Laboratory Exercises

- **Set 1 (08-08-2019): Basic Plotting Concepts**

  1. With the help of a single code, plot the following functions:
     A. $y = e^x$ B. $y = x$ C. $y = \ln x$
     Use suitable ranges of x for each of the functions and judge their properties on various scales of x. Extending this exercise, plot $e^{\pm x}$ on the same graph and compare them.
  2. For a fixed parameter k, plot the function $y = \sin(kx)$ for a few suitably chosen values of k. What is the role of k in determining the profile of the function? Thereafter, for $k = 1$, plot $\sin x$ and $\sin^2 x$ on the same graph within $-\pi < x < \pi$. Compare both.
  3. Plot the Gaussian function $y = y_0 e^{-a(x-\mu)^2}$ for a few suitably chosen values of the fixed parameters y_0, a and μ. Examine the shifting profile of the function, with changes in the parameters ($\mu = vt$ simulates a single wave pulse, like a tsunami, travelling with a velocity v). Then for $y_0 = a = 1$ and $\mu = 0$, consider a first-order expansion of the Gaussian function to obtain the Lorentz function. Plot both of them together and compare their behaviour. For every value of x take the difference between the two functions and plot it against x over $0 < x < 10$.
  4. Plot $y = x \ln x$ and carefully examine it for $0 < x < 2$. Provide an analytical justification for what you observe. Also note the growth of the function for very large x.
  5. Plot $y(x)$, $y'(x)$ and $y''(x)$ for the following polynomial functions:
     A. $y = -ax + x^3$ B. $y = -ax^2 + x^4$
     Change a continuously over a suitable range of values ($a \geq 0$) to observe the shift in the function profiles and their two derivatives. Carefully check all conditions for $a = 0$.

- **Set 2 (08-08-2019): Taylor Polynomials**

  1. Consider the following functions, $y = f(x)$:
     A. $y = e^x$ B. $y = \ln x$ C. $y = \sin x$ D. $y = \cos x$
     Produce the first, the second and the third-degree Taylor polynomials for each of the foregoing functions, using $a = 1$ as the point of approximation for $\ln x$ and $a = 0$ for the rest. In a suitably chosen neighborhood of a, follow how the accuracy of a Taylor polynomial improves with its increasing degree. For this you will have to estimate the difference between $f(x)$ and its Taylor polynomials in a code. Present your results graphically for each function along with its Taylor polynomials of all three degrees.

- **Set 3 (22-08-2019): The Bisection Method**

  1. Write a code, applying the algorithm of the bisection method to determine both the real roots of $f(x) = x^6 - x - 1 = 0$.
  2. Numerically implement the bisection algorithm to solve all the problems given in the theory exercises on bisection.
     (*Note*: Provide a plot of each function, and also plot the convergence towards the root with every iteration in the bisection table.)

[1]Statutory Warning: Numerics without proper mathematical judgement is injurious to health.

- **Set 4 (22-08-2019): The Newton-Raphson Method**

  1. Write a code, applying the algorithm of the Newton-Raphson method to determine both the real roots of $f(x) = x^6 - x - 1 = 0$.
  2. Numerically implement the Newton-Raphson algorithm to solve all the problems given in the theory exercises on bisection.

     (*Note*: Plot the convergence towards the root and compare the efficiency of the convergence here with the bisection method.)
  3. Numerically test the convergence of the problem given in **Question 2** in the theory exercises on the Newton-Raphson method.
  4. The function $y = f(x) = a + x(x-1)^2$, with $0 \leq a \leq 0.1$. When $a \neq 0$, there is only one real root of $f(x) = 0$, with the root being negative. Analytically check how many roots are obtained for $a = 0$, and what is the nature of the roots. Thereafter, using the Newton-Raphson method, test for the convergence towards the negative real root, through a series of suitably chosen a values going right down to $a = 0$ (the most important case). In every case your initial guess value should be slightly larger than 1, say 1.01, and slightly smaller than 1, say 0.99. For every value of a, starting from both sides of $x = 1$, check how quickly the convergence happens.

- **Set 5 (29-08-2019): The Secant Method**

  1. Write a code, applying the algorithm of the secant method to determine both the real roots of $f(x) = x^6 - x - 1 = 0$.
  2. Numerically implement the secant method to solve all the problems given in the theory exercises on bisection.

     (*Note*: Since by now, through the bisection and the Newton-Raphson exercises, you know the values of the roots in your given problems, experiment with initial guess values on both sides of the actual root and on the same side of it. Plot the convergence towards the root in both the cases to check whether the convergence is monotonic or not. Also compare the efficiency of the convergence with both the bisection and the Newton-Raphson methods.)

- **Set 6 (10-09-2019): Lagrange and Newton Interpolation**

  1. Carry out the Lagrange linear interpolation between $(1, 1)$ and $(4, 2)$. Plot your interpolation function together with $y = \sqrt{x}$ for comparison.
  2. Carry out a Lagrange linear interpolation for $(0.82, 2.270500)$ and $(0.83, 2.293319)$. Extend your study with a Lagrange quadratic polynomial using $(0.84, 2.316367)$. Compare your polynomials with the function $y = e^x$, plotting all of them on the same graph.
  3. Construct a quadratic Lagrange polynomial using the points $(0, -1)$, $(1, -1)$ and $(2, 7)$. Plot your result. Extend this entire exercise with Newton's divided-difference quadratic polynomial and compare the two methods.
  4. With the data in the following table:

x	3.35	3.40	3.50	3.60
$f(x)$	0.298507	0.294118	0.285714	0.277778

     (a) Produce Lagrange polynomials of the linear, quadratic and cubic orders with increasing values of x.

     (b) Produce Newton's divided-difference polynomials of all the three foregoing orders.

(c) Plot the results of both methods on the same graph and compare them with the function $y = 1/x$. Also comment on the respective computational advantages of the two methods above.

5. With the data in the following table:

x	0	1	2	2.5	3	3.5	4
y	2.5	0.5	0.5	1.5	1.5	1.125	0

(a) Interpolate successive points by straight line segments. This is known as piecewise linear interpolation.

(b) On each of the three following subintervals of x $[0, 2]$, $[2, 3]$ and $[3, 4]$ interpolate using both Lagrange's quadratic polynomial and Newton's divided-difference interpolation polynomial.

(c) Plot the results of both methods covering all the three subintervals on the same graph and compare them.