Low temperature Resistance

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Abstract

This experiment investigated the current against voltage graphs for two conductors, copper and constantan and a diode at different temperatures. The range of temperatures were achieved using a heater, liquid nitrogen and helium as an exchange gas. The resistance of each conductor was calculated for each temperature and a relationship between temperature and resistance was quantitatively described by the temperature coefficient of resistance. This was calculated as $\alpha_{copper} = 4.314 \pm 0.021$ mK^{-1} for copper and as $\alpha_{const} = 59 \pm 22$ μK^{-1} for constantan. The energy gap of the diode was calculated to be $E_g = 1.289 \pm 0.003$ eV by analysing the voltage at a chosen current over a range of temperatures.

1. Introduction

The resistance of a material is a measure of the opposition to the flow of current in an electrical circuit; however, it is not a constant of a material, it is affected by external factors such as temperature. By varying the temperature of the material, a relationship between temperature and resistance can be determined and quantitively described by the temperature coefficient of resistance, TCR. Metals have a positive TCR, but this can be close to zero. Metals with a large TCR are used as thermistors, first made by Faraday in 1833 [1], and metals with a negligible TCR are used in thermocouples, first made by Seebeck in 1821 [2]. In this experiment the resistances of two conductors, copper and constantan were calculated for a range of temperatures. This was then used to calculate the temperature coefficient of resistance for each specimen. Also, by measuring the voltage and current through a diode at varying temperatures the energy gap was calculated.

2. Theory

2.1. Conductors

The fermi level is the highest energy level that an electron occupies in a material at 0K [3]. In a conductor the fermi level lies within a region where the conduction band and valence band overlap. Therefore, valence electrons can move into the conduction band with the addition of small amounts of energy. The electrical conductivity of a metal is governed mainly by the scattering of electrons from lattice vibrations [4]. A lattice vibration mode of frequency, ω, will behave as a simple harmonic oscillator and will thus have energy levels, E_m, given by

$$E_m = \left(m + \tfrac{1}{2}\right)\hbar\omega, \tag{1}$$

where $\hbar$ is reduced Planck's constant and m is a positive integer representing the energy level. An excited state is constructed using m excitation quanta, phonons, each with energy $\hbar\omega$. The energy of the lattice vibration, and thus the amplitude and frequency of a phonon, increases with temperature. Therefore, in a conductor there will be more interference between the conducting electrons and phonons at higher temperature and thus higher resistance, however, due to different lattice structures, the relationship between temperature and resistance varies with the material. This is described by

$$\frac{dR}{R} = \alpha dT, \tag{2}$$

where R is resistance, T is temperature and α is the temperature coefficient of resistance. By assuming that α does not vary with temperature and the change in temperature is small, this equation can be rearranged into

$$\frac{R}{R_{ref}} = 1 + \alpha(T - T_{ref}), \tag{3}$$

where R_{ref} is the resistance of the conductor at temperature T_{ref}.

2.2. The Diode

A diode in forward bias will allow positive current to flow. Below the threshold voltage, the current is small and constant; however, for small voltage changes above the threshold voltage the current increases exponentially. This relationship for an ideal diode is represented by

$$I = I_0 e^{-\frac{E_g}{kT}}\left(e^{\frac{qV}{kT}} - 1\right), \tag{4}$$

where I is the current, I_0 is the saturation current, E_g is the energy gap, k is the Boltzmann constant, T is the absolute temperature, q is the elementary charge and V is the voltage. For the range of temperatures in this experiment, $e^{\frac{qV}{kT}} \gg 1$ so Equation (4) reduces to

$$I = I_0 e^{\frac{qV - E_g}{kT}}. \tag{5}$$

This can be rearranged to model a straight-line fit given by

$$V = \frac{k \ln\frac{I}{I_0}}{q} T + \frac{E_g}{q}. \tag{6}$$

3. Experimental Approach

3.1. Initial set-up

Using the set-up shown in Figure 1, the temperature of the specimen was varied using a heater or liquid nitrogen via an exchange gas. A 4-wire circuit was used to remove measurements of resistances of current carrying wires. This reduced the systematic error on the specimen resistance. A large range of current was achieved by switching between four different precision resistors, shown in Figure 2. A thermocouple was used to measure the temperature. One wire was submerged in melting ice as a reference point, and the other was connected to the specimen holder.

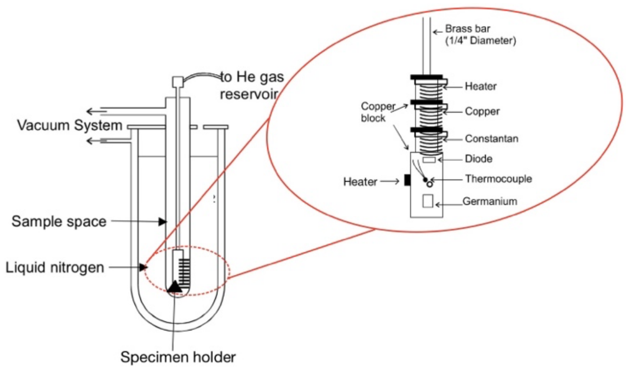

Figure 1. A schematic diagram of the equipment used to vary the temperature of the specimens. This includes a specimen holder in a sample place filled with helium as an exchange gas for low temperature readings. The specimen holder is a brass bar containing three different specimens along with a heater and thermocouple to increase and record temperatures. This was connected to a circuit to measure resistances.

From the thermocouple dataset provided, two functions were fitted as shown in Figure 3. This allowed for accurate voltage to temperature conversions. At room temperature the thermocouple gave a voltage reading of 0.87 ± 0.01 mV which corresponded to a temperature of 294.6 ± 0.2 K. To check for systematic error on temperature readings, a thermometer was used to measure room temperature. This was measured to be 294 ± 0.5 K so this error was assumed to be negligible as the temperature readings were consistent.

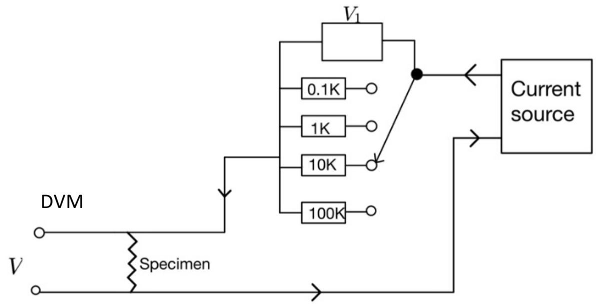

Figure 2. A simplified circuit diagram including a voltmeter measuring the voltage drop, V_1, across a chosen resistor from a set of four precision resistors, and a DVM measuring the voltage across the specimen.

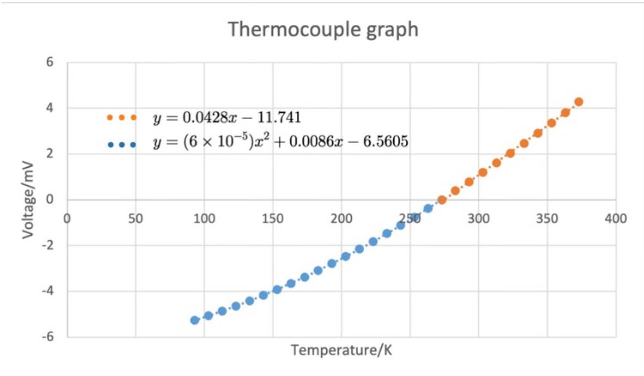

Figure 3. A plot of the thermocouple data, with two different functions fitted for above and below 273 K. The function above 273 K was a linear relationship, whereas the function below was a second-order polynomial.

3.2. Resistances at room temperature and above

A range of currents through each specimen were calculated using Ohm's law on the voltage drop, V_1 in Figure 2, across a precision resistor of known resistance. Voltages through the specimen, V, were then measured using an accurate digital voltmeter. The uncertainties on V_1 and V were taken to be the smallest increment on the voltmeters which was 1 mV for V_1 and ranged from 1 mV to 0.01 mV for V. The calculated current was then multiplied by the fractional error on V1 to give the uncertainty on each value of current. The voltages across the specimen were higher when using a lower resistance precision resistor, which reduced the fractional error on the voltage. However, lower resistance resulted in a larger fractional error on the current. Thus, the 1 kΩ precision resistor was chosen, as this reduced the error on the current and voltage. As precision resistors were used the uncertainty was assumed to be negligible and was thus neglected. The measurements were checked for hysteresis error by repeating voltage readings at constant temperature. The readings only differed twice by 0.01 mV; therefore, this error was assumed to be negligible and thus neglected for the whole experiment. Current was plotted against V and through Ohm's law, the gradient was equal to the reciprocal of the specimen resistance. The error on the resistance was therefore equal to the resistance multiplied by the fractional error on the gradient. This method was repeated at nine different temperatures, shown in Table 1. The reference temperature for plotting the TCR graph was 294.6 ± 0.2K and the reference resistance was 38.02 ± 0.02 Ω for copper and 19.38 ± 0.01 Ω for constantan. A graph of the ratio of resistance to reference resistance against the difference in temperature and reference temperature was then plotted. From Equation (3) the gradient of the graph was equal to the temperature coefficient of resistance α, and the uncertainty on the gradient was equal to the uncertainty on α.

3.3. Resistances below room temperature

The specimen was cooled below room temperature by pouring liquid nitrogen into a dewar. Helium was pumped into a sample space to act as an exchange gas. Helium was chosen as it has a lower boiling point than nitrogen so remained a gas. The temperature of the specimen was then controlled by varying either the volume of the exchange gas or by using the heater. For four temperatures ranging from 108.4 ± 0.3 K to 258.2 ± 0.3 K the method described in Section 3.2 was repeated. A summary of the results can be seen in Table 1. The method for plotting the TCR graph in Section 3.2 was repeated for this data.

3.4. Diode

The process described in Section 3.2 was also followed to plot current against voltage graphs for a diode. However, these measurements were recorded over a larger range of currents to ensure the whole characteristic diode curve was observed. A current range of 1 μA to 10 mA was obtained by switching between the precision resistors and corresponding voltage measurements were taken. This was repeated for the same temperatures used in Sections 3.2 and 3.3. For each voltage against current graph the voltages corresponding to 0.004 A were recorded, as shown in Figure 6. The uncertainty on the voltage readings were taken to be 0.002 V, which was a fifth of the smallest increment on the graph and was constant for each graph. Voltage against temperature was then plotted as shown in Figure 6 and compared to Equation (7) to calculate the energy gap of the diode.

4. Experimental Results

4.1. Copper

The results for copper were consistent with theory Section 2.1 as the resistance increased with temperature, displayed in Table 1. As described in Section 3.2 the temperature coefficient of resistance was equal to the gradient of the graph in Figure 4. This gave $\alpha_{copper} = 4.314 \pm 0.021$ mK^{-1}. The true value is $\alpha = 4.293 \pm 0.004$ mK^{-1} [5] and thus our calculated value is consistent. Values of resistance below room temperature were neglected as α was not constant.

		Resistance / Ω			
Temperature / K	Error / K	Copper	Error	Constantan	Error
108	0.3	5.68	0.01	18.86	0.01
154	0.3	16.77	0.01	19.12	0.01
201	0.3	26.06	0.02	19.24	0.01
258	0.3	35.80	0.02	19.34	0.01
294	0.2	38.02	0.02	19.38	0.01
315	0.2	42.02	0.03	19.38	0.01
323	0.2	42.92	0.03	19.42	0.01
343	0.2	46.08	0.03	19.42	0.02
353	0.2	47.62	0.03	19.42	0.01

Table 1. A table showing the resistances of each specimen at different temperatures and corresponding errors.

The dominant error on the calculations for resistance was 1 mV from the reading of V_1. The dominant error on the calculation for α was from the error on the ratio of resistances, and thus also from V_1. This was the dominant error throughout the experiment.

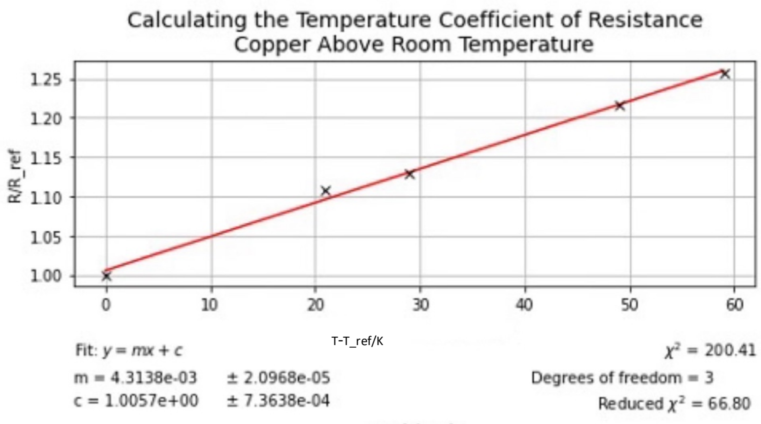

Figure 4. A graph of the ratio of the resistances against the difference in temperatures for copper. The least squares fit line has a gradient m = 4.31 ± 0.02 mK^{-1}, $\chi^2_{reduced} = 66.80$ and y-intercept of $c = 1.005 \pm 0.001$. Error bars are plotted but are too small to be seen.

The reduced chi squared was equal to 66.80 which suggests that the errors on V_1 were underestimated. This also suggests that the data did not fit the straight-line model, which fits the theory as α is not expected to be constant over a large range of temperatures and therefore a smaller range of temperatures should have been used.

4.2. Constantan

As shown in Table 1, as temperatures increase the resistance remains constant. The temperature coefficient of resistance was also calculated for to be $\alpha_{const} = 59 \pm 22$ μK^{-1}, which did overlap with the true value of $\alpha = 50 \pm 2$ μK^{-1} [6]. Values of resistance below room temperature were neglected as α was not constant.

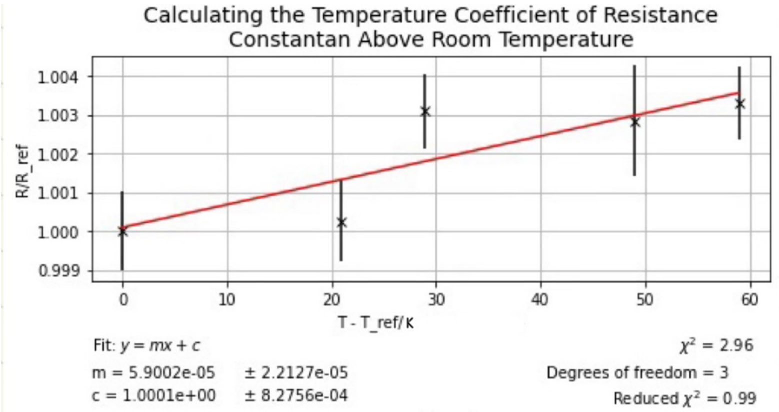

Figure 5. A graph of the ratio of the resistances against the difference in temperatures for constantan. The least squares fit line has a gradient $m = 5.9 \pm 22$ μK^{-1}, $\chi^2_{reduced} = 0.99$ and a y-intercept of $c = 1.000 \pm 0.001$.

The reduced chi squared was 0.99 which indicates that the straight-line fit was suitable for the data, as a reduced chi squared of 1 indicates the data fits with the model. In accordance with the theory, the straight-line fit intercepts the y-axis at 1. This reinforces that the data is a good fit to the straight-line model.

4.3. Diode

An example of a current against voltage curve for the diode at 323 ± 0.3 K is shown in Figure 6. The shape of the curve fits the theory from Section 2.2.

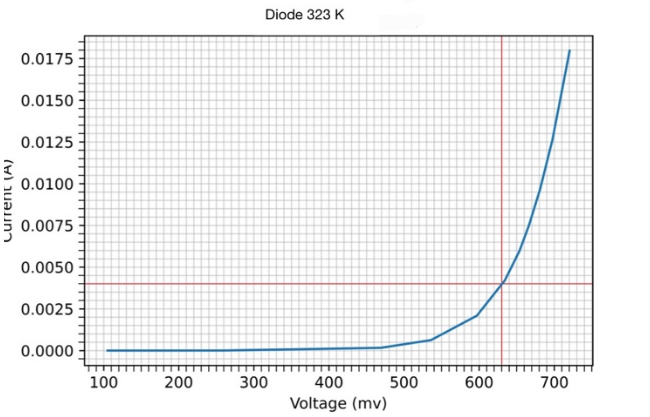

Figure 6. An example of a current against voltage graph at 323K. The red lines indicate the voltage at 0.004A. Error bars are plotted but are too small to be seen.

When plotting the voltage against temperature graph to calculate the energy gap, the voltage measurements had an error of 2mV from reading the graph as before. Using Figure 7 and Equation (6) the energy gap calculated to be $E_g = 1.289 \pm 0.003$ eV, which did not overlap with the true energy gap of $E_{g\ true} = 1.111 \pm 0.001$ eV [7]. This is because the energy gap was assumed to be linear from 153 ± 0.3 K to 350 ± 0.2 K; however, comparing to theory this is not true over a wide range of temperatures [7].

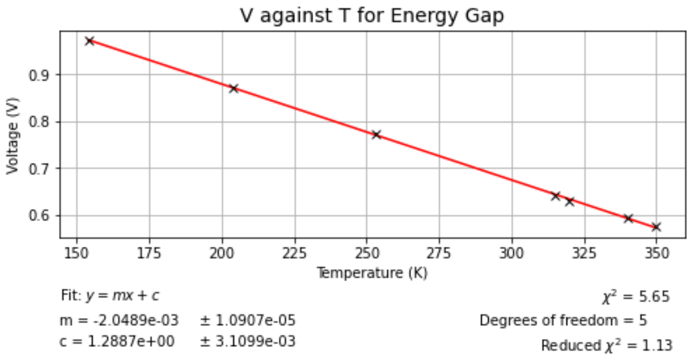

Figure 7. A plot of temperature against voltage at 0.004 A for a diode. The least squares fit line has a $\chi^2_{reduced} = 1.13$ and a y-intercept of $c = 1.289 \pm 0.003$ V. The error bars are plotted but are too small to be seen.

5. Conclusion

The resistance of copper was shown to increase with temperature, in accordance with theory. The TCR for copper was calculated to be $\alpha_{copper} = 4.314 \pm 0.021$ mK^{-1} overlapping with true value of $\alpha = 4.293 \pm 0.004$ mK^{-1}. The TCR for constantan was calculated as $\alpha_{const} = 59 \pm 22$ μK^{-1}, which also overlapped with the true value of $\alpha_{const_{true}} = 50 \pm 2$ μK^{-1}. The voltage against current graphs for the diode fitted with its expected characteristic equation. However, the energy gap of the silicon diode was calculated as $E_g = 1.289 \pm 0.003$ eV, which did not overlap with the true energy gap of $E_{g\ true} = 1.111 \pm 0.001$ eV. The error on the voltage drop, V_1, across the precision resistors was the

dominant error throughout the experiment.

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