Homework 2

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Problem 3:

Part i

```
x = c(0.5, 1.2, -0.5, 0.9, -0.7, 1.5, -1, 2.5, 3.25, -1.6, 0.2, 2.75)
mu = 1
sigma_squared = 2
a = sum((x - mu)^2) / sigma_squared
n = length(x)
df = n - 1
probabilitya = pchisq(a, df, lower.tail = TRUE)
```

```
## The value of a is: 13.7325
```

```
## The distribution of the static is a chi-square distritubtion.
```

```
## The probability is: 0.7518497
```

Part ii

```
x_bar = mean(x)
x_bar
```

```
## [1] 0.75
```

```
b = sum((x-x_bar)^2) / sigma_squared
probabilityb = pchisq(b,df,lower.tail = TRUE)
```

```
## The value of b is: 13.3575
```

```
## The distribution of the static is a chi-square distritubtion.
```

```
## The probability is: 0.7293972
```

Part iii

```
s_squared = var(x)
c = (x_bar - mu) / sqrt(s_squared/n)
probabilityc = pt(c,df)
probabilityc
```

```
## [1] 0.2947659
```

```
## The value of c is: -0.5557115
```

```
## The distribution of the static is a Student's T distribution.
```

```
## The probability is: 0.2947659
```

Problem 4:

Part i

```
P_X = pnorm(1,-1,2) - pnorm(0,-1,2)
P_Y = pchisq(13,10) - pchisq(2,10)
P_T = pt(2,11) - pt(0,11)
P_F = pf(3,8,11) - pf(1,8,11)
```

```
## The value of P(X ∈ (0, 1)) is: 0.1498823
```

```
## The value of P(Y ∈ (2, 13)) is: 0.7726683
```

```
## The value of P(T ∈ (0, 2)) is: 0.464598
```

```
## The value of P(F ∈ (1, 3)) is: 0.4385959
```

Part ii

```
alpha = 0.05
X_quant1 = qnorm(alpha/2,-1,2)
X_quant2 = qnorm(alpha,-1,2)
X_quant3 = qnorm(1-alpha,-1,2)
X_quant4 = qnorm(1 - alpha/2,-1,2)

Y_quant1 = qchisq(alpha/2,10)
Y_quant2 = qchisq(alpha,10)
Y_quant3 = qchisq(1 - alpha,10)
Y_quant4 = qchisq(1 - alpha/2,10)

T_quant1 = qt(alpha/2,11)
T_quant2 = qt(alpha,11)
T_quant3 = qt(1-alpha,11)
T_quant4 = qt(1-alpha/2,11)

F_quant1 = qf(alpha/2,8,11)
F_quant2 = qf(alpha,8,11)
F_quant3 = qf(1 -alpha,8,11)
F_quant4 = qf(1-alpha/2,8,11)
```

```
## The value of α/2 for X is: -4.919928
```

```
## The value of α for X is: -4.289707
```

```
## The value of 1 - α for X is: 2.289707
```

```
## The value of 1 - α/2 for X is: 2.919928
```

```
## The value of α/2 for Y is: 3.246973
```

```
## The value of α for Y is: 3.940299
```

```
## The value of 1 − α for Y is: 18.30704
```

```
## The value of 1 − α/2 for Y is: 20.48318
```

```
## The value of α/2 for T is: −2.200985
```

```
## The value of α for T is: −1.795885
```

```
## The value of 1 − α for T is: 1.795885
```

```
## The value of 1 − α/2 for T is: 2.200985
```

```
## The value of α/2 for F is: 0.2356594
```

```
## The value of α for F is: 0.3018457
```

```
## The value of 1 − α for F is: 2.94799
```

```
## The value of 1 − α/2 for F is: 3.663819
```

Problem 5:

Part i

E[N] = np

The expectation of a Binomially distributed random variable can be derived from the sum of expectations of individual Bernoulli trials. Since each trial has a success probability p, the expected value of each trial is p. For n trials, the total expected value is:

E[N] = E[X1 + X2 + X3 + ….. + XN] = E[X1] + E[X2] + E[X3] + … + E[N] = np

E[N] = $\sum_{k=0}^{n} k * (n,k)p^k(1-p)^(n-k)$ = np $\sum_{k=1}^{n} p^(k-1) * (nk)p^(k-1)(1-p)^(n-k)-(k-1)$

$p + (1-p)^(n-1) = 1$ This is (1-p) to the (n-1) power

$\sum_{k=1}^{n} p^(k-1) * (nk)p^(k-1)(1-p)^(n-k)-(k-1)$ = $E[N] = np$

This result aligns with our initial definition of E[N] for a binomial distribution, achieved through a process analogous to integration, focusing on the summation of discrete probabilities.

Var[N] = np(1-p)

The variance of a binomially distributed random variable is given by the formula Var[N] = np(1-p). This formula comes from the fact that variance measures the spread of the distribution around the mean (expected value), and in each trial, the probability of not succeeding (failure) is 1-p . Since the outcome of each trial is independent, the total variance is the product of the number of trials , the probability of success p , and the probability of failure 1-p.

**Part ii*

E[T] = 0

The PDF of a Student's t-distribution with n degrees:

$f(t) = \frac{T}{sqrt(n*pi)*T*(n/2)} * (\frac{n+1}{2}) * (1 + \frac{t^2}{n})^{-\frac{n+1}{2}}$

The expected value of E[T] of a continuous random variable T is defined as:

E[T] =

$$\int_{\infty}^{\infty} t * f(t)\,dt$$

(negative inf to inf)

Symmetry of PDF: f(t) = f(-t) for all t.

Expected Value Definition: E[T] =

$$\int_{\infty}^{\infty} t * f(t)\,dt$$

(negative inf to inf)

Splitting the Integral:

Positive side:

$$\int_0^{\infty} t * f(t)\,dt$$

Negative side:

$$\int_{\infty}^{0} t * f(t)\,dt$$

(negative inf to 0)

Symmetry Application: The positive and negative sides cancel out due to symmetry.

Conclusion: E[T] = 0

−F −1(α) = F −1(1 − α)

Given: A symmetric distribution around zero, and its CDF F(x).

Prove: F −1(α) = F −1(1 − α)

Let x = F^-1(α). This means F(x) = α, the probability of the random variable being less than or equal to x is α .

Due to the symmetry of the distribution, the value at -x will have the cumulative probability of 1 - α. F(-x) = 1 - α

Since F(-x) = 1 - α is the inverse CDF -x = F-1(1-α), and since x = F-1(α), we can sub and get -F −1(α) = F −1(1 − α).