Question:

Suppose we are interested in testing the null hypothesis that the mean of a normal population is 10 against the alternative that it is greater than 10. A random sample of size 20 from this population gives 9.02 as the sample mean and 2.22 as the sample standard deviation.

(a) Set up the null and alternative hypotheses.

(b) Which test would you use? What is the test statistic? What is the null distribution of the test statistic?

(c) Compute the observed value of the test statistic.

(d) Compute the p-value of the test using the usual way.

(e) Estimate the p-value of the test using Monte Carlo simulation. How do your answers in (d) and(e) compare?

(f) State your conclusion at 5% level of significance.

Report:

(a) The null hypothesis H0: true population mean is equal to 10.

The alternative hypothesis H1: true population mean is greater than 10.

(b) The test I will use one sample T-test because we don't know variance of population. Also it is a one-sided test. The test statistic is defined by $t = (\bar{x} - \mu) / (s/\sqrt{n})$. The null distribution of the test statistic is T-distribution.

(c)

```
t<-(9.02-10)*sqrt(20)/2.22
t
```

The answer is -1.974186

(d)

I will choose 'pt' function to get value of F(x) in R function.

```
F(tobs)<-pt(-abs(t),df=19)
```

The answer is [1] 0.03153941

Because in this question, H1: $\mu > \mu_0$, so that P-value = 1-F(tobs).

```
P-value<-1- F(tobs)
```

The answer is 0.96846.

(e)

I do 10000 times operation in order to realize Monte Carlo simulation. In this 'for' loop, we calculate the times of mean value more than 10 (reject H0). The 'for' loop is shown below:

```
count=0
for (i in 1:10000){
  onetime=rnorm(n=20,mean=9.02,sd=2.22)
  onemean=mean(onetime)
```

```
    if (onemean>10){
      count=count+1
    }
  }
```

And then I calculate the percent of the mean is greater than 10.

```
percent<-count/10000
```

And then the p-value is shown below:

```
pvalue<-1-percent
```

The answer is 0.974.

(f)

At 5% level of significance, we accept H0 because P-value is greater than 0.05. So that reject H1.