Cepheid Variables

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Abstract

This experiment calculated the distance to spiral galaxy NGC 4258 by measuring the intensity variation of 15 variable stars in the outer regions of the galaxy. Light curves were constructed by writing a phase folding algorithm and an experimentally determined relationship between the period of intensity variation and absolute magnitude was used to calculate the distance to each star. The distance to NGC 4258 was calculated to be $d = 8.65 \pm 1.06$ Mpc by taking the average distance to each star. Then using the recession velocity of the galaxy, the Hubble's constant was calculated to be $H_0 = 51.68 \pm 6.34$ kms^{-1}Mpc^{-1}.

1. Introduction

A Cepheid variable star has an apparent magnitude that varies with a regular period [1]. In 1904 Henrietta Leavitt first published a relationship between this period and the intrinsic luminosity of the star after studying variable stars in the Magellanic clouds [2]. This was a key result in astronomy as the size of Cepheid variables means they can be seen from a large distance. Therefore, they provide a method of measuring distances to astronomical objects outside the range of parallax [3]. In this experiment, the periods of intensity variation of 15 Cepheids in the NGC 4258 spiral galaxy were measured using data from the Hubble Space Telescope. From this the distance to NGC 4258 was subsequently calculated and compared to other calculated values. Also using the known recession velocity of this galaxy, the Hubble's constant was calculated and compared to the true value.

2. Theory

Cepheid variable stars are found in the instability strip of the Hertzsprung-Russell diagram. Most stars that have a mass greater than the solar mass cross this region after leaving the main sequence. Whilst in the instability strip the star contains helium. In high temperature conditions this helium is double ionised, He^{2+}. In this state it is highly opaque as the free electrons can absorb photons with a wide range of energies. Therefore, the star is less bright as photons get trapped inside. This subsequently causes the star to heat up and expand. As the outer layers are now further from the stars core, they decease in temperature. The helium ions can accept an electron due to the lower temperature and becomes single ionised helium, He^{+}. This does not absorb as much light as He^{2+}as electrons are bound and can only absorb photons with a small range of energies. Therefore, photons can escape the star which causes the brightness to increase and the temperature to decrease. The lower temperature leads to contraction of the star which means the outer layers are closer to the core, so they heat up. This creates the conditions for double ionised helium again and the cycle repeats [4]. This process explains why Cepheid variable stars have an apparent magnitude that varies with a regular period. When magnitude is plotted against time, it increases sharply and falls slowly as shown in Figure 1.

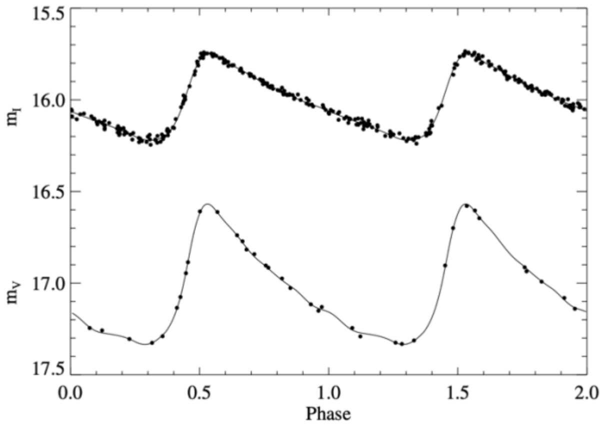

Figure 1: An example of a Cepheid light curve. Phase is plotted against magnitude for infrared (top) and visible light (bottom). Data points are joined with a fit function [5; p4699]

The relationship between period and apparent magnitude determined in the revised Key Project is

$$\overline{M}_V = -2.760(\pm 0.03)(\log_{10} P - 1) - 4.218(\pm 0.02), \tag{1}$$

where $\overline{M}_V$ is the mean absolute magnitude in visible light and P is the period of the Cepheid variable in days [6; PAGE]. This equation was found experimentally using Cepheids with a known distance, so the coefficients have uncertainties. However, these are small compared to the other errors in the experiment.

The average apparent magnitude, $\overline{m}$, of a star is calculated using

$$\overline{m} = m_r - 2.5 \log_{10} \frac{\overline{I_C}}{I_r}, \tag{2}$$

where $\overline{I_C}$ is the average intensity and I_r is the intensity of a reference star with apparent magnitude m_r [7]. The distance to an astronomical object in parsecs, d, is found using

$$d = 10^{\frac{5+(m-M)}{5}}, \tag{3}$$

where m is the apparent magnitude and M is the absolute magnitude [7].

The Hubble's constant is a measure of the expansion of the universe. This is calculated using the Hubble-Lemaître law

$$H_0 = \frac{v}{D}, \tag{4}$$

where v is the recession velocity of the galaxy and D is the proper distance to the galaxy [8].

3. Experimental Approach

Observations from the outer regions of the NGC 4258 spiral galaxy taken from the Hubble Space Telescope were provided in the form of .fits files. Outer regions were used as the density of stars decreases with distance from the centre of the galaxy so there is less background noise. This data was imported into a python terminal as an 800x800 array of intensity values. These values were measured in counts of photons collected by the CCD per second. Therefore, following Poisson statistics, the error associated with these measurements was $\sqrt{n}$ where n is the number of counts. This error was random and was dominant throughout the experiment. The intensity data was plotted on a logarithm scale as shown in Figure 2. The Julian date at which the data was taken was also extracted from the .fits files and displayed on graph. The coordinates of Cepheids in this region were taken from Newman et al's paper [6; p566]. However, these were adjusted as they were not accurate for all Cepheids and times. The intensity of the points surrounding the Cepheid in a circle of radius 3 pixels were summed. This size was chosen as it included the cepheid and minimum background data. To account for background noise, this was repeated with a second region with radius 7 pixels around the initial region as shown for Cepheid 1 in Figure 2. The difference between the intensity per pixel of this region and the original region gave the background intensity per pixel. This was then taken from the intensity per pixel of the Cepheid region and multiplied by the number of pixels in this region to get the final intensity of the Cepheid. The error of this was calculated by combining the fractional errors of the intensity per pixel of each region in quadrature. Some Cepheids were close to other bright stars, such as Cepheid 4 in Figure 2. As these stars didn't contribute to the background intensity, the background regions were moved to the side. For these regions the difference between the intensities did not need to be calculated as the intensity per pixel of the whole region was used. The errors were propagated using a similar method to before.

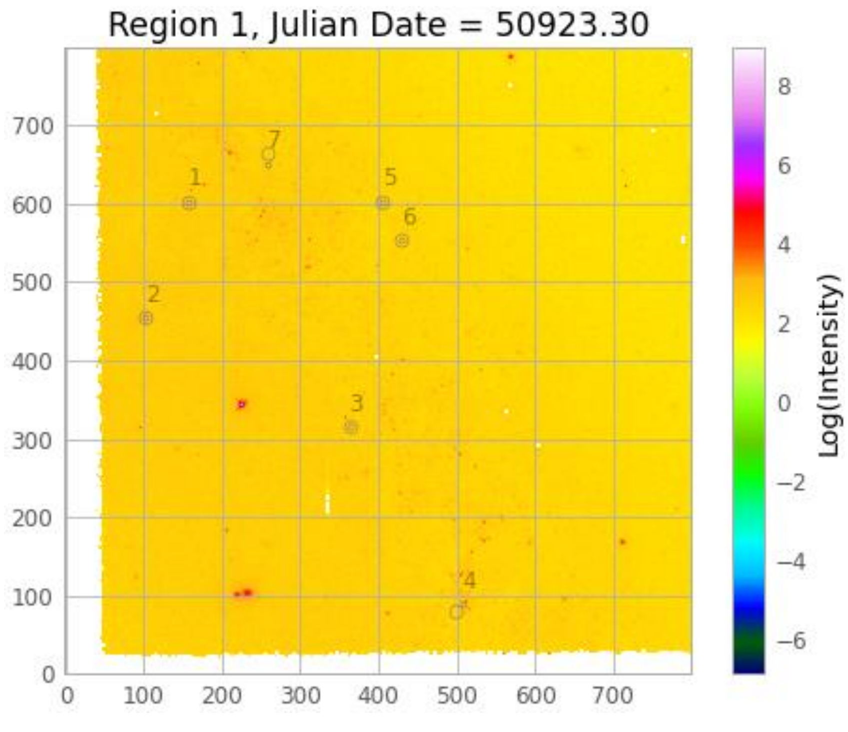

Figure 2: A logged intensity plot of a region in NGC 4258 used to illustrate the intensity of Cepheids. Central and background regions that were used in calculated are drawn and labelled with Cepheid number.

This method was repeated for all 15 known Cepheids in the regions and at 12 different points in time. However, the last measurement date was far in the future so was excluded from all data analysis as it added error to the period calculation. Background measurements were taken from multiple regions around the first Cepheid to reduce the systematic error from the background noise. However, the reduction in the error was negligible so this was not repeated for subsequent Cepheids.

Graphs of intensity against time were plotted to construct light curves. The period of varying intensity was estimated by taking the difference in time values of the maxima. To improve the calculated period a shortest string algorithm was written to phase fold the data. This moved points that were greater than one period backwards by 1 period unit such that the distance between all points was minimised [5;p4697]. The error on the period was the increment of period values tested in the algorithm. This was chosen to be the smallest increment for which a difference on the graph could be seen when the period changes.

4. Experimental Results

An example of a light curve after phase folding for Cepheid 1 is shown in Figure 3. The shape of this graph matches theoretical expectations. The period was found to be $P_1 = 23.73 \pm 0.04$ days. Using Equation (1), this corresponds to a mean absolute magnitude of $\overline{M_{V,1}} = -5.254 \pm 0.023$. The error on the logarithm was found by dividing the fractional error on period by the $\ln 10$. This was then combined in fractional quadrature with the error of the first coefficient in Equation (1). The error on $\bar{M}$ was calculated by combining this with the error on the second coefficient. The mean average of intensity was calculated with an error equal to the sum of the fractional errors in quadrature divided by the square root of the number of data points. This was then used in Equation (2) with reference values $I_r = 1000$ counts s^{-1} and $m_r = 22.5$ to calculate a mean apparent magnitude of $\overline{m_1} = 24.922 \pm 0.246$. The error was calculated by multiplying the fractional error of the mean intensity by 2.5 and dividing by $\ln 10$. The distance to the Cepheid was calculated to be $d_1 = 10.842 \pm 1.235$ Mpc using the mean absolute and apparent magnitudes and Equation (3). This fractional error was found by multiplying the error on the exponent by the natural log of 10.

This method was repeated for each of the 15 Cepheids in order to calculate a mean average of the distances. As the stars are all in NGC 4258 this is a good estimate for the distance to the galaxy. This was calculated to be $d_g = 11.67 \pm 2.79$ Mpc. The true distance to NGC 4258 is $d_{true} = 7.8 \pm 0.8$ Mpc [6; p562]. Therefore, the calculated value does not overlap with the true distance. However, some of the Cepheid light curves were inconclusive due to small variations in intensities, as shown in Figure 4. This could have been caused by noisy background regions or the size of the increments in time of data points in comparison to the period. After removing these Cepheids from the average, the calculated distance was $d_g = 10.57 \pm 1.99$ Mpc. This does overlap with the value in Newman et al's paper but has a percentage difference of 35.5%. To decrease the percentage difference the quantum efficiency of the CCD was considered. This is the percentage of photons that are counted as a measurement by the telescope. At a wavelength of $\lambda = 6000$ Å, this is approximately 67% [9]. Therefore, by increasing all intensity values by a factor of $\frac{1}{0.67}$, the average distance was calculated to be $d_g = 8.65 \pm 1.06$ Mpc. This has a percentage difference of 10.9 % compared to the true distance.

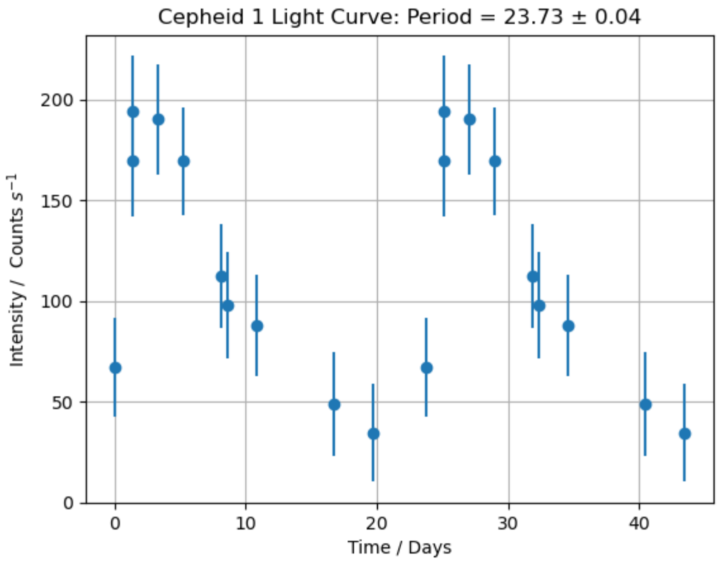

Figure 3: An example of a Cepheid light curve with phase folded data for Cepheid 1. Relative Intensity is plotted against the phase. The period of the varying intensity is 23.73 ± 0.04 days.

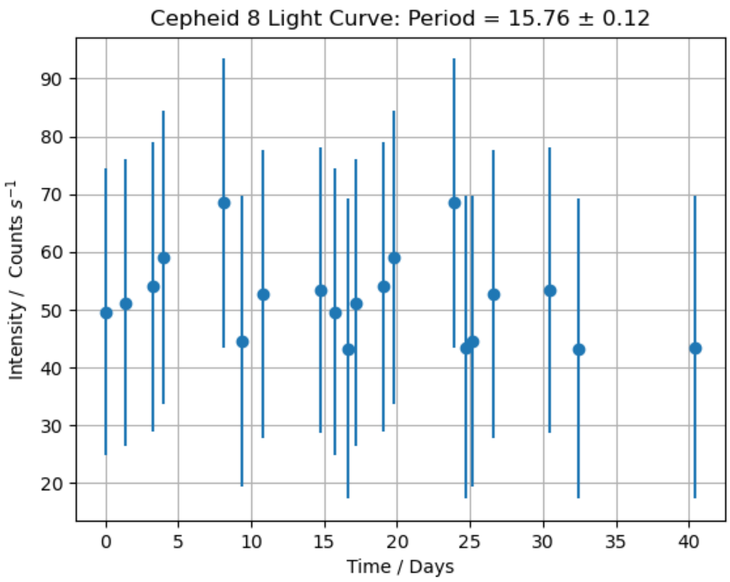

Figure 4: An example of a Cepheid light curve with phase folded data for Cepheid 8. Relative Intensity is plotted against the phase. The period of the varying intensity is 15.76 ± 0.12 days.

The calculated distance to NGC 4258 was used in Equation (4) to calculate the Hubble's constant. The recession velocity of NGC 4258 is $v = 447.9 \pm 3.0$ kms^{-1} [10]. Therefore, the calculated value of the Hubble's constant was $H_0 = 51.68 \pm 6.34$ kms^{-1}Mpc^{-1}. The true value of the Hubble's constant is $H_{0,true} = 72.86^{+0.94}_{-1.06}$ kms^{-1}Mpc^{-1} [11] which does not overlap with the calculated value. The calculated value could have been improved by repeating the experiment for multiple galaxies and plotting recession velocity against distance. This would have decreased the random error by calculating an average. Another improvement to the experiment is accounting for reddening. Dust particles in the interstellar medium are of the same size as the wavelength of blue light. Therefore, blue light gets scattered and absorbed more than red light so light reaching the telescope is redder than photons leaving the star. As light has been scattered and absorbed, the measured intensity is less that the intrinsic intensity of the star. This wasn't accounted for due to a lack of information about the stars. Another source of error in the experiment is the difference between the dates of the readings and the dates that the Hubble's constant and recession velocity correspond to as these values vary over time. However, the time difference is small, so this error is negligible compared to the sizes of other errors in the experiment.

5. Conclusion

The distance to NGC 4258 was calculated to be $d = 11.67 \pm 2.79$ Mpc. This was calculated by investigating light curves of 15 Cepheids in the galaxy. The distance was calculated using a similar method by Newman et al to be $d_{true} = 7.8 \pm 0.8$ Mpc. This was not consistent with the calculated value. After removing light curves that did not fit theoretical expectations and accounting for the CCD efficiency the calculated distance was improved to be $d = 8.65 \pm 1.06$ Mpc. This was consistent with Newman's calculated value. However, reddening was not accounted for, and this would have decreased the difference between results. The Hubble's constant was calculated to be $H_0 = 51.68 \pm 6.34$ $\mathrm{kms^{-1}Mpc^{-1}}$ which did not overlap with the true value of $H_{0,true} = 72.86^{+0.94}_{-1.06}$ $\mathrm{kms^{-1}Mpc^{-1}}$. Improvements to the experiment include, accounting for reddening and increasing the sample size of galaxies for a more precise Hubble's constant.

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