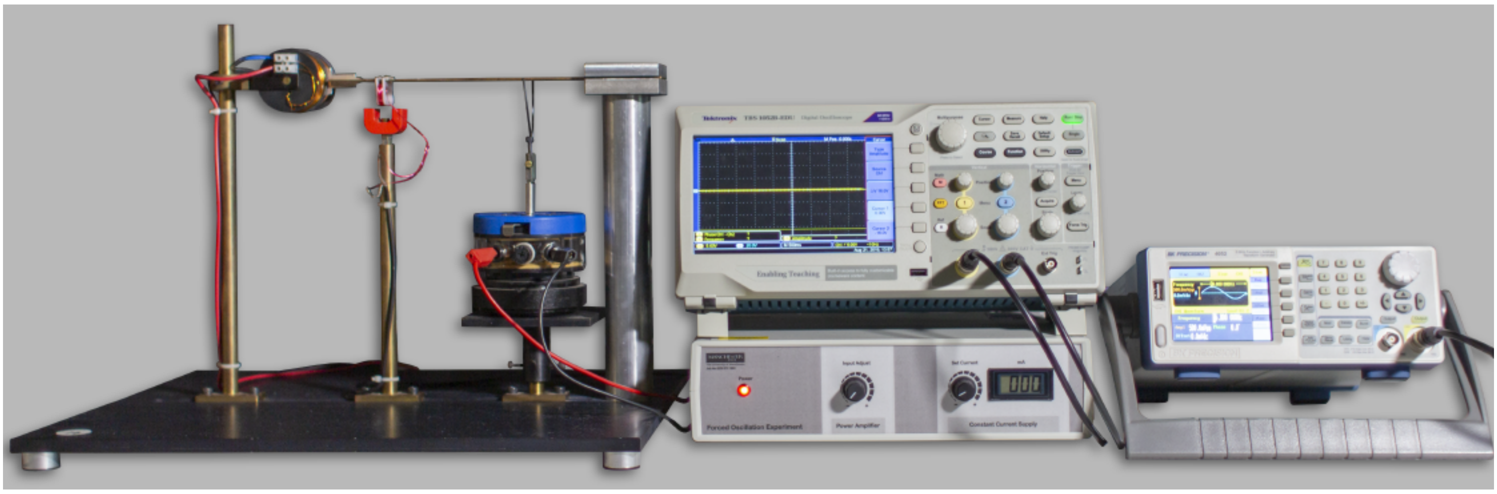
PHYS10180: Prelab Information

Forced Oscillations

Introduction



Equipment used in the Forced Oscillations experiment

In this experiment you are going to study the resonant behaviour of a simple mechanical system. The central part of the system is a metal bar which can be made to oscillate. A digital oscilloscope connected to a coil sensor on the bar is used to measure the motion of the bar. You will investigate two cases of oscillating behaviour: free oscillations, where the bar is set in motion and the oscillations die down naturally due to damping, mainly from air resistance, and forced oscillations, where the bar is forced to oscillate at a particular frequency. A signal generator is used to drive the forced oscillations.

There are two forms of energy involved in the experiment: the potential energy stored in the bar as it is bent away from the equilibrium position and the kinetic energy of the bar moving. We may start the system in motion either by tapping the bar to add kinetic energy, or by pulling it to one side to put in potential energy. Either way, the subsequent behaviour of the system is identical. It trades energy back and forth between kinetic and potential energy. You are probably already familiar with the standard example of an oscillating system, a mass on a spring, where the same exchange between potential and kinetic energy occurs as the mass oscillates.

This experiment has three important components

- The study of resonance and the effect of damping;
- Gaining experience in using a function generator;
- Learning how to make measurements with a digital oscilloscope.

Resonance

All systems capable of vibration have a natural frequency that they vibrate at if no external forces are present. Consider a mass connected to a spring: as it vibrates energy is transferred from kinetic to potential energy in a way that is controlled by how stiff the spring is and how large the mass is. A quartz crystal driven by an electrical signal in a quartz watch is just the same, it has a mass and a stiffness determined by the nature of quartz. If you drive a system at a specific frequency the amplitude of the vibrations will be largest when the driving frequency is at the resonant frequency of the system as energy is given to it in a way that helps build a large amplitude of vibration.

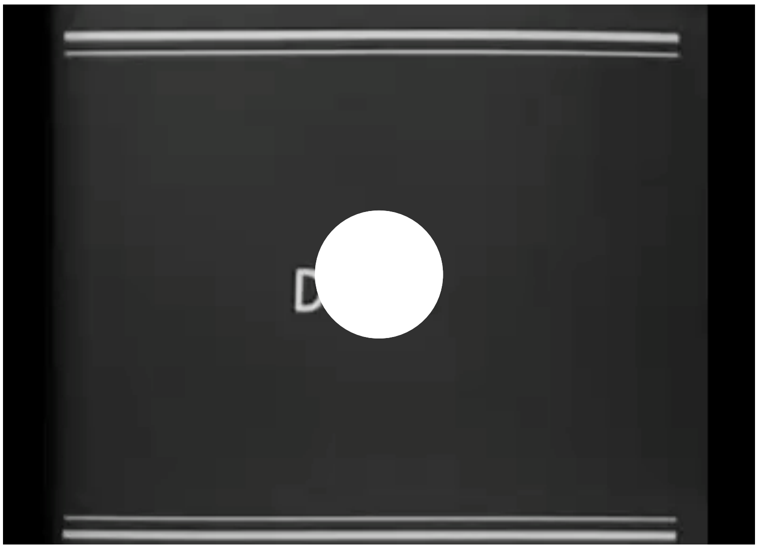


Figure 1. Video clip of the Tacoma Bridge collapse.

Perhaps the most famous example of resonance is shown in Figure 1, the film of the Tacoma Narrows Bridge vibrating on the day it collapsed [1]. You may have seen this film already? This bridge collapsed because the wind allowed the bridge to reach one of its resonant frequencies, with little or no damping to stop really large amplitude vibrations.

Another famous example of resonance even closer to home was the breaking of Broughton suspension bridge over the river Irwell in April 1831. A group of soldiers from the 60th Rifle Corps (Figure 2) supposedly marched in time to the bridge's sway. Unfortunately the frequency of their marching step was close to the natural frequency of the bridge, which therefore underwent a severe mechanical oscillation leading to its collapse. Ever since, soldiers have been told to break step when crossing bridges. Clearly the designers of the Millenium Bridge in London had not counted on the natural habit of people to walk in step!

Figure 2. Soldiers from the King's Royal Rifle Corps. Image by Harry Payne.

Breaking glasses (Figure 3) by singing the right note is a similar resonance phenomenon. If the note (sound wave) has frequency close to the resonant frequency of the glass, then the amplitude of the oscillations is increased, which can cause the class to shatter. It is a slightly too dangerous activity to test in the lab with other students around.

Figure 3. Shattering Glass.

References

1. Tacoma Bridge Collapse video, available from commons.wikimedia.org/wiki/File:Tacoma_Narrows_Bridge_destruction.ogv (Accessed 19 Dec 2019)

PHYS10180: Prelab Information

Forced Oscillations

Equipment and Experimental Method

For this experiment you will study the oscillating behaviour of a thin phosphor-bronze bar, rectangular in cross-section, and clamped at one end. In addition to the labelled photograph below, an interactive diagram which enables you to identify each component of the experimental set-up and also to see the mechanical and electrical connections between components is available here.

[Open the interactive diagram]

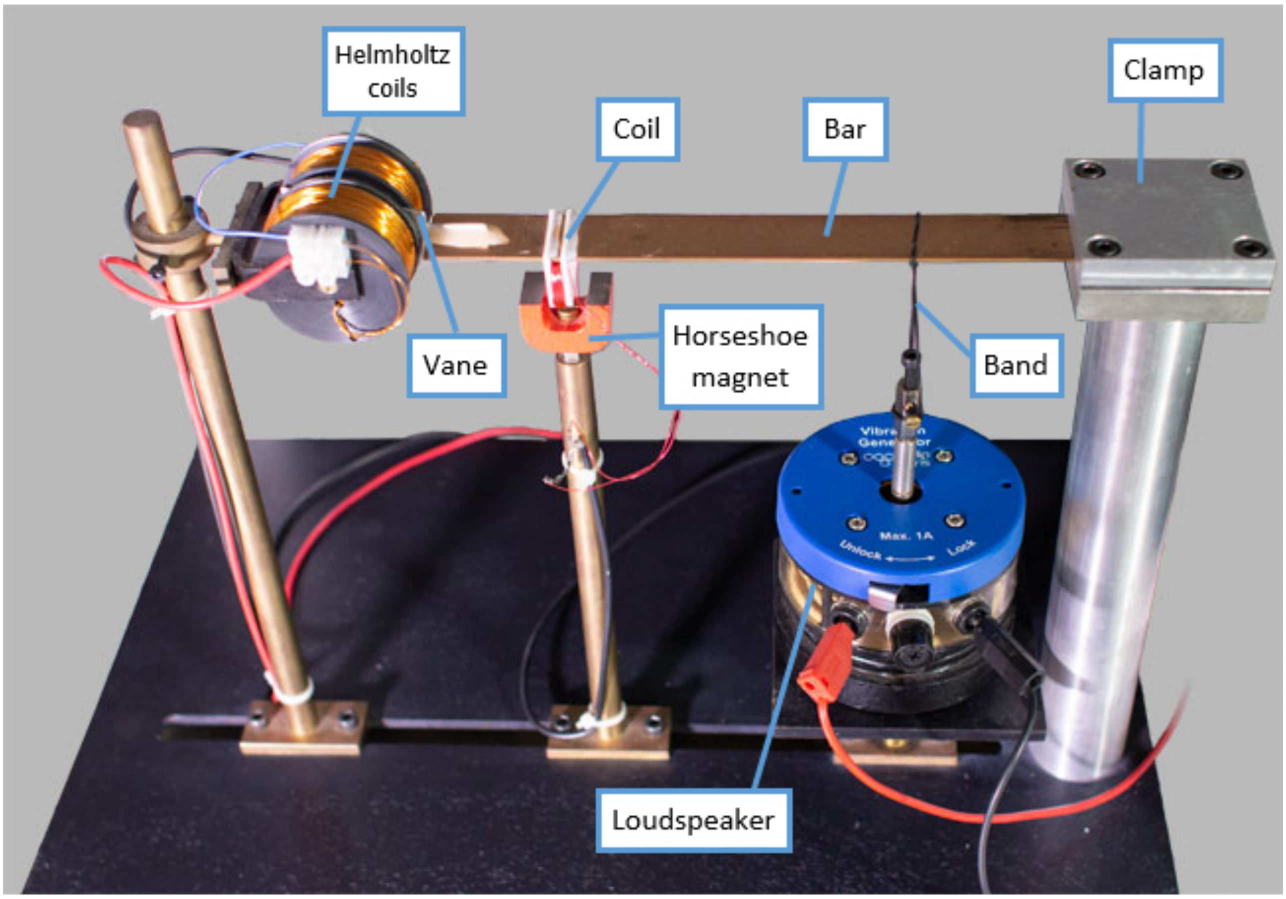


Figure 4. A photograph showing the phospher-bronze bar and associated equipment.

The phosphor-bronze bar (Figure 4) can be made to vibrate at its resonant frequency by displacing it and allowing it to spring back, or by tapping it.

Alternatively, it can be forced to oscillate at a specific frequency by connecting it to a mechanical oscillator which is powered by the amplified signal from a function generator (Figure 5). The driving oscillator consists of a loudspeaker coil which drives a rod up and down. The rod is connected by an elastic band to the bar, causing the bar to vibrate at the same frequency as the rod. The frequency of the driving sine wave can be varied using the function generator.

Mounted on the bar is a

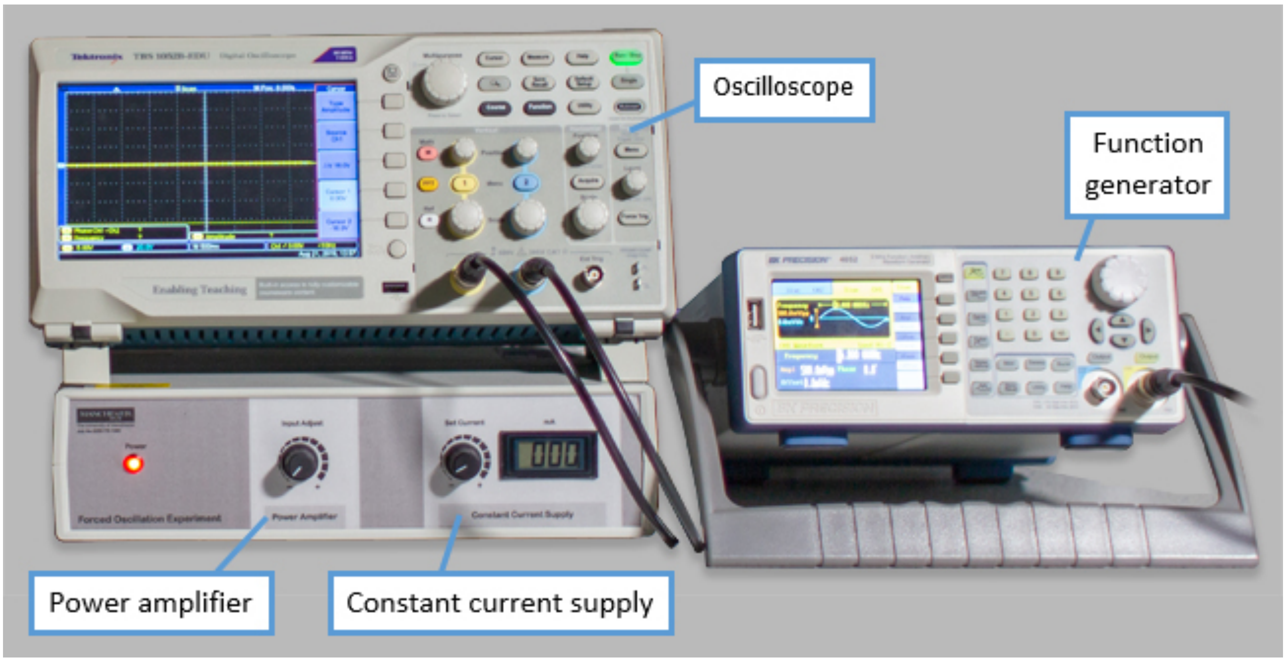


Figure 5. A photograph showing the signal generator, oscilloscope and amplifier boxes.

rectangular coil consisting of 20 turns of insulated copper wire. As the bar oscillates the coil moves between the poles of a fixed horseshoe magnet (Figure 4). The motion of the coil through the magnetic field produced by the magnet induces a current to flow, which is proportional to the velocity of the bar. The resulting voltage difference between the two ends of the wire can be viewed on the oscilloscope (Figure 5).

A light, vertical aluminium plate is attached to the free end of the phosphor-bronze bar and as the bar oscillates the aluminium plate moves between a pair of Helmholtz coils (Figure 4). When a current is passed through the Helmholtz coils, a magnetic field is induced between them. This field then causes eddy currents to flow in the aluminium plate which in turn gives rise to velocity dependent damping. Thus you can investigate the effects of increased damping on the system by increasing the current through the Helmholtz coils.

Experimental Method

As this is a very lightly damped system, the resonant frequency of the bar is close to its natural frequency. Tap the bar to make it oscillate and measure the resonant frequency of the bar by analyzing the voltage signal from the coil using the oscilloscope.

To begin with, run the experiment without adding any extra damping via the Helmholtz coils.

Investigating Free Oscillations

To investigate free oscillations, set up the oscilloscope to show the motion of the bar and set the time base so that you can see several oscillations on the screen. You should be able to see the amplitude of the oscillations decaying exponentially with time. By measuring the decay constant, you can calculate the Quality factor, Q, of the bar.

You can download a screenshot of the oscilloscope display so that you can measure the amplitude as a function of time, in order to find the decay constant. Use a USB stick to copy the screenshot from the oscilloscope to your computer. You should have a USB stick from your Welcome Week pack, which is useful for this purpose, otherwise you may be able to borrow one from the demonstrators' desk on the first floor of the lab.

Investigating Forced Oscillations

To investigate forced oscillations, use the function generator to drive the loudspeaker with a sinusoidal signal which has a frequency equal to the resonant frequency of the bar. You already have a good estimate of this frequency, from your measurement of the frequency of free oscillations. The amplitude of the vibrations will be largest when the bar is driven at its resonant frequency. Set the peak-to-peak voltage produced by the function generator to be no greater than 0.5 V, and then check that the bar is not being driven too strongly by the amplified signal from the signal generator. You should see a clear sinusoidal signal on the oscilloscope and you should not hear any tapping or buzzing sound from the bar. When you are satisfied that you have a strong, clear, sinusoidal signal you are ready to start taking measurements.

You should make a table in your lab book, similar to the one shown in the video, and record your measurements of the amplitude of the signal produced by the bar, and its phase shift from the driving signal, as a function of the driving frequency. Don't forget to include a note of your measurement uncertainties. You should start with the driving frequency set to around 10 Hz, and increase the driving frequency in steps up to about 20 Hz. Think about how large, or small, you should make the steps in driving frequency and whether or not to use the same step size throughout. You will need enough measurements to enable you to plot the response of the system as a function of driving frequency.

You will then use these measurements to make three plots, to show:

1. amplitude of oscillation (arbitrary units)
2. phase angle (radians or degrees)
3. mean power absorbed (arbitrary units)

as a function of driving frequency. You can compare your plots to the predicted behaviour of a driven oscillating system and use the third (power resonance) plot to calculate the Quality factor Q for the system.

Remember that the oscilloscope is measuring the voltage induced by the coil as it moves through the magnetic field from the horseshoe magnet. The voltage is directly proportional to the velocity of the coil, which varies sinusoidally. The amplitude of this sinusoidal variation corresponds to the maximum velocity of the bar, which depends on the **amplitude** of the bar's displacement and on the **frequency** of oscillation.

The introductory video for this experiment shows the setting up procedure in more detail.

More Information About Setting Up Your Equipment

It is important to ensure that your equipment is set up to produce sinusoidal oscillations which are not so large that they become distorted when the system is driven close to the resonant frequency but are large enough to be measurable at driving frequencies away from the resonant frequency.

If the amplitude of oscillation is too large, such that the bar is not vibrating sinusoidally, you will be able to see this on the oscilloscope and sometimes also hear it as a buzzing or knocking sound. To reduce the amplitude of your oscillations, either reduce the output voltage of the function generator or adjust the amplification of the signal which is sent to the loudspeaker.

After completing your initial set-up to ensure that vibrations at the resonant frequency are sinusoidal, make sure that you keep the amplitude of the signal sent to the mechanical oscillator constant while you are taking your measurements. The only parameter of the system you are studying which you should change is the driving frequency. You will notice that the amplitude of the signal from the bar shown on the oscilloscope decreases as you move away from the resonant frequency. You can adjust the Volts per division setting on the oscilloscope to show this signal at as large a scale as possible so that you can measure the amplitude and phase shift more easily and precisely. This adjustment does not change the signal itself, just the scale at which it is displayed.

The Oscilloscope

The oscilloscope (Figure 6) allows you to display waveforms and to measure various quantities associated with them such as frequency and amplitude. If two waveforms are displayed together you can also measure the phase difference between them. The Additional Information section for this experiment includes a short introductory video describing the basic use of an oscilloscope, which will help you to set up and use the oscilloscope properly, as well as a user note and manufacturer's information sheets for the TBS1052B oscilloscope you will be using.

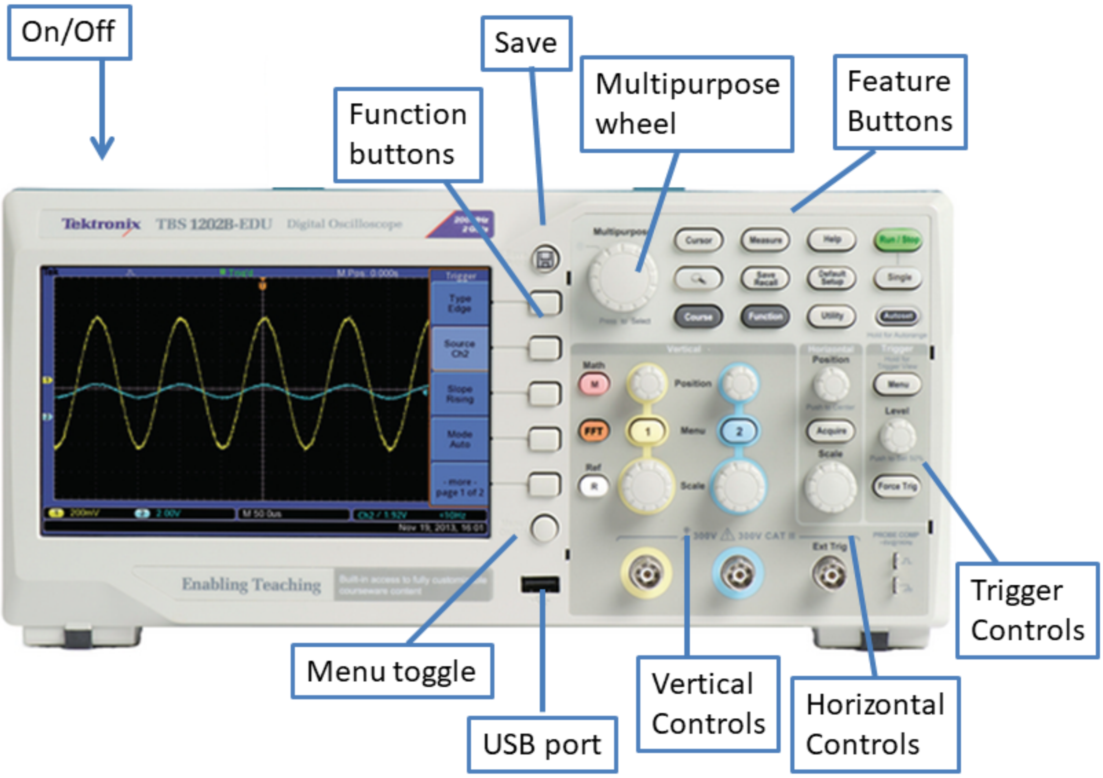


Figure 6. A labelled diagram of the oscilloscope you will use in this experiment.

You should make sure that you know about the following important controls:

- **Horizontal Controls** - use the Scale knob to change the time duration displayed per division on the horizontal axis. Increasing the timebase will allow you to display more oscillations on the screen at once.
- **Vertical Controls** - there are two input channels for the oscillosope and you can adjust each channel independently. Use the Scale knobs to change the number of volts displayed per division on the vertical axis. Reducing the number of volts per division will allow you to zoom in vertically on a weaker signal.
- **Trigger Controls** - used to set the oscilloscope to refresh the display in such a way that the waveform appears steady on the screen. It is usually much easier to measure the properties of the signal if it is properly triggered.
- **Autoset** (bottom right Feature Button) - useful to initially set up the oscilloscope.
- **Measure Functions** (top, second from left Feature Button) - you can set up to six quantities to be measured by the oscilloscope.
- **Multipurpose wheel and Function buttons** - use these to select between the various options available for each setting.

Note: Many oscilloscopes have a coupling control that can be set as DC or AC. The DC setting passes both AC and DC components of the signal and the AC one only allows AC signals. For the oscilloscope you are using, AC coupling attenuates the input signal below 10 Hz. Since this experiment involves using signal freqencies close to this cut-off value; using AC coupling could lead to inaccurate measurements which would affect your final results. Therefore you should choose the DC coupling option. If you use the AUTOSET button on the oscilloscope this will use DC coupling as a default.

The Function Generator

The function generator (Figure 7) can be used to produce a signal of specified shape, frequency and amplitude. You will use it to send a sinusoidal signal to the loudspeaker. The Additional Information section for this experiment, includes a general introductory video about signal generators as well as a user note and manufacturer's information sheets for the BK Precision 4050 signal generator you will be using.

You should make sure that you know about the following important controls:

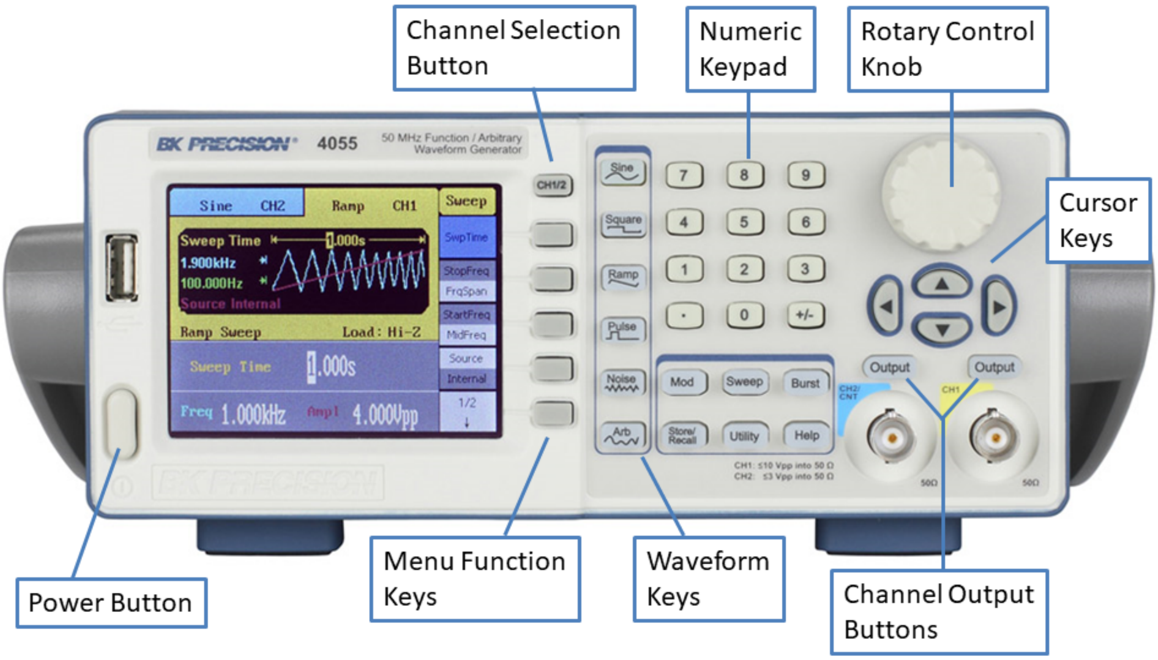


Figure 7. A labelled diagram of the function generator you will use in this experiment.

- **Channel Selection Button** - The Function Generator has two outputs, Channel 1 and Channel 2. Use the Channel Selection button to set up the signal on the correct output channel.
- **Menu Function Keys** - Use these keys to select the various parameters of the function you want to generate.
- **Numeric Keypad, Rotary Control Knob and Cursor Key**s - use whichever you prefer to set the values of the parameters of your function.
- **Output Button** - the signal is only sent from the function generator when the output button has been pressed and is illuminated green.

PHYS10180: Prelab Information

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Theory

This experiment involves the analysis of a simple oscillating system. The physics of such systems can be found in any good textbook on vibrations and waves. Here, we follow the treatment given in King [1]. Note that for this experiment, it is not necessary for you to be able to derive the solutions to the differential equations which describe the motion of the oscillator under different circumstances. The solutions you will need, along with other useful results, are all given in this pre-lab.

To understand the expected behaviour of the system, we begin by considering the case of simple harmonic motion (SHM) [1; ch. 1].

Simple harmonic motion

You should already be familiar with the idea of SHM, which happens when the force on a body is proportional to its displacement and acts in the opposite direction. A common example is a mass on a spring, where this restoring force is described by Hooke's Law. If the mass is displaced from equilibrium and then released, its motion is periodic, sinusoidal and has a constant amplitude. There is no external driving force, and the restoring force is proportional to the displacement from equilibrium.

Newton's Second Law states that a body of mass m will have acceleration a which depends on the sum of the forces acting on it. A body undergoing SHM will experience a force $-kx$ where k is a spring constant and x is the displacement of the body. Thus we can write Newton's Second Law as

$$m\frac{d^2x}{dt^2} = -kx \tag{1}$$

where we have written the acceleration as the second derivative of displacement. This is a simple second order differential equation which has a general solution

$$x(t) = A\cos(\omega t + \phi)$$

where $\omega = \sqrt{(k/m)}$ is the angular frequency of the system, and A and ϕ are constants which describe the initial conditions: A is the amplitude of the oscillations and ϕ is the phase angle which specifies the relative position of the mass through its cycle at time $t = 0$

The damped oscillator

So far, we have considered a frictionless system, however, it is more realistic to assume that there will be frictional, and possibly other dissipative forces which act to damp the motion of the oscillator. Following [1; ch 2], we include a damping force $F_d = -bv$ in the equation of motion for the oscillator which thus becomes

$$m\,\frac{d^2x}{dt^2} + b\,\frac{dx}{dt} + kx = 0\,.$$

The damping force is proportional to the velocity v of the oscillating body and the constant of proportionality b depends on the properties of the system. This second order differential equation is often rewritten in the form

$$\frac{d^2x}{dt^2} + \gamma \frac{dx}{dt} + \omega_0^2 x = 0 \qquad (2)$$

where $\gamma = b/m$ and the ratio k/m is now written as ω_0^2. We call ω_0 the natural frequency of the oscillator. Provided the damping is not too large, we would expect the system to oscillate with decreasing amplitude, because the system is losing energy due to the damping. It turns out that equation 2 has a solution

$$x(t) = A \exp(-\gamma t/2)\ \cos(\omega t + \phi)$$

where ω is the angular frequency of the oscillations. It can be shown that $\omega^2 = \omega_0^2 - \gamma^2/4$ and the system is said to be lightly damped as long as $\gamma < 2\omega_0$

Note that if $\gamma \geq 2\omega_0$ then $\omega \leq 0$ and so the system will return to equilibrium without oscillating. For the special case $\gamma = 2\omega_0$ the system is said to be critically damped and for larger values of γ the system is said to be heavily damped. The larger the value of γ, the longer the system will take to relax to the equilibrium position.

Energy Loss and the Quality Factor

The total energy E in an oscillating system is given by the sum of the kinetic (K) and potential (U) energies. In the case of no damping, E is constant, although K and U both vary with time.

However, a damped oscillator will lose energy, generally through dissipation of heat into its surroundings [1; ch 2.3]. In the case of very light damping, i.e. $\gamma \ll 2\omega_0$ and $\omega \approx \omega_0$, it can be shown that E decreases exponentially with time from its initial value E_0:

$$E(t) = E_0 \exp(-\gamma t)\ .$$

The rate of energy loss can be found by differentiating E:

$$\frac{dE}{dt} = \frac{d}{dt}\left(\frac{1}{2}mv^2 + \frac{1}{2}kx^2\right) = -bv^2 \qquad (3)$$

showing that it is proportional to the rate of work done against the damping force (bv).

The **Quality Factor** Q is a dimensionless number which characterizes how quickly or slowly the amplitude of the oscillations dies down relative to the number of cycles completed. It can be calculated for any oscillating system and is defined as

$$Q = \frac{\omega_0}{\gamma}\ .$$

A *good* oscillator will have a large value of Q indicating that the oscillations are sustained for a long time relative to the period of oscillation. A *poor* oscillator will have a low value of Q indicating that any oscillations die down rapidly. For a given system, if the amount of damping is increased, we would expect to see a decrease in the Q value.

Quality factor and decay

The quality factor Q can also be interpreted as the number of cycles required for the energy in the oscillating system to fall by a factor of $1/e^{2\pi}$ which is about $1/535$. The "quality" terminology arises from the fact that friction and its associated loss of energy is often considered a bad thing, so a mechanical device that can vibrate for many oscillations before it loses a significant fraction of its energy would be considered a high-quality device.

Example: Decay of a saxophone tone

A typical saxophone has a Q-factor of about 10. How long will it take for a 100-Hz note played on a baritone saxophone to die down by a factor of 535 in energy, after the player suddenly stops blowing?

A Q factor of 10 means that it takes 10 cycles for the vibrations to die down in energy by a factor of 535. Ten cycles at a frequency of 100 Hz would correspond to a time of 0.1 seconds, which is not very long. This is why a saxophone note doesn't "ring" like a note played on a piano or an electric guitar.

The driven oscillator

A driving force in the form of a sinusoidal signal can be applied to the system, which forces it to oscillate at the applied signal frequency and with constant amplitude [1; ch 3]. When such a driving force is applied, the equation of motion of the oscillating mass becomes

$$\frac{d^2x}{dt^2} + \gamma\,\frac{dx}{dt} + \omega_0^2\,x = \frac{F_0\cos(\omega t)}{m}\,, \tag{4}$$

where the external driving force has amplitude F_0 and angular frequency ω.

This second order differential equation has two parts to its solution: a transient part [1; ch 3.5] which is the same as the solution to equation 2 and dies away quickly, and a steady state solution of the form

$$x(t) = A(\omega)\cos(\omega t - \delta)\,.$$

We will consider the behaviour of the system when it is in its steady state, ignoring the transient behaviour. The phase shift δ in the steady state solution is the angle by which the displacement lags behind the driving force. Figure 8 shows how the driving force and the resulting displacement vary with time; in this example the driving frequency is less than the natural frequency of the system.

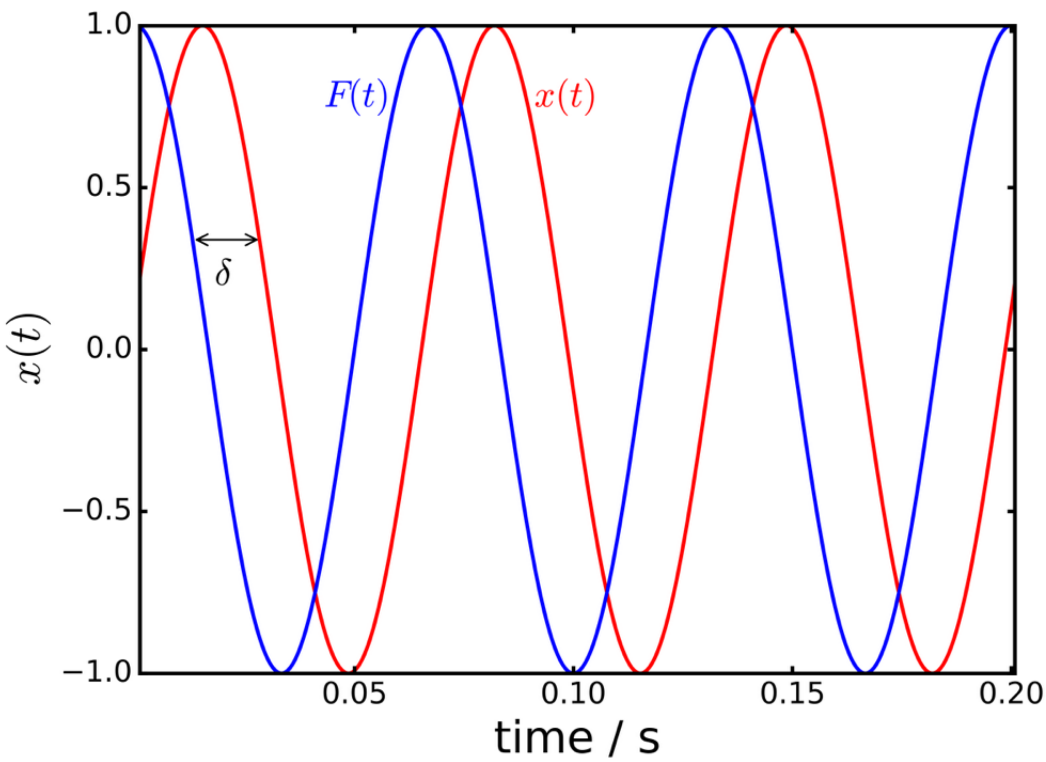

Figure 8. A plot showing the displacement $x(t)$ of an oscillating system caused by a sinusoidal driving force $F(t)$. The displacement and driving force have each been scaled to have amplitude equal to 1.

The amplitude and phase shift of the oscillations both depend on the angular driving frequency and it can be shown that

$$A(\omega) = \frac{F_0/m}{[(\omega_0^2 - \omega^2)^2 + \omega^2\gamma^2]^{1/2}}$$

and

$$\tan(\delta) = \frac{\omega\gamma}{(\omega_0^2 - \omega^2)} .$$

Note that frequency is directly proportional to angular frequency ($f = \omega/2\pi$) and therefore the equations for A and $\tan\delta$ can easily be rewritten in terms of driving frequency f and natural frequency f_0.

Plots of A and δ as a function of f are shown in Figure 9. It can be seen that the amplitude of the vibrations reaches a maximum for a particular value of driving frequency, called the resonant frequency f_r. This peak in the amplitude response is called a **resonance**. Note that $f_r < f_0$ but for very lightly damped systems these two values will be very close. When the system is driven at its resonant frequency, $\delta = \pi/2$

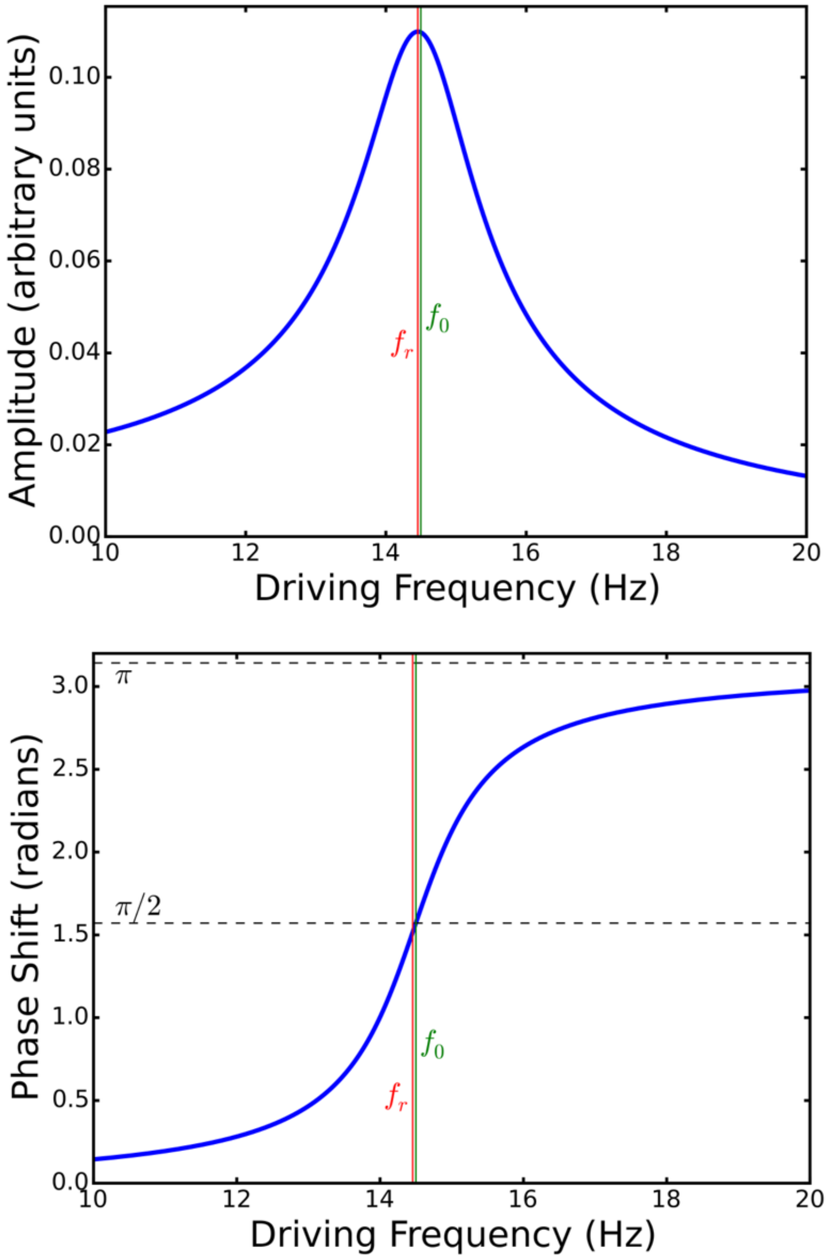

Figure 9. The amplitude and phase shift of a driven oscillator, with very light damping, plotted as a function of driving frequency. The shapes of the plots would be exactly the same if they were plotted as a function of ω instead of f, the only difference would be a scaling of the x-axis by a factor 2π.

Power Absorbed by a Driven Oscillator

We have seen that a damped oscillator loses energy, and therefore the amplitude of the oscillations decreases with time. In a driven oscillator, after any transients have died away and the system is in its steady state, then the driving force supplies energy to the system at exactly the same rate as that at which the system loses energy. Therefore the amplitude of a driven oscillator does not change with time.

The average power $\bar{P}$ absorbed over one cycle can be found by integrating the instantaneous power (eqation 3) over one period [1; ch 3.3]:

$$\bar{P}(\omega) = \frac{1}{T}\int_{t_o}^{t_o+T} P(t)\,dt = \frac{b[v_0(\omega)]^2}{2}$$

where t_o is the time at the start of the cycle, the period is T, and $v_0(\omega) = A(\omega)\omega$, which is the maximum value of the velocity of the oscillator. This can be re-written as

$$\bar{P}(\omega) = \frac{\omega^2 F_0^2 \gamma}{2m\left[(\omega_0^2 - \omega^2)^2 + \gamma^2\omega^2\right]}\ . \qquad (5)$$

A plot of $\bar{P}$ as function of f is shown in Figure 10. Such a plot is called a Power Resonance Curve. It can be seen that the $P_{\rm max}$ occurs when $f = f_0$. The sharpness of the peak can be characterised by its width $f_{\rm fwhm}$ at half the maximum value of $\bar{P}$.

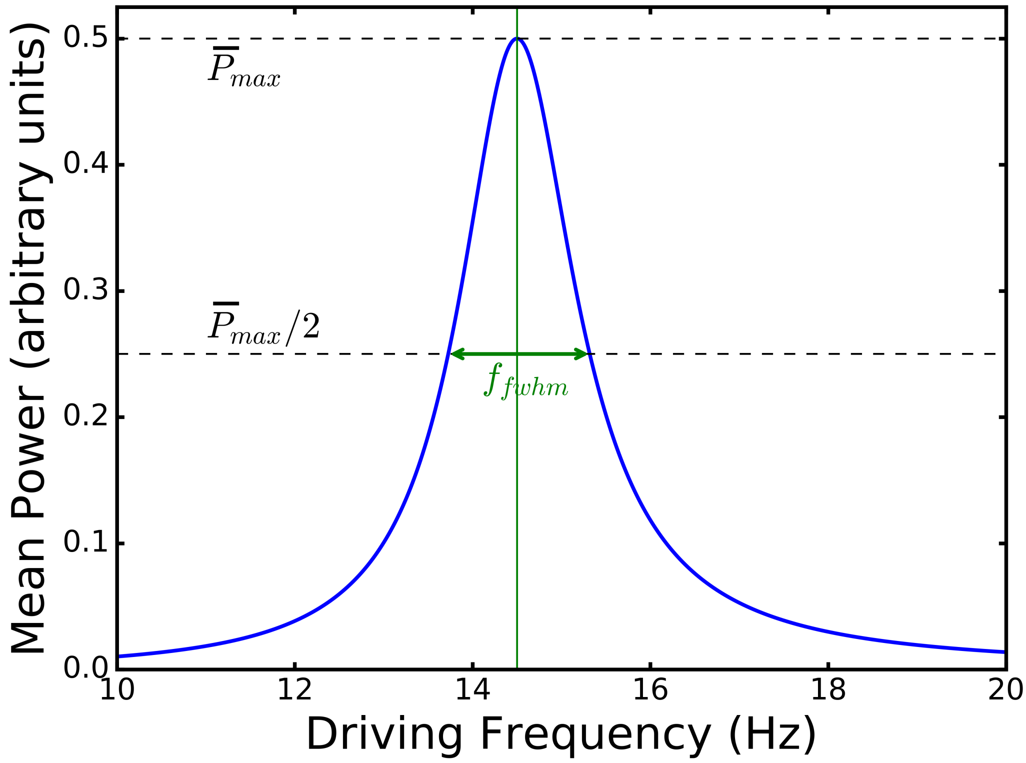

Figure 10. The power resonance curve of a lightly damped driven oscillator.

The Quality Factor of a Lightly Damped Resonant System

The shape of the power resonance curve reveals something about the quality of the oscillator. A system with a sharply peaked power resonance curve will absorb power strongly at driving frequencies close to its natural frequency, so it will maintain strong oscillations at these frequencies but the oscillations will be much smaller at other frequencies. However, for a system where the peak is less sharply defined, oscillations can be maintained over a wider range of driving frequencies. In general, as the amount of damping is increased, the mean power absorbed decreases and the resonance peak becomes smaller and wider.

In the case of very light damping, when $\omega \approx \omega_0$, Equation (5) can be re-written as

$$\bar{P}(\omega) = \frac{F_0^2}{2m\gamma\,(4\Delta\omega^2/\gamma^2 + 1)}\ ,$$

where $\Delta\omega = \omega - \omega_0$. This version of the power resonance curve also has its maximum value at ω_0 and it is symmetric about this value. The half power points of the curve occur when $2\Delta\omega = \gamma$, thus $\omega_{\text{fwhm}} = \gamma$ and we can write the expression for the quality factor in terms of the resonance frequency and full width at half maximum of the power resonance curve:

$$Q = \frac{\omega_0}{\gamma} = \frac{\omega_0}{\omega_{\text{fwhm}}}$$

and since $f = \omega/2\pi$ then we can write

$$Q = \frac{\omega_0}{\gamma} = \frac{f_0}{f_{\text{fwhm}}} \, .$$

Thus it does not matter whether the resonance curve is plotted in terms of driving frequency or angular driving frequency, the Q factor can be measured in the same way.

Example: Analogue Radio Receivers

A good analogue radio receiver allows you to listen to a particular radio station (frequency) with minimal interference from other radio stations broadcasting at different frequencies. Thus the Q factor of the oscillator at the heart of the receiver should be very high, so that the power resonance curve is sharply peaked around the desired frequency. Strong signals at frequencies very close to the resonant frequency will be produced, however, any signals at other frequencies will effectively be filtered out. This also means that a good analogue radio receiver can be tricky to tune precisely, as there is only a very narrow range of frequencies over which the desired signal will be heard.

References

1. G.C. King, 2009, "Vibrations and Waves", Manchester Physics Series, John Wiley & Sons Ltd, UK.
