The Franck-Hertz Experiment

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Abstract

This experiment investigated the discrete electron energy levels of atoms. Electrons were accelerated through gases of mercury and neon using a varying electric potential towards a collecting anode with a retarding voltage. The current from collected electrons was plotted against the accelerating voltage. From this the excitation energies from the ground state to the 6^3P_1 state for mercury and to the 3^2P_1 state for neon were calculated to be $\Delta E_{Mercury} = 5.0 \pm 0.3$ eV and $\Delta E_{Neon} = 17.45 \pm 0.28$ eV. The temperature dependence of the collected current graph for mercury was also investigated and explained qualitatively.

1. Introduction

In 1913 Ernst Rutherford and Niels Bohr proposed that electrons occupy discrete energy orbitals around the nucleus of an atom [1]. This was subsequently proven for mercury in The Franck-Hertz Experiment in 1914 by Gustav Hertz and James Franck which they proceeded to receive the Nobel prize for in 1925 [2, p. 423]. This was one of the first experiments that gave proof for quantum mechanics. In this modification of the experiment, electrons were accelerated using a varying electric potential through mercury and neon gas towards a collecting anode with a decelerating voltage. These electrons collided inelastically at distinct accelerating voltages which is proves Bohr's atomic model. The difference between these voltages were then used to calculate the excitation energies of transitions in mercury and neon. The temperature dependence of the current from the electrons collected was also investigated and described qualitatively.

2. Theory

A charged particle in a linear electric field produced by two plates experiences a force given by

$$F = \frac{qV}{d}, \tag{1}$$

where q is the charge of the particle, V is the voltage generating the field and d is the distance between the plates. As this is a constant force, the work done by the field on the particle is equal to the kinetic energy of the particle, T, and is given by

$$T(x) = \frac{qVx}{d}, \tag{2}$$

where x is the distance travelled in the electric field. Therefore, the kinetic energy of the particle is a function of distance travelled.

The Bohr model of the atom states that electrons occupy quantised energy levels. Due to this, electrons can only move between orbitals by absorbing or emitting discrete values of energy. Therefore, an electron travelling through an electric field will collide elastically with an atom if it has enough energy to excite the atom to a higher state. If it doesn't have enough energy, it will collide inelastically, maintaining its kinetic energy. Therefore, due to Equation (2), at a fixed voltage the accelerated electrons must travel far enough to have a kinetic energy large enough to excite an atom.

The theoretically expected graph of collected current against accelerating voltage from accelerating electrons in a gas is an increasing periodic function. The collected current increases with voltage until electrons are given enough energy to collide inelastically with the atoms. Above this voltage, electrons lose energy in collisions and do not overcome the retarding potential; therefore, current decreases. As voltage increases further, current increases as electrons have enough energy to collide and overcome the retarding potential. This increases until the next local maximum at which the electrons have enough

energy to inelastically collide twice. Therefore, the difference between the voltages of the peaks, ΔV, is used to calculate the excitation energy by

$$E_g = e\Delta V, \tag{3}$$

where e is the elementary charge.

A Grotrian diagram shows the possible states that an atom can be excited to. An example of this for mercury is shown in Figure 1. Transitions between states with the same parity are forbidden which have been omitted in Figure 1. This explains why the 6^3P_0 state is not excited in this experiment despite a lower excitation energy. When an electron moves down an energy level it emits a photon. The difference in energies between the states, ΔE, is related to the wavelength of photon, λ, by

$$\Delta E = \frac{hc}{\lambda}, \tag{4}$$

where h is Planck's constant and c is the speed of light in a vacuum.

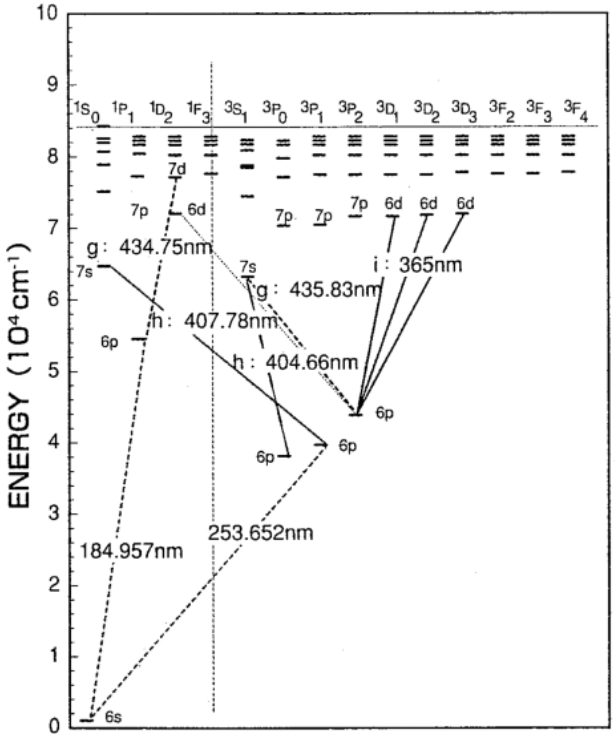

Figure 1: An example of a Grotrian diagram for mercury showing the allowed transitions between atomic energy levels along with the wavelength of photons emitted. [3]

The states in Figure 1 are written in spectroscopic notation, this is of the form $n\ ^{2S+1}L_J$, where n, S and J are quantum numbers that describe energy, total electron spin and total angular momentum respectively. L is a placeholder for the letter representing the total orbital angular momentum, these are (S, P, D, F) for L from 0 to 3 and is alphabetical for higher L.

3. Experimental Approach

Using the setup shown in Figure 2, electrons were emitted from a cathode, C, into the tube. For the mercury experiment, the tube was heated to 150.0 ± 0.1°C to ensure the mercury was in a gaseous state. A current is formed between the cathode and the first grid plate due to a potential difference U_1. The electrons then accelerated between the grid plates due to a varying accelerating potential U_2. Electrons that have enough energy to overcome the retarding potential U_3 reach a detector, this current is then recorded with the corresponding accelerating potential using a data logger. Initially, U_1 and U_3 were set to 1.50 ± 0.01V but were changed to $U_1 = 1.53 \pm 0.01$V and $U_3 = 1.94 \pm 0.01$V. These were random errors, but no effort was made to reduce them as they were not used in calculation. This maximised the number of peaks on the current against voltage graph which meant more readings could be taken, also this increased the heights of the peaks which decreased the errors on reading the excitation energies. This method was repeated 3 times for the same values of U_1 and U_3 in order to increase the number of measurements and calculate a better mean average.

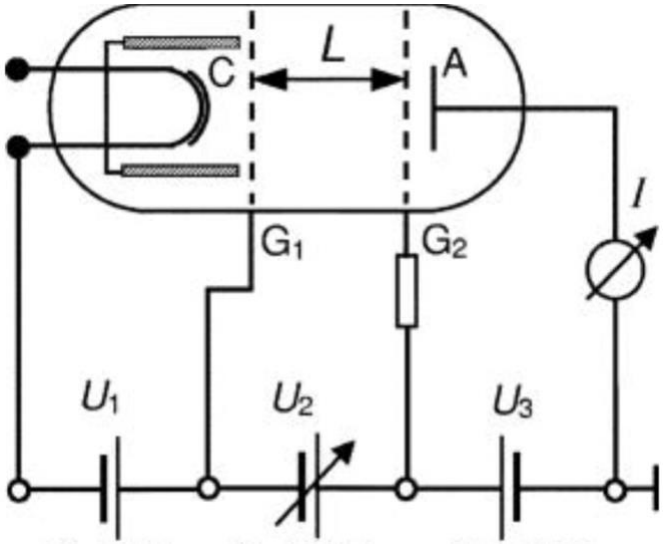

Figure 2: A schematic diagram for the equipment used to accelerate electrons through mercury and neon gas. Where C is the cathode, A is the anode, G_1 and G_2 are grid plates, U_1 is the potential between C and G_1, U_2 is the accelerating voltage between G_1 and G_2 and U_3 is the retarding potential between A and G_2. [2, Fig. 1]

After plotting current against accelerating voltage, the values of voltage corresponding to the maxima were estimated by eye with an error of a fifth of the smallest increment of the scale. The excitation energy corresponded to the difference between the voltages of the peaks, the error of this was calculated by combining the individual voltage errors in quadrature. A mean average of the excitation energy was then calculated from each individual reading to decrease the random error on measurements, the error on this was calculated by adding the individual errors in quadrature and dividing by the number of readings. To decrease the random error on estimating the peaks, the data was sorted into minima and maxima and second order polynomials were fit to the maxima. The voltages corresponding to the peaks were then calculated by differentiating the equation for each maximum and equating to zero. The errors

on the coefficients of the polynomials were given in a covariance matrix from the python module. The error on the voltage of the peak was calculated by summing the fractional errors of the coefficients of the quadratic and linear term in quadrature. The mercury tube was replaced with a tube containing neon gas at room temperature and the same method was repeated. The temperature dependence of the current against voltage curve was investigated by varying the temperature of the mercury sample from 150.0 ± 0.1ºC to 180.0 ± 0.1ºC in 10ºC intervals.

4. Experimental Results

4.1. Mercury Results

An example of an experimental graph for the experiment with mercury is shown in Figure 3. It agrees with the theoretical expectation. By estimating the voltages of the peaks, the average excitation energy from the 3 graphs was calculated to be $\Delta E_{mercury} = 5.0 \pm 0.3$ eV. This was calculated using the mean average of the 12 differences in voltages and Equation (3). Using Equation (4), this corresponds to a photon with wavelength $\lambda_{mercury} = 249 \pm 15$ nm being emitted when the mercury atom de-excites to the ground state. By fitting second order polynomials as described in Section 3, the excitation energy was calculated to be $\Delta E_{mercury} = 5.07 \pm 0.11$ eV. This corresponds to a photon with wavelength of $\lambda_{mercury} = 245 \pm 5$ nm being emitted. The polynomial equations with corresponding voltages of the peaks for the fit functions in Figure 3 are shown in Table 1.

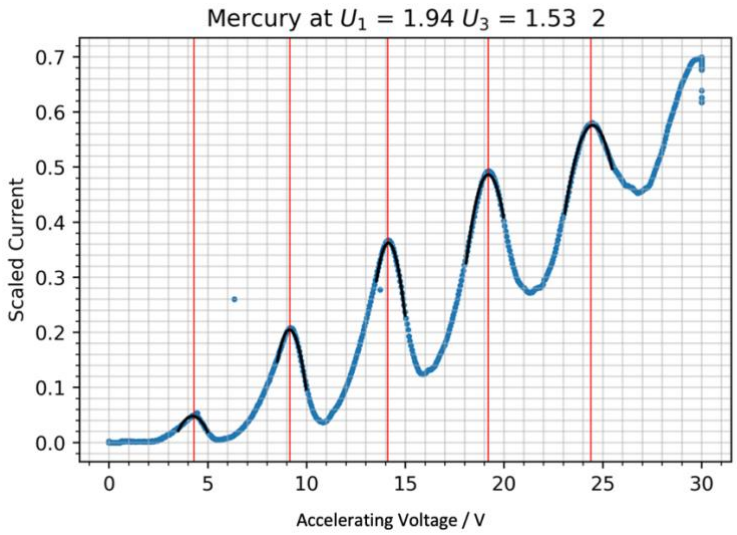

Figure 3: An example of a graph of scaled current against accelerating voltage for mercury at 150 ± 0.1 ºC. Second order polynomials are plotted in black, and the locations of maxima indicated by red lines. Error bars on current are plotted but are too small to be seen.

The true excitation energy from the 6^1P_0 state to the 6^3P_1 state is $\Delta E_{mercury,true} = 4.89 \pm (9 \times 10^{-7})$ eV [4]. This corresponds to a photon with wavelength $\lambda_{mercury} = 255 \pm (5 \times 10^{-5})$ nm being emitted when a mercury atom de-excites from the 6^3P_1 state.

This is the transition that is closest to the experimental results, so it was assumed that this was the observed transition. The wavelength from estimating the peaks by eye is consistent with the true value, but the wavelength found from the polynomial fitting is not. However, the fitting method has a smaller error and can be given to a greater precision. The dominant error in the first method is from estimating the voltages of the peaks, this could be reduced by using a smaller grid spacing. The dominant error from the fitting procedure is from the residuals between the fit and data. This could be reduced by fitting a higher order polynomial to the maxima.

Polynomial	Voltage at Peak (V)
$y_1 = -0.05\,x^2 + 0.42\,x - 0.85$	$V_1 = 4.23 \pm 0.03$
$y_2 = -0.15\,x^2 + 2.77\,x - 12.44$	$V_2 = 9.14 \pm 0.07$
$y_3 = -0.19\,x^2 + 5.27\,x - 36.90$	$V_3 = 14.14 \pm 0.09$
$y_4 = -0.13\,x^2 + 4.90\,x - 46.52$	$V_4 = 19.20 \pm 0.10$
$y_5 = -0.08\,x^2 + 3.93\,x - 47.53$	$V_5 = 24.48 \pm 0.06$

Table 1: A table showing the equations and the corresponding voltages at the peaks for the fitted second order polynomials in Figure 3.

4.2. Neon Results

An example of an experimental graph for neon is shown in Figure 4. Following the same method as mercury, the average excitation energy was calculated to be $\Delta E_{neon} = 17.0 \pm 0.2$ eV. This corresponds to a photon with wavelength $\lambda = 73.1 \pm 0.9$ nm being emitted when the neon atom de-excites to the ground state. As described in section 3, second order polynomials were fitted to the data; however, they were fitted to the minima for neon as the maxima did not represent parabolas. The excitation energy was calculated to be $\Delta E_{neon} = 17.45 \pm 0.28$ eV. This corresponds to a photon with wavelength $\lambda_{neon} = 71.2 \pm 1.1$ nm being emitted. The polynomials and corresponding voltages of the peaks for the fit functions in Figure 4 are shown in Table 2.

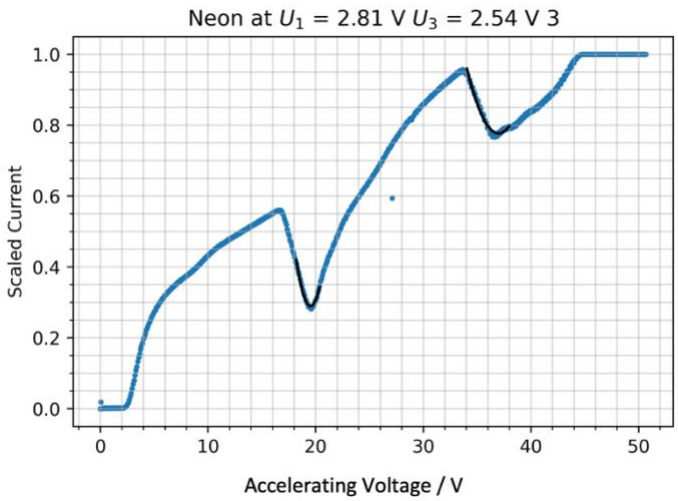

Figure 4: An example of a graph of scaled current against accelerating voltage for neon at 21 ± 0.1 ºC. Second order polynomials are plotted in black. Error bars on current are plotted but are too small to be seen.

The true excitation energy from the 2^1S_0 state to the 3^2P_1 state is $\Delta E_{neon,true} = 18.38 \pm (1.20 \times 10^{-7})$eV [5]. This corresponds to a photon with wavelength $\lambda_{neon} = 67.6 \pm (4 \times 10^{-5})$ nm being emitted when a neon atom de-excites from the 3^2P_1 state. Therefore, neither of the calculated values overlap with the true value. However, despite a large error, the excitation energy calculated from the fitting procedure is closer to the true value. The dominant error for the fitting method originates from the residuals between the data and the fit function. This could be reduced by fitting higher order polynomials to the data as it doesn't resemble a parabola. The dominant error for the estimation by eye method is from reading the graph. This could be reduced by using a smaller grid spacing.

Polynomial	Voltage at Peak (V)
$y_1 = 0.08\,x^2 - 2.95\,x + 29.13$	$V_1 = 19.52 \pm 0.12$
$y_2 = 0.02\,x^2 - 1.56\,x + 29.68$	$V_2 = 36.99 \pm 0.08$

Table 2: A table showing the equations and the corresponding voltages at the peaks for the fitted second order polynomials in Figure 4.

4.1. Mercury Temperature Dependence

A graph showing the temperature dependence of current against accelerating voltage is shown in Figure 5. It is seen that the voltages of the minima and maxima are constant with temperature and thus so is the excitation energy. However, the amplitude of current decreases with temperature. This is explained by a higher temperature corresponding to more random motion in the mercury gas. Therefore, there are more elastic collisions between electrons and atoms so scattered electrons travel a longer path. Thus, the time taken to reach the detector is higher and as current is inversely proportional to time, current in lower.

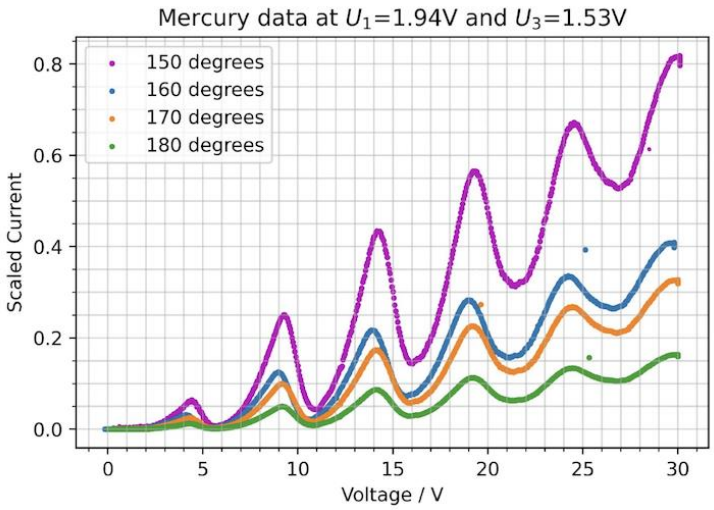

Figure 5: A graph of scaled current against voltage for mercury at a range of temperatures. Errors on scaled current are plotted but are too small to be seen.

5. Conclusion

The excitation energy of mercury from the 6^1P_0 state to the 6^3P_1 state was calculated to be $\Delta E_{mercury} = 5.0 \pm 0.3$ eV. This was calculated by estimating the locations of the peaks by eye; therefore, the dominant error was from reading the graph. The calculated value is consistent with the true value of $\Delta E_{mercury,true} = 4.89 \pm (9 \times 10^{-7})$ eV. The excitation energy for neon from the 2^1S_0 state to the 3^2P_1 state was calculated to be $\Delta E_{neon} = 17.45 \pm 0.28$ eV. This was calculated by fitting second order polynomials to the data; therefore, the dominant error was from the residuals between the data and the fit function. The calculated value is not consistent with the true value of $\Delta E_{neon,true} = 18.38 \pm (1.20 \times 10^{-7})$ eV, but it is closer than the value calculated from estimating the peaks by eye. The temperature dependence graphs agreed with the theoretical expectation.

References

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