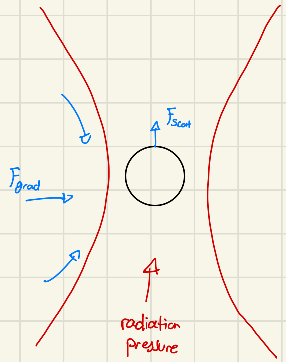
# Derivations

momentum of a photon → $P = \hbar k = \dfrac{h}{\lambda_0} = \dfrac{E}{c} = \dfrac{hf}{c}$

wavelength in a vaccum

in a medium, $\lambda = \dfrac{\lambda_0}{n}$




change in momentum: $|\Delta P| = \dfrac{2hfn}{c}$

a beam of Power P, emits N photons per second : $N = \dfrac{P}{hf}$

force due to photons: $F = \dfrac{\Delta P}{\Delta t} = \dfrac{2hfnP}{hfc} = \underline{\underline{\dfrac{2nP}{c}}}$

⟹ If surface is not a perfect reflector : $F = \dfrac{2nP}{c} \times \Gamma$

↳ reflection coefficient

To account for a Spherical surface: $\underline{\underline{|F| = \dfrac{QnP}{c}}}$ (7) $Q$ = trap efficiency.

| Mie regime: $2r \lesssim \dfrac{\lambda}{\pi}$ |

| Rayleigh Regime $2r \gg \dfrac{\lambda}{\pi}$ |

$$|F_{scat}| = \frac{I_0 \sigma n_m}{c}$$

Scattering Cross section → $\sigma = \dfrac{128 \pi^5 r^6}{3 \lambda^4} \left(\dfrac{m^2 - 1}{m^2+1}\right)^2$ $m = \dfrac{n_p}{n_m}$

$$|F_{grad}| = \frac{2\pi \alpha}{c\, n_m^2} \nabla I_0$$

Polarizability → $\alpha = n_m^2 r^3 \left(\dfrac{m^2-1}{m^2+1}\right)^2$

most particles used fall inbetween the regimes so eq. 7 is used and $Q$ accounts for uncertainties.

# Trapping Force:

Treating the trap as a Hookean Spring → particle is an over-damped oscillator:

$$m\ddot{x}(t) + \gamma\dot{x}(t) + k x(t) = F_B(t)$$

- $m$ → particle mass
- $\gamma$ → viscous drag coefficient of medium
- $k$ → optical trap stiffness
- $F_B$ → force due to thermal fluctuations

$m\ddot{x}(t)$ is the inertial force. Trapping is in low Reynolds number regime, $Re \ll 1$, viscous forces dominate. $\therefore$ $m\ddot{x}(t)$ is neglected

## Stoke's Drag Method

Stokes law

$$\gamma = 6\pi r \eta$$

Stokes viscous force:

$$F_s = 6\pi r \eta \, \dot{x}(t)$$

Therefore, if the thermal fluctuations force is small (assume $= 0$):

$$\underbrace{-kx(t)}_{F_T} = \underbrace{\gamma\dot{x}(t)}_{F_s}$$

$F_T$ → Trapping force

$F_s$ Stoke's force

- move container at increasing velocities until $F_s > F_T$ → particle leaves trap
- velocity at which the particle leaves the trap → $F_s = \gamma\dot{x}(t)$ → $F_T$

## Equipartion method

as the optical trap is 1D

$$\frac{1}{2}k_BT = \frac{1}{2}k\langle x^2\rangle$$

→ positional variance

Probability of finding the particle at $x$ in a potential $U(x)$ is given by a boltzmann distribution:

$$P(x) \propto \exp\left(-\frac{U(x)}{k_BT}\right) \propto H(x)$$

→ Histogram of particle position

$$U(x) = \frac{1}{2}k(x-x_0)^2$$

$$H(x) \propto P(x) = \underbrace{C\exp\left(-\frac{k}{2k_BT}(x-x_0)^2\right)}_{\text{gaussian}}$$

- measure position (T) → calculate $H(x)$
- fit Gaussian to determine $\langle x^2\rangle$ then $k$ ⇒ $k = \frac{k_BT}{\langle x^2\rangle}$ (goal)

in both $x$ and $y$

# ACF and MSD

Overdamped Langevin eqn:

$$m\ddot{x}(t) + \gamma\dot{x}(t) + kx(t) = F_B(t)$$

Solving for $x(t)$ gives the autocorrelation function (ACF):

$$\langle x(t+\tau)\,x(t)\rangle = \langle x^2\rangle e^{-\tau/\tau_c}$$

$\tau_c$ → decay time; auto correlation time

$$\tau_c = \frac{\gamma}{k} \quad \gamma \to 6\pi r\eta, \quad k \to \frac{k_BT}{\langle x^2\rangle}$$

$$\eta = \frac{\tau_c k_BT}{6\pi r\langle x^2\rangle}$$

MSD of $x(t)$:

$$\langle\Delta x^2(\tau)\rangle = \langle(x(t+\tau) - x(t))^2\rangle$$

related to ACF by:

$$\langle\Delta x^2(\tau)\rangle = 2\langle x^2\rangle - 2\langle x(t+\tau)x(t)\rangle$$

$\therefore$

$$\langle\Delta x^2(\tau)\rangle = \frac{2k_BT}{k}\left[1 - \exp\left(-\frac{k\tau}{\gamma}\right)\right]$$

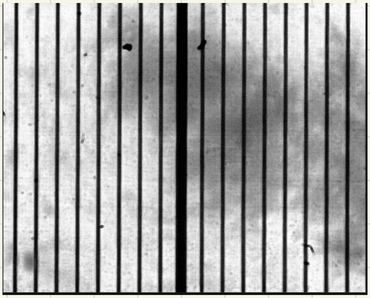
So ACF and MSD are complimentary

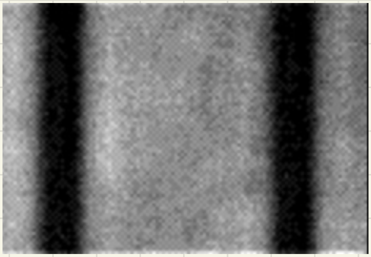
# Day 1:

## Calibration

Started with the 2 $\mu m$ silica beads. We concluded that the lens was dirty, therefore we cleaned it with methanol. The light was also dirty, this was cleaned to the best of our abilities but still shows in photos.

We calibrated the magnification of the microscope.





This is a graticule seen under the microscope.

Each increment is 10 $\mu m$.

Measured the graticule in both $x$ and $y$ to account for any camera distortion. However, both directions measured the same:

$10\,\mu m = (75 \pm 2\sqrt{2})$ px in $x$

$10\,\mu m = (75 \pm 2\sqrt{2})$ px in $y$

$\sigma_f = f \frac{\sigma_x}{x}$

$1$ px $= 0.133\,\mu m \pm 0.005\,\mu m$

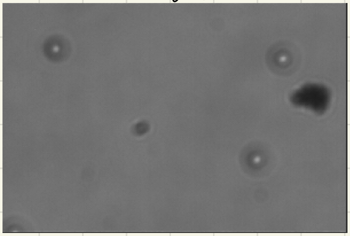
# Brownian motion

Day 2-3

diameter

Prepared a Sample of diluted 2$\mu m$ ± 5% microbeads.

Tracked the motion of particles. Made an effort to maximise fps and minimise lost frames. Also tried to keep ROI bright

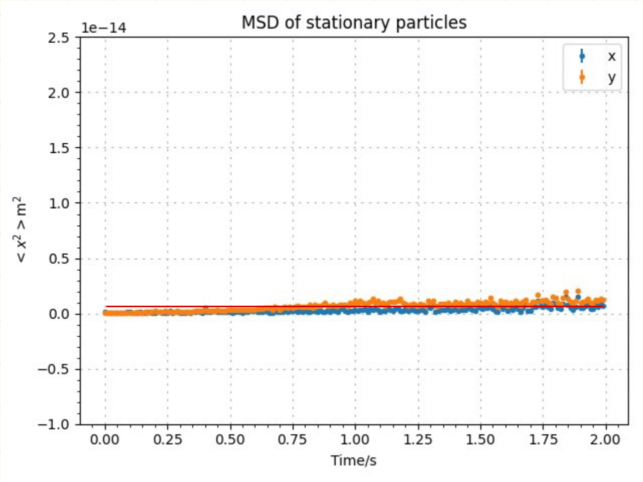


Dried a particle on a slide

The track of a stationary particle was measured to account for thermal vibrations

A mean-squared displacement was plotted for this stationary particle.

An average value was taken of this graph. This average was subtracted from all subsequent data, this accounted for the Systematic error.



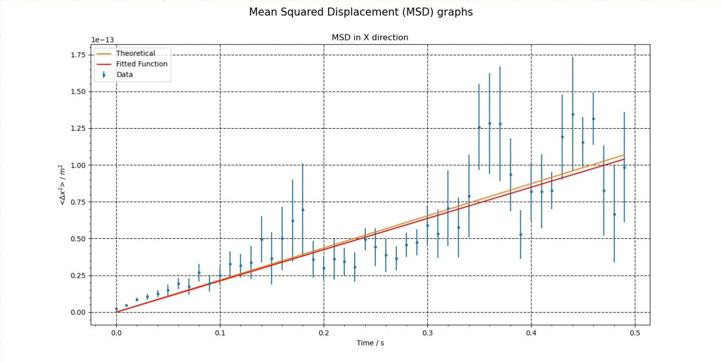

However this error was between 2 and 3 orders of magnitude smaller than data so was neglected.

MSD was calculated by finding the squared difference between each data point and finding the mean, the next data point is found by taking every other data point, third, every 3 data points...

The particle should take larger steps over larger time steps.

Error on each data point is the SD on the mean.

Then plotted MSD for a series of particles. An example plot shown below



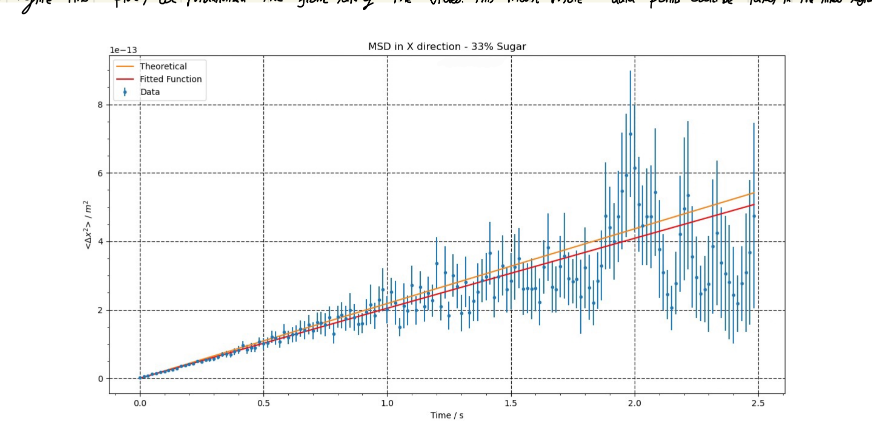

This is a small part of the MSD.

The large data points have large errors due to the nature of the MSD.

Therefore, the first few data points were taken.

This part of the plot is linear.

To refine this plot, we maximised the frame rate of the video. This meant more data points could be taken in the linear region.




The equation for the MSD is

$$\langle \Delta r^2 \rangle = 2nD\tau$$

where $n$ is the number of dimensions

$D$ is the diffusion constant.

$$\langle \Delta x^2 \rangle = 2D\tau$$

Therefore the gradient of the MSD is: $m = 2D$

$$D = \frac{k_B T}{6\pi\eta r}$$

So the viscosity of the medium is: $\eta = \frac{k_B T}{6\pi D r} = \frac{k_B T}{3\pi m r}$ $T = 298.0 \pm 0.5$

$r = 2.06\,\mu m \pm 5\%$

Error on the gradient found from polyfit.

$$\sigma_\eta = \eta\sqrt{\left(\frac{\sigma_T}{T}\right)^2 + \left(\frac{\sigma_r}{r}\right)^2 + \left(\frac{\sigma_m}{m}\right)^2}$$

The values of viscosity calculated are as follows:

| Viscosity / mPaS | Viscosity Uncertainty / mPaS |
|---|---|
| 0.611 | 0.083 |
| 0.660 | 0.049 |
| 1.003 | 0.144 |
| 0.836 | 0.057 |
| 0.903 | 0.075 |
| 0.905 | 0.091 |
| 0.818 | 0.103 |
| 1.063 | 0.133 |
| 0.911 | 0.125 |
| 0.610 | 0.055 |
| 0.813 | 0.079 |
| 0.570 | 0.038 |
| 0.301 | 0.029 |
| 0.906 | 0.139 |
| 1.097 | 0.140 |
| 1.302 | 0.182 |

This gives a viscosity of $\eta = 0.832 \pm 0.104$ mPaS

However if we calculate the std

$\sigma = 0.242$

and remove any data points further than $2\sigma$ away from the mean

Then the table becomes:

| Viscosity / mPaS | Viscosity Uncertainty / mPaS |
|---|---|
| 0.611 | 0.083 |
| 0.660 | 0.049 |
| 1.003 | 0.144 |
| 0.836 | 0.057 |
| 0.903 | 0.075 |
| 0.905 | 0.091 |
| 0.818 | 0.103 |
| 1.063 | 0.133 |
| 0.911 | 0.125 |
| 0.610 | 0.055 |
| 0.813 | 0.079 |
| 0.570 | 0.038 |
| 0.906 | 0.139 |
| 1.097 | 0.140 |
| 1.302 | 0.182 |

This gives a viscosity of

$\eta = 0.867 \pm 0.107$ mPaS

Which is consistent with the viscosity of water at $T = 295K$

$\eta_{true} = 0.958$ mPas

# Optical trap Stiffness - Equipartition method

day 4 - 5

Trapped particles using the laser.

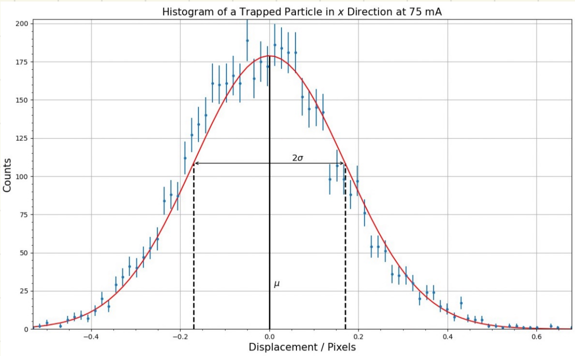
Tracked the particles movement using blender.

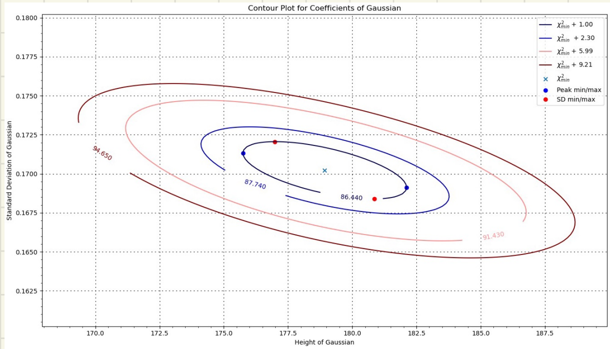
Took the mean of all position in $x$ and $y$ and took this away from all data points to normalise the data around (0,0).

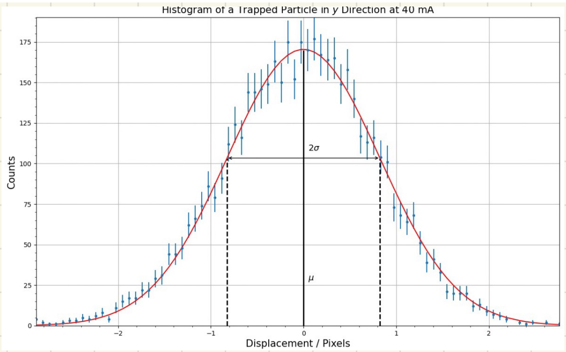
Particles position in a potential well is given by a Boltzmann distribution

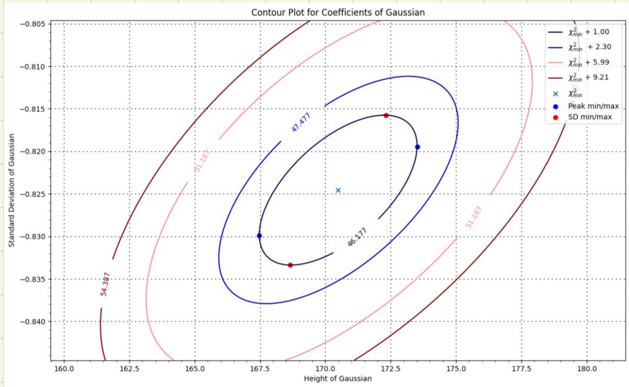
$$P(x) \propto \exp\left(-\frac{U(x)}{k_B T}\right)$$

Plotted the position of the particle in a histogram as we expect this to be gaussian. Example shown below:




As seen above, we fitted a Gaussian to the data. We did this by taking the equation for a Gaussian $G(x) = A\exp\left(\frac{(x-\mu)^2}{2c^2}\right)$ and varying $A$ and $C$ whilst minimising the chi square. $A$ is the peak and $C$ is the standard deviation.

We also tested the Kurtosis and Skewness of the data to evaluate whether a gaussian could be fitted.

Kurtosis $\simeq 0$ for all gaussians and skewness also $\simeq 0$

$$\text{Kurtosis} = n\frac{\sum(x_i - \bar{x})^4}{\left(\sum(x_i-\bar{x})^2\right)^2} \quad ; \quad \text{Skewness} = \frac{1}{n\sigma}\sum(x_i - \bar{x})^3$$

Data is all mesokurtic with no skew so all suitable for Gaussians

The potential caused by the laser has a 'harmonic' form : $U(x) = \frac{1}{2}k(x-x_0)^2$

and $P(x) \propto$ to the particles position (Histogram)

$H(x) = A\exp\left(\frac{-k}{2k_BT}(x-x_0)^2\right)$ $x_0 = 0$ here as we centered at 0.

Comparing this to the Gaussian we fit: $\frac{k}{k_BT} = \frac{1}{c^2}$

$$k = \frac{k_BT}{c^2}$$

where $k$ is the optical trap stiffness and $C$ is the standard deviation

After fitting a gaussian we varied the Coefficients (A and c) and calculated the $\chi^2$ for different values.

From this we plotted Contour plots.

The Contour corresponding to $\chi^2 = \chi^2_{min} + 1$ has max and min coefficient values that are a standard deviation away from the values that give $\chi^2_{min}$.

Therefore finding the difference between the max and min values of each coefficient on this contour corresponds to $2\sigma$ on that coefficient. Half of this is the associated error.

This method gives an error associated with C.

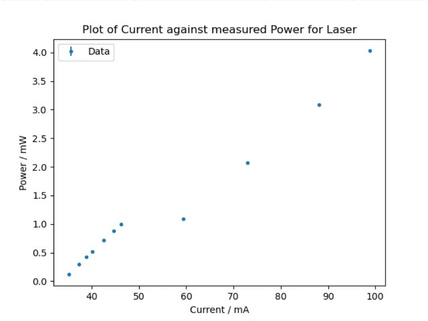
| Current / mA | Kappa / pN microm^-1 | Kappa Error / pN microm^-1 |
|---|---|---|
| 35 | 0.083 | 0.004 |
| 40 | 0.302 | 0.017 |
| 45 | 0.462 | 0.026 |
| 50 | 0.710 | 0.039 |
| 55 | 1.069 | 0.055 |
| 60 | 1.382 | 0.075 |
| 65 | 1.411 | 0.075 |
| 70 | 1.595 | 0.087 |
| 75 | 1.947 | 0.091 |
| 80 | 2.713 | 0.143 |
| 85 | 2.514 | 0.134 |
| 99 | 2.631 | 0.169 |

Now using $k = \frac{k_B T}{C^2}$ with $T = 298.5 \pm 0.5$

So $\sigma_k = k\sqrt{\left(\frac{\sigma_T}{T}\right)^2 + \left(2\frac{\sigma_C}{C}\right)^2}$

the optical trap stiffness, k, was calculated with an associated error for a range of currents supplied to the laser.

As expected, k increased (c decreased) as I increased. This is because the laser is weaker so the particle can move more from the equilibrium position.



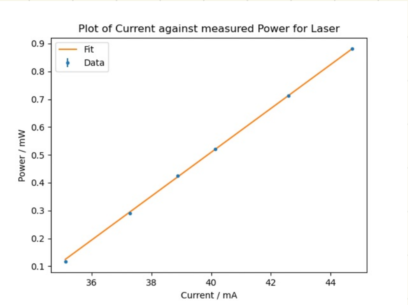

We wanted to plot power against k as this is more physical than current.

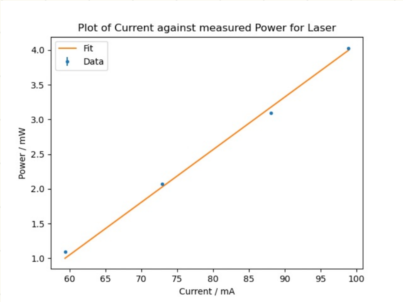
To do this, we measured the power output of the laser after the lens for a range of currents.

We plotted current against power. However, due to the resolution of the equipment the graph needed to be split into 2 regions of high and low current.

The equations of the fitted lines were then used to relate power to current.

Error bars too small to be seen



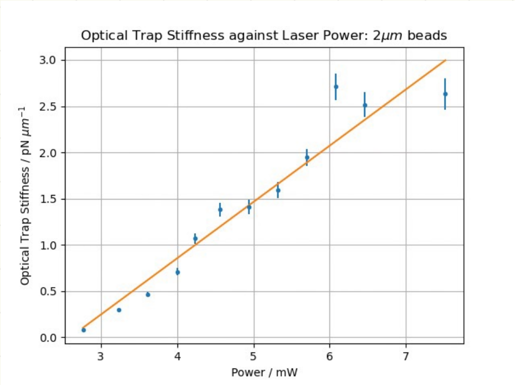




$y = 0.079x - 2.642$

$y = 0.076x - 3.500$

As the intercept should be 0 we take the relationship to be $P = 0.0775 I$

Now finally, optical trap stiffness was plotted against laser power.




$$\chi^2_{red} = 10.94$$

$$y = (0.608 \pm 0.003)\,x - (1.579 \pm 0.069)$$

This follows a linear relationship as expected. Trap stiffness increases linearly with laser power. Although the relationship depends on the laser parameters, the values of k were of the same order as values from literature.

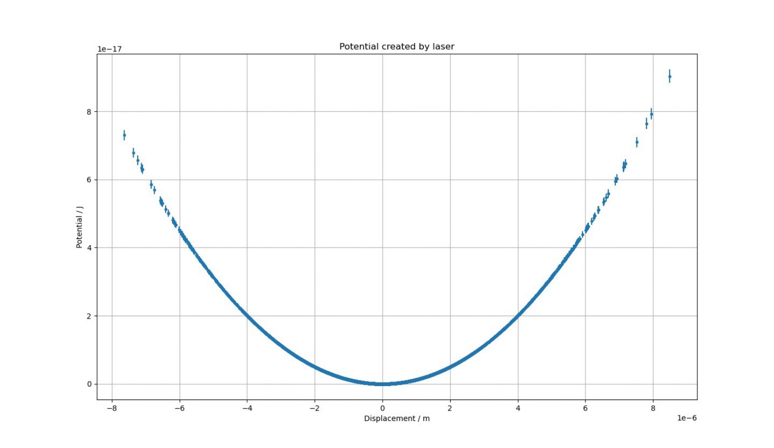
The $\chi^2_{red}$ is too big, this is because errors were underestimated or some of the data points didn't fit the data. Taking more repeats or omitting data points would help with this.

Also, despite doing no analysis on it, we plotted the potential caused by the laser by assuming a harmonic potential:

$$U = \frac{1}{2}k(x - x_0)^2$$

$$x_0 = 0$$

$$\sigma_U = U\frac{\sigma_k}{k}$$



# Optical Trap Stiffness - Stokes method

day 6

By accelerating a trapped particle through the medium the drag force increases as $F_s = \gamma \dot{x}$. Therefore, the particle will reach a point where $F_s > F_T$ and the particle leaves the trap. The velocity at which this occurs can be used to calculate the optical trap stiffness.

$$F_T = F_s$$

$$kx = \gamma \dot{x}$$

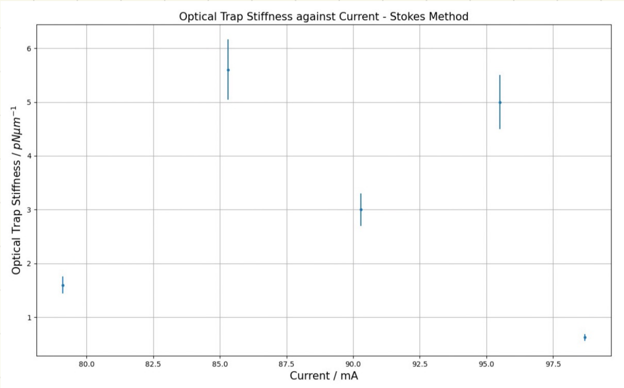
$$k = \frac{\gamma \dot{x}}{x}$$

where $\gamma = 6\pi r n$

and $x$ is the displacement from the equilibrium point.

$$\sigma_k = k\sqrt{\left(\frac{\sigma_r}{r}\right)^2 + \left(\frac{\sigma_{\dot{x}}}{\dot{x}}\right)^2 + \left(\frac{\sigma_x}{x}\right)^2}$$

We repeated this for a range of laser powers; however, there was no correlation between the data so this was not pursued.




One reason this didn't work is it is not possible to see all particles in the different planes of the sample. However, the trapped particle can still collide with this.
Despite taking repeats it appears this was happening more than we thought.
A more dilute sample would help to combat this.

# Optical trap Stiffness - MSD

day 6

The equation of an MSD of a particle in an optical trap with stiffness $k$ is given by:

$$\langle \Delta x^2(\tau) \rangle = \frac{2k_BT}{k}\left[1 - \exp\left(-\frac{k\tau}{\gamma}\right)\right]$$

Taking the log of both sides:

$$\log(\Delta x^2) = \log\left(\frac{2k_BT}{k}\left(1 - \exp\left(-\frac{k\tau}{\gamma}\right)\right)\right)$$

$$\log(\Delta x^2) = \log\left(\frac{2k_BT}{k}\right) + \underbrace{\log\left(1 - \exp\left(-\frac{k\tau}{\gamma}\right)\right)}$$

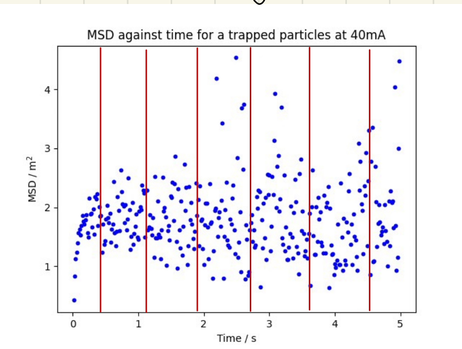
$k \simeq 10^{-6}$
$\gamma \simeq 10^{-3}$
so $e^{-x} \simeq 1 - x$ for small $x$

$$\log(\Delta x^2) = \log\left(\frac{2k_BT}{k}\right) + \frac{k\tau}{\gamma} \quad \text{for small } x.$$

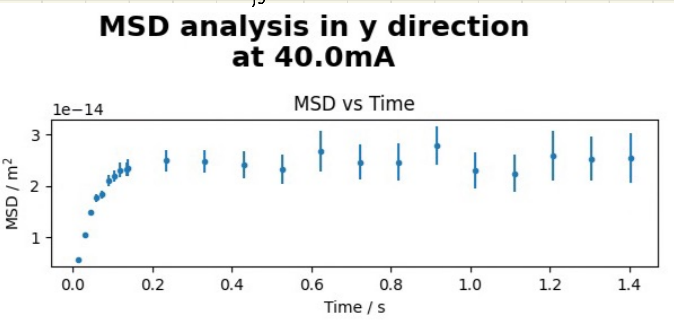
Although we are only interested in the intercept.

When plotting the MSD the data was very noisy so we divided it into bins. In each bin we took a mean and SD. We removed any data points more than 2 SD away from the mean. A new mean was then found in each bin, the error on these data points (RH plot) were $\sigma = \sqrt{\frac{\Sigma(\sigma_i)^2}{N}}$

Before Binning




After Binning




We then took the log:

error on $\log(\Delta x^2)$:

$$\sigma_f = \sqrt{\left(\frac{df}{dz}\right)^2 \sigma_z^2} \qquad f = \ln z$$

$$= \frac{df}{dz}\sigma_z$$

$$= \frac{1}{z}\sigma_z$$

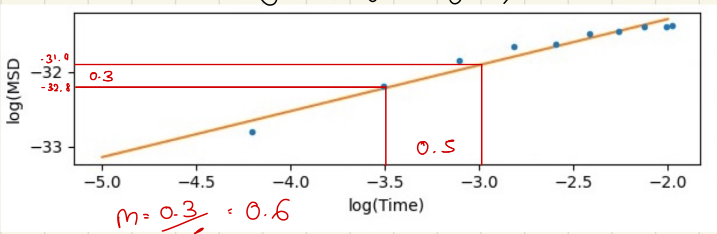
$$\sigma_{\log(\Delta x^2)} = \frac{\sigma_{\Delta x^2}}{\Delta x^2}$$

$y = 0.6x + C$

$-32.8 = 0.6(-3.5) + C$

$-30.7 = C$

log(MSD) against log(time) at 40.0 mA




$m = \frac{0.3}{0.5} = 0.6$

We then used polyfit to fit a linear function to this data before the Cornering frequency.

The y intercept is given by:

$$C = \ln\left(\frac{2k_BT}{k}\right)$$

$$e^c = \frac{2k_BT}{k}$$

$$k = \frac{2k_BT}{e^c}$$

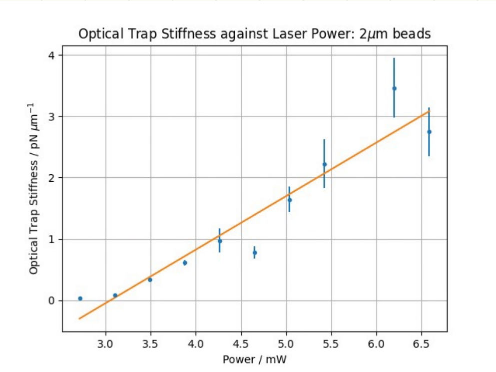
$\sigma_c$ Comes from the Covariance matrix of the fit $\begin{pmatrix} \sigma_m^2 & \sigma_{mc} \\ \sigma_{mc} & \sigma_c^2 \end{pmatrix}$

$$\sigma_f = \frac{df}{dx}\sigma_x$$

$$= -f\sigma_x \qquad f = e^{-x}$$

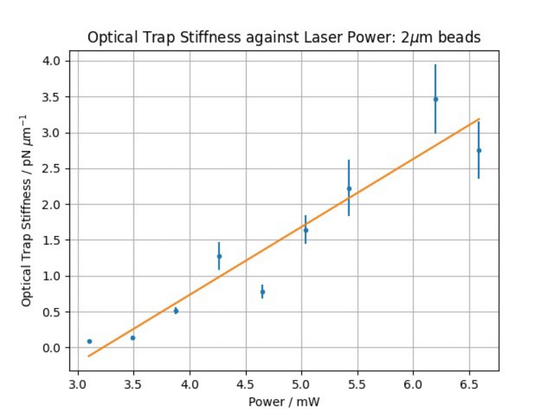
$$\underline{\sigma_k = k\,\sigma_c}$$

So we Calculated k and its error for a range of laser powers and plotted as a straight line. Repeated measurements at the same current so used $\sigma_N = \sqrt{\frac{\sum(\sigma_i)^2}{N}}$ for error on the mean.




The errors on the first data points are small so they inflate the $\chi^2$.

Removing the first data point as an anomaly to the linear fit improved the $\chi^2_{red}$ to 49.6




$$y = (0.95 \pm 0.12)x - (3.06 \pm 0.58)$$

This method does not produce a line equation that is consistent with the equipartition method.

The ep method has $\chi^2_{red}$ of $\simeq 10$ whereas the log(msd) method has $\chi^2_{red} \simeq 50$. This means the ep method is more suitable for a linear fit.

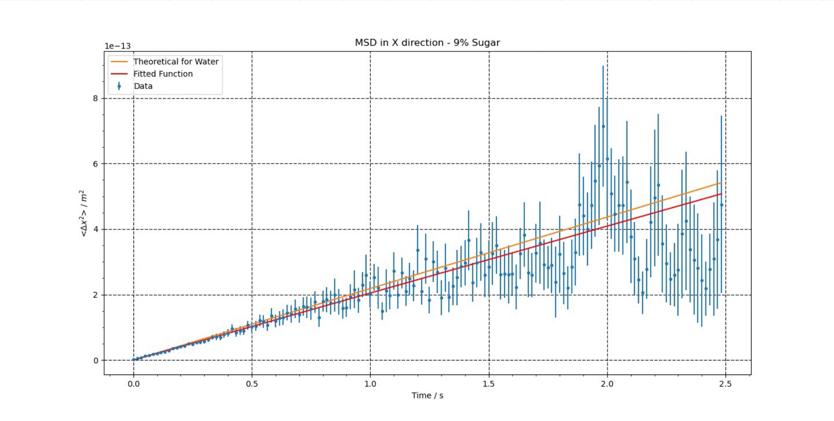
Also the errors on k are smaller in ep. Therefore, the equipartition method is more suitable for calculating k.

# Varying Concentration of Sugar

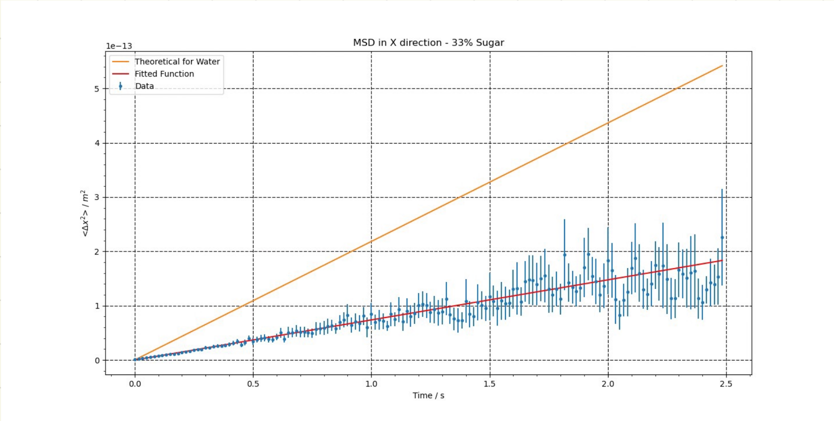
day 7

As an extension we tracked the Brownian motion of particles in solutions of varying sugar concentration.

For this we followed the same method as before to calculate the viscosity.



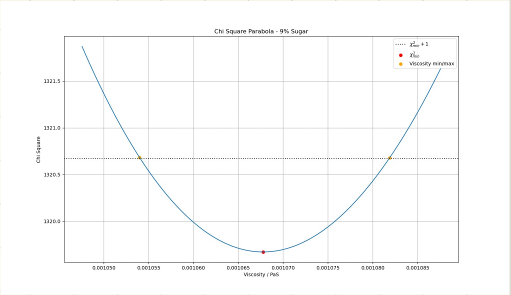

Example of an MSD for 9% Sugar




Example of an MSD for 33% Sugar.

The orange line is has a gradient corresponding to $\eta = 0.001$ Pa s (water). As $m = \frac{2k_BT}{6\pi r\eta}$ then we expect that solutions with higher viscosities than water have a shallower gradient.

After fitting a straight line to the MSD we performed a 1 dimensional varying chi square plot as shown below.




We varied the viscosity around the value that gave the minimum chi square.

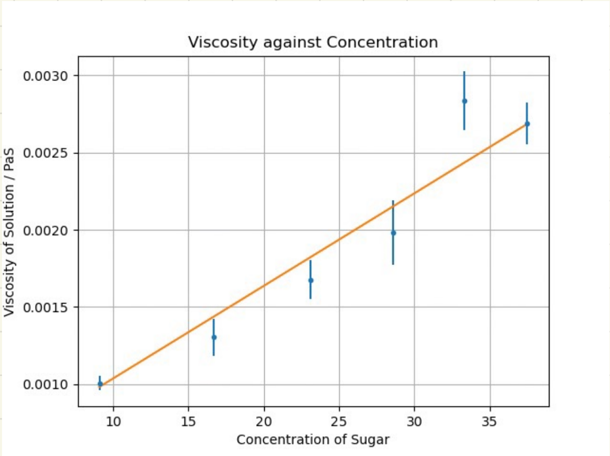
The chi square was calculated for each of these viscosities.

These were plotted against each other.

The values of viscosity that have a $\chi^2 = \chi^2_{min} + 1$ are $1\sigma$ away from the calculated viscosity.

This is therefore the error on viscosity.

So after finding errors for each viscosity, viscosity was plotted as a function of Sugar Concentration




This data appeared linear to us so we fitted a linear function to it

This gave a chisquare of $\chi^2_{red} = 1.31$

and

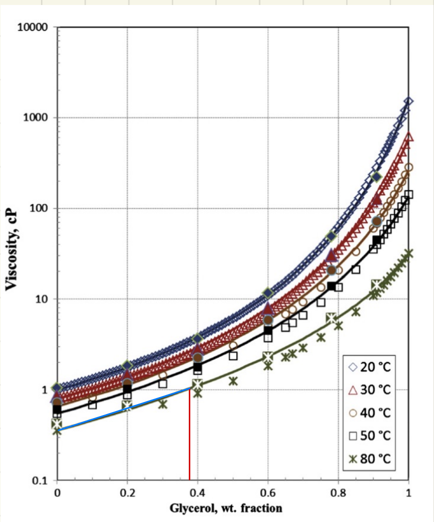
$$y = (5.99\times10^{-5} \pm 3.24\times10^{-11})x + (4.37\times10^{-4} \pm 1.04\times10^{-5})$$

The maximum concentration of sugar that we could measure brownian motion for was 37.5%.

Anything above this and we found that the viscosity was too high for Brownian motion for 2µm beads.

Note that if we had used smaller beads we could have tested higher concentrations

In theory the Viscosity is not linear with Concentration. It follows a somewhat exponential relationship.




However, we only tested up to a concentration of 37.5%

Looking at the graph the relationship is linear up to this concentration

So our data matches theory.

" Physical Properties of aqueous glycerol Solutions"

Takamura, Fischer, Morrow, Journal of Petroleum Science and Engineering (98-99) 2012.

# Simulating Brownian motion

day 8...

Due to limitations of the equipment (limited particle sizes, fixed temperature, can only track in 2 dimensions) we wanted to simulate brownian motion to investigate these variables.

To start we consider the Langevin equation:

$$m\frac{d^2x}{dt^2} = -\gamma\frac{dx}{dt} + kx + \sqrt{2k_BT\gamma}\,W(t)$$

$\sqrt{2k_BT\gamma}\,W(t)$ is the force arising from random impulses (Brownian Motion)

This is a 'white noise' term making the ODE stochastic

For a free particle $k=0$ so

$$m\frac{d^2x}{dt^2} = -\gamma\frac{dx}{dt} + \sqrt{2k_BT\gamma}\,W(t)$$

White noise is discontinuous and has infinite variation

Using finite difference:

$$m\left[\frac{x_i - 2x_{i-1} + x_{i-2}}{\Delta t^2}\right] = -\gamma\left[\frac{x_i - x_{i-1}}{\Delta t}\right] + \sqrt{2k_BT\gamma}\,W(t)$$

Properties of $W(t)$: $\langle W(t)\rangle = 0$ $\langle W(t)^2\rangle = 1$ $W(t_1)$ independent of $W(t_2)$. Cannot approximate with instantaneous values at $t_i$

It is convenient and valid to ignore the inertial term $(m=0)$

(replace $W(t)$ with $\frac{w_i}{\sqrt{\Delta t}}$)

$$\gamma\dot{x} = \sqrt{2k_BT\gamma}\,W(t)$$

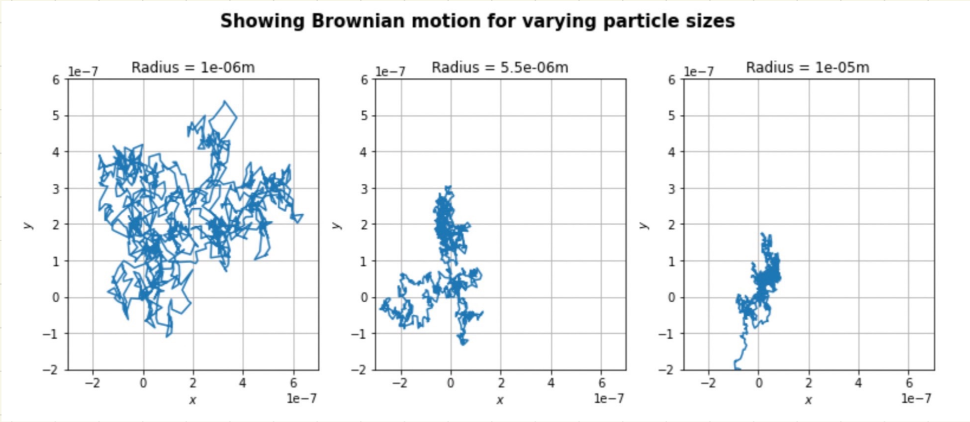
$$\dot{x} = \sqrt{2D}\,W(t)$$

where $w_i$ are normally distributed random numbers with unit variance.

$$\frac{x_i - x_{i-1}}{\Delta t} = \sqrt{2D}\,\frac{w_i}{\sqrt{\Delta t}}$$

$$x_i = x_{i-1} + \sqrt{2D\Delta t}\,w_i$$

This equation was then used to simulate Brownian motion for a range of particle sizes

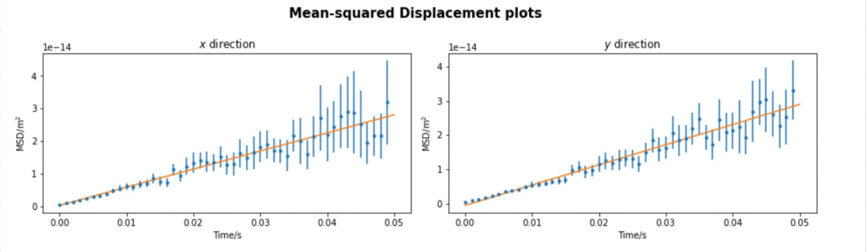


It can be seen from these plots that a particle of larger radius takes larger steps than a particle of smaller radius.

This is because the drag force is $F_D = 6\pi r \eta v$, larger particles experience a larger drag so move less for after a collision.

These Brownian motion paths were then used to plot an MSD using the method used originally.

An example of this is shown below.




The gradient of this was used to calculate the viscosity (used as a check for code)

This returned a viscosity $\simeq$ to the input viscosity.

# Simulating optical trap

In an optical trap the Langevin equation becomes

$$\dot{\underline{r}}(t) = -\frac{1}{\gamma}\,\underline{k}\cdot\underline{r}(t) + \sqrt{2D}\,W(t) \qquad \text{where } \underline{k} = \begin{pmatrix} k_x \\ k_y \\ k_z \end{pmatrix}$$

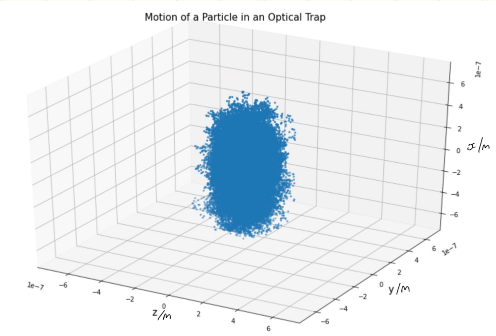
$k_x = k_y \neq k_z$ due to scattering forces and gravity in the z direction and a different intensity profile.

Using the same method to solve this:

$$\underline{r}_i = \underline{r}_{i-1} - \frac{1}{\gamma}\,\underline{k}\cdot\underline{r}_{i-1}\,\Delta t + \sqrt{2D\Delta t}\,\underline{\omega}_i$$

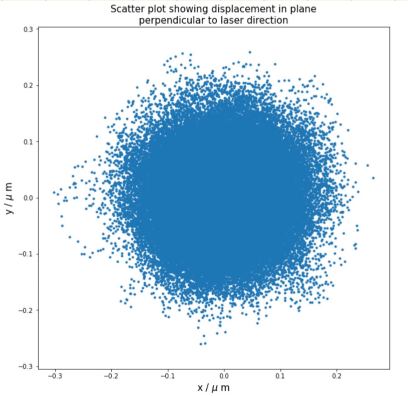
this was then used to simulate particles in an optical trap in 3 dimensions

for $k_x = k_y = 10^{-6}$ and $k_y = 10^{-7}$ and $R = 2\mu m$.



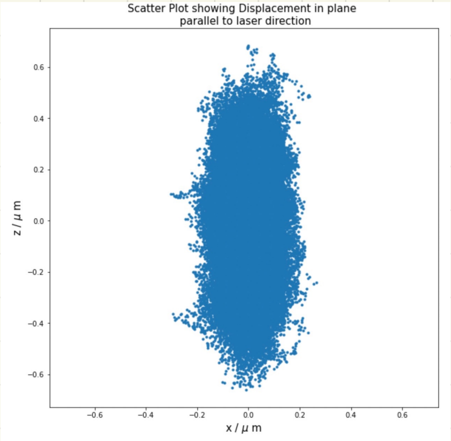

The particle has more variation in the z direction as expected.

In the x-y plane (perpendicular to the propagation of the laser):




This is symmetric as expected, $k_x = k_y$.
The particle has a higher probability of being in the centre and lower as $\underline{r}$ increases.

In the $x$-$z$ plane:

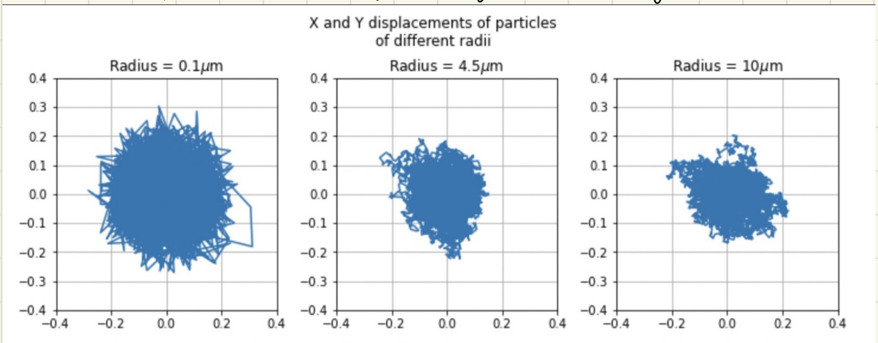



Again, as $k_z < k_x$ the particle is more free to move in the $z$ direction

This is shown by a higher variation in position in the $z$ direction

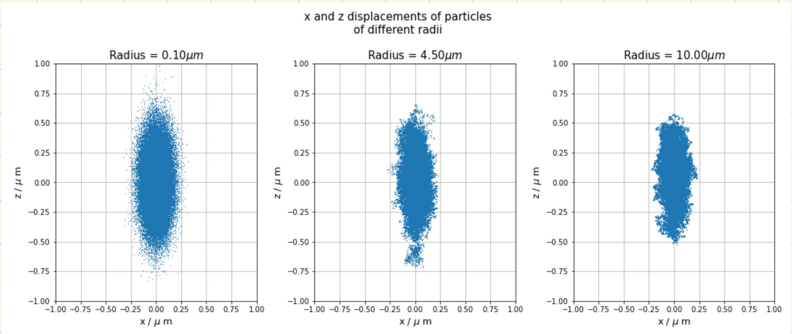
## Varying R in an optical trap

We repeated the process above for 3 different radius of beads to investigate how different sized beads behave in optical traps.



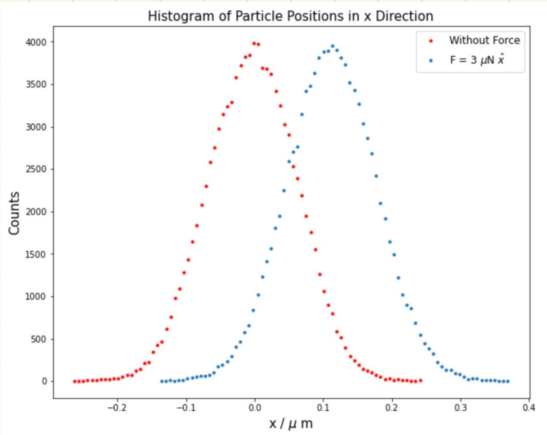

The particles variation decreases as radius increases. This makes intuitive sense as the step size is smaller (from before) so remain close to the equilibrium position. This is because a smaller particles are subject to a smaller drag force $\left(F_D = 6\pi r\eta v\right)$ So will move further for a given impulse.

In the $x$-$z$ plane:



# Adding a Constant force

Adding a Constant force to the System will Shift the equilibrium position of the particle.
If we plot a histogram of the particles position we expect a gaussian.
By applying a Constant force the mean of this gaussian will shift.



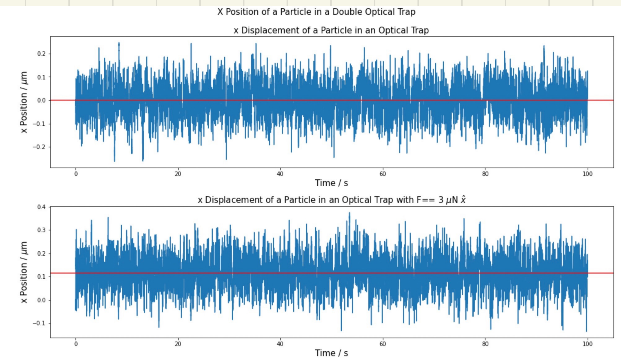

As expected the equilibrium position shifted by about $0.11\,\mu m$ with a force of $\underline{F} = 3\mu N\,\hat{\underline{x}}$

The total force in the $x$ direction here is

$$\underline{F} = \left(-k_x x + 3\mu N\right)\hat{\underline{x}}$$

The motion is unchanged in the $y$ and $z$ direction.

Another way of visualising this:




Again, it can be seen that the equilibrium position has shifted by $1.1\,\mu m$.

# Double Optical trap

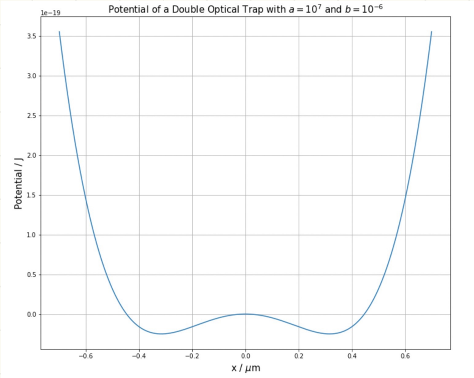
$$x_i = x_{i-1} + \frac{1}{\gamma}\left[-a x_{i-1}^3 + b x_{i-1}\right]\Delta t + \sqrt{2D\Delta t}\, w_i$$

Here we Study the effect of a Single particle in a double optical trap:

The potential Caused by a double optical trap is of the form $U(x) = \frac{a x^4}{4} - \frac{b x^2}{2}$

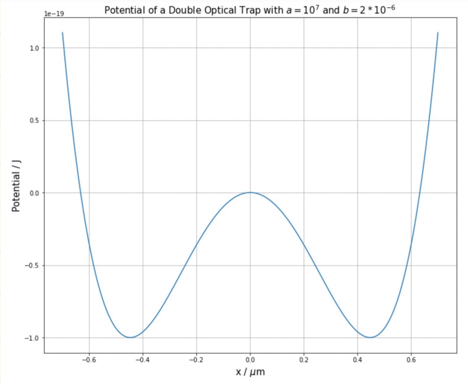
This means the force of the optical trap in $x$ is $F = -ax^3 + bx$

Graphing this:



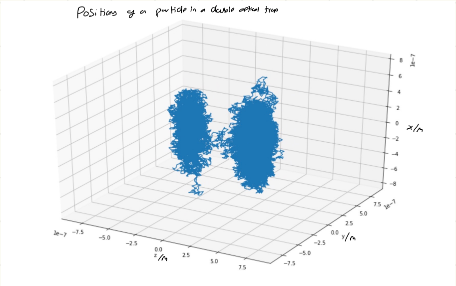

Just by looking at this it is clear to See that there are 2 wells that the particle Can sit in.

If the barrier between the wells is low enough that random Brownian motion fluctuations Can push the particle into another well

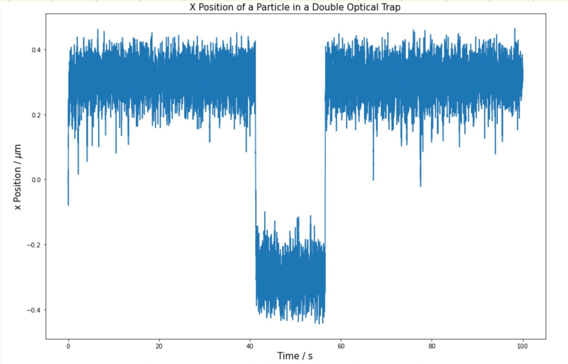



Here b is such that the size of the barrier is larger so the chance of a particle switching wells is unlikely.

By Refining the values of a and b So the particle Can jump between the wells (Kramers transitions) we produced this plot of particle positions:

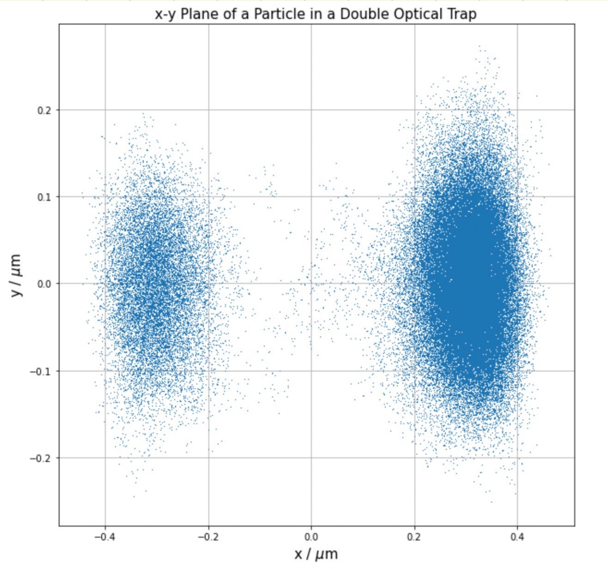


Plotting only the $x$ positions:




This makes it clear when the particle jumps between traps.

And plotting the $x$-$y$ plane




<u>Note</u>

- Now two equilibrium positions
- No Hookes law in $x$ direction
- Force in $x$ is stronger than with 1 trap
  ↳ more spread in $y$.

# Viscosity Gradient

Here we consider when the viscosity of the medium is not constant.

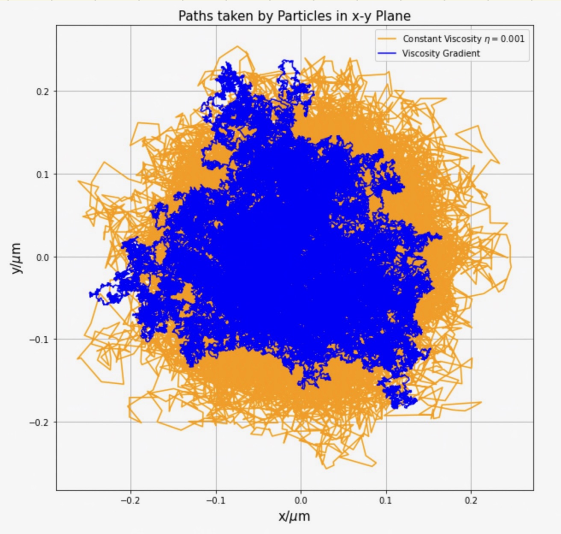
We use the viscosity gradient:

$$\gamma = \gamma_i + 4\times10^{8}\left(x + y\right) \qquad \text{where } \gamma_i = 0.001 \text{ Pa s (water)}$$

This means that as the particle gets further from the origin the viscosity of the medium increases

This means that the step size of the particle decreases as it gets further from the origin.

This, in turn, means the particle should be confined to a smaller area for a constant $k$




This matches all expectations.

More landscapes could have been explored such as the trap being on an interface of two mediums with different viscosities.

# Rotational Force

$$x_i = x_{i-1} + \frac{1}{\gamma}\left[-k_x x_{i-1} - \gamma\Omega y_{i-1}\right]\Delta t + \sqrt{2D\Delta t}\, \omega_i$$
$$y_i = y_{i-1} + \frac{1}{\gamma}\left[-k_y y_{i-1} + \gamma\Omega x_{i-1}\right]\Delta t + \sqrt{2D\Delta t}\, \omega_i$$

We investigated the motion of a particle in an optical trap but also with a rotational force.

The origin of this force could be some type of fluid flow or the incident light having some angular momentum component.

The form of the force we used is:

$$\underline{F} = -\begin{bmatrix} k_x & \gamma\Omega \\ -\gamma\Omega & k_y \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$
$$= \left(-k_x x - \gamma\Omega y\right)\underline{\hat{x}} + \left(\gamma\Omega x - k_y y\right)\underline{\hat{y}}$$

Also we chose $\Omega$ such that $\gamma\Omega \sim k_x$ so all effects could still be observed.

Therefore as $\gamma \sim 10^{-8}$ and $k_x \sim 10^{-6}$ we chose $\Omega = 10^{2}$

$\gamma = 6\pi r \eta$

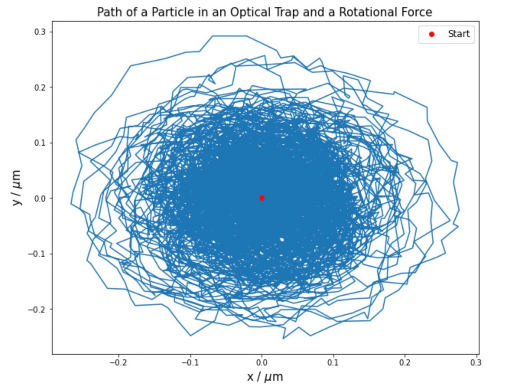
$10\ 10^{-6}\ 10^{-3}$

$10^{-8}$

$\gamma\Omega x = [N]$

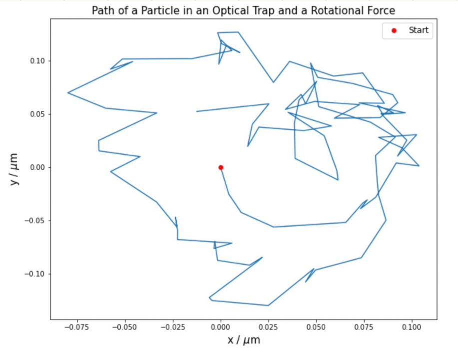
$N s m^{-1} [\gamma]\, m = N$

$s\, \cancel{m}^{-1}[\gamma] = 1 \qquad \gamma = \cancel{m}\, s^{-1}$



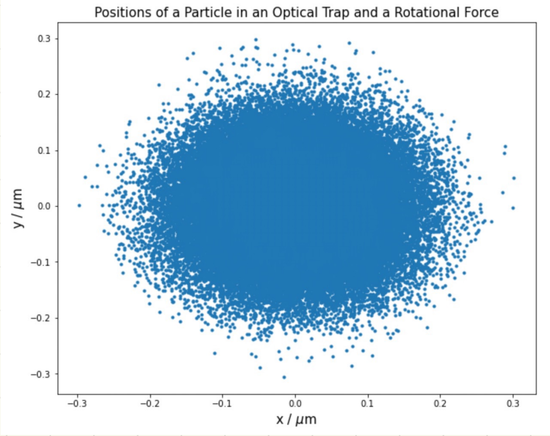

This is a plot of the first 1000 steps.

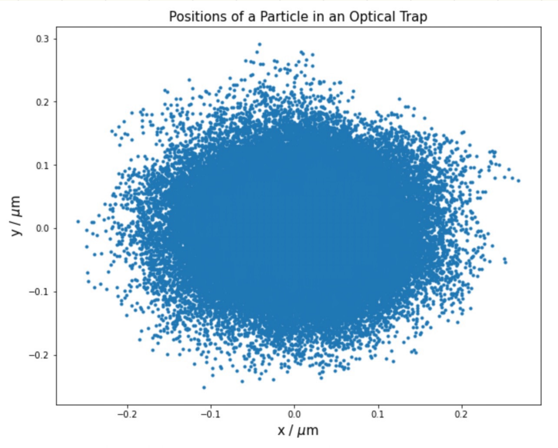
It is difficult to see the effect of the rotational force with so many data points but it does show how the optical trap still works as a trap in the presense of a rotational force.




Plotting the same data but only the first 100 points shows the effect of the rotational force more clearly.

The particle gets pushed around anticlockwise but still taking random brownian motion steps. Also still confined by the trapping force.




The addition of a rotational force does not seem to have an effect on the effectiveness of the optical trap → the equilibrium position remains the same, the variation of the data is the same in $x$ and $y$.

# File Directories

All in folder : /Users/harrytabb/Desktop/Uni/uni year 3/labs/laser tweezers/

- Figures/tables used in lab interview ... /lab interview/
- Water Brownian motion ... /BM water/
- Equipartition method ... /EPmethod/
- Stokes method ... /Stokes/
- MSD method ... /Stiffness MSD/
- Varying Sugar concentration ... /Sugar/

All Simulation code + figures:

/Users/harrytabb/documents/labs/laser tweezers/simulation/