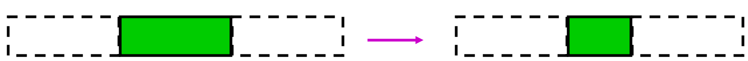
Amdahl's Law, Comparing Performance

Amdahl's Law

Speedup due to enhancement E:

$$\texttt{Speedup(E)} = \frac{\texttt{ExTime w/o E}}{\texttt{ExTime w/ E}} = \frac{\texttt{Performance w/ E}}{\texttt{Performance w/o E}}$$



Suppose that enhancement **E** accelerates a fraction $\texttt{Fraction}_{\text{enhanced}}$ of the task by a factor of $\texttt{Speedup}_{\text{enhanced}}$ and the remainder of the task is unaffected.

What are the new execution time and the overall speedup due to the enhancement?

Amdahl's Law

$$ExTime_{new} = ExTime_{old} * \left[(1 - Fraction_{enhanced}) + \frac{Fraction_{enhanced}}{Speedup_{enhanced}} \right]$$

$$Speedup_{overall} = \frac{ExTime_{old}}{ExTime_{new}} =$$

$$= \frac{1}{(1 - Fraction_{enhanced}) + \frac{Fraction_{enhanced}}{Speedup_{enhanced}}}$$

Example of Amdahl's Law

Floating point instructions improved to run 2X; but only 10% of the time was spent on these instructions

$ExTime_{new} =$

$Speedup_{overall} =$

Example of Amdahl's Law

Floating point instructions improved to run 2X; but only 10% of the time was spent on these instructions

$$\begin{aligned} \text{ExTime}_{new} &= \text{ExTime}_{old} * (0.9 + 0.1/2) \\ &= 0.95 * \text{ExTime}_{old} \end{aligned}$$

$$\text{Speedup}_{overall} = \frac{\text{ExTime}_{old}}{\text{ExTime}_{new}} = \frac{1}{0.95} = 1.053$$

The new machine is 5.3% faster <u>for this mix of instructions</u>

Comparing/Summarizing Performance

* ***Arithmetic mean*** – average of execution time that tracks total execution time

$$1/n \sum_{i=1}^{n} \text{time}_i$$

where n is total number of programs and i is the i^{th} program

* ***Harmonic mean*** – average of execution expressed as a rate that tracks total execution time

$$\frac{n}{\sum_{i=1}^{n} 1/\text{time}_i}$$

Comparing/Summarizing Performance

* ***Weighted arithmetic mean*** – uses a weighting factor in attaining the arithmetic mean

$$\sum_{i=1}^{n} \text{weight}_i * \text{time}_i$$

where weight_i is the relative frequency of program i in the workload of n programs

* ***Weighted harmonic mean*** – uses a weighting factor to attain performance as a rate of the harmonic mean

$$\frac{1}{\sum_{i=1}^{n} \text{weight}_i / \text{time}_i}$$

Comparing/Summarizing Performance

* ***Normalized Geometric mean*** – execution times normalized to a reference machine and then taken as an average

$$\left[\prod_{i=1}^{n} \text{execution time ratio}_i\right]^{1/n}$$

where $\text{execution time ratio}_i$ is the execution time of program i after it has been normalized to a reference machine

Comparing/Summarizing Performance

In order to obtain equal time weightings, w_i, on a machine with $time_i$ we can use the following formula:

$$w_i = \frac{1}{time_i * \sum_{k=1}^{n} 1/time_k}$$

Comparing/Summarizing Performance

		Computer A	Computer B	Computer C
	P1	30	20	75
	P2	47	25	60
	P3	78	95	55
	P4	90	80	50
	P5	130	40	100
AM		75.00	52.00	68.00
HM		57.98	36.22	63.95

Comparing/Summarizing Performance

		Computer A	Computer B	Computer C
	P1	30	20	75
	P2	47	25	60
	P3	78	95	55
	P4	90	80	50
	P5	130	40	100
WAM		58.00	36.24	63.94
WHM		45.11	27.64	60.80

Note: The "weights" used to calculate the WAM and WHM were obtained by using the formula on slide 9. These respective weights are shown in the next slide.

Comparing/Summarizing Performance

		Computer A	Computer B	Computer C
	P1	30	20	75
	P2	47	25	60
	P3	78	95	55
	P4	90	80	50
	P5	130	40	100
w1		0.3867	0.3623	0.1705
w2		0.2468	0.2899	0.2131
w3		0.1487	0.0763	0.2325
w4		0.1289	0.0906	0.2558
w5		0.0892	0.1812	0.1279

weights calculated by formula on slide 9

Comparing/Summarizing Performance

	A	B	C
P1	1	0.6666	2.5000
P2	1	0.5319	1.2766
P3	1	1.2180	0.7051
P4	1	0.8888	0.5555
P5	1	0.3077	0.7692
GM	1	0.6523	0.9922

	A	**B**	C
P1	1.5000	1	3.7500
P2	1.8800	1	2.4000
P3	0.8211	1	0.5789
P4	1.1250	1	0.6250
P5	3.2500	1	2.5000
GM	1.5330	1	1.5211

	A	B	**C**
P1	0.4000	0.2666	1
P2	0.7833	0.4167	1
P3	1.4182	1.7272	1
P4	1.8000	1.6000	1
P5	1.3000	0.4000	1
GM	1.0079	0.6574	1