Mutually Unbiased Bases (MUBs)

Mutually Unbiased Bases: If A is a matrix then A^* denotes the conjugate transpose of A. Let $A, B \in U(d)$. As elements in $U(d)$, the columns of A and the columns of B each form orthonormal bases for $\mathbb{C}^d$. The bases are said to be mutually unbiased if A^*B is flat. What this means is that the entries of A^*B are all of equal magnitude (of size $\sqrt{1/d}$). In $U(4)$ there exists a family of 5 mutually unbiased bases. In $U(6)$, a family of 3 mutually unbiased bases is known. It is an open problem whether there exists a family of 4 mutually unbiased bases in $U(6)$. Note that if A, B, C are mutually unbiased and flat then I, A, B, C will be mutually unbiased bases.

Note: If A is a square matrix with Singular Value Decomposition $A = U\Sigma V^*$ then the closest unitary matrix to A with respect to the Frobenius norm is UV^*.

Problem: Given a unitary matrix A, can you find and implement an algorithm to determine the closest flat unitary matrix to A with respect to the Frobenius norm on matrices.

General Open Problem: What is the maximum number of MUBs in $U(d)$ where d is not a prime power?

First Open Case - Equivalent Problem: Do there exist flat matrices $A, B, C \in U(6)$ such that A^*B, A^*C, B^*C are flat?

General Problem: Produce a set of three flat matrices $A, B, C \in U(6)$ such that A^*B, A^*C, B^*C are "as flat as possible".

Problem: Let $V \subset U(d)$ denote the subset of unitary matrices that are flat. Define a combinatorial graph G whose vertices are the elements in V and whose edges are the set of pairs $\{A, B\} \subset V$ satisfying $A^*B \in V$. What can one say about the graph G?

`https://en.wikipedia.org/wiki/Mutually_unbiased_bases`