

Free single pendulum and damped harmonic oscillator - Problem VII and VIII

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1 Single pendulum

We solve the following system of ordinary differential equations

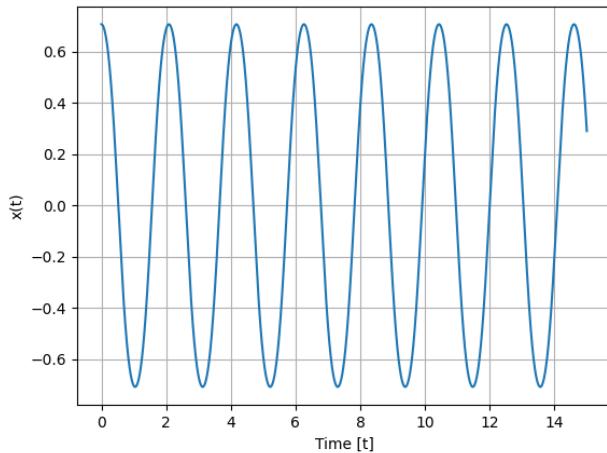
$$\begin{aligned}\frac{d\theta}{dt} &= y \\ \frac{dy}{dt} &= -\frac{g}{l} \sin \theta\end{aligned}$$

where $l = 1\text{ m}$ is the string length and $g = 9.81\text{ m/s}^2$ is the acceleration of gravity. Initial conditions: $\theta(0) = \frac{\pi}{4}$, $\frac{d\theta}{dt}|_{t=0} = 0$.

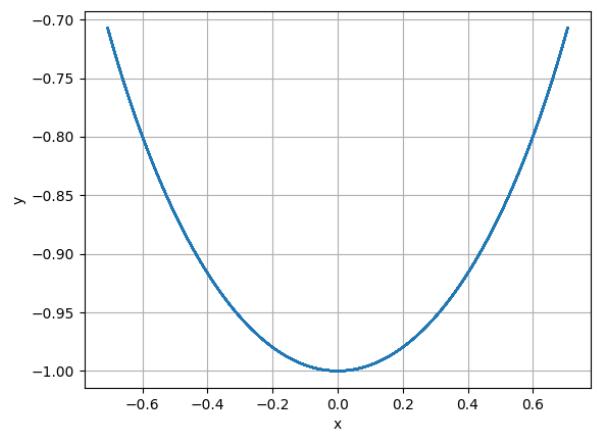
We decide to use the RK4 method to solve the system of ODEs.

1.1 Code structure

- Function
 - for the system of ODEs ("pendulum_eq")
 - performing one "RK4 step" in vectorial form ("rk4_step")
 - performing all the RK4 steps ("solve_pendulum")
- We set initially $t_start=0$, $t_end=15$, and $n_points=15000$ ($\delta t = 0.001$).
- We solve the system and plot $x = x(t)$ and the phase diagram $x(t)$ vs $y(t)$.



(a) Time evolution of the Cartesian coordinate x



(b) Phase diagram

- We also create the animated plot (with the help of chat GPT).

2 Damped harmonic oscillator

We solve the following system of ODEs:

$$\begin{aligned}\frac{dx}{dt} &= y \\ \frac{dy}{dt} &= -\frac{1}{m}(kx + by)\end{aligned}$$

where m is the mass of the oscillator, k is the spring's constant, and $by = b\dot{x}$ is the damping term. Initial conditions: $x(0) = 1$ and $\dot{x}(0) = 0$. We set $b = 1$, $k = 300$, and $m = 1$.

2.1 Code structure

- Function
 - for the system of ODEs ("pendulum_eq")
 - performing one "RK4 step" in vectorial form ("rk4_step")
 - performing all the RK4 steps ("solve_pendulum")
- We set initially $t_start=0$, $t_end=15$, and $n_points=15000$ ($\delta t = 0.001$).
- We make an animated plot as well as a plot for $x = x(t)$.

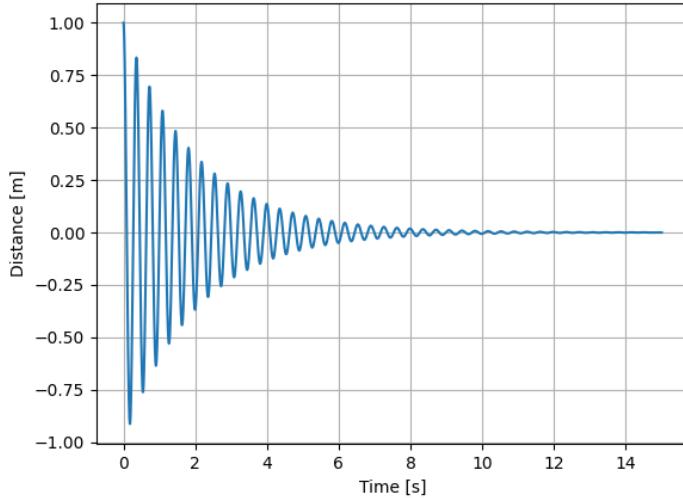


Figure 2: Cartesian coordinate x against time.