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Discrete Mathematics
Assignment - 3.

Ques 1. What do you mean by SOP and POS. Interpret the following SOP expression and convert it to an equivalent POS expression.

$$AB'C' + A'B'C' + AB'C + ABC'$$

Solution - SOP stands for Sum of Product. It is a set of product (AND) terms that are summed (OR) together. When an expression or term is represented in a sum of binary terms known as minterms and sum of products

Example → $A'B'C + A'BC$.

POS stands for product of sum. A technique of explaining a Boolean expression through a set of max terms or sum terms, is known as POS.

Example → $(A+B+C).(A'+B'+C)$.

⟹ $(AB'C') + (A'B'C') + (AB'C) + ABC'$

Complete SOP will be

$A'B'C' + A'B'C + A'BC' + A'BC + AB'C' + AB'C + ABC' + ABC$

POS expression will be -

$(A'+B'+C').(A+B+C').(A+B'+C).(A+B'+C')$

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Ques 2) $(p \leftrightarrow q) \leftrightarrow (q \leftrightarrow p)$. Consider this is a tautology.

Solution

p	q	$p \leftrightarrow q$	$q \leftrightarrow p$	$(p \leftrightarrow q) \leftrightarrow (q \leftrightarrow p)$
T	T	T	T	T
T	F	F	F	T
F	T	F	F	T
F	F	T	T	T

Resultant is always true. Therefore it a tautology.

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Ques 3. Show using a truth table that the bi conditional is equivalent to contrapositive

Solution Biconditional is represented as $p \leftrightarrow q$
Contrapositive of biconditional can be represented as
$\sim q \leftrightarrow \sim p$

Truth table.

p	q	$\sim p$	$\sim q$	$p \leftrightarrow q$	$\sim q \leftrightarrow \sim p$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

Hence, we can say that biconditional is equivalent to contrapositive. Proved

Ques 4. State and prove the validity of the following theorems of Boolean Algebra by means of truth table.
1. Distributive law
2. Absorption law.

Ques: 4B) i) Distributive law.

$\Rightarrow a.(b+c) = (a.b) + (a.c)$

Solution

Truth table -

a	b	c	a.b	a.c	b+c	a.(b+c)	(a.b)+(a.c)
T	T	T	T	T	T	T	T
T	T	F	T	F	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	F	F	F	F
F	T	T	F	F	T	F	F
F	T	F	F	F	T	F	F
F	F	T	F	F	T	F	F
F	F	F	F	F	F	F	F

Hence, $a.(b+c) = (a.b) + (a.c)$
distributive law is proved.

ii) Absorption law.

$\Rightarrow a+(a.b) = a$

Truth table -

a	b	a.b	a+a.b
T	T	T	T
T	F	F	T
F	T	F	F
F	F	F	F

$\therefore a+(a.b) = a$

Hence, absorption law Proved

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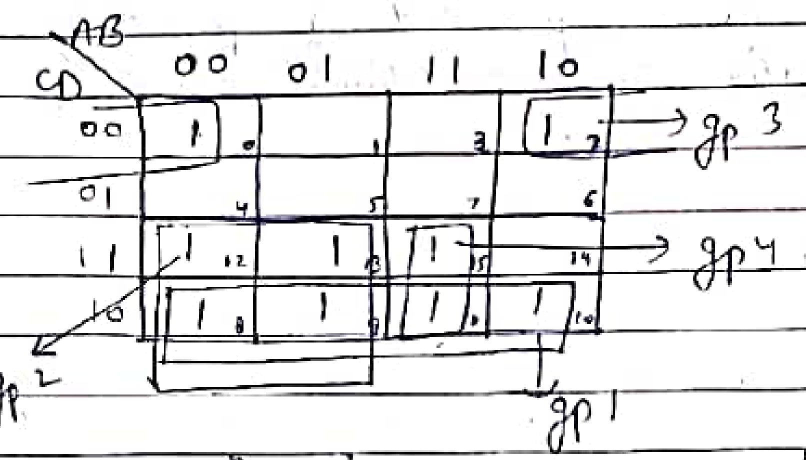
Ques 5. Simplify the Boolean function by means of K-Map.

$$f(A,B,C,D) = \Sigma m(0,2,8,9,10,11,12,13,15)$$



gp 1 → CD'
gp 2 → $A'C$

gp 3 → $B'C'D'$
gp 4 → ABC

Final answer = $A'C + CD' + B'C'D' + ABC$

Ques-6. Write all rules of inference and by using these rules check whether the following arguments are logically correct or not ?

Premises : There are men who are soldiers. All soldiers are strong. All soldiers are brave

Conclusion : Therefore some strong men are brave

Solution: Rules of inference

Name	Rule	
① Addition	ⓐ $\dfrac{p}{p \vee q}$	ⓑ $\dfrac{q}{p \vee q}$
② Simplification	ⓐ $\dfrac{p \wedge q}{p}$	ⓑ $\dfrac{p \wedge q}{q}$

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③ Conjunction
$$\frac{\begin{array}{c} p \\ q \end{array}}{p \wedge q}$$

④ Modus Ponens
$$\frac{\begin{array}{c} p \to q \\ p \end{array}}{q}$$

⑤ Modus Tollens
$$\frac{\begin{array}{c} p \to q \\ \sim q \end{array}}{\sim p}$$

⑥ Hypothetical Syllogism
$$\frac{\begin{array}{c} p \to q \\ q \to r \end{array}}{p \to r}$$

⑦ Disjunctive
$$\frac{\begin{array}{c} p \vee q \\ \sim p \end{array}}{q}$$

⑧ Constructive dillema
$$\frac{\begin{array}{c} (p \to q) \wedge (r \to s) \\ p \vee r \end{array}}{q \vee s}$$

⑨ Destructive dillema
$$\frac{\begin{array}{c} (p \to q) \wedge (r \to s) \\ \sim q \vee \sim s \end{array}}{\sim p \vee \sim r}$$

Premises : There are men who are soldiers.
All soldiers are strong. All soldiers are brave

~~P(x) ... P(x)~~

P(x) : x is a man
Q(x) : x is a soldier
R(x) : x is strong
S(x) : x is brave.

The premises and the conclusions in terms of the predicate defined above are -

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P1: $\exists x\,[P(x) \wedge Q(x)]$
P2: $\forall x\,[Q(x) \rightarrow R(x)]$
P3: $\forall x\,[Q(x) \rightarrow S(x)]$

C: $\exists x\,[P(x) \wedge R(x) \wedge S(x)]$

Therefore, the conclusion "Some strong men are brave" is logically correct based on the given premises and the rules of inference.

Ques 7. How many 7 digit numbers can be formed using digits 1, 7, 2, 7, 6, 7, 6 ?

Solution 7 digit numbers can be formed in

$$\frac{7!}{1! \times 1! \times 3! \times 2!} = \frac{7 \times 6 \times 5 \times 4 \times 3 \times 2}{3 \times 2 \times 2}$$

$$= 7 \times 5 \times 4 \times 3$$
$$= 420$$

420 different 7-digit numbers can be formed using digits 1, 7, 2, 7, 6, 7, 6.

Ques 8. How many permutations can be made out of the letter of word "COMPUTER"?
How many of these -

~~Solution~~.

a) Begin with C
b) End with R
c) Begin with C and end with R
d) C and R occupy the end places.