Gas Flow

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Abstract

The viscosity of a fluid is a measure of the resistance to movement of neighbouring portions of the fluid relative to each other. In this experiment the viscosities of helium and argon were calculated and compared by measuring the pressure difference of each gas along a long narrow tube. The results for the viscosities were 9.2 ± 1.9 μPa s for helium and 16.2 ± 3.3 μPa s for argon. From this, the radius of each atom was calculated to be 260 ± 26 pm for helium and 348 ± 36 pm for argon.

1. Introduction

The viscosity of a fluid is a measure of the resistance to movement of neighbouring portions of the fluid relative to each other [1]. A fluid with a large viscosity will resist motion more than a fluid with a small viscosity. The concept of viscosity was first theorised by Newton in the 17th century [2] and was later amended by Poiseuille and Hagen independently in the 19th century [3]. In this experiment, the difference in pressure of gases across a narrow tube, a macroscopic quantity, was used to estimate the radius, a microscopic quantity, of two ideal gases. This was achieved by plotting a function of the flow rate against time. Using the gradient of this plot and the Hagen-Poiseuille equation, the co-efficient of viscosity for helium and argon were calculated and compared. These gases were used as they are monoatomic and inert, so they are assumed to behave as an ideal gas. The mean free path, collision cross section, mean speed of the atoms and radius were then calculated.

2. Theory

The derivation in this section follows the Gas Flow lab pre-lab [4]. When a fluid is in laminar flow the velocity is constant along the fluid, but neighbouring layers move at different velocities. Due to this, there is a shearing force between the layers acting as a frictional force. This is quantified by the co-efficient of viscosity, η. The flow rate of a fluid, Q, in laminar flow along a narrow tube is

$$Q = \frac{\pi a^4 (p_{in}^2(t) - p_{out}^2)}{16\eta l}, \tag{1}$$

where a is the radius of a tube, $p_{in}(t)$ is the pressure at the start of the tube, t is the time, p_{out} is the pressure at the other end of the tube and l is the length of the tube. This is the Hagen-Poiseuille equation. The flow rate has units of $\mathrm{m^3\ Pa\ s^{-1}}$ so is also written as

$$Q = -V_{in}\frac{dp_{in}}{dt}, \tag{2}$$

where V_{in} is the volume of the system before the tube. Therefore, equating and rearranging Equations (1) and (2) gives

$$\frac{dp_{in}}{dt} = -\frac{\pi a^4 (p_{in}^2(t) - p_{out}^2}{16\eta l V_{in}}. \tag{3}$$

By keeping p_{out} constant, Equation (3) then integrates to

$$\ln\left(\frac{p_{in}(t) - p_{out}}{p_{in}(t) + p_{out}}\right) = -\frac{\pi a^4 p_{out} t}{8\eta l V_{in}} + c. \tag{4}$$

Therefore, a plot of $\ln\left(\frac{\mathrm{p_{in}(t)} - \mathrm{p_{out}}}{\mathrm{p_{in}(t)} + \mathrm{p_{out}}}\right)$ against time will have a gradient, m, of

$$m = -\frac{\pi a^4 p_{out}}{8 l V_{in}\eta}, \tag{5}$$

which is rearranged to calculate the viscosity. The mean speed of the atoms, $\bar{c}$, is defined as

$$\bar{c} = \sqrt{\frac{8kT}{\pi M}}, \tag{6}$$

where k is the Boltzmann constant, T is the temperature and M is the atomic mass. This is used in calculating the mean free path of the atom, λ, by using

$$\lambda = \frac{3\eta}{n_v M \bar{c}}, \tag{7}$$

where n_v is the number density and is defined using kinetic theory as

$$n_v = \frac{P_1 N_a}{RT}, \tag{8}$$

where N_a is Avogadro's constant and R is the molar gas constant. The cross section of the collision, σ, is

$$\sigma = \frac{1}{\sqrt{2} n_v \lambda}. \tag{9}$$

Then the radius of the atom, r, is approximately equal to

$$r = \frac{1}{2}\sqrt{\frac{\sigma}{\pi}}. \tag{10}$$

3. Experimental method

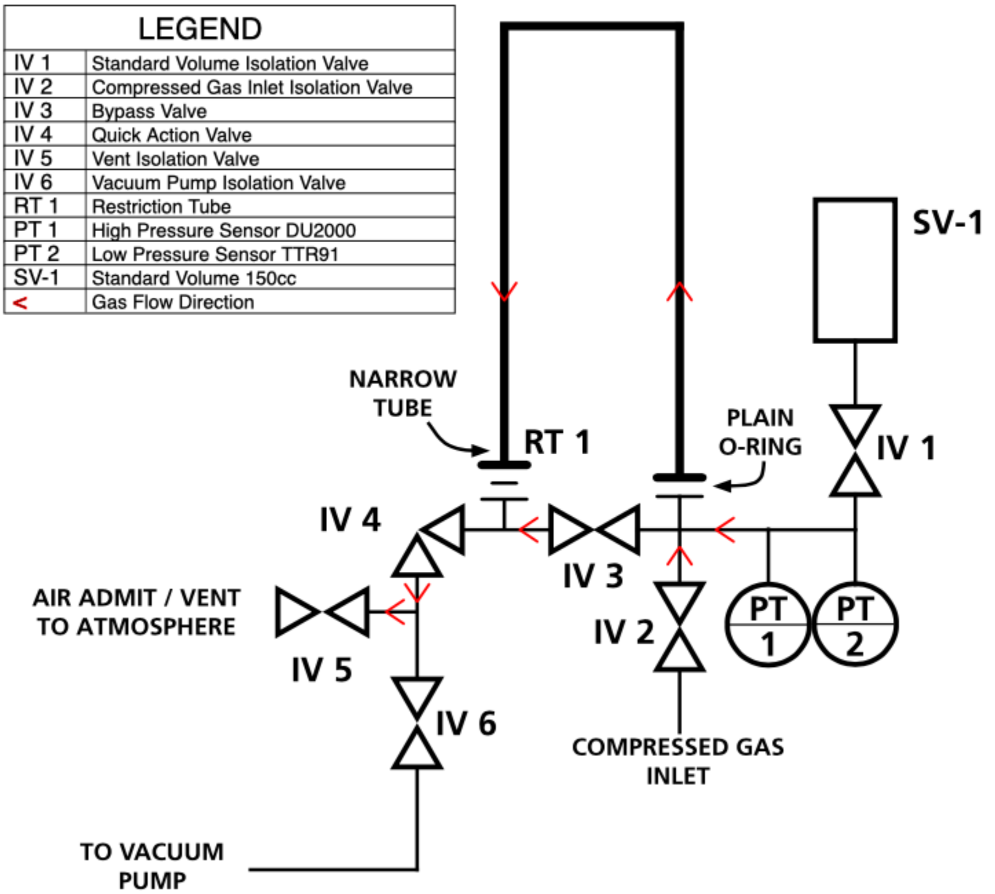

Figure 1. A diagram of equipment used which shows the series of pipes, valves and pressure sensors used to control and measure the flow of the gas. A legend is given to label key pieces of equipment. Reproduced with permission from the Gas Flow Pre-Lab.

Using the setup shown in Figure 1, the system was evacuated to below 0.1 mbar, measured with a low-pressure sensor, using a vacuum pump and opening IV3, IV4 and IV6. The right side of the system was then isolated by closing IV3 and filled with helium to a pressure of 2000 mbar, measured using a high-pressure sensor. The left side of the system was set to atmospheric pressure by opening IV5. This pressure was calculated by measuring the average pressure outside the system from 10 readings. Each individual measurement had an uncertainty of 0.25% given by the pressure sensor. Every measurement of pressure for the rest of the experiment also had this uncertainty. The atmospheric pressure was calculated to be 1033 ± 8.94 mbar. A narrow tube with radius $a = 0.1 \pm 0.005$ mm and length $l = 30 \pm 0.5$ mm connected the two sides of the system. Helium flowed through this tube due to a pressure difference caused by opening IV4. The pressure was recorded in increments of 10 seconds using the high-pressure sensor until the system was in equilibrium with atmospheric pressure. This was repeated 3 times to increase the reliability of the experiment as the readings rely on reaction time. Then, $\ln(\frac{p_{in} - p_{out}}{p_{in} + p_{out}})$ was calculated for each set of data and plotted against time. Using the 3 plots, an average gradient was calculated and used in Equation (5) to calculate the viscosity. The system was then evacuated again, ensuring all valves were open apart from IV1 and IV5, to remove any excess helium. The same procedure was repeated using argon. This was performed at a different time and the atmospheric pressure was measured to be 1031 ± 9.98 mbar using the same process as before. The pressure, p_1, of the standard volume canister, volume $V_1 = 1.5 \times 10^{-4}$ m^3, was measured to be 2022 ± 20.2 mbar. This was released into the right side of the system with new volume $V_2 = V_{in} + V_1$. The new pressure, p_2, was measured to be 435 ± 2.9 mbar. As $P_1V_1 = P_2V_2$, V_{in} is calculated using

$$V_{in} = \frac{p_1V_1}{p_2} - V_1, \tag{11}$$

to give 547 ± 7 cm^3 which was used in Equation (5). The uncertainty was calculated by combining the fractional errors in quadrature. The temperature was measured to be 292.4 ± 0.2 K for use in Equation (7).

4. Experimental results

The data sets were plotted against time using LSFR.py. The start of this data was not linear due to leaking at the start of the flow, so the linear segment was extracted and was used in the analysis. Examples of this is are shown in Figures 1 and 2 for helium and argon respectively.

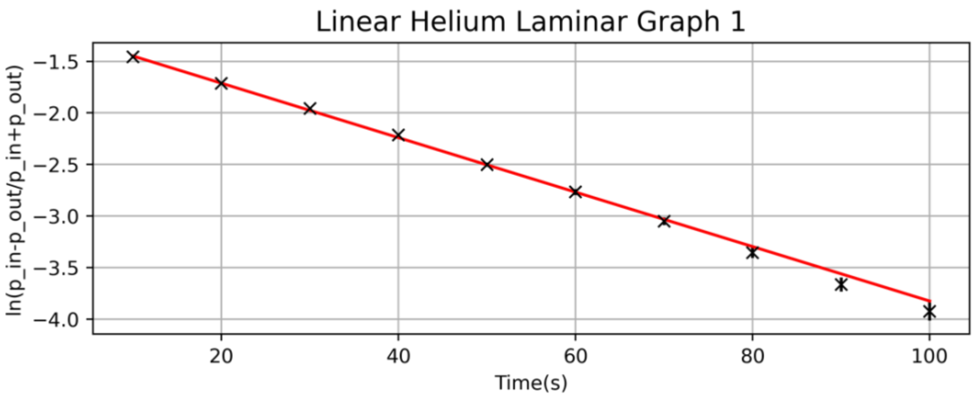

Figure 2. An example of a $\ln(\frac{p_{in} - p_{out}}{p_{in} + p_{out}})$ against time/s plot for helium in laminar flow through a narrow tube. The least squares fit line has $\chi^2 = 10.69$, $\chi^2_{reduced} = 1.34$, a gradient of $m = -0.0264 \pm 0.0003$ s^{-1} and an intercept of $c = -1.18 \pm 0.01$. The error bars are too small to be seen.

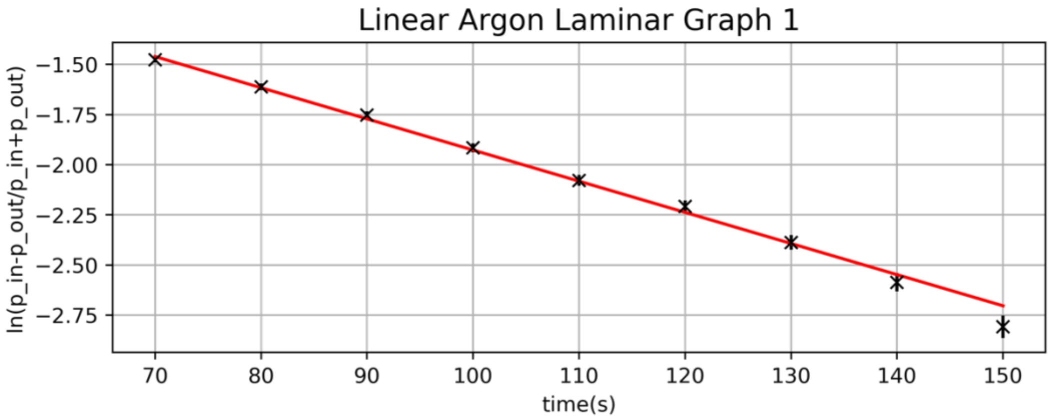

Figure 3. An example of a $\ln(\frac{p_{in} - p_{out}}{p_{in} + p_{out}})$ against time/s plot for argon in laminar flow through a narrow tube. The least squares fit line has $\chi^2 = 7.55$, $\chi^2_{reduced} = 1.08$, a gradient of $m = -0.0155 \pm 0.0004$ s^{-1} and an intercept of $c = -0.37 \pm 0.03$. The error bars are too small to be seen.

This experiment was repeated 3 times for each gas. The intercept is stated for each graph but was not used in any calculation for this experiment. The average gradients were -0.0269 ± 0.0007 s^{-1} for helium and -0.0152 ± 0.0006 s^{-1} for argon. The errors on the individual gradients were added in quadrature to find an absolute uncertainty for the average. The average gradient was then used in Equation (5) to calculate the viscosity to be 9.19 ± 1.87 μPa s for helium and 16.2 ± 3.3 μPa s for argon. The ratio between these was calculated to be 0.57 ± 0.16. The uncertainties were calculated by combining the individual fractional errors in quadrature. The largest contributor to the absolute uncertainties was the error on the radius of the tube at 20% as this was raised to the fourth power in Equation (5). Although the linear region was estimated by eye, the reduced chi squared was 1.34. This indicated the fit was suitable as it lies in the range of 0.5 to 2. Also, the reduced chi squared for each individual graph fell into this range. The calculated values of viscosity are not consistent with the true values which are 19.9 ± 0.6 μPa s for helium and 22.7 ± 0.7 μPa s for argon [5; p229]. The ratio of these is 0.88 ± 0.04 which is also not consistent. These values of viscosity are for 300 K which is higher than the temperature measured. However, this difference would not be measurable with the precision of this experiment. Using Equations (6), (7), (8), (9) and (10) the radii of the atoms were calculated to be 130 ± 13 pm for helium and 174 ± 18 pm for argon with a ratio of 0.747 ± 0.107. The uncertainties were calculated by adding the individual errors in quadrature and when a quantity was raised to a power the fractional uncertainty was multiplied by the modulus of the power. The highest contributor to the absolute uncertainty was the error on the radius of the tube. These were compared to the Van Der Waals radius of each atom which is 140 pm for helium and 188 pm for argon [6] which is consistent with the calculated values. The ratio between the Van Der Waals radii is 0.745 which is also consistent with results. The calculated value for helium is closer to the true value than argon. This is due to helium behaving more like an ideal gas than argon as it has a smaller radius.

5. Conclusion

The final calculated value for the viscosities were 9.19 ± 1.87 μPa s for helium and 16.2 ± 3.3 μPa s for argon. This was not consistent with the true values which are 19.9 ± 0.6 μPa s for helium and 22.7 ± 0.7 μPa s for argon. However, this is consistent with theory as it is expected that a larger atom has a higher viscosity. From this, the calculated values of the radii were 130 ± 13 pm for helium and 174 ±18 pm for argon. This was consistent with the Van Der Waals radii which are 140 pm for helium and 188 pm for argon. This is also consistent with theory as helium has a lower atomic number than argon, so the radius is expected to be smaller. It was also expected that the radius of helium would be calculated closer to the true value. This is due to helium being more ideal than argon and an underlying assumption of the experiment was that the gases were ideal. The largest contributor to the uncertainties on the calculated viscosities and radii was the error on the radius of the narrow tube which was 20% due to the fourth power in Equation (5).

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