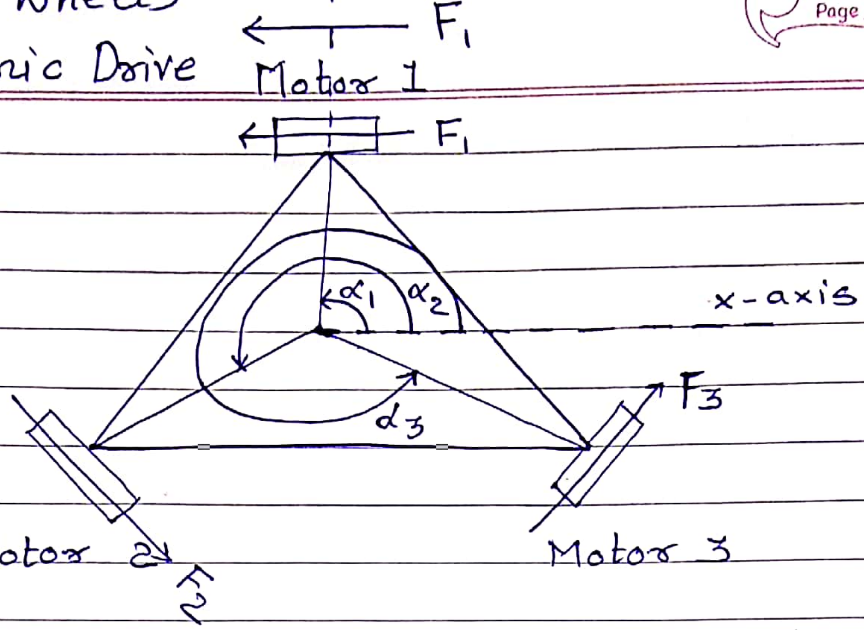
Calculation For 3-Wheels Holonomic Drive



Each (motor-wheel) mounting is 120° apart. As drawn in figure

Value of $\alpha_1 = 90^\circ \longrightarrow (1)$

$\alpha_2 = 90^\circ + 120^\circ = 210^\circ \longrightarrow (2)$

$\alpha_3 = 90^\circ + 2 \cdot 120^\circ = 330^\circ \longrightarrow (3)$

As shown in the figure F_1, F_2 and F_3 are the velocity given to the motors and wheels.

So, the vector acceleration given to the robot. (a)

$$a = F_1 + F_2 + F_3 = a_x + a_y \longrightarrow (4)$$

(Which includes both x- and y-direction)

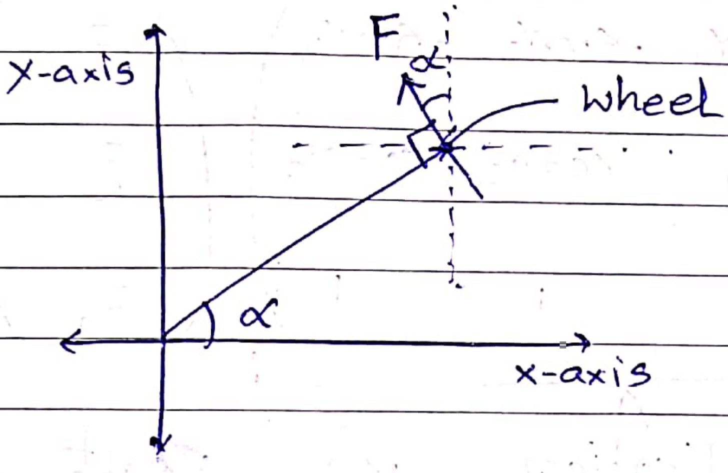
Angular acc. of the robot (ω)

$$\omega = |F_1| + |F_2| + |F_3|$$

Suppose $f_1 = |F_1|$ $f_2 = |F_2|$ $f_3 = |F_3|$

$\therefore \omega = F_1 + F_2 + F_3 \longrightarrow (5)$

Now calculating the effect of the speed given to each motor.



So effect of the speed F given to the wheel in x-direction.
$= -F \sin\alpha$

$\therefore a_x = -F \sin\alpha \qquad a_y = F\cos\alpha$

doing same for all the three (motor-wheel) mounting.

$\therefore a_{1x} = -F_1 \sin\alpha_1 \qquad a_{1y} = F_1 \cos\alpha_1$

$a_{2x} = -F_2 \sin\alpha_2 \qquad a_{2y} = F_2 \cos\alpha_2$

$a_{3x} = -F_3 \sin\alpha_3 \qquad a_{3y} = F_3 \cos\alpha_3$

acc. of robot in x-direction

$\therefore a_x = -F_1 \sin\alpha_1 - F_2 \sin\alpha_2 - F_3 \sin\alpha_3 \longrightarrow (6)$

y-direction

$\therefore a_y = F_1 \cos\alpha_1 + F_2 \cos\alpha_2 + F_3 \cos\alpha_3 \longrightarrow (7)$

From (5)

$$w = f_1 + f_2 + f_3$$

So converting eq^n into the matrix

$$\begin{pmatrix} a_x \\ a_y \\ w \end{pmatrix} = \underbrace{\begin{pmatrix} -\sin\alpha_1 & -\sin\alpha_2 & -\sin\alpha_3 \\ \cos\alpha_1 & \cos\alpha_2 & \cos\alpha_3 \\ 1 & 1 & 1 \end{pmatrix}}_{A} \begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix}$$

For our application we want f_1, f_2 and f_3 as per the (a_x, a_y, w).

So, by multiplying both side with the inverse of A

Firstly calculating matrix

$$-\sin\alpha_1 = -\sin(90^\circ) = -1$$
$$-\sin\alpha_2 = -\sin(180^\circ + 30^\circ) = 1/2$$
$$-\sin\alpha_3 = -\sin(360^\circ - 30^\circ) = 1/2$$
$$\cos\alpha_1 = \cos(90^\circ) = 0$$
$$\cos\alpha_2 = \cos(180^\circ + 30^\circ) = -\sqrt{3}/2$$
$$\cos\alpha_3 = \cos(360^\circ - 30^\circ) = \sqrt{3}/2$$

putting values ~~im~~ in matrix and calculating the inverse of that.

$$A = \begin{pmatrix} -1 & 1/2 & 1/2 \\ 0 & -\sqrt{3}/2 & \sqrt{3}/2 \\ 1 & 1 & 1 \end{pmatrix} \qquad A^{-1} = \frac{adj(A)}{|A|}$$

$$cofactor(A) = \begin{pmatrix} -\sqrt{3} & +\sqrt{3}/2 & \sqrt{3}/2 \\ -0 & -3/2 & +3/2 \\ \sqrt{3}/2 & +\sqrt{3}/2 & \sqrt{3}/2 \end{pmatrix}$$

$$|A| = -1\left(\frac{-\sqrt{3}}{2} - \frac{\sqrt{3}}{2}\right) - \frac{1}{2}\left(0 - \sqrt{3}/2\right) + \frac{1}{2}\left(0 - \left(-\sqrt{3}/2\right)\right)$$

So $adj(A) = cofactor(A)^T$

$$|A| = -1(-\sqrt{3}) + \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4}$$

$$= \begin{pmatrix} -\sqrt{3} & 0 & \sqrt{3}/2 \\ \sqrt{3}/2 & -3/2 & \sqrt{3}/2 \\ \sqrt{3}/2 & 3/2 & \sqrt{3}/2 \end{pmatrix}$$

$$|A| = \sqrt{3} + \frac{\sqrt{3}}{2}$$

$$|A| = \frac{3\sqrt{3}}{2}$$

$$\therefore A^{-1} = \frac{adj(A)}{|A|} = \frac{adj(A)}{3\sqrt{3}/2} = \frac{2}{3\sqrt{3}} \cdot adj(A)$$

$$\therefore A^{-1} = \frac{2}{3\sqrt{3}} \begin{pmatrix} -\sqrt{3} & 0 & \sqrt{3}/2 \\ \sqrt{3}/2 & -3/2 & \sqrt{3}/2 \\ \sqrt{3}/2 & 3/2 & \sqrt{3}/2 \end{pmatrix} = \begin{pmatrix} -2/3 & 0 & 1/3 \\ 1/3 & -1/\sqrt{3} & 1/3 \\ 1/3 & 1/\sqrt{3} & 1/3 \end{pmatrix}$$

So.

$$\begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix} = \begin{pmatrix} -2/3 & 0 & 1/3 \\ 1/3 & -1/\sqrt{3} & 1/3 \\ 1/3 & 1/\sqrt{3} & 1/3 \end{pmatrix} \begin{pmatrix} a_x \\ a_y \\ \omega \end{pmatrix}$$

$$\therefore \boxed{\begin{aligned} f_1 &= -2/3 \cdot a_x + 0 \cdot a_y + 1/3\, \omega \\ f_2 &= 1/3 \cdot a_x + (-1/\sqrt{3}) \cdot a_y + 1/3\, \omega \\ f_3 &= 1/3 \cdot a_x + (1/\sqrt{3}) \cdot a_y + 1/3\, \omega \end{aligned}}$$