Diffraction of Light

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Abstract

This experiment investigated the diffraction and interference of coherent, monochromatic light through single and double slits. This was accomplished by using a moving photodiode to measure the interference patterns. These were then used to calculate the slit width of the single slits and the slit separation for the double slit. These values were compared to values measured from a vernier microscope. The slit widths calculated from the diffraction pattern were $a_{narrow} = 86.4 \pm 13.3$ µm, $a_{medium} = 150.8 \pm 31.9$ µm and $a_{wide} = 194.1 \pm 53.1$ µm. The slit widths from the vernier microscope were $a_{m,narrow} = 80 \pm 10$ µm, $a_{m,medium} = 100 \pm 10$ µm and $a_{m,wide} = 310 \pm 10$ µm. The slit separation of the double slit was calculated to be $d = 212 \pm 14$µm from the interference pattern and $d_m = 190 \pm 10$ µm from the vernier microscope. Also, the intensity distributions recorded from the single and the double slit were compared to the theoretical intensity distributions.

1. Introduction

Diffraction of coherent monochromatic light occurs when finite wavefronts disperse after a narrow slit divides a theoretically infinite wavefront [1]. This follows Huygens principle [2]. These wavefronts superpose with each other due to the wave nature of light. A distribution of the amplitude of the diffracted light can be analysed to deduce dimensions of the slit. This experiment was first conducted by Young in 1801 to demonstrate the wave theory of light. However, since then, it has been reproduced using electrons and atoms to show they can behave as waves as well as particles. These properties are used to measure small distances such as interatomic spacing of molecules and to observe the structure of proteins [3]. In this experiment the width of three single slits were measured using a vernier microscope, along with the slit separation for the double slit. These were compared to calculated values from the diffraction pattern of light when shone through the slits. The intensity distributions were plotted and compared to the theoretical intensity distributions.

2. Theory

2.1. Single Slit

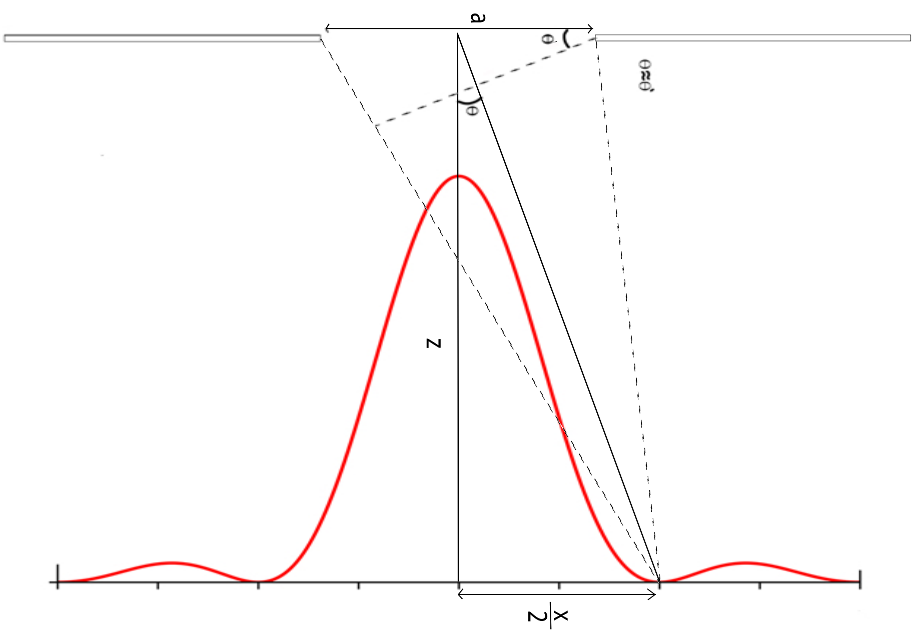

Figure 1. A diagram to illustrate the theoretical diffraction pattern from a single slit and the trigonometry used for the amplitude distribution function [4].

The condition for minima due to wavelets destructively interfering for single slit diffraction is given by

$$a\sin(\theta) = n\lambda, \tag{1}$$

where a is the slit width, n is the order of the minima, θ is the angle deviation from the central maxima and λ is the wavelength of light [5]. From Figure 1, through trigonometry and the small angle approximation $\sin(\theta) \approx \tan(\theta)$,

$$\sin(\theta) = \frac{x}{2z}, \tag{2}$$

where $\frac{x}{2}$ is the distance between the central maxima and the first minima and z is the distance between the slit to the screen. Substituting this into Equation (1) and taking $n = 1$ for the first minima results in

$$a = \frac{2\lambda z}{\Delta x}, \tag{3}$$

where Δx is the width of the central fringe. This directly relates the slit width to the width of the first maxima. The wavefront passing through the slit is represented by infinitesimally small segments. Each segment reaches some point on the detector with a different phase difference. A phasor vector with magnitude and direction given by the amplitude and phase difference is constructed for each segment. These vectors are then combined linearly. Therefore, at maxima, phasor vectors have the same direction so individual segments will constructively interfere. At minima, phasor vectors form a full circle, so the resultant vector has zero amplitude. The amplitude at an angle θ from the central maximum is a combination of these effects resulting in

$$A(\theta) = A_0 \frac{\sin\left(\phi_a/_2\right)}{\left(\phi/_2\right)}, \tag{4}$$

where A_0 is the amplitude at the central maxima and $\phi_a = \frac{2\pi}{\lambda} a \sin(\theta)$. As intensity is proportional to the square of amplitude, the intensity as a function of theta is

$$I(\theta) = I_0 sinc^2(\phi_a/_2). \tag{5}$$

2.2. Double Slit

The condition for a maxima of a double slit interference pattern is

$$d sin(\theta) = m\lambda, \tag{6}$$

where d is the slit separation and m is the order of the maxima. The slit separation is much smaller than the distance between the slit and the detector. Therefore, following theory from Section 2.1 gives

$$\sin(\theta) = \frac{\Delta y}{z}, \tag{7}$$

where Δy is the distance between the central and the mth maxima. Combining Equations (6) and (7) results in

$$d = \frac{m\lambda z}{\Delta y}. \tag{8}$$

The Fraunhofer diffraction pattern describes the amplitude distribution for a double slit, given by

$$A(\theta) = \cos\left(\frac{\pi d}{\lambda} sin\,\theta\right). \tag{9}$$

The overall intensity distribution is the product of the interference pattern of a double slit and the diffraction pattern of a single slit. This is because the double slit is the convolution of a finite single slit and two infinitesimally small slits. Therefore, due to the convolution theorem, the resultant intensity distribution is the product of the Fourier transforms of these two functions [6]. As intensity is proportional to the squared amplitude the intensity distribution for a double slit is given by

$$I(\theta) = I_0 cos^2\,(\phi_d) sinc^2(\phi_a), \tag{10}$$

where $\phi_d = \frac{2\pi}{\lambda} d\,\sin\,(\theta)$.

3. Experimental approach

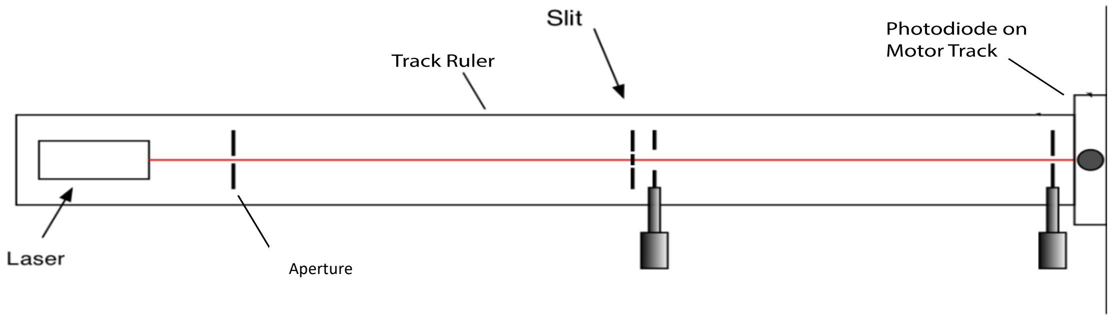

Figure 2. A schematic diagram of the equipment used to produce a diffraction pattern of light through slits. This includes the laser, aperture, slit and photodiode detector mounted on a track ruler. The laser beam is normal to both the slit and the photodiode [7].

3.1. Initial set-up

Using the set-up shown in Figure 2, apertures at different points on a track reduced the width of a beam of light and ensured the beam was perpendicular to a detector. The wavelength of light used was $\lambda = 670$nm and was constant throughout the experiment. The detector was moved on its track by a motor with a speed calculated by recording the time taken to travel a known distance. The errors on these were the resolution of the timer and half of the smallest increment of the ruler respectively. This was repeated three times for different distances to reduce the error from the varying speed of the detector. An average speed was calculated to be 0.15±0.006mms^{-1} which was constant throughout the experiment. This error was calculated by combining the absolute errors on each repetition in quadrature. The sampling interval was set at 50ms and duration of one run was 300s to ensure the whole diffraction pattern was measured. Background noise was measured to be $V = 0.001$V and was subtracted from each reading to eliminate this systematic error.

3.2. Single Slit

The widths of three slits were measured using a vernier microscope. Each slit was fixed onto the track and the distance from the slit to the detector, z, was measured using the track ruler. The error on this was the smallest increment of the track ruler scale. Perpendicular rulers were used to line up the slit and detector with the track ruler. This reduced the parallax error from measuring distance. This distance was $z = 32.3 \pm 0.1$ cm for the single slit measurements. The detector was then moved through the diffraction pattern as PicoLog software recorded the voltage readings against time. From the speed of the detector the time was converted to distance. This distance was centred at the central maxima and converted to an angle using the distance between the slit and screen. Therefore, voltage was plotted against distance, and intensity was plotted against angle from the centre maxima, as shown in Figures 3 and 4. This was repeated three times per slit, from which, the widths of central maxima were measured, and an average was calculated. The slit widths were then calculated using Equation (3). These values were compared to values measured from the vernier microscope. The intensity distributions for each slit were compared to the function given by Equation (5).

3.3. Double Slit

The same method as Section 3.2 was followed, whereby the width and slit spacing of a double slit were measured using a vernier microscope. The distance between the slit and the screen was measured to be $z = 16.0 \pm 0.1$cm. The data collected was used in Equation (8) to calculate the slit separation, d. This was then compared to the vernier microscope measurements. The intensity distribution from the data was compared to the theoretical intensity distribution from Equation (10).

4. Experimental Results

4.1. Single Slit Diffraction

An example of the amplitude distribution for a single slit is shown in Figure 3. Two vertical lines were drawn at the first minima with the error on each reading equal to half of the smallest increment on the horizontal axis. As two measurements were taken from the graph, the total error was equal to the two individual errors added in quadrature. The random errors from the calculation of distance were two orders of magnitude smaller than the error from reading the graph, so were neglected. For Figure 3 the width of the centre fringe was calculated to be $\Delta x = 4.12 \pm 0.28$mm. The average slit widths were then calculated to be $\Delta x_{narrow} = 5.01 \pm 0.77$ mm, $\Delta x_{medium} = 2.87 \pm 0.61$ mm and $\Delta x_{wide} = 2.23 \pm 0.61$ mm. The errors on these averages were found by adding the individual errors of the widths of the centre fringes in quadrature. Equation (3) and the average widths of the centre fringes were used to calculate the slit widths to be $a_{narrow} = 86.4 \pm 13.3$ μm, $a_{medium} = 150.8 \pm 31.9$ μm and $a_{wide} = 194.1 \pm 53.1$ μm. The error on the slit width was calculated by combining in quadrature, the fractional errors of the average width of the centre fringe and the distance between the slit and the screen. The dominant contributor to the overall error was reading values from the amplitude distribution. This was reduced by increasing the scale of the graph. The calculated slit widths were then compared to the values measured using the vernier microscope, these were $a_{m,narrow} = 80 \pm 10$ μm, $a_{m,medium} = 100 \pm 10$ μm and $a_{m,wide} = 310 \pm 10$ μm. The errors on these were found from the smallest increment of the microscope's scale. The measurement of the narrow slit was consistent with the calculated value; however, the values for the other two slits were not. Measuring the slit width with a microscope was more suitable for larger slits due to a smaller percentage error on larger microscope measurements. Conversely, diffracting light through the slit was a more suitable method for narrower slits. This is because smaller slit widths produce a wider central fringe so the percentage error from reading the graph is smaller. However, for the slits used in this experiment the microscope gave a consistently smaller percentage error.

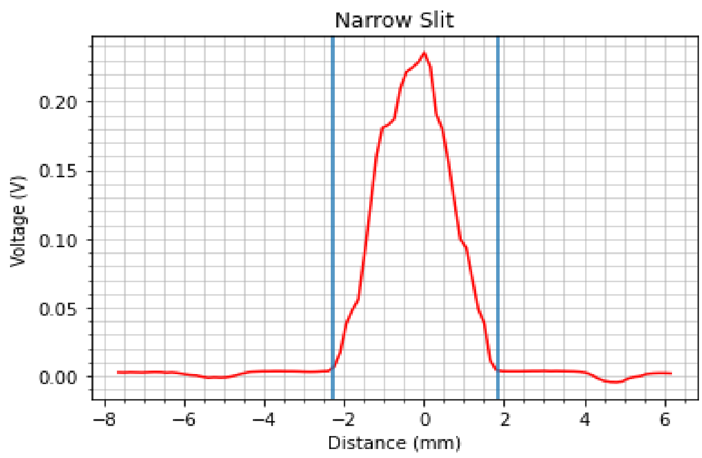

Figure 3. An example of a voltage/V against distance/mm plot for the narrow slit. The centre fringe width was estimated to be Δx =4.12±0.28 mm. The error bars are too small to be seen.

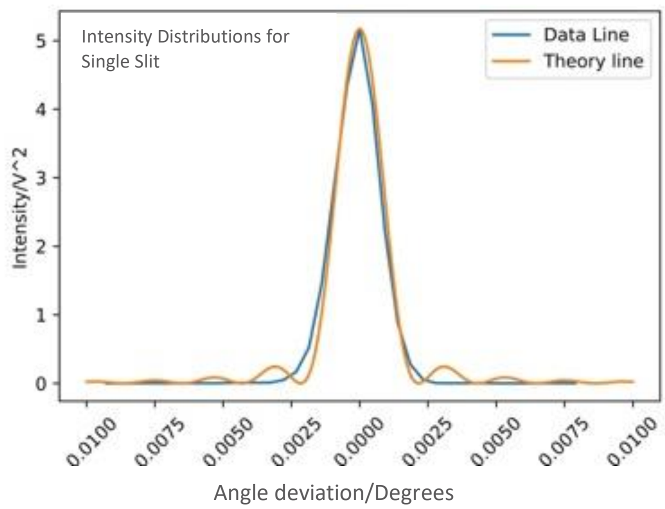

Figure 4. A plot of the theoretical intensity distribution compared to the intensity distribution of the data from a single slit. The errors bars on the data line are too small to be seen.

The theoretical intensity distribution from Equation (5) was compared to the intensity distribution for each slit. An example of this is shown in Figure 4. It is seen from the graph that the data fits theory for small angle deviations however, it does not for large angle deviations.

4.2. Double Slit Diffraction

An example of the amplitude distribution for the double slit is shown in Figure 5. Lines were drawn at the central and third maximum. The third maximum was used as it was the highest distinguishable maxima and this decreased the percentage of the random error as the measurement was larger. For Figure 5 the distance between the central and third maximum was $\Delta y = 1.49 \pm 0.14$ mm, whereby the error was propagated as in Section 4.1.

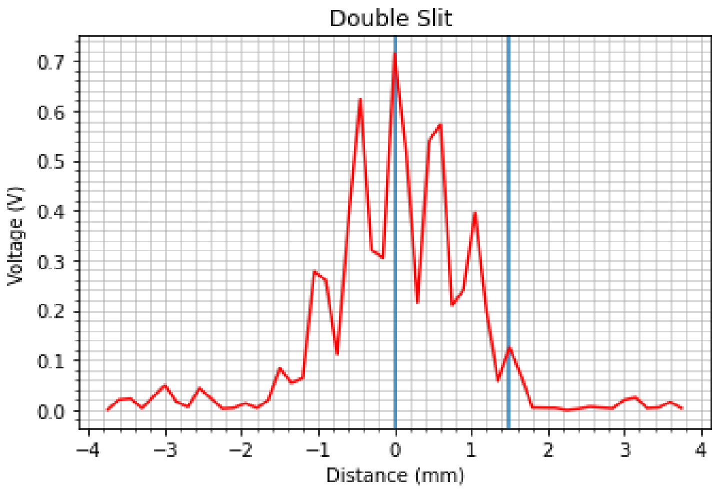

Figure 5. An example of voltage/V against distance/mm from light shone through the double slit. The distance between the central and third maximum was $\Delta y = 1.49 \pm 0.14$ mm. The error bars are too small to be seen.

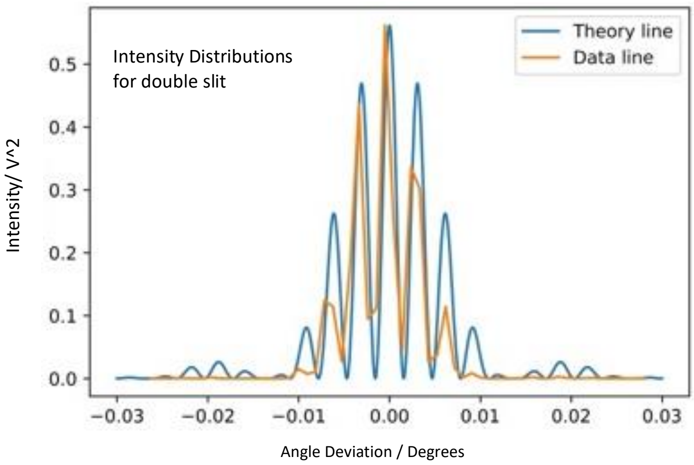

Figure 6. This is a plot of the theoretical intensity distribution compared to the intensity distribution of the data for the double slit. The error bars on the data line are too small to be seen.

The experiment was repeated three times for the double slit. A mean average of the distance between the central and third maximum was calculated to be $\Delta y = 1.52 \pm 0.14$mm. This error was determined by adding the errors on the individual measurements in quadrature. Then using Equation (8) the slit separation was calculated to be $d = 0.212 \pm 0.014$mm. The error was calculated by combining, in quadrature, the fractional errors on the distance between the slit to the screen and the distance between the centre and third maxima. The dominant error in this calculation was the error on reading the graph. The error on the distances were an order of magnitude smaller than this so were neglected. The calculated slit separation was then compared to the measured value from the microscope. This was

$d_m = 0.190 \pm 0.010$mm with an error calculated from the smallest increment on the microscope scale. This is consistent with the value calculated from the amplitude distribution. The value from the microscope had a smaller percentage error so was more accurate for this slit separation. The theoretical intensity distribution from Equation (10) was compared to the distribution of the double slit in Figure 6. It is seen from the graph that the data fits with theory for small angle deviations but does not for larger angles.

5. Conclusion

The values of slit width calculated from the diffraction pattern were $a_{narrow} = 86.4 \pm 13.3$ µm, $a_{medium} = 150.8 \pm 31.9$ µm and $a_{wide} = 194.1 \pm 53.1$ µm. The corresponding measurements from the vernier microscope were $a_{m,narrow} = 80 \pm 10$ µm, $a_{m,medium} = 100 \pm 10$ µm and $a_{m,wide} = 310 \pm 10$ µm. The values for the narrow slit overlapped; however, they did not for the other slit widths. It was seen from the measurements that the microscope method had a smaller percentage error than the calculations from the diffraction pattern. Therefore, it was more suitable for the slit widths used in this experiment. The intensity distribution fitted the theoretical distribution for small angles. For the double slit, the separation was measured to be $d_m = 190 \pm 10$ µm from the microscope and was calculated to be $d = 212 \pm 14$µm from the amplitude distribution. These values overlapped. The percentage error from the microscope method had a smaller percentage error than the diffraction calculation so was more suitable for this slit separation. The intensity distribution also fitted the theoretical distribution for small angles.

References

[1] F.G Smith and J.H Thomson, *Optics*, John Wiley & Sons Ltd, Second Edition, 1971, page 127-133.

[2] https://www.britannica.com/science/Huygens-principle, *Huygens' principle*, Britannica, Date accessed (20/11/2021).

[3] https://www.diamond.ac.uk/industry/Industry-News/Latest-News/Synchrotron-Industry-News-Focus-Diffraction.html, *Diffraction for Industry*, Industrial Liaison Group, Date accessed (20/11/2021).

[4] E. Hecht, *Optics*, Pearson Education Limited, Fifth Edition, 2016, page 467.

[5] https://courses.lumenlearning.com/austincc-physics2/chapter/27-5-single-slit-diffraction/, *Single Slit Diffraction*, OpenStax College, Date accessed (23/11/2021).

[6]https://phys.libretexts.org/Bookshelves/University_Physics/Book%3A_University_Physics_(OpenStax)/Book%3A_University_Physics_III_-_Optics_and_Modern_Physics_(OpenStax)/04%3A_Diffraction/4.04%3A_Double-Slit_Diffraction, *Double-Slit Diffraction*, OpenStax University Physics, Date accessed (23/11/2021).

[7] https://wanda.fiu.edu/boeglinw/courses/Modern_lab_manual3/interference.html, *Two-Slit Interference: Young's Experiment,* Werner U. Boeglin, Date accessed (24/11/2021).

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