Quadrotor

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1 Quodrotor

Consider a simplified two-dimensional quadrotor model

$$\begin{cases} m\ddot{h} = (\tau_R + \tau_L)\sin\theta - \beta\dot{h} \\ m\ddot{v} = -mg + (\tau_R + \tau_L)\cos\theta - \beta\dot{v} \\ I\ddot{\theta} = \tau_R - \tau_L \end{cases}$$

where h, v describe the horizontal and vertical positions of the quadrotor. θ denotes the quadrotor's angle of rotation. m and I represent the mass and moment of inertia of the quadrotor. g is the acceleration due to gravity and (u_1, u_2) denote the net (thrust, torque) applied by spinning rotors.

Here we want to observe positions (h, v) by GPS, for example. Therefore, the dynamic system can be constructed as the following. Let

$$\mathbf{q} = \begin{bmatrix} h & v & \theta \end{bmatrix}^T \in \mathbb{R}^3.$$

Now, define the state, input and output vectors of this system as

$$\mathbf{x} = \begin{bmatrix} \mathbf{q} \\ \dot{\mathbf{q}} \end{bmatrix}, \mathbf{u} = \begin{bmatrix} \tau_R + \tau_L \\ \tau_R - \tau_L \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}, \mathbf{y} = \begin{bmatrix} h \\ v \end{bmatrix}.$$

Next, the system can be described as

$$\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u}), \mathbf{y} = h(\mathbf{x}),$$

where

$$f(\mathbf{x}, \mathbf{u}) = \begin{bmatrix} \dot{h} \\ \dot{v} \\ \dot{\theta} \\ \frac{u_1}{m}\sin\theta - \frac{\beta\dot{h}}{m} \\ \frac{u_1}{m}\cos\theta - g - \frac{\beta\dot{v}}{m} \\ \frac{u_2}{I} \end{bmatrix}, h(\mathbf{x}) = \begin{bmatrix} \frac{h}{\sqrt{h^2+v^2}} \\ \frac{v}{\sqrt{h^2+v^2}} \end{bmatrix}.$$