# Projections of $n$-cubes, $n$-simplices, and $n$-cross polytopes

### $n$-cubes

An $n$-cube is the convex hull of the points in $\mathbb{R}^n$ with coordinates $(\pm 1, \ldots, \pm 1)$.

A zonotope is the projection of an $n$-cube.

### $n$-simplex

Let $I_n$ denote the $n \times n$ identity matrix. Each row of $I_n$ corresponds to a point in $\mathbb{R}^n$.

The standard $n$-simplex in $\mathbb{R}^{n+1}$ is the convex hull of the rows of $I_{n+1}$.

Subtracting $(1/n, \ldots, 1/n)$ from each row centers this $n$-simplex about the origin.

The centered $n$-simplex lies in the vector space orthogonal to the vector $[1, 1, \ldots, 1]$.

A polytope is the convex hull of a set of points in $\mathbb{R}^k$.

Every non-degenerate polytope is the projection of a scaled/translated regular simplex.

### $n$-cross polytope

The standard $n$-cross polytope is the convex hull of the rows of $I_n$ and $-I_n$.

Problem 1: Experimentally find the expected number of vertices and edges of the shadow of a 3-cube to $\mathbb{R}^2$ under a random projection.

Problem 2: Experimentally find the expected number of vertices and edges of the shadow of an $n$-cube to $\mathbb{R}^2$ under a random projection.

Problem 3: Experimentally find the expected area of the shadow of a 3-cube in $\mathbb{R}^2$ under a random projection.

Problem 4: Experimentally find the expected area of the shadow of an $n$-cube in $\mathbb{R}^2$ under a random projection.

Problem 5: Repeat Problems 1-4 for the $n$-simplex and the $n$-cross polytope.

Problem 6: Repeat Problems 1-5 for projections to $\mathbb{R}^3$ and $\mathbb{R}^k$.

You might enjoy: `https://www.youtube.com/watch?v=ltLUadnCyi0`