FIRST YEAR LABORATORY

GAS FLOW THROUGH NARROW TUBES

1 Aims

1. To determine and compare the viscosity of two ideal gases (helium and argon) by measuring their flow rates.
2. To observe, compare and understand laminar, molecular and turbulent flow on both a micro and macroscopic basis.

2 Objectives

1. To measure the rate of flow of helium and argon through narrow metal tubes from a constant volume to a constant pressure environment.
2. To obtain a measurement of the viscosity of each gas and from these obtain their respective mean free paths and collision cross-sections.
3. To become familiar with automated data logging.

3 Introduction

The properties of materials - gases, liquids and solids - are determined by the behaviour of their constituent atoms. In this experiment the rates of flow of two monatomic and inert gases, argon and helium, will be measured. The rate at which gas flows is a *macroscopic* (i.e. large-scale) quantity which can easily be measured in the laboratory by observing pressure changes in volumes large enough to contain an enormous number of atoms. These *macroscopic* quantities are related to *microscopic* properties of the gas atoms, the speed of individual atoms and the effect of the collisions they make with other atoms and with the walls containing them.

There are several different regimes which may occur in the flow of gas though a tube:

- In *laminar* flow the gas moves smoothly and the rate of flow is determined by the *viscosity* of the gas. In Experiment 1 you will determine the viscosities of argon and helium.

- If the pressure difference across the ends of the tube is too large, eddies form which reduce the flow rate - the flow is then described as *turbulent*. In Experiment 1 you will find the point at which the flow becomes turbulent for each gas.

- At very low pressures (Experiment 2) the atoms are more likely to collide with the walls than with each other. In this regime - called *molecular flow* - the flow rate is completely determined by the speed of the atoms and the geometry of the tube.

This experiment is either a two week or three week experiment. For students doing the two week experiment you are expected to do the laminar flow part of Experiment 1 and molecular flow (Experiment 2). For students doing the three week experiment you are expected to complete all parts of Experiments 1 and 2.

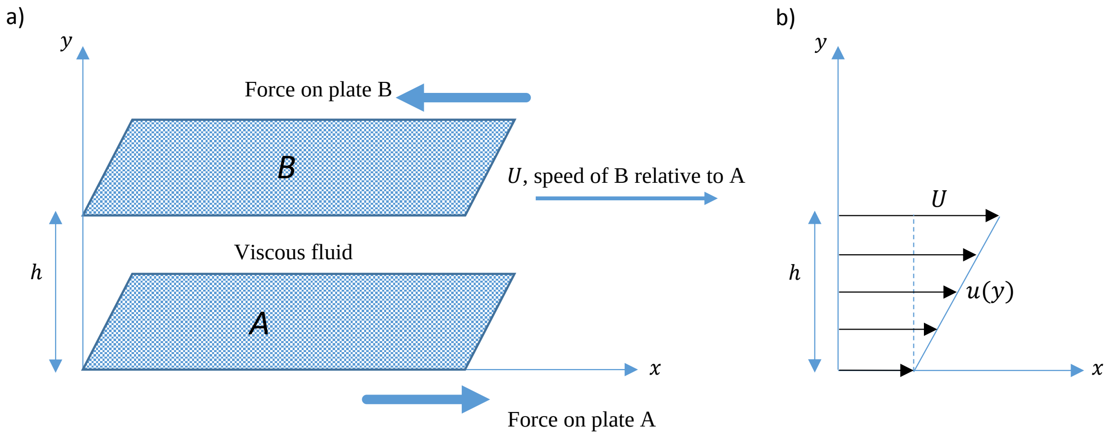

Figure 1: (a) Illustration of the relative direction of the motion and forces due to viscosity. (b) Illustration of the velocity gradient, $\frac{du}{dy}$, which is given by $\frac{U}{y}$.

3.1 Viscosity

The forces due to viscosity are illustrated in Figure 1. Plate B (at $y = h$) is moving in the x-direction at speed U relative to the plate A (at $y = 0$). The force (in the x-direction) on plate A (is equal and opposite to the force on plate B) and is proportional to the area of the plates, to their relative speeds and inversely to their separation. Accordingly, viscosity, η (the Greek letter, *eta*.), is defined by the relationship:

$$F_\mathrm{a} = \eta \frac{\mathrm{d}u}{\mathrm{d}y}, \tag{1}$$

where F_a is the force per unit area and $\frac{\mathrm{d}u}{\mathrm{d}y}$ is the velocity gradient [2] . Viscosity, (technically the dynamic viscosity), has units kg m^{-1} s^{-1} and the SI unit is called a Poisseuile (Pl), but often it is written as Pa s. Thick liquids such as treacle have a high viscosity whereas runny liquids such as water have a lower viscosity. (N.B. the SI unit of pressure is the Pascal and 1 Pa $\equiv$ 1 N m^{-2} $\equiv$ 0.01 mbar.)

4 Apparatus

Two narrow tubes are provided for the gas to flow through. The long one, suitable for studying laminar and turbulent flow, (Experiment 1), is 30.0 $\pm$ 0.5 mm long (check and measure more accurately if needed). According to the manufacturer, the radius is 0.100 $\pm$ 0.005 mm (again check and measure more accurately if needed). The shorter one, suitable for studying molecular flow, (Experiment 2), is 10.0 $\pm$ 0.5 mm long and has the same radius.

Low pressures are achieved in the apparatus by using a rotary pump; you don't need to know the details of how this pump works. There are two pressure gauges in the apparatus, both are digital and are readout by the PC. PT 1 (DU2000) covers the range from 1 mb, through 1000 mb (i.e. atmospheric pressure) and up to about 2000 mb and is relevant for laminar (and turbulent) flow. It estimates pressure by measuring the displacement of a diaphragm. The calibration is independent of the gas. In the range 1000 to 2000 mb PT 1 readings are accurate to 0.25%. PT 2 (TTR91), relevant for molecular flow, covers pressures from 5 $\times$ 10^{-4} to 100 mb. It measures the thermal conductivity of the gas and

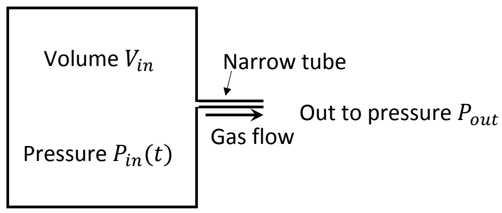

Figure 2: Illustration of the volume and narrow tube.

therefore the calibration is dependent on the gas. This needs correcting, see Appendix A2. In the range 10^{-3} to 100 mb PT 2 readings are accurate to 15%.

Figure 2 shows a schematic diagram of the volume and the narrow tube and Figure 3 shows a diagram of the equipment: the layout of the pipes and valves and the position of the narrow tube.

5 Experiment 1

5.1 Laminar flow theory

In *laminar* flow (also called *streamline* flow) the gas is in a steady state, moving smoothly through the tube. At any point in the tube the velocity of the gas remains constant, although neighbouring layers of gas are moving at different speeds, giving the velocity gradient, du/dy. The differing speeds cause a shearing force in the gas which is described by the viscosity, η. According to kinetic theory, the viscosity, depends on the temperature of the gas but not on its pressure. The rate of flow of gas through a long cylindrical tube with one end at a pressure $p_{\text{in}}(t)$ and the other at a lower pressure p_{out} is

$$Q = \frac{\pi a^4 (p_{\text{in}}^2(t) - p_{\text{out}}^2)}{16\eta l}, \tag{2}$$

where a is the radius of the tube and l its length. This equation, which is known as Poiseuille's equation, is derived in Appendix A1.
The units of flow rate in Equation (2) are [volume per second times pressure]. As the gas moves along the tube, the volume (of a small element of gas) increases, but the pressure falls, so that the flow, Q, (which is proportional to the mass flow) is constant.
The rate of fall of pressure in the volume V_{in} at the high pressure end of the tube is just

$$Q = -V_{\text{in}} \frac{dp_{\text{in}}(t)}{dt},$$

or

$$\frac{dp_{\text{in}}(t)}{dt} = -\frac{\pi a^4 (p_{\text{in}}^2(t) - p_{\text{out}}^2)}{16\eta l V_{\text{in}}}.$$

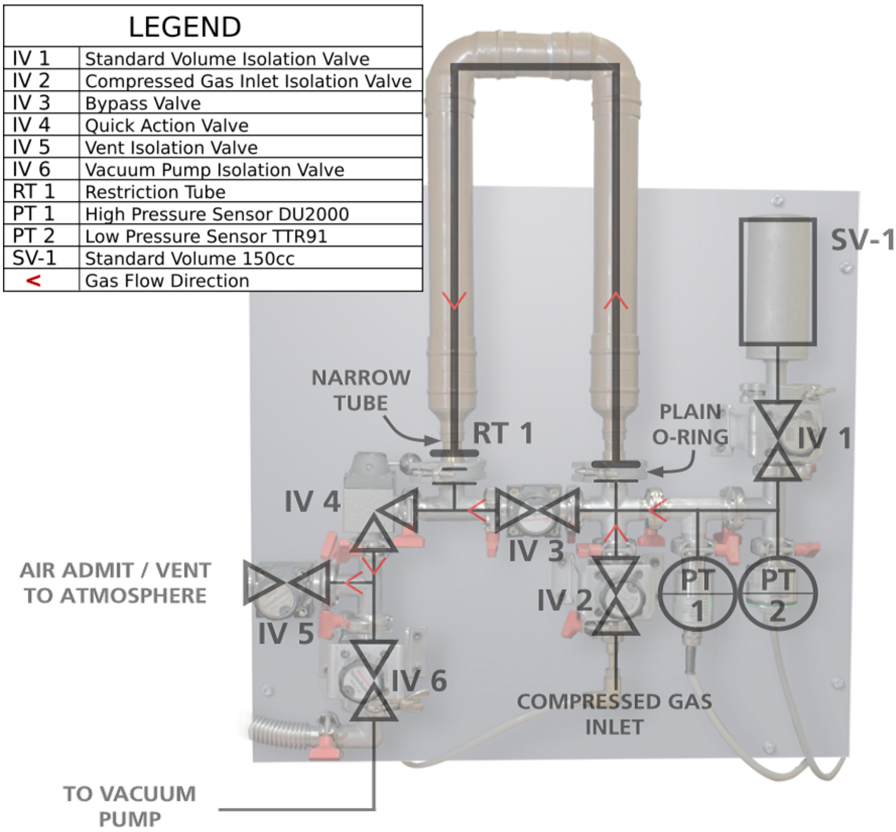

Figure 3: Diagram of experimental setup showing pipes and valves. IV 3 is the by-pass valve. PT 1 and PT 2 are the pressure gauges which are read out by the PC for automatic data recording.

Hence

$$\int \frac{dp}{(p_{\text{in}}^2(t) - p_{\text{out}}^2)} = -\frac{\pi a^4}{16\eta l V_{\text{in}}} \int dt.$$

Treating p_{out} as constant, this equation integrates to give

$$\ln\left(\frac{p_{\text{in}}(t) - p_{\text{out}}}{p_{\text{in}}(t) + p_{\text{out}}}\right) = -\frac{\pi a^4 p_{\text{out}} t}{8\eta l V_{\text{in}}} + C. \quad (3)$$

5.2 Experimental procedure

To measure the viscosity of argon, first evacuate the apparatus and then fill the volume of the pipes, V_{in}, with argon to a pressure of about 2000 mbar as measured by the digital pressure gauge. Use the valves so that the low pressure side of the narrow tube is at atmospheric pressure, p_{out}. The pressure, $p_{\text{in}}(t)$, measured on the digital gauge falls as gas flows out through the narrow tube. Use the PC to record the pressure, $p_{\text{in}}(t)$, as a function of time, until atmospheric pressure is almost reached. You need to choose a sensible time interval to save the readings. Finally, remember to open the by-pass valve to equalise the pressures on the ends of the tube, in order to measure p_{out}. Plot a graph of $p_{\text{in}}(t)$ and of $\ln\frac{(p_{\text{in}}(t)-p_{\text{out}})}{(p_{\text{in}}(t)+p_{\text{out}})}$ against time. Use the standard volume to devise a means of deducing the volume, V_{in}. From the slope of the linear portion of the graph and Equation

(3) determine the viscosity of argon. Estimate the uncertainty in this result remembering to include the uncertainties in all the relevant quantities.

Repeat this entire procedure for helium. See refs [3, 4, 5] for typical viscosity values.

5.3 Kinetic theory

In the kinetic theory of gases, the viscosity of a gas is given in terms of its microscopic parameters, see ref [1], by the expressions

$$\eta = \frac{1}{3} n_v m \lambda \bar{c}, \tag{4}$$

where n_v is the number density, m the mass of an atom, λ the mean free path. The mean speed of the atoms, $\bar{c}$, depends on the mass and temperature and is given by :

$$\bar{c} = \sqrt{\frac{8kT}{\pi m}}. \tag{5}$$

The mean free path is related to the collision cross-section, σ, by:

$$\lambda = \frac{1}{\sqrt{2}} \frac{1}{n_v \sigma}. \tag{6}$$

Finally, the collision cross-section (for two atoms to collide) is given approximately by

$$\sigma = \pi d^2,$$

where d is the diameter of an atom.
Equations (4) , (5) and (6) are equivalent to equations (6.20), (5.10) and (6.17a) in ref [1], a book available from the technician or from the School Library. Read the sections of the book leading to these equations.

5.3.1 Calculations

From your experimentally measured viscosity of argon, estimate the mean free path of the argon atoms, their collision cross-section and their diameter. Of course, assign uncertainties to all these quantities.

Repeat the experiment for helium.

Evaluate also the ratio of the viscosities, $\frac{\eta_{\mathrm{He}}}{\eta_{\mathrm{Ar}}}$, and the ratio of the atomic diameters, $\frac{d_{\mathrm{He}}}{d_{\mathrm{Ar}}}$. These ratios are pure numbers and do not depend for instance on the length and radius of the tube. Assign uncertainties.

5.4 Turbulence

When the gas flow through the tube is very high, laminar flow cannot be sustained, and turbulent motion occurs. Eddies swirl about in the tube, and the gas is no longer moving steadily parallel to the tube's axis. Turbulence is an example of chaos, in the sense that it is not possible to predict the distribution of gas velocities at any time. However, the onset of turbulence is fairly well defined, and is determined by a dimensionless quantity called the Reynolds number. This number is named after Osborne Reynolds, a professor of engineering at Manchester University who was one of the first people to study turbulence. The Reynolds number, R, for flow in a cylindrical tube of radius a is

$$R = 2U\rho a/\eta,$$

where ρ is the density of the gas and U is the bulk speed of the gas (not equal to the thermal mean speed of individual atoms, $\bar{c}$).
Turbulence reduces the rate of flow compared with laminar flow, so at the onset of turbulence there is a change in the slope of the graph of Equation (3). Calculate the Reynolds number corresponding to the start and end of your data and, if the data indicates a change in slope, the Reynolds number at the transition.

6 Experiment 2

6.1 Molecular flow theory

When the pressure in the tube is sufficiently low, the mean free path of the gas atoms or molecules is large compared with the diameter of the tube. This is the regime called molecular flow. Collisions are then almost all with the walls of the tube rather than with other atoms or molecules. Once a gas atom has entered the tube, the probability that it will pass through and leave at the far end is therefore determined by the geometry of the tube. Assuming that after each collision with the wall a gas atom leaves the surface at an angle uncorrelated with its direction of arrival, this probability is; $\xi = 8a/3l$ for a cylindrical tube of radius a and length l (ξ is a Greek letter, called *xi*.). Assuming that the pressure at the far end of the tube is zero, the flow rate though the tube is the rate at which molecules arrive at the near end, times the probability, ξ . The rate of arrival at a tube of area A is $\frac{1}{4}n_v\bar{c}A$, see ref [1] equation (6.18), and the number of atoms passing through the tube per second is $\frac{1}{4}n_v\bar{c}A\xi$. Using the ideal gas equation, the throughput (as previously this is [pressure $\times$ volume per second]) is therefore:

$$Q = \frac{1}{4}\xi\pi a^2\bar{c}p_{\text{in}}(t).$$

Substituting for $\bar{c}$ and ξ gives

$$Q = Sp_{\text{in}}(t),$$

where S is called the *pumping speed* and is given by

$$S = \frac{2\pi a^3}{3l}\sqrt{\frac{8kT}{\pi m}}. \tag{7}$$

Even after pumping for a very long time, the pressure in the volume V_{in} does not fall to zero, but approaches a limiting residual pressure, p_{r}, maintained by small leaks and by outgassing from the surface of the apparatus which contribute a flow, Sp_{r}, at all times. Including these effects, the throughput is given by

$$Q = Sp_{\text{in}}(t) = -V_{\text{in}}\frac{dp_{\text{in}}(t)}{dt} + Sp_{\text{r}}. \tag{8}$$

Integrating this equation leads to

$$\ln(p_{\text{in}}(t) - p_{\text{r}}) = -\frac{St}{V_{\text{in}}} + C. \tag{9}$$

6.2 Experimental procedure

Investigate the molecular flow regime using the short tube. As before, pump out the volume, V_{in}, but fill it (with argon or helium) to a pressure of about 5-8 mbar. Use the valves to connect the output end of the narrow tube to the vacuum pump, so that the pressure is less than 10^{-1} mb. Use the PC to record the pressure as a function of time and plot the data according to Equation (9). You need to choose a sensible time interval to save the readings.
Estimate (using the equations for the mean free path given in Experiment 1) the pressure below which the flow is expected to be molecular.
Determine the pumping speeds for argon and helium. Estimate uncertainties. Compare with the values given by Equation (7) and consider the ratio, which from Equation (7) should be:

$$\frac{S_{\text{He}}}{S_{\text{Ar}}} = \sqrt{\frac{m_{\text{Ar}}}{m_{\text{He}}}}.$$

Appendix A

A1 Poiseuille's formula for laminar flow through a cylindrical tube

The arrows in Figure 4 represent the gas velocity at different distances from the axis of the tube. The velocity is assumed to be zero for the gas in contact with the walls. This is a reasonable assumption, because atoms which collide with the walls usually stick to the surface for some time, and then leave at a random angle with respect to the axis.

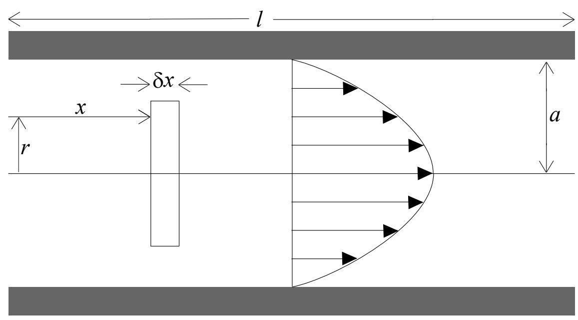

Figure 4: Schematic of laminar flow.

Viscous forces act where there is a velocity gradient in the gas. The force per unit area in the x-direction is ,

$$F_{\text{ax}} = -\eta \frac{\mathrm{d}u_x}{\mathrm{d}r},$$

where $\frac{\mathrm{d}u_{\text{x}}}{\mathrm{d}r}$ is the velocity gradient and η is the viscosity, see ref [1] section (6.1.5). The minus sign shows that the viscous force is tending to reduce the velocity gradient, which can only be maintained by an external force. For the gas flow through a tube, the external force is provided by a pressure difference between the ends of the tube. Lets call the pressures p_{in} and p_{out} at the high and low pressure ends of the tube respectively.
The total force on the cylinder of gas of radius r and length δx is

$$\pi r^2 \frac{dp}{dx} \delta x,$$
this is balanced by the viscous force on the cylindrical area $2\pi r \delta x$, i.e.

$$\pi r^2 p'(x) \delta x = -2\pi r \delta \eta \frac{du_x}{dr},$$
where $p'(x) = dp/dx$ is the rate of change of pressure along the tube.
Integrating and requiring $u_x(r,x)$ to be zero on the tube walls at $r = a$,

$$u_x(r,x) = -\frac{(a^2 - r^2)p'(x)}{4\eta}. \tag{10}$$
The speed of the gas varies quadratically across the tube, with the maximum speed naturally being in the middle at $r = 0$.
As the gas moves along the tube it expands, and since the mass flow is constant the volume and hence the speed u_x of the gas increases along the tube. In a narrow tube the gas remains at the same temperature as the walls, so that pV is constant. u_x is proportional to V, i.e. to $1/p$, and using Equation (10) gives:

$$p'(x) = \frac{dp}{dx} = p'(0)x \frac{p_{\text{in}}}{p(x)},$$
or

$$pdp = p_{\text{in}} p'(0) dx.$$
Integrating and requiring $p = p_{\text{in}}$ at $x = 0$,

$$p^2(x) = 2p_{\text{in}} p'(0)x + p_{\text{in}}^2,$$
and since $p(l) = p_{\text{out}}$,

$$p'(0) = -\frac{(p_{\text{in}}^2 - p_{\text{out}}^2)}{2p_{\text{in}} l}.$$
From Equation (9) the speed of the gas at the end of the tube where $x = 0$ is

$$u_x(r,0) = \frac{(p_{\text{in}}^2 - p_{\text{out}}^2)(a^2 - r^2)}{8\eta p_{\text{in}} l},$$
and the throughput of gas is

$$Q = \int p_{\text{in}} u_x(r,0) \times 2\pi r dr = \int \frac{(p_{\text{in}}^2 - p_{\text{out}}^2)}{8\eta l}(a^2 - r^2) \times 2\pi r dr = \frac{\pi a^4}{16\eta l}(p_{\text{in}}^2 - p_{\text{out}}^2).$$
This is Poiseuille's formula.

A2 Pressure gauge PT 1; technical data

Manufacturer Oelikon Leybold Vacuum
Model DU12000
Measuring range 2000 - 1 mbar
Uncertainty 0.25 % FS (linearity, hysteresis, reproducibility)
Reproducibility 0.05 % FS

A3 Vacuum gauge PT 2; technical data

Manufacturer Leybold
Model TTR91

Technical Data

Measurement principle	thermal conductivity according to Pirani
Measurement range	5×10^{-4}...1000 mbar
(air, O_2, CO, N_2)	
Accuracy (N_2)	
1×10^{-3}...100 mbar	$\pm 15\%$ of reading
5×10^{-4}...1×10^{-3} mbar	$\pm 50\%$ of reading
100...1000 mbar	$\pm 50\%$ of reading
Resolution	1% of reading
Repeatability	
1×10^{-3}...100 mbar	2% of reading

Gas Type Dependence

The actual pressure, P, is given by $P = C_{\mathrm{gas}} \times P^*$ where P^* is the pressure calculated assuming the gas is air.

Gas type	Calibration factor, C_{gas}
He	0.8
Ne	1.4
Ar	1.7
Kr	2.4
Xe	3.0
H_2	0.5
air, O_2, CO, N_2	1.0
CO_2	0.9
water vapour	0.5
freon 12	0.7

History

I Grant (1990's)
I Duerdoth and A Russell (2018 Oct) Re-written to follow house style (where appropriate) and to include new digital pressure sensors and auto data logging and converted to LaTeX from Word.

References

[1] Mendoza, E. and Flowers, B. H. 1970. Properties of Matter, Manchester Physics Series, John Wiley.

[2] Young, H.D. and Freedman, R. A. 2012. University Physics, 13th Ed, Adison-Wesley.

[3] Kaye, G. W. C. and Laby, T. H. 1986. Tables of Physical and Chemical Constants, 15th Ed, Longman (1986) and available on-line at ref [4, 5].

[4] National Physical Laboratory, `http://www.npl.co.uk/reference/kaye-laby/`, accessed 28/03/17.

[5] National Institute of Standards and Technology, `www.nist.gov`, accessed 28/03/17.

This script was prepared using LaTeX.