Midterm

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I. Problem Statement and Motivation

The concept of gravity and spacetime curvature is commonly visualized in media through the use of an elastic sheet with a heavy object like a bowling ball in the center and smaller orbs revolving around it. While not a perfect representation of general relativity, they can be a good analogy for illustrating to the average person the concept of relativity and in a way how gravity itself works. While this can be something easily assembled in the classroom, it also lends itself to being simulated through real world testing.

II. Background and Literature Review

The use of a rubber sheet is derived from the theory of relativity. It is used in schools to teach general relativity to students, teach them about orbits, gravitational waves, and the relation of geodesics to this concept. In the concept of general relativity, the path of a freely falling object is a geodesic path through spacetime. This spacetime is curved by massive objects such as planets and stars. In a similar manner, we can represent the curvature of spacetime through the use of an elastic sheet that is deformed by a large mass.

Research that has been done on this analogy and modeling it in the real world has been done through either the use of trampolines or stretched latex. Batlle et al. analyzed the movement of a small ball orbiting a heavy mass placed in the center of a trampoline (a bowling ball with additional weights added. They modeled the trampoline shape and both a point particle model and a rolling sphere model to account for torque and moment of inertia. The experimental data found that the rolling sphere model significantly outperformed the simpler model in accurately representing the trajectories. Critically, though, they found that in representing the true model of relativity, the model failed giving apsidal precession values that were negative compared to the positive values predicted by general relativity. In this case Apsidal precision is the rotation of the elliptical orbit so that its apoapsis and periapsis (peak and valley) process around the body being orbited.

One of several methods evaluated for the validation of a ball on sphere model is the use of a Kinematic Match method which was specifically developed for the modelling of an elastic surface undergoing deformation under the impact of a sphere. This method uses the matching of kinematic constraints to the coinciding surfaces, matching velocities and imposing a tangency condition at the boundary of the contact. It allows for a detailed method of calculating the sphere's trajectory. This method however is not a nodal method and more complicated than would be desired in the universe model where new objects are not appearing in spacetime.

Another method that is better applied to the nodal contact solution is the use of implicit material point method (MPM) with the contact dynamics (CD) method to simulate soft particle interactions between the surface and the orbs. This method allows for the use of arbitrary particle shapes and assumes that deformations are localized to the contact points. It is able to create frictional contacts without the use of artificial damping or regularization. The use of friction is crucial in this method as the friction strongly impacts the deformations in the model due to the formation of stress chains which are greatly increased in strength.

III. Approach and Contributions

The project will have three main aspects.

(1). Implementation of a simple rolling sphere model over a deformed membrane. The membrane will be modeled as a discretized elastic element with a mesh of points and springs used. This is similar to how the mesh of points were made by Batlle et al.

(2). Implementation of contact dynamics. This will rely on the use of implicit material methods with the elastic being a large deformable surface that is deformed by what will be assumed to be perfectly hard spheres. When the spheres contact the surface they will use an iterative process to calculate their applied contact area at each step so that the tangency error will be minimized during the tracking.

(3). The last and final step is the simulation of the gravitational analogy. This will be taking the applied steps and putting them together to simulate the motion of the orbs across the surface.

The main contribution that this project will apply to this field is the use of different methods for calculating the surface dynamics as well as the use of a simulated elastic surface in the model to account for the gravitational effect of the orbiting body.

IV. Progresss to Date and Upcoming work

Most of the progress so far on this project has been made to developing the flat sheet mesh of the model as seen in figure 1. This was done through the automation of the points and their boundary conditions and a simple spring mesh model connecting each of the nodes.

The next phase of the project is to match the characteristics of the surface when it has a large mass at the center to make sure the sheet droops as expected. Afterwards, the most crucial aspect will be the implementation of the contact mechanics where the sphere will be able to roll. This will be done through the implementation of implicit frictional mechanics of the surface with spheres that are treated as rigid on this surface. Once this is done the trajectories of the balls will be simulated rolling across the surface.

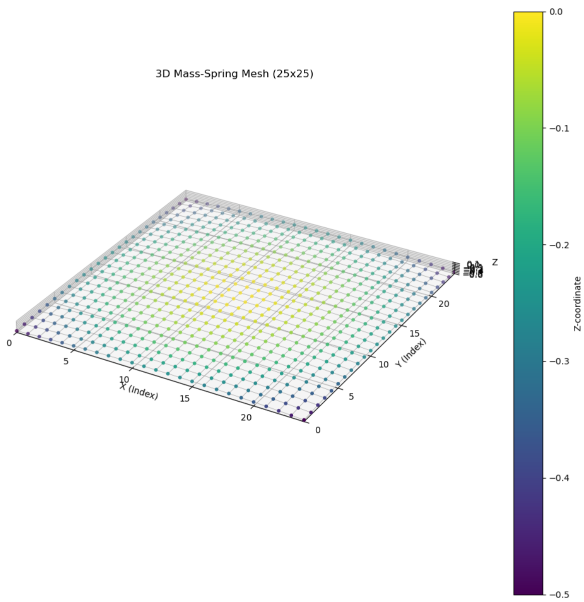

Fig. 1. Flat sheet mesh with gravity

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