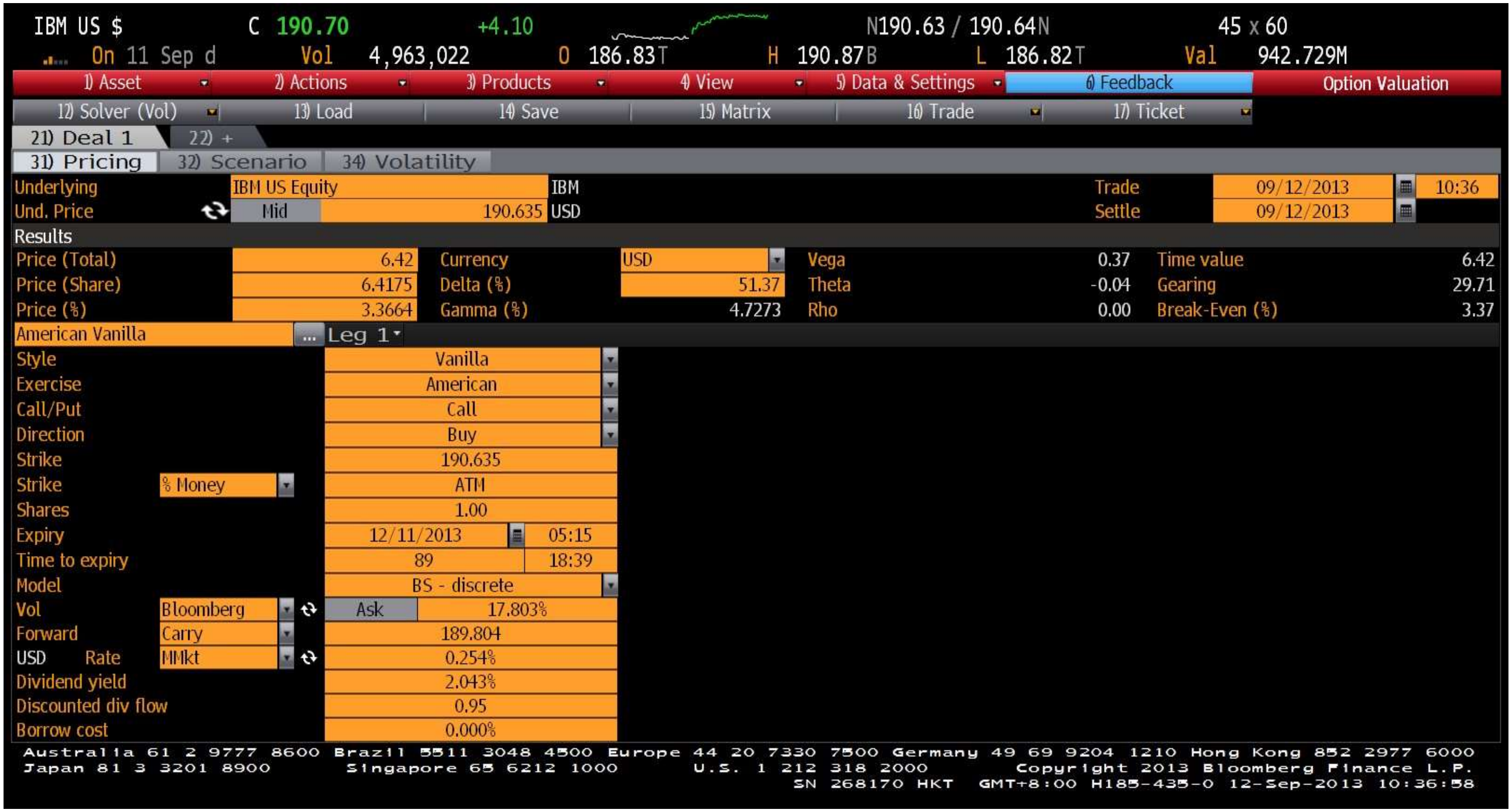
Option Contract

- European-style option may only exercised on date of expiration
- American-style option may be exercised at any time up to date of expiration
- Bermudan-style option may be exercised at fixed times up to day of expiration (i.e., half way between European and American!)

Option Contract

- Option may be exchange-traded or OTC
- Exchange-traded options tend to be more standardized and liquid then OTC options
- Long party may be exposed to default risk for OTC options (but not short party)

Example: Option on IBM



IBM stock prices

Exchange-Traded Option

Premium Quoted European Style Options on Euro/US Dollar Futures Contract Specs

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Futures | Options

Type: Premium-Quoted European Options

Contract Unit	One futures contract for 125,000 euro	
Minimum Price Fluctuation	$.0001 per euro increments ($12.50/contract). Also, trades may occur at $.00005 ($6.25), $.00015 ($18.75), $.00025 ($31.25), $.00035 ($43.75), and $.00045 ($56.25), when price is below five ticks of premium.	
Trading Hours	CME Globex:	Sunday - Friday 5:00 p.m. - 4:00 p.m. CT with a 60-minute break each day beginning at 4:00 p.m. CT
	Open Outcry:	Monday - Friday 7:20 a.m. – 2:00 p.m. Central Time (CT)
	CME ClearPort:	Sunday - Friday 5:00 p.m. - 4:00 p.m. CT with a 60-minute break each day beginning at 4:00 p.m. CT

Listed Contracts	Four months in the March quarterly cycle (Mar, Jun, Sep, Dec) and 3 serial months
Settlement Procedures	Option on physical delivery futures contract
Termination Of Trading	Trading terminates on the second Friday immediately preceding the third Wednesday of the contract month. (2:00 p.m. CT)
Position Limits	CME Position Limits
Exchange Rulebook	CME 261A
Block Minimum	Block Minimum Thresholds
Price Limit Or Circuit	Price Limits
Vendor Codes	Quote Vendor Symbols Listing
Exercise Style	European
Settlement Method	Deliverable
Underlying	Euro FX Futures

EURUSD FX Futures

Euro FX Volume

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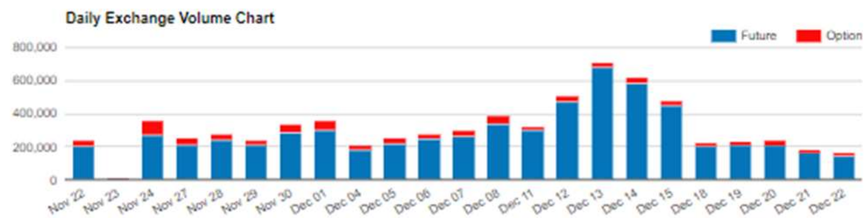
View FX Asset Class Volume and Open Interest

Trade Date: Friday, 22 Dec 2017 (PRELIMINARY

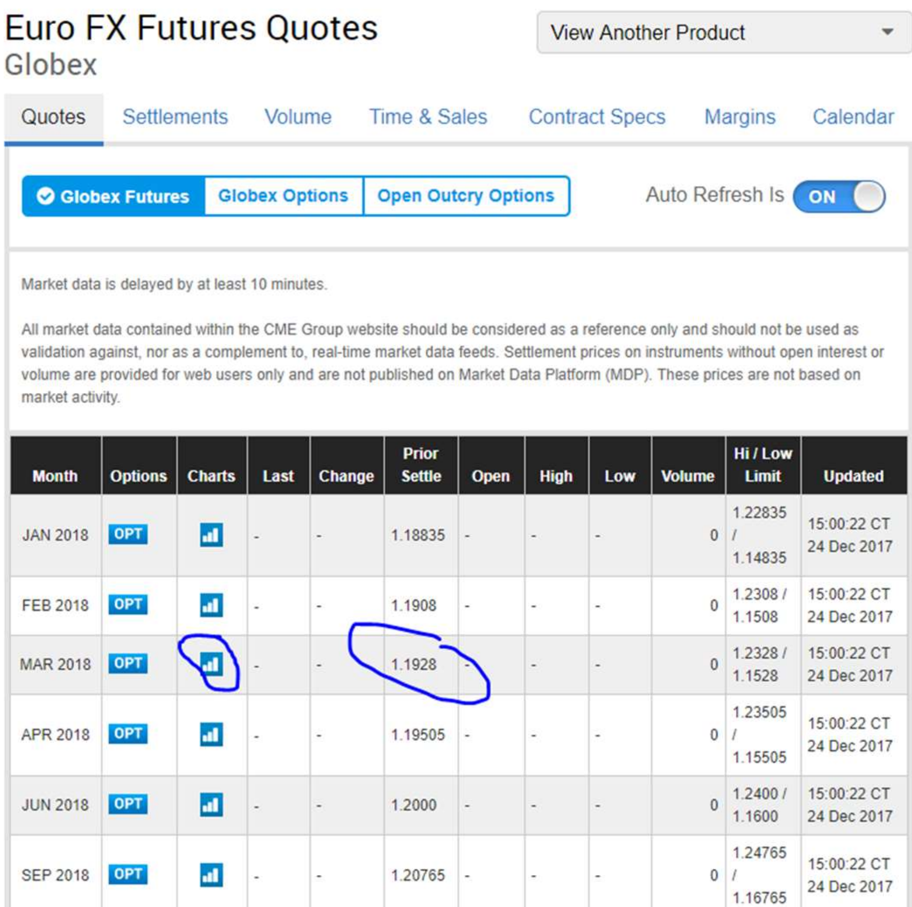
Download Data



	Volume										Open Interest	
	Venue Detail				Trade Type Detail							
Month	Globex	Open Outcry	PNT / ClearPort	Total Volume	Block Trades	EFP	EFR	EFS	TAS	Deliveries	At Close	Change
JAN 18	1,277	0	0	1,277	0	0	0	0	0	0	7,678	-123
FEB 18	618	0	0	618	0	0	0	0	0	0	2,044	-35
MAR 18	136,530	0	2,691	139,221	2,691	0	0	0	0	0	463,968	3,446
APR 18	660	0	0	660	0	0	0	0	0	0	637	-38
JUN 18	81	0	0	81	0	0	0	0	0	0	18,383	13
SEP 18	1	0	0	1	0	0	0	0	0	0	1,225	0
DEC 18	0	0	0	0	0	0	0	0	0	0	560	0
MAR 19	0	0	0	0	0	0	0	0	0	0	53	0
JUN 19	3	0	0	3	0	0	0	0	0	0	44	3
SEP 19	0	0	0	0	0	0	0	0	0	0	138	0
Totals	139,170	0	2,691	141,861	2,691	0	0	0	0	0	494,730	3,266

About This Report

EURUSD FX Futures



Euro FX Futures Quotes
Globex

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Globex Futures | Globex Options | Open Outcry Options

Auto Refresh Is ON

Market data is delayed by at least 10 minutes.

All market data contained within the CME Group website should be considered as a reference only and should not be used as validation against, nor as a complement to, real-time market data feeds. Settlement prices on instruments without open interest or volume are provided for web users only and are not published on Market Data Platform (MDP). These prices are not based on market activity.

Month	Options	Charts	Last	Change	Prior Settle	Open	High	Low	Volume	Hi / Low Limit	Updated
JAN 2018	OPT		-	-	1.18835	-	-	-	0	1.22835 / 1.14835	15:00:22 CT 24 Dec 2017
FEB 2018	OPT		-	-	1.1908	-	-	-	0	1.2308 / 1.1508	15:00:22 CT 24 Dec 2017
MAR 2018	OPT		-	-	1.1928	-	-	-	0	1.2328 / 1.1528	15:00:22 CT 24 Dec 2017
APR 2018	OPT		-	-	1.19505	-	-	-	0	1.23505 / 1.15505	15:00:22 CT 24 Dec 2017
JUN 2018	OPT		-	-	1.2000	-	-	-	0	1.2400 / 1.1600	15:00:22 CT 24 Dec 2017
SEP 2018	OPT		-	-	1.20765	-	-	-	0	1.24765 / 1.16765	15:00:22 CT 24 Dec 2017

Moneyness

- Let S(t) be spot price of underlying asset at time t
- Let K be exercise price
- Option is in-the-money when exercise is profitable: $S(t) > K$ for call and $S(t) < K$ for put
- Option is out-of-the-money when exercise is not profitable: $S(t) < K$ for call and $S(t) > K$ for put
- Option is at-the-money when $S(t) = K$

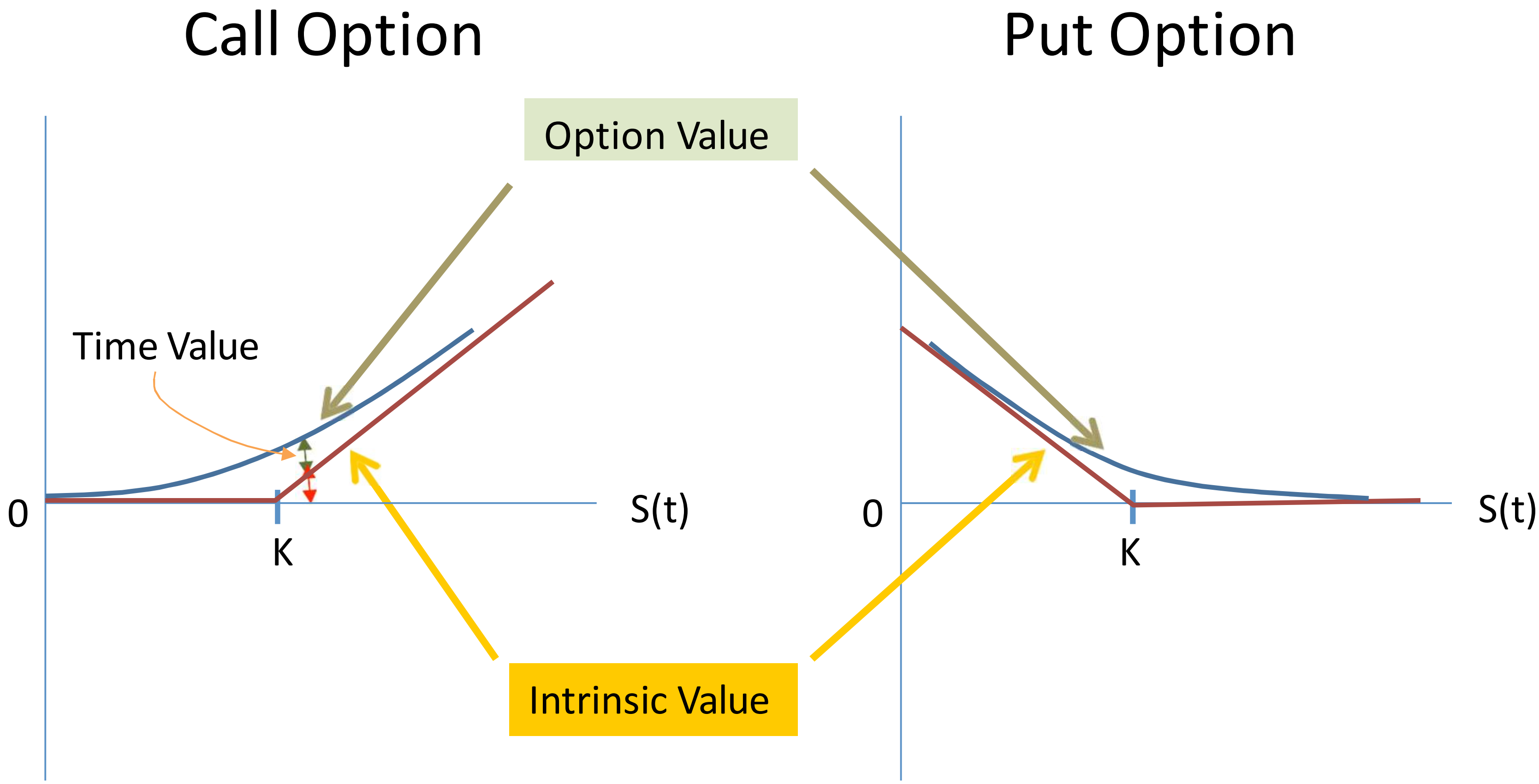
Moneyness

- Note that this is not the only definition of moneyness.
- Moneyness can be set with respective to forward in some market like rates option.
- For FX option, ATM strike is the strike such that a straddle has no delta. See chapter 3.5 in Foreign Exchange Option Pricing: A Practitioner's Guide by Clark.

Option Value

- Option Value = Intrinsic Value + Time Value
- Intrinsic value is payoff from immediate exercise: max(S(t) – K, 0) for call option and max(K – S(t), 0) for put option
- Time value is value of not exercising, since intrinsic value may increase if we don't exercise.

Option Value



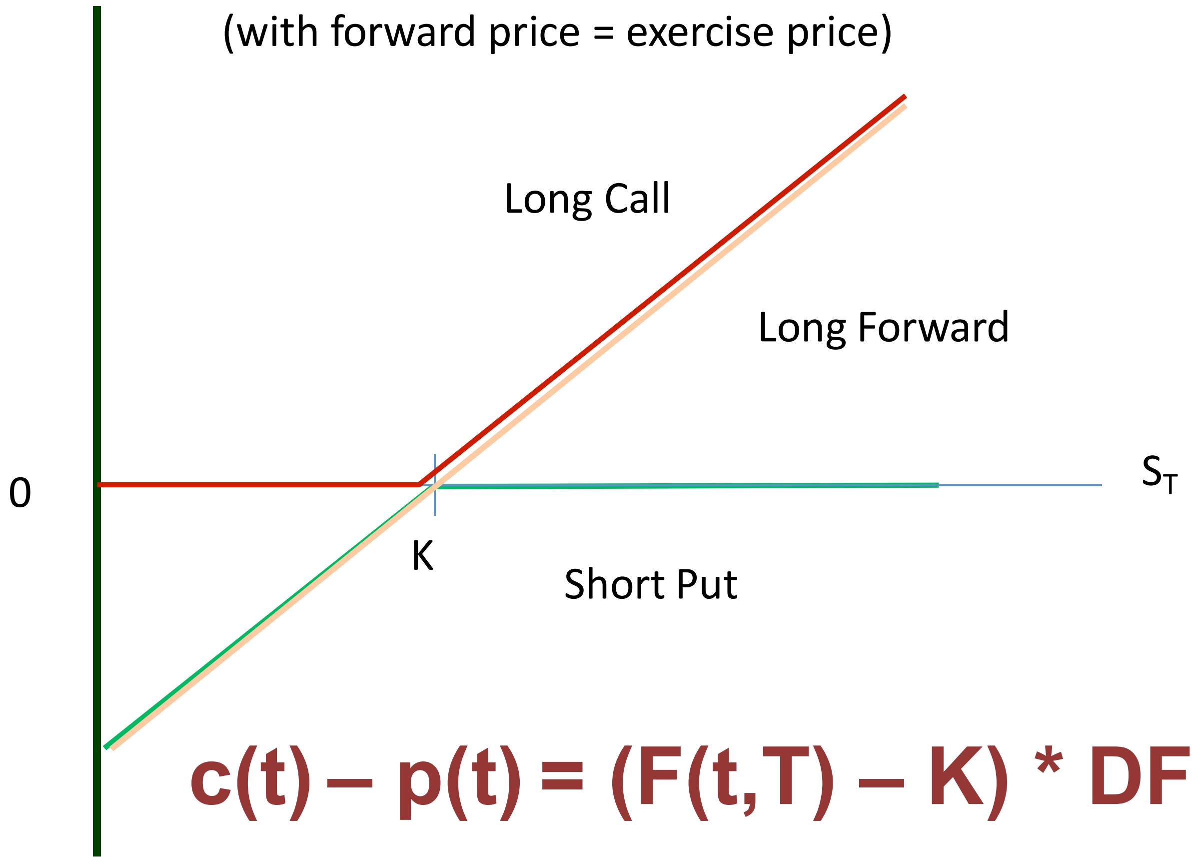
Put–Call Parity

- Let c(t) and p(t) be value of European call and put, respectively, with same underlying, exercise price, and maturity T
- Combination of long call and short put at the same strike delivers same payoff as long forward contract
- If no cash flows for underlying asset, then no-arbitrage relation for put–call parity:

$$c(t) - p(t) = (F(t,T) - K) * DF(T-t)$$

Put–Call Parity

Long Call + Short Put = Long Forward
(with forward price = exercise price)



$$c(t) - p(t) = (F(t,T) - K) * DF(T\text{-}t)$$

Put–Call Parity

- Rearrange to get different interpretation:
  $c(t) + K * DF(T-t) = p(t) + F(t,T) * DF(T-t)$
- Left side represents a portfolio A, where enough money is deposited into interest-bearing account to cover exercise of call
- Right side represents a portfolio B, where put provides protection against drop in value of underlying asset

Example: Put–Call Parity

- Call price: \$7.50
- Put price: \$4.25
- Exercise price: \$100
- Underlying price: \$99
- Time to maturity: 6 months
- Risk-free interest rate: 10% p.a., DF(6m) = 0.9512
- Forward price = \$99 / 0.9512 = \$104.08
- Dividend yield and repo rate = 0

Example: Put–Call Parity

- Value of portfolio A:
  \$7.50 + \$100 * 0.9512 = \$102.62
- Value of portfolio B:
  \$4.25 + \$104.08 * 0.9512 = \$103.25
- B is worth more than A, so arbitrage opportunity exists
- Buy low, sell high: short B and buy A for immediate profit of 63¢

American option

- European option can only be exercised at the maturity.
- American option can be exercised at any time before the maturity.
- Let $c(t)$, $p(t)$ be European call and put and $C(t)$ and $P(t)$ be American call and put.
- It is obvious that $C(t) >= c(t)$ and $P(t) >= p(t)$ if all other terms are the same.
- Is there a situation that the $C(t) = c(t)$ and $P(t) = p(t)$?

American option

- Without dividends, never exercise an American call early.
- Exercise early requires paying the exercise price early, hence loses the time value of money because he doesn’t receive interest on this cash amount.
- On the other hand, he would receive future dividends for holding the stock.
- If dividend yield is higher than the interest rate until maturity then it is optimal to exercise.

American option

- Without dividends, it can be optimal to exercise an American put early.
- Consider a put with K = 100 on a stock with S(t) = 0.
- S(t) cannot go any lower and this is the max one can earn for holding a put option.
- Exercise now gives $100 today.
- Exercise later gives $100 later.

Black Scholes Merton

- Key assumptions:
- Volatility is constant over time.
- Underlying is traded continuously and is log-normally distributed.
- One can always short sell.
- No transaction costs.
- One can sell any fraction of a share.
- One can borrow and lend cash at a constant risk free rate.
- Stock pays a constant dividend yield.

Risk neutral pricing

- The fundamental assumption behind risk-neutral pricing is to use a replicating portfolio of assets with known prices to remove any risk.
- In BSM world, options are considered to be redundant in the sense that one can replicate the payoff of an European option on stock using the stock itself and risk-free bonds.
- Since options can be replicated and their theoretical values do not depend upon investors’ risk preferences.
- The idea of replication is one of the most important contributions by Black, Scholes and Merton.

Black Scholes Formula

- Let S(0) be the spot price at time 0.
- σ be the volatility of the underlying log return.
- r and q be the interest rate and dividend yield respectively.
- Forward price at time 0 with maturity T is

$$F(0,T) = S(0)e^{(r-q)T}$$

- The price of an European call option is given by

$$DF(T)(F(0,T)N(d1) - KN(d2))$$

$$d1 = \frac{\ln\left(\frac{F}{K}\right)+\frac{\sigma^2 T}{2}}{\sigma\sqrt{T}}, d2 = d1 - \sigma\sqrt{T}, N(a) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{a} e^{-u^2/2}du$$

Black Scholes Formula

- The price of an European put option is given by

$$DF(T)(KN(-d2) - F(0,T)N(-d1))$$

- Note that $N(-a) = 1 - N(a)$
- It is easy to show that BS Call(K) - BS Put(K) becomes

$$DF(T)(F(0,T) - K)$$

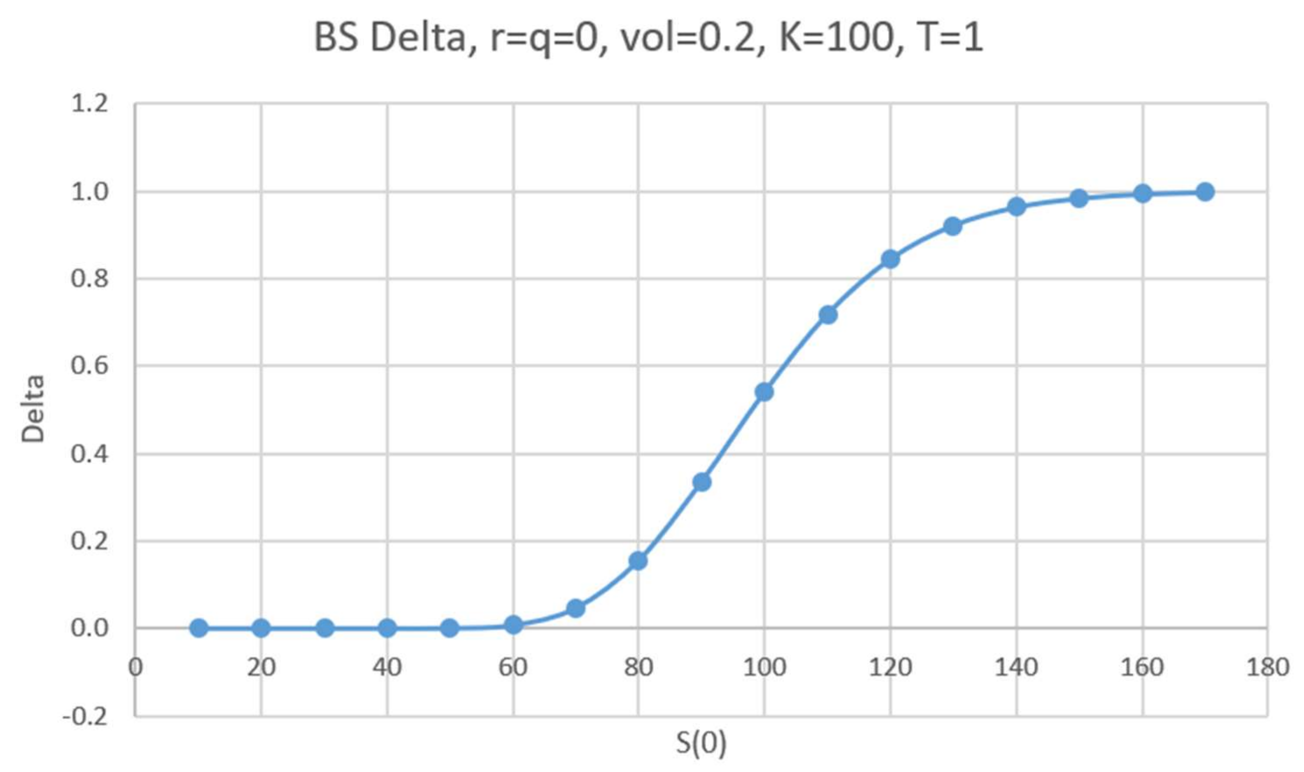
- Put Call parity works!!!!!
- In fact, this is a model-free result and must be satisfied by any models.

Black Scholes Formula

- We briefly introduce “Greeks” in this lecture and will come back in more details later.
- Call option price is a function of:
  - Spot
  - Interest rate
  - Dividend yield
  - Time to maturity
  - Volatility
- Their sensitivities can be computed analytically in the BS model.

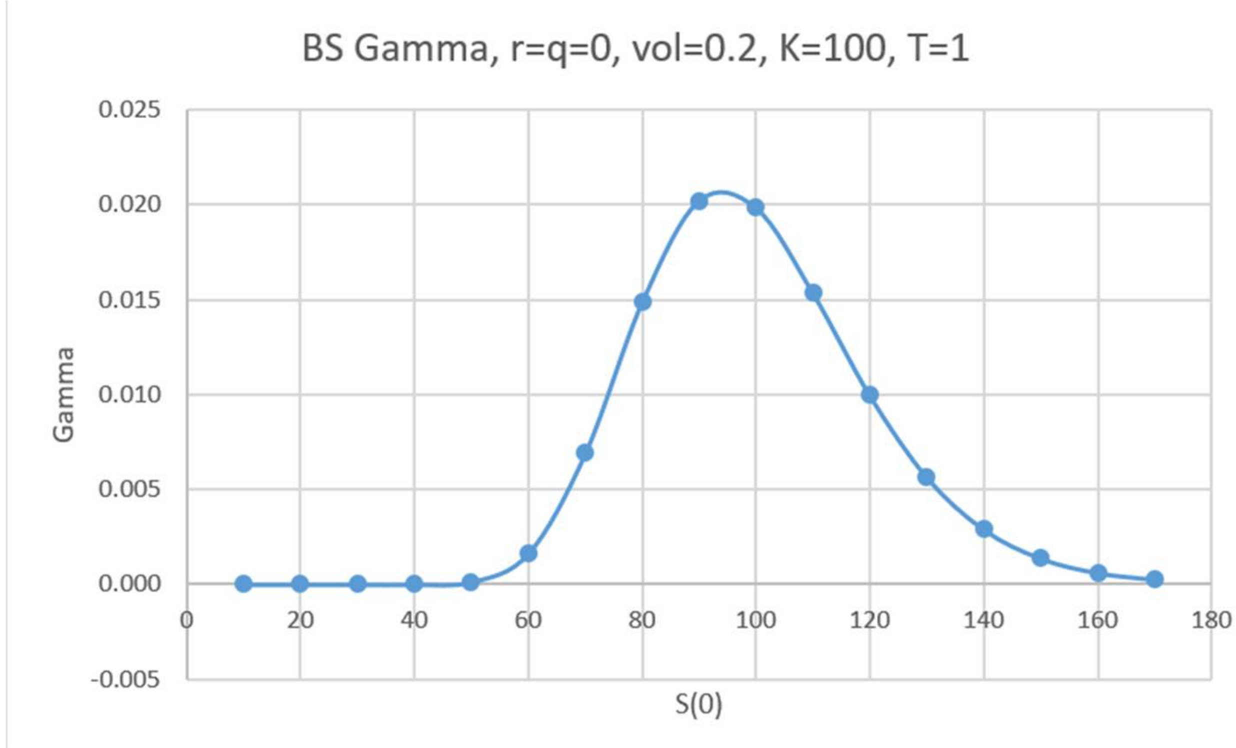
Black Scholes Formula

- Delta is defined as $\frac{\partial Call}{\partial S(0)}$, 1st order sensitivity to spot



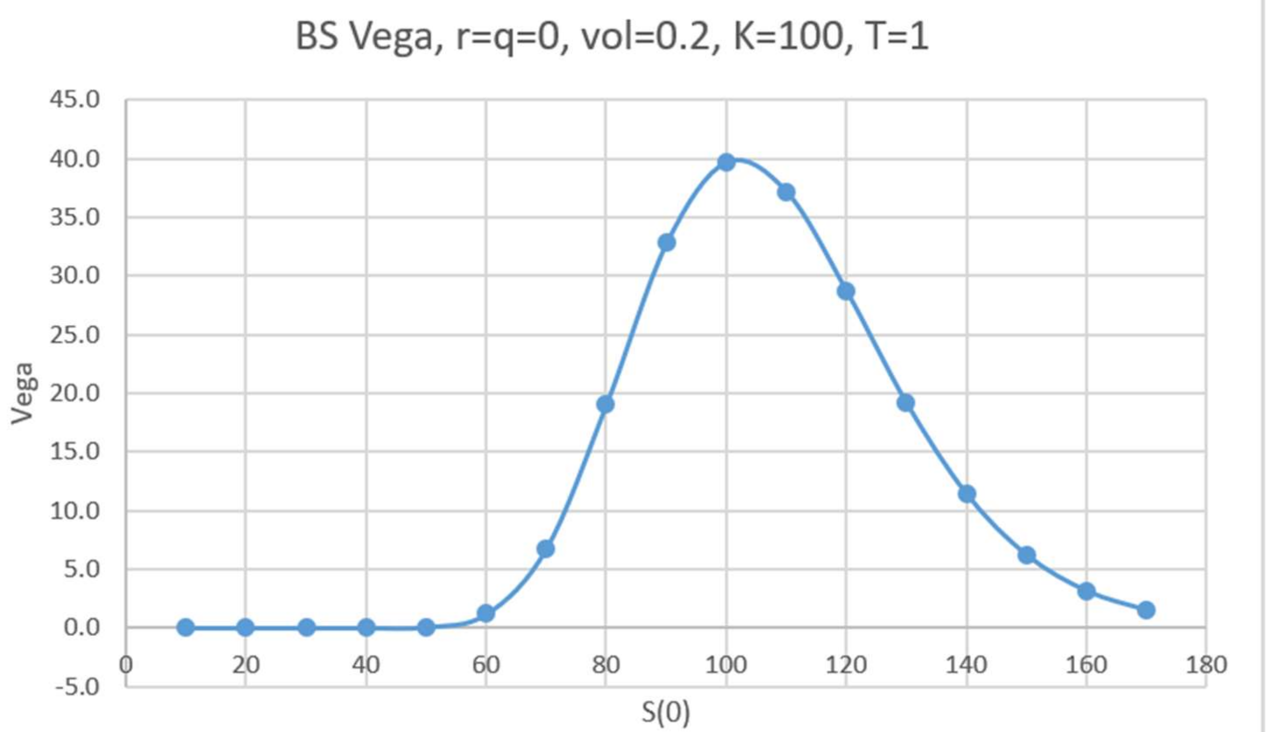
Black Scholes Formula

- Gamma is defined as $\frac{\partial^2 Call}{\partial S(0)^2}$, 2nd order sensitivity to spot



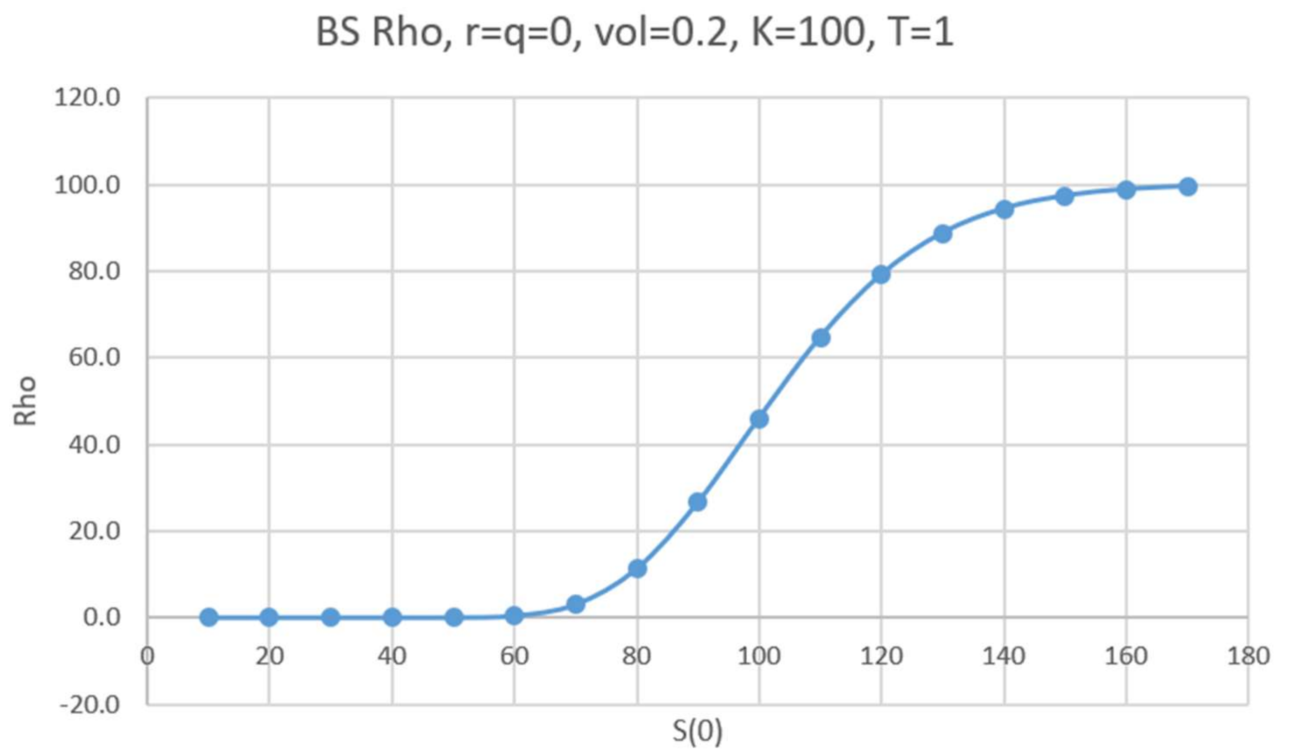
Black Scholes Formula

- Vega is defined as $\frac{\partial Call}{\partial vol}$, 1st order sensitivity to implied vol



Black Scholes Formula

- Rho is defined as $\frac{\partial Call}{\partial r}$, 1st order sensitivity to interest rate



Black Scholes Formula

- Theta is defined as $\frac{\partial Cal}{\partial t}$, 1st order sensitive to "time".