

Preliminary

A review of and HPS discovery potential for True Muonium

Emrys Peets

Preliminary

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Road Map

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- What is True Muonium?
- Calculating Primary Production Cross Section
 - Radiative
 - Dissociation and Ideal Targets
 - Calculating Yield
 - Three Photon Production Mechanism
- HPS-MC Study
 - 4.55 GeV Acceptance Result
 - 6.6 GeV Roadblock
- Next Steps

True Muonium



- True muonium is the exotic “fermionium” atom consisting of a bound ($\mu^+ \mu^-$)
- $M_{TM} \sim 2 M\mu$ (2 $M\mu$ - Binding)
- Has yet to be experimentally verified
- True muonium decays sooner than constituent muons
- Offers unique precision test of QED
- 512 fm Bohr radius smaller than that of hydrogen
- Can be used to verify robustness of existing HPS software

Production of True Muonium

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$$\pi^- p \rightarrow (\text{TM})n$$

$$\gamma Z \rightarrow (\text{TM})Z$$

$$e^- Z \rightarrow e^- (\text{TM})Z$$

$$Z_1 Z_2 \rightarrow Z_1 Z_2 (\text{TM})$$

$$\eta \rightarrow (\text{TM})\gamma$$

$$e^+ e^- \rightarrow (\text{TM})$$

$$\mu^+ \mu^- \rightarrow (\text{TM})$$

Production mechanism we care about

Mechanism Considered for LHCb run 3
search: [Discovering True Muonium at LHCb](#)

Production of True Muonium

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True Muonium Level Diagram

Electro Production of Dimuonium

$$e^- Z \rightarrow e^- (\text{TM}) Z$$

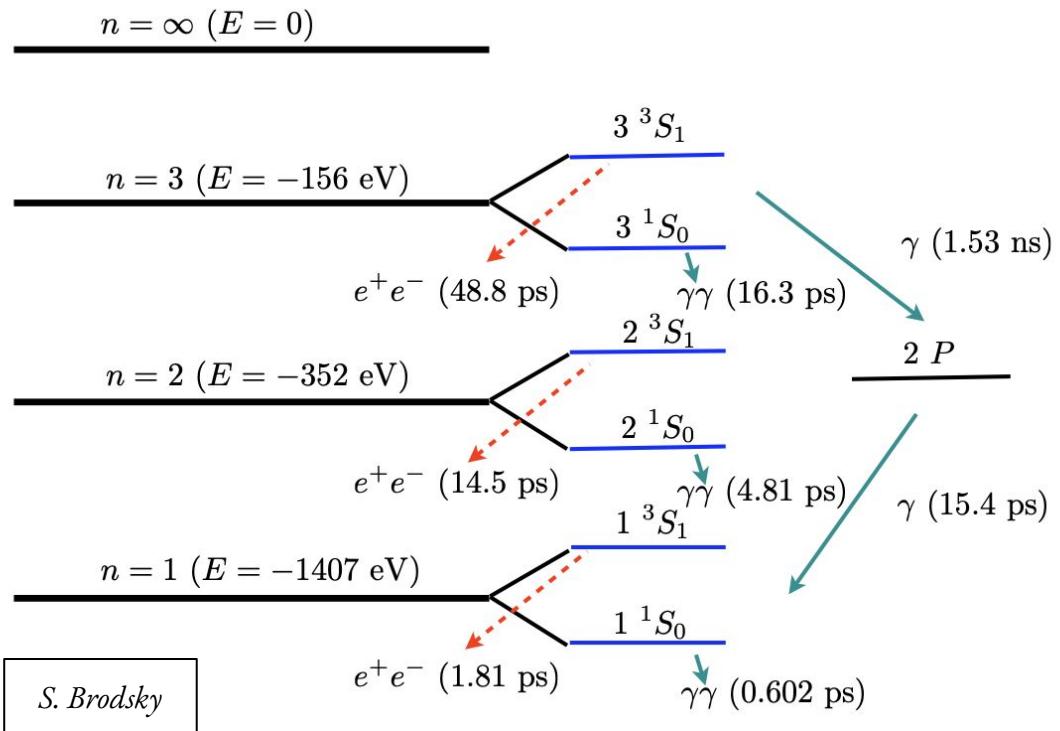
TABLE I: True fermionium decay times and their ratios.

$$\tau(n^3S_1 \rightarrow e^+ e^-) = \frac{6\hbar n^3}{\alpha^5 mc^2}, \quad \tau(n^1S_0 \rightarrow \gamma\gamma) = \frac{2\hbar n^3}{\alpha^5 mc^2},$$

$$\tau(2P \rightarrow 1S) = \left(\frac{3}{2}\right)^8 \frac{2\hbar}{\alpha^5 mc^2}, \quad \tau(3S \rightarrow 2P) = \left(\frac{5}{2}\right)^9 \frac{4\hbar}{3\alpha^5 mc^2},$$

$$\frac{\tau(n^3S_1 \rightarrow e^+ e^-)}{\tau(n^1S_0 \rightarrow \gamma\gamma)} = 3, \quad \frac{\tau(2P \rightarrow 1S)}{\tau(n^1S_0 \rightarrow \gamma\gamma)} = \left(\frac{3}{2}\right)^8 \frac{1}{n^3} = \frac{25.6}{n^3},$$

$$\frac{\tau(3S \rightarrow 2P)}{\tau(2P \rightarrow 1S)} = \left(\frac{5}{3}\right)^9 = 99.2.$$



S. Brodsky

Production of True Muonium

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True Muonium Level Diagram

Electro Production of Dimuonium

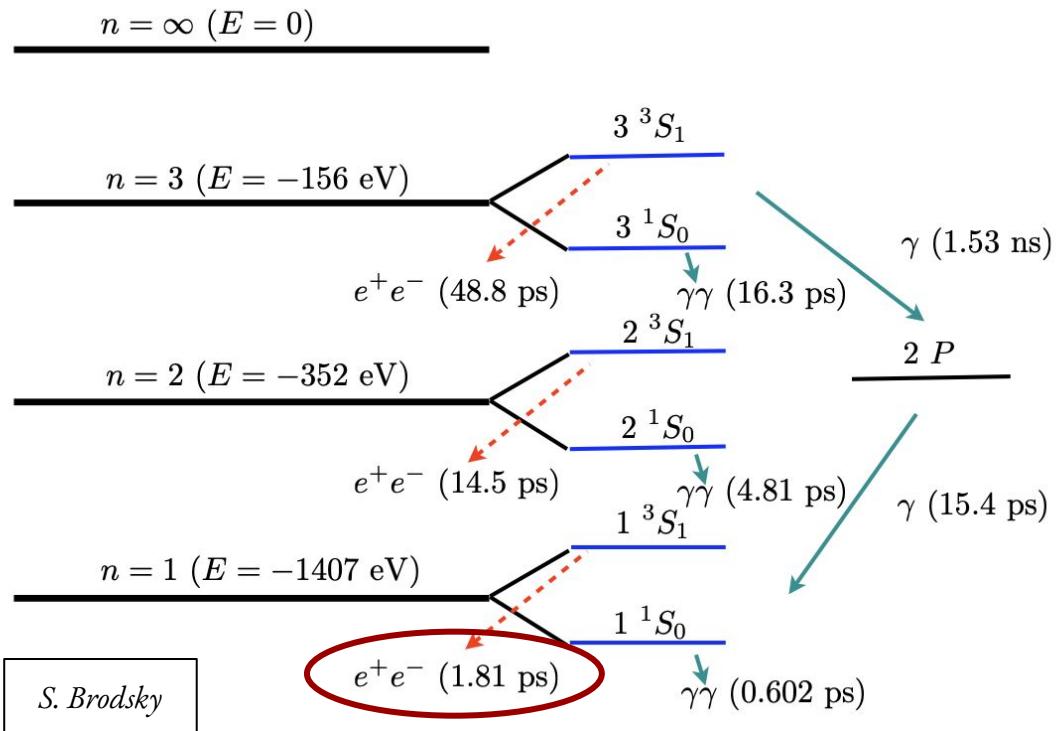
$$e^- Z \rightarrow e^- (\text{TM}) Z$$

Decay Length of Interest

$$l_{dec} = \gamma \beta c \tau_{(1^3S_1)}$$

$$l_{dec}(4.55 \text{ GeV}) = 1.17 \text{ cm}$$

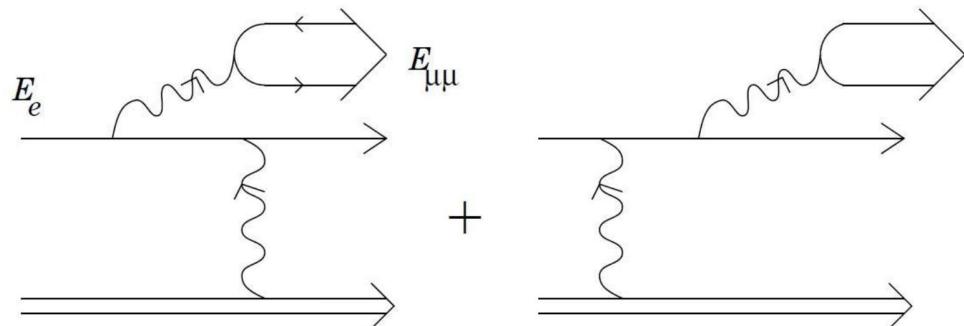
$$l_{dec}(6.60 \text{ GeV}) = 1.69 \text{ cm}$$



Primary Production Modes of Orthodimuonium

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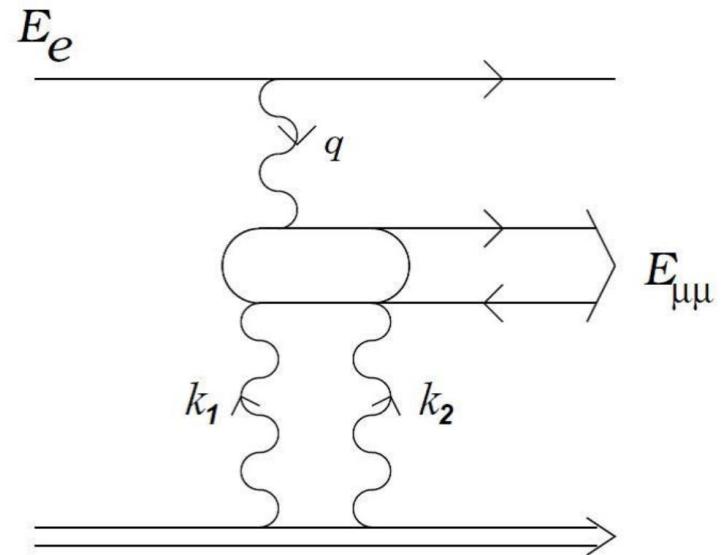
One photon (radiative) mode



Similar Kinematics to A' !

$$\text{Scattering Angle: } \theta_{1\gamma} \sim \frac{m_\mu}{E_0}$$

Three photon mode



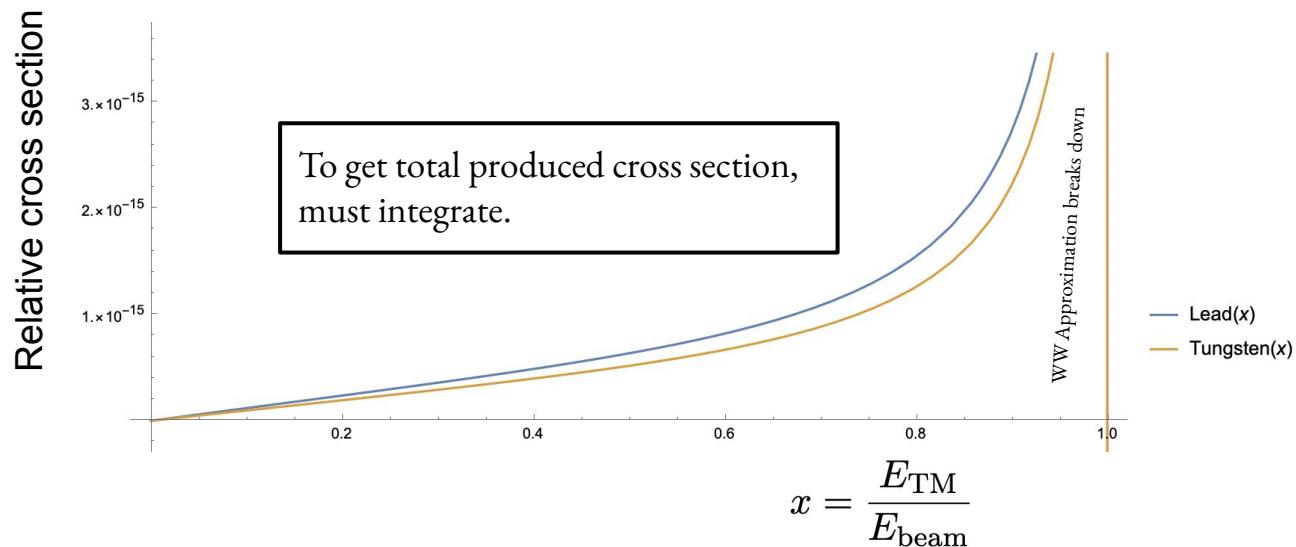
Oppressed by Bethe-Heitler Background

$$\text{Scattering Angle: } \theta_{3\gamma} \sim \frac{\Lambda}{xE_0} \quad \Lambda = \frac{405}{A^{1/3}} \text{ MeV}$$

Calculating Radiative Production Cross Section

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$$d\sigma = \frac{1}{4n^3} \frac{Z^2 \alpha^7}{m_\mu^2} \frac{x(1-x)(1-x+\frac{1}{3}x^2)dx}{[1-x+(m_e/m_{\mu\bar{\mu}})^2]^2} \times \left(\ln \left[\frac{(E_{beam}/m_\mu)^2 (1-x)^2}{1-x+(m_e/m_{\mu\bar{\mu}})^2} \right] - 1 \right)$$



Integrated Cross Sections

Pb (4.55 GeV) = 0.16 pb
Pb (6.6 GeV) = 0.33 pb
W (4.55 GeV) = 0.13 pb
W (6.6 GeV) = 0.27 pb

For more information on WW approximation: [Validity of the Weizsäcker-Williams Approximation and the Analysis of Beam Dump Experiments: Production of an axion, a dark photon, or a new axial-vector boson](#) (tldr: good for >100 MeV invariant mass, can be meh for low mass)

Total Electric Differential Cross Section and Dissociation

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$$d\sigma_{\text{tot}}^{nl} = Z^2 \frac{\alpha^2}{\pi} \left(1 - F^{nl0,n'l'0}(q)\right) \times \frac{1}{a^2} |\Delta(q, Z)|^2 q dq.$$

Thomas-Fermi-Moliere

$$F^{nlm,n'l'm'}(q) = \int_0^\infty \int_0^\pi \int_0^{2\pi} x^2 \sin(\theta) e^{iqx \cos(\theta)} \times \psi^{n'l'm'}(x, \theta, \phi)^* \psi^{nlm}(x, \theta, \phi) d\phi d\theta dx$$

$$\Delta(q, Z) = 4\pi \sum_{i=1}^3 \frac{\alpha_i}{q^2 + \beta_i^2}$$

$$\beta_i = \frac{m_e b_i}{121} Z^{1/3}$$

$$b_1 = 6.0, b_2 = 1.2, b_3 = 0.3$$

$$\alpha_1 = 0.10, \alpha_2 = 0.55, \alpha_3 = 0.35$$

- Includes transition and dissociation cross sections
- Doesn't include full magnetic differential cross section (~1% difference for Pb)

Note: more complete calculation in [Contribution of \$\alpha 2\$ -terms to the total interaction cross sections of relativistic elementary atoms with atoms of matter](#)

Total Electric Differential Cross Section and Dissociation

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Dissociation Length of Interest

$$l_{1S \rightarrow X} = \frac{A}{N_A \rho \sigma(1S \rightarrow X)} = \frac{1}{N \sigma(1S \rightarrow X)}$$

Note: more complete calculation in [Contribution of \$\alpha_2\$ -terms to the total interaction cross sections of relativistic elementary atoms with atoms of matter](#)

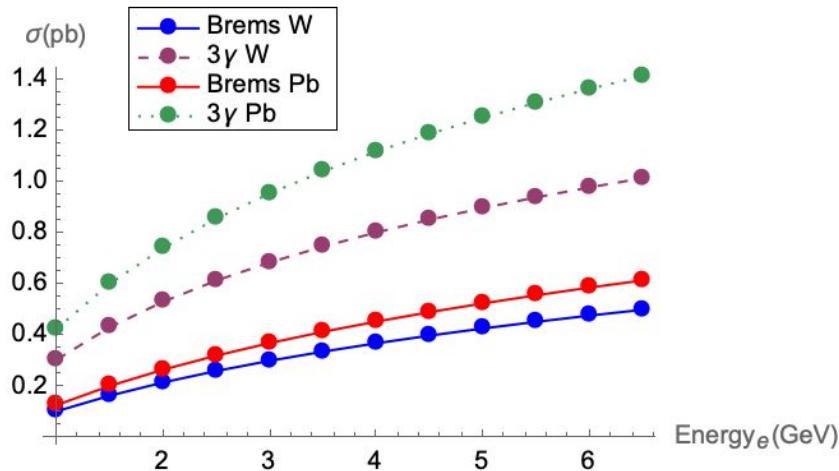
Total Dissociation Length and Cross Section for Different Z

(brem 13S1)

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Element	Atomic number	Atomic Density (g/cm^3)	Atomic Number Density	TM (13S1-->X) cross section (pb)	Dissociation Length (micron)
Tungsten	74	19.4	6.34E+22	5.68E+16	2.78
Gold	79	19.3	5.90E+22	6.43E+16	2.64
Lead	82	11.4	3.30E+22	6.90E+16	4.39
Bismuth	83	9.75	2.81E+22	7.06E+16	5.04

- Changing the target would lead to an increased yield by changing the dissociation length and increasing overall integrated cross section.
- Still need to calculate temperature at target.



B. Yale. Integrated cross section for Lead and Tungsten compared at different beam energies.

Cross Section for Specific Transitions

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$$d\sigma^{nl,n'l'} = \left(1 - (-1)^{l-l'}\right) Z^2 \frac{\alpha^2}{\pi} \frac{1}{a^2} q |\Delta(q, Z)| \times F^{nl0,n'l'0} \left(\frac{q}{2}\right) |^2 dq$$

	from 1S	from 2S	from 2P
to 1S	0	0	3.20×10^{-20}
to 2S	0	0	3.82×10^{-19}
to 2P	3.20×10^{-20}	3.82×10^{-19}	0
to 3S	0	0	9.53×10^{-21}
to e^+e^-	2.65×10^{-20}	7.52×10^{-20}	3.34×10^{-19}
to X	6.89×10^{-20}	5.92×10^{-19}	7.61×10^{-19}

- To calculate total cross-sections for each transition must integrate total cross section and subtract relative dissociation.

Schuster. Relative Differential cross section for different transitions.

Calculating Yield (in progress)

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Primary Contribution

$$\frac{dN_{1S}}{dz} = N_e \frac{\sigma(e^- \rightarrow 1S)}{\sigma(1S \rightarrow X)} - N_{1S}$$

$$z = \frac{l}{l_{1S \rightarrow X}}$$

Secondary Contributions

$$\frac{dN_{2S}}{dz} = N_e \frac{\sigma(e^- \rightarrow 2S)}{\sigma(1S \rightarrow X)} - N_{2S} \frac{\sigma(2S \rightarrow X)}{\sigma(1S \rightarrow X)} + N_{2P} \frac{\sigma(2P \rightarrow 2S)}{\sigma(1S \rightarrow X)}$$

$$\frac{dN_{2P}}{dz} = N_e \frac{\sigma(e^- \rightarrow 2P)}{\sigma(1S \rightarrow X)} - N_{2P} \frac{\sigma(2P \rightarrow X)}{\sigma(1S \rightarrow X)} + N_{1S} \frac{\sigma(1S \rightarrow 2P)}{\sigma(1S \rightarrow X)} + N_{2S} \frac{\sigma(2S \rightarrow 2P)}{\sigma(1S \rightarrow X)}$$

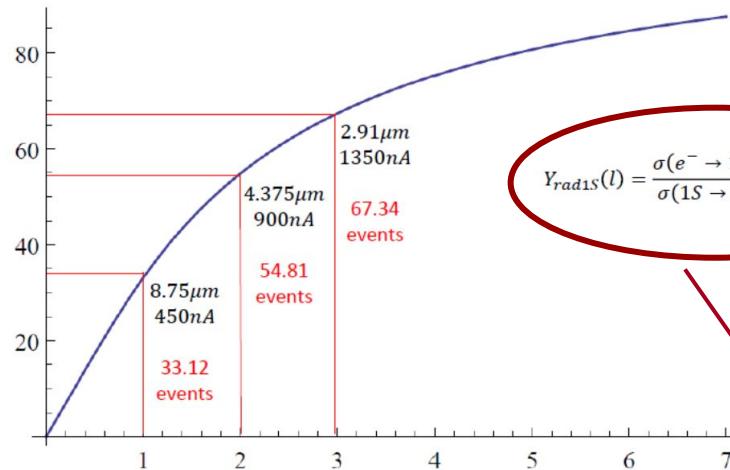
Calculating Yield (in progress)

Primary Contribution

$$\frac{dN_{1S}}{dz} = N_e \frac{\sigma(e^- \rightarrow 1S)}{\sigma(1S \rightarrow X)} - N_{1S}$$

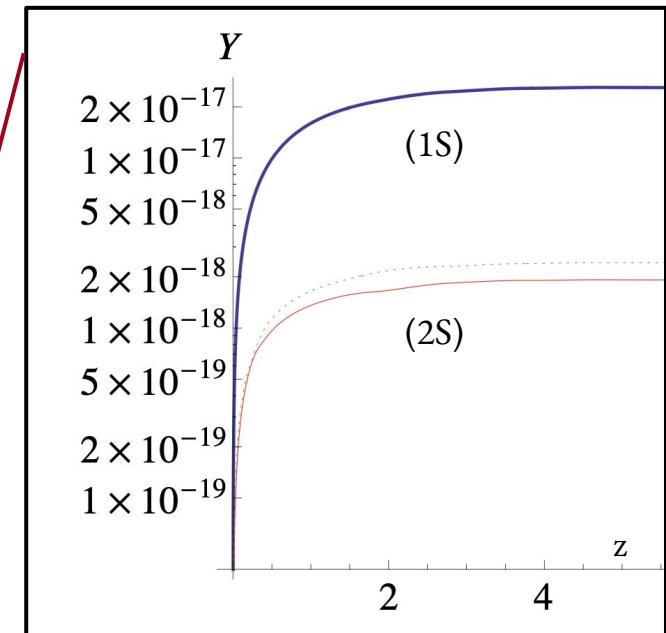
Rad. 1S generated per month

$$N * 450nA * Y_{rad1S}(8.75\mu m/N)$$



$$Y_{rad1S}(l) = \frac{\sigma(e^- \rightarrow 1S)}{\sigma(1S \rightarrow X)} \left(1 - e^{-\frac{l}{l_{1S-X}}} \right)$$

Relative Yield



B. Yale. Optimizing events by maximizing luminosity for a tungsten target

P. Schuster. Relative abundance of TM as a function of distance traversed through target measured in units of dissociation length.

Calculating Yield (in progress)

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- Assuming 450 nA current at JLAB, 6.6 GeV beam energy, 8.75 micron tungsten target
- Total **$1S_{(brem)}$** yield for 1 month of beam ~ 33.13
- **To-do:** Finish yield calculations and contributions from (2S, 2P)
- Agrees mostly with Yale yield calculations
 - Yale calculated **3.94 analyzable TM events per month**
 - With detector acceptance discussed later we expect **(2.58 - 3.15)** analyzable events per month

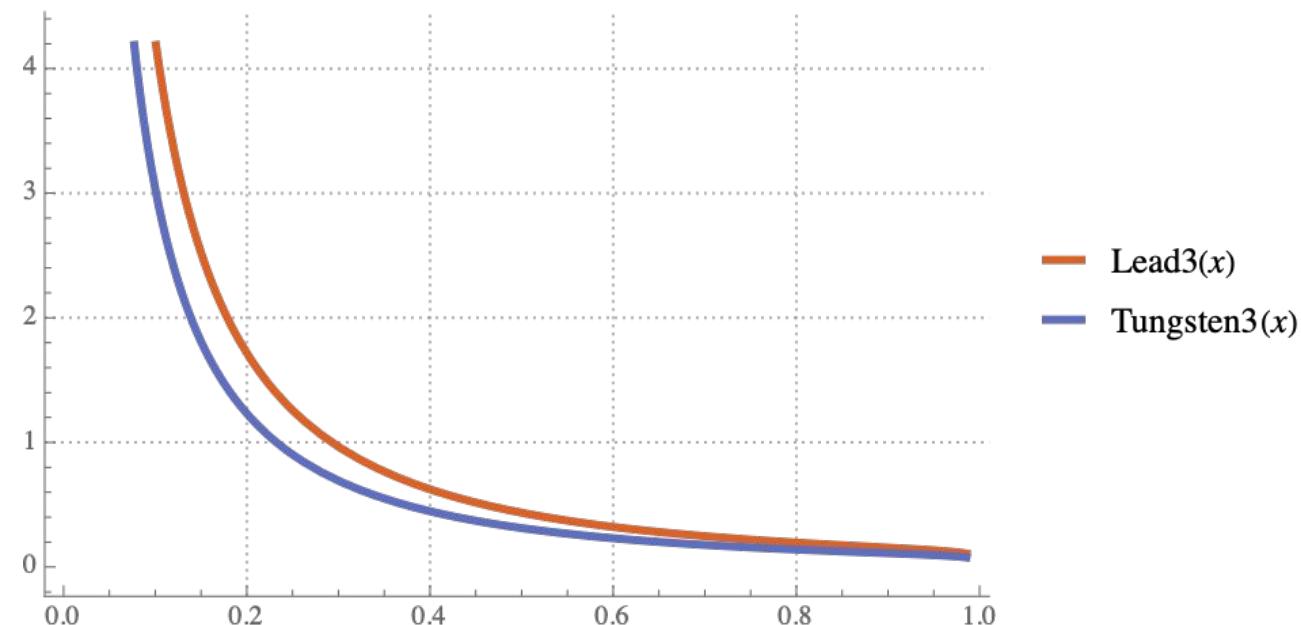
Calculating 3 Photon Production Cross Section

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Taking into account atomic screening

$$\frac{d\sigma}{dx} = \frac{1}{4n^3} Z^2 \frac{\alpha^7}{m_\mu^2} \left(\frac{Z\alpha\Lambda}{m_\mu} \right)^2 \frac{4B}{x} \times \left(\left(1 - x + \frac{x^2}{2} \right) \text{Log} \left[\frac{(1-x)(m_\mu)^2}{x^2(m_e)^2} \right] - 1 + x \right)$$

Three Photon Production Cross Section



- More dependent on energy fraction than beam energy
- Calculating Total yield is equivalent process to radiative

Three photon mechanism is derived fully here: [Production of the bound triplet \$\mu^+ \mu^-\$ system in collisions of electrons with atoms](#)

Prospects of 3 Photon Events in Previous Data (Without Acceptance Cuts)

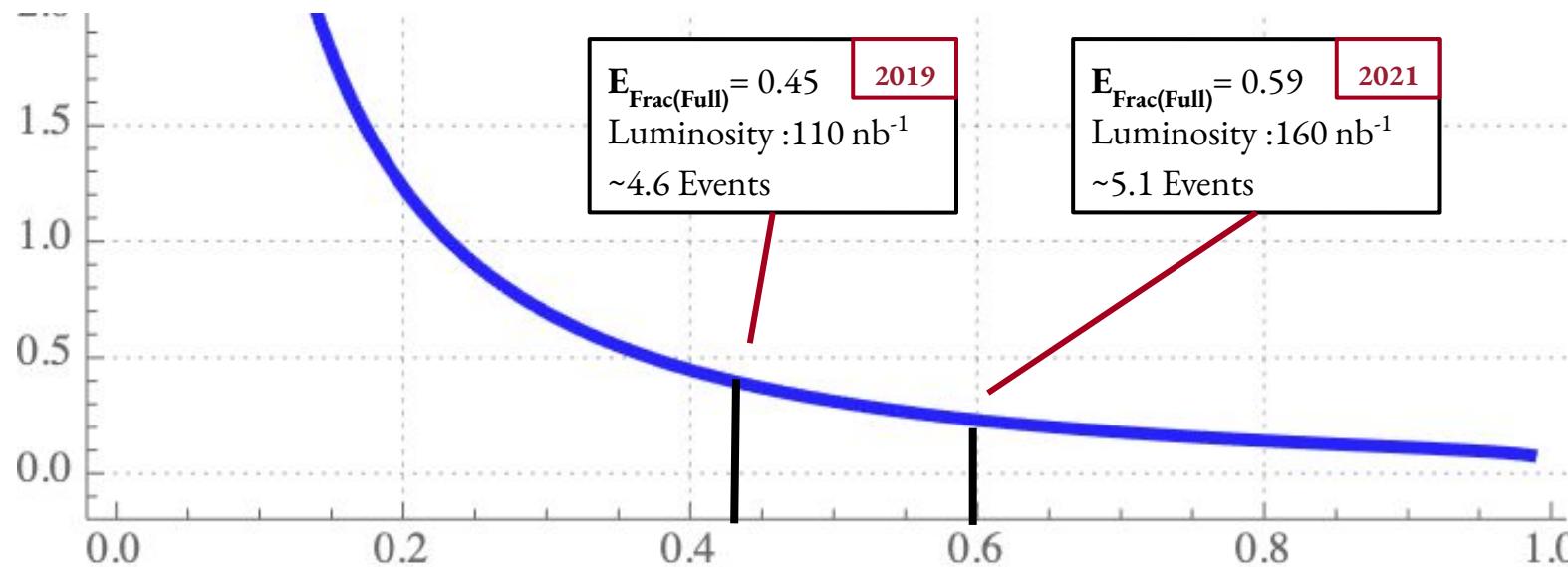
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Triggers used during runs limit number of potential events.

- 2019 and 2021 trigger reach full sensitivity at ~ 2.2 GeV,
- we are sensitive to $\sim \frac{1}{2}$ events from 1 GeV - 2.2 GeV

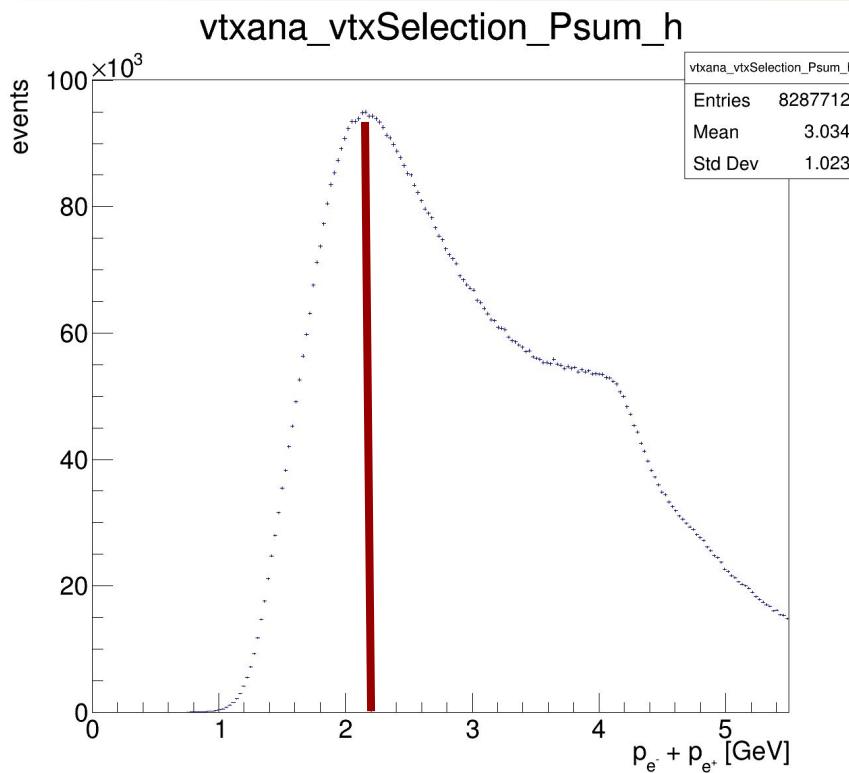
Year	$E_{Frac(partial)}$	$E_{Frac(Full)}$
2019	0.22	0.46
2021	0.27	0.59

Run Year	Total Number of Electrons	Int Cross section at Efrac (full) [pb]	Int Cross section at Efrac (partial) [pb]	Full Sensitivity Yield	Half Sensitivity Yield
2019	1.56E+18		0.099	0.147	2.72
2021	2.27E+18		0.059	0.141	2.35

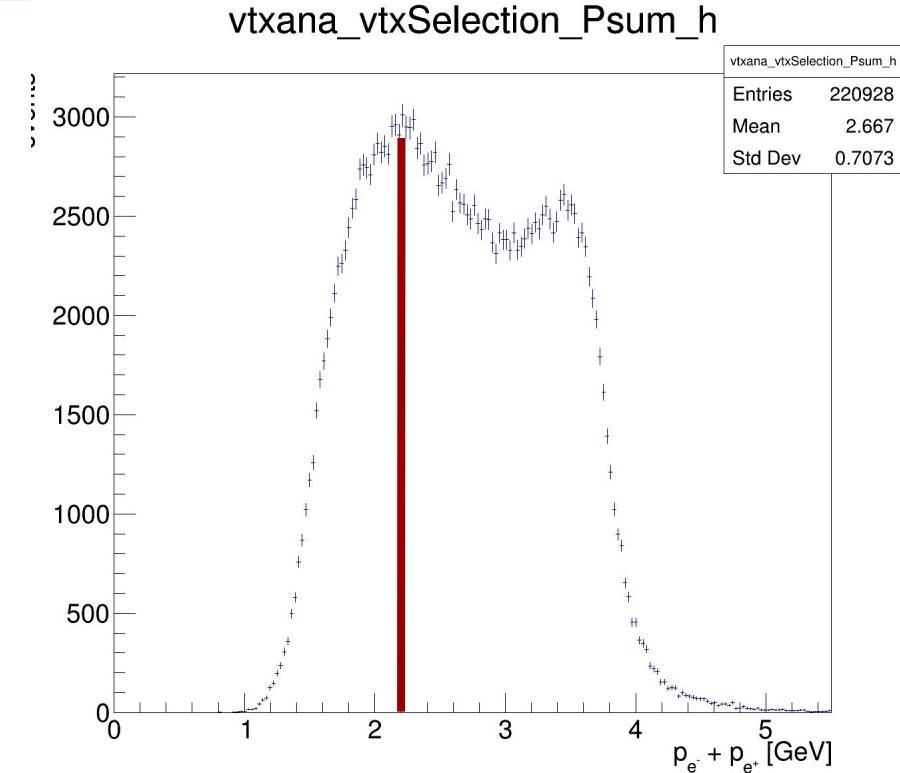


Psum Plots illustrating optimal Efrac and detector sensitivity for 2019/2021

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Full detector efficiency: 2019



Full detector efficiency: 2021

Note: previously mentioned partial sensitivity to events before reaching full efficiency. Total number slightly overestimated as more events are produced at lower E_{Frac}

Summarizing Three Photon Production

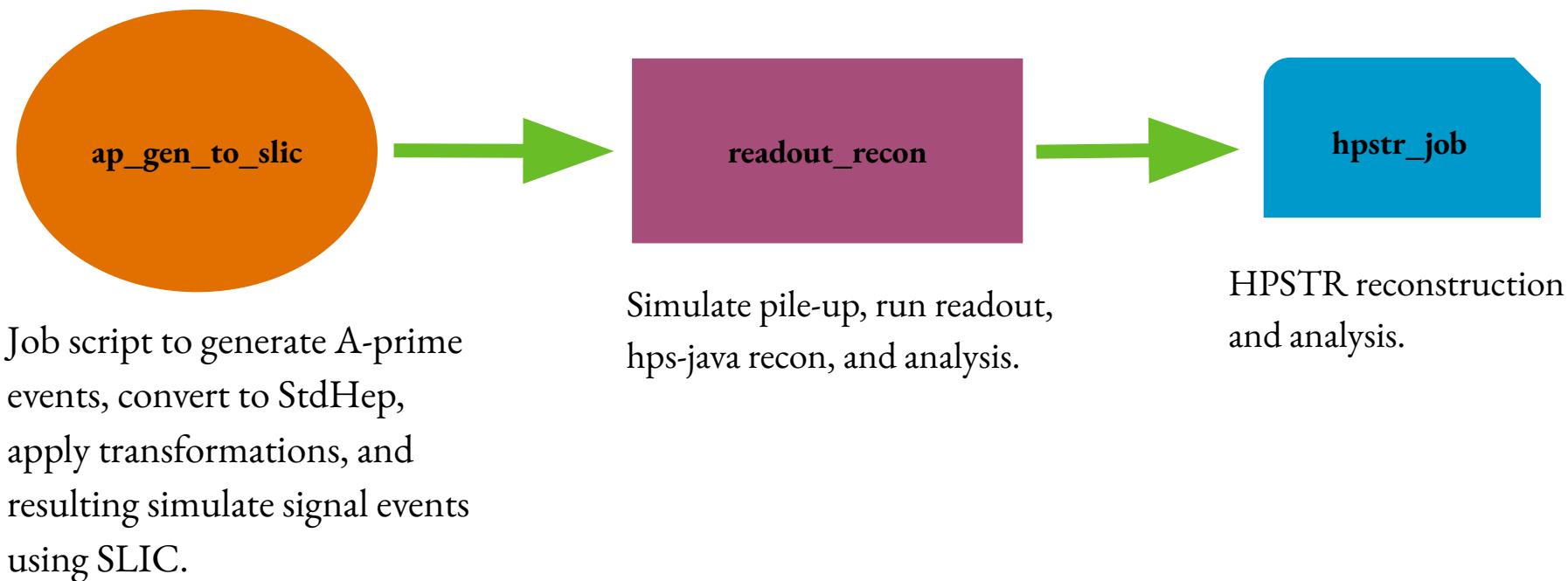
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- Total Events from 2019/2021 ~ 9.7
 - Motivating toy monte carlo study to more concretely determine kinematics and determine detector efficiency for these events.
- If these particles are traveling through detector, would then need to determine if a lower PSum trigger would help for future runs

- Use existing infrastructure for a scan at a single mass (Assume $A' = 211$ MeV) and two beam energies:
 - **4.55 GeV, 6.6 GeV.**
- Detector used for 4.55 GeV → HPS_Nominal_I0
- Made Detector for 6.6 GeV → HPS_Nominal_6pt6GeV
 - Used field map from detector 2017-nominal-6pt6
 - Changed bfield to -1.5T

Flow using /hps-mc/python/jobs

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Dedicated TM Scripts



TM_gen_to_slic

- job.json.tpl
- mkjobs.sh
- jars.json
- vars.json
- subArray.sh (using ap_gen_to_slic type)
- .hpsmc

run : 10666
apmass: 211.0 MeV
detector: HPS_Nominal_iter0
target_z: 0.0
ctau: 160.0 events: 10000

- output/HPS_Nominal_iter0/lhe/TM_4pt55_TMm211pt0_100.lhe.gz
- output/HPS_Nominal_iter0/slic/TM_4pt55_TMm211pt0_100.slcio

readout_recon_TM

- subArray.sh (using readout_recon type)
- **hpstr Ana**

Config files: **recon - kalSimpTuple_cfg.py, ana - anaVtxTuple_cfg.py**

Steering files:

- readout: "PhysicsRun2019TrigPulse", "PhysicsRun2019TrigMultiSingles"
- recon: /org/hps/steering/recon/PhysicsRun2019MCRecon.lcsim

- /HPS_Nominal_iter0/PhysicsRun2019TrigPulse/TM_reco_100(.root, .slcio)
- /HPS_Nominal_iter0/PhysicsRun2019TrigMultiSingles/TM_reco_100(.root, .slcio)

hpstr_ana

- subArray.sh (using hpstr type)

Added following files to hpstr:

- /analysis/selections/vertexSelection_TM_4pt55.json
- /analysis/selections/Tight_TM_4pt55.json
- /analysis/selections/Tight_pTop_TM_4pt55.json
- /analysis/selections/Tight_pBot_TM_4pt55.json
- /analysis/plotconfigs/tracking/vtxAnalysis_TM_4pt55.json

Config: anaTMTuple.py (added into hpstr/processors/config)

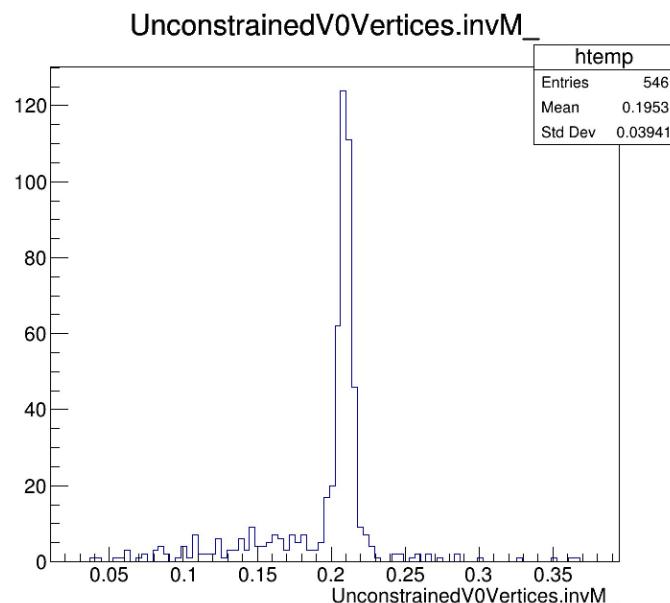
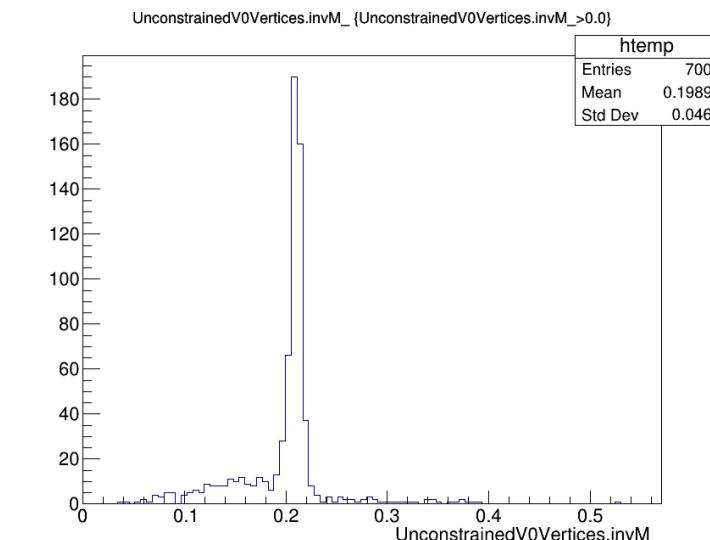
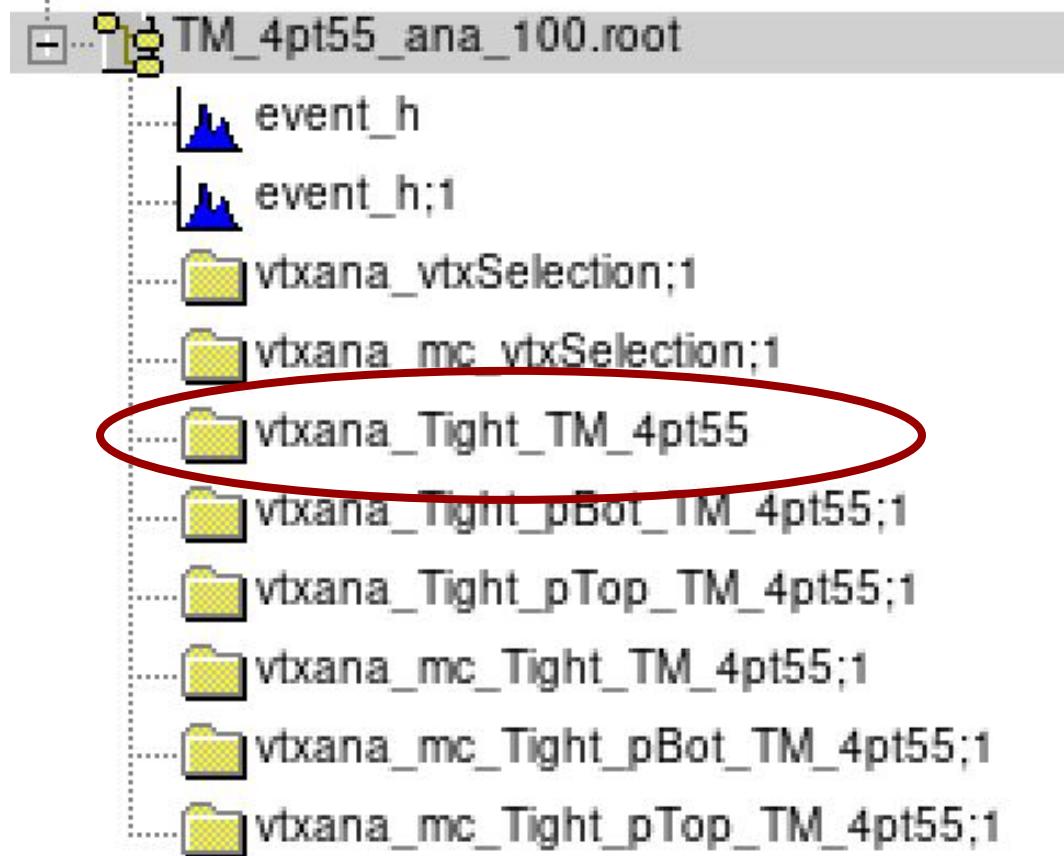
Run_mode: 0

Year: 2019

- /HPS_Nominal_iter0/PhysicsRun2019TrigPulse/TM_4pt55_ana_100.root
- /HPS_Nominal_iter0/PhysicsRun2019TrigMultiSingles/TM_4pt55_ana_100.root

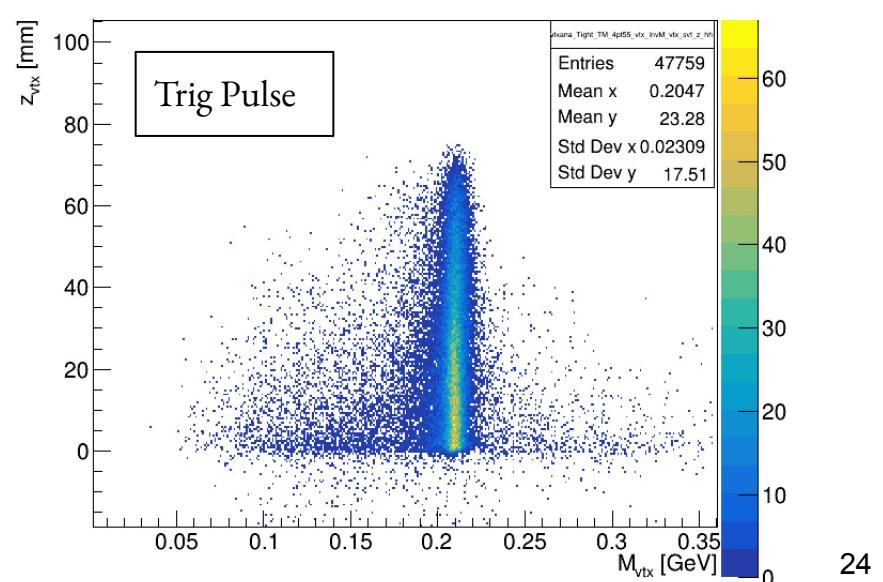
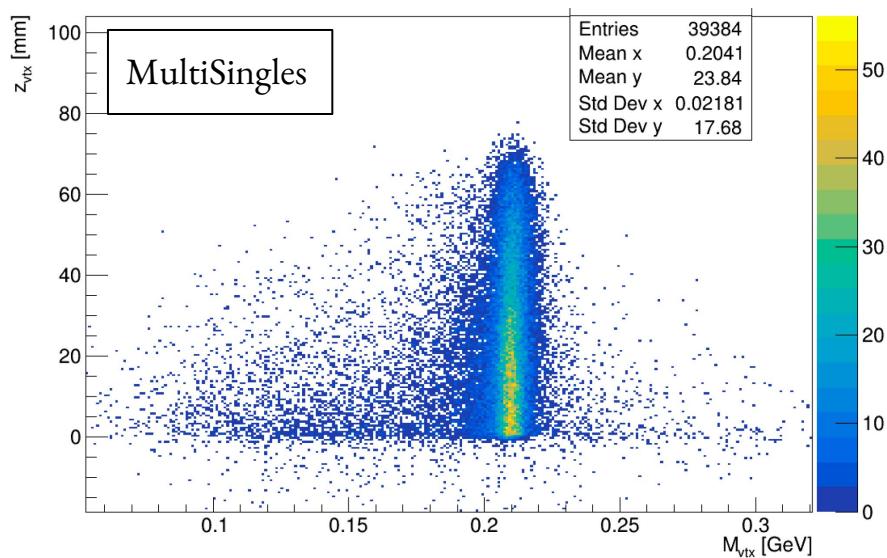
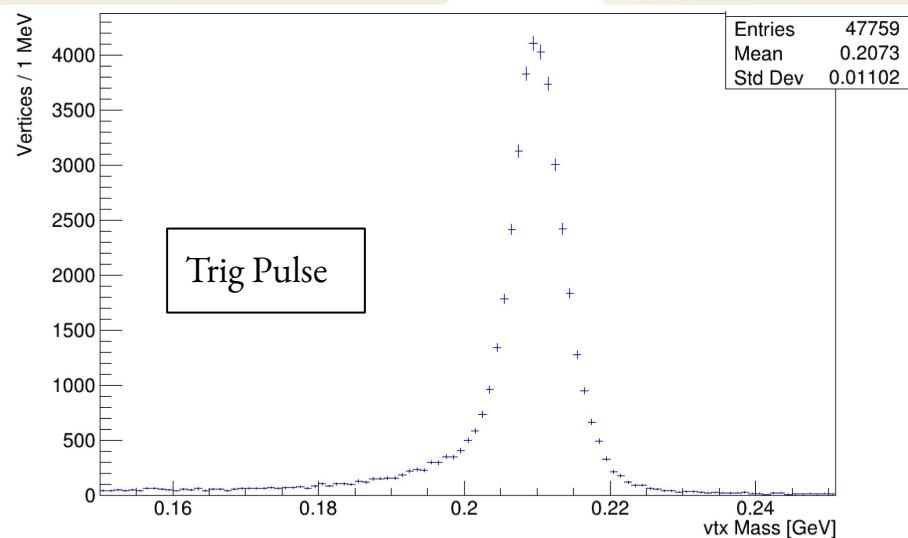
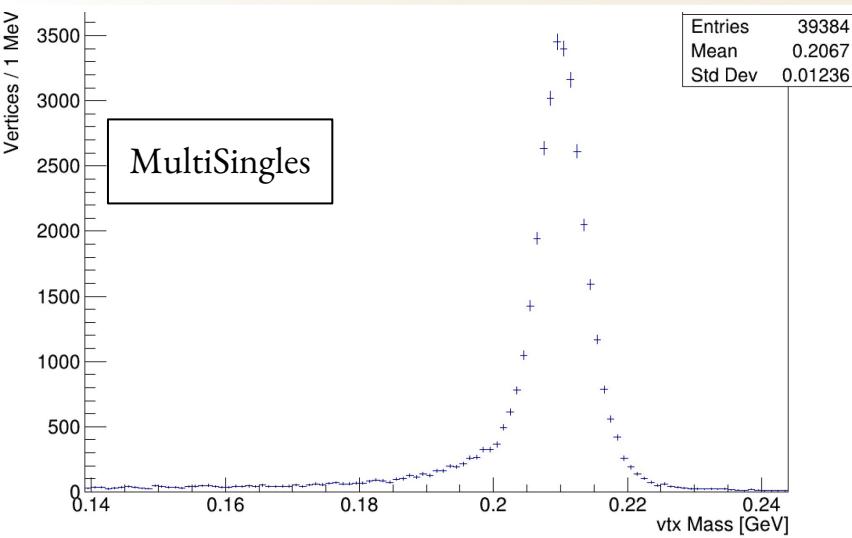
4.55 GeV (Preliminary)

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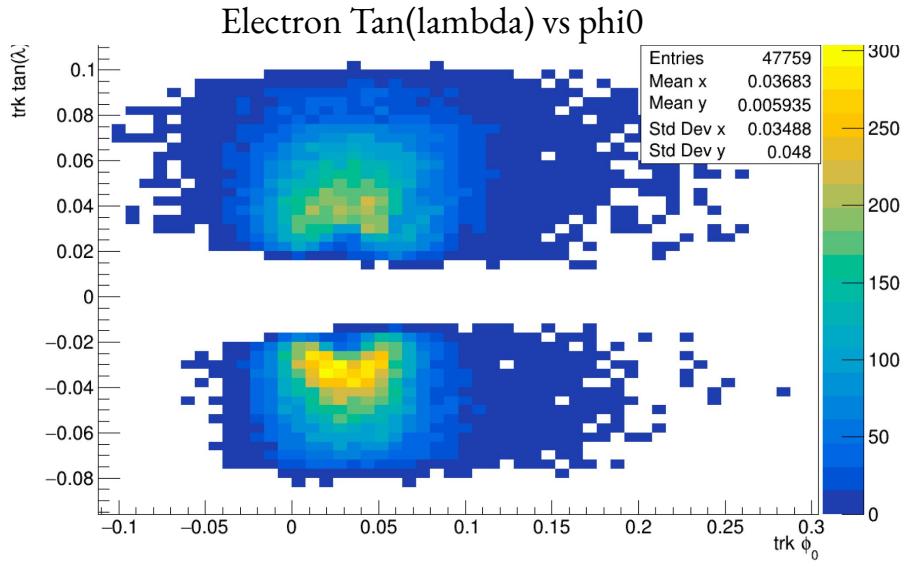
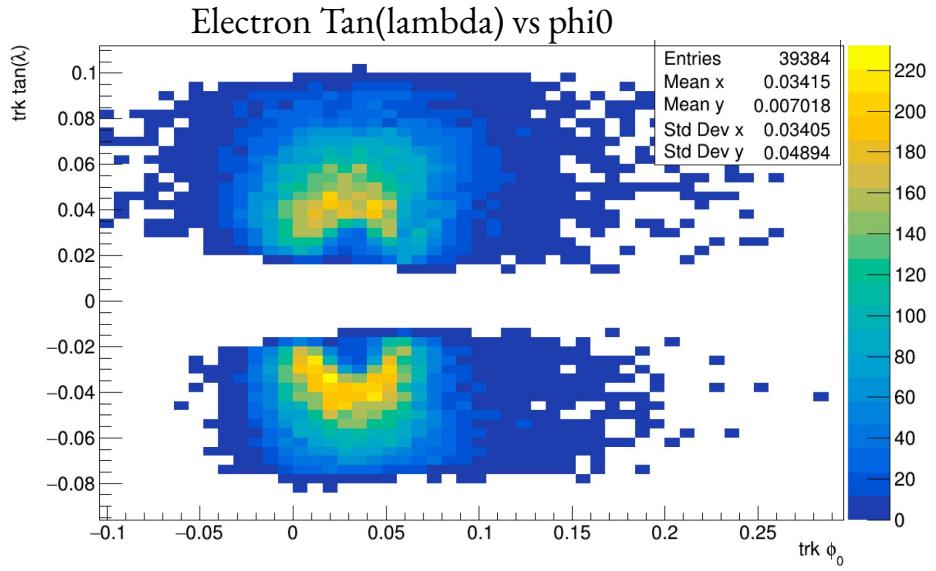
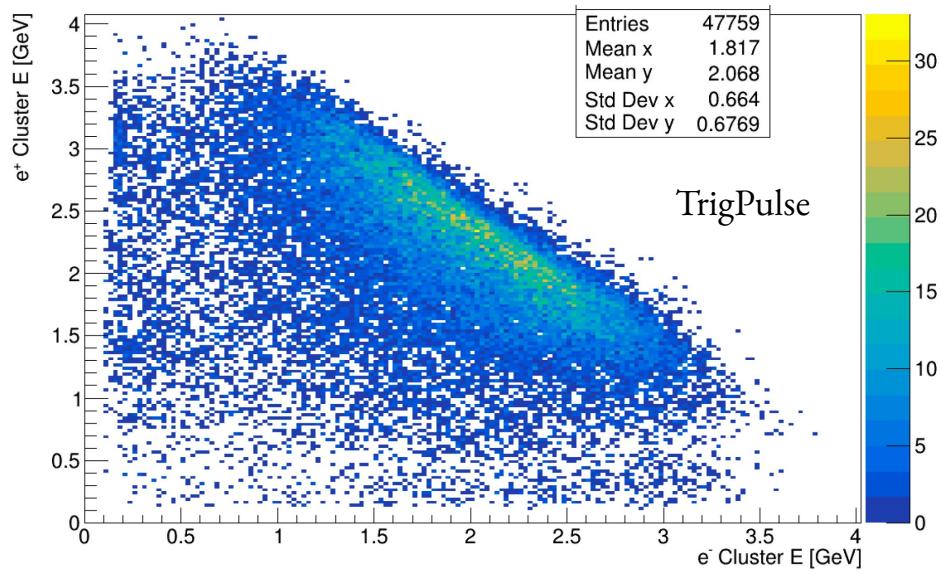
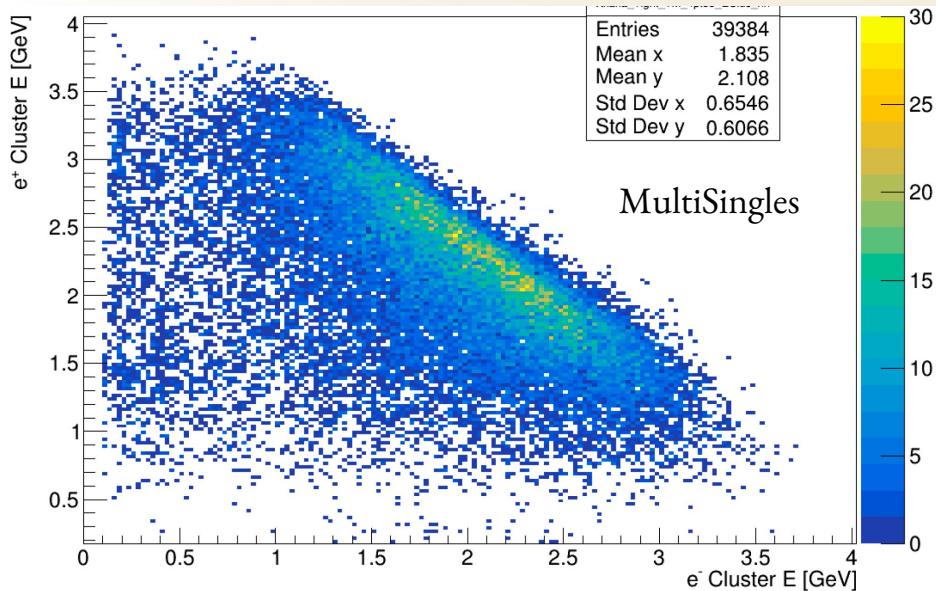
4.55 GeV Plots (1/2)

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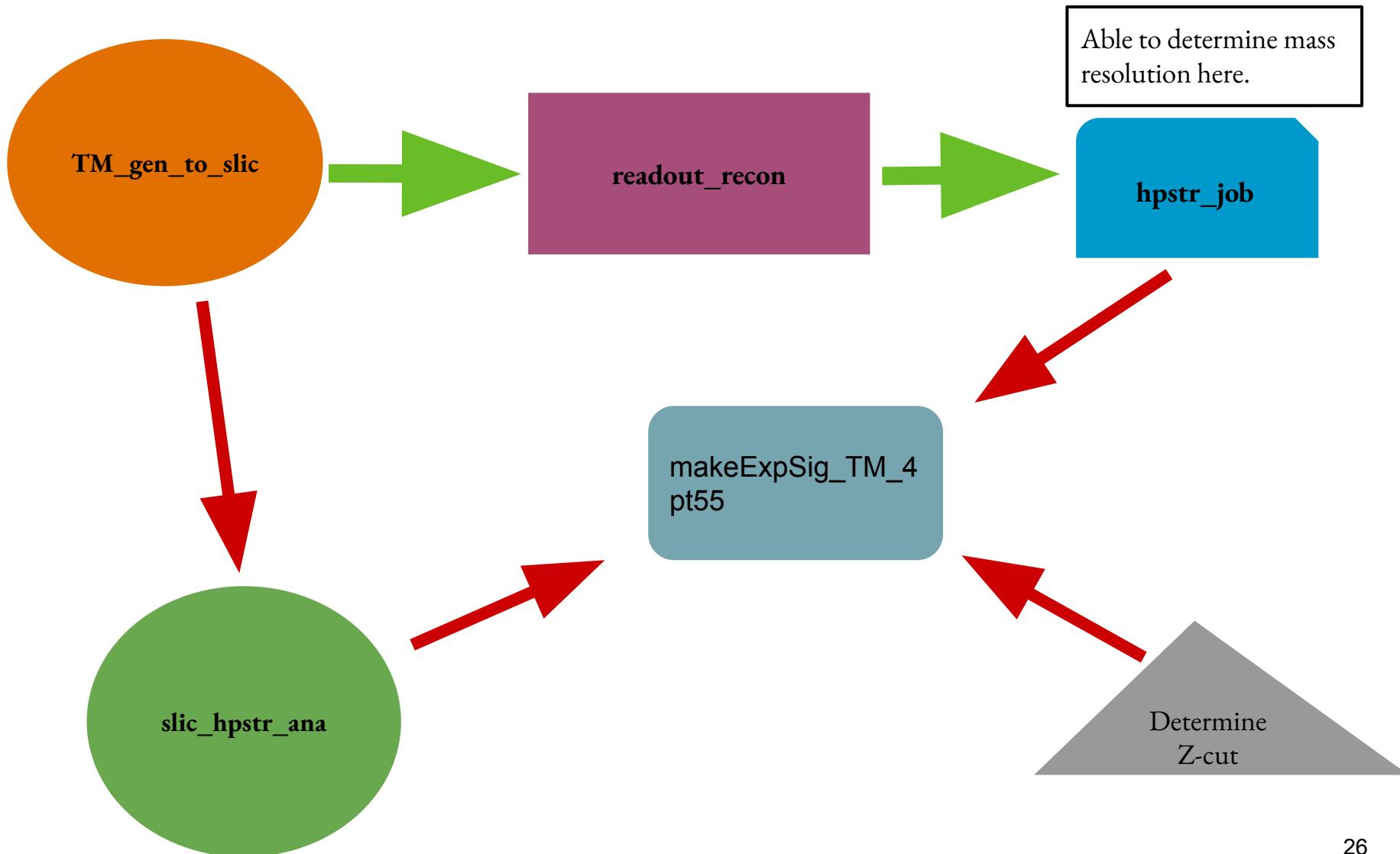
4.55 GeV Plots (1/2)

SLAC



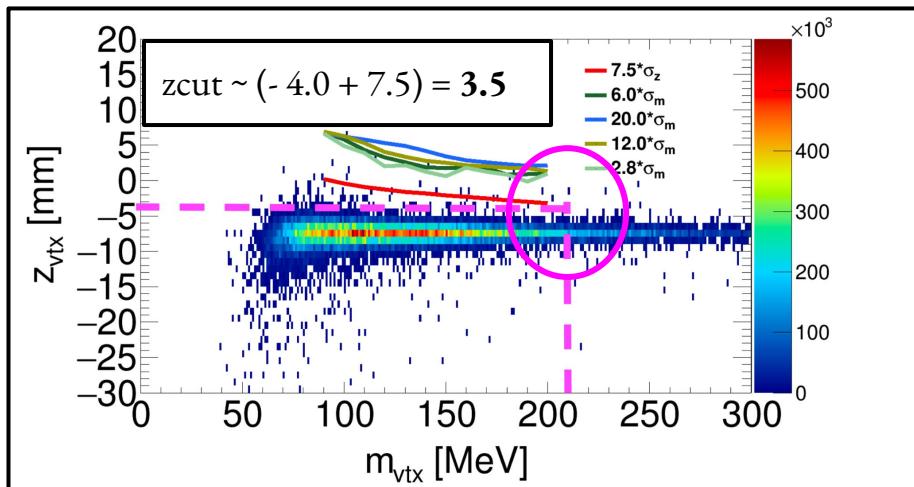
Vertex acceptance flow using /hps-mc/python/jobs

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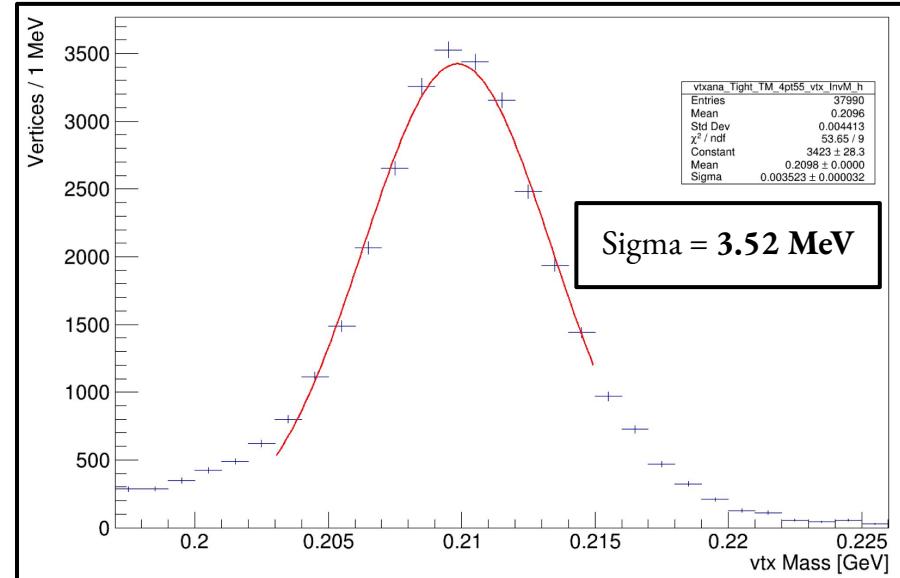


Calculating Vertex Acceptance for Radiative (1S) [1/2]

- Determine mass resolution of TM
 - a. Fit gaussian to invariant Mass plot of hpstr TrigPulse output and determine sigma
- Determine ZCut for TM for each beam energy (using 0.0 as origin)
 - a. If good stats for WAB and tritrig, use `hpstr/plotUtils/reach/makeZcuts.py`
 - b. For 2019, sufficient to use [Cam's reach estimate](#) to determine zcut



C. Bravo. Comparison of Zcuts for 2019 Reach.



Using `makeZcuts.py` gave a zcut of 0.89, but input file stats were low and ultimately need to be redone to be thorough.

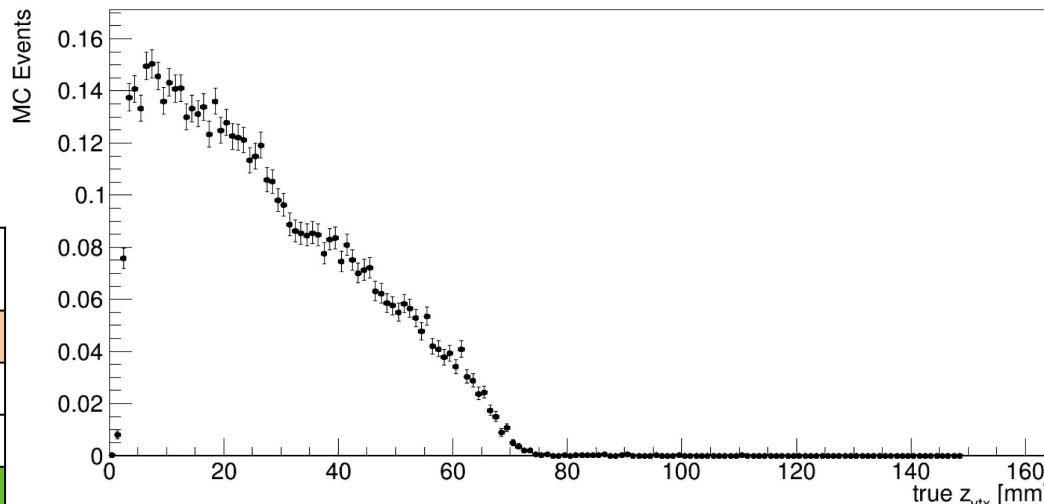
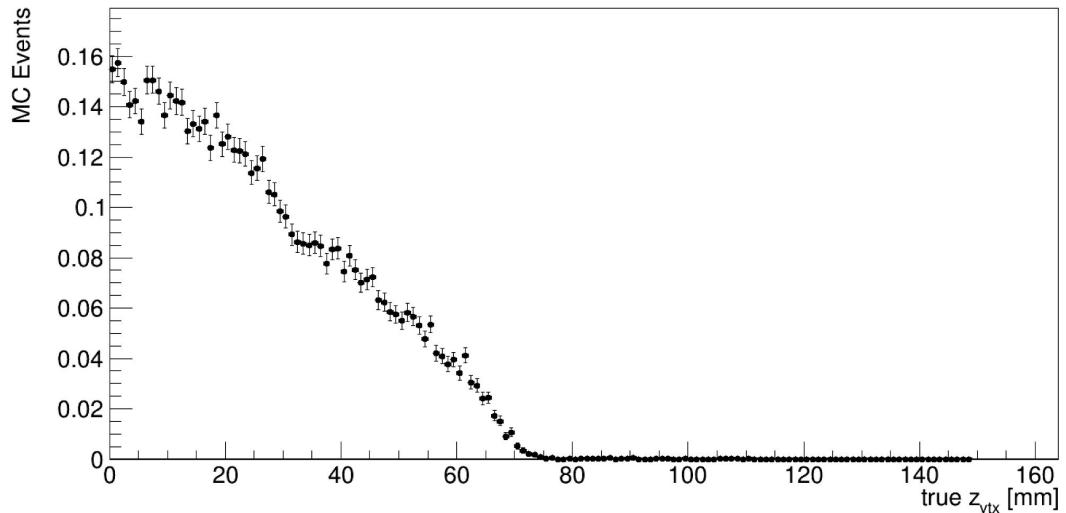
Sigma without PSum cuts was improved at 3.39 MeV.

Calculating Vertex Acceptance for Radiative (1S) [2/2]

- Reformat / repurpose
`/hpstr/plotUtils/reach/summer20/makeExpSi
g2019.py` to only use a single invariant mass
(211 MeV) and
 - use mass resolution and zcut determined previously
 - Input files:
 - hpstr output of the slic data
 - hpstr output of readout_recon
`(vtxana_Tight_TM_4pt55/vtxan
a_Tight_TM_4pt55_tree)`
- Signal efficiency and normalized vertex selection gives vertex acceptance

Vertex Acceptance for Radiative (1S)

ZCut Applied	Momentum Cuts Applied?	Vertex Acceptance
0.89	No	12.50%
2.50	Yes	8.80%
2.50	No	10.50%
3.50	No	9.50%
4.50	No	8.60%



Radiative (1S) True Muonium Event (s) from Previous Data

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Integrated Cross Section	Total Radiative Events	Zcut	Vertex Acceptance	Accepted Events
0.126 pb	3.5	3.5	(7.8 - 9.5)%	0.27 - 0.33

This should motivate an acceptance study on 3.7 GeV.

- Radiative events from 2021: 1.92 anticipate accepting (0.61 - 0.77) events.
- Lower energy seems to marginally help with acceptance of TM as the on average decay length is shorter

Must still determine consequences of removing psum cuts from Tight.json files to determine this.

Total Radiative Events	Total 3Photon Events	Anticipated Yield	Optimistic Accepted Events****
5.37	9.82	15.19	(1.18 - 1.44)

****Need to determine efficiency for radiative and 3 photon events to rigorously determine this.

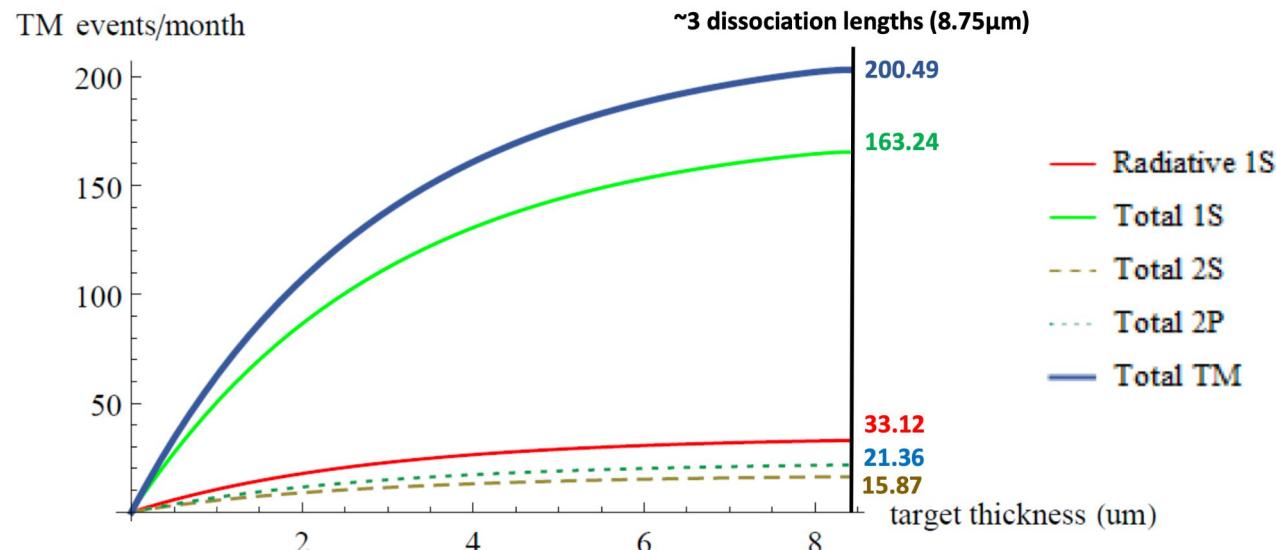
6.6 GeV Future run?

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True Muonium Generated (450nA)

Assuming current trigger cuts on three photon events and equivalent target, HPS would likely generate:

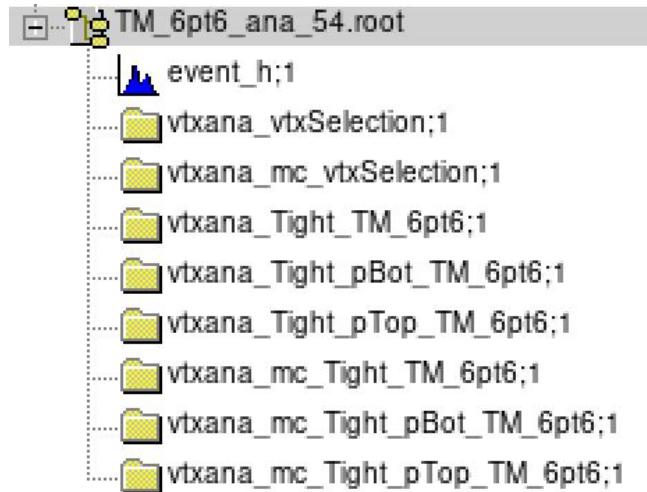
- 32 Three photon events
- 31 Radiative events



B. Yale. On the generation of true muonium.

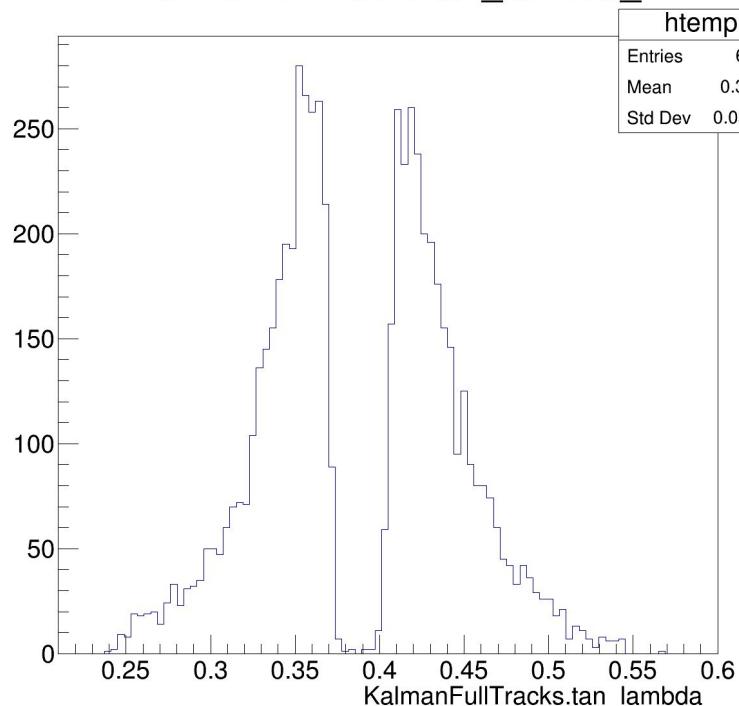
6.6 GeV (Preliminary)

- Tongtong made a new steering file that set DAQ configuration for a 6.6 GeV Beam
Future6pt6TrigSingles.lcsim – thanks!
- Was able to go through the same procedure as for the 4.55 GeV
 - Tm_gen_to_slic → readout_recon → hpstr_ana
- Noticed a print statement still set to 4.55 GeV (need to figure out where this is coming from)
- Have been stonewalled, next slide illustrates dilemma

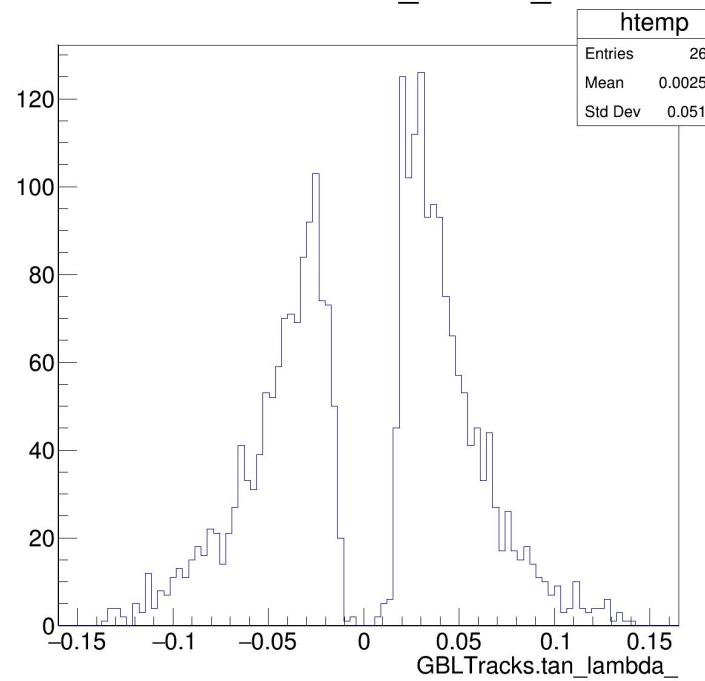


6.6 GeV Plots

KalmanFullTracks.tan_lambda_

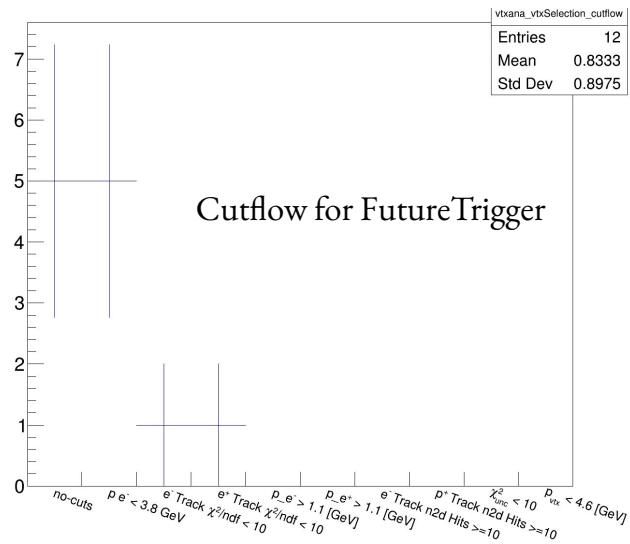


GBLTracks.tan_lambda_

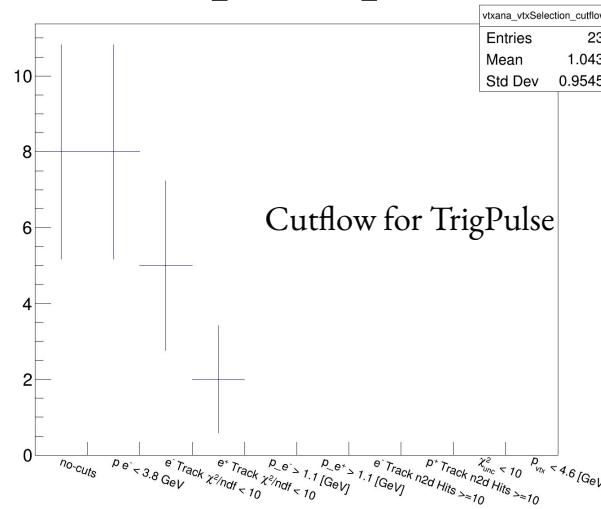


SLAC

Cutflow for FutureTrigger



vtxana_vtxSelection_cutflow



Next Steps

SLAC

- Finish acceptance study for 6.6 GeV
- Study kinematics of three photon mechanism
 - Then determine efficiency if these particles go through detector
- Determine temp of target during run (dependent on E)
- Determine optimal run parameters for 6.6 GeV beam
 - lead and tungsten comparative study
 - optimized trigger?

Worthwhile Resources (incomplete)



- Schuster. [The Production and Discovery of True Muonium in Fixed-Target Experiments](#).
- Holvik E, Olsen HA. [Creation of relativistic fermionium in collisions of electrons with atoms](#). Phys Rev D Part Fields. 1987 Apr 1;35(7):2124-2129. doi: 10.1103/physrevd.35.2124. PMID: 9957899.
- N. Arteaga-Romero, C. Carimalo, and V. G. Serbo. [Production of the bound triplet \$\mu+\mu-\$ system in collisions of electrons with atoms](#). Physical Review A, 62(3), Aug 2000.
- Stanley J. Brodsky and Richard F. Lebed. [Production of the smallest qed atom: True muonium\(\$\mu+\mu-\$ \)](#). Physical Review Letters, 102(21), May 2009.
- B. Yale. [HEAVY PHOTON DISPLACED VERTEX SEARCH AT 2.3GeV WITH PROSPECTS FOR TRUE MUONIUM DISCOVERY](#) (Chapter 8)