Circuits

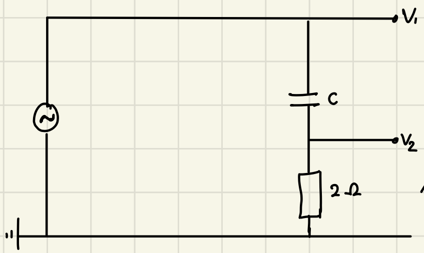
22/03/21

Objectives:

- Analyse simple electronic circuits and understand their response to sinusoidal waveforms.
- Understand and use complex representation of voltage, current and impedance.
- Understand phase difference of sinusoidal voltages and currents.
- Understand low pass and high pass filters.
- Understand decibel notation for the gain of an electrical filter.
- Understand the response of simple circuits to a step input.

Experiment 1A:

The circuit was set up as shown:



Ammeter

Using 'Aquire' button to take a moving window average. This removes noise on the signals on oscilloscope.

The aim of 1A was to find the unknown capacitance, C, from values of V_1 and V_2 (amplitude and phase difference) for different values of generator frequency. The circuit was set up as shown in the diagram. The wave generator was checked by comparing expected amplitudes with readings. V_1, V_2 and I were displayed on oscilloscope. The peak to peak voltage for each voltage was recorded and halved for calculations. The measurements were taken using the y cursors on the oscilloscope taking the maximum and minimum and finding the difference. The error on these readings are therefore the smallest step size of the cursor. When taking peak voltage insure that the scale on the y axis is such that the wave stretches across the height of the screen. This gives the highest accuracy of measurement. Switch to the x cursors and changing the scale on the x axis so only 1 x intercept for each graph is on the screen. With this the phase difference between the current and the voltage across the capacitor was found by the difference in x values. This was at zero voltage point as there is too much error at the peak value. The error on this reading was the smallest step size of the cursor. This was repeated for 2 values of frequency (5 kHz and 10 kHz) and 2 values of input voltage (2.5 V and 1.25 V)

The tolerance on the resistance was 5% and the error on frequency was 130 ppm

Readings for 1A

Wave Gen Amp	V1	Uncert in V1	V2	Uncert in V2	Resistance	Tolerance	Freq	Uncert in f	Ang Freq	Uncert in AF	Phase Diff	Uncert in Phase	Capacitance	Uncert on Cap
V	V	V	V	V	Ohms	Ohms	Hz	Hz	Rad/s	Rad/s	s	s	nF	nF
1.25	1.21375	0.00625	0.0075	0.0000125	2	0.1	5000	0.65	31415.92654	4.08407045	0.0000468	0.000002	98.34496998	4.945989693
2.5	2.4625	0.00625	0.014965	0.0000125	2	0.1	5000	0.65	31415.92654	4.08407045	0.0000476	0.000002	96.72096339	4.842964963
1.25	1.173125	0.00625	0.01425	0.0000125	2	0.1	10000	1.3	62831.85307	8.168140899	0.0000246	0.000002	96.6630981	4.861269976
2.5	2.34375	0.00625	0.02825	0.0000125	2	0.1	10000	1.3	62831.85307	8.168140899	0.0000246	0.000002	95.91737904	4.802888609

$\omega = 2\pi f$

Uncertainty on F was 130 PPM

So: $\frac{130}{10^6} \times 5000 = 0.6\,Hz$

$\frac{130}{10^6} \times 10000 = 1.3\,Hz$

Tolerance on R was 5% so:

$0.05 \times 2 = 0.1\,\Omega$

From the equation for a Capacitor $Q = C\tilde{V}$ and the voltage is sinusoidal so $\tilde{V} = V_0 \sin(\omega t + \phi)$

$\tilde{I} = I_0 \sin \omega t$

Differentiating w.R.T time: $\tilde{I} = \frac{dQ}{dt} = C\frac{d\tilde{V}}{dt}$ (C is constant) $\frac{d\tilde{V}}{dt} = V_0 \omega \cos(\omega t + \phi)$

$\tilde{I} = C V_0 \omega \cos(\omega t + \phi)$ But from CIVIL, for a Capacitor current leads voltage by $\frac{\pi}{2} = \phi$ and Sin and Cos have a phase difference of $\frac{\pi}{2}$

$\tilde{I} = I_0 \cancel{\sin \omega t} = C V_0 \omega \cancel{\sin \omega t}$

$C = \frac{I_0}{V_0 \omega}$

I_0 can be found using Ohm's law across the resistor $I_0 = \frac{V_2}{R}$

$$C = \frac{V_2}{V_0 R \omega}$$

The error on C was calculated by:

$$\sigma_C = C\sqrt{\left(\frac{\sigma_{V_2}}{V_2}\right)^2 + \left(\frac{\sigma_{V_0}}{V_0}\right)^2 + \left(\frac{\sigma_R}{R}\right)^2 + \left(\frac{\sigma_\omega}{\omega}\right)^2}$$

Average value for C: $\frac{\Sigma C_i}{4} = 96.9\,nF$ The error on the average was found by: $\sqrt{\Sigma \sigma_{C_i}^2}$

$= 9.73\,nF$

So $C = (96.9 \pm 9.7)\,nF$

This experiment was repeated on Falstad

Wave Gen Amp	V1	Uncert in V1	V2	Uncert in V2	Resistance	Tolerance	Freq	Uncert in f	Ang Freq	Uncert in AF	Phase Diff	Uncert in Phase	Capacitance	Uncert on Cap
V	V	V	V	V	Ohms	Ohms	Hz	Hz	Rad/s	Rad/s	s	s	nF	nF
1.25	1.25	0.015	0.00787	0.001267	2	0	5000	0	31415.92654	0	0.00005	0.000005	100.2039522	16.32222913
2.5	2.5	0.031	0.01574	0.001267	2	0	5000	0	31415.92654	0	0.00005	0.000005	100.2039522	8.088348569
1.25	1.25	0.015	0.015836	0.001267	2	0	10000	0	62831.85307	0	0.000025	0.000005	100.8151072	8.156192947
2.5	2.5	0.031	0.031672	0.001267	2	0	10000	0	62831.85307	0	0.000025	0.000005	100.8151072	4.222291616

The same method was carried out to give: $C = (100.5 \pm 20)\,nF$

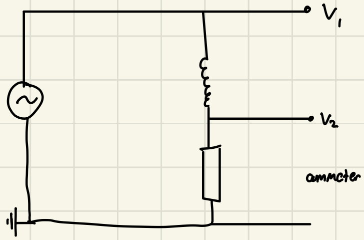
The error on the Falstad calculation was higher as the cursor precision was lower.
The Falstad calculation gave a closer result but the error was larger due to a large error on V_2. The largest source of error on the real circuit was the resistor tolerance at 5%. To improve this the resistor box could be used (0.1% tolerance).

Actual value = 100nF

The error on F and R were 0 for falstad as they were inputs.

Experiment 1B

The exact same method was used but with this circuit. (unknown inductor)



Results:

Wave Gen Amp	V1	Uncert in V1	V2	Uncert in V2	Resistance	Tolerance	Freq	Uncert in f	Ang Freq	Uncert in AF	Phase Diff	Uncert in Phase	Inductance	Uncert on Induc
V	V	V	V	V	Ohms	Ohms	Hz	Hz	Rad/s	Rad/s	s	s	mH	mH
1.25	0.777185	0.00625	0.034375	0.0000125	2	0.1	5000	0.65	31415.92654	4.08407045	0.000045	0.000002	1.439334801	0.07289375179
2.5	1.5375	0.00625	0.067875	0.0000125	2	0.1	5000	0.65	31415.92654	4.08407045	0.000045	0.000002	1.442066888	0.07234197743
1.25	1.04375	0.00625	0.023375	0.0000125	2	0.1	10000	1.3	62831.85307	8.168140899	0.0000242	0.000002	1.42133024	0.07157861
2.5	2.09375	0.00625	0.046625	0.0000125	2	0.1	10000	1.3	62831.85307	8.168140899	0.0000242	0.000002	1.429407666	0.07159890651

Using the same derivation method as 1A

$$L = \frac{V_0}{I_0 \omega} \quad \text{where } I_0 = \frac{V_2}{R}$$

$$L = \frac{R V_0}{V_2 \omega}$$

$$\sigma_L = L\sqrt{\left(\frac{\sigma_R}{R}\right)^2 + \left(\frac{\sigma_{V_0}}{V_0}\right)^2 + \left(\frac{\sigma_{V_2}}{V_2}\right)^2 + \left(\frac{\sigma_\omega}{\omega}\right)^2}$$

Note that errors on R and F are the same.

Taking an average gave: $L = (1.43 \pm 0.14)\,\text{mH}$

Repeating on Falstad:

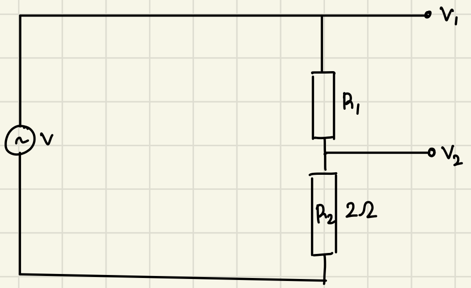
Wave Gen Amp	V1	Uncert in V1	V2	Uncert in V2	Resistance	Tolerance	Freq	Uncert in f	Ang Freq	Uncert in AF	Phase Diff	Uncert in Phase	Inductance	Uncert on Induc
V	V	V	V	V	Ohms	Ohms	Hz	Hz	Rad/s	Rad/s	s	s	mH	mH
1.25	1.25	0.015	0.052848	0.001267	2	0	5000	0	31415.92654	0	0.000045	0.000005	1.505780191	0.04036987088
2.5	2.5	0.031	0.105695	0.001267	2	0	5000	0	31415.92654	0	0.000045	0.000005	1.505794438	0.02597030047
1.25	1.25	0.015	0.026296	0.001267	2	0	10000	0	62831.85307	0	0.0000242	0.000005	1.513109818	0.07513207473
2.5	2.5	0.031	0.052591	0.001267	2	0	10000	0	62831.85307	0	0.0000242	0.000005	1.513138589	0.04099918652

Using the method above $L = (1.51 \pm 0.09)\,\text{mH}$

Both methods gave an answer equal to the true value of 15 mH (with error).
The step size error on falstad here was less important as the values of V_2 were higher so the overall error was less. But this was still the largest source of error.
For the real circuit the resistor's tolerance was the highest source of error at 5%. Improved by resistor box.

Experiment 1C:

Replacing the inductor with an unknown resistor gives:



The aim was to find the Resistance of the unknown resistor (R_1)

$$V_2 = IR_2 \qquad V_1 = I(R_1 + R_2) \qquad \Delta V = V_1 - V_2 = IR_1$$

$$I = \frac{V_1}{R_1 + R_2} \Rightarrow \Delta V = \frac{V_1 R_1}{R_1 + R_2}$$

$$R_2 = \frac{\Delta V R_2}{V_1 - \Delta V} = \frac{(V_1 - V_2) R_2}{V_2}$$

The same method for 1A and B was conducted to give these results:

Wave Gen Amp	V1	Uncert in V1	V2	Uncert in V2	v1-v2	Uncert V1-v2	Freq	Uncert in f	Ang Freq	Uncert in AF	Phase Diff	Uncert in Phase	Resistance	Uncert on R
V	V	V	V	V			Hz	Hz	Rad/s	Rad/s	s	s	Ohms	Ohms
1.25	1.178125	0.00625	0.00235	0.0000125	1.175775	0.0062500125	10000	1.3	62831.85307	8.168140899	0	0	1000.659574	26.12372254
2.5	2.3625	0.00625	0.004685	0.0000125	2.357815	0.0062500125	10000	1.3	62831.85307	8.168140899	0	0	1006.537887	25.44660869

$$\sigma_{V_1 - V_2} = \sqrt{(\sigma_{V_1})^2 + (\sigma_{V_2})^2} \qquad \sigma_R = R\sqrt{\left(\frac{\sigma_{V_2}}{V_2}\right)^2 + \left(\frac{\sigma_{R_1}}{R_1}\right)^2 + \left(\frac{\sigma_{V_1 - V_2}}{V_1 - V_2}\right)^2}$$

Taking a mean average: $R = (1003.6 \pm 36.5)\,\Omega$ $\Rightarrow$ Using the method shown in 1A to find error.

Then repeating on falstad:

Wave Gen Amp	V1	Uncert in V1	V2	Uncert in V2	v1-v2	Uncert V1-v2	Freq	Uncert in f	Ang Freq	Uncert in AF	Phase Diff	Uncert in Phase	Resistance	Uncert on R
V	V	V	V	V			Hz	Hz	Rad/s	Rad/s	s	s	Ohms	Ohms
1.25	1.25	0.015	0.002496	0.000112	1.247504	0.01500041813	10000	0	62831.85307	8.168140899	0	0	999.6025641	46.4364921
2.5	2.5	0.031	0.00499	0.000224	2.49501	0.03100080928	10000	0	62831.85307	8.168140899	0	0	1000.004008	46.57782103

$R = (999.8 \pm 65.8)\,\Omega$

The true value of resistance was $1000\,\Omega$. This lies in the range of both measurements.

The biggest source of error on the falstad experiment was the measurement on V_2 at about 5%. This was due to the step size of the cursor. There is no real way of improving this.

For the real circuit the errors on V_1, V_2, $V_1 - V_2$ and R were all $\approx$ 5%. The voltage readings cannot be improved but a resistor with a lower tolerance can be used.

Summarising:

	Capacitance/F	Inductance/H	Resistance/Ohms
Value_real	96.91160263	0.001433034899	1003.598731
Error	9.72711717	0.000144210823	36.46887403
Amplitude Ratio	2	2	2
Phase/rad	-1.514247659	1.467123769	0

=> amplitude ratio $\left(\frac{V}{I}\right)$ is just R for each as L and C have negligable resistance so $\frac{V}{I} = R$.

For a capacitor the current is $-\frac{\pi}{2}$ behind voltage

For an inductor the current is $\frac{\pi}{2}$ ahead of voltage

For a resistor I and V are in phase.

Calculating the error on the phase:

$$\sigma_{\phi c} = \phi \sqrt{\left(\frac{\sigma_{\phi s}}{\phi_s}\right)^2 + \left(\frac{\sigma_f}{f}\right)^2}$$

As $\underset{\text{rad}}{\phi} = 2\pi f \times \phi_s$ ← phase in seconds

And then an average: $\sigma_{\phi c} = 0.198$

so

$$\phi_c = (-1.514 \pm 0.198) \text{ rad}$$

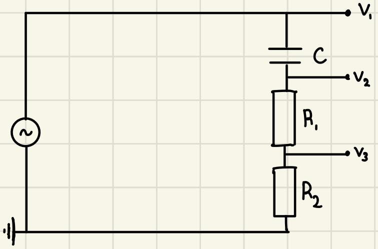
Following the same method for ϕ_I:

$$\phi_I = (1.467 \pm 0.198)$$

Therefore $\frac{\pi}{2}$ lies in the range of both values.

Experiment 2A:

- The capacitor from Circuit 1A is replaced with a capacitor and resistor in series:



This circuit was run for $R_1 = 10k\Omega$ and $R_1 = 1k\Omega$.
$R_2 = 2\,\Omega$ and $C = 100nF$ and were kept constant.

The impedance of the resistor and capacitor is: $Z = Z_R + Z_C$ $\quad Z_R = R$

$Z = R + \frac{1}{j\omega C}$ $\quad Z_C = \frac{1}{j\omega C}$

$= R - j\left(\frac{1}{\omega C}\right)$

$$\tan(\phi_z) = \tan(\phi_V - \phi_I) = \frac{\text{Im}\,Z}{\text{Re}\,Z}$$

$$\tan(\Delta\phi) = \frac{-1/\omega C}{R} = \frac{-1}{\omega RC}$$

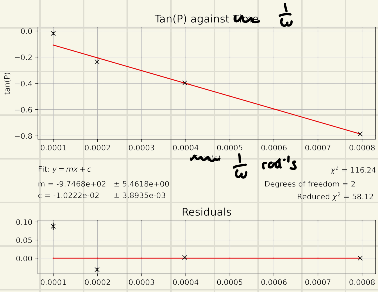
RC is the time constant of the RC circuit. This is the time taken for the capacitor to charge to $\frac{e-1}{e}$ of its maximum voltage.

If $\tan\phi$ is plotted on the y axis and $\frac{1}{\omega}$ is plotted on the x. Then the gradient of the line will be $\frac{-1}{RC}$.

Following the Standard method for taking measurements from 1A gave these results:

Wave Gen Amp	Freq	Uncert in F	Angular f	Error	1/w	V1	V2	V3	Phase Diff V1 and V2	Uncert on Phase V1 and V2	Phase Shift	Error	Tan(L)	Error	Resistance	Capactitance
V	Hz	Hz				V	V	V	s	s	rad				Ohms	F
5	100	0.013	628.3185307	0.013	0.001591549431	4.95	0.0005	4.1625	-0.00188	0.00001	-1.181238838	-0.006285061543	-2.435829717	-0.04357600609	10000	0.0000001
5	200	0.026	1256.637061	0.026	0.000795774715	4.95	0.000829	3.125	-0.00053	0.000001	-0.6660176426	-0.00125961629	-0.7857925767	-0.002037391527	10000	0.0000001
5	400	0.052	2513.274123	0.052	0.000397887357	4.95	0.000925	1.8625	-0.00015	0.000001	-0.3769911184	-0.002513751914	-0.3959280088	-0.00290780512	10000	0.0000001
5	800	0.104	5026.548246	0.104	0.000198943678	4.95	0.001	0.9725	-0.000046	0.000001	-0.2312212193	-0.005026638121	-0.2354319018	-0.005305255525	10000	0.0000001
5	1600	0.208	10053.09649	0.208	0.000099471839	4.9875	0.001	0.50125	-0.0000019	0.000001	-0.01910088333	-0.0100530968	-0.01910320662	-0.0100567655	10000	0.0000001

From this, $\tan\phi$ was plotted against $\frac{1}{\omega}$ on LSFR to give this graph:



The gradient of this graph was $m = -974.68 \pm 5.46$

The expected value is $-\frac{1}{RC} = -1000$

This does not lie in the range of error but $\chi^2_{reduced} = 58.12$ which is very large. This indicates that the errors were underestimated. With higher errors the true value would lie in range.

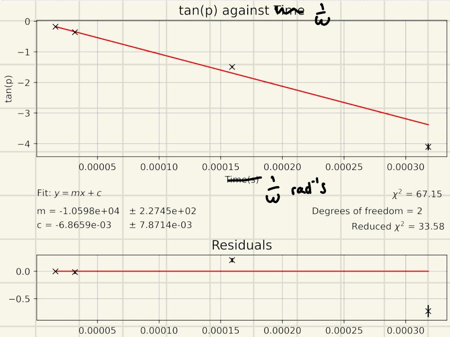
The largest source of error is in the $\tan\phi$ at about 2%. This was propagated to be $\sec^2(\phi) \times \sigma_\phi$
↳ increased to 50% for smaller reading.

The 100Hz reading was omitted as it was an anomaly.

The Same was done for the $1k\Omega$ resistor:

$= \sqrt{\sigma_{v_1}^2 + \sigma_{v_2}^2}$ ϕ_R $= \phi_R \sqrt{\left(\frac{\sigma_f}{f}\right)^2 + \left(\frac{\sigma_{\phi_R}}{\phi_R}\right)^2}$ $\sec^2\phi_R \times \sigma_{\phi_R}$

Wave Gen Amp	Freq	Uncert in F	Angular f	Error	1/w	V1	V2	V3	Phase Diff V1 and V2	Uncert on Phase V1 and V2	Phase Shift	Error	Tan(L)	Error	Resistance	Capactitance
V	Hz	Hz				V	V	V	s	s	rad				Ohms	F
5	100	0.013	628.3185307	0.013	0.001591549431	4.925	0.0006	4.925	-0.00256	0.00001	-1.608495439	-0.006286663842	26.51325629	-4.425514356	1000	0.0000001
5	500	0.065	3141.592654	0.065	0.000318309886	4.925	0.00292	4.725	-0.000424	0.000002	-1.332035285	-0.006285571062	-4.108396844	-0.1123792512	1000	0.0000001
5	1000	0.13	6283.185307	0.13	0.000159154943	4.8625	0.0051	4.1125	-0.000156	0.000002	-0.9801769079	-0.01256701663	-1.491528583	-0.04052432461	1000	0.0000001
5	5000	0.65	31415.92654	0.65	0.000031830988	4.8	0.00907	1.48125	-0.000011	0.000001	-0.3455751919	-0.03141595866	-0.3600221531	-0.03548796801	1000	0.0000001
5	10000	1.3	62831.85307	1.3	0.000015915494	4.775	0.00937	0.76	-0.0000028	0.0000001	-0.1759291886	-0.006283226932	-0.1777670111	-0.006481783879	1000	0.0000001



$m = -10598 \pm 227.5$

Expected value was $-\frac{1}{RC} = -10000$

Using the Same arguments as before: This isn't in range but $\chi^2_{reduced} = 33.58$

So higher errors might include 10000 in range.

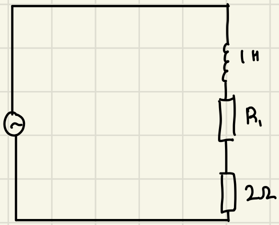
100Hz reading was an anomaly so removed.

Largest Source of error was $\tan\phi$.

Both parts could be improved by more measurements, apart anomaly 4 points isn't enough to get an accurate gradient.

Experiment 2B:

This is the Same as 2A but Simulated on falstad. The Capacitor is replaced by a 15mH inductor.



The values of inductance and R_2 are a constant $L = 1H$ and $R_2 = 2\Omega$

Using the Same method:

$$Z = R + jL\omega$$

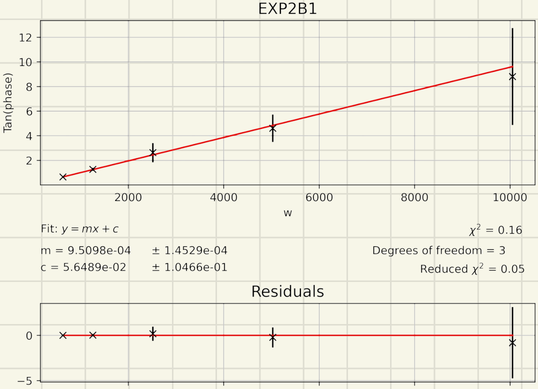
$$\tan\phi = \frac{\text{Im } Z}{\text{Re } Z} = \frac{L\omega}{R}$$

So plotting $\tan\phi$ against ω should give a graph with gradient $\frac{L}{R}$ which is the time Constant of an LR Circuit.

Using falstad to Simulate and collect results:

Wave Gen Amp	Resistance	Inductance	Freq	Uncert in F	w	Error	V1	V2	Error	Phase Diff V1 and V2	Uncert on Phase	Phase Shift	Error	Tan(N)	Error
V	Ohms	H	Hz	Hz			V	V		s	s	rad			
2.5	1000	1	100	0	628.3185307	0	2.5	0.004228	0.000002	0.00092	0.00004	0.5780530483	0.02513274123	0.6523893179	0.03582953302
2.5	1000	1	200	0	1256.637061	0	2.5	0.003111	0.000001	0.00072	0.00004	0.9047786842	0.05026548246	1.272600467	0.1316710319
2.5	1000	1	400	0	2513.274123	0	2.5	0.001848	0.000001	0.00048	0.00004	1.206371579	0.1005309649	2.621486539	0.7913990251
2.5	1000	1	800	0	5026.548246	0	2.5	0.000975467	0.000001444	0.00027	0.00001	1.357168026	0.05026548246	4.609600706	1.118327498
2.5	1000	1	1600	0	10053.09649	0	2.5	0.000494803	0.000000594	0.000145	0.000005	1.457698991	0.05026548246	8.804209985	3.946549792
2.5	1000	1	3200	0	20106.19298	0	2.5	0.000248163	0.000001266	0.000075	0.000005	1.507964474	0.1005309649	15.89454484	25.49832769

From this data, $\tan\phi$ was plotted against ω on LSFR (next page)



The gradient of this is $m = 0.000951 0 \pm 0.000145$

The true value is $\frac{L}{R} = 0.001$

This lies within the range of error.

The error on the gradient is large (about 15%).

The highest contribute to this is the uncertainty in $\tan\phi$

↳ found by: $\sigma_{\tan\phi} = \sec^2(\phi) \times \sigma_\phi$

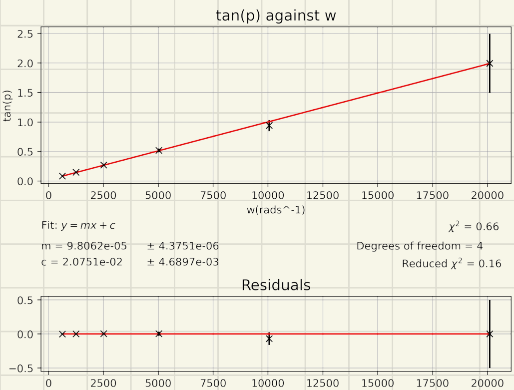
This error increased to 40% when ϕ got smaller.

↳ Smaller measurements will have higher error.

Very low $\chi^2_{reduced}$ which indicates errors were overestimated as more than 68% of the error bars cross line of best fit.

This was then repeated with a $10k\Omega$ resistor:

Wave Gen Amp	Resistance	Inductance	Freq	Uncert in F	w	Error	V1	V2	Error	Phase Diff V1 and V2	Uncert on Phase	Phase Shift	Error	Tan(N)	Error
V	Ohms	H	Hz	Hz			V	V		s	s	rad			
2.5	10000	1	100	0	628.3185307	0	2.5	0.000498917	0.000000011	0.00013	0.000005	0.08168140899	0.003141592654	0.08186355054	0.00316264647
2.5	10000	1	200	0	1256.637061	0	2.5	0.000496001	0.000000012	0.000115	0.000005	0.1445132621	0.006283185307	0.1455277465	0.00641625264
2.5	10000	1	400	0	2513.274123	0	2.5	0.000484821	0.000000069	0.000105	0.000005	0.2638937829	0.01256637061	0.2701952221	0.0134837842
2.5	10000	1	800	0	5026.548246	0	2.5	0.000446662	0.000000133	0.000095	0.000005	0.4775220833	0.02513274123	0.5174653751	0.0318625457
2.5	10000	1	1600	0	10053.09649	0	2.5	0.000352462	0.000000064	0.000075	0.000005	0.7539822369	0.05026548246	0.9390625058	0.0945915145
2.5	10000	1	3200	0	20106.19298	0	2.5	0.000222492	0.000001321	0.000055	0.000005	1.105840614	0.1005309649	1.993476544	0.500035865



$m = 0.000098 \pm 0.000044$

True value is $\frac{L}{R} = 0.0001$ => lies within range of error.

Error on gradient is less (5%)

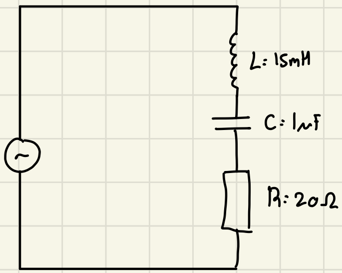
Highest contribute is $\tan\phi$

Very low $\chi^2_{reduced}$ which indicates overestimated errors.

As this experiment was all done on Falstad there will be mathmatical errors in the program that were not accounted for. This will increase absolute error.

Experiment 3

The aim was to find the Q factor of an LCR (falstad).



The average power is zero for an inductor and capacitor as $\phi = \frac{\pi}{2}$ and $\langle P \rangle = VI\cos\phi$ and $\cos\phi = 0$

For the resistor (and thus the circuit) $\langle P \rangle = \langle IV \rangle = V_0 I_0 \langle\cos^2\omega t\rangle$

$= \frac{1}{2} V_0 I_0 \qquad \langle \cos^2 x \rangle = \frac{1}{2}$

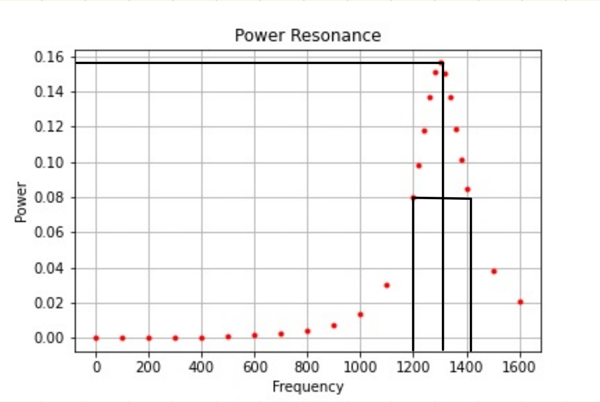
As $I_0 = \frac{V_0}{R} \Rightarrow \langle P \rangle = \frac{V^2}{2R}$

This circuit was simulated for varying frequency. The resonant frequency was found experimentally beforehand using trial and error

Generator Voltage	Frequency	Resistance	Current	Error	Voltage(R)	Error	Phase	Error	Phase Shift	Power	Error
V	Hz	Ohms	A	A	V	V	s	s	Rad	W	
2.5	0	20	0	0	0	0	0	0	0	0	
2.5	100	20	0.00158	0.000003	0.031601	0.000009	0.00256	0.00008	1.608495439	0.00002496558	0.00000001422
2.5	200	20	0.003217	0.000001	0.064335	0.000011	0.00128	0.00004	1.608495439	0.000103474805	0.000000035384
2.5	300	20	0.004974	0.000002	0.099475	0.000026	0.00084	0.00002	1.583362697	0.000247381890	0.000000129317
2.5	400	20	0.00693	0.000001	0.138609	0.000002	0.00066	0.00002	1.658760921	0.000480311372	0.000000013860
2.5	500	20	0.009194	0.000006	0.183879	0.000119	0.00052	0.00001	1.63362818	0.000845287166	0.00000109408
2.5	600	20	0.011924	0.000005	0.238486	0.000008	0.00044	0.000005	1.658760921	0.001421889305	0.000000095394
2.5	700	20	1.54E-02	1.70E-05	3.07E-01	3.30E-04	3.90E-04	1.00E-05	1.72E+00	0.002363552638	5.07E-06
2.5	800	20	1.99770E-02	2.00000E-06	3.99535E-01	3.00000E-05	3.30000E-04	1.00000E-05	1.65876E+00	0.003990705406	5.99303E-07
2.5	900	20	0.026554	0.000013	0.531482	0.000253	0.000315	0.000005	1.781283035	0.007061827908	0.000006723247
2.5	1000	20	0.036815	0.000035	0.736306	0.000714	0.0003	0.000005	1.884955592	0.01355366314	0.000026286124
2.5	1100	20	0.054815	0.000024	1.096	0.002	0.000295	0.000005	2.038893632	0.0300304	0.0001096
2.5	1200	20	0.089492	0.000084	1.79	0.003	0.000315	0.000005	2.375044046	0.0801025	0.0002685
2.5	1220	20	0.098939	0.00006	1.979	0.001	0.000325	0.000005	2.491282974	0.097911025	0.00009895
2.5	1240	20	1.08E-01	9.80E-05	2.169	0.005	3.30E-04	1.00E-05	2.57E+00	0.117614025	0.00054225
2.5	1260	20	1.17E-01	1.01E-04	2.34	0.01	3.50E-04	1.00E-05	2.77E+00	0.13689	0.00117
2.5	1280	20	1.23E-01	4.27E-04	2.459	0.002	3.60E-04	1.00E-05	2.90E+00	0.151167025	0.0002459
2.5	1300	20	1.25E-01	4.15E-04	2.5	0.013	3.80E-04	1.00E-05	3.104E+00	0.15625	0.001625
2.5	1320	20	1.23E-01	1.11E-04	2.454	0.005	3.50E-04	1.00E-05	2.90E+00	0.1505529	0.0006135
2.5	1340	20	1.17E-01	1.16E-04	2.339	0.003	3.30E-04	1.00E-05	2.78E+00	0.136773025	0.00035085
2.5	1360	20	1.09E-01	3.53E-04	2.182	0.007	3.10E-04	1.00E-05	2.65E+00	0.1190281	0.0007637
2.5	1380	20	1.01E-01	3.81E-04	2.011	0.002	2.80E-04	1.00E-05	2.43E+00	0.101103025	0.0002011
2.5	1400	20	9.22E-02	3.59E-04	1.844	0.002	2.70E-04	1.00E-05	2.38E+00	0.0850084	0.0001844
2.5	1500	20	6.16E-02	3.52E-04	1.232	0.007	2.30E-04	1.00E-05	2.17E+00	0.0379456	0.0004312
2.5	1600	20	4.53E-02	1.02E-04	9.07E-01	2.80E-04	1.90E-04	1.00E-05	1.91E+00	0.02055402766	1.27E-05

$\sigma_P = P \times 2 \times \left(\frac{\sigma_V}{V}\right)$

From this a graph of power against frequency was plotted (power resonance curve)



The resonant frequency is when the power is maximum.

$f_0 = 1300\ \mathrm{Hz}$

$\Delta f = 210\ \mathrm{Hz}$

$Q = \frac{f_0}{\Delta f} = 6.19$ This has no error as f was an input.

An error could be calculated by calculating a max and min value of Q from the graph.

Error bars are plotted but too small to be seen.

This can be checked by: $\Delta f = \frac{R}{2\pi L}$ So the true value of Δf is $212.2\ Hz$

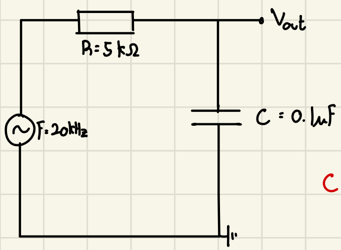
Calculated value is close.

With a min and max this might be in range.

Experiment 4

The aim was to determine the output waveform of an integrating circuit and find the output voltage.

Using this circuit:



It can be shown that this is an integrating circuit:

$V_{in} - V_{out} = IR$

$$C(V_{out} - V_{grd}) = Q \Rightarrow C\left(\frac{dV_{out}}{dt} - \frac{dV_{grd}}{dt}^{=0}\right) = \frac{dQ}{dt} = I$$

$$\frac{V_{in} - V_{out}}{R} = C\frac{dV_{out}}{dt}$$

$V_{out} << V_{in}$
(This is assumed for all parts of this notebook)

$$\frac{dV_{out}}{dt} = \frac{V_{in}}{RC}$$

$$V_{out} = \frac{1}{RC}\int V_{in}\ dt \qquad V_{in} = V_0 \cos(\omega t)$$

$$V_{out} = \frac{V_0}{\omega RC} \sin \omega t$$

← time constant.

Therefore it is expected that V_{out} will be a factor of $\frac{1}{\omega RC}$ smaller than V_{in}. It will have a sinusoidal shape and will be $\frac{\pi}{2}$ out of phase of V_{in}.

This circuit was run and results were taken:

Note that a resistor box was used so tolerance = 0.1%

Wave Gen Amp/V	Freq/Hz	Error Hz	V_in/V	Error/V	V_out/V	Error	Phase	Error	Expected V_out
2.5	20000	1	2.44	0.008	0.03885	0.00015	0.00001198	0.00000002	0.03883380611
Resistance Ω	Capacitance F	↑ 5ppm					↓		
5000	0.0000001								

error: 5 Ω

in rad: $\phi \times 2\pi \times F = 1.505 \pm 0.00251$

$$\sigma = 1.505\sqrt{\left(\frac{2\times10^{-8}}{1.198\times10^{-6}}\right)^2 + \left(\frac{1}{20000}\right)^2} = 2.51\times10^{-3}$$

$\approx \frac{\pi}{2}$ Maybe unaccounted errors

V_{out} was measured to be: $V_{out} = (0.03885 \pm 0.00015)\,V$

The expected value was: $\frac{V_0}{\omega RC} = \frac{2.44}{2\pi\times20000\times5000\times0.1\times10^{-6}} = 0.0388\ V$

$V_{out, expected} = (0.0388 \pm 0.00194)\ V$

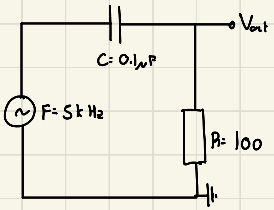
$$\sigma_{V_{out,e}} = 0.0388\sqrt{\left(\frac{0.008}{2.44}\right)^2 + \left(\frac{1}{20000}\right)^2 + \left(\frac{5}{5000}\right)^2 + (0.05)^2}$$

$= 0.00194$

The measured V_{out} is very close to the expected value.
This is expected as all the errors are very low
↳ highest is capacitance at 5%
Therefore the best way to improve would be to have a capacitor with a lower tolerance.

Experiment 5A:

This part follows the same procedure as 4 but R and C are interchanged:



This was simulated on Falstad so it can be expected that there will be unaccounted mathematical errors.

$$C(V_{in}-V_{out}) = Q \Rightarrow C\left(\frac{dV_{in}}{dt} - \frac{dV_{out}}{dt}\right) = \frac{dQ}{dt} = I$$

$$\frac{V_{out} - V_{grd}}{R} = I$$

F and R were decreased as original values gave $V_{out} \approx V_{in}$

$$\frac{dV_{in}}{dt} - \frac{dV_{out}}{dt} = \frac{V_{out}}{RC} \qquad \frac{dV_{out}}{dt} \ll \frac{dV_{in}}{dt}$$

$$RC\frac{dV_{in}}{dt} = V_{out}$$

$$V_{in} = V_0 \sin\omega t$$

$$\therefore V_{out} = \omega R C V_0 \cos\omega t$$

V_{out} is expected to be a factor of ωRC of V_{in}, sinusoidal and $\frac{\pi}{2}$ out of phase.

Using Falstad:

input

Wave Gen Amp/V	Freq/Hz	Error	V_in/V	Error/V	V_out/V	Error	Phase/s	Error	Expected V_out
2.5	5000	0	2.5	0.001	0.7506	0.000009	0.00006	0.000005	0.7853981634
Resistance/Ohms	Capacitance/F								
100	0.0000001								

error = 0

$\left.\begin{array}{l}\phi = 2\pi \times 0.00006 \times 5000 \\ = 1.885 \pm 0.157\end{array}\right\} \approx \frac{\pi}{2}$, large unaccounted errors.

$$V_{out,\,expected} = V_0 \omega R C$$

$$= 2.5 \times 2\pi \times 5000 \times 100 \times 1\times10^{-7}$$

$$= (0.785 \pm 0.001)\,V$$

$\sigma = 0.001$ (from V_{in})

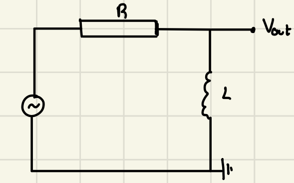
$$V_{out,\,Reading} = (0.7506 \pm 0.000009)\,V$$

The expected value is not in the range of the error of the reading.
The most likely reason for this is that there are unaccounted for errors in Falstad. If these were included it may lie in range.
The highest source of error was on V_i but this was still small.

Experiment 5B:

The aim was to consider how RL and LR Circuits behave. (integrating and differentiating)

RL Circuit:



$$V_{in} - V_{out} = IR \Rightarrow \frac{1}{R}\left(\frac{dV_{in}}{dt} - \frac{dV_{out}}{dt}\right) = \frac{dI}{dt}$$

$$V_{out} - V_{grd} = L\frac{dI}{dt}$$

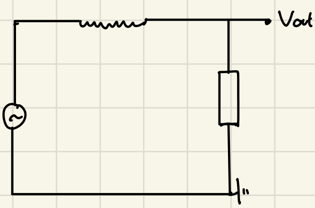
$$\frac{1}{R}\left(\frac{dV_{in}}{dt} - \frac{dV_{out}}{dt}\right) = \frac{1}{L}\left(V_{out} - \underbrace{\cancel{V_{grd}}}_{=0}\right) \qquad \frac{dV_{out}}{dt} \ll \frac{dV_{in}}{dt}$$

$$\frac{1}{R}\frac{dV_{in}}{dt} = \frac{1}{L}V_{out}$$

$$V_{out} = \frac{L}{R}\frac{dV_{in}}{dt}$$

Therefore an RL Circuit is a differentiating Circuit with time constant $\frac{L}{R}$

LR Circuit:



$$(V_{in} - V_{out}) = L\frac{dI}{dt}$$

$$V_{out} - V_{grd} = IR$$

$$\frac{1}{R}\left(\frac{dV_{out}}{dt} - \underbrace{\cancel{\frac{dV_{grd}}{dt}}}_{=0}\right) = \frac{dI}{dt}$$

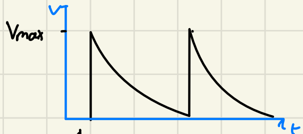
$$\frac{1}{R}\frac{dV_{out}}{dt} = \frac{1}{L}\left(V_{in} - \cancel{V_{out}}\right) \qquad V_{out} \ll V_{in}$$

$$V_{out} = \frac{R}{L}\int V_{in}$$

LR Circuit is integrating, time constant $\frac{L}{R}$ as for an integrating circuit is $V_{out} = \frac{1}{\tau}\int V_{in}$

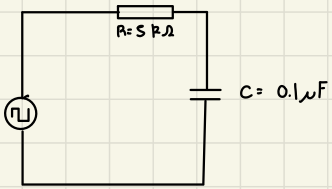
Experiment 6.

Aim was to investigate an RC Circuit with a step input.
The step input was achieved by switching the oscilloscope input to a square wave.
Then using the standard method of taking results it was evident that the voltage across the capacitor decayed after the voltage step (see diagram).



The tip of the curve is $V_{max} = V_{in}$. This should be equal to the step input voltage minus the voltage lost to internal resistance.

The time taken for the voltage to reach $\frac{1}{10}$th of its initial value was measured at 6 equally spaced points.



As this is a differentiating circuit:

$$\frac{dV_{in}}{dt} - \frac{dV_{out}}{dt} = \frac{V_{out}}{RC}$$

For $t > 0$, V_{in} = Constant

$$\frac{dV_{in}}{dt} = 0$$

$$-\frac{dV_{out}}{dt} = \frac{V_{out}}{RC}$$

$$\int -\frac{1}{V_{out}} \frac{dV_{out}}{dt} = \int \frac{1}{RC}\, dt$$

$$\ln V_{out} = -\frac{t}{RC} + k \quad ①$$

$$V_{out} = V_s \exp\left(-\frac{t}{RC}\right) \quad ②$$

This agrees with experiment. Voltage across C decays exponentially with time constant RC.

It is evident that if ① is compared to $y = mx + c$

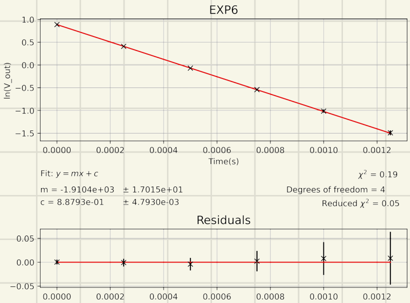
$$\ln V_{out} = -\frac{t}{RC} + k$$

then a plot of $\ln V_{out}$ against against t will give a graph with gradient $-\frac{1}{RC}$ and intercept of $\ln V_{out}$ at $t = 0$.

Results:

V_in/V	Error	V_out/ V	Error	ln(v_out)	Error	t/s	Error
2.46975	0.0125	2.43125	0.0125	0.8884055284	0.005141388175	0	0.000002
2.46975	0.0125	1.50625	0.0125	0.4096231183	0.008298755187	0.00025	0.000002
2.46975	0.0125	0.93125	0.0125	-0.07122750929	0.01342281879	0.0005	0.000002
2.46975	0.0125	0.58125	0.0125	-0.5425743221	0.02150537634	0.00075	0.000002
2.46975	0.0125	0.3625	0.0125	-1.014730805	0.03448275862	0.001	0.000002
2.46975	0.0125	0.225	0.0125	-1.491654877	0.05555555556	0.00125	0.000002

graph:



$m = (-1910.4 \pm 17.0)$

The expected value was $-\frac{1}{RC} = -2000 \pm 100$

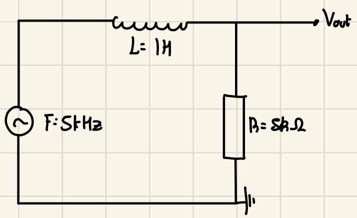
$$\sigma_{\frac{1}{RC}} = 2000\sqrt{(0.001)^2 + (0.05)^2} = 100.02$$

With the errors the expected value is in the range of the calculated value.
However $\chi^2_{reduced} = 0.05$ which indicates that the errors were overestimated.
The largest contributor to error was the capacitor tolerance (5%) and the error on V_{out} (5%) but this was less (3%) when the ln was taken.

Experiment 7

Aim was to simulate an LR circuit (integrating) and show it integrates. Then apply a step input and measure the time constant as in Exp 6.

The circuit was set up as shown:



It was shown in 5B that

$$\frac{1}{R}\frac{dV_{out}}{dt} = \frac{1}{L}(V_{in} - V_{out})$$

and thus when $V_{out} \ll V_{in}$

$$V_{out} = \frac{R}{L}\int V_{in}$$

$V_{in} = V_0 \cos\omega t$

$\frac{L}{R} = \tau$

as $V_{out} = \frac{1}{\tau}\int V_{in}$

$$V_{out} = \frac{RV_0}{L\omega}\cos\omega t$$

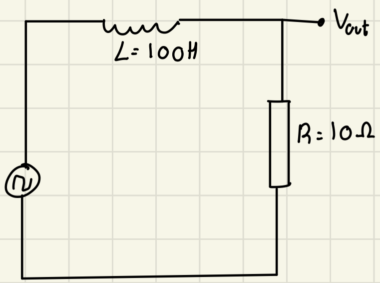
So amplitude of V_{out} should $= \frac{RV_0}{L\omega}$

To test this, the circuit was set up on falstad and these readings were taken as in exp5A:

Wave Gen Amp/V	Freq/Hz	Error	V_in/V	Error/V	V_out/V	Error	Phase/s	Error	Expected V_out	Error
2.5	5000	0	2.5	0.031	0.392143	0.004801	0.000055	0.000005	0.3978873577	0.004933803236
Resistance/Ohms	Inductance/H									
5000	1									

The measured value was $V_{out} = (0.392 \pm 0.005)\,V$
and the expected value was $V_{out} = (0.397 \pm 0.005)\,V$
From this, including errors, the measured V_{out} = expected V_{out}. Therefore, as this is consistent with theory, the LR circuit integrates.

Then, by changing the sinusoidal input to a step input, the decay of V_{out}:



$V_{out} = V_{in} - A e^{-\frac{R}{L}t}$

So rearranging into $y = mx + c$:

$\ln(V_{in} - V_{out}) = -\frac{R}{L}t + C$

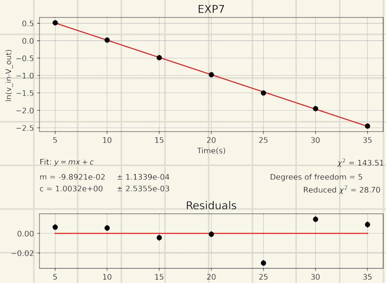
$y \quad = \quad m x \quad + c$

$\therefore$ It is expected that $m = -\frac{R}{L}$

And $\tau = \frac{L}{R}$ so $\tau = \frac{-1}{m}$

Data: (reduced)

time	ln(V_s-V_out)	Error
5	0.515	0.003
10	0.0195	0.003
15	-0.485	0.003
20	-0.976	0.003
25	-1.5	0.003
30	-1.95	0.003
35	-2.45	0.003



$m = (-0.0989 \pm 0.000134)\,s^{-1}$ The expected value for m was $-\frac{10}{100} = -0.1$

This doesn't lie in uncertainty range but is close.

Calculated $\tau = \frac{-1}{m} = 10.1 \pm 0.01\,s$

Expected $\tau = \frac{L}{R} = \frac{100}{10} = 10$

Again, not in range but due - falstad math errors.

Experiment 8A:

The aim of this experiment was to investigate light damping for an RLC circuit.
In this case, $R = 20\,\Omega$, $C = 0.1\,\mu F$ and $L = 1.5\,mH$.
This satisfies light damping as $\gamma < \omega_0$

$$\Downarrow$$

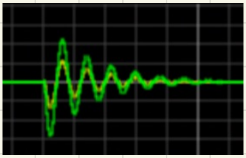
$$\gamma < \omega_0$$

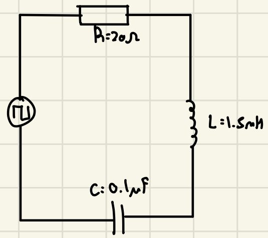
$$\frac{R}{2L} < \sqrt{\frac{1}{LC}}$$

$$\frac{20}{3\times10^{-3}} < \sqrt{\frac{1}{1.5\times10^{-3}\times10^{-7}}}$$

$$6666 < 81649 \quad \therefore \text{ light damping.}$$

When a step input is applied across the circuit the resistor 'rings' shown:





The frequency of this oscillation should be:

$$\omega = \sqrt{\omega_0^2 - \gamma^2} \qquad \omega_0 = \sqrt{\frac{1}{LC}} \qquad \gamma = \frac{R}{2L}$$

$$\omega = 81377 \text{ rad s}^{-1}$$

By measuring the period of oscillation to be $T = (0.00008 \pm 0.000005)\,s$

$$\frac{1}{T} = f = 12500 \pm 781.25 \qquad \sigma_f = 12500 \times \frac{0.000005}{0.00008}$$

$$\omega = 2\pi f = (78539 \pm 4908) \text{ rads}^{-1} \qquad \sigma_\omega = \frac{781.25}{12500} \times 78539$$

Therefore the theory result agrees with calculation (within range of error).

The resistance is oscillating with SHM. The solution to $\ddot{x} + 2\gamma\dot{x} + \omega_0^2 x = 0$ in this case is $I = Ae^{-\gamma t}$ where $A = \frac{V_s}{\omega L}\sin\omega t$

$$\therefore \ln I = \ln A - \gamma t \qquad \gamma = \frac{R}{2L}$$

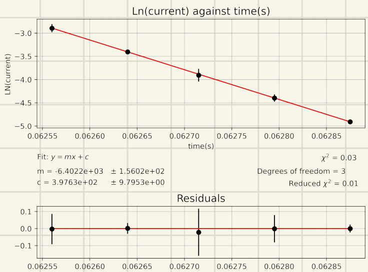
$$y = c + mx$$

So if $\ln I$ is plotted against t, m should equal $-\frac{R}{2L}$

Results:

I /A	Error /A	ln(I)	Error	time /S
0.055294	0.004941	-2.895090875	0.08935870076	0.06256
0.033262	0.001076	-3.403339675	0.03234922735	0.06264
0.020111	0.002793	-3.90648835	0.1388792203	0.062715
0.012305	0.000992	-4.397749595	0.08061763511	0.062795
0.00738	0.000175	-4.90898164	0.02371273713	0.062875

graph:



$m = -6402.2 \pm 156$

The expected gradient is $\frac{-R}{2L} = \frac{-20}{2\times1.5\times10^{-3}} = -6666$

This isn't in error range but as this was simulated there are mathmatical errors that can't be accounted for.

$\chi^2_{red} = 0.01$ which implies errors on current were overestimated.

The amplitude decreases by a factor of about 0.60 per cycle. $\Rightarrow \frac{0.033262}{0.055294} = 0.604$

The time taken for the amplitude to decay to 37% of its initial value is τ

This is roughly the 3rd peak. $\Rightarrow \Delta t = 1.55\times10^{-4}$ s

If the negative reciprocal of m is taken: $\frac{-1}{-6402.2} = 1.56\times10^{-4} \pm 3.81\times10^{-6}$

Therefore, the negative inverse gradient $\left(\frac{2L}{R}\right)$ is the time constant of the decay.

Experiment 8B:

The aim was to increase R until the circuit is heavily damped. It can be seen from a simple calculation that this will be when $R > 245\,\Omega$:

$$\frac{R}{2L} > \sqrt{\frac{1}{LC}}$$

$$R > 2L\sqrt{\frac{1}{LC}}$$

$$R > 245\,\Omega$$

- So set $R = 300\,\Omega$
- Now measure the time for the transient to reach its maximum and the time for it to decay by $1/e$.

Time taken to reach max: $(15.015)-(14.995)$ms $= 0.00002$ s $= 2\times10^{-5}$ s $\quad \sigma_{t_{max}} = \sqrt{2(0.005\times10^{-3})^2}$

$= (2\times10^{-5} \pm 7.07\times10^{-6})$ s

$= (2 \pm 0.707)\times10^{-5}$ s

max: 26.352 mA

$\frac{1}{e}$ max: 9.69 mA

$\Delta t = 15.05 - 15.015 = 0.000035$ s

$= (3.5 \pm 0.707)\times10^{-5}$ s

According to theory: $I \approx V_s R\left[e^{-\frac{t}{RC}} - e^{-\frac{Rt}{L}}\right]$

$$\frac{dI}{dt} = 0 = V_s R\left[-\frac{1}{RC}e^{-\frac{t}{RC}} + \frac{R}{L}e^{-\frac{Rt}{L}}\right]$$

$$\frac{R}{L}e^{-\frac{Rt}{L}} = \frac{1}{RC}e^{-\frac{t}{RC}}$$

$$\frac{R^2C}{L}e^{-\frac{Rt}{L}} = e^{-\frac{t}{RC}}$$

$$\frac{R^2C}{L} = e^{\frac{Rt}{L} - \frac{t}{RC}}$$

$$\ln\frac{R^2C}{L} = \frac{Rt}{L} - \frac{t}{RC}$$

$$t = \left(\frac{1}{\frac{R}{L} - \frac{1}{RC}}\right)\ln\left(\frac{R^2C}{L}\right)$$

$$t = 1.08\times10^{-5}$$

This value is very close to the calculated ($t = (2 \pm 0.707)\times10^{-5}s$). The discrepancy could be in falstad maths errors.

In theory the time taken to decay by $\frac{1}{e}$ is τ. In this case $\tau = RC$ as the $e^{-\frac{Rt}{L}}$ becomes small quickly. Therefore the expected time for $\frac{I_{max}}{e} = RC = 3\times10^{-5}$ s. This lies in the range of error of calculated value.

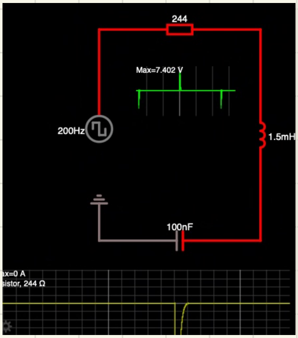
Experiment 8A:

The aim of this experiment was to find the value for R that causes critical damping.
Repeating the calculation in 8B: $\frac{R}{2L} = \sqrt{\frac{1}{2C}}$

$$R = 2L\sqrt{\frac{1}{2C}}$$

$$R = 245\,\Omega$$

Using falstad, the value for R for which there is critical damping is $244\,\Omega$:



There is no positive amplitude of the graph so no oscillation.
The difference in $1\,\Omega$ will be due to mathmatical falstad error.