Implications of Narrow Framing for Firm Dynamics *

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Abstract

Large firms are subject to internal information and communication frictions. In this paper, I propose a novel framework to incorporate both of them in firm's choices of inputs. I model the firm as a team of input managers who dynamically maximize a common objective, but makes decisions individually, based on noisy, different and non-nested information, which I refer to as "narrow framing". Using this framework, I provide a novel approach to test whether a firm's investment and hiring decision are made under same information. Applying it to a merged belief-outcome dataset of US public firms, I document strong evidence for the existence of information asymmetry within the firms. Quantitatively, I show that manager-level private information is necessary to match the low responsiveness of capital and labor to firm fundamentals and the low correlation between investment and hiring in US public firms. Controlling for adjustment frictions, narrow framing is responsible for about a third of capital and labor misallocation, and leads to a sizable amount (15%) of TFP loss.

1 Introduction

Standard firm dynamic models in macroeconomics posit that firm's input decisions are chosen under same, potentially noisy information. The underlying assumption of this formulation

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is that information is processed and communicated frictionlessly within the firm, so that each managers in the firm are "on the same page". However, given the delegation nature of firm's decisions, the validity of this premise is questionable. There is a large organizational economics literature showing that the firm's internal communication is far from perfect (Malenko (2022)): factors such as allocation of power (Aghion and Tirole (1997)), conflict of interest (Dessein (2002)), geographical and cultural barriers (Mian (2006) and Giroud (2013)), relationship between managers (Qian, Strahan, and Yang (2015)) can all lead to imperfect information flow within the firm. As a result, large firm's decisions are often made by managers with dispersed, potentially imperfect information. This obvious tension between the micro and macro firm literature raises the following questions. First, how can we test whether the decisions of a firm (e.g. capital and labor) are made under imperfect communication with a representative firm-level dataset? Second, what aspect of firm dynamics can the communication friction help to explain quantitatively? Lastly, what are the aggregate implications of imperfect communication?

In this paper, I answer all the three questions with a novel theoretical framework that incorporates both imperfect information and imperfect communication in an otherwise standard dynamic inputs choice problem with convex adjustment cost. To allow for communication frictions, I follow the approach by Lian (2020) and model the firm as a team of two managers, one in charge of capital and the other in charge of labor. They share a common objective of maximizing firm's profit but make input decisions based on their own information. Under this framework, imperfect information is modeled as the managers receiving noisy signals about the firm fundamentals, and imperfect communication is modeled as the managers' information sets being different and non-nested with each other. In other words, imperfect communication is equivalent to asymmetric information among the managers. The firm's behavior is therefore akin to a behavioral household who has trouble coordinating his multiple selves, and therefore I use the terminology of Lian (2020) and term it as "narrow framing".

To compare and contrast the role of information, communication and adjustment friction play in firm dynamics, I extend the static narrow framing framework in Lian (2020) to a dynamic one to allow for the existence of adjustment costs on both inputs. As a result, the firm's input choices are characterized by the Markov Perfect Bayesian Equilibrium (MPBE) of this dynamic team production problem with incomplete information. To simplify the analysis and computation of firm dynamics, I do a Woodford (2003b)-type of log quadratic approximation to the profit function and transform the problem into an easier-to-solve Linear-Quadratic-Gaussian game between two managers, with each manager learning about the unknown firm fundamentals from his/her own signals and the other manager's action in the previous period.

Applying the methods in Hambly, Xu, and Yang (2023), I characterize the linear Markov Perfect Bayesian Equilibrium of the LQG game as a law of motion for firm fundamental, capital, labor and managers' beliefs and compute a stationary equilibrium by iterating it forward.

A Direct Test of Narrow Framing A direct empirical test to the within-firm mechanism of information flow using firm-level data is challenging because (1) it is costly to conduct firm-level surveys to elicit beliefs of different managers within the firm, and (2) specific management practices, such as internal information flows, are usually confidential and difficult to be quantified. For that reason, the empirical studies from organizational economics usually focus on a particular type of firms and organizations (e.g. Dessein, Lo, and Minami (2022)) and do not provide evidence on a representative sample of large firms. Macroeconomic firm expectations, such as Coibion, Gorodnichenko, and Kumar (2018) and Coibion, Georgarakos, Gorodnichenko, Kenny, and Weber (2024), focus mainly on the expectation of firms on macroeconomic variables such as inflation and pay less attention to the expectation on firms' own information or decisions.

Using the dynamic narrow framing model, I fill the gap in the literature by providing a direct test on the hypothesis that all decisions within a firm are based on the same information. The idea is to regress the forecast error of one particular decision (e.g. investment) on the actual decision taken by the other manager (e.g. hiring) in the previous period, and use the regression coefficient to identify the information asymmetry among the managers. Intuitively, if investment and hiring decisions are based on the same information, then there should be no strategic uncertainty between capital and labor managers: each manager knows exactly the other manager's decision in each period. Since the labor choice is already contained in capital manager's information set for investment, it should not be able to predict the forecast error for investment as it contains no additional information about the unknown firm fundamental. Instead, if labor manager bases her hiring decision on some information not known by the capital manager, then the revelation of hiring decision to the capital manager at the end of each period gives the capital manager an additional source of signal extraction, and as a result, the forecast error of investment can be predicted by the lagged hiring of the labor manager.

I apply this simple test to a merged expectation-outcome dataset for US public firms. I merge the IBES Guidance dataset, which contains US public firm's self-reported expectation on the capital expenditure, with the Compustat Fundamental Annual, which contains the actual capital expenditure and hiring for all US public firms, and run the regression on the IBES/Compustat merged sample. I show that the regression results overwhelmingly reject the common information hypothesis: the coefficient on lagged hiring is nonzero and significant at 1 percent level for all the specifications that I consider, meaning that the lagged hiring decisions can predict the forecast error of investment and indicates that hiring contains information that the capital manager does not know when he is making the investment decisions.

Quantitative Analysis of Narrow Framing I then use the dynamic narrow framing framework to quantify the aggregate effect of narrow framing and show that narrow framing is a quantitatively promising mechanism and sheds new lights on the role of information frictions on firm dynamics. In the model, there are three types of frictions: adjustment frictions on both capital and labor, firm-level commonly known noisy information, and manager-level private noisy information. I apply the method in David and Venkateswaran (2019) and target a variety of moments to identify and discipline the sizes of each forces.

The canonical way of introducing information friction to firm dynamics is to endow the firm with a firm-level public noisy information. I claim that this approach is misleading as manager-level private noisy information is a different type of friction from firm-level noisy information. I show that the firm-level and manager-level information frictions have different implications for the investment-hiring correlation: a more precise manager-level private signal lowers the correlation between firm-level investment and hiring decisions, while a more precise firm-level public signal increases the investment-hiring correlation. Intuitively, if the firm-level public information is more precise, then both labor and capital manager will put more weight on it than on their own private signals, and as a result, the role of firm-level public information as a coordination device between managers becomes more pronounced, which drives up the correlation between the managers' actions. Instead, if the manager-level information becomes more precise, then each manager would depend more on his/her own information rather than the public information, which makes the coordination device weaker and drives down the investment-hiring correlations. From the perspective of Hsieh and Klenow (2009), manager-level private noisy information should be regarded as a factor-specific distortion that alters the capital-labor ratio and is a different type of distortion from firm-level noisy information.

Additionally, the leading explanation to sluggish adjustment of capital and labor in firm dynamics is adjustment cost. In my quantification, I show that without the help of information frictions, models with convex adjustment cost cannot fit the data well. To distinguish the adjustment frictions from information frictions, I use the serial correlation of investment and hiring. Intuitively, higher adjustment costs smooth the investment and hiring over time in response to shocks in firm fundamentals and hence lead to a larger serial correlation of actions, while information friction attenuates the responsiveness concurrently. Hence, serial correlation of investment and hiring helps us identify the size of adjustment frictions and information frictions in the economy.

I use a simulated method of moments algorithm to pin down the adjustment cost and information frictions parameters by targeting the serial correlation of investment and hiring, the correlation of investment and hiring with firm fundamentals, and the correlation between investment and hiring. My baseline quantification shows that that asymmetric information among managers is necessary for matching jointly the low correlation between manager actions and firm fundamentals and the low correlation between manager actions, while alternative models under firm-level public noisy information tend to miss the investment-hiring action badly. Additionally, information frictions in the form of narrow framing are quantitatively more promising than adjustment cost models in the ability of matching the targeted moments. In order to match the low responsiveness of investment and hiring to firm fundamentals, absent narrow framing, we need an unrealistically high capital adjustment cost and labor adjustment cost, which leads to an unrealistically high serial correlations of investment and hiring. I show that a hybrid model with capital and labor adjustment cost, noisy firm-level public information and noisy manager-level private information best matches the targeted moments.

I then use the calibrated model to quantify the aggregate consequences of all the frictions in the dynamic narrow framing model. I find that adjustment cost and narrow framing jointly explains about 28 percent of capital misallocation (defined as the MRPK dispersion) and 36 percent of labor misallocation (defined as the MRPN dispersion) we observe in the data, which translate to a sizable 16% of aggregate TFP loss. I find that the majority of this huge aggregate effect comes from information frictions/narrow framing rather than adjustment frictions. Lastly, I find that narrow framing helps match the relative misallocation between labor and capital as an untargeted moment, which the existing literature on capital and labor adjustment (e.g. David et al. (2016)) fails to match.

Related Literature This paper belongs to the literature that introduces information frictions to households' or firms' decision-making processes (Woodford, 2003a; Maćkowiak and

Wiederholt, 2009; Matějka, 2016; Gabaix, 2014; Flynn and Sastry, 2020). These papers typically assume that firms make decisions subject to a cognitive or attentional cost. Hence, in these papers, decisions within the same firm are determined under same, imperfect information. My paper differs by assuming that decisions within a firm are made under different, non-nested, imperfect information.

In terms of the modeling technique, my paper belongs to the literature that introduces incomplete information into macroeconomic models. The idea of modeling coordination and communication frictions using imperfect common knowledge is inherited directly from Angeletos and Lian (2016),Angeletos and Lian (2018),Angeletos and Huo (2021) and Lian (2021). Most of these works introduce incomplete information to a static problem of households or firms, and my work extends the analysis of incomplete information to dynamic decision-making problems. Formulating the firm's input choice problem as a dynamic team production problem is inherited from Marschak and Radner (1972). My theoretical work formalizes the insights in the book.

My empirical test of narrow framing is largely in the spirit of Hall (1978), Coibion and Gorodnichenko (2012) and Coibion and Gorodnichenko (2015). This empirical literature shows that the structure of forecast error is highly related to the expectation formation process of the agent. The intuition of my test is similar to theirs: one cannot predict the forecast error using something already in the information set. My work also contributes to the empirical organizational economics literature (Malenko (2022)) as it provides an investigation of firm's internal information structure to a wider range of firms. Previous works in this literature (such as Mian (2006), Qian et al. (2015) and Dessein et al. (2022)) seek to document direct evidence of internal information and coordination frictions affecting the firm's organization features and they typically require direct observations of the organizational practices for the firms in the sample. As a result, the scope and size of the firm sample are restricted, and most of these papers focus on one particular type of firms or organizations (e.g. multinational banks or large retailers). My empirical test, which does not necessarily rely on manager-level data, can be applied to more representative firm-level datasets to make arguments at the macro level.

My quantitative analysis is closely related to the literature that studies the linkage between information friction/uncertainty and firm dynamics. Previous work, such as David, Hopenhayn, and Venkateswaran (2016) and David and Venkateswaran (2019), quantifies the contribution of information friction to misallocation under two types of internal information structure: either there is a firm-level noisy signal that is commonly known by the managers, or the labor decision is made *ex post* and only capital decisions are subject to information frictions. My paper does not make such assumptions on firm's internal information structures and can accommodate a much richer information structure within the firm. Previous works on uncertainty-driven firm dynamics, such as Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2018), focus on firm-level uncertainty. My paper shows the importance of subordinates' uncertainty in explaining firm dynamics (which echoes the large within-firm variation in management practices documented by Bloom, Brynjolfsson, Foster, Jarmin, Patnaik, Saporta-Eksten, and Van Reenen (2019)) and highlights the difference between information and communication friction (Bloom, Garicano, Sadun, and Van Reenen, 2014) in firm dynamics.

Layout The layout of this paper is as follows. Section 2 sketches general framework of dynamic narrow framing. Section 3 discusses the test of narrow framing using expectation data. Section 4 details my approach to quantify the macroeconomic implications of the dynamic narrow framing model. A conclusion follows.

2 Model

In this section, I describe the model of dynamic input choices under narrow framing. Throughout the paper, there is no aggregate uncertainty, so that all the aggregate variables are fixed at their steady-state values in equilibrium and are common knowledge to all the agents in the economy.

Household There is a representative household in the economy, who has standard preference on final-good consumption, discounts future consumption utility by a discount factor β and supplies a fixed amount of labor $\bar{N}$ to the firms at a wage rate W_t. He can save in a risk-free bond at a real interest rate r in each period. The household chooses how much to save and consume subject to a budget constraint. In a stationary equilibrium, consumption is constant over time, and by the Euler equation, we have $\beta(1+r) = 1$. The only role of the household sector is to close the model, and it is not essential to the analysis.

Final-Good Producer There is a final good producer who combines intermediate goods $\{Y_{it}\}$ into final good Y_t by a CES technology

$$Y_t = \left(\int_0^1 (A_{it} Y_{it})^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}} \tag{1}$$

where A_{it} can be interpreted as an idiosyncratic productivity shock on intermediate-good producer i or a demand shock on the product i, and $\epsilon > 1$ is the parameter for elasticity of substitution.

The price of final good is normalized to one. Given price $\{P_{it}\}$, the final good producers choose how much intermediate goods to purchase from intermediate-good producers. This yields a standard demand curve

$$P_{it} = \left(\frac{Y_t}{Y_{it}} \right)^{\frac{1}{\epsilon}} A_{it}^{1-\frac{1}{\epsilon}} \tag{2}$$

Intermediate-Good Producer There is a continuum $i \in [0, 1]$ of intermediate-good producers in the economy. Each producer i has a constant-return-to-scale production function to combine capital and labor inputs into intermediate goods:

$$Y_{it} = K_{it}^{\alpha} N_{it}^{1-\alpha} \tag{3}$$

where $\alpha \in (0, 1)$ is the parameter for capital share. Each producer i operates in a standard monopolistic competition environment: it internalizes the demand function (2), so that its revenue can be expressed as a decreasing-return-to-scale function of capital and labor, denoted as $R(K_{it}, N_{it}; A_{it})$:

$$R(K_{it}, N_{it}; A_{it}) = P_{it} Y_{it} = Y_t^{\frac{1}{\epsilon}} A_{it}^{1-\frac{1}{\epsilon}} Y_{it}^{1-\frac{1}{\epsilon}} = Y_t^{\frac{1}{\epsilon}} A_{it}^{1-\frac{1}{\epsilon}} K_{it}^{\hat{\alpha}_1} N_{it}^{\hat{\alpha}_2}$$

where $\hat{\alpha}_1 \equiv \alpha(1 - 1/\epsilon)$ and $\hat{\alpha}_2 \equiv (1 - \alpha)(1 - 1/\epsilon)$.

Producer i's cost of production includes two parts. First, the firm pays the rental cost of capital and hiring cost of labor. Additionally, the firm faces convex adjustment costs

$$\frac{\xi_k}{2} \left(\frac{I_{it}}{K_{i,t-1}} - \delta \right)^2 K_{i,t-1} \text{ and } \frac{\xi_n}{2} \left(\frac{H_{it}}{N_{i,t-1}} \right)^2 N_{i,t-1}$$

where $I_{it} \equiv K_{it} - (1 - \delta) K_{i,t-1}$ is investment and $H_{it} \equiv N_{it} - N_{i,t-1}$ is hiring. ξ_k and

ξ_n are parameters that control the slope of marginal adjustment cost for capital and labor respectively. Putting everything together, the cost function $\mathcal{C}(K_{it}, N_{it}; K_{i,t-1}, N_{i,t-1})$ is

$$\mathcal{C}(K_{it}, N_{it}; K_{i,t-1}, N_{i,t-1}) = R_t K_{it} + W_t N_{it} + \frac{\xi_k}{2}\left(\frac{I_{it}}{K_{i,t-1}} - \delta\right)^2 K_{i,t-1} + \frac{\xi_n}{2}\left(\frac{H_{it}}{N_{i,t-1}}\right)^2 N_{i,t-1}$$

where δ is the deprecation rate, and R_t is the rental rate of capital, which is equal to $r + \delta$ in a stationary equilibrium.

Denote profit function $\Pi(K_{it}, N_{it}; K_{i,t-1}, N_{i,t-1}, A_{it}) \equiv R(K_{it}, N_{it}; A_{it}) - \mathcal{C}(K_{it}, N_{it}; K_{i,t-1}, N_{i,t-1})$. Under full information, the firm's input choices are characterized by the Bellman equation

$$V(K_{i,t-1}, N_{i,t-1}; A_{it}) = \max_{K_{it}, N_{it}} \Pi(K_{it}, N_{it}; K_{i,t-1}, N_{i,t-1}, A_{it}) + \frac{1}{1+r}\mathbb{E}[V(K_{it}, N_{it}; A_{i,t+1})|A_{it}] \tag{4}$$

To facilitate my later discussions, it is useful to think of the firm as a team of two managers, one in charge of capital decisions (I call him the Capital Manager) and the other in charge of labor decisions (I call her the Labor Manager), who share the common objective of maximizing the firm's lifetime profit. Hence, the capital and labor policy function that solves the *firm's* Bellman equation (4) are equivalent to a Markov Perfect Equilibrium (MPE) of a dynamic team production game (Marschak and Radner (1972), Chapter 7), in which

- Given the labor manager's policy N_{it}, the capital manager's strategy K_{it} is the one that solves the Bellman equation

$$V^k(K_{i,t-1}, N_{i,t-1}; A_{it}) = \max_{K} \Pi(K_{it}, N_{it}; K_{i,t-1}, N_{i,t-1}, A_{it}) + \frac{1}{1+r}\mathbb{E}[V^k(K, N_{it}; A_{i,t+1})|A_{it}] \tag{5}$$

- Given the capital manager's policy K_{it}, the capital manager's strategy N_{it} is the one that solves the Bellman equation

$$V^n(K_{i,t-1}, N_{i,t-1}; A_{it}) = \max_{N} \Pi(K_{it}, N_{it}; K_{i,t-1}, N_{i,t-1}, A_{it}) + \frac{1}{1+r}\mathbb{E}[V^n(K_{it}, N; A_{i,t+1})|A_{it}] \tag{6}$$

Equation (5) and (6) are the *manager*-level Bellman equations that characterize the firm's input decisions. This reformulation gives us more flexibility to accommodate incomplete/asymmetric information between managers within the firm and makes it easier for us to analyze a richer internal information structure within the firm.

Information The only source of uncertainty in this simple model is the idiosyncratic firm fundamentals A_{it}. I assume that the log of A_{it}, denoted as a_{it}, follows an AR(1) process

$$a_{it} = \rho a_{i,t-1} + \mu_{it}, \text{ with } \mu_{it} \overset{iid}{\sim} N(0, \sigma_\mu^2) \tag{7}$$

The firm managers know the data generating process in (7) and treat the objective distribution of μ_{it} or a_{it} as the prior. However, the true value of a_{it} is never fully revealed to the firm and its managers.[1] Hence, when making decisions, the capital and labor manager of firm i need to learn dynamically from the Gaussian noisy signals they receive to form expectations about the "latent variable" a_{it} over time using Kalman filters, which I will specify later in detail.

The key friction that I introduce, which I call "narrow framing" (as in Lian (2020)), is that I allow each manager's decision to be dependent on different, non-nested information sets. Formally, let $\mathcal{I}_{it}^k$ denote capital manager's information set in period t, and $\mathcal{I}_{it}^n$ denote the labor manager's information set in period t. In particular, I allow the possibility that $\mathcal{I}_{it}^k \neq \mathcal{I}_{it}^n$ and are not a subset to each other. This captures the notion of imperfect communication within the organization: with imperfect information sharing, the managers' information sets do not perfectly overlap.

I now specify the elements in the managers' information sets $\mathcal{I}_{it}^k$ and $\mathcal{I}_{it}^n$. I assume that there is a piece of firm-level information a_{it}^p, which is a noisy signal of the firm's fundamental a_{it} and is commonly known by the managers (i.e. $a_{it}^p \in \mathcal{I}_{it}^k \cup \mathcal{I}_{it}^n$):

$$a_{it}^p = a_{it} + \epsilon_{it}^p, \text{ with } \epsilon_{it}^p \overset{iid}{\sim} N(0, \sigma_{\epsilon,p}^2) \tag{8}$$

For the non-nested part of the information set, I assume that each manager $m \in \{k, n\}$ is endowed with a private, noisy signal a_{it}^m about a_{it}:

$$a_{it}^m = a_{it} + \epsilon_{it}^m, \text{ with } \epsilon_{it}^m \overset{iid}{\sim} N(0, \sigma_{\epsilon,m}^2) \text{ for } m \in \{k, n\}. \tag{9}$$

The signal noises $\epsilon_{it}^k, \epsilon_{it}^n, \epsilon_{it}^p$ and the innovation in the firm fundamental μ_{it} are independent to each other. In each period, the realization of a_{it}^k and a_{it}^n are private to the capital and labor manager respectively, but the data generating process of the signals, (8) and (9), are commonly known by the managers.

[1] This is different from the setup in David, Hopenhayn, and Venkateswaran (2016) and David and Venkateswaran (2019), where the lagged firm fundamental $a_{i,t-1}$ is fully revealed to the firm in period t.

The standard deviations of the signal noises, $(\sigma_{\epsilon,p}, \sigma_{\epsilon,k}, \sigma_{\epsilon,n})$, define the internal information structure of the firm and are the key parameters to pin down in the quantitative analysis. This parametrization gives us a parsimonious way to model the degree of information segmentation within the firm. For example, if we make the private signals to be extremely noisy, i.e. $\sigma_{\epsilon,m} \to \infty$ for $m \in \{k, n\}$, then the managers will put essentially zero weight on private signals when they apply Bayes's rule to update their beliefs, and the firm's behavior is as if the firm is endowed with a public information only and the managers' information set perfectly overlaps. On the other hand, if $\sigma_{\epsilon,p} \to \infty$ while $\sigma_{\epsilon,m} > 0$, then the managers will not rely on the firm-level signal to update their beliefs, and it's as if the firm is operating on a non-nested information structure.

Since the realization of signal a_{it}^m is private to manager m and both managers choose their decisions simultaneously, the action that manager m takes in period t is not known by the other manager m', and vice versa. However, I assume that at the end of each period t, the actions taken by the managers in period t, K_{it} and N_{it}, are revealed to both managers and become public information. This assumption allows the managers to use the observed actions in the previous period to update their beliefs about the unknown fundamental a_{it} and the other manager's action in the current period, which simplifies the characterization of the equilibrium concept. Formally, putting everything together, we can express the law of motion for the managers' information sets $\mathcal{I}_{it}^k$ and $\mathcal{I}_{it}^n$ as

$$\mathcal{I}_{it}^k = \mathcal{I}_{i,t-1}^k \cup \{a_{it}^p, a_{it}^k, N_{i,t-1}\} \text{ and } \mathcal{I}_{it}^n = \mathcal{I}_{i,t-1}^n \cup \{a_{it}^p, a_{it}^n, K_{i,t-1}\} \tag{10}$$

Equilibrium Under narrow framing, the firm faces a dynamic team production problem with incomplete information. The firm's input decisions can therefore be interpreted as a Markov Perfect Bayesian Equilibrium (MPBE), in which

- Given labor manager's policy function $N_{it}(\cdot)$, capital manager's policy function $K_{it} : \mathcal{I}_{it}^k \to \mathbb{R}^+$ solves Bellman equation

  $$V^k(K_{i,t-1}, N_{i,t-1}; \mathcal{I}_{it}^k) = \max_K \mathbb{E}\left[\Pi(K, N_{it}(s_{it}^n); K_{i,t-1}, N_{i,t-1}, A_{it}) + \beta V^k(K, N_{it}(s_{it}^n); \mathcal{I}_{i,t+1}^k) \middle| \mathcal{I}_{it}^k\right] \tag{11}$$

  where s_{it}^n is the signal realization of the labor manager in period t, and

- Given capital manager's policy function $K_{it}(\cdot)$, labor manager's policy function N_{it} :

$\mathcal{I}^n_{it} \to \mathbb{R}^+$ solves Bellman equation

$$V^n(K_{i,t-1}, N_{i,t-1}; \mathcal{I}^n_{it}) = \max_N E_t \left[\Pi(K_{it}(s^k_{it}), N; K_{i,t-1}, N_{i,t-1}, A_{it}) + \beta V^n(K_{it}(s^k_{it}), N; \mathcal{I}^n_{i,t+1}) \middle| \mathcal{I}^n_{it} \right] \tag{12}$$

where s^k_{it} is the signal realization of the capital manager in period t, and

- In period t, each manager updates his/her belief about a_{it} using his/her observed signals and the past actions $K_{i,t-1}, N_{i,t-1}$, consistent with the policy function $K_{it}(\cdot), N_{it}(\cdot)$ and the Bayes's rule.

The advantage of using MPBE in (11) and (12) to characterize firm's behavior lies in its flexibility of accommodating essentially any possible internal information structure of the firm. This flexibility makes it a more general model and nests two classes of extensively studied models in the firm dynamics literature. One is the models based on full-information rational expectation (FIRE), as in (4) or (5) and (6) above. They can be regarded as a special case of (11) and (12), with $\mathcal{I}^k_{it} = \mathcal{I}^n_{it} = \{a_{it}, a_{i,t-1}, \cdots, a_{i0}\}$, i.e. the true firm fundamental is always revealed to both managers. Using our parameterization (8) and (9) above, this full-information benchmark maps to the limit $\sigma_{\epsilon,p} \to 0$ and $\sigma_{\epsilon,m} \to \infty$ for $m \in \{k, n\}$. Another class of models are the ones based on common, noisy information, as in David, Hopenhayn, and Venkateswaran (2016).[2] Although it is a useful benchmark to analyze the effect of uncertainty and imperfect information on firm dynamics, it presumes that that imperfectly observed information is frictionlessly shared and communicated within the organization. From the above discussion, this is again a special case of the current framework, with $\mathcal{I}^k_{it} = \mathcal{I}^n_{it} = \{a^p_{it}, \cdots, a^p_{i0}\}$. Using our parameterization (8) and (9) above, this noisy, common information benchmark maps to the limit $\sigma_{\epsilon,p} \in (0, \infty)$ and $\sigma_{\epsilon,m} \to \infty$ for $m \in \{k, n\}$. Hence, the firm dynamics implied by noisy, common information models is also contained in (11) and (12).

Equation (11) and (12) illustrate that the managers face two types of uncertainty when they are making their own decisions. For the capital manager, apart from the fundamental uncertainty about a_{it} or A_{it}, he faces a strategic uncertainty about the action of the labor manager N_{it}, which is measurable to labor manager's information set only. The source of this strategic uncertainty is the fact that information is not perfectly shared within the firm,

[2] More precisely, David et al. (2016) studies two extreme cases, where (1) imperfectly observed information is perfectly shared among capital and labor managers, and (2) labor decisions are made *ex post* and frictionlessly, and only capital decisions are subject to information friction (which is also the main specification of David and Venkateswaran (2019)). The latter case is also a special case of the current framework, with the capital manager's information set is a subset of the labor manager's information set. My paper can be regarded as providing a smooth version of information friction between these two extreme cases.

so that there is labor-specific information that is only known to the labor manager and not to the capital manager. This strategic uncertainty affects capital manager's decision because labor manager's action affects both the marginal revenue product of capital and the continuation value of the capital manager, and therefore capital manager has an incentive to form expectation about N_{it} to guide his own decision about K_{it}.

Log-Quadratic Approximation The third requirement of an MPBE requires that the belief dynamics of both managers are consistent with the policy function and Bayes's rule. It is a difficult task to analytically characterize or compute the evolution of beliefs since it is an infinite-dimensional object.

Here, I get around this technical challenge by doing a Woodford (2003b)-type log-quadratic approximation on the profit function with respect to $(K_{it}, N_{it}, K_{i,t-1}, N_{i,t-1}, A_{it})$ around their frictionless steady-state values.[3] I can therefore express the log-quadratic approximated profit function π_{it} as a quadratic function in terms of the log deviations of capital, labor and firm fundamental from their own frictionless steady-state value. Denote these log-deviation values as k_{it}, n_{it} and a_{it} respectively, and define investment (rate) as $\iota_{it} \equiv k_{it} - k_{i,t-1}$ and hiring (rate) as $h_{it} \equiv n_{it} - n_{i,t-1}$. The approximated profit function can be written as

$$\pi(\iota_{it}, h_{it}; x_{it}) = x_{it}' P x_{it} + x_{it}' Q \iota_{it} + x_{it}' R h_{it} + H_\iota \iota_{it}^2 + H_{\iota h} \iota_{it} h_{it} + H_h h_{it}^2 \tag{13}$$

where $x_{it} = [k_{i,t-1}, n_{i,t-1}, a_{it}]'$ is the state vector of the firm with a law of motion

$$x_{it} = A x_{i,t-1} + B \iota_{it} + C h_{it} + D \mu_{it} \tag{14}$$

and matrices $A, B, C, D, P, Q, R, H_\iota, H_h$ and $H_{\iota h}$ are constants imputed from the frictionless steady-state values and model parameters. The simple but tedious derivations of (13) are left to Appendix A.1.

Characterization of Linear MPBE The log-quadratic approximation of profit function allows us to characterize a linear MPBE analytically.

Plugging (13) and (14) back into the manager-level Bellman equations (11) and (12) transforms the original problem into a linear-quadratic (LQ) control problem, with investment ι_{it}

[3] By frictionless, I mean the case where there is no information friction and adjustment friction, i.e. a_{it} is fully revealed to the firm, and $\xi_k = \xi_n = 0$.

and hiring h_{it} as control variables and x_{it} as the state vector. An immediate implication of the LQ control is that the policy function is linear and certainty equivalence holds:

Proposition 1. *With the log-quadratic-approximated profit function (13) and the law of motion for state (14), we have*

1. *The policy function for the full-information MPE in (5) and (6) takes the form*

$$\begin{aligned}
\iota_{it} &= F_\iota x_{it} = F_\iota^k k_{i,t-1} + F_\iota^n n_{i,t-1} + F_\iota^a a_{it} \\
h_{it} &= F_h x_{it} = F_h^k k_{i,t-1} + F_h^n n_{i,t-1} + F_h^a a_{it}
\end{aligned}$$

   where the policy matrix F_ι, F_h can be solved by iterating forward a pair of Ricatti equations;

2. *The narrow-framing policy function for (11) and (12) takes the form*

$$\iota_{it} = F_\iota \mathbb{E}[x_{it}|\mathcal{I}_{it}^k] = F_\iota^k k_{i,t-1} + F_\iota^n n_{i,t-1} + F_\iota^a \hat{a}_{it}^k \tag{15}$$

$$h_{it} = F_h \mathbb{E}[x_{it}|\mathcal{I}_{it}^n] = F_h^k k_{i,t-1} + F_h^n n_{i,t-1} + F_h^a \hat{a}_{it}^n \tag{16}$$

   where F_ι, F_h are the same policy matrix as in the full-information MPE, and $\hat{a}_{it}^k \equiv \mathbb{E}[a_{it}|\mathcal{I}_{it}^k]$ and $\hat{a}_{it}^n \equiv \mathbb{E}[a_{it}|\mathcal{I}_{it}^n]$ are the posterior mean of capital and labor manager's first-order belief on a_{it}.

Proof. See Appendix A.3. □

The linearity of (15) and (16), together with the Gaussian structure of μ_{it} and signal noises, significantly simplifies the characterization of the belief updating process for two reasons. First, the Gaussian shocks and noises imply that tracking the first and second moment is sufficient for the characterization of belief dynamics, which significantly lowers the dimensions needed for the computation. Moreover, the linearity of policy functions makes actions jointly Gaussian with firm fundamentals and signals, and the managers can simply treat the other manager's lagged action as a noisy signal of the firm fundamentals and learn from it in the same manner as from other signals. For instance, in period t, with (16), the capital manager forms belief about the labor manager's action h_{it} by

$$\hat{h}_{it}^k \equiv \mathbb{E}[h_{it}|\mathcal{I}_{it}^k] = F_h^k k_{i,t-1} + F_h^n n_{i,t-1} + F_h^a \mathbb{E}[\hat{a}_{it}^n|\mathcal{I}_{it}^k] \tag{17}$$

Equation (17) is an important equation. It sends us two important messages. First, it tells us that the capital manager's belief about the labor manager's action is linear on his *high-order belief* about labor manager's nowcast on a_{it}, and therefore it is necessary for him to track the evolution of both first-order beliefs $a_{it}|\mathcal{I}_{it}^k \sim N(\hat{a}_{it}^k, \hat{\Sigma}_t^k)$ and high-order beliefs $\hat{a}_{it}^n|\mathcal{I}_{it}^k \sim N(\hat{a}_{it}^{(k,n)}, \hat{\Sigma}_t^{(k,n)})$ along the belief updating process. Second, it tells us that the elasticity of hiring to firm fundamental, F_h^a, directly enters the belief formation process by affecting the nowcast error $h_{it} - \hat{h}_{it}^k$. This channel of policy matrix affecting beliefs will be in effect if and only if there is information asymmetry between the managers: if the information is symmetric within the firm, the capital manager knows that the labor manager shares the same information with him, which allows him to know the labor manager's action exactly, i.e. $h_{it} = \hat{h}_{it}^k$. Similarly, the labor manager updates her beliefs using capital manager's past action and policy function (15) in a MPBE. For that purpose, she will need to track her first-order belief $a_{it}|\mathcal{I}_{it}^n \sim N(\hat{a}_{it}^n, \hat{\Sigma}_t^n)$ and high-order belief $\hat{a}_{it}^k|\mathcal{I}_{it}^n \sim N(\hat{a}_{it}^{(n,k)}, \hat{\Sigma}_t^{(n,k)})$ dynamically.

The following proposition characterizes the evolution of the posterior mean $\hat{a}_{it}^k, \hat{a}_{it}^{(k,n)}$ and $\hat{a}_{it}^n, \hat{a}_{it}^{(n.k)}$ for a given pair of policy matrices (F_ι, F_h).

Proposition 2. *For a given pair of policy matrices F_ι and F_h, the posterior first-order and high-order mean $\hat{a}_{it}^k, \hat{a}_{it}^{(k,n)}$, $\hat{a}_{it}^n, \hat{a}_{it}^{(n.k)}$ evolves by*

$$\begin{aligned} \hat{a}_{it}^k &= \hat{a}_{it}^{(k,n)} = (\hat{a}_{it}^k)^- + G_t^k \left(z_{it}^k - H(\hat{a}_{it}^k)^- \right) && (18) \\ \hat{a}_{it}^n &= \hat{a}_{it}^{(n,k)} = (\hat{a}_{it}^n)^- + G_t^n \left(z_{it}^n - H(\hat{a}_{it}^n)^- \right) && (19) \end{aligned}$$

where $z_{it}^k = [a_{it}^p, a_{it}^k]'$, $z_{it}^n = [a_{it}^p, a_{it}^n]'$, $H = [1,1]'$, with the Kalman gains being

$$G_t^k = (\hat{\Sigma}_t^k)^- H' \left[H(\hat{\Sigma}_t^k)^- H' + W^k \right]^{-1} \text{ and } G_t^n = (\hat{\Sigma}_t^n)^- H' \left[H(\hat{\Sigma}_t^n)^- H' + W^n \right]^{-1}$$

and the pre-estimates $(\hat{a}_{it}^k)^-, (\hat{a}_{it}^n)^-$ are given by

$$\begin{aligned} (\hat{a}_{it}^k)^- &= \rho(\hat{a}_{i,t-1}^k)^+ = \rho \left[\hat{a}_{i,t-1}^k + J_{t-1}^k (h_{i,t-1} - \hat{h}_{i,t-1}^k) \right] && (20) \\ (\hat{a}_{it}^n)^- &= \rho(\hat{a}_{i,t-1}^n)^+ = \rho \left[\hat{a}_{i,t-1}^n + J_{t-1}^n (\iota_{i,t-1} - \hat{\iota}_{i,t-1}^n) \right] && (21) \end{aligned}$$

with the Kalman gain being

$$J_{t-1}^k = (\hat{\Sigma}_{t-1}^k - \tilde{\Sigma}_{t-1}^{(k,n)}) \left[\hat{\Sigma}_{t-1}^{(k,n)} F_h^a \right]^{-1} \text{ and } J_{t-1}^n = (\hat{\Sigma}_{t-1}^n - \tilde{\Sigma}_{t-1}^{(k,n)}) \left[\hat{\Sigma}_{t-1}^{(k,n)} F_\iota^a \right]^{-1}$$

The evolution of the first-order and higher order uncertainty $(\hat{\Sigma}_t^k, \hat{\Sigma}_t^n, \hat{\Sigma}_t^{(k,n)})$ is defined by a triplet of Riccati equations.

Proof. See Appendix 2. □

In Proposition 2, the belief updating from period $t-1$ to t is done in two steps. In the first step, the managers observe the other manager's action in the previous period, $h_{i,t-1}$ and $\iota_{i,t-1}$, and use them to form a pre-estimate $(\hat{a}^k_{it})^-$ and $(\hat{a}^n_{it})^-$, according to (20) and (21). In the second step, the managers use the signal realization in period t to update their beliefs on top of these pre-estimates, following equation (18) and (19). Note that the first step of belief updating exists if and only if the managers' information sets are non-nested. Absent asymmetric information, there would be no strategic uncertainty between managers, so that $h_{i,t-1} = \hat{h}^k_{i,t-1}$ and the equation (20) degenerates to $(\hat{a}^k_{it})^- = \rho \hat{a}^k_{i,t-1}$. In that case, the labor manager's action $h_{i,t-1}$ is already contained in the capital manager's information set in period $t-1$, and hence revealing $h_{i,t-1}$ to the capital manager doesn't give him additional information about the unobserved firm fundamental.

Now we have all the ingredients to characterize a linear MPBE associated with (13) and (14).

Definition 1. A linear Markov Perfect Bayesian Equilibrium is a set of policy (F_ι, F_h) and beliefs $(\hat{a}^k_{it}, \hat{\Sigma}^k_t)$, $(\hat{a}^n_{it},, \hat{\Sigma}^n_t)$, $(\hat{a}^{(k,n)}_{it}, \hat{\Sigma}^{(k,n)}_t)$, and $(\hat{a}^{(n,k)}_{it}, \hat{\Sigma}^{(n,k)}_t)$ such that

- Beliefs are consistent with strategies: Given policy (F_ι, F_h), the capital and labor managers' update their beliefs $(\hat{a}^k_{it}, \hat{\Sigma}^k_t)$, $(\hat{a}^n_{it}, \hat{\Sigma}^n_t)$, $(\hat{a}^{(k,n)}_{it}, \hat{\Sigma}^{(k,n)}_t)$, and $(\hat{a}^{(n,k)}_{it}, \hat{\Sigma}^{(n,k)}_t)$ by the Bayes rule, according to Equations (18) to (19) in Proposition 2;
- Strategies are consistent with beliefs: Given the beliefs $(\hat{a}^k_{it}, \hat{\Sigma}^k_t)$, $(\hat{a}^n_{it},, \hat{\Sigma}^n_t)$, $(\hat{a}^{(k,n)}_{it}, \hat{\Sigma}^{(k,n)}_t)$, and $(\hat{a}^{(n,k)}_{it}, \hat{\Sigma}^{(n,k)}_t)$, the policy matrices (F_ι, F_h) solves the Bellman equations (5) and (6), and the policy function for investment and hiring satisfy (15) and (16), as in Proposition 1.

The linear MPBE in Definition 1 implies a law of motion for the joint distribution of action, expectation and state, namely, the vector $\equiv [k_{it}, n_{it}, \hat{a}^k_{it}, \hat{a}^n_{it}, a_{it}]'$. A stationary equilibrium requires this distribution to be at its stationary distribution. The computation of this stationary distribution is essential to the quantitative analysis in Section 4 and the details of computation are left to Appendix A.4.

3 Testing Narrow Framing

I now illustrate how I use the above framework to test the hypothesis that the decisions within a firm are made under the same information (I call it the "common information hypothesis"). The key variables that I will use in the test are the capital manager's forecast error of investment $\iota_{it} - \mathbb{E}[\iota_{it}|\mathcal{I}^k_{i,t-1}]$, the lagged hiring decisions h_{t-1}, and the lagged nowcast error of sales $y_{i,t-1} - \hat{y}^k_{i,t-1}$ for the capital manager. In this section, I will first show the relationship between these variables under the narrow framing model, and then propose and implement the test for the common information hypothesis.

3.1 Implication of Narrow Framing on Investment Forecast Error

I first characterize the investment forecast error under the above narrow framing framework. From the linear investment decision rule (15) and the belief updating equation (18) and (20) for the capital manager, the investment forecast error can be written as

$$\iota_{it} - \mathbb{E}^k_{i,t-1}[\iota_{it}] = \beta_1(h_{i,t-1} - \hat{h}^k_{i,t-1}) + \beta_2(a_{i,t-1} - \hat{a}^k_{i,t-1}) + \beta_3(H\mu_{it} + w^k_{it}) \tag{22}$$

where $\beta_1, \beta_2 \in \mathbb{R}$ and $\beta_3 \in \mathbb{R}^{1\times 2}$ are constants in terms of the policy matrices F_ι, F_h as in Proposition 1 and Kalman gain matrices G^k, G^n, J^k, J^n in Proposition 2. The detail of the derivation is in Appendix A.5.

Equation (22) tells us that the capital manager's investment forecast error consists of three components. The first component is the nowcast error of the labor manager's hiring decision in period $t-1$, which results from the strategic uncertainty of between the managers under asymmetric information. The second term is the capital manager's nowcast error about the unknown firm fundamental $a_{i,t-1}$, which reflects the posterior first-order uncertainty of the capital manager due to the information frictions. The last term is a linear combination of period-t firm fundamental innovation μ_{it} and the noises $w^k_t = [\epsilon^p_{it}, \epsilon^k_{it}]$ from the capital manager's period-t noisy signals. They are orthogonal to the rest of period-$(t-1)$ variables on the right hand side of (22).

The following result is immediate from (22):

Proposition 3. *If information sharing within the firm is frictionless, so that there is no asymmetric information between the capital and labor manager, then the investment forecast* *error $\iota_{it} - \mathbb{E}^k_{i,t-1}[\iota_{it}]$ should be independent of lagged hiring $h_{i,t-1}$, after controlling for the lagged fundamental nowcast error $a_{i,t-1} - \hat{a}^k_{i,t-1}$.*

Intuitively, absent asymmetric information between capital and labor manager, then the labor manager's past action $h_{i,t-1}$ should already be in the capital manager's information set $\mathcal{I}^k_{i,t-1}$, and the revelation of $h_{i,t-1}$ should add no more information on top of $\hat{h}_{i,t-1}$. In that case, $h_{i,t-1} = \hat{h}^k_{i,t-1}$, and equation (22) degenerates to

$$\iota_{it} - \mathbb{E}^k_{i,t-1}[\iota_{it}] = \beta_2(a_{i,t-1} - \hat{a}^k_{i,t-1}) + \nu_{it}$$

where $\nu_{it} = \beta_3(H\mu_{it} + w^k_{it})$. In other words, without asymmetric information, the only period-$(t-1)$ information that the capital manager did not pick up in his period-$(t-1)$ dataset is the actual realization of the fundamental $a_{i,t-1}$.

Proposition 3 alludes to an approach to directly test the existence of asymmetric information among managers within the firm. If we regress the capital manager's investment forecast error on the lagged hiring and lagged fundamental nowcast error, from Proposition 3, we know that the coefficient on the lagged hiring will be 0 if and only if the investment and hiring were based on the same information in period $t-1$, and be nonzero as long as there is information asymmetry within the firm.

3.2 Mapping Theory to Data

Although Proposition 3 suggests a promising way to test the common information hypothesis, it cannot be directly brought to data because (1) the expectation of firm fundamentals a_{it} is unavailable from most of the firm-level expectation data (such as IBES),[4] and (2) the measurement of the firm fundamentals a_{it} in the firm-level outcome dataset (such as Compustat) is indirect and subject to noises. Hence, it is difficult to directly control for the fundamental nowcast error $a_{i,t-1} - \hat{a}^k_{i,t-1}$ in (22). I get around this issue by replacing $a_{i,t-1} - \hat{a}^k_{i,t-1}$ with the widely reported sales expectation and sales outcome. We know that the logged sales $y_{i,t-1}$ satisfy

$$y_{i,t-1} = (1 - 1/\epsilon)a_{i,t-1} + \hat{\alpha}_1 k_{i,t-1} + \hat{\alpha}_2 n_{i,t-1}$$

[4] An exception is the Duke/Fed Richmond CFO survey Graham (2022), which elicits CFO's one-year-ahead expectations about wage, productivity, interest rate as well as their decisions in each quarter. However, that dataset does not include time-consistent firm ID, which leads to huge sample loss in the analysis. Hence, I refrain from using it for the analysis here.

and therefore the sales nowcast error in period $t-1$ can be written as

$$y_{i,t-1} - \hat{y}^k_{i,t-1} = (1 - 1/\epsilon)(a_{i,t-1} - \hat{a}^k_{i,t-1}) + \hat{\alpha}_2(h_{i,t-1} - \hat{h}^k_{i,t-1})$$

i.e. it is a linear combination of the fundamental nowcast error and the hiring nowcast error. Plug it into (22), we get a model-implied relationship between investment forecast error, lag fundamental nowcast error, lag fundamental hiring nowcast error, and time-t innovations/noises:

$$\iota_{it} - \mathbb{E}^k_{i,t-1}[\iota_{it}] = \beta_h(h_{i,t-1} - \hat{h}^k_{i,t-1}) + \beta_y(y_{i,t-1} - \hat{y}^k_{i,t-1}) + \nu_{it} \tag{23}$$

Equation (23) is a model-implied reduced-form relationship between $\iota_{it} - \mathbb{E}^k_{i,t-1}[\iota_{it}]$, h_{it} and $y_{i,t-1} - \hat{y}^k_{i,t-1}$, all of which are available or can be proxied from the firm-level expectation and outcome data.

3.3 Testing Common Information Hypothesis

In a merged firm-level expectation-outcome dataset with $\iota_{it} - \mathbb{E}^k_{i,t-1}[\iota_{it}]$, h_{it} and $y_{i,t-1} - \hat{y}^k_{i,t-1}$, I propose the following regression

$$\iota_{it} - \mathbb{E}^k_{i,t-1}[\iota_{it}] = \tilde{\beta}_h h_{i,t-1} + \beta_y(y_{i,t-1} - \hat{y}^k_{i,t-1}) + \nu_{it} \tag{24}$$

to test the common information hypothesis directly. The coefficient of interest is $\tilde{\beta}_h$. From the lens of the model-implied reduced-form equation (23), the regression coefficient $\tilde{\beta}_h$ picks up both the direct effect of $h_{i,t-1}$ on the investment forecast error and the indirect effect of $h_{i,t-1}$ via the expectation $\hat{h}^k_{i,t-1}$, holding $y_{i,t-1} - \hat{y}^k_{i,t-1}$ fixed:

$$\tilde{\beta}_h = \left.\frac{\partial(\iota_{it} - \mathbb{E}^k_{t-1}[\iota_{it}])}{\partial h_{it}}\right|_{y_{i,t-1} - \hat{y}^k_{i,t-1} \text{ is fixed}} = \beta_h \left(1 - \underbrace{\frac{\partial \hat{h}^k_{i,t-1}}{\partial h_{i,t-1}}}_{\equiv \zeta_h}\right) \tag{25}$$

As long as $\beta_h \neq 0$, we can see that (25) provides a valid test for the common information hypothesis:

- If the investment and hiring decisions are based upon same information in period $t-1$, then there is no strategic uncertainty for the capital manager, so that $\hat{h}^k_{i,t-1} = h_{i,t-1}$

and hence $\zeta_h = 1$ and $\tilde{\beta}_h = 0$.

- If the investment and hiring decisions are based on different, non-nested information in period $t-1$, then $\frac{\partial \hat{h}^k_{i,t-1}}{\partial h_{i,t-1}} \neq 1$ since the capital manager's expectation about the labor manager's action in period $t-1$ is not perfect, and, as a result $\zeta_h \neq 1$ and $\tilde{\beta}_h \neq 0$.

Hence, if we are able to reject the null hypothesis that $\tilde{\beta}_h = 0$, we can conclude that the investment and hiring decisions are made under different, non-nested information within the firm.

More generally, test (24) can be regarded as a test of whether hiring is contained in the information set of the capital manager when he is making the investment decisions. It echos the insights from the literature of forecast error regressions (e.g. Hall (1978), Coibion and Gorodnichenko (2012, 2015)) in the test of full-information rational expectation (FIRE): any information already contained in the period $t-1$ information set should not be able to help us predict the forecast error in period t.

3.4 Application to Data

I now apply the test (24) to a merged firm-level expectation-outcome dataset for US public firms. For the expectation data, I use the IBES Guidance dataset, which contains all the guidance activities (i.e. the firm's public announcement on its own expectations about sales, capital expenditure, earnings per share, among many others, for the current fiscal year) of US public firms since 1992. Since all measurements in the guidance are financial indicators, I assume that the expectations posted in IBES Guidance mainly reflect the capital manager's view about the firm's performance.

According to (24), the firm-level expectation that we are interested in is (1) the forecast of investment $\mathbb{E}^k_{i,t-1}[\iota_{it}]$ and (2) the nowcast of sales in period $t-1$, $\mathbb{E}^k_{i,t-1}[y_{i,t-1}]$. Hence, I focus on the US public firms in the IBES dataset that post both capital expedenditure and sales guidance in a given year. Fortunately, IBES dataset has a wide coverage of sales and capital expenditure guidance,[5] as they are the two most reported measurements in guidance activities. However, since guidance activities are self-reported and can happen in any time of the fiscal year, it is difficult to get a notion for the time horizon of the forecast

[5]Here, I regard capital expenditure (CPX) as a proxy for investment, although it is not exactly the same as the ι_{it} throughout the paper.

from the announcement date. Here, I restrict my attention to the firms that have multiple guidance activities within the same year, and in that given year, I take the first posted capital expenditure guidance as a proxy for the one-period-ahead forecast of investment $\mathbb{E}^k_{i,t-1}[\iota_{it}]$, and I take the last posted sales guidance as a proxy for the nowcast of sales $\mathbb{E}^k_{i,t-1}[y_{i,t-1}]$.

I then merge the IBES Guidance dataset to CRSP/Compustat Fundamental Annual for the realized outcome for each firm-year expectations in IBES. For each firm-year observation, I take the realized sales and capital expenditure from Compustat and impute the forecast error as the log difference between the realized outcome and the first reported guidance in the fiscal year, and the nowcast error as the log difference between the realized outcome and the last reported guidance in the fiscal year. I trim the 3% tails for both the sales nowcast errors and capital expenditure forecast errors to remove the outliers. The final merged sample covers 6,839 firm-year observations spanning over 2004 to 2024.

Table 1 tabulates the regression result for the tests of common information hypothesis. Column (1) and (3) uses our baseline regression (24), with Column (3) controlling for the sector-year fixed effect to absorb the sector-year trends. Column (2) and (4) uses a modified specification that replaces the nowcast error of sales with the lagged sales on the right hand side of (24) for the concern that $\hat{y}^k_{t-1}$ may not be properly proxied by the last posted guidance of the year.

We can see from the first row of Table 1 that in all specifications, the common information hypothesis is clearly rejected: the estimated coefficient $\tilde{\beta}_h$ in all the specifications are nonzero and statistically significant at 1% level. The fact that lagged hiring $h_{i,t-1}$ has statistically significant predictive power on investment forecast error confirms that hiring is not fully internalized in the capital manager's information set in period $t-1$ and is consistent with narrow framing rather than standard models of input choices under symmetric information.

The coefficient estimates for $\tilde{\beta}_h$ are similar across the specifications. To interpret the magnitude of this coefficient, recall from (25) that $\tilde{\beta}_h = \beta_h(1-\zeta_h)$. Using the calibrated parameters in Section 4, $\beta_h \approx 0.09$, and hence an estimate of $\tilde{\beta}_h \approx 0.06$ means that for each 1 percent growth in hiring, the capital manager's expectation on hiring increases by only a third percentage point. The severe information rigidity of hiring expectation indicates that a substantial part of labor manager's information is not integrated in the capital manager's information set.

	Dependent variable: $\iota_t - \mathbb{E}^k_{t-1}[\iota_t]$			
	(1)	(2)	(3)	(4)
h_{t-1}	0.057***	0.062***	0.055***	0.058***
	(0.020)	(0.020)	(0.020)	(0.020)
$y_{t-1} - \hat{y}^k_{t-1}$	0.051**		0.054**	
	(0.024)		(0.023)	
y_{t-1}		0.010***		0.013***
		(0.003)		(0.003)
Constant	−0.092***	−0.171***		
	(0.004)	(0.024)		
Fixed Effects	No	No	Sector-Year	Sector-Year
Observations	3,809	3,809	3,809	3,809
R^2	0.004	0.005	0.051	0.053
Residual Std. Error	0.266 (df = 3806)	0.266 (df = 3806)	0.261 (df = 3749)	0.261 (df = 3749)
F Statistic	6.909***	10.079***	6.914***	11.270***
Note:				*p<0.1; **p<0.05; ***p<0.01

Table 1: Tests of Common Information Hypothesis

4 Quantitative Analysis

After establishing the theory and evidence of the narrow framing at the firm level, I quantify the aggregate effect of narrow framing in this section. I will first discuss the calibration strategy to separate narrow framing from adjustment frictions, and then use the calibrated parameters to investigate the pattern of learning and quantify the capital and labor misallocation and aggregate TFP loss from information and adjustment frictions. Lastly, I show that narrow framing can be a promising source that distorts the capital-labor ratio and help match relative misallocation between labor and capital as an untargeted moment.

4.1 Identification

Now I discuss how I pick the targeted moments to identify and discipline the adjustment and information friction parameters in the model. There are two key challenges that I need to address in the quantitative exercise. First, I need to find moments that speaks to the difference between the firm-level and manager-level information in firm dynamics, since they have similar attenuation forces on action-fundamental correlations. Additionally, I need to find moments that help me separate the role of adjustment and information friction on dampening the action-fundamental correlations.

Firm-level vs Manager-level Information The novel part of narrow framing from the standard firm dynamics models with uncertainty/information frictions is the existence of asymmetric information among managers, and one may wonder whether and how noisy firm-level information and noisy manager-level information have different implications for firm dynamics.

Figure 1 illustrates how we can distinguish labor manager's private information from the firm-level information by jointly targeting two moments: the action-state correlation ($\text{corr}(h, \Delta a)$) and correlation between actions ($\text{corr}(h, \iota)$). On each panel, I fix all other parameters at their estimated values, vary only the parameter on the x-axis and plot the model-implied moments. It is not surprising to see that the action-state correlation decreases with the variances of both firm-level and manager-level information, because both types of noises make signal extraction more difficult. What makes manager-level information different from firm-level information is its implication on the cross-action correlation $\text{corr}(\iota, h)$. As firm-level information becomes more precise (so that $\sigma_{\epsilon,p}$ becomes lower), we can see that the capital-labor correlation goes up as shown on the left panel of Figure 1. In contrast, as manager-level information gets more precise (so that $\sigma_{\epsilon,n}$ becomes smaller), the capital-labor correlation goes down. Intuitively, as the firm-level information gets more precise, both the capital and labor manager observe a more precise signal about the firm fundamental and will put more weight on it in their decision making. Additionally, the more precise firm-level signal is also shared by the capital manager, and his decision will be more responsive to the firm-level signal as well. As a result, variations in the firm-level signal creates a stronger comovement between capital and labor when it becomes more precise. In fact, absent adjustment frictions, we know that the capital and labor decision rules are $k_{it} = n_{it} = (\epsilon - 1)\mathbb{E}_{it}[a_{it}]$, in which case the investment and hiring are always perfectly coordinated by the common firm-level information they observe, regardless of its precision. Instead, if the manager-level information becomes more precise, the labor manager will put more weights on her private signal than the firm-level signal. As a result, the comovement between capital and labor generated by the common firm-level information is weakened, which leads to a lower capital-labor correlation. The role of firm-level information being a internal coordination device is smaller.

The different implications of firm-level and manager-level information on the capital-labor

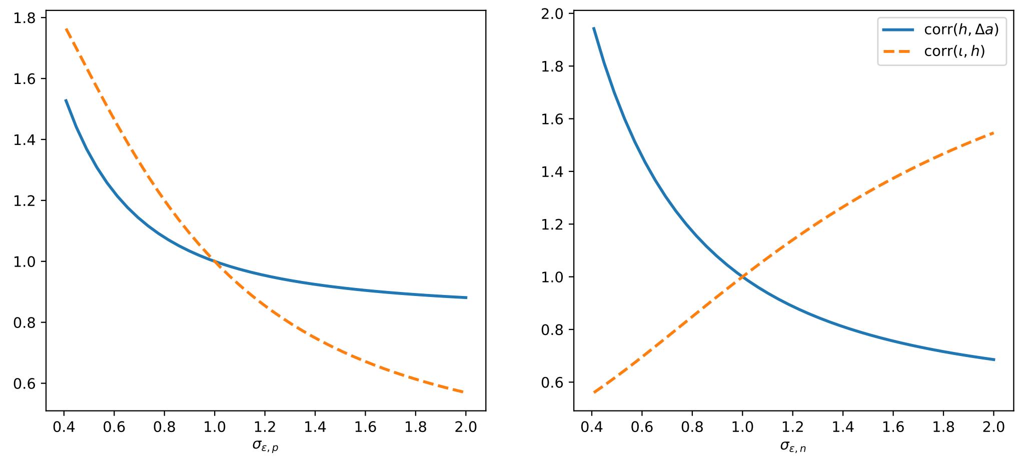

Figure 1: Firm-level vs. Manager-level Information

Note: The parameter values on the x-axis are rescaled as multiples of the estimated value. The y-axis is the ratio of moment implied by the parameter value on the x-axis to the model-estimated moments.

correlation indicate that they are essentially different types of frictions. In the language of Hsieh and Klenow (2009), firm-level noisy signals that are perfectly shared among all decision-making managers are akin to "τ_Y", the distortions that affect the marginal revenue product of capital and labor jointly without distorting the capital-labor ratio, while the manager-level private noisy signals can be interpreted as "τ_K", the distortions that affects the marginal revenue product of one input more than the other and distorts the capital-labor ratio. We see that the capital-labor ratio corr(ι, h) is an informative moment that helps us distinguish between these types of distortions.

Adjustment vs Information Friction It is also important to distinguish the adjustment frictions and information frictions as they both help dampen the responsiveness of capital and labor to firm fundamentals and hence lower corr$(\iota, \Delta a)$ and corr$(h, \Delta a)$. Figure 2 illustrates how we can distinguish adjustment frictions from information frictions through the lens of two moments: serial correlation of actions (corr(ι, ι_{-1})) and action-fundamental correlation (corr$(\iota, \Delta a)$). We can see that larger adjustment friction lowers the investment-fundamental correlation and increases the serial correlation, while larger degree of information friction (here, higher variance for firm-level information $\sigma_{\epsilon,p}$) lowers the investment-fundamental correlation without significantly altering the serial correlation of investment. This echoes the insights in David and Venkateswaran (2019): convex adjustment cost attenuates the responsiveness by making firms intertemporally smooth their reactions, which necessarily enlarges the persistence of actions and leads to higher serial correlation of actions. Information friction, on the other hand, is a mechanism of "concurrent" attenuation: it dampens the responsiveness by the difficulty of distinguishing between the fundamental shocks and the i.i.d. signal noises in the current period. A similar pattern holds for labor decisions as well.

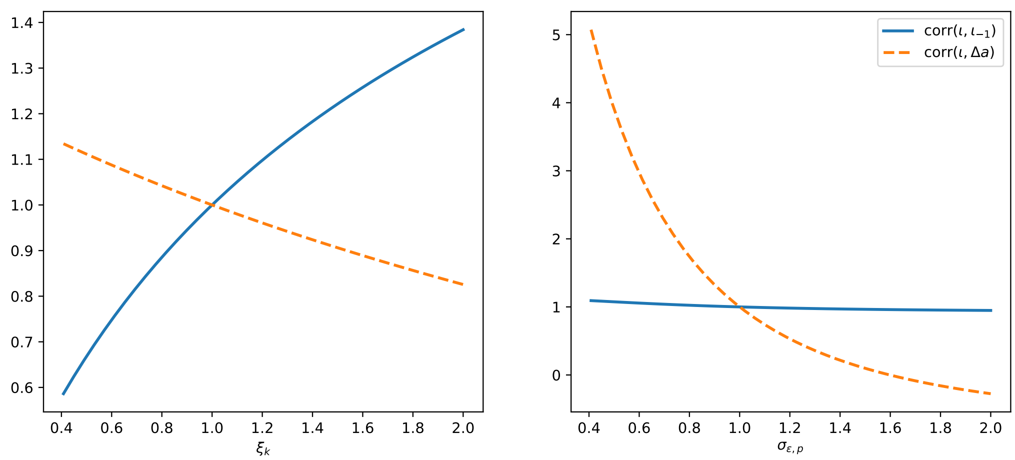

Figure 2: Adjustment vs. Information Friction

Note: The parameter values on the x-axis are rescaled as multiples of the estimated value. The y-axis is the ratio of moment implied by the parameter value on the x-axis to the model-estimated moments.

Lessons for Choosing Targeted Moments The previous discussions imply that it is necessary to jointly target the action-fundamental correlation with

- cross-action correlation, to distinguish between manager-level and firm-level information (or, more generally, joint distortion vs. input-specific distortion), and
- serial correlations of actions, to distinguish between adjustment frictions and information frictions.

Overlooking cross-action correlation will lead us to misunderstand the nature of information friction in effect, and overlooking serial correlations of action will make us overstate the significance of information friction in attenuating the action-fundamental correlations.

4.2 Data and Parametrization

Now I discuss the details of the quantitative analysis. I will first summarize how I construct the dataset in the analysis, and then I will show my parametrization and calibration strategy.

Data For the quantitative analysis, I use Compustat Fundamentals Annual, which provides financial information (such as balance sheets, earnings statements, etc.) at the firm level for all US public firms. It provides a historically consistent classification of industry by SIC or NAICS codes, as well as measures of the capital stock and employment. My Compustat sample spans from 1998 to 2021 and exclude firms that are not incorporated in the US, do not use the US dollar as currency, belong to the utility or finance sector, experience large mergers and acquisitions, or have missing values in key variables of interest.

From the Compustat sample, I take the book value of property, plant and equipment (PPE) as the measure for capital stock.[6] Investment rate ι and hiring rate h are obtained by first-differencing the logged capital and employment series respectively. Value-added is constructed by sales minus the material expenditure.[7] I then construct the total revenue factor productivity (TFPR) or sales Solow residual as the firm fundamental a_{it} by

$$a_{it} = \log(\text{Value-added}) - \alpha(1 - 1/\epsilon)\log(\text{PPE}) - (1 - \alpha)(1 - 1/\epsilon)\log(\text{Employment})$$

where the capital share α is chosen to match the cost share of labor in the sample, and the elasticity of substitution $\epsilon = 4$ is taken from the literature (Hsieh and Klenow (2009)). As standard in the literature, the measure for capital and labor misallocation is the dispersion of the average/marginal revenue product of capital and labor (MRPK or MRPN). I define MRPK as the difference between logged value-added and logged capital and MRPN as the difference between logged value-added and logged employment. Following David and Venkateswaran (2019), I regress the MPRK, MRPN, investment, hiring and firm fundamental series on a sector-year fixed effect and keep the residual to isolate the firm-specific variation in each series.[8] I trim 3% tails of each series to exclude the outliers in the analysis.

[6]The quantitative results are robust to alternative definitions of capital stock, e.g. the ones constructed by the perpetual inventory method.

[7]Material expenditure is defined as Cost of Goods Sold (COGS) + Selling, General & Administrative Expenses (XSGA) - Depreciation (DP) - Wage Bill (XLR), following the approach by Keller and Yeaple (2009) and Flynn and Sastry (2020). Since wage bill is missing for about 90% of the firm, I use the two-digit industry-level wages in the Census Bureau County Business Patterns dataset to impute the wage bill for the firm. See Appendix for details.

[8]Here, sector or industry is defined as 2-digit NAICS code for non-manufacturing industries and 3-digit

The final Compustat sample consists of 42,837 firm-year observations.

Parametrization There are in total 10 parameters in the model. Table 2 summarizes the strategy of parameterization.

I start with the externally calibrated parameters. Depreciation rate δ is set to 0.1, which is standard in the misallocation literature (David and Venkateswaran (2019)). Elasticity of substitution ϵ is set to 4. This is on the lower end of the literature, and the purpose of doing so is twofold. First, as pointed out by Hsieh and Klenow (2009), a higher ϵ translates the dispersion in marginal revenue products to aggregate TFP loss more promptly, and hence choosing a lower ϵ gives us a conservative quantification of TFP loss to the mechanisms in the model. Additionally, in our Cobb-Douglas revenue function, a lower elasticity of substitution implies a lower degree of strategic complementarity between capital and labor decisions. From the noisy information literature (e.g. Morris and Shin (2002)), having strong strategic complementarity amplifies the role of firm-level public information as coordinating device between capital and labor, and can hence lead to an overstate of precision of private signals to match the low cross-action correlation in the data.

Parameter	Description	Target
ϵ	Elasticity of substitution	External
δ	Depreciation Rate	External
α	Capital share of income	Share of labor cost in value-added
ρ	Persistence of profit fundamental a_{it}	Estimates of $a_{it} = \rho a_{i,t-1} + \mu_{it}$.
σ_μ^2	Variance of profit shock μ_{it}	
ξ_k	Capital Adjustment Cost	$\text{corr}(\iota, \iota_{-1})$
ξ_n	Labor Adjustment Cost	$\text{corr}(h, h_{-1})$
$\sigma_{\epsilon,p}$	Noise of within-firm public signal	$\text{corr}(\iota, \Delta a)$
$\sigma_{\epsilon,k}$	Noise of capital manager's private signal	$\text{corr}(h, \Delta a)$
$\sigma_{\epsilon,n}$	Noise of labor manager's private signal	$\text{corr}(\iota, h)$

Table 2: Summary of Parametrization

Capital share of income α and the AR(1) parameters for the firm fundamentals are chosen to match the features in the sample. The capital share α is chosen to match the share of total labor expenditure in total value-added. In the Compustat sample, $\alpha \approx 0.42$ and is higher than the value (0.33) usually taken for the US economy. The persistence ρ and standard deviation of fundamental innovations σ_μ are chosen to match the persistence and standard deviation of shocks in the sales Solow residual. The parameter values for the parameters calibrated outside the estimation are summarized in Table 3.

NAICS code for manufacturing industries.

Parameter	Description	Value
α	Capital share of income	0.42
δ	Capital depreciation rate	0.1
ϵ	Elasticity of substitution	4
ρ	Persistence of Firm Fundamental a_{it}	0.92
σ_μ	Std. Dev of Firm Fundamental Innovation μ_{it}	0.22

Table 3: Parameters Fixed outside the Estimation

For the rest five parameters, namely, the adjustment cost parameters for capital ξ_k and for labor ξ_n, the standard deviation of firm-level signal noise $\sigma_{\epsilon,p}$ and the standard deviations of manager-level signal noises $\sigma_{\epsilon,k}, \sigma_{\epsilon,n}$, I use a simulated method of moments algorithm to match five moments: serial correlation of investment $\text{corr}(\iota, \iota_{-1})$, serial correlation of hiring $\text{corr}(h, h_{-1})$, investment-fundamental correlation $\text{corr}(\iota, \Delta a)$, hiring-fundamental correlation $\text{corr}(h, \Delta a)$ and investment-hiring correlation $\text{corr}(\iota, h)$. I search for the parameters $(\xi_k, \xi_n, \sigma_{\epsilon,p}, \sigma_{\epsilon,k}, \sigma_{\epsilon,n})$ that minimizes the unweighted sum of quadratic deviations of model-implied moments from data moments. The computation algorithm is summarized in Appendix A.4.

4.3 Results

Table 4 tabulates the parameter estimates and compares the model-implied moments with the data moments. The numbers in parenthesis are bootstrapped standard error from the data. The small standard errors show that these targeted moments are accurately estimated in the data. To make the parameters easier to interpret, I report the adjustment cost parameters in terms of its fraction in the marginal cost of capital (i.e. the steady-state rental rate) and labor (e.g. the steady state wage rate), denoted as $\hat{\xi}_k$ and $\hat{\xi}_n$, and I report the capital manager's posterior uncertainty $\hat{\Sigma}_k$, labor manager's posterior uncertainty $\hat{\Sigma}_n$, and the high-order uncertainty between the managers $\hat{\Sigma}_{(k,n)}$ implied by the estimated information parameters $(\sigma_{\epsilon,p}, \sigma_{\epsilon,k}, \sigma_{\epsilon,n})$.

We can see from Table 4 that the dynamic narrow framing model with adjustment cost, firm-level public information and manager-level private information can almost match the data moments. The estimation results are consistent with the intuition we built in the identification subsection. We can see that capital is estimated to have a higher adjustment cost (as a fraction of its marginal cost) than labor, which is consistent with the fact that the serial correlation of investment is higher than the serial correlation of hiring. The estimated

Moments	corr(ι, ι_{-1})	corr(h, h_{-1})	corr$(\iota, \Delta a)$	corr$(h, \Delta a)$	corr(ι, h)
Data	0.2808	0.1838	-0.0045	0.2362	0.493
	(0.0115)	(0.0108)	(0.01)	(0.01)	(0.0087)
Model-implied Values	0.2842	0.1945	0.0493	0.2189	0.4752
Parameter Estimates	$\hat{\xi}_k$	$\hat{\xi}_n$	$\hat{\Sigma}_k$	$\hat{\Sigma}_n$	$\hat{\Sigma}_{(k,n)}$
	0.2845	0.1272	0.1421	0.1219	0.0202
	(0.0216)	(0.0112)	(0.0049)	(0.0054)	(0.0007)

Table 4: Parametrization of Estimated Models

parameters also show that in order to match the investment-fundamental correlation being much lower than the hiring-fundamental correlation, we need the capital manager to subject from more severe information friction, so that his posterior uncertainty is higher than the labor manager's. Lastly, we can see that the high-order uncertainty $\hat{\Sigma}_{(k,n)}$ is non-zero. It indicates that introducing asymmetric information between managers is necessary to match the low investment-hiring correlation in the data.

Source of Learning We are interested in how the firm and managers learn from the firm-level and manager-level noisy signals. Table 5 shows that the capital manager's posterior uncertainty is 42.4% of prior uncertainty (i.e. the variance of the objective distribution of a_{it}), and the labor manager's posterior uncertainty is 36.3% of prior uncertainty, which shows that the noisy signals help reduce subjective uncertainty substantially for both managers. From the estimation, we can also investigate the source of learning for both managers by comparing their Kalman gains on the firm-level public signal and the manager-level private signal. We can see from Table 5 that the labor manager relies primarily on her manager-level private signal in the learning process, while the capital manager learns mainly from the firm-level public signal. Intuitively, since investment has an extremely weak correlation with the firm fundamental, an extremely noisy signal for the capital manager is necessary to attenuate its responsiveness to a_{it}. Additionally, since hiring has a much higher correlation with firm fundamentals in the data than investment, hiring is a more informative signal of the firm fundamental to the capital manager than investment is to the labor manager. As a result, revealing the labor manager's decision in the previous period to the capital manager as part of the firm-level information gives him a better source to learn from the firm fundamental than his own signals. It is also worth noting that the inferred pattern of learning in this structural quantitative analysis is consistent with the reduced-form evidence in 3: due to the dependence of labor manager on her own private signals, there is substantial information content in labor decisions that cannot be internalized by the capital manager in each period.

	Uncertainty		Source of Learning	
	Posterior	Posterior/Prior	Firm-level	Manager-level
Capital Manager	0.142	42.4%	100%	0%
Labor Manager	0.122	36.4%	31.1%	68.9%

Table 5: Posterior Uncertainty and Source of Learning

Investigation of Mechanisms To further understand how adjustment frictions, firm-level public information and manager-level private information affect the firm dynamics, I make use of the rich structure of the dynamic narrow framing framework to recalibrate four alternative specifications to the same set of targeted moments. These alternative specifications include

1. Adjustment Cost only (AC only), where all the information frictions are muted and only capital and labor adjustment costs are in effect. This maps to the parameterization with $\xi_k, \xi_n \geq 0$ and $\sigma_{\epsilon,p} = 0, \sigma_{\epsilon,k} = \sigma_{\epsilon,n} = \infty$.

2. Narrow Framing only (NF only), where all the adjustment frictions and muted and only information frictions are kept. This maps to the parametrization with $\xi_k = \xi_n = 0$ and $0 < \sigma_{\epsilon,p}, \sigma_{\epsilon,k}, \sigma_{\epsilon,n} < \infty$.

3. Adjustment Cost with firm-level public information (AC + firm info), where all the manager-level private information are muted and adjustment frictions and firm-level public noisy information are in effect. This maps to the parametrization with $\xi_k, \xi_n \geq 0$, $0 < \sigma_{\epsilon,p} < \infty$ and $\sigma_{\epsilon,k} = \sigma_{\epsilon,n} = \infty$.

4. Adjustment Cost with manager-level private information (AC + manager info), where there is no firm-level public information available in each period. This maps to the parametrization with $\xi_k, \xi_n \geq 0$, $\sigma_{\epsilon,p} = \infty$ and $0 < \sigma_{\epsilon,k}, \sigma_{\epsilon,n} < \infty$.

As we can see, all the alternative specifications are nested by the general dynamic narrow framing framework and can therefore be readily parameterized by imposing restrictions on a subset of dynamic narrow framing parameters. I recalibrate these alternative specifications, compute the implied targeted and untargeted moments and compare them with ones

Moments	Data	Model-Implied Values				
		Dynamic NF	AC only	NF only	AC + firm info	AC + manager info
$\text{corr}(\iota, \iota_{-1})$	0.2808	0.2842	0.9286	-0.0388	0.4257	0.2885
$\text{corr}(h, h_{-1})$	0.1838	0.1945	0.6626	-0.0388	0.1452	0.3329
$\text{corr}(\iota, \Delta a)$	-0.0045	0.0493	0.1906	0.0819	0.1107	-0.0457
$\text{corr}(h, \Delta a)$	0.2362	0.219	0.6831	0.1958	0.135	0.2344
$\text{corr}(\iota, h)$	0.493	0.4752	0.5016	0.4742	0.956	0.3323
Loss	-	0.0036	0.8866	0.1611	0.2603	0.0498

Table 6: Targeted Moments, Data vs Models

implied by the parameterization in Table 4 for the dynamic narrow framing model (labeled as Dynamic NF).

We first look into the performance of different specifications in their ability to match the targeted moments. Figure 6 tabulates the model-implied targeted moments, the data moments and the sum of quadratic loss from data moments. As we can see, the dynamic narrow framing model with adjustment frictions, firm-level and manager-level noisy information performs the best as it can almost match the data moments. None of the alternative specifications is able to match the five targeted moments simultaneously.

The reasons why the alternative models miss the targets give us important lessons about the multiple mechanisms at play in the dynamic narrow framing framework. The first takeaway is that convex adjustment cost is not a convincing modeling choice for attenuating action-state responsiveness. As we can see from the "AC only" column in Table 6, in order to match the low correlation between capital and labor to firm fundamentals, we need unrealistically high adjustment cost parameters ξ_k, ξ_n for both capital and labor adjustment, which generates unrealistically high serial correlations in investment and hiring. Therefore, it is unlikely that convex adjustment cost is the main driving force of attenuating the action-state correlations, and introducing concurrent dampening forces, such as information frictions or correlated distortions, is necessary and more promising to match the pattern.

The second takeaway is that it's misleading to model information friction using only the firm-level public noisy information, as it is inconsistent with the targeted moments. To see that, we look into the "AC + firm info" column in Table 6. We can see that compared with the "AC only" specification, introducing firm-level noisy information helps attenuate the action-state correlations without scaling up the adjustment cost parameters and the serial correlation of actions. However, the investment-hiring correlation implied by the "AC + firm info" is equal to 0.96, which is much higher than the investment-hiring correlation in the data. Despite being a popular benchmark to introduce information friction to firm dynamics, the

"AC + firm info" specification makes the capital and labor decisions excessively coordinated by the firm-level information commonly known by the managers. In comparison, manager-level private noisy information seems a promising force of generating both low action-state correlations and low cross-action correlations. From the "NF only" column of Table 6, we can see that without the help of adjustment frictions, the firm-level and manager-level information frictions can already reproduce a decent match in action-state correlations and the investment-hiring correlation.

The results in Table 6 confirm that the best way to model information frictions for the firm is to incorporate both firm-level and manager-level noisy signals. Comparing the column "Dynamic NF" and "AC + manager info", we can see that completely muting the public signal leads the model to have too low investment-hiring correlation and hence overstating the degree of information segmentation or communication frictions within the firm. Essentially, a combination of firm-level public information and manager-level private information provides a unique bundle of "joint distortion" τ_Y and "input-specific distortion" τ_K in Hsieh and Klenow (2009).

		Model-Implied Values				
Moments	Data	Dynamic NF	AC only	NF only	AC + common info	AC + private info
var(ι)	0.0249	0.1186	0.0	0.2455	0.0617	0.113
var(h)	0.024	0.1357	0.0086	0.2664	0.1102	0.0979
$\frac{\text{var}(mrpn)}{\text{var}(mrpk)}$	0.4314	0.575	0.0799	0.6682	0.8845	0.4552

Table 7: Untargeted Moments, Data vs Models

Relative Misallocation between Inputs I now compare the performance of the models on several important untargeted moments in Table 7.

An important untargeted moment is the relative misallocation between labor and capital, $\frac{\text{var}(mrpn)}{\text{var}(mrpk)}$. According to David et al. (2016), it is also a moment sensitive to the nature of distortion. If the face mainly faces "joint distortions", such as a firm-level noisy signal, then the capital and labor will be affected by the same proportion, and hence the resulting MPRN and MPRK dispersion are similar and the ratio between them will be close to 1. On the other hand, if the firm mainly faces "input-specific distortions", such as noisy, private information at the manager level or different degree of capital and labor adjustment costs, then the relative misallocation will be different from 1. Absent adjustment frictions, David et al. (2016) studies two extreme cases of information friction: (1) both capital and labor are chosen under same, imperfect information, and (2) the labor is chosen *ex post* under perfect information and only capital decision is subject to noisy information. In case (1), the relative misallocation is equal to 1 since a commonly shared imperfect information affects both capital and labor by the same proportion. In case (2), the relative misallocation is equal to 0, as labor is chosen frictionlessly.

According to Table 7, the relative misallocation between labor and capital is about 0.43 in the data, falling in between these two extreme cases. Narrow framing can be considered a promising way to provide a smooth model of relative misallocation. Using our estimated value in Table 4, we get a model-implied relative misallocation at 0.58 as an untargeted moment. We can also see from Table 7 that alternative models with segmented information generally outperforms those without information segmentation: the "NF only" model predicts a slightly higher relative misallocation at 0.66, and the "AC + manager info" model gets 0.46, very close to the data counterpart. The other models misses the relative misallocation badly. The "AC only" model predicts a close-to-zero relative misallocation. Although introducing adjustment costs on both inputs can qualitatively give us relative misallocation between 0 and 1, if we discipline the adjustment cost parameters using the action-fundamental correlations, we need an extremely strong capital adjustment cost and an relatively weak labor adjustment cost to match the targeted moments, which makes the relative misallocation falls short of the data. The "AC + common" model, on the other hand, predicts a relative misallocation close to 1. This is because the commonly known noisy signal dominates the adjustment frictions when matching the low action-fundamental correlations and noises up both the capital and labor decisions by the same proportion.

Potential Omitted Variable Bias In Table 7, we can see that our dynamic narrow framing model implies an excessive level of investment and hiring volatility. It is a by-product of severe information friction in the estimates: while information friction dampens the correlation between action and the firm fundamental, it makes the actions series more volatile because the actions are now dependent upon the realized noisy signals, which inserts additional volatility from the signal noises or subjective uncertainty on top of the fundamental uncertainty. Other specifications with information frictions also observe the same pattern. This is a warning flag that indicates mechanisms attenuating the action-fundamental correlation without making the investment and hiring series more volatile may be important yet ignored in the current analysis.

	Data	All Frictions	Adjust. Cost			Info. Frictions			
			ξ_k	ξ_n	Joint	$\sigma_{\epsilon,p}$	$\sigma_{\epsilon,k}$	$\sigma_{\epsilon,n}$	Joint
var(MRPK)	0.4477	0.1255	0.0106	0.0	0.0079	0.1075	0.2051	0.0769	0.1517
var(MRPN)	0.1931	0.0722	0.0	0.0034	0.0021	0.1075	0.0473	0.14	0.0877
$a - a^*$	-	-15.97%	-0.53%	-0.26%	-0.85%	-21.51%	-6.56%	-6.92%	-16.37%

Table 8: Size and Source of Misallocation and Aggregate TFP Loss

Aggregate Effect of Narrow Framing Lastly, I quantify the aggregate effect of the frictions in the dynamic narrow framing model in Table 8. Using the parameters estimated in Table 4, we get a model-implied MRPK dispersion of 0.13 and a model-implied MRPN dispersion of 0.19. Adjustment frictions and information frictions jointly account for about 29% of capital misallocation and about 37% of labor misallocation observed in the data. These MRPK and MRPN dispersions translate mechanically into a sizable 16% of aggregate TFP loss, i.e. the aggregate TFP would be 16% higher if all the frictions in the model were removed.

The right two panels in Table 8 quantify the contribution of adjustment and information frictions to misallocation and aggregate TFP loss. The numbers are calculated under the assumption that there is only one factor (e.g. ξ_k) is in effect and all other frictions are muted. Due to the complicated interactions between factors, these numbers do not sum up to the aggregate quantifications on the second column. The takeaway from this decomposition exercise is that the contribution of information frictions to factor misallocation and aggregate TFP losses is much larger than that of adjustment cost. If we mute the information friction completely (i.e. $\sigma_{\epsilon,p} = 0$) and keep only the labor and capital adjustment costs, the capital and labor misallocation from adjustment frictions can only explain less than 2 percent of MRPK and MRPN dispersions in the data and yields a small TFP loss of less than 1%. The information friction, on the other hand, is much more inductive to MRPK and MRPN dispersions and TFP losses. It is noticeable that the firm-level public noisy information is a particularly strong force in generating factor misallocation and TFP loss. This is because as long as the firm-level commonly known information is noisy, it affects the capital and labor manager by the same proportion and introduce misallocation to both inputs simultaneously, while the other frictions are akin to factor-specific distortions and generate much milder aggregate inefficiency.

5 Conclusion

This paper develops a parsimonious framework, namely "dynamic narrow framing", to incorporate both imperfect observation and imperfect communication within the firm into the analysis of firm's dynamic input choices. The framework makes it possible for us to provide a direct test on whether multiple input decisions of the firm is made under same information. Using a merged expectation-outcome dataset for the US public firms, I reject the null hypothesis and conclude that the investment decisions are not based on the same information as the hiring decisions in the firm. I then use the framework to back out the average adjustment friction and the internal information structure for the US public firms, and quantify their aggregate implications on firm dynamics. I confirm that narrow framing is quantitatively promising. Modeling the information friction as a combination of noisy firm-level and manager-level information is the key to match both the low action-fundamental correlations and the low investment-hiring correlation in the data. Hence, information friction should not only be considered as distorting capital and labor jointly but also an input-specific distortion that alters the capital-labor ratio. Controlling for convex adjustment costs, I find that narrow framing generates strong aggregate effects, explaining about a third of MRPK and MRPN dispersion in the data, and implying a sizable 16% aggregate TFP loss.

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A Theory Appendix

A.1 Second-order Approximation of the Profit Function

Denote the frictionless steady state value of variable X_t as $\bar{X}$, and denote the log deviation of X_t from $\bar{X}$ as x_t.

Frictionless Steady State Fix $\bar{A} = 1$. The profit maximization problem without information and adjustment frictions is to choose K and N to maximize

$$\Pi(K, N) = Y^{1/\epsilon} A^{1-1/\epsilon} (K^{\alpha} N^{1-\alpha})^{1-1/\epsilon} - RK - WN$$

FOC:

$$Y^{\epsilon} A^{1-1/\epsilon} K^{\alpha(1-1/\epsilon)-1} N^{(1-\alpha)(1-1/\epsilon)} \alpha(1 - 1/\epsilon) = R$$

$$Y^{\epsilon} A^{1-1/\epsilon} K^{\alpha(1-1/\epsilon)} N^{(1-\alpha)(1-1/\epsilon)-1} (1 - \alpha)(1 - 1/\epsilon) = W$$

In the quantitative exercise, $\alpha, \epsilon, \rho, \sigma_u, \delta, \beta$ are calibrated outside the model. Given these parameters, the frictionless steady-state values for rental rate, capital, labor and wage can be expressed analytically. First, rental price $\bar{R}$ is

$$\bar{R} = 1/\gamma - 1 + \delta = \alpha(1 - 1/\epsilon)\bar{K}^{\alpha(1-1/\epsilon)-1} N^{(1-\alpha)(1-1/\epsilon)}$$

Normalize $\bar{Y} = 1, \bar{A} = 1$, we get

$$\bar{K}^{\alpha(1-1/\epsilon)} \bar{N}^{(1-\alpha)(1-1/\epsilon)} = 1$$

We can use the above two equations, together with FOCs, to back out steady state $\bar{K}, \bar{N}, \bar{W}$:

$$\bar{K} = \frac{\alpha(1 - 1/\epsilon)}{1/\gamma - 1 + \delta}$$

$$\bar{N} = \left(\frac{1/\gamma - 1 + \delta}{\alpha(1 - 1/\epsilon)} \right)^{\alpha/(1-\alpha)}$$

$$\bar{W} = \bar{K}^{\alpha} \bar{N}^{-\alpha} (1 - \alpha)(1 - 1/\epsilon)$$

Second-order Approximation We have

$$
\begin{aligned}
\Pi \approx\ & \tilde{A}^{1/\epsilon}\bar{K}^{\alpha(1-1/\epsilon)}\bar{N}^{(1-\alpha)(1-1/\epsilon)} + \bar{R}\bar{K}k_t + \bar{W}\bar{N}n_t \\
& + \frac{1}{2}\bar{R}\bar{K}(1-1+\alpha(1-1/\epsilon))k_t^2 + \frac{1}{2}\bar{W}\bar{N}(1-1+(1-\alpha)(1-1/\epsilon))n_t^2 \\
& + \tilde{A}^{1/\epsilon}\alpha(1-\alpha)(1-1/\epsilon)^2\bar{K}^{\alpha(1-1/\epsilon)}\bar{N}^{(1-\alpha)(1-1/\epsilon)}k_t n_t \\
& - \bar{R}\bar{K} - \bar{R}\bar{K}k_t - \bar{R}\bar{K}\frac{1}{2}k_t^2 \\
& - \bar{W}\bar{N} - \bar{W}\bar{N}n_t - \bar{W}\bar{N}\frac{1}{2}n_t^2 \\
& - \frac{\xi_k}{2}\bar{K}(k_t-k_{t-1})^2 - \frac{\xi_n}{2}\bar{N}(n_t-n_{t-1})^2 \\
& + \frac{1}{\epsilon}\tilde{A}^{1/\epsilon}\bar{K}^{\alpha(1-1/\epsilon)}\bar{N}^{(1-\alpha)(1-1/\epsilon)}(a_t + \frac{1}{2}a_t^2) + \frac{1}{2}\frac{1}{\epsilon}(\frac{1}{\epsilon}-1)\tilde{A}^{1/\epsilon}\bar{K}^{\alpha(1-1/\epsilon)}\bar{N}^{(1-\alpha)(1-1/\epsilon)}a_t^2 \\
& + \frac{1}{\epsilon}\tilde{A}^{1/\epsilon}\alpha(1-1/\epsilon)\bar{K}^{\alpha(1-1/\epsilon)}\bar{N}^{(1-\alpha)(1-1/\epsilon)}k_t a_t + \frac{1}{\epsilon}\tilde{A}^{1/\epsilon}(1-\alpha)(1-1/\epsilon)\bar{K}^{\alpha(1-1/\epsilon)}\bar{N}^{(1-\alpha)(1-1/\epsilon)}n_t a_t
\end{aligned}
$$

where $\tilde{A} \equiv \bar{Y}\bar{A}^{\epsilon-1}$, and the sixth line is an approximation of adjustment cost for capital:

$$
\begin{aligned}
\frac{\xi_k}{2}\left(\frac{K_t-(1-\delta)K_{t-1}}{K_{t-1}}-\delta\right)^2 K_{t-1} &= \frac{\xi_k}{2}\left(\exp(k_t-k_{t-1})-1\right)^2\bar{K}\exp(k_{t-1}) \\
&= \frac{\xi_k\bar{K}}{2}(k_t-k_{t-1})^2(1+k_{t-1}+...) = \frac{\xi_k\bar{K}}{2}(k_t-k_{t-1})^2
\end{aligned}
$$

Define investment $\iota_t \equiv k_t - k_{t-1}$ and hiring $h_t \equiv n_t - n_{t-1}$.

Replace $k_t = k_{t-1} + \iota_t$ and $n_t = h_t + n_{t-1}$:

$$
\begin{aligned}
\pi =\ & \frac{1}{\epsilon}\bar{A}^{1/\epsilon}\bar{K}^{\alpha(1-1/\epsilon)}\bar{N}^{(1-\alpha)(1-1/\epsilon)} \\
& + 0k_t + 0n_t + \frac{1}{\epsilon}\bar{A}^{1/\epsilon}\bar{K}^{\alpha(1-1/\epsilon)}\bar{N}^{(1-\alpha)(1-1/\epsilon)}a_t \\
& + \frac{1}{2}\bar{R}\bar{K}(\alpha(1-1/\epsilon)-1)k_t^2 + \frac{1}{2}\bar{W}\bar{N}((1-\alpha)(1-1/\epsilon)-1)n_t^2 + \frac{1}{2}\frac{1}{\epsilon^2}\bar{Y}a_t^2 \\
& + \alpha(1-\alpha)(1-1/\epsilon)^2\bar{Y}k_t n_t + \frac{1}{\epsilon}\alpha(1-1/\epsilon)\bar{Y}k_t a_t + \frac{1}{\epsilon}(1-\alpha)(1-1/\epsilon)\bar{Y}n_t a_t \\
& - \frac{\xi_k}{2}\bar{K}(k_t-k_{t-1})^2 - \frac{\xi_n}{2}\bar{N}(n_t-n_{t-1})^2 \qquad \text{(A1)} \\
=\ & \frac{1}{\epsilon}\bar{Y} + \frac{1}{\epsilon}\bar{Y}a_t + \frac{1}{2}\bar{R}\bar{K}(\alpha(1-1/\epsilon)-1)(k_{t-1}+\iota_t)^2 + \frac{1}{2}\bar{W}\bar{N}((1-\alpha)(1-1/\epsilon)-1)(n_{t-1}+h_t)^2 \\
& + \frac{1}{2}\frac{1}{\epsilon^2}\bar{Y}a_t^2 \\
& + \alpha(1-\alpha)(1-1/\epsilon)^2\bar{Y}(k_{t-1}+\iota_t)(n_{t-1}+h_t) + \frac{1}{\epsilon}\alpha(1-1/\epsilon)\bar{Y}(k_{t-1}+\iota_t)a_t
\end{aligned}
$$

$$+\frac{1}{\epsilon}(1-\alpha)(1-1/\epsilon)\bar{Y}(n_{t-1}+h_t)a_t-\frac{\xi_k\bar{K}}{2}\iota_t^2-\frac{\xi_n\bar{N}}{2}h_t^2$$

Hence

$$\begin{aligned}\pi=&\frac{1}{\epsilon}\bar{Y}+\frac{1}{\epsilon}\bar{Y}a_t+\frac{1}{2}\frac{1}{\epsilon^2}\bar{Y}a_t^2\\&+\frac{1}{2}\bar{R}\bar{K}(\alpha(1-1/\epsilon)-1)\left(k_{t-1}^2+2k_{t-1}\iota_t+\iota_t^2\right)\\&+\frac{1}{2}\bar{W}\bar{N}((1-\alpha)(1-1/\epsilon)-1)(n_{t-1}^2+2n_{t-1}h_t+h_t^2)\\&+\alpha(1-\alpha)(1-1/\epsilon)^2\bar{Y}(k_{t-1}n_{t-1}+\iota_tn_{t-1}+k_{t-1}h_t+\iota_th_t)\\&+\frac{1}{\epsilon}\alpha(1-1/\epsilon)\bar{Y}(k_{t-1}a_t+\iota_ta_t)+\frac{1}{\epsilon}(1-\alpha)(1-1/\epsilon)\bar{Y}(n_{t-1}a_t+h_ta_t)\\&-\frac{\xi_k}{2}\bar{K}\iota_t^2-\frac{\xi_n}{2}\bar{N}h_t^2\end{aligned}\tag{A2}$$

Hence The log-quadratic profit function takes the form

$$x_t'Px_t+x_t'Q\iota_t+x_t'Rh_t+H_\iota\iota_t^2+H_{\iota h}\iota_th_t+H_hh_t^2$$

where state vector $x_t=[1,k_{t-1},n_{t-1},a_t]$, and the matrices are

$$P=\begin{bmatrix}\frac{1}{\epsilon}\bar{Y} & 0 & 0 & \frac{\epsilon-1}{2\epsilon}\bar{Y}\\ * & \frac{1}{2}\bar{R}\bar{K}(\alpha(1-1/\epsilon)-1) & \frac{1}{2}\alpha(1-\alpha)(1-1/\epsilon)^2\bar{Y} & \frac{\epsilon-1}{2\epsilon}\alpha(1-1/\epsilon)\bar{Y}\\ * & * & \frac{1}{2}\bar{W}\bar{N}((1-\alpha)(1-1/\epsilon)-1) & \frac{\epsilon-1}{2\epsilon}(1-\alpha)(1-1/\epsilon)\bar{Y}\\ * & * & * & \frac{1}{2}\frac{(\epsilon-1)^2}{\epsilon^2}\bar{Y}\end{bmatrix}$$

$$Q=\begin{bmatrix}0\\ \bar{R}\bar{K}(\alpha(1-1/\epsilon)-1)\\ \alpha(1-\alpha)(1-1/\epsilon)^2\bar{Y}\\ \frac{1}{\epsilon}\alpha(1-1/\epsilon)\bar{Y}\end{bmatrix}$$

$$R=\begin{bmatrix}0\\ \alpha(1-\alpha)(1-1/\epsilon)^2\bar{Y}\\ \bar{W}\bar{N}((1-\alpha)(1-1/\epsilon)-1)\\ \frac{1}{\epsilon}(1-\alpha)(1-1/\epsilon)\bar{Y}\end{bmatrix}$$

and $H_\iota = \frac{1}{2}\bar{R}\bar{K}(\alpha(1-1/\epsilon)-1) - \frac{1}{2}\xi_k\bar{K}, H_h = \frac{1}{2}\bar{W}\bar{N}((1-\alpha)(1-1/\epsilon)-1) - \frac{1}{2}\xi_n\bar{N}, H_{\iota h} = \alpha(1-\alpha)(1-1/\epsilon)^2\bar{Y}$.

The law of motion for the state vector x_t is given by

$$x_{t+1} \equiv \begin{bmatrix} 1 \\ k_{t-1}+\iota_t \\ n_{t-1}+h_t \\ \rho a_t + \mu_{t+1} \end{bmatrix} = Ax_t + B\iota_t + Ch_t + D\mu_{t+1}$$

where

$$A \equiv \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \rho \end{bmatrix}, B \equiv \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, C \equiv \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, D \equiv \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

A.2 Proof of Proposition 2

To save some notations, let

$$\hat{a}_t^k \equiv E\left[a_{it} \middle| \mathcal{I}_{it}^k\right], \hat{\Sigma}_t^k \equiv E\left[(a_{it}-\hat{a}_t^k)(a_{it}-\hat{a}_t^k)'\right]$$

$$\hat{a}_t^n \equiv E\left[a_{it} \middle| \mathcal{I}_{it}^n\right], \hat{\Sigma}_t^n \equiv E\left[(a_{it}-\hat{a}_t^n)(a_{it}-\hat{a}_t^n)'\right]$$

and

$$\hat{a}_t^{(k,n)} \equiv E\left[\hat{a}_t^n \middle| \mathcal{I}_{it}^k\right], \hat{\Sigma}_t^{(k,n)} \equiv E\left[(\hat{a}_t^{(k,n)}-\hat{a}_t^n)(\hat{a}_t^{(k,n)}-\hat{a}_t^n)'\right]$$

$$\hat{a}_t^{(n,k)} \equiv E\left[\hat{a}_t^k \middle| \mathcal{I}_{it}^n\right], \hat{\Sigma}_t^{(n,k)} \equiv E\left[(\hat{a}_t^{(n,k)}-\hat{a}_t^k)(\hat{a}_t^{(n,k)}-\hat{a}_t^k)'\right]$$

so that

$$a_{it} \middle| \mathcal{I}_{it}^k \sim N(\hat{a}_t^k, \hat{\Sigma}_t^k) \text{ and } a_{it} \middle| \mathcal{I}_{it}^n \sim N(\hat{a}_t^n, \hat{\Sigma}_t^n) \tag{A3}$$

characterize the first-order beliefs of capital and labor manager on the state a_{it}, and

$$\hat{a}_t^n \middle| \mathcal{I}_{it}^k \sim N(\hat{a}_t^{(k,n)}, \hat{\Sigma}_t^{(k,n)}) \text{ and } \hat{a}_t^k \middle| \mathcal{I}_{it}^n \sim N(\hat{a}_t^{(n,k)}, \hat{\Sigma}_t^{(n,k)}) \tag{A4}$$

characterize the high-order beliefs of capital and labor manager on each other's beliefs about a_{it}.

Initial Beliefs To fully specify the belief dynamics, I assume that the capital manager believes that the initial state $a_{i,0}$ is drawn from a Gaussian distribution

$$a_{i,0} \sim N(\hat{a}_0^k, W_0^k)$$

and that the initial information set $\mathcal{I}_0^k = \{\hat{a}_0^k\}$. Similarly, for the labor manager, she believes that the initial state is drawn from a Gaussian distribution

$$a_{i,0} \sim N(\hat{a}_0^n, W_0^n)$$

and her initial information set is $\mathcal{I}_0^n = \{\hat{a}_t^n\}$. For simplicity, I assume that the two initial distributions are independent to each other.

For simplicity, I assume that W_0^k and W_0^n are common knowledge. To characterize the initial higher-order belief $\hat{a}_0^n|\mathcal{I}_0^k$ and $\hat{a}_0^k|\mathcal{I}_0^n$, it is perhaps more convenient to define the initial forecast errors e_0^k, e_0^n as

$$e_0^k \equiv \hat{a}_0^k - a_{i,0} \text{ and } e_0^n \equiv \hat{a}_0^n - a_{i,0}$$

with $e_0^k \sim N(0, W_0^k)$ and $e_0^n \sim N(0, W_0^n)$ being independent to each other. Therefore

$$\hat{a}_0^n = \hat{a}_0^k - e_0^k + e_0^n$$

From the capital manager's perspective,

$$E\left[\hat{a}_0^n|\mathcal{I}_0^k\right] = \hat{a}_0^k$$

$$var\left[\hat{a}_0^n|\mathcal{I}_0^k\right] = var(-e_0^k + e_0^n) = W_0^n + W_0^k$$

Therefore the initial high-order beliefs are

$$\hat{a}_0^n|\mathcal{I}_0^k \sim N(\hat{a}_0^k, W_0^n + W_0^k) \text{ and } \hat{a}_0^k|\mathcal{I}_0^n \sim N(\hat{a}_0^n, W_0^n + W_0^k)$$

A first-glance at HOB at period t The same method can be used to characterize the first moment of high-order beliefs at period t. At period t, from the capital manager's perspective,

$$\hat{a}_t^n = \hat{a}_t^k - e_t^k + e_t^n$$

where e_t^k and e_t^n are forecast errors of the capital and labor manager. Hence

$$\begin{aligned} \hat{a}_t^{(k,n)} &= E\left[\hat{a}_t^n \middle| \mathcal{I}_t^k\right] = \hat{a}_t^k && \text{(A5)} \\ \hat{a}_t^{(n,k)} &= E\left[\hat{a}_t^k \middle| \mathcal{I}_t^n\right] = \hat{a}_t^n && \text{(A6)} \end{aligned}$$

Characterizing the second moment of high-order belief is a bit more involved since the forecast errors e_t^k and e_t^n are correlated with each other. For simplicity, I denote the correlation $E[e_t^k e_t^n]$ as $\tilde{\Sigma}_t^{(k,n)}$. Immediately, we have

$$\hat{\Sigma}_t^{(k,n)} = E\left[(\hat{a}_t^n - \hat{a}_t^{(k,n)})^2\right] = var\left[-e_t^k + e_t^n\right] = \hat{\Sigma}_t^k + \hat{\Sigma}_t^n - 2\tilde{\Sigma}_t^{(k,n)} \tag{A7}$$

Partitioning the State Vector Note that our state vector $x_t \equiv [1, k_{t-1}, n_{t-1}, a_{it}]'$ can be partitioned into a perfectly observed part $s_t \equiv [1, k_{i,t-1}, n_{i,t-1}]'$ and an imperfectly observed part a_{it}. Hence, the linear policy

$$\iota_t = F_\iota E\left[x_t \middle| \mathcal{I}_t^k\right], h_t = F_h E\left[x_t \middle| \mathcal{I}_t^n\right]$$

can be rewritten as

$$\begin{aligned} \iota_t &= F_\iota^s s_t + F_\iota^a \hat{a}_t^k && \text{(A8)} \\ h_t &= F_h^s s_t + F_h^a \hat{a}_t^n && \text{(A9)} \end{aligned}$$

Now I specify the observation equations for the partitioned state vector. The capital manager's signals about a_{it} are summarized by

$$z_t^k \equiv \begin{bmatrix} a_t^p \\ a_t^k \end{bmatrix} = Ha_t + w_t^k \tag{A10}$$

where $H = [1, 1]'$ and $w_t^k = [\epsilon_t^p, \epsilon_t^k]' \sim N(0, diag(\sigma_p^2, \sigma_{\epsilon,k}^2))$. Similarly, the labor manager's signals about a_{it} are summarized by

$$z_t^n \equiv \begin{bmatrix} a_t^p \\ a_t^n \end{bmatrix} = Ha_t + w_t^n \tag{A11}$$

where $H = [1,1]'$ and $w_t^k = [\epsilon_t^p, \epsilon_t^n]' \sim N(0, diag(\sigma_p^2, \sigma_{\epsilon,n}^2))$.

Updating FOB and HOBs Now, suppose we are at the end of period $t-1$, with information set $\mathcal{I}_{t-1}^k$ and $\mathcal{I}_{t-1}^n$. From now on I will only focus on how the capital manager updates beliefs, given he knows that the labor manager's action takes the form

$$h_t = F_h^s s_t + F_h^a \hat{a}_t^n$$

the arguments for the labor manager are basically the same.

At period t, the belief updating takes two steps:

1. The capital manager observes the labor manager's action $h_{t-1} = F_h^s s_{t-1} + F_h^a \hat{a}_{t-1}^n$ and updates the estimates about a_{t-1} as well as a_t. Formally, after observing h_{t-1}, the capital manager computes

$$a_{t-1} \big| \mathcal{I}_{t-1}^k \cup \{h_{t-1}\} \sim N\left((\hat{a}_{t-1}^k)^+, (\hat{\Sigma}_{t-1}^k)^+ \right)$$

   and

$$a_t \big| \mathcal{I}_{t-1}^k \cup \{h_{t-1}\} \sim N\left((\hat{a}_t^k)^-, (\hat{\Sigma}_t^k)^- \right)$$

2. Then the capital manager uses the private signal z_t^k to further update the state estimate a_t on top of the pre-estimate $a_t \big| \mathcal{I}_{t-1}^k \cup \{h_{t-1}\}$ and get

$$a_t \big| \mathcal{I}_{t-1}^k \cup \{h_{t-1}\} \cup \{z_t^k\} = a_t \big| \mathcal{I}_t^k \sim N\left(\hat{a}_t^k, \hat{\Sigma}_t^k \right)$$

We do it step-by-step.

The first step: note that

$$\begin{bmatrix} a_{t-1} \\ h_{t-1} \end{bmatrix} \Bigg| \mathcal{I}_{t-1}^k = \begin{bmatrix} a_{t-1} \\ F_h^s s_{t-1} + F_h^a \hat{a}_{t-1}^n \end{bmatrix} \Bigg| \mathcal{I}_{t-1}^k \sim N\left(\begin{bmatrix} \hat{a}_{t-1}^k \\ F_h^s s_{t-1} + F_h^a \hat{a}_{t-1}^{(k,n)} \end{bmatrix}, \begin{bmatrix} \hat{\Sigma}_{t-1}^k & * \\ * & F_h^a \hat{\Sigma}_{t-1}^{(k,n)} (F_h^a)' \end{bmatrix} \right) \tag{A12}$$

where $*$ is equal to

$$\begin{aligned} * &= E\left[(a_{t-1} - \hat{a}_{t-1}^k)(\hat{a}_{t-1}^n - \hat{a}_{t-1}^{(k,n)})'(F_h^a)' \right] = E\left[(a_{t-1} - \hat{a}_{t-1}^k)(\hat{a}_{t-1}^n - \hat{a}_{t-1}^k)'(F_h^a)' \right] \\ &= E\left[-e_{t-1}^k (\hat{a}_{t-1}^n - a_{t-1} + a_{t-1} - \hat{a}_{t-1}^k)'(F_h^a)' \right] = E\left[-e_{t-1}^k (e_{t-1}^n - e_{t-1}^k)'(F_h^a)' \right] \end{aligned}$$

$$= (\hat{\Sigma}^k_{t-1} - \tilde{\Sigma}^{(k,n)}_{t-1})(F^a_h)' \tag{A13}$$

Therefore the capital manager updates his belief about a_{t-1} using the labor manager's action by

$$(\hat{a}^k_{t-1})^+ = \hat{a}^k_{t-1} + (\hat{\Sigma}^k_{t-1} - \tilde{\Sigma}^{(k,n)}_{t-1})(F^a_h)' \left[F^a_h \hat{\Sigma}^{(k,n)}_{t-1} (F^a_h)'\right]^{-1} F^a_h (\hat{a}^n_{t-1} - \hat{a}^k_{t-1}) \tag{A14}$$

$$(\hat{\Sigma}^k_{t-1})^+ = \hat{\Sigma}^k_{t-1} - (\hat{\Sigma}^k_{t-1} - \tilde{\Sigma}^{(k,n)}_{t-1})(F^a_h)' \left[F^a_h \hat{\Sigma}^{(k,n)}_{t-1} (F^a_h)'\right]^{-1} F^a_h (\hat{\Sigma}^k_{t-1} - \tilde{\Sigma}^{(k,n)}_{t-1})' \tag{A15}$$

Since F^a_h is a scalar, these can be simplified as

$$(\hat{a}^k_{t-1})^+ = \hat{a}^k_{t-1} + (\hat{\Sigma}^k_{t-1} - \tilde{\Sigma}^{(k,n)}_{t-1}) \left[\hat{\Sigma}^{(k,n)}_{t-1}\right]^{-1} (\hat{a}^n_{t-1} - \hat{a}^k_{t-1}) \tag{A16}$$

$$(\hat{\Sigma}^k_{t-1})^+ = \hat{\Sigma}^k_{t-1} - (\hat{\Sigma}^k_{t-1} - \tilde{\Sigma}^{(k,n)}_{t-1}) \left[\hat{\Sigma}^{(k,n)}_{t-1}\right]^{-1} (\hat{\Sigma}^k_{t-1} - \tilde{\Sigma}^{(k,n)}_{t-1})' \tag{A17}$$

and therefore, the pre-estimates for a_t after observing the opponent's action is given by

$$(\hat{a}^k_t)^- = \rho(\hat{a}^k_{t-1})^+ \tag{A18}$$

$$(\hat{\Sigma}^k_t)^- = \rho^2(\hat{\Sigma}^k_{t-1})^+ + \sigma^2_\mu \tag{A19}$$

Now we incorporate the signal z^k_t into the information set and update the belief about a_t. We have

$$\begin{bmatrix} a_t \\ z^k_t \end{bmatrix} \Bigg| \mathcal{I}^k_{t-1} \cup \{h_{t-1}\} \sim N\left(\begin{bmatrix} (\hat{a}^k_t)^- \\ H(\hat{a}^k_t)^- \end{bmatrix}, \begin{bmatrix} (\hat{\Sigma}^k_t)^- & (\hat{\Sigma}^k_t)^- H' \\ H(\hat{\Sigma}^k_t)^- & H(\hat{\Sigma}^k_t)^- H' + W^k \end{bmatrix} \right)$$

where $W^k \equiv diag(\sigma^2_p, \sigma^2_{\epsilon,k})$. Therefore,

$$\hat{a}^k_t = (\hat{a}^k_t)^- + (\hat{\Sigma}^k_t)^- H' \left[H(\hat{\Sigma}^k_t)^- H' + W^k\right]^{-1} \left(z^k_t - H(\hat{a}^k_t)^-\right) \tag{A20}$$

$$\hat{\Sigma}^k_t = (\hat{\Sigma}^k_t)^- - (\hat{\Sigma}^k_t)^- H' \left[H(\hat{\Sigma}^k_t)^- H' + W^k\right]^{-1} H(\hat{\Sigma}^k_t)^- \tag{A21}$$

To close the analysis, I need to specify how high-order beliefs evolve. According to (A7), the uncertainty of the capital manager on the labor manager's expectation about the productivity is a function of (1) the capital manager's posterior uncertainty $\hat{\Sigma}^k_t$ on the productivity at period t, (2) the labor manager's posterior uncertainty $\hat{\Sigma}^n_t$ on the productivity at period t, and (3) the covariance between capital manager and labor manager's forecast errors $\tilde{\Sigma}^{(k,n)}_t$. We already have the law of motion for $\hat{\Sigma}^k_t$ given by (A21) and a similar law of motion for

$\hat{\Sigma}_t^n$. It remains to characterize the covariance for forecast errors $\tilde{\Sigma}_t^{(k,n)}$.

According to (A20), the forecast error at period t is given by

$$\hat{a}_t^k - a_t = \left(I - (\hat{\Sigma}_t^k)^- H' \left[H(\hat{\Sigma}_t^k)^- H' + W^k\right]^{-1} H\right)\left((\hat{a}_t^k)^- - a_t\right) + (\hat{\Sigma}_t^k)^- H' \left[H(\hat{\Sigma}_t^k)^- H' + W^k\right]^{-1} w_t^k \tag{A22}$$

and similarly

$$\hat{a}_t^n - a_t = \left(I - (\hat{\Sigma}_t^n)^- H' \left[H(\hat{\Sigma}_t^n)^- H' + W^n\right]^{-1} H\right)\left((\hat{a}_t^n)^- - a_t\right) + (\hat{\Sigma}_t^n)^- H' \left[H(\hat{\Sigma}_t^n)^- H' + W^n\right]^{-1} w_t^n \tag{A23}$$

Therefore

$$\begin{aligned}\tilde{\Sigma}_t^{(k,n)} = &\left(I - (\hat{\Sigma}_t^k)^- H' \left[H(\hat{\Sigma}_t^k)^- H' + W^k\right]^{-1} H\right) \Lambda_t \left(I - (\hat{\Sigma}_t^n)^- H' \left[H(\hat{\Sigma}_t^n)^- H' + W^n\right]^{-1} H\right)' \\ &+ (\hat{\Sigma}_t^k)^- H' \left[H(\hat{\Sigma}_t^k)^- H' + W^k\right]^{-1} \Omega_t \left[H(\hat{\Sigma}_t^n)^- H' + W^n\right]^{-1} H(\hat{\Sigma}_t^n)^-\end{aligned} \tag{A24}$$

where $\Lambda_t \equiv E\left[(a_t - (\hat{a}_t^k)^-)(a_t - (\hat{a}_t^n)^-)'\right]$ and $\Omega_t = E\left[w_t^k (w_t^n)'\right] = diag(\sigma_p^2, 0)$.

We know that

$$a_t - (\hat{a}_t^k)^- = \rho a_{t-1} + \mu_t - \rho(\hat{a}_{t-1}^k)^+ = \rho(a_{t-1} - (\hat{a}_{t-1}^k)^+) + \mu_t$$

$$a_t - (\hat{a}_t^n)^- = \rho a_{t-1} + \mu_t - \rho(\hat{a}_{t-1}^n)^+ = \rho(a_{t-1} - (\hat{a}_{t-1}^n)^+) + \mu_t$$

and therefore

$$\Lambda_t = E\left[(a_t - (\hat{a}_t^k)^-)(a_t - (\hat{a}_t^n)^-)'\right] = \rho^2 E\left[(a_{t-1} - (\hat{a}_{t-1}^k)^+)(a_{t-1} - (\hat{a}_{t-1}^n)^+)'\right] + \sigma_\mu^2$$

It remains to compute $E\left[(a_{t-1} - (\hat{a}_{t-1}^k)^+)(a_{t-1} - (\hat{a}_{t-1}^n)^+)'\right]$. Note that

$$a_{t-1} - (\hat{a}_{t-1}^k)^+ = -\left(I - (\hat{\Sigma}_{t-1}^k - \tilde{\Sigma}_{t-1}^{(k,n)}) \left[\hat{\Sigma}_{t-1}^{(k,n)}\right]^{-1}\right) e_{t-1}^k - (\hat{\Sigma}_{t-1}^k - \tilde{\Sigma}_{t-1}^{(k,n)}) \left[\hat{\Sigma}_{t-1}^{(k,n)}\right]^{-1} e_{t-1}^n$$

$$a_{t-1} - (\hat{a}_{t-1}^n)^+ = -\left(I - (\hat{\Sigma}_{t-1}^n - \tilde{\Sigma}_{t-1}^{(k,n)}) \left[\hat{\Sigma}_{t-1}^{(k,n)}\right]^{-1}\right) e_{t-1}^n - (\hat{\Sigma}_{t-1}^n - \tilde{\Sigma}_{t-1}^{(k,n)}) \left[\hat{\Sigma}_{t-1}^{(k,n)}\right]^{-1} e_{t-1}^k$$

Hence

$$E\left[(a_{t-1} - (\hat{a}_{t-1}^k)^+)(a_{t-1} - (\hat{a}_{t-1}^n)^+)'\right]$$

$$
\begin{aligned}
&= \left(I - (\hat{\Sigma}^k_{t-1} - \tilde{\Sigma}^{(k,n)}_{t-1}) \left[\hat{\Sigma}^{(k,n)}_{t-1}\right]^{-1}\right) \tilde{\Sigma}^{(k,n)}_{t-1} \left(I - (\hat{\Sigma}^n_{t-1} - \tilde{\Sigma}^{(k,n)}_{t-1}) \left[\hat{\Sigma}^{(k,n)}_{t-1}\right]^{-1}\right)' \\
&+ (\hat{\Sigma}^k_{t-1} - \tilde{\Sigma}^{(k,n)}_{t-1}) \left[\hat{\Sigma}^{(k,n)}_{t-1}\right]^{-1} \hat{\Sigma}^n_{t-1} \left(I - (\hat{\Sigma}^n_{t-1} - \tilde{\Sigma}^{(k,n)}_{t-1}) \left[\hat{\Sigma}^{(k,n)}_{t-1}\right]^{-1}\right)' \\
&+ \left(I - (\hat{\Sigma}^k_{t-1} - \tilde{\Sigma}^{(k,n)}_{t-1}) \left[\hat{\Sigma}^{(k,n)}_{t-1}\right]^{-1}\right) \hat{\Sigma}^k_{t-1} \left[\hat{\Sigma}^{(k,n)}_{t-1}\right]^{-1} (\hat{\Sigma}^n_{t-1} - \tilde{\Sigma}^{(k,n)}_{t-1})' \\
&+ (\hat{\Sigma}^k_{t-1} - \tilde{\Sigma}^{(k,n)}_{t-1}) \left[\hat{\Sigma}^{(k,n)}_{t-1}\right]^{-1} \hat{\Sigma}^{(k,n)}_{t-1} \left[\hat{\Sigma}^{(k,n)}_{t-1}\right]^{-1} (\hat{\Sigma}^n_{t-1} - \tilde{\Sigma}^{(k,n)}_{t-1})' \\
&= \tilde{\Sigma}^{(k,n)}_{t-1} + (\hat{\Sigma}^k_{t-1} - \tilde{\Sigma}^{(k,n)}_{t-1}) \left[\hat{\Sigma}^{(k,n)}_{t-1}\right]^{-1} (\hat{\Sigma}^n_{t-1} - \tilde{\Sigma}^{(k,n)}_{t-1}) \qquad \text{(A25)}
\end{aligned}
$$

and therefore

$$
\Lambda_t = \rho^2 \left\{ \tilde{\Sigma}^{(k,n)}_{t-1} + (\hat{\Sigma}^k_{t-1} - \tilde{\Sigma}^{(k,n)}_{t-1}) \left[\hat{\Sigma}^{(k,n)}_{t-1}\right]^{-1} (\hat{\Sigma}^n_{t-1} - \tilde{\Sigma}^{(k,n)}_{t-1}) \right\} + \sigma^2_\mu \qquad \text{(A26)}
$$

and we finally complete our characterization of belief updates.

A.3 Proof of Proposition 1

A.3.1 Part I: linearity of MPE

Guess that ι and h are linear:

$$
\iota_t = F_\iota x_t, h_t = F_h x_t
$$

Hence, the MPE is defined as

- Given F_h, the capital manager's strategy $\iota_t = F_\iota x_t$ solves the Bellman equation

$$
\begin{aligned}
V^k(x_t) &= \max_{\iota_t} x_t' P x_t + x_t' Q \iota_t + x_t' R F_h x_t + H_\iota \iota_t^2 + H_{\iota h} \iota_t F_h x_t + x_t' F_h' H_h F_h x_t + \gamma E\left[V^k(x_{t+1})\right] \\
&= \max_{\iota_t} x_t' \left(P + R F_h + F_h' H_h F_h\right) x_t + x_t' \left(Q + F_h' H_{\iota h}\right) \iota_t + H_\iota \iota_t^2 + \gamma E\left[V^k(x_{t+1})\right]
\end{aligned} \qquad \text{(A27)}
$$

  subject to the law of motion (14) with h_t replaced by $F_h x_t$:

$$
x_{t+1} = (A + C F_h) x_t + B \iota_t + D \mu_{i,t+1} \qquad \text{(A28)}
$$

- Given F_ι, the labor manager's strategy $h_t = F_h x_t$ solves the Bellman equation

$$\begin{aligned}
V^n(x_t) &= \max_{h_t} x_t' P x_t + x_t' Q F_\iota x_t + x_t' R h_t + x_t' F_\iota' H_\iota F_\iota x_t + x_t' F_\iota' H_{\iota h} h_t + H_h h_t^2 + \gamma E\left[V^n(x_{t+1})\right] \\
&= \max_{h_t} x_t' \left(P + Q F_\iota + F_\iota' H_\iota F_\iota\right) x_t + x_t' \left(R + F_\iota' H_{\iota h}\right) h_t + H_h h_t^2 + \gamma E\left[V^n(x_{t+1})\right]
\end{aligned} \tag{A29}$$

subject to the law of motion (14), with ι_t replaced by $F_\iota x_t$:

$$x_{t+1} = (A + BF_\iota) x_t + C h_t + D\mu_{i,t+1} \tag{A30}$$

With the linear-quadratic setting, we know that the value functions V^k and V^n takes the form

$$V^k(x_t) = x_t' \hat{P}_k x_t + \hat{Q}_k, V^n(x_t) = x_t' \hat{P}_n x_t + \hat{Q}_n$$

Plug it into the Bellman equations: capital manager's problem becomes

$$\begin{aligned}
& x_t' \left[P + RF_h + F_h' H_h F_h + \gamma (A + CF_h)' \hat{P}_k (A + CF_h)\right] x_t + i_t' (H_i + \gamma B' \hat{P}_k B) i_t \\
& + i_t' (H_{ih} F_h + Q' + \gamma B' \hat{P}_k (A + CF_h)) x_t + \gamma x_t' (A + CF_h)' \hat{P}_k B i_t \\
& = \cdots + i_t' \left[H_{ih} F_h + Q' + \gamma B' (\hat{P}_k + \hat{P}_k')(A + CF_h)\right] x_t
\end{aligned}$$

take first order condition on ι:

$$2(H_i + \gamma B' \hat{P}_k B)\iota_t = -H_{ih} F_h + Q' + \gamma B' (\hat{P}_k + \hat{P}_k')(A + CF_h)$$

we get the policy matrix F_ι that solves (A27):

$$F_\iota = -\frac{1}{2}\left(H_\iota + \gamma B' \hat{P}_k B\right)^{-1} \left[Q' + H_{\iota h} F_h + \gamma B' (\hat{P}_k + \hat{P}_k')(A + CF_h)\right] \tag{A31}$$

where $\hat{P}_k$ is defined by Riccati equation

$$\begin{aligned}
\hat{P}_k &= P + RF_h + F_h' H_h F_h + \gamma (A + CF_h)' \hat{P}_k (A + CF_h) \\
& - \frac{1}{4}\left[Q' + H_{\iota h} F_h + 2\gamma B' \hat{P}_k (A + CF_h)\right]' \left(H_\iota + \gamma B' \hat{P}_k B\right)^{-1} \left[Q' + H_{\iota h} F_h + 2\gamma B' \hat{P}_k (A + CF_h)\right]
\end{aligned} \tag{A32}$$

and similarly the solution to (A29) is

$$F_h = -\frac{1}{2}\left(H_h + \gamma C' \hat{P}_n C\right)^{-1} \left[R' + H_{\iota h} F_\iota + 2\gamma C' \hat{P}_n (A + BF_\iota)\right] \tag{A33}$$

where $\hat{P}_n$ is defined by Riccati equation

$$\begin{aligned}\hat{P}_n &= P + QF_\iota + F_\iota' H_\iota F_\iota + (A+BF_\iota)'\hat{P}_n(A+BF_\iota) \\ &\quad - \frac{1}{4}\left[R' + H_{\iota h}F_\iota + 2\gamma C'\hat{P}_n(A+BF_\iota)\right]'\left(H_h + \gamma C'\hat{P}_n C\right)^{-1}\left[R' + H_{\iota h}F_\iota + 2\gamma C'\hat{P}_n(A+BF_\iota)\right]\end{aligned} \tag{A34}$$

A.3.2 Part II: linearity of MPBE

Now that we have characterized beliefs given policy F_ι, F_h, I go back to the Bellman equations and derive these policy matrices F_ι, F_h. Under incomplete information, the capital manager's Bellman equation is

$$V^k(x_t) = \max_{\iota_t} E\left[x_t'\left(P + RF_h + F_h' H_h F_h\right)x_t + x_t'\left(Q + F_h' H_{\iota h}\right)\iota_t + H_\iota \iota_t^2 + \gamma V^k(x_{t+1})\middle|\mathcal{I}_t^k\right] \tag{A35}$$

To save some notation, let $\tilde{P}^k \equiv P + RF_h + F_h' H_h F_h$ and $\tilde{Q}^k \equiv Q + F_h' H_{\iota h}$.

From the previous section, we know that $x_t = [s_t, a_t]'$ where $s_t \equiv [1, k_{i,t-1}, n_{i,t-1}]'$ is the perfectly observable part, and a_t is the partially observed productivity. We know that the time-t estimate of x_t for the capital manager is

$$\hat{x}_t^k \equiv [s_t, \hat{a}_t^k]'$$

To see how the noisy information changes the Bellman equation, we first look at the first term $x_t'\left(P + RF_h + F_h' H_h F_h\right)x_t$. Put it in the conditional expectation, we get

$$\begin{aligned}E\left[x_t'\left(P + RF_h + F_h' H_h F_h\right)x_t\middle|\mathcal{I}_t^k\right] &= E\left[x_t'\tilde{P}^k x_t\middle|\mathcal{I}_t^k\right] \\ &= E\left[[s_t', a_t']\begin{bmatrix}\tilde{P}_{11}^k & \tilde{P}_{12}^k \\ \tilde{P}_{21}^k & \tilde{P}_{22}^k\end{bmatrix}\begin{bmatrix}s_t \\ a_t\end{bmatrix}\middle|\mathcal{I}_t^k\right] \\ &= E\left[s_t'\tilde{P}_{11}^k s_t + s_t'\tilde{P}_{12}^k a_t + a_t'\tilde{P}_{21}^k s_t + a_t'\tilde{P}_{22}^k a_t\middle|\mathcal{I}_t^k\right] \\ &= s_t'\tilde{P}_{11}^k s_t + s_t'\tilde{P}_{12}^k \hat{a}_t^k + (\hat{a}_t^k)'\tilde{P}_{21}^k s_t + E\left[a_t'\tilde{P}_{22}^k a_t\middle|\mathcal{I}_t^k\right]\end{aligned}$$

We knwo that

$$
\begin{aligned}
E\left[a_t'\tilde{P}_{22}^k a_t \middle| \mathcal{I}_t^k\right] &= E\left[tr\left(a_t'\tilde{P}_{22}^k a_t\right)\middle| \mathcal{I}_t^k\right] \\
&= E\left[tr\left(\tilde{P}_{22}^k a_t a_t'\right)\middle| \mathcal{I}_t^k\right] = tr\left(\tilde{P}_{22}^k E\left[a_t a_t' \middle| \mathcal{I}_t^k\right]\right) \\
&= tr\left(\tilde{P}_{22}^k\left[\hat{a}_t^k(\hat{a}_t^k)' + \hat{\Sigma}_t^k\right]\right) = tr\left(\tilde{P}_{22}^k\hat{\Sigma}_t^k\right) + tr\left(\tilde{P}_{22}^k\hat{a}_t^k(\hat{a}_t^k)'\right) \\
&= tr\left(\tilde{P}_{22}^k\hat{\Sigma}_t^k\right) + (\hat{a}_t^k)'\tilde{P}_{22}^k\hat{a}_t^k
\end{aligned}
$$

Plug it back:

$$
\begin{aligned}
E\left[x_t'\left(P + RF_h + F_h'H_hF_h\right)x_t \middle| \mathcal{I}_t^k\right] &= s_t'\tilde{P}_{11}^k s_t + s_t'\tilde{P}_{12}^k\hat{a}_t^k + (\hat{a}_t^k)'\tilde{P}_{21}^k s_t + E\left[a_t'\tilde{P}_{22}^k a_t\middle|\mathcal{I}_t^k\right] \\
&= s_t'\tilde{P}_{11}^k s_t + s_t'\tilde{P}_{12}^k\hat{a}_t^k + (\hat{a}_t^k)'\tilde{P}_{21}^k s_t + tr\left(\tilde{P}_{22}^k\hat{\Sigma}_t^k\right) + (\hat{a}_t^k)'\tilde{P}_{22}^k\hat{a}_t^k \\
&= tr\left(\tilde{P}_{22}^k\hat{\Sigma}_t^k\right) + [s_t', (\hat{a}_t^k)'] \begin{bmatrix} \tilde{P}_{11}^k & \tilde{P}_{12}^k \\ \tilde{P}_{21}^k & \tilde{P}_{22}^k \end{bmatrix} \begin{bmatrix} s_t \\ \hat{a}_t^k \end{bmatrix} \\
&= tr\left(\tilde{P}_{22}^k\hat{\Sigma}_t^k\right) + (\hat{x}_t^k)'\tilde{P}^k\hat{x}_t^k
\end{aligned}
$$

The rest of terms in the quadratic profit function is straight forward. For the second term,

$$
E\left[x_t'\left(Q + F_h'H_{\iota h}\right)\iota_t\middle|\mathcal{I}_t^k\right] = (\hat{x}_t^k)'\tilde{Q}^k\iota_t
$$

and the third term can be taken out of the expectation directly since ι_t is a function of elements in $\mathcal{I}_t^k$.

As a result, we have

$$
\begin{aligned}
V^k(x_t) = \max_{\iota_t} tr\left(\tilde{P}_{22}^k\hat{\Sigma}_t^k\right) + (\hat{x}_t^k)'\left(P + RF_h + F_h'H_hF_h\right)\hat{x}_t^k + (\hat{x}_t^k)'\left(Q + F_h'H_{\iota h}\right)\iota_t + H_\iota\iota_t^2 \\
+ \gamma E\left[V^k(x_{t+1})\middle|\mathcal{I}_t^k\right] \qquad \text{(A36)}
\end{aligned}
$$

Compared with the complete-information case, we can see that the expected profit under incomplete information is different in the following aspects:

- replacing the true state variable x_t by the belief $\hat{x}_t^k$;
- adding a constant term that contains the posterior uncertainty $\hat{\Sigma}_t^k$.

Notably, the second moment of state estimate $\hat{\Sigma}_t^k$ enters the value function only via the constant term (in the trace) and does not become part of the coefficient on terms with $\hat{x}_t^k$ or ι_t. Hence, $\hat{\Sigma}_t^k$ will not affect the first-order conditions of the Bellman equation — this should not be surprising since we are doing linear-quadratic control and certainty equivalence holds.

Again, I guess that the value function takes the form

$$(\hat{x}_t^k)'\hat{P}_k\hat{x}_t^k + \hat{Q}_k$$

so that

$$\begin{aligned} E\left[V^k(x_{t+1})\middle|\mathcal{I}_t^k\right] &= E\left[(\hat{x}_{t+1}^k)'\hat{P}_k\hat{x}_{t+1}^k + \hat{Q}_k\middle|\mathcal{I}_t^k\right] \\ &= tr\left(\hat{P}_k\hat{\Sigma}_{t+1}^k\right) + (\hat{x}_{t+1}^k)'\hat{P}_k\hat{x}_{t+1}^k + \hat{Q}_k \end{aligned}$$

We plug it into the (A36) and take first order condition, we will end up with exactly the same F_ι that we had in (A31), the full-information benchmark. The undertermined coefficient matrix $\hat{P}_k$ in the value function is pinned down by exactly the same Riccatti equation (A32) in the full-information benchmark. The constant matrix $\hat{Q}^k$ will be different, since now it has to contain the trace term with posterior uncertainty.

Similarly, for the labor manager, the policy matrix F_h will be exactly the same as the (A33), with Riccatti equation defined by (A34).

A.4 Computation of Joint Distribution and Estimation

The computation of SS Joint Distribution of $[k_{it}, n_{it}, \hat{a}_{it}^k, \hat{a}_{it}^n, a_{it}]' \sim N(0, \mathbf{\Sigma})$ takes the following steps:

1. Compute the decision rule F_ι, F_h for capital and labor under the complete-info Linear MPE. Denote $F_\iota = [F_\iota^k, F_\iota^n, F_\iota^a]$ and $F_h = [F_h^k, F_h^n, F_h^a]$.
2. Given F_ι, F_h, use (18) - (**??**) to get the SS beliefs $\hat{\Sigma}_k = \lim \hat{\Sigma}_t^k, \hat{\Sigma}_n = \lim \hat{\Sigma}_t^n$ and $\hat{\Sigma}_{(k,n)} = \lim \hat{\Sigma}_t^{(k,n)}$.

3. Stack the decision rule and the belief dynamics into the following recursion:

$$B \begin{bmatrix} \iota_t \\ h_t \\ \hat{a}_t^k \\ \hat{a}_t^n \\ a_t \end{bmatrix} = C \begin{bmatrix} k_{t-1} \\ n_{t-1} \\ \hat{a}_{t-1}^k \\ \hat{a}_{t-1}^n \\ a_{t-1} \end{bmatrix} + D \begin{bmatrix} \epsilon_t^p \\ \epsilon_t^k \\ \epsilon_t^n \\ \mu_t \end{bmatrix}$$

where elements in B, C and D can be computed from $F_\iota, F_h, \hat{\Sigma}_k, \hat{\Sigma}_n, \hat{\Sigma}_{(k,n)}$.

4. Rewrite the above equation in terms of $[k_{t-1}, n_{t-1}, \hat{a}_{t-1}^k, \hat{a}_{t-1}^n, a_{t-1}]'$:

$$\begin{bmatrix} k_t \\ n_t \\ \hat{a}_t^k \\ \hat{a}_t^n \\ a_t \end{bmatrix} = \tilde{C} \begin{bmatrix} k_{t-1} \\ n_{t-1} \\ \hat{a}_{t-1}^k \\ \hat{a}_{t-1}^n \\ a_{t-1} \end{bmatrix} + \tilde{D} \begin{bmatrix} \epsilon_t^p \\ \epsilon_t^k \\ \epsilon_t^n \\ \mu_t \end{bmatrix}$$

where $\tilde{C} \equiv B^{-1}C + diag(1, 1, 0, 0, 0), \tilde{D} \equiv B^{-1}D$.

5. Compute the variance-covariance matrix of $[k_{t-1}, n_{t-1}, \hat{a}_{t-1}^k, \hat{a}_{t-1}^n, a_{t-1}]'$, denoted as $\mathbf{\Sigma}$, by Lyapunov equation

$$\mathbf{\Sigma} = \tilde{C}\mathbf{\Sigma}\tilde{C}' + D\Sigma_\epsilon D'$$

solve this Lyapunov equation and we obtain $\mathbf{\Sigma}$.

All the five targeted moments can be computed from the matrix $\mathbf{\Sigma}$. Let the vector of parameters $\mathbf{x} \equiv (\xi_k, \xi_n, \sigma_{\epsilon,p}, \sigma_{\epsilon,k}, \sigma_{\epsilon,n}) \in (\mathbb{R}^5)^+$ and the vector of targeted moments implied by parameter values $\mathbf{x}$ be $\mathbf{m}(\mathbf{x})$. Let the corresponding data moments be $\mathbf{m}^{\text{data}}$. The simulated method of moments search over the space of parameters $\mathbf{x}$ to minimize

$$(\mathbf{m}(\mathbf{x}) - \mathbf{m}^{\text{data}})'(\mathbf{m}(\mathbf{x}) - \mathbf{m}^{\text{data}})$$

i.e. the unweighed sum of the quadratic deviations from model-implied moments and data moments.

Figure 3 illustrates the identification of parameters.

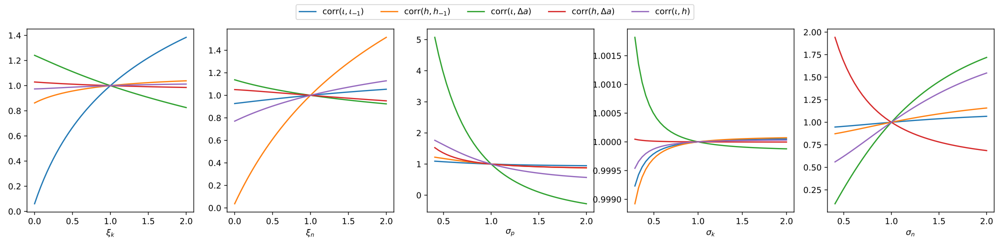

Figure 3: Identification of Parameters

A.5 Analysis of the Model-implied Investment Forecast Error

We know that

$$\begin{aligned} \iota_t &= F_\iota^k k_{t-1} + F_\iota^n n_{t-1} + F_\iota^a \hat{a}_t^k \\ \mathbb{E}_{t-1}^k[\iota_t] &= F_\iota^k k_{t-1} + F_\iota^n \mathbb{E}_{t-1}^k[n_{t-1}] + F_\iota^a \mathbb{E}_{t-1}^k[a_t] \end{aligned}$$

Hence

$$\begin{aligned} \iota_t - \mathbb{E}_{t-1}^k[\iota_t] &= F_\iota^n(n_{t-1} - \mathbb{E}_{t-1}^k[n_{t-1}]) + F_\iota^a(\hat{a}_t^k - \mathbb{E}_{t-1}^k[a_t]) \\ &= F_\iota^n(h_{t-1} - \hat{h}_{t-1}^k) + F_\iota^a((I - G^k H)\varrho J^k(h_{t-1} - \hat{h}_{t-1}^k) + G^k(z_t^k - H\varrho\hat{a}_{t-1}^k)) \\ &= \left[F_\iota^n + F_\iota^a(I - G^k H)\varrho J^k\right](h_{t-1} - \hat{h}_{t-1}^k) + F_\iota^a G^k(z_t^k - H\varrho\hat{a}_{t-1}^k) \end{aligned} \tag{A37}$$

Since

$$\begin{aligned} F_\iota^a G^k(z_t^k - H\varrho\hat{a}_{t-1}^k) &= F_\iota^a G^k(Ha_t + w_t^k - H\varrho\hat{a}_{t-1}^k) \\ &= F_\iota^a G^k(H\varrho a_{t-1} + H\mu_t + w_t^k - H\varrho\hat{a}_{t-1}^k) \\ &= \underbrace{F_\iota^a G^k(H\mu_t + w_t^k)}_{\text{orthogonal to } h_{t-1}} + \underbrace{F_\iota^a G^k H\varrho(a_{t-1} - \hat{a}_{t-1}^k)}_{\text{correlates with } h_{t-1} - \hat{h}_{t-1}^k} \end{aligned}$$

we have

$$\begin{aligned} \iota_t - \mathbb{E}_{t-1}^k[\iota_t] &= \left[F_\iota^n + F_\iota^a(I - G^k H)\varrho J^k\right](h_{t-1} - \hat{h}_{t-1}^k) + F_\iota^a G^k(H\varrho a_{t-1} + H\mu_t + w_t^k - H\varrho\hat{a}_{t-1}^k) \\ &= \left[F_\iota^n + F_\iota^a(I - G^k H)\varrho J^k\right](h_{t-1} - \hat{h}_{t-1}^k) + F_\iota^a G^k H\varrho(a_{t-1} - \hat{a}_{t-1}^k) + F_\iota^a G^k(H\mu_t + w_t^k) \end{aligned} \tag{A38}$$

B Data Appendix

B.1 Steps of Data Cleaning

After obtaining COMPUSTAT Fundamentals Annual dataset from WRDS, I applied the following filter:

1. Exclude the observations before 1998;
2. Exclude the firms incorporated or legally registered outside the US (filter by COMPUSTAT terminology `fic`);
3. Exclude the firms that do not use US dollar as their native currency (filter by COMPUSTAT terminology `curncd`);
4. Following the literature (e.g. De Loecker, Eeckhout, and Unger (2020), Flynn and Sastry (2020), Afrouzi, Drenik, and Kim (2023), among many others), exclude firms in the following sectors
   (a) financial sector (SIC code 6000-6999, or 2-digit NAICS 52);
   (b) utility sector (SIC code 4900-4999, or 2-digit NAICS 22);
   (c) non-operating sector (SIC code 9995).

   It is well documented that financial firms are different from non-financial firms in many aspects (e.g. production function, business cycle properties, etc). The input and output prices of the utility sector is heavily regulated.
5. Exclude observations with missing SIC or NAICS codes;
6. Exclude observations with missing total assets (COMPUSTAT terminology `at`);
7. Exclude observations with acquisitions (COMPUSTAT terminology `aq`) exceeding 5 percent of total assets (COMPUSTAT terminology `at`);
   This is a common practice in the literature because firms that went through large mergers and acquisitions tend to have very large changes in input choices and measured productivity. [9]

[9] I may slack this filter if one believes that major M&As are (exogenous) variations to the management style and hence information friction.

8. Exclude observations with non-positive sales (COMPUSTAT terminology `sale`);

9. Exclude observations with non-positive employments (COMPUSTAT terminology `emp`);

10. (Optional; for robustness check) Construct the capital stock series by perpetual inventory method. As pointed out in Becker et al. (2005) and Bachmann and Bayer (2014), the capital measure in COMPUSTAT, namely PPEGT, is book value. It cannot be directly used because it is valued at historical prices and the accounting depreciation in the balance sheet is usually higher than the economic depreciation for tax benefit. I construct the capital stock series as follows:

    (a) For each firm, the initial capital stock K_{i,t_0} is obtained by deflating the first observation of total gross property, plant and equipment (COMPUSTAT terminology `ppegt`), denoted BV_{i,t_0}; for the deflator P_{t_0}, I use the non-residential gross private domestic investment deflator (row 9) as in Implicit Price Deflators for Gross Domestic Product (NIPA Table 1.1.9) from BEA website [in this table, the base year is 2017]. Hence, initial capital stock is

    $$K_{i,t_0} = \frac{P_{2017}}{P_{t_0}} BV_{i,t_0}$$

    (b) For each firm, build up the capital recursively by

    $$K_{i,t} = I_t - \delta K_{i,t-1} + K_{i,t-1} = NI_{i,t} + K_{i,t-1} \tag{B1}$$

    where NI denotes net investment. There are multiple ways of measuring net investment. Here I use the approach in Ottonello and Winberry (2020): deflate the series of total net property, plant and equipment (COMPUSTAT terminlogy `ppent`) and use $\Delta PPENT_{i,t} \equiv PPENT_{i,t} - PPENT_{i,t-1}$ as a measure of (undeflated) net investment. Deflate $\Delta PPENT_{i,t}$ by the corresponding deflator P_t, we obtain

    $$K_{i,t} = K_{i,t-1} + \frac{P_{2017}}{P_t} \Delta PPENT_{i,t} \tag{B2}$$

11. Exclude observations with missing cost of goods sold.

12. The following steps are essential to backing out the capital share α:

    (a) merge compustat with County Business Pattern (CBP) from Census (which contains industry-level wages - here, industry = 3-digit NAICS for manufacturing and 2-digit for other industries);

(b) fill up the missing values in wage bill for Compustat firms: wage bill = emp * CBP$_{wages}$;

(c) compute the material expenditure = cogs + xsga - dp - wage bill (see Keller and Yeaple (2009) and Flynn and Sastry (2020)).

(d) compute value-added = sales - material expenditure;

(e) compute the labor share in sales:

$$\text{rshare}_{\text{labor}} = \sum(wagebill)/\sum(\text{value-added})$$

Here, I sum up all the firm-year observations to get an aggregate share; in an alternative exercise I do the summation up to the industry level and get an industry-specific labor share for all the industries;

(f) back out the capital share of income:

$$\alpha = 1 - rshare_{labor}/(1 - 1/\epsilon)$$

13. The remaining steps:

(a) Define capital stock as the book value of PPE (PPEGT);

(b) take log of value-added, employment and capital stock, get the lagged and first-differenced log employment and log capital stock;

(c) Define the profit shock/Sales Solow residual as

$$a = \log(valueadded) - \alpha * (1 - 1/\epsilon) \log(capitalstock) - (1 - \alpha) * (1 - 1/\epsilon) * \log(emp)$$

(d) get the lagged and first-differenced series for a;

(e) define mrpk = log(value added) - log(capital stock) and mrpn = log(value added) - log (emp)

(f) regress $k, n, a, \Delta k, \Delta n, \Delta a, mrpk, mrpn$ on sector-year fixed effects and keep the residuals (this is to remove the trends in the data);

(g) trim the 3% tail of the above residuals;

(h) remove the observations with investment rate higher than 100% (dk ¿ 1 or dk ¡-1);

We end up with a sample spanning from 1998 to 2021, with 43,152 firm-year observations. All the moments are computed using from this sample.