第二次作业

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考虑欧拉-伯努利梁直梁单元，x 为梁的方向，2 节点单元，扭转转角、拉伸位移采用拉格朗日插值，弯曲位移与弯曲转角共同通过 Hermite 三次多项式插值及其导数取得，单元自由度记为:

$$\boldsymbol{a_e} = [u^1 \ v^1 \ w^1 \ \theta_x^1 \ \theta_y^1 \ \theta_z^1 \ u^2 \ v^2 \ w^2 \ \theta_x^2 \ \theta_y^2 \ \theta_z^2]^T \tag{1}$$

拉格朗日 2 节点一维插值为:

$$N^1(\xi) = 1 + \xi, \ N^2(\xi) = 1 - \xi \tag{2}$$

Hermite 插值为:

$$\begin{aligned} N^1(\xi) = & \ \frac{1}{2} - \frac{3}{4}\xi + \frac{1}{4}\xi^3 \\ N^2(\xi) = & \ \frac{1}{4} - \frac{1}{4}\xi - \frac{1}{4}\xi^2 + \frac{1}{4}\xi^3 \\ N^3(\xi) = & \ \frac{1}{2} + \frac{3}{4}\xi - \frac{1}{4}\xi^3 \\ N^4(\xi) = & -\frac{1}{4} - \frac{1}{4}\xi + \frac{1}{4}\xi^2 + \frac{1}{4}\xi^3 \end{aligned} \tag{3}$$

其中，$1,2$ 对应 $\xi = -1$ 的函数值和一阶导，$3,4$ 对应 $\xi = 1$。

欧拉-伯努利直梁内的总势能为：

$$\begin{aligned} \Pi_p = & \int_l \left[\frac{1}{2} EI_y \left(\frac{d^2 w}{dx^2} \right)^2 + \frac{1}{2} EI_z \left(\frac{d^2 v}{dx^2} \right)^2 + \frac{1}{2} EA \left(\frac{du}{dx} \right)^2 + \frac{1}{2} GI_x \left(\frac{d\theta_x}{dx} \right)^2 + \right] dl + \\ & + \int_l [uP_x + vP_y + wP_z + \theta_x Mx + \theta_y My + \theta_z Mz] \, dl \\ & + \sum_{\forall i_c} [u_{i_c} P_{xi_c} + v_{i_c} P_{vi_c} + w_{i_c} P_{zi_c} + \theta_{xi_c} M_{xi_c} + \theta_{yi_c} M_{yi_c} + \theta_{zi_c} M_{zi_c}] \end{aligned} \tag{4}$$

势能变分原理为

$$\delta \Pi_p = 0$$

其中要求位移满足可能，即边界满足位移约束，转角满足:

$$\theta_y = -\frac{dw}{dx}, \quad \theta_z = \frac{dv}{dx} \tag{5}$$