Forced oscillations

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Abstract

This experiment investigated both forced and free oscillating systems by measuring the resonant frequencies of a thin phosphor bronze bar with different levels of damping. The quality factor for each different system was calculated. With no damping force, the forced oscillation yielded $Q = 157 \pm 14$ and the free oscillation yielded $Q = 388 \pm 3$. With a damping force, the forced oscillation yielded $Q = 44 \pm 1$ and the free oscillation yielded $Q = 96 \pm 2$. This illustrated that as damping increased the quality factor decreased which is consistent with the theory.

1. Introduction

The properties of an oscillating system change depending on the driving frequency and the magnitude of the damping force. The quality factor is a dimensionless parameter used to give a figure of merit to how good an oscillator is [1;p40]. It is a relation between the initial energy and the energy lost in one radian of an oscillation [2;p42]. The Q-factor can also be used in optical, acoustical, and electrical systems [3]. In this experiment the quality factor can be determined through the Power Resonance curve of a forced oscillator, Figures 2 and 4, or through the analysis of the decay of the amplitude of a freely oscillating system, Figures 3 and 5. The aim of the experiment was to find how the Q-factor changed with a damping force.

2. Experimental method

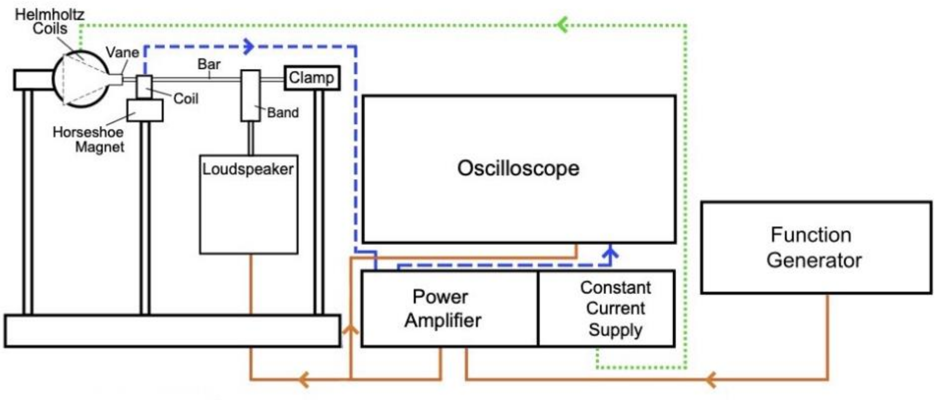

Figure 1. A diagram of equipment used to create an oscillating system which was driven by a function generator and a power amplifier. Data was then recorded from this system through the use of an oscilloscope. Reproduced with permission from Forced Oscillation pre-lab.

2.1. Forced Oscillations: No Damping

Using the setup shown in Figure 1, the function generator was used to supply a sinusoidal driving frequency to a loudspeaker. This oscillated a bronze-phosphate bar via the connection with a band. There was no current through a pair of Helmholtz coils, so the only damping was negligible air resistance. A coil on the bar passed through a horseshoe magnet which induced a current in the coil proportional to the velocity of the bar from Faraday's Law [4;ch16]. Signals of the voltage and function generator were amplified and displayed on the oscilloscope. The amplitude of the voltage signal was taken for varying driving frequencies from 17.35 Hz to 18.75 Hz in increments of 0.02 Hz. The amplitude of the voltage squared, proportional to power shown by Equation 7, was calculated and plotted as a function of driving frequency, giving a Power Resonance Curve. The resonant frequency was found by taking the value of the driving frequency corresponding to the maximum amplitude of the curve. The width of this graph at half of the peak power was recorded as f_{fwhm}, shown in Figure 2. Then the equation

$$Q = \frac{f_0}{f_{fwhm}}, \quad (1)$$

where Q is the quality factor of the bar and f_0 is the resonant frequency, was used to find Q.

2.2. Free Oscillations: No Damping

The same setup was used but with no driving frequency from the loudspeaker and the bar was tapped into free oscillations. The voltage signal displayed on the oscilloscope showed the decay of the velocity of the bar. Extracting the peaks values of voltage and taking the logarithm produced a linear set of data, which allowed the data to be used in the LSFR code and Figure 3 was plotted. The decay constant of the oscillation was found through

$$\gamma = -2m, \tag{2}$$

where γ is the decay constant and m is the gradient of the graph. Then by applying

$$Q = \frac{2\pi f_0}{\gamma}, \tag{3}$$

Q was calculated.

2.3. Forced Oscillations: Damping

The same method as Section 2.1 was used but a uniform magnetic field was induced by the Helmholtz coils [5] with a current of 272 mA. This caused very light damping. Figure 4 was plotted, and Q was calculated using Equation (1).

2.4. Free Oscillations: Damping

The same method as Section 2.2 was used but the Helmholtz coils were supplied with a current of 272 mA to produce a damping. The same measurements were taken, and Figure 5 was plotted. Using Equations (2) and (3), γ and Q were calculated.

3. Theory

In this experiment the bar oscillates in simple harmonic motion given by

$$\frac{d^2x}{dt^2}+\gamma\frac{dx}{dt}+\omega_0^2 x = 0, \tag{4}$$

where x is the displacement of the bar, t is time and ω_0 is the natural frequency of the oscillation, derived from Newton's second law [1;p34]. The quality factor of such an oscillator is given by Equation (3) [1;p40]. The amplitude of vibration will be maximum when the frequency of the driving force is equal to the resonant frequency of the bar. The energy of a damped oscillator will decay and for very light damping

$$E = E_0 \exp(-\gamma t), \tag{5}$$

where E is the energy and E_0 is the initial energy [1;p39]. The rate of energy loss, or power, can then be calculated as

$$\frac{dE}{dt} = \frac{d}{dt}\left(\frac{1}{2}mv^2 + \frac{1}{2}kx^2\right) = -bv^2, \tag{7}$$

where m is the mass of the bar, v is the velocity of the bar, k is the spring constant of the bar, x is the displacement and b is a constant [1;p40]. Therefore, power is proportional to velocity squared. A driven oscillator will have energy supplied at the same rate it is lost due to damping, the average energy lost is found by integrating Equation (7) over one period. For light damping this reduces to

$$\bar{P} = \frac{f_0^2}{2m\gamma(\frac{4\Delta\omega^2}{\gamma^2}+1)}, \tag{8}$$

where $\bar{P}$ is the average energy lost for one period and $\Delta\omega$ is the difference between the angular frequency and the natural frequency of the system. The half power point of this graph is when $\omega_{fwhm} = \gamma$ and therefore Equation (3) can be written as Equation (1) [1; Sec. 3.3].

4. Experimental Results and Data Analysis

4.1. Forced Oscillations: No Damping

The data from Section 2.1 can be summarised in Figure 1. The graphs maximum was estimated by eye and halved. This was then used to find f_{fwhm}.

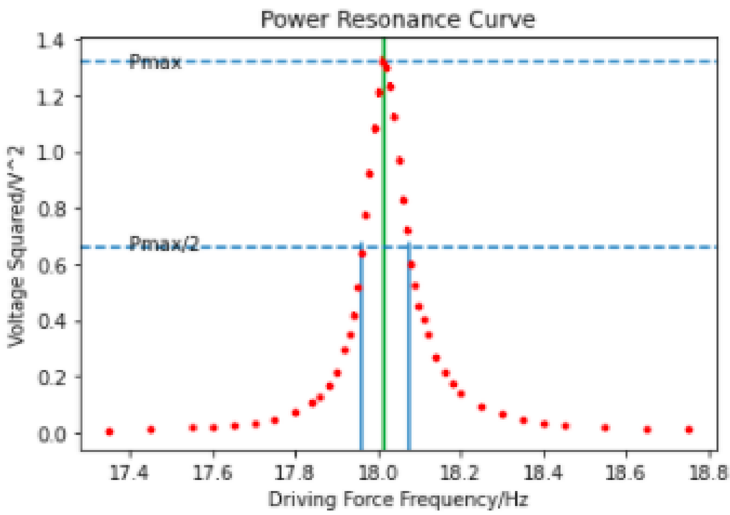

Figure 2. Voltage squared/V^2 against driving force frequency/Hz for forced oscillations with no damping force. From this graph and using Equation (1) it is found that $Q = 157 \pm 14$. The error bars are too small to be seen.

Using $f_{fwhm} = 0.115$ Hz and $f_0 = 18.0$ Hz the Q-factor was calculated. Errors in voltage were 0.01 V, and 0.01 Hz for frequency. The uncertainties in frequency were added in quadrature to calculate the absolute error shown in the caption of Figure 2. However, the true absolute error will be higher as the maximum of the graph was estimated by eye thus readings from Figure 2 were less accurate. The main contributor to the uncertainty was the error in the frequency. Errors on the voltage were not taken into consideration as they were not used in Equation (1).

4.2. Free Oscillations: No Damping

The logged data of the peak amplitudes of voltage were plotted using LSFR.py giving the gradient, m, shown in Figure 3. Equations (2) and (3) were used to determine Q for this system.

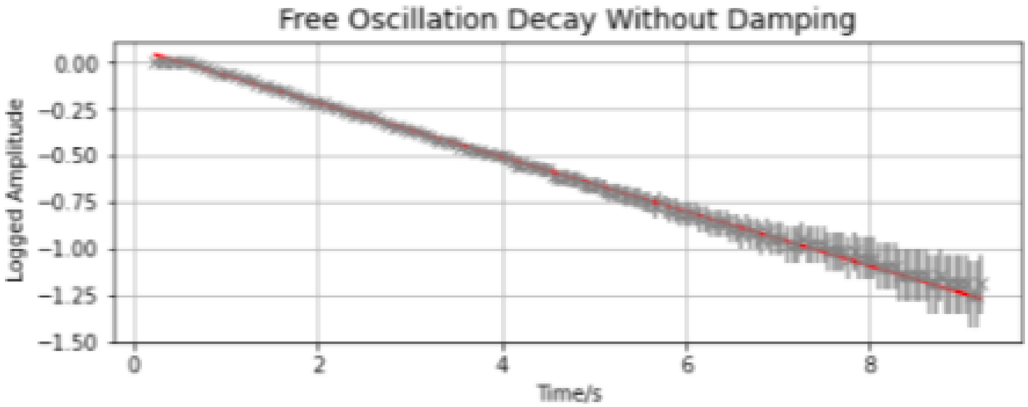

Figure 3. Logged amplitude of voltage against time. The graph shows the data from Section 2.2 and a least squares fit line with $\chi^2 = 55$ and $\chi^2_{reduced} = 0.34$. The line has a gradient of $m = (-0.146 \pm 0.001)\text{s}^{-1}$ and an intercept $c = 0.074 \pm 0.003$.

From Figure 3 the quality factor was found to be $Q = 388 \pm 3$, where f_0 is the same as Section 4.1. The error on the gradient, which was the highest contributor, and the frequency were combined in quadrature for the absolute error. However, the $\chi^2_{reduced}$ was lower than 0.5 meaning that more than 68% of the error bars crossed the line of best fit indicating that the original errors were overestimated. The intercept, c, represented the logarithm of the initial peak amplitude but was not used in analysis.

4.3. Forced Oscillation: Damping

The data from Section 2.3 is shown in a Power Resonance Curve in Figure 4.

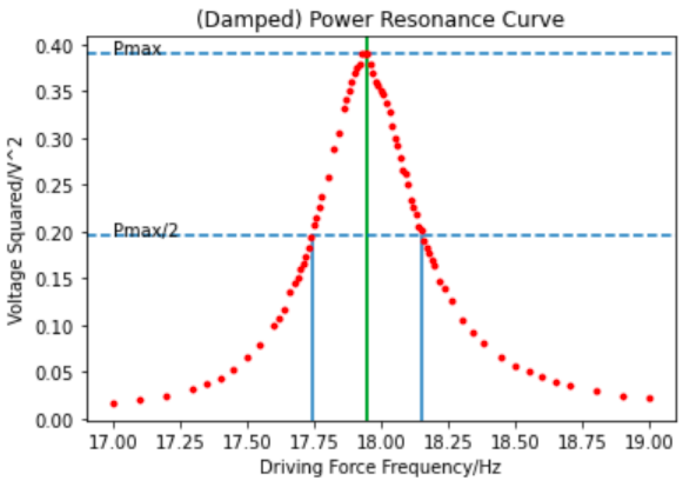

Figure 4. Voltage squared/V^2 against driving force frequency/Hz for forced oscillations with damping. The Q-factor was found to be, $Q = 44 \pm 1$. The error bars are too small to be seen.

The method to find the overall error on Q was the same as Section 4.1. From Figure 4, $f_0 = 17.9$ and $f_{fwhm} = 0.405$, which were used to calculate the Q-factor. The main contributor to the absolute uncertainty is the error on frequency due to Equation (1) and the error on voltage was not used. The true absolute error will be larger as f_{fwhm} was estimated by eye so it is less accurate. This Q-factor is lower than the value with no damping which is consistent with theory.

4.4. Free Oscillations: Damping

The method from Section 4.2 was used to plot Figure 5. The Q-factor was found by Equations (2) and (3).

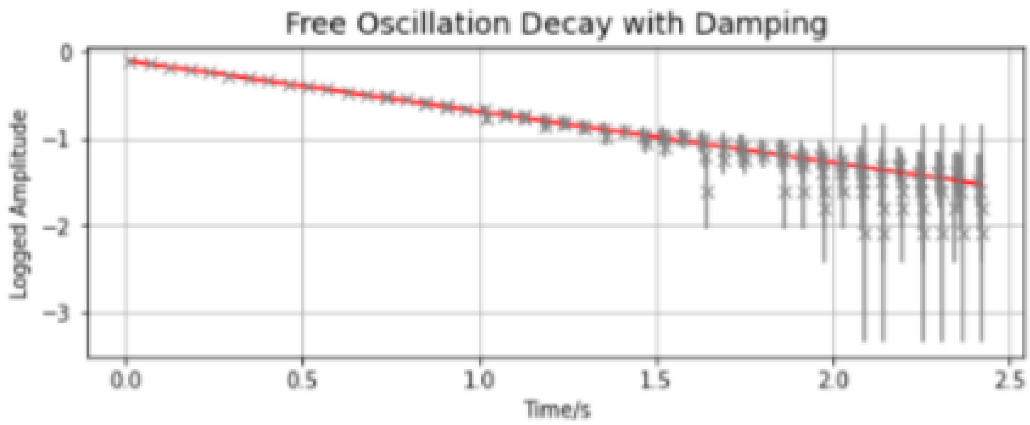

Figure 5. Logged peak amplitudes against time/s. The graph shows data from Section 2.4 and a least squares fit line with $\chi^2 = 22$ and $\chi^2_{reduced} = 0.20$. The line has a gradient of $m = (-0.585 \pm 0.010)$s^{-1} and an intercept $c = -0.112 \pm 0.007$.

The Q-factor was calculated in the same way as Section 4.2 and found to be $Q = 96 \pm 2$, where f_0 is the same as in Section 4.3. Error bars are larger for smaller amplitudes as they are a larger percent of the reading. The uncertainty on the gradient was the largest contributor to the absolute error. The $\chi^2_{reduced}$ value was below 0.5, which again suggests that the original errors were overestimated. As expected from theory, the Q-factor is lower for the damped system.

5. Conclusion

The final calculated value of the Q-factor for undamped systems were $Q = 157 \pm 14$ and $Q = 388 \pm 3$ for forced and freely oscillating systems respectively. For damped systems, $Q = 44 \pm 1$ for forced oscillations and $Q = 96 \pm 2$ for free oscillations. In each system the Q-factors approximately agree as for both cases the calculated values are within a factor of 2.5. These results are consistent with theory as a lower Q-factor indicates a higher energy loss which is caused by the damping force. The error on the gradient was the highest contributing factor to the absolute uncertainty on the freely oscillating system. The error on the frequency was the largest contributing factor to the absolute uncertainty on the forced oscillating systems. However, there was a large error on readings from the power resonance curves as it was done by eye and as a result the Q-factors would agree to a larger extent if a better technique was used. For example, code to determine the maximum point and width of the graph.

References

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