Comparison of *Mineos* and *RadialPNM*

Harry Matchette-Downes

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1 Models and parameters

We use *RadialPNM_py*, version 4, with 700 elements. We compare with *Mineos*, version 1.0.2, with an `eps` parameter of 10^{-10}. Our input models are based on the PREM model `prem_noocean.txt` included in the *Mineos* package. This model has radial anisotropy, which we remove by taking the mean of the vertically- and horizontally-polarised wavespeeds. We ignore attenuation by setting Q_μ and Q_κ to 0, and setting `tref` to -1. The model created in this fashion is called `prem_noocean_at_1_s_noq.txt`, acknowledging the 1-second reference period of the PREM model. We also test a second model, which is corrected to make it more appropriate to the low frequencies which interest us. In this model, the elastic moduli are reduced using the formulae 9.50 and 9.51 of Dahlen and Tromp (1998) combined with the Q-values of the original `prem_noocean.txt` model. We use a new reference frequency of 3 mHz, so thenew model is called `prem_noocean_at_03.000_mHz_noq.txt`.

2 Known issues in *RadialPNM*

We observe some instabilities in a few of the eigenfunctions, which are more obvious when then gradient of the eigenfunctions is plotted, as in Figure 1. We have not explored fully which modes are affected by this, or whether it is confined to the fluid outer core. It is thought this is caused by incorrect choice of finite-order element and we are currently investigating this issue.

3 Known issues in *Mineos*

We have found issues with inner-core toroidal modes, and also in the summation of radial modes, but both of these are outside the scope of this report. Stoneley modes have been reported to be problematic, for example on the CIG boards.

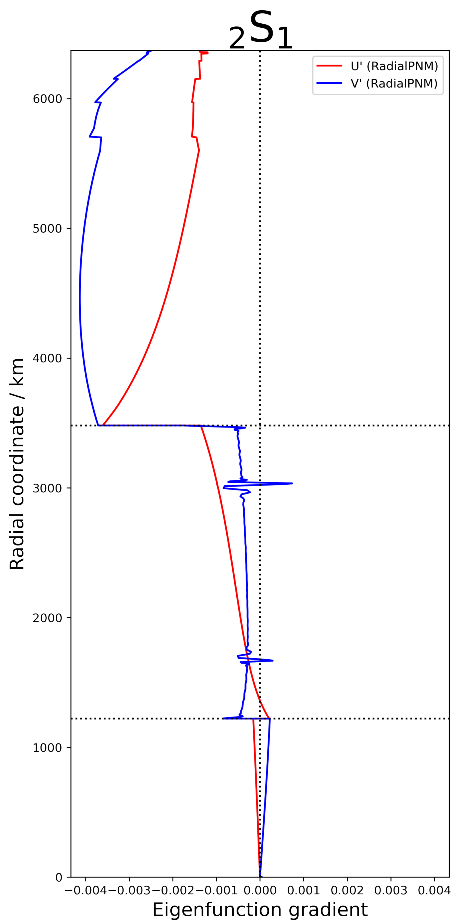

Figure 1: Gradient of eigenfunctions for mode $_2\mathrm{S}_1$, including gravity and gravity perturbation terms.

4 Differences between *Mineos* and *RadialPNM*

In *Mineos*, it is not possible to neglect gravity. Therefore we can only consider two cases, the first with gravity but no perturbation, and the second with both gravity and perturbation.

4.1 Mode ${}_2\mathrm{S}_1$

When *Mineos* is used to calculate the mode ${}_2\mathrm{S}_1$, it is dramatically different depending on whether the gravity perturbation is included or not. On the other hand, if *RadialPNM* is used, the result is more similar. We do not know why this is.

4.2 Gravity with no perturbation

To ignore the gravity perturbation, we run *RadialPNM* in '*G*' mode and set the *Mineos* gravity cut-off frequency to 0.

4.2.1 PREM-1s reference model

The frequency differences are shown in Figure 2. We see that in almost all cases, *RadialPNM* gives higher frequencies, with differences of up to 0.47 %. The Stoneley branches are notably more similar in frequency.

The eigenfunction differences are shown in Figure 3 (see Appendix A for definition of the differences between eigenfunctions). Differences are generally small, except near the intersections of branches. For example, mode ${}_2\mathrm{S}_{25}$ shows a significant difference between the two codes, as can be seen in Figure 4.

4.2.2 PREM-3mHz reference model

The frequency differences are shown in Figure 5. We see that the frequencies are in better agreement with this model, differing by no more than 0.04 % and showing less of a systematic offset than in the previous case.

The eigenfunction differences are shown in Figure 6 with the same scale. The differences are much smaller and are probably mostly related to poor sampling near the ICB. The differences at the branch quasi-intersection points vanish; for example, the eigenfunction of mode ${}_2\mathrm{S}_{25}$ is indistinguishable between the two methods, as shown in Figure 7.

4.3 Gravity with perturbation

To ignore the gravity perturbation, we run *RadialPNM* in '*GP*' mode and set the *Mineos* gravity cut-off frequency to a large arbitrary value. We find that the comparison between the two codes yields almost identical results as the case with no perturbation, so we do not include them here.

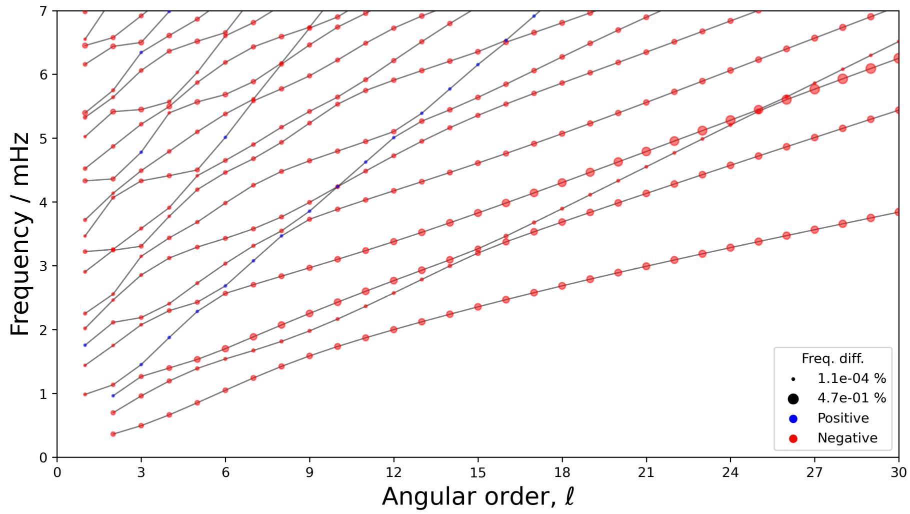

Figure 2: Frequency differences for modes calculated using *Mineos* and *RadialPNM* with a 1-second reference period, including gravity but neglecting the gravity perturbation. A value is 'positive' if the *Mineos* frequency is larger.

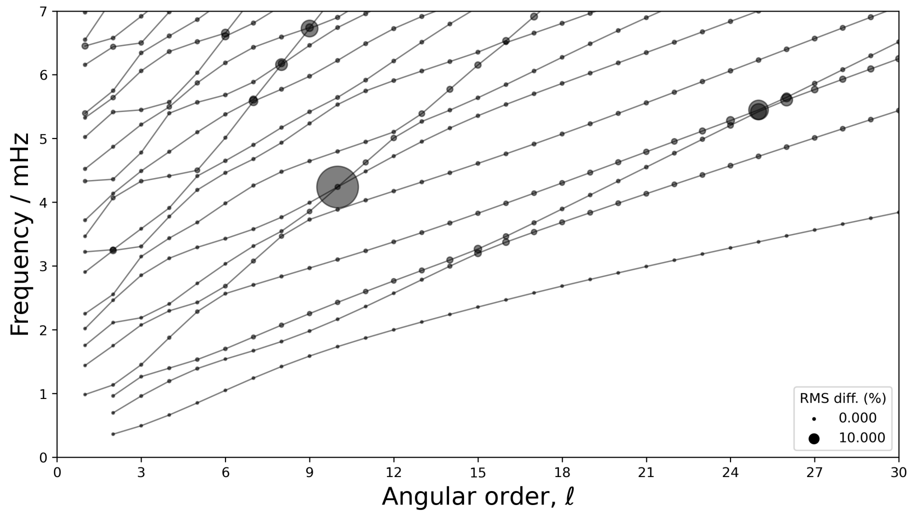

Figure 3: Eigenfunction differences for modes calculated using *Mineos* and *RadialPNM* with a 1-second reference period, including gravity but neglecting the gravity perturbation.

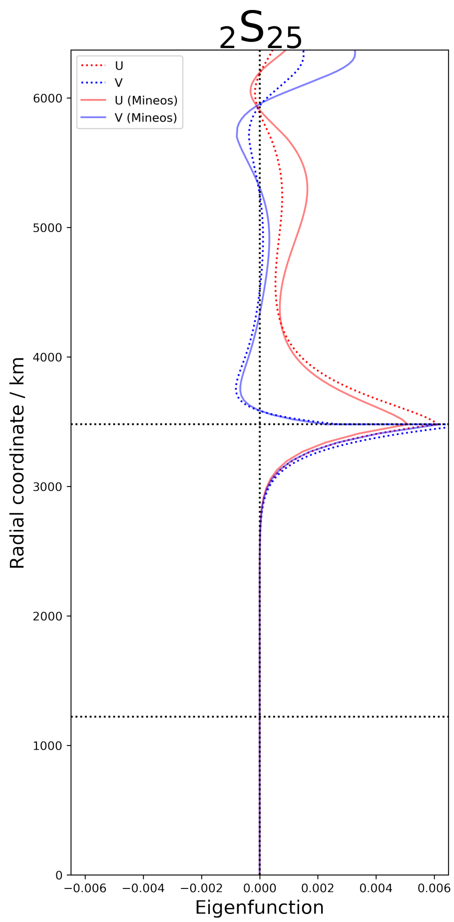

Figure 4: Comparison of eigenfunction of mode $_2S_{25}$ calculated with two methods: *Mineos* and *RadialPNM*. The model has a 1-s reference period and the calculation includes gravity but neglects the gravity perturbation.

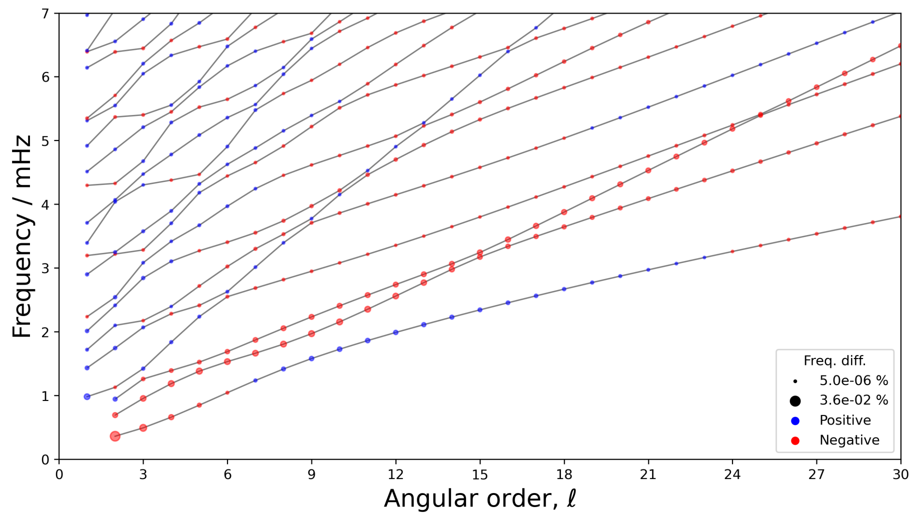

Figure 5: Frequency differences for modes calculated using *Mineos* and *RadialPNM* with a 3-mHz reference frequency, including gravity but neglecting the gravity perturbation. A value is 'positive' if the *Mineos* frequency is larger.

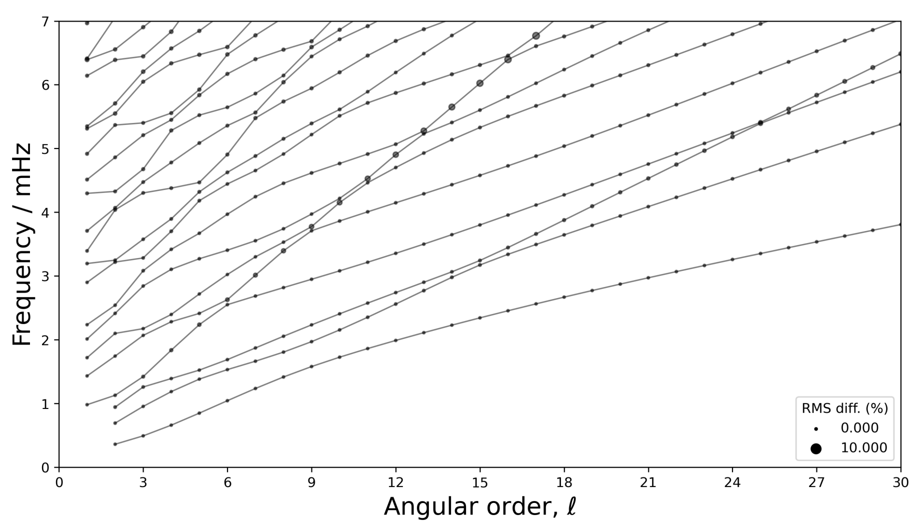

Figure 6: Eigenfunction differences for modes calculated using *Mineos* and *RadialPNM* with a 3-mHz reference period, including gravity but neglecting the gravity perturbation.

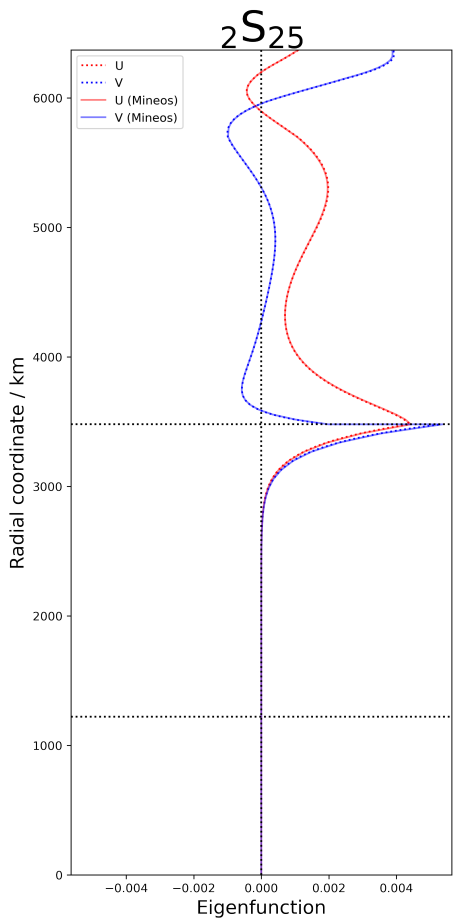

Figure 7: Comparison of eigenfunction of mode $_2S_{25}$ calculated with two methods: *Mineos* and *RadialPNM*. The model has a 3-mHz reference period and the calculation includes gravity but neglects the gravity perturbation.

5 Discussion

We see that the two codes disagree when using the standard 1-s PREM model (Figures 2 and 3), especially for the quasi-intersection modes. However, when the model is corrected to 3 mHz, the two codes agree quite well (Figures 5 and 6); in this case the frequency differences are on the order of µHz, which is probably smaller than experimental uncertainties.

One explanation could be that *Mineos* tacitly applies some sort of anelastic correction, which would be larger if the mode frequencies were far from the model reference frequency. However, the model reference frequency is specified as -1, so the anelastic correction should not be applied, as we can see in `minos_bran.f`:

```
if(tref.gt.0.d0) fct=2.d0*dlog(tref*wdim)/pi
```

Therefore, it seems that both codes are treating the calculation as fully elastic, and one code is incorrect for the 1-s reference model. One test of 'correctness' is that all of the eigenfunctions should be orthogonal, so we evaluated the scalar product of all eigenfunction pairs. This is shown for *Mineos* in Figure 8. We see that the troublesome quasi-intersection modes give the largest deviations from orthogonality; the ${}_1S_{15}$–${}_2S_{15}$ pair has a scalar product 0.9 % and the ${}_2S_{25}$–${}_3S_{25}$ pair has a scalar product of 2.0 %. By contrast, *RadialPNM* gives eigenfunctions which are much closer to orthogonal, as shown in Figure 9 (note the difference in colour scale).

This seems to suggest that *Mineos* is incorrect, although it may be that the relatively coarse grid is causing the scalar product to be calculated inaccurately (we use the trapezium rule). Guy Masters pointed out that the *Mineos* calculations are much more consistent between the two models (which can be seen by carefully comparing Figures 4 and 7), which he cited as evidence that *Mineos* is more likely to be correct.

We consider this issue to be unresolved. Fortunately, it is not germane to our study, because we are interested in low-frequency modes, and after the frequency correction is applied, to two methods give very similar results. However, we would still like to resolve the issue. One possible test would be to compare with an independent method such as the Haskell-Thomson propagator matrix technique combined with the Earth-flattening transform, however this technique does not include gravity and so could not be used to test *Mineos*.

Appendix A: Definition of difference between eigenfunctions

Appendix B: Commands used to make these plots

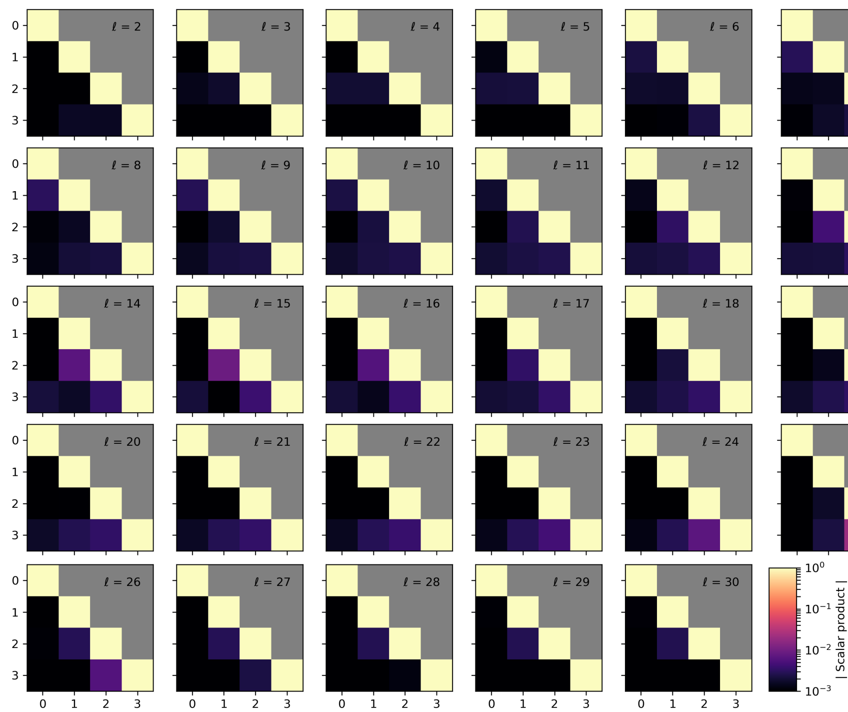

Figure 8: Scalar product of eigenfunctions for pairs of modes calculated using the *Mineos* code, using a model with a reference period of 1s, including gravity but neglecting gravitational potential. Each coloured square indicates one mode pair, with the x-axis showing the n-value of the first mode, and the y-axis showing the n-value of the second mode; only modes with n less than or equal to 4 are shown. Only modes with the same value of ℓ are compared, because other mode pairs are automatically orthogonal due to orthogonality of the spherical harmonic basis functions. The ℓ-value is indicated for each panel.

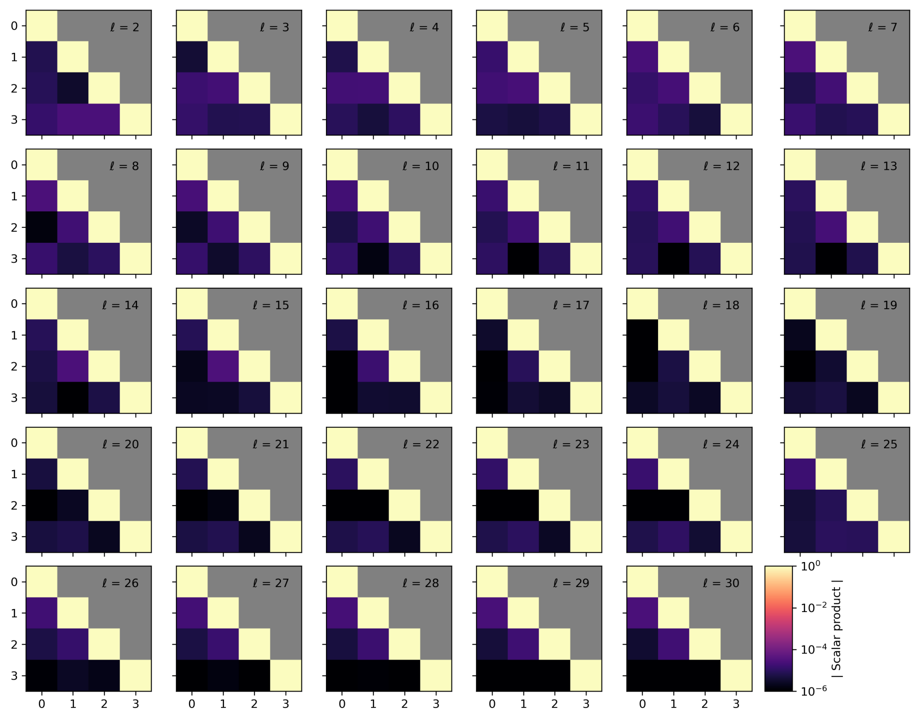

Figure 9: Scalar product of eigenfunctions for pairs of modes calculated using the *RadialPNM* code, using a model with a reference period of 1s, including gravity but neglecting gravitational potential. See the caption of Figure 8 for description of the plot.