TOV equations - Problem VI

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1 TOV equations

We integrate the TOV equations:

$$\frac{dP}{dr} = -G\left(\rho + \frac{P}{c^2}\right)\frac{m + \frac{4\pi r^3 P}{c^2}}{r\left(r - \frac{2Gm}{c^2}\right)}$$
$$\frac{dm}{dr} = 4\pi r^2 \rho$$

where $\rho = \rho_0(1+\epsilon/c^2)$ is the energy density, ρ_0 is the rest mass, and ϵ is the specific internal energy of the fluid and can be computed with the following expression:

$$\epsilon = \frac{P}{\rho_0(\Gamma - 1)}.$$

The initial value of the pressure can be computed with the EoS: $P = \kappa\rho_0^\Gamma$. We initially set $\rho_{0,c} = 5 \times 10^{14}\,\mathrm{g/cm^{-3}}$ and $\kappa = 3000$. Initial conditions: $P(r=0) = P_c(\rho_c)$, $m(r=0) = 0$.

We decide to use the RK4 method to solve the system of ODEs and to write things in $c = G = M_\odot = 1$ units.

1.1 Code structure

- Function
  - for the EoS ("EoS")
  - for the system of ODEs ("TOV_eq")
  - for constructing the stellar model ("make_star")
    - It takes the central value of the rest mass density $\rho_{0,c}$, the parameters of the polytropic EoS and performs the "RK4 steps". We compute the initial value of the pressure with the EoS. At each step, we compute ρ with the following expression:

$$\rho = \rho_0 + \frac{current\_P}{\Gamma - 1}, \tag{1}$$

where

$$\rho_0 = \left(\frac{current\_P}{\kappa}\right)^{\frac{1}{\Gamma}} \tag{2}$$

- The integration of the TOV equations stops when $P < 10^{-7}$.
- Then we find the radius $R = r[current\_index]$ and the mass $M = m[current\_index]$.
- Define the conversion factor for the density and the distance.
- We set $r\_start\_IS = 10^{-5}$ km and $r\_end\_IS = 16$ km and we convert these values in $c = G = M_\odot = 1$ units.
- We compute the mass and the radius of the related neutron star: $M = 0.859$ and $R = 8.116$ in $c = G = M_\odot = 1$ units as expected.
- We then vary $\rho_{0,c}$ and plot the M-R diagram and the mass of the star against $\rho_{0,c}$ ($\mathrm{g/cm^3}$):

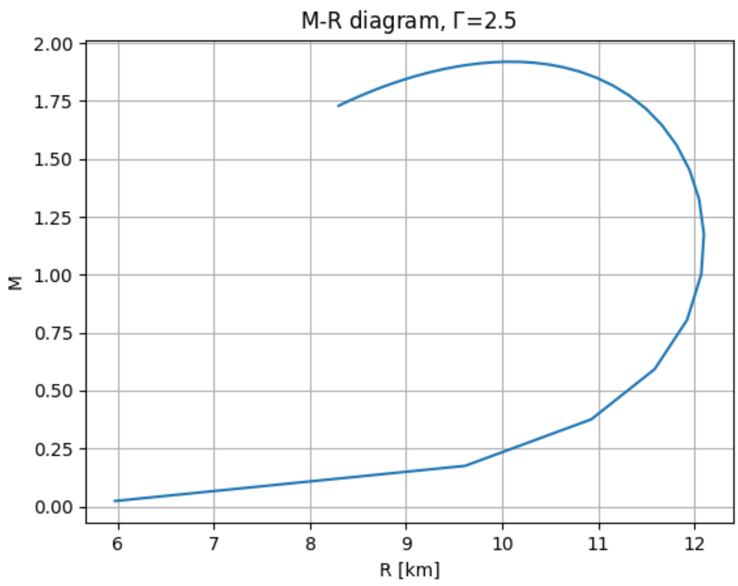
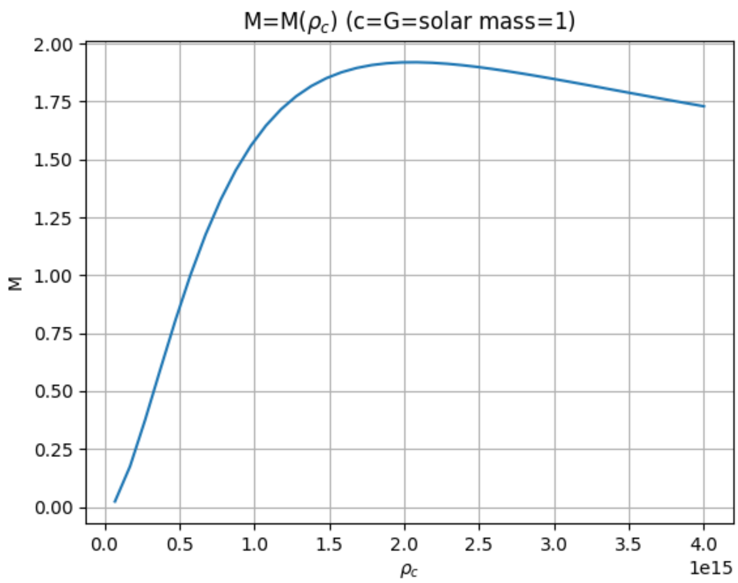


Figure 1: M-R diagram for $8 \times 10^{13}\,\mathrm{g/cm^3} < \rho_{0,c} < 4 \times 10^{15}\,\mathrm{g/cm^3}$



We find $M_{\mathrm{TOV}} = 1.92$ in this case (related radius $R = 10.04\,\mathrm{km}$).

- We consider a different polytropic EoS: $\Gamma = 2$ and $\kappa = 100$.
- M-R diagram:

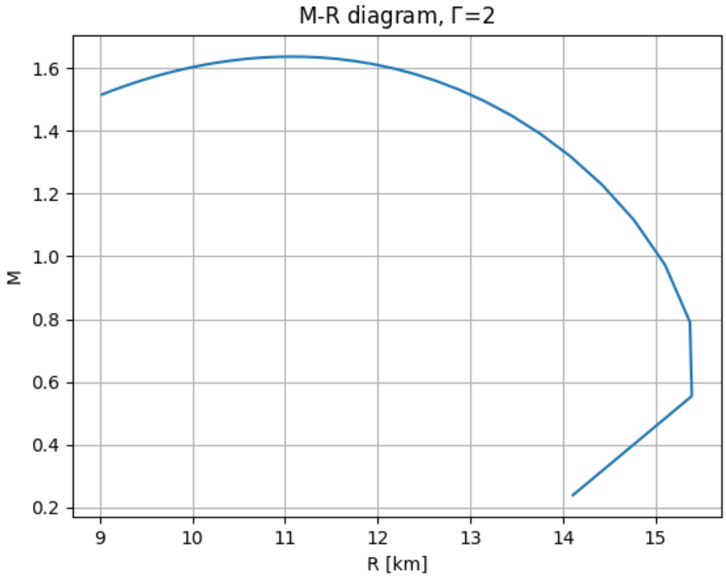
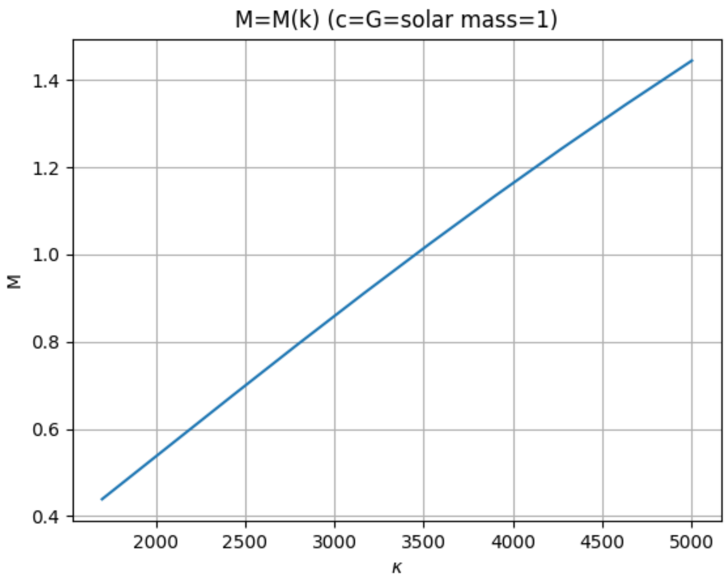
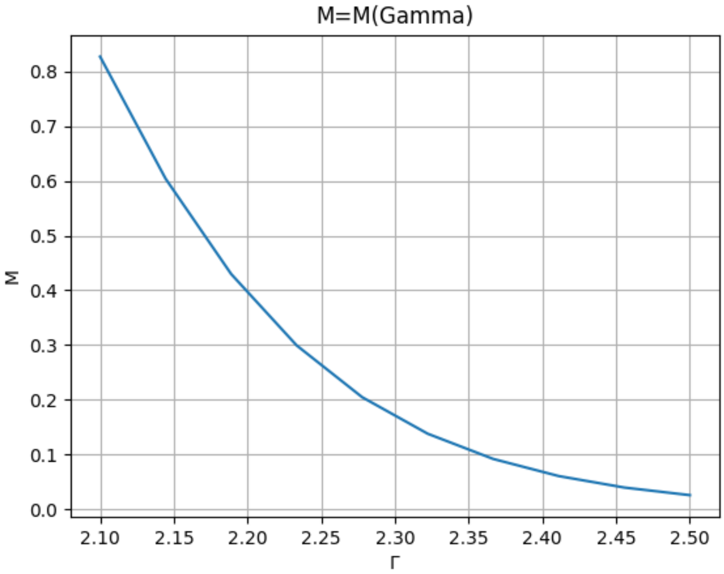


Figure 2: M-R diagram for $8 \times 10^{13}\,\mathrm{g/cm^3} < \rho_{0,c} < 4 \times 10^{15}\,\mathrm{g/cm^3}$

- Then, we vary κ by keeping $\Gamma = 2.5$ fixed. For each κ we find the stellar model.
- Plot of mass against κ:



- Then, we vary Γ by keeping $\kappa = 100$ fixed. For each Γ we find the stellar model.
- Plot of mass against Γ:


M=M(Gamma)
M
Γ
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
2.10
2.15
2.20
2.25
2.30
2.35
2.40
2.45
2.50