School of Electronics and Computer Science
University of Southampton

COMP6216: Report on Coursework Assignment II

Worth: 70%

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Contents

1 Introduction

This report is about solving a modeling problem by using both agent based model and differential equation based approach. The problem is called "Group Dynamics of Hard Workers and Lazy Workers", it is described in the section below.

1.1 Problem Description

In one university, all the students is working on group projects for every courses, they randomly form into groups and regroup after finish projects then prepare for next course:

1. Student can follow two strategies (S) in the beginning : work hard ($S = H$) or lazy ($S = L$).
2. The effort (e) of the group is the sum of the strategies for all group members (n).
3. When course finished, each group member gets the same mark (m) as $m = e/n$.
4. Then every student rethink his strategy by randomly select another student and comparing a measure (π) based on $\pi = m - a * S$. If he had a lower π than the another student, he will imitate the another student's strategy S with a probability proportional to the difference in measures π, otherwise stay the same strategy.

If all the students take courses forever and follow the same procedure above, then we have a dynamic model of hard and lazy students working in groups.

The research question is to assess how these parameters below affect the results, such as how long it takes to reach equilibrium state.

- Initial composition of the population
- Group size (n)
- Cost of effort a
- Contribution of hard workers to group effort (H *and* L)

2 Analytical Solution

This section mainly include how to develop a model based on differential equations addressing this problem, and also explored the equilibrium solutions and its stability. Then, try to mathematically solve the differential equation by integration.

2.1 Differential Equations

As it described in the section above, there is only one variable needed for this particular problem which is the population with strategy H or L, we use the density ζ_H to represents the portion of H's population, therefore the density of L's population will be $1 - \zeta_L$.

For the evolution of ζ_H, certain rule is needed. lets say we have a population of u agents $\{\zeta_i\}_{i=1}^{u}$ in which each agents makes payoff π_i and agents evolve according to the update rule below:

$$
\begin{aligned}
\dot{\zeta}_i &= \zeta_i \Big\{ -\sum_{j,\pi_j>\pi_i} \zeta_j(\pi_j - \pi_i) + \sum_{j,\pi_i>\pi_j} \zeta_j(\pi_i - \pi_j) \Big\} \\
&= \zeta_i \sum_j \zeta_j(\pi_i - \pi_j) \\
&= \zeta_i(\pi_i \sum_j \zeta_j - \sum_j \zeta_j \pi_j) \quad where \sum_j \zeta_j = 1 \ and \ \sum_j \zeta_j \pi_j = \bar{\pi} \\
&= \zeta_i(\pi_i - \bar{\pi})
\end{aligned} \tag{1}
$$

In our problem, the $\bar{\pi}$ can be obtained as the equation (2) below:

$$
\begin{aligned}
\bar{\pi} &= \zeta_H \pi_H + \zeta_L \pi_L \\
&= \zeta_H \pi_H + (1 - \zeta_H)\pi_L \\
&= \zeta_H(\pi_H - \pi_L) + \pi_L
\end{aligned} \tag{2}
$$

Next, we substitute the equation (2) into (1) given:

$$
\begin{aligned}
\dot{\zeta}_H &= \zeta_H \{\pi_H - (\zeta_H(\pi_H - \pi_L) + \pi_L)\} \\
&= \zeta_H \{\pi_H - \pi_L - \zeta_H(\pi_H - \pi_L)\} \\
&= \zeta_H(\pi_H - \pi_L)(1 - \zeta_H)
\end{aligned} \tag{3}
$$

The equation above can still be simplified by the function below as it described in the problem:

$$
\begin{aligned}
\pi_H - \pi_L &= (m - a * H) - (m - a * L) \\
&= a(L - H)
\end{aligned} \tag{4}
$$

Finally, the differential equation for the problem is obtained as this function below:

$$
\dot{\zeta}_H = a(L - H)(\zeta_H - \zeta_H^2) \tag{5}
$$

2.2 Analyses System Behavior

2.3 Integration

$$
\begin{aligned}
\int_{\zeta_0}^{\zeta} \frac{d\zeta}{\zeta(1-\zeta)} &= \int_{t_0}^{t} a(L-H)dt \\
\int_{\zeta_0}^{\zeta} \frac{(1-\zeta)+\zeta}{\zeta(1-\zeta)} d\zeta &= a(L-H)t \\
\int_{\zeta_0}^{\zeta} (\frac{1}{\zeta} + \frac{1}{1-\zeta}) d\zeta &= a(L-H)t \\
ln\frac{\zeta}{\zeta_0} + ln\frac{1-\zeta_0}{1-\zeta} &= a(L-H)t \\
ln\frac{\zeta/(1-\zeta)}{\zeta_0/(1-\zeta_0)} &= a(L-H)t \\
\frac{\zeta/(1-\zeta)}{\zeta_0/(1-\zeta_0)} &= e^{a(L-H)t} \\
\frac{\zeta}{1-\zeta} &= \frac{\zeta_0}{1-\zeta_0} e^{a(L-H)t}
\end{aligned} \tag{6}
$$

We replace the RHS in equation (6) as A, then the function obtained can be derived as:

$$
\begin{aligned}
\frac{\zeta}{1-\zeta} &= A \\
\zeta &= A(1-\zeta) \\
\zeta(1+A) &= A \\
\zeta &= \frac{A}{1-A} \\
\zeta &= \frac{1}{1-\frac{1}{A}}
\end{aligned} \tag{7}
$$

Then, we substitute the A into equation (7):

$$
\begin{aligned}
\zeta &= \frac{1}{1-\frac{1}{\frac{\zeta_0}{1-\zeta_0} e^{a(L-H)t}}} \\
\zeta &= \frac{1}{1-\frac{1-\zeta_0}{\zeta_0}\frac{1}{e^{a(L-H)t}}} \\
\zeta &= \frac{1}{1-\frac{1-\zeta_0}{\zeta_0} e^{a(H-L)t}}
\end{aligned} \tag{8}
$$

3 Program Implementation

3.1 Numerically Integration

3.2 Agent-based Model

4 Comparison