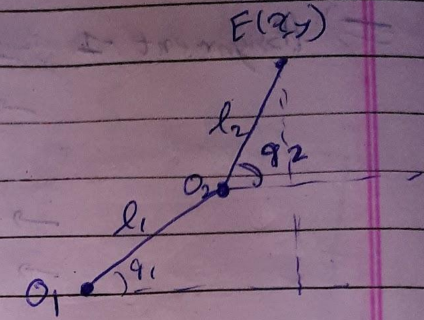
Name: Harsh Mandalia
Roll No. 19110186

General instructions to run the code:

1. Each task has one or more than one python file that you need to run to see the animation.
2. Press “q” while “window” is selected to close the window. Or press ctrl+c while the terminal is selected to force quit.
3. Feel free to change the parameters in the __init__(self) function.
4. Task1.py involved kinematic approach where calculated q1 and q2 are directly animated, but task1_dynamics.py involves dynamics approach where tau1 and tau2 (torques) are calculated.
5. In task2.py, self.wall1 and self.wall2 are points of two ends of the wall. The wall is being shown in animation between those two points only but actually, the wall is taken as an infinite line for the 2R manipulator.
6. In task3.py self.x0 and self.y0 are the mean position of the spring(bot).
7. In task4.py in line 47 (mybot.trace(5)) change the value 5 to something else (integer) to change the speed at with the simulation draws the workspace.

Notes are on the next page.

Mini Project



⇒ here from the right angle triangles,

$$x = l_1 \cos q_1 + l_2 \cos q_2$$
$$y = l_1 \sin q_1 + l_2 \sin q_2 \quad \text{--- (1)}$$

⇒ to get the end effector's velocity $E(\dot{x}, \dot{y})$,

$$\dot{x} = -l_1 \dot{q}_1 \sin q_1 - l_2 \dot{q}_2 \sin q_2$$
$$\dot{y} = l_1 \dot{q}_1 \cos q_1 + l_2 \dot{q}_2 \cos q_2$$

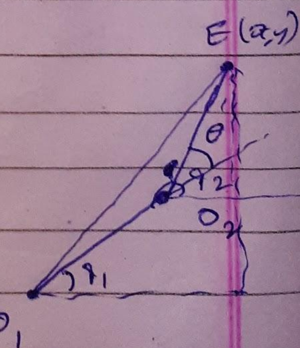
in vector-matrix form,

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -l_1 \sin q_1 & -l_2 \sin q_2 \\ l_1 \cos q_1 & l_2 \cos q_2 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} \quad \text{--- (2)}$$

⇒ find q_1 & q_2 given x & y using eq^n (1),

here, $\theta = (q_2 - q_1)$

from cosine law,

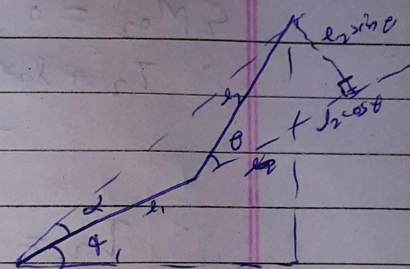


$$x^2 + y^2 = l_1^2 + l_2^2 - 2 l_1 l_2 \cos(\pi - \theta)$$

$$\therefore \cos\theta = \frac{x^2 + y^2 - l_1^2 - l_2^2}{2 l_1 l_2}$$

$$\therefore \theta = \cos^{-1}\left(\frac{x^2+y^2-l_1^2-l_2^2}{2l_1l_2}\right)$$

here, $\tan(\alpha+q_1) = \frac{y}{x}$



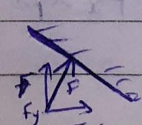
$$\therefore \alpha + q_1 = \tan^{-1}\left(\frac{y}{x}\right)$$

where, $\tan\alpha = \frac{l_2\sin\theta}{l_1+l_2\cos\theta}$

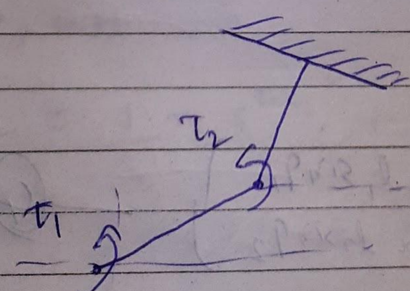
$$\therefore \alpha = \tan^{-1}\left(\frac{l_2\sin\theta}{l_1+l_2\cos\theta}\right)$$

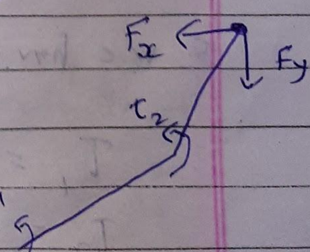
$$\left.\begin{aligned} \therefore q_1 &= \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{l_2\sin\theta}{l_1+l_2\cos\theta}\right) \\ \text{and } q_2 &= \theta + q_1 \end{aligned}\right\} \quad \text{——} \ 3$$

$\Rightarrow$ to apply constant normal force on the wall (ignoring gravity)



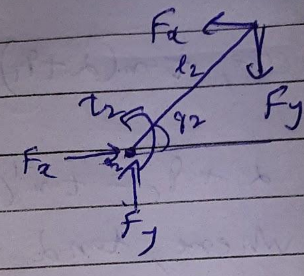
and





→ for link-2 to be in equilibrium,

~~$\sum F_x = 0$~~



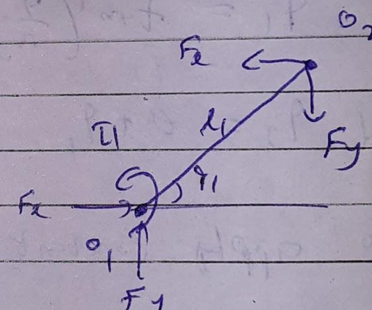
$\sum M_{o_2} = 0$

$\therefore T_2 + l_2 \sin q_2 F_x - l_2 \cos q_2 F_y = 0$

$\therefore T_2 = F_y l_2 \cos q_2 - F_x l_2 \sin q_2$

→ for link-1 to be in equilibrium,

$\sum M_{o_1} = 0$



$\therefore T_1 + F_x l_1 \sin q_1 - F_y l_1 \cos q_1 = 0$

$\therefore T_1 = F_y l_1 \cos q_1 - F_x l_1 \sin q_1$

(assuming remately driven joins)

So we have,

$$\left.\begin{aligned} T_1 &= F_y l_1 \cos q_1 - F_x l_1 \sin q_1 \\ T_2 &= F_y l_2 \cos q_2 - F_x l_2 \sin q_2 \end{aligned}\right\} \quad \text{— (4)}$$

⇒ equation of motion of 2R Manipulator :-

→ Langrangian :- $L = K - V$

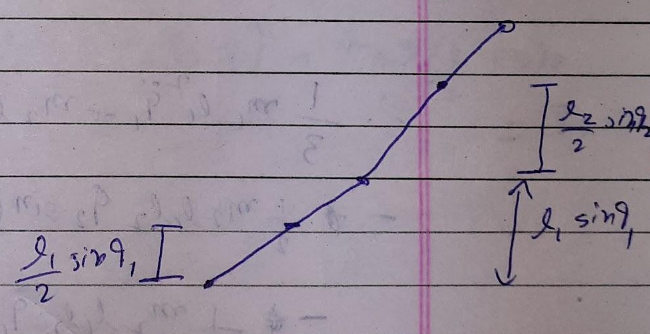
Where, $K = K_{1r} + K_{2r} + K_{2t}$

$$= \frac{1}{2}\left(\frac{1}{3} m_1 l_1^2\right) \dot{q}_1^2$$

$$+ \frac{1}{2} m_2 v_{2c}^2 + \frac{1}{2}\left(\frac{1}{12} m_2 l_2^2\right) \dot{q}_2^2$$

here, $v_{2c}^2 = l_1^2 \dot{q}_1^2 + \frac{1}{4} l_2^2 \dot{q}_2^2 + l_1 l_2 \dot{q}_1 \dot{q}_2 \cos(q_2 - q_1)$

and $V = m_1 g \frac{l_1}{2} \sin q_1 + m_2 g \left(l_1 \sin q_1 + \frac{l_2}{2} \sin q_2\right)$



So, Langrangian,

$$L = \frac{1}{6} m_1 l_1^2 \dot{q}_1^2 + \frac{1}{2} m_2 l_1^2 \dot{q}_1^2 + \frac{1}{8} m_2 l_2^2 \dot{q}_2^2 +$$

$$\frac{1}{2} m_2 l_1 l_2 \dot{q}_1 \dot{q}_2 \cos(q_2 - q_1) + \frac{1}{24} m_2 l_2^2 \dot{q}_2^2$$

$$- \frac{1}{2} m_1 g l_1 \sin q_1 - m_2 g \left(l_1 \sin q_1 + \frac{l_2}{2} \sin q_2\right)$$

⇒ equation of motion,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i \qquad \text{(5)}$$

for $i=1$,

$$\frac{d}{dt}\left[\frac{1}{3} m_1 l_1^2 \dot{q}_1 + m_2 l_1^2 \dot{q}_1 + \frac{1}{2} m_2 l_1 l_2 \dot{q}_2 \cos(q_2-q_1)\right]$$

$$- \left[\frac{1}{2} m_2 l_1 l_2 \dot{q}_1 \dot{q}_2 \{-\sin(q_2-q_1)\}(-1) - \frac{1}{2} m_1 g l_1 \cos q_1 - m_2 g l_1 \cos q_1\right]$$

$$= \tau_1$$

$$\therefore \frac{1}{3} m_1 l_1^2 \ddot{q}_1 + m_2 l_1^2 \ddot{q}_1 + \frac{1}{2} m_2 l_1 l_2 \ddot{q}_2 \cos(q_2-q_1)$$

$$- \frac{1}{2} m_2 l_1 l_2 q_2 \sin(q_2-q_1)\left[\dot{q}_2-\dot{q}_1\right]$$

$$- \frac{1}{2} m_2 l_1 l_2 q_1 q_2 \sin(q_2-q_1) + \frac{1}{2} m_1 g l_1 \cos q_1$$

$$+ m_2 g l_1 \cos q_1$$

$$= \tau_1$$

$$\text{(6a)}$$

for $i=2$,

$$\frac{d}{dt}\left[\frac{1}{4} m_2 l_2^2 \dot{q}_2 + \frac{1}{2} m_2 l_1 l_2 \dot{q}_1 \cos(q_2-q_1) + \frac{1}{12} m_2 l_1^2 \dot{q}_2\right]$$

$$-\left\{\frac{1}{2} m_2 l_1 l_2 \dot{q}_1 \dot{q}_2 \left[-\sin(q_2-q_1)\right] - m_2 g \frac{l_2}{2} \cos q_2\right] = \tau_2$$

$$\therefore \frac{1}{4} m_2 l_2^2 \ddot{q}_2 + \frac{1}{2} m_2 l_1 l_2 \ddot{q}_1 \cos(q_2-q_1) + \frac{1}{12} m_2 l_1^2 \ddot{q}_2$$

$$- \frac{1}{2} m_2 l_1 l_2 \dot{q}_1 \sin(q_2-q_1)\left[\dot{q}_2-\dot{q}_1\right]$$

$$+ \frac{1}{2} m_2 l_1 l_2 \dot{q}_1 \dot{q}_2 \sin(q_2-q_1) + m_2 g \frac{l_2}{2} \cos q_2 = \tau_2$$