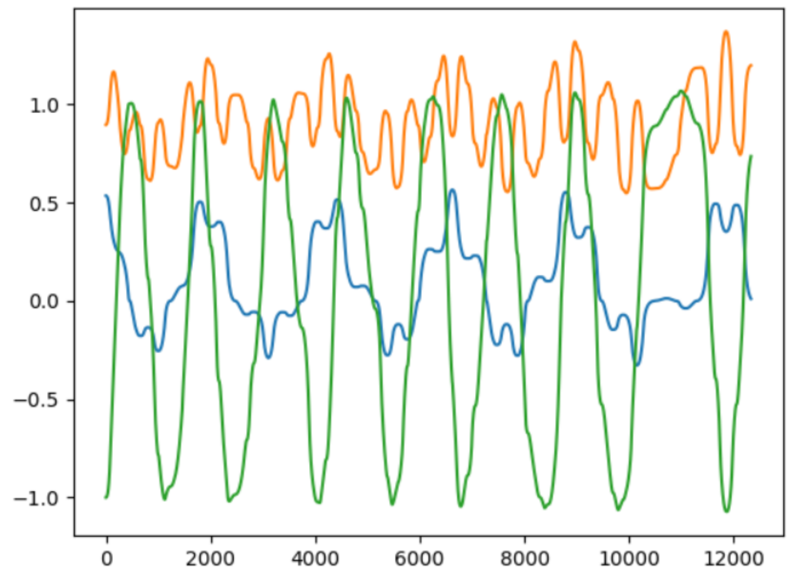
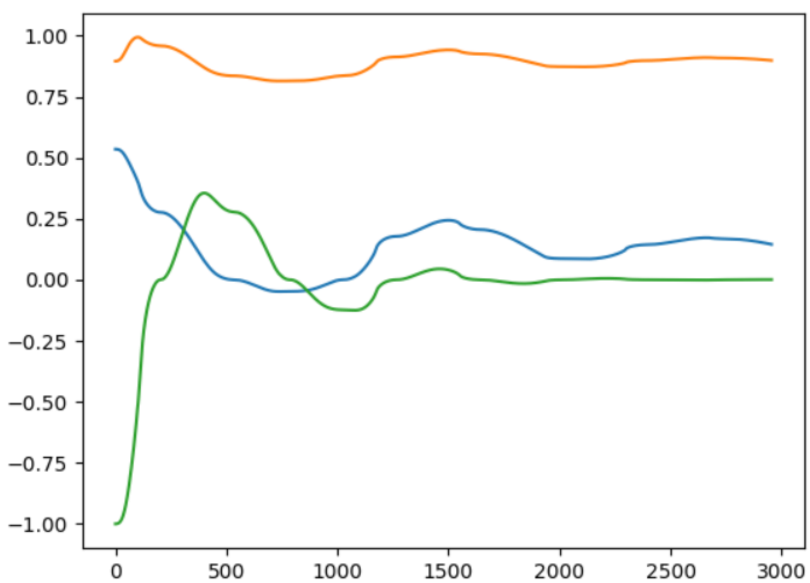
For the PI controller, the oscillations were not decaying and due to larger simulation time, I was not able to tune it properly. So, I instead implemented a PID controller as the differential term will decay the oscillation of join parameters.



(PI controller for SCARA manipulator)



(PID controller for SCARA manipulator)

Run the code pre-calculated.py and change the variable name “bot_name” to one of the bots to see the pre-calculated joint animation and plot for that bot.

The jacobians and potential energy were hard coded to find the D matrix. Below is the small derivation of it.

Assignment →

⇒ Scara:-

$$P_{c_1} = \begin{bmatrix} \frac{l_2}{2}\cos q_1 \\ \frac{l_2}{2}\sin q_1 \\ l_1 \end{bmatrix} \Rightarrow v_{c_1} = \begin{bmatrix} -\frac{l_2}{2}\sin q_1 & 0 & 0 \\ \frac{l_2}{2}\cos q_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{d}_3 \end{bmatrix}$$

$$P_{c_2} = \begin{bmatrix} l_2\cos q_1 + \frac{l_3}{2}\cos(q_1+q_2) \\ l_2\sin q_1 + \frac{l_3}{2}\sin(q_1+q_2) \\ l_1 \end{bmatrix}$$

⇓

$$V_{c_2} = \begin{bmatrix} -l_2\sin q_1 - \frac{l_3}{2}\sin(q_1+q_2) & -\frac{l_3}{2}\sin(q_1+q_2) & 0 \\ l_2\cos q_1 + \frac{l_3}{2}\cos(q_1+q_2) & \frac{l_3}{2}\cos(q_1+q_2) & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{d}_3 \end{bmatrix}$$

$$P_{c_3} = \begin{bmatrix} l_2\cos q_1 + l_3\cos(q_1+q_2) \\ l_2\sin q_1 + l_3\sin(q_1+q_2) \\ l_1 + \frac{d_3}{2} \end{bmatrix}$$

⇓

$$V_{c_3} = \begin{bmatrix} -l_2\sin q_1 - l_3\sin(q_1+q_2) & -l_3\sin(q_1+q_2) & 0 \\ l_2\cos q_1 + l_3\cos(q_1+q_2) & l_3\cos(q_1+q_2) & 0 \\ 0 & 0 & 1/2 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{d}_3 \end{bmatrix}$$

→ Potential energy, $V = m_1 g l_1 + m_2 g l_1 + m_3 g (l_1 + d_3/2)$

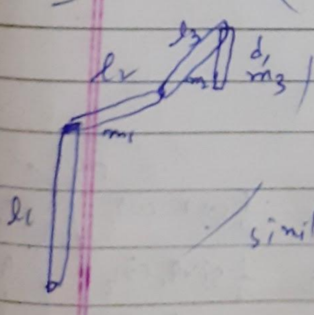
$$W_1 = \begin{bmatrix} 0 \\ 0 \\ \dot{q}_1 \end{bmatrix}, \quad W_2 = \begin{bmatrix} 0 \\ 0 \\ \dot{q}_1 + \dot{q}_2 \end{bmatrix}$$

$$\cancel{w_1^T I_1 w_1 = \dot{q}^T I}$$

$$\therefore W_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix}, \quad W_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix}$$

so, $$w_1^T I_1 w_1 = \dot{q}^T \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \dot{q}$$

$$= \dot{q}^T \begin{bmatrix} I_{zz} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \dot{q}$$



when, $I_{zz} = I_1$ (in code)

$= \frac{1}{12} m_1 l_2^2$

similarly, $w_2^T I_2 w_2 = \dot{q}^T \begin{bmatrix} I_{zz} & I_{zz} & 0 \\ I_{zz} & I_{zz} & 0 \\ 0 & 0 & 0 \end{bmatrix} \dot{q}$, $I_{zz} = I_2$ (in code)

$= \frac{1}{12} m_2 l_3^2$

$\Rightarrow$ Stanford:-

$$P_{c_1} = \begin{bmatrix} 0 \\ 0 \\ l_1/2 \end{bmatrix} \Rightarrow V_{c_1} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{d}_3 \end{pmatrix}$$

$$P_{c_2} = \begin{bmatrix} \frac{l_2}{2}\cos q_1 \cos q_2 \\ \frac{l_2}{2}\sin q_1 \cos q_2 \\ l_1 + \frac{l_2}{2}\sin q_2 \end{bmatrix}$$

$$\Rightarrow \cancel{V_{c_2} = \begin{bmatrix} -\frac{l_2}{2}\sin q_1 & 0 & 0 \\ \frac{l_2}{2}\cos q_1 & 0 & 0 \\ 0 & \frac{l_2}{2}\cos q_2 & 0 \end{bmatrix} \begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{d}_3 \end{pmatrix}}$$

$\cancel{P_{c_3}}$

$$\text{so, } V_{c_2} = \begin{bmatrix} -\frac{l_2}{2}\sin q_1 \cos q_2 & -\frac{l_2}{2}\cos q_1 \sin q_2 & 0 \\ \frac{l_2}{2}\cos q_1 \cos q_2 & -\frac{l_2}{2}\sin q_1 \sin q_2 & 0 \\ 0 & \frac{l_2}{2}\cos q_2 & 0 \end{bmatrix} \begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{d}_3 \end{pmatrix}$$

$$P_{c_3} = \begin{bmatrix} \left(l_2 + \frac{d_3}{2}\right)\cos q_1 \cos q_2 \\ \left(l_2 + \frac{d_3}{2}\right)\sin q_1 \cos q_2 \\ l_1 + \left(l_2 + \frac{d_3}{2}\right)\sin q_2 \end{bmatrix}$$

$$\Downarrow$$

$$V_{c_3} = \begin{bmatrix} -\left(l_2 + \frac{d_3}{2}\right)\sin q_1 \cos q_2 & -\left(l_2 + \frac{d_3}{2}\right)\cos q_1 \sin q_2 & \frac{1}{2}\cos q_1 \cos q_2 \\ \left(l_2 + \frac{d_3}{2}\right)\cos q_1 \cos q_2 & -\left(l_2 + \frac{d_3}{2}\right)\sin q_1 \sin q_2 & \frac{1}{2}\sin q_1 \cos q_2 \\ 0 & \left(l_2 + \frac{d_3}{2}\right)\sin q_2 & \frac{1}{2}\sin q_2 \end{bmatrix} \begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{d}_3 \end{pmatrix}$$

$\Rightarrow V = m_1 g \frac{l_1}{2} + m_2 g \left(l_1 + \frac{l_2}{2} \sin q_1\right) + m_3 g \left[l_1 + \left(l_2 + \frac{l_3}{2}\right) \sin q_2\right]$

$\Rightarrow \cancel{w_1 = \begin{bmatrix} 0 \\ 0 \\ \dot{q}_1 \end{bmatrix}}, \cancel{w_2 =} \Big[$

$\Rightarrow$ for puma,

$$v_{c_1} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix}$$

$$J_{c_2} = \begin{bmatrix} -\frac{l_2}{2} \sin q_1 \cos q_2 & -\frac{l_2}{2} \sin q_2 \cos q_1 & 0 \\ \frac{l_2}{2} \cos q_1 \cos q_2 & -\frac{l_2}{2} \sin q_1 \sin q_2 & 0 \\ 0 & \frac{l_2}{2} \cos q_2 & 0 \end{bmatrix}$$

$$J_{c_3} = \begin{bmatrix} -l_2 \sin q_1 \cos q_2 - \frac{l_3}{2} \sin q_1 \cos(q_2 + q_3) & -l_2 \sin q_2 \cos q_1 - \frac{l_3}{2} \sin(q_2+q_3) \cos q_1 & -\frac{l_3}{2} \sin(q_2+q_3) \cos q_1 \\ l_2 \cos q_1 \cos q_2 + \frac{l_3}{2} \cos q_1 \cos(q_2+q_3) & -l_2 \sin q_1 \sin q_2 - \frac{l_3}{2} \sin q_1 \sin(q_2+q_3) & -\frac{l_3}{2} \sin q_1 \sin(q_2+q_3) \\ 0 & l_2 \cos q_2 + \frac{l_3}{2} \cos(q_2+q_3) & \frac{l_3}{2} \cos(q_2+q_3) \end{bmatrix}$$

$\Rightarrow V = m_1 g \frac{l_1}{2} + m_2 g \left(l_1 + \frac{l_2}{2} \sin q_1\right) + m_3 g \left[l_1 + l_2 \sin q_2 + \frac{l_3}{2} \sin(q_2+q_3)\right]$