# Forced oscillations

08/02/21

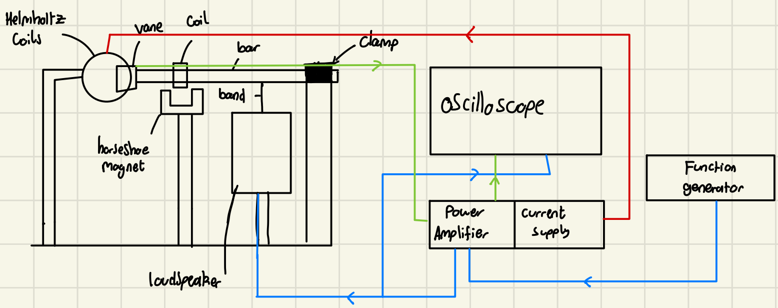
## Aim:

- Study how an oscillating system changes with driving frequency.
- Study how the system behaves without a driving force.

## Objectives:

- make measurements of the amplitude of the oscillation and the phase as a function of driving frequency.
- From this, plot a power resonance curve.
- Study how damping effects free oscillations (no driving force) and forced oscillations.
- Measure the resonant frequency and the quality factor of the system for free oscillations and forced oscillations with damping and no damping.

## Diagram




- Green wires carry current generated in the coil from moving through the magnet to the power amplifier. It is amplified and displayed on oscilloscope

- Signal from function generator is amplified and sent to loudspeaker causing the bar to oscillate. Also displayed on oscilloscope

- Current through the helmholtz coils creates a magnetic field. This induces eddy currents in the vane resulting in a damping force on the bar.

# Forced Oscillations

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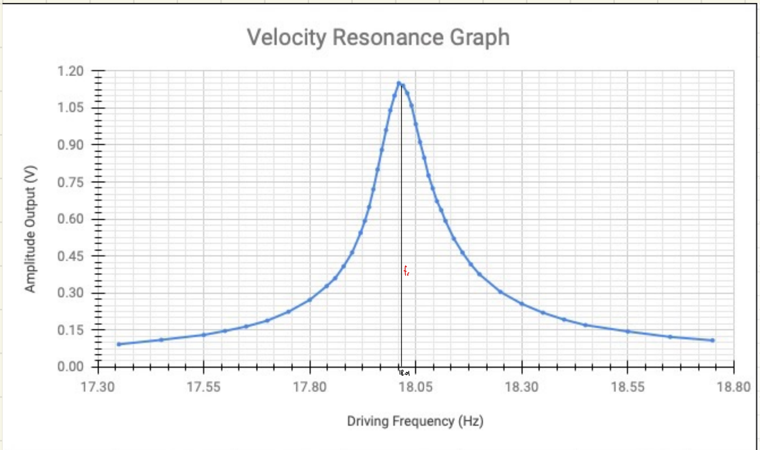
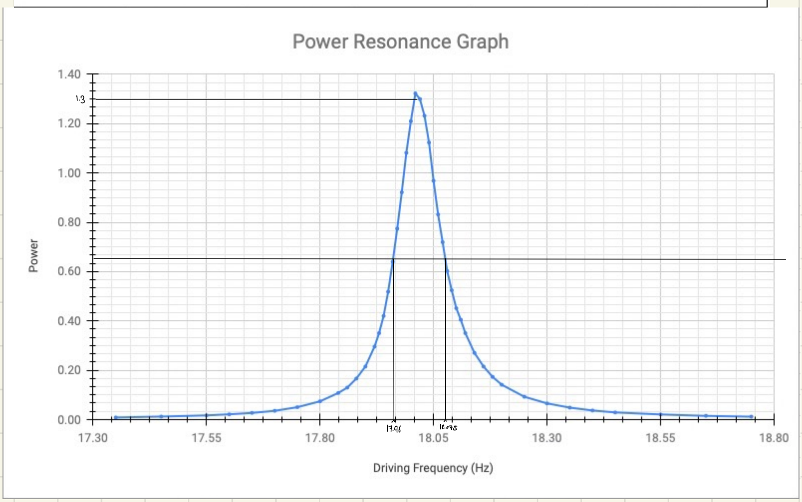
## Forced Oscillation, no damping:




Using a function generator to drive the loudspeaker, we took readings of the amplitude of oscillation and phase angle as a function of driving frequency. With this I plotted amplitude (which is proportional to velocity from Faraday's law) against driving frequency. The amplitude is maximum when the driving frequency is equal to the resonant frequency. Therefore, reading the peak of the graph: $f_r = 18.01\ Hz$

The data is shown on the next page.



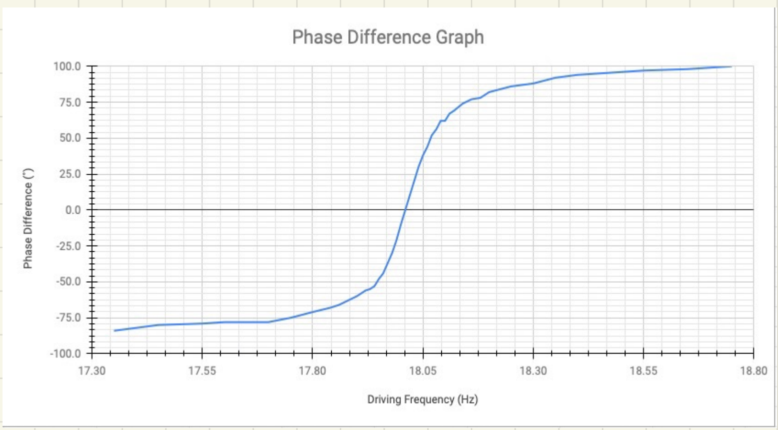

Now a plot of amplitude squared (proportional to power) against driving frequency can be used to find the peak power. Reading this from the graph:

$P_{max} = 1.3$

Halving this value and reading the width of the graph at this point:

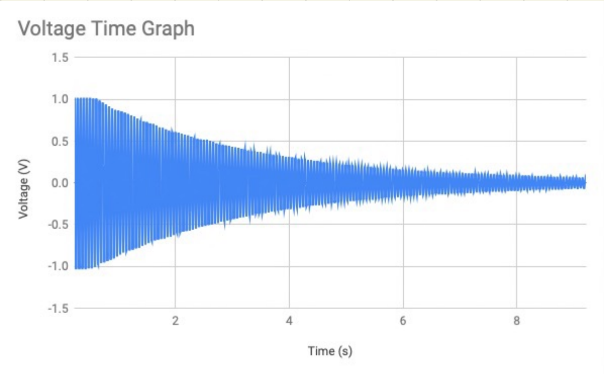
$\frac{P_{max}}{2} = 0.65 \Rightarrow \Delta f = 0.115 = f_{fwhm}$

Then calculating the Q factor of the bar for a forced oscillation with no damping: $Q = \frac{f_0}{f_{fwhm}} = \frac{18.01}{0.115} = 156$ (3sf)

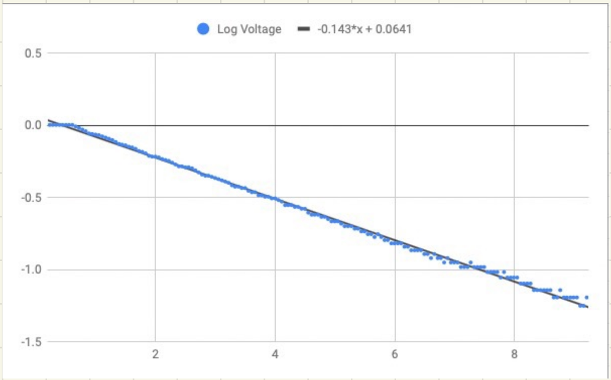


| Frequency of Driving Voltage (Hz) | Amplitude Output Voltage (V) | Phase Difference (˚) |
|---|---|---|
| 17.35 | 0.092 | -84.0 |
| 17.45 | 0.110 | -80.0 |
| 17.55 | 0.130 | -79.0 |
| 17.60 | 0.146 | -78.0 |
| 17.65 | 0.164 | -78.0 |
| 17.70 | 0.188 | -78.0 |
| 17.75 | 0.224 | -75.0 |
| 17.80 | 0.272 | -71.0 |
| 17.84 | 0.328 | -68.0 |
| 17.86 | 0.360 | -66.0 |
| 17.88 | 0.408 | -63.0 |
| 17.90 | 0.464 | -60.0 |
| 17.92 | 0.544 | -56.0 |
| 17.93 | 0.592 | -55.0 |
| 17.94 | 0.648 | -53.0 |
| 17.95 | 0.720 | -48.0 |
| 17.96 | 0.800 | -44.0 |
| 17.97 | 0.880 | -37.0 |
| 17.98 | 0.960 | -30.0 |
| 17.99 | 1.040 | -21.0 |
| 18.00 | 1.100 | -10.0 |
| 18.01 | 1.150 | 0.0 |
| 18.02 | 1.140 | 10.0 |
| 18.03 | 1.110 | 20.0 |
| 18.04 | 1.060 | 30.0 |
| 18.05 | 0.984 | 38.0 |
| 18.06 | 0.912 | 44.0 |
| 18.07 | 0.848 | 52.0 |
| 18.08 | 0.776 | 56.0 |
| 18.09 | 0.724 | 62.0 |
| 18.10 | 0.672 | 62.0 |
| 18.11 | 0.636 | 67.0 |
| 18.12 | 0.592 | 69.0 |
| 18.14 | 0.520 | 74.0 |
| 18.16 | 0.464 | 77.0 |
| 18.18 | 0.416 | 78.0 |
| 18.20 | 0.376 | 82.0 |
| 18.25 | 0.304 | 86.0 |
| 18.30 | 0.256 | 88.0 |
| 18.35 | 0.220 | 92.0 |
| 18.40 | 0.192 | 94.0 |
| 18.45 | 0.170 | 95.0 |
| 18.55 | 0.144 | 97.0 |
| 18.65 | 0.122 | 98.0 |
| 18.75 | 0.108 | 100.0 |

## Free oscillation, no Damping:




Disconnecting the loudspeaker and tapping the bar sends the bar into a free oscillation The graph shown is the decay of the velocity of the bar over time. Extracting the peaks of the graph and taking the log to produce linear data gives the graph below.




The gradient of this graph can be used to find the decay constant.

$m = -0.143 \qquad m = -\frac{\gamma}{2} \Rightarrow \gamma = 0.286$

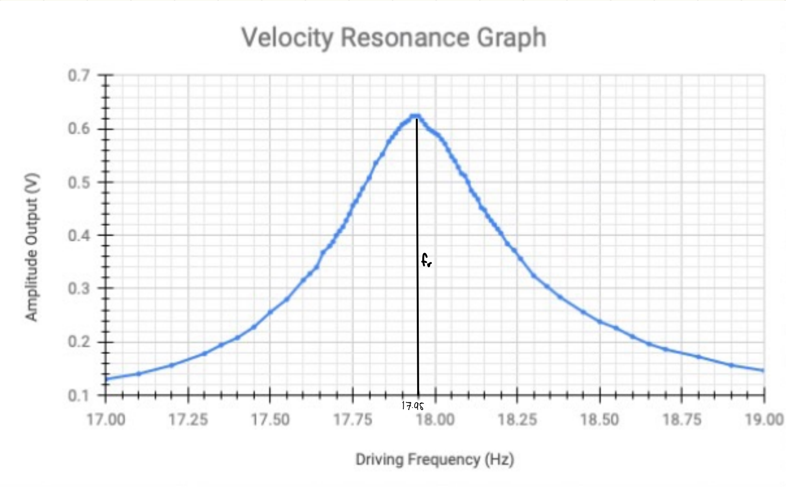
Then using the decay constant to find the Q factor for free oscillation with no damping

$$Q = \frac{\omega_0}{\gamma} = \frac{2\pi f}{\gamma} = \frac{2\pi(18.01)}{0.286} = 396 \ (3sf)$$

The data for this could not be shown as it had 2000 data points

# Forced oscillations, damping:

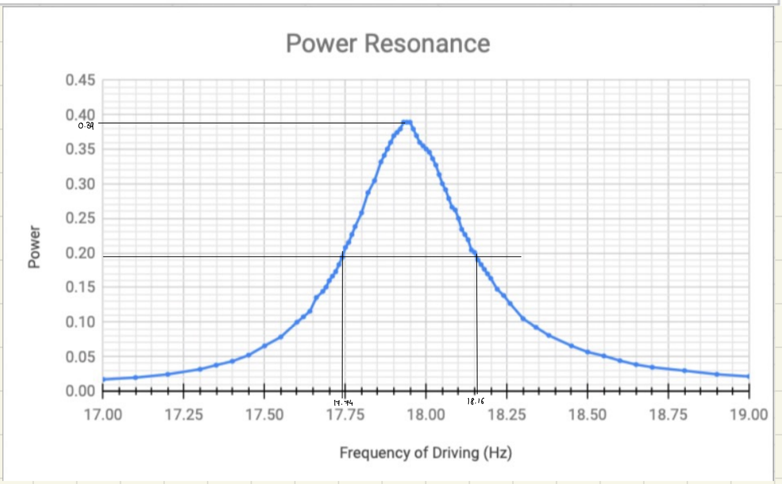
15/02/21




Using the same method as Forced oscillations with no damping but with 272 mA through the Helmholtz Coils to produce damping. Taking the same measurements I plotted velocity against driving frequency. The peak is the new resonant frequency.

$f_r = 17.95\ Hz$

The data is shown on the next page.




Now plotting Amplitude Squared against driving frequency giving a power resonance curve:

$P_{max} = 0.39$

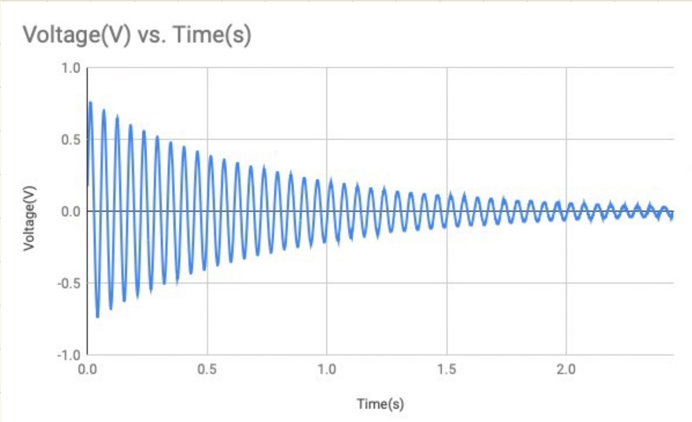
$\frac{P_{max}}{2} = 0.195$

$\Delta f = f_{fwhm} = 0.42$

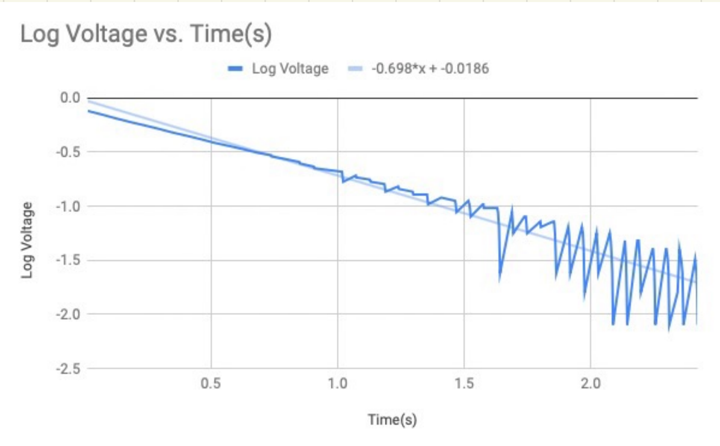
$Q = \frac{f_0}{f_{fwhm}} = \frac{17.95}{0.42} = 42.7$ (3sf)

| Frequency of Driving (Hz) | Voltage (V) | Phase Difference (°) |
|---|---|---|
| 17 | 0.13 | -56 |
| 17.1 | 0.14 | -54 |
| 17.2 | 0.156 | -50 |
| 17.3 | 0.178 | -47.5 |
| 17.35 | 0.194 | -46 |
| 17.4 | 0.208 | -43 |
| 17.45 | 0.228 | -40 |
| 17.5 | 0.256 | -37 |
| 17.55 | 0.28 | -36 |
| 17.6 | 0.316 | -31.5 |
| 17.62 | 0.328 | -30 |
| 17.64 | 0.34 | -28.5 |
| 17.66 | 0.368 | -25.5 |
| 17.68 | 0.38 | -24 |
| 17.69 | 0.388 | -22.5 |
| 17.7 | 0.4 | -21 |
| 17.71 | 0.408 | -20 |
| 17.72 | 0.416 | -17.5 |
| 17.73 | 0.428 | -16.5 |
| 17.74 | 0.44 | -15 |
| 17.75 | 0.456 | -13.5 |
| 17.76 | 0.464 | -11 |
| 17.77 | 0.476 | -9 |
| 17.78 | 0.488 | -8 |
| 17.8 | 0.508 | -5 |
| 17.82 | 0.536 | -0.5 |
| 17.84 | 0.552 | 4 |
| 17.86 | 0.576 | 9.5 |
| 18.12 | 0.476 | 72 |
| 18.13 | 0.468 | 74 |
| 18.14 | 0.452 | 76 |
| 18.15 | 0.448 | 77.5 |
| 18.16 | 0.436 | 79 |
| 18.17 | 0.428 | 80 |
| 18.18 | 0.42 | 82 |
| 18.19 | 0.412 | 83.5 |
| 18.2 | 0.404 | 85 |
| 18.22 | 0.384 | 87 |
| 18.24 | 0.372 | 88.5 |
| 18.26 | 0.356 | 89.5 |
| 18.3 | 0.324 | 94 |
| 18.34 | 0.304 | 97 |
| 18.38 | 0.284 | 100 |
| 18.45 | 0.256 | 103 |
| 18.5 | 0.238 | 106 |
| 18.55 | 0.226 | 107 |
| 18.6 | 0.21 | 110 |
| 18.65 | 0.196 | 111 |
| 18.7 | 0.186 | 112 |
| 18.8 | 0.172 | 114 |
| 18.9 | 0.156 | 117 |
| 19 | 0.146 | 118 |
| 17.87 | 0.584 | 11.5 |
| 17.88 | 0.592 | 14.5 |
| 17.89 | 0.6 | 17 |
| 17.9 | 0.608 | 19.5 |
| 17.91 | 0.612 | 22.5 |
| 17.92 | 0.616 | 25 |
| 17.93 | 0.624 | 29 |
| 17.94 | 0.624 | 32 |
| 17.95 | 0.624 | 35 |
| 17.96 | 0.616 | 37 |
| 17.97 | 0.608 | 41 |
| 17.98 | 0.6 | 43.5 |
| 17.99 | 0.596 | 46 |
| 18 | 0.592 | 47 |
| 18.01 | 0.588 | 49 |
| 18.02 | 0.58 | 52 |
| 18.03 | 0.572 | 55 |
| 18.04 | 0.56 | 57 |
| 18.05 | 0.548 | 59 |
| 18.06 | 0.54 | 61 |
| 18.07 | 0.528 | 63 |
| 18.08 | 0.516 | 65 |
| 18.09 | 0.512 | 66.5 |
| 18.1 | 0.5 | 69 |
| 18.11 | 0.484 | 70.5 |

# Free oscillations, damping:




Setting the bar into a free oscillation but with the Same current through the Helmholtz coils gives this decay graph. Following the Same method as the previous free oscillation gives the graph below.



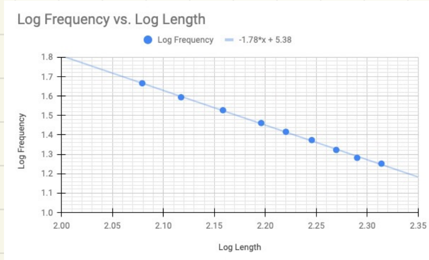

Now using the gradient of the graph to find the decay constant and thus the Q factor:

$m = -0.698 \quad m = -\frac{\gamma}{2} \quad \gamma = 1.396$

$$Q = \frac{\omega}{\gamma} = \frac{2\pi f}{\gamma} = \frac{2\pi(17.95)}{1.396} = 80.8$$

The data for this could not be shown as it had 2000 data points

| Length(mm) | Resonant Frequency (Hz) | Uncertainty on F | Log Length | Log Frequency |
|---|---|---|---|---|
| 120 | 46.3 | 0.1 | 2.079181246 | 1.665580991 |
| 131 | 39.24 | 0.1 | 2.117271296 | 1.593728999 |
| 144 | 33.64 | 0.05 | 2.158362492 | 1.526855987 |
| 157 | 28.9 | 0.05 | 2.195899652 | 1.460897843 |
| 166 | 26.04 | 0.02 | 2.220108088 | 1.41564098 |
| 176 | 23.61 | 0.02 | 2.245512668 | 1.373095987 |
| 186 | 21 | 0.01 | 2.269512944 | 1.322219295 |
| 195 | 19.12 | 0.01 | 2.290034611 | 1.281487888 |
| 206 | 17.85 | 0.01 | 2.31386722 | 1.25163822 |




We also studied how changing a mechanical property (in this case length of the bar). We did this by recording the resonant frequency as a function of length. By taking the log of both of these values and plotting it, we find that the gradient is -1.78. If we sum the uncertainties on f, the total is 0.37. Therefore the range of values for the gradient is $-1.78 \pm 0.37$

-2 is within this range which is the expected value for the power of the length.

Using LSFR, $\chi^2 = 0.97$ at 7 degrees of freedom. This is lower than the critical value of 14.07 so we can claim there is no significant difference between the expected and actual data.

Summarising: Forced oscillations, no damping: $Q = 156$
Free oscillations, no damping: $Q = 396$
Forced oscillation, damping: $Q = 42.7$
Free oscillations, damping: $Q = 80.8$

The Q factor represents how good of an oscillator the system is. A higher Q indicates a slower rate of energy loss; therefore for an undamped oscillator it is expected that the Q factor is higher. This matches the data. For the undamped forced oscillations the Q factor is approximately 3 times larger which matches expectation. Also for the free oscillations the Q factor is roughly 5 times larger for the undamped system, this also fits expectations.

The chi square for the first 4 experiments were very high, this could mean that the linear fit (required for LSFR) is not suitable or the errors were underestimated.

The uncertainties for frequency was negligable, for the voltage 0.01V and for phase difference between 1 and 3 degrees. The uncertainty on phase was due to air currents in the room, unfortunately this could not be prevented. But to improve the experiment these currents can be prevented by closing window and reducing people walking around the lab.

It can also be concluded that the resonant frequency is proportional to the length to the power -2.