Normalisation notes

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1 Normalisation in *Ouroboros*

In *Ouroboros*, the following normalisations are used for radial, spheroidal and toroidal modes, respectively:

$$\omega^2 \int U^2 \rho r^2 \,\mathrm{d}r = 1$$
$$\omega^2 \int \left(U^2 + k^2 V^2\right) \rho r^2 \,\mathrm{d}r = 1$$
$$\omega^2 \int k^2 W^2 \rho r^2 \,\mathrm{d}r = 1$$

where ω is the angular frequency measured in $\mathrm{rad\,s^{-1}}$, ρ is the density measured in $\mathrm{kg\,m^{-3}}$, r is the radial coordinate measured in km, and $k = \sqrt{\ell(\ell+1)}$. If SI units are used instead (i.e. r is measured in m), the eigenfunctions U, V and W must be divided by $10^{9/2}$ for the above normalisation to be correct.

2 Comparison with *Mineos*

The *Mineos* manual describes two different normalisation conventions. The first is given in their section 3.1.3.2 and applies to the binary file output of `minos_bran`. The *Ouroboros* package does not work with these files directly, so we do not discuss this normalisation. The second normalisation applies to output of `eigcon`, which is used by *Ouroboros*. This normalisation, discussed in section 3.2.3.2, has the same form as the *Ouroboros* normalisation written above, however the units are different, as described in section 3.1.3.2. The density is divided by a reference density ρ_n of $5515\,\mathrm{kg\,m^{-3}}$, the radius is divided by the planetary radius (6371 km), and the frequency is divided by $\omega_n \equiv (\pi G \rho_n)^{1/2} \approx 1.0754\,\mathrm{mHz}$ (note there is a small error in the *Mineos* manual so that the inverse of the frequency normalisation is given), where G is Newton's constant. Therefore, to convert from *Ouroboros* eigenfunctions to *Mineos* eigenfunctions, it is necessary to multiply by

$$
\begin{aligned}
k &= \omega_n \sqrt{10^{-9} \rho_n r_n^3} \\
&= \rho_n \sqrt{10^{-9} \pi G r_n^3} \\
&\approx 40\,610\,\mathrm{kg}^{1/2}\,\mathrm{s}^{-1}
\end{aligned}
$$

where the constants are expressed in SI units.

3 Comparison with textbook

In Dahlen and Tromp (1998), the normalisation used is given by their equations 8.107–8, which can be written for radial, spheroidal and toroidal modes as:

$$
\begin{aligned}
\int U^2 \rho r^2 \,\mathrm{d}r &= 1 \\
\int \left(U^2 + V^2\right) \rho r^2 \,\mathrm{d}r &= 1 \\
\int W^2 \rho r^2 \,\mathrm{d}r &= 1
\end{aligned}
$$

Therefore, to use *Ouroboros* eigenfunctions in any formula from that textbook, the substitutions

$$
\begin{aligned}
U &\to \omega U \\
V &\to \omega k V \\
W &\to \omega k W
\end{aligned}
$$

must be made. For example, the sensitivity kernel (D&T eq. 9.15)

$$2\omega K_\rho = -\omega^2 r^2 U^2$$

becomes

$$2\omega K_\rho = -\omega^2 r^2 \omega^2 U^2$$

or, more simply,

$$2K_\rho = -\omega^3 r^2 U^2$$