

# Graph 1 : Introduction, DFS & Cycle Detection

## # INTRODUCTION TO GRAPHS

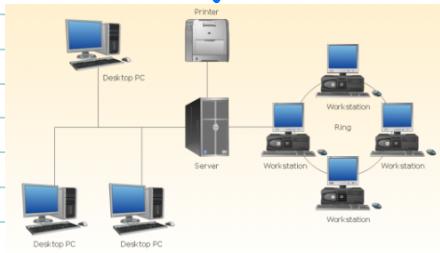
GRAPH :- It is simply a collection of nodes connected to each other using edges.

⇒ Real life use cases

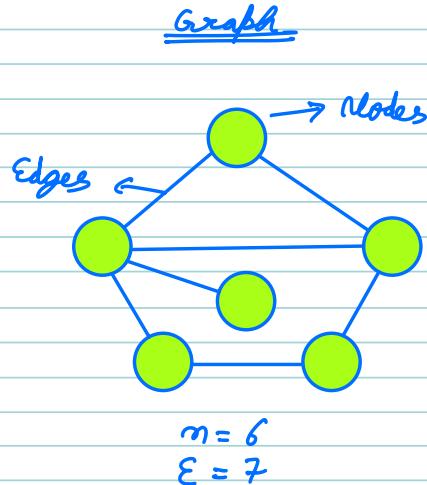
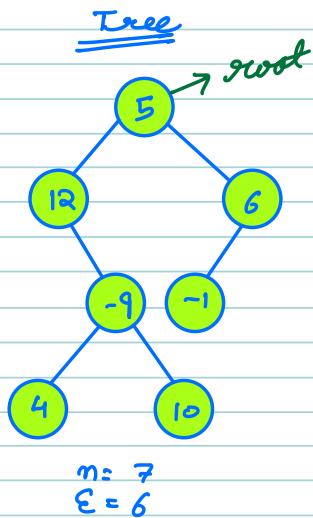
Ex 1 Network of Roads



Ex 2 Network of Computers



## # Difference b/w Trees & Graph



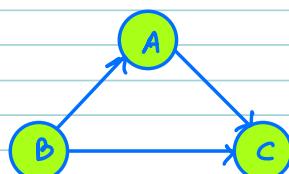
\* In Graph we can start from any Node while in trees there is a root to start.

\* Tree is a Hierarchical Data Structure

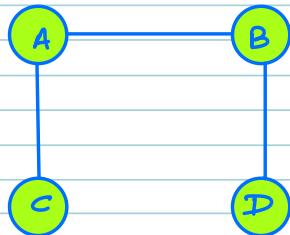
\* In a Tree, No. of Edges  $\{N-1\}$

## # CLASSIFICATION OF GRAPHS

### Case I

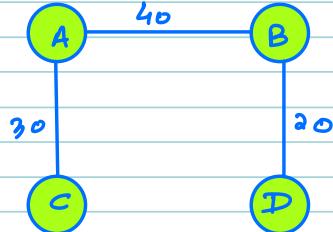


A Graph with directional edge is called  
DIRECTED GRAPH

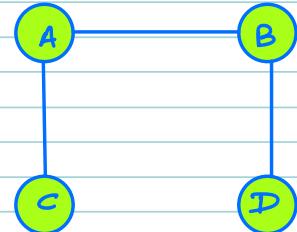


$\Rightarrow$  UNDIRECTED GRAPH

Case II

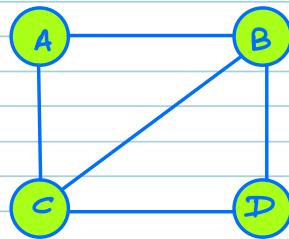


$\Rightarrow$  WEIGHTED GRAPH

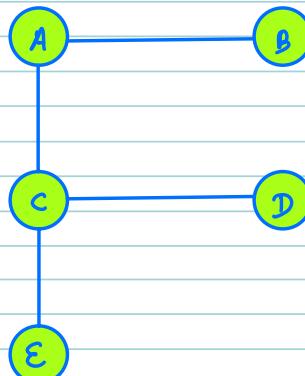


$\Rightarrow$  UNWEIGHTED GRAPH

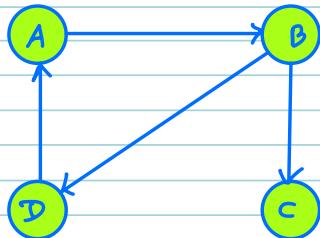
Case III



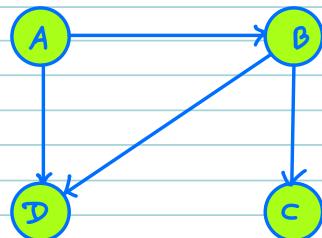
undirected cyclic graph



undirected acyclic graph



Directed cyclic graph



Directed acyclic graph.

## # How Graph is given as Input?

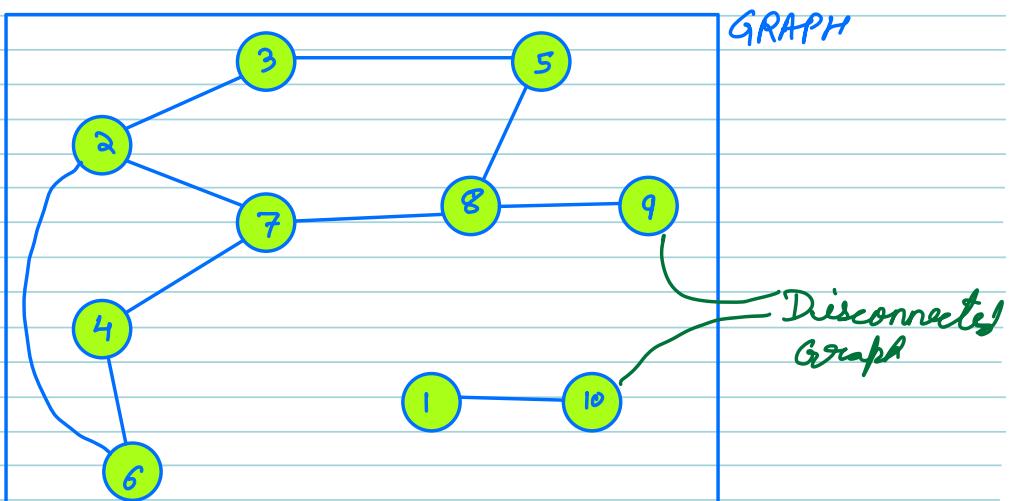
↪ Any Graph is Collection of Nodes & Edges.

Q Given a Undirected Graph with N Nodes & E Edges.

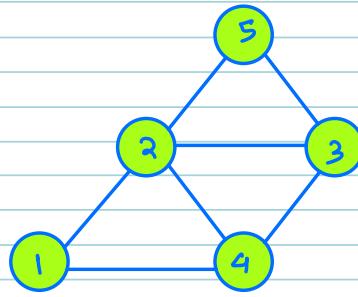
// Input

→ 1<sup>st</sup> Line N & E  
→ Followed by E Lines  
Each line contains  
u & v indicating  
edge b/w u & v nodes

N	E
10	10
u	v
2	3
4	7
8	9
2	7
7	8
10	1
4	6
5	8
2	6
3	5



## # STORING A GRAPH



INPUT :-

Nodes Count	Edges Count
5	7

1	4
2	5
3	2
4	3
5	4
6	5
7	2

APPROACH 1 :- Adjacency Matrix

int [ ] [ ] mat = new int [6] [6];

	0	1	2	3	4	5
0						
1			1		1	
2		1		1	1	1
3			1		1	1
4		1	1		1	
5			1	1		

For edge  $i, j$  we need to mark  
 $\text{mat}[i][j] = 1$  &  
 $\text{mat}[j][i] = 1$  because  
of undirected graph

$\Rightarrow$  Given  $N \& E$ ,  $\text{Mat} [N+1] [N+1]$

$\text{# } u$	unweighted	weighted
undirected	$\text{mat}[u][v] = 1$ $\text{mat}[v][u] = 1$	$\text{mat}[u][v] = \text{weight}$ $\text{mat}[v][u] = \text{weight}$
directed	$\text{mat}[u][v] = 1$	$\text{mat}[u][v] = \text{weight}$

$SC \rightarrow O(N^2)$

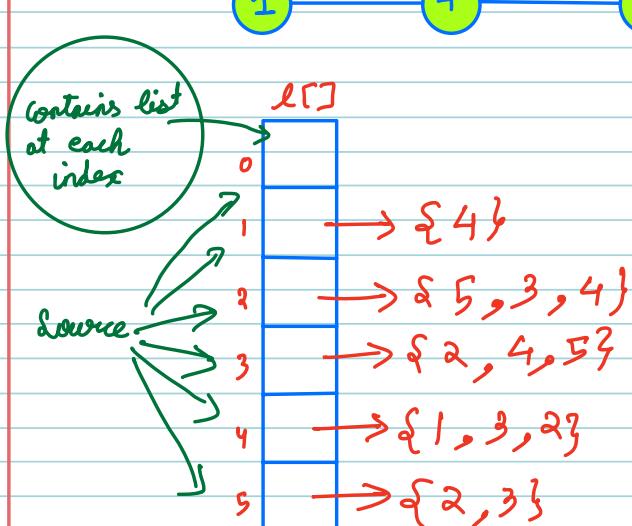
Q Any problem in storing Graph using Adjacency Matrix?  
↳ Space wastage

APPROACH :- Adjacency list

$list<int> l[]$  arr = new list[int][N+1]

Input :-

N	E
5	6
1	4
2	5
3	2
4	3
5	4
3	5



	unweighted	weighted
undirected	$l[u]. add(v)$ $l[v]. add(u)$	$l[u]. add(\{v, w\})$ $l[v]. add(\{u, w\})$
directed	$l[u]. add(v)$	$l[u]. add(\{v, w\})$

Ques 1 :- Consider a graph contains  $V$  vertices &  $E$  edges . what is the SC of adjacency list

$$SC \rightarrow O(V + 2E)$$

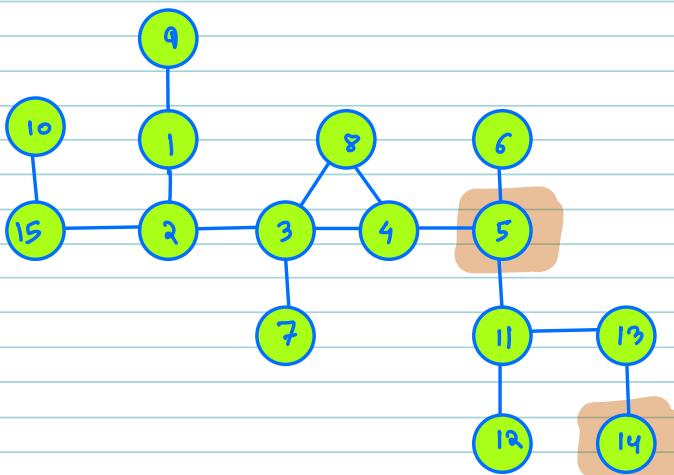
$$\approx O(V + E)$$

## # TRAVERSAL OVER GRAPH

### ⇒ DEPTH FIRST SEARCH

Source = 5 , Destination = 14

Need to tell whether there is path from source to destination

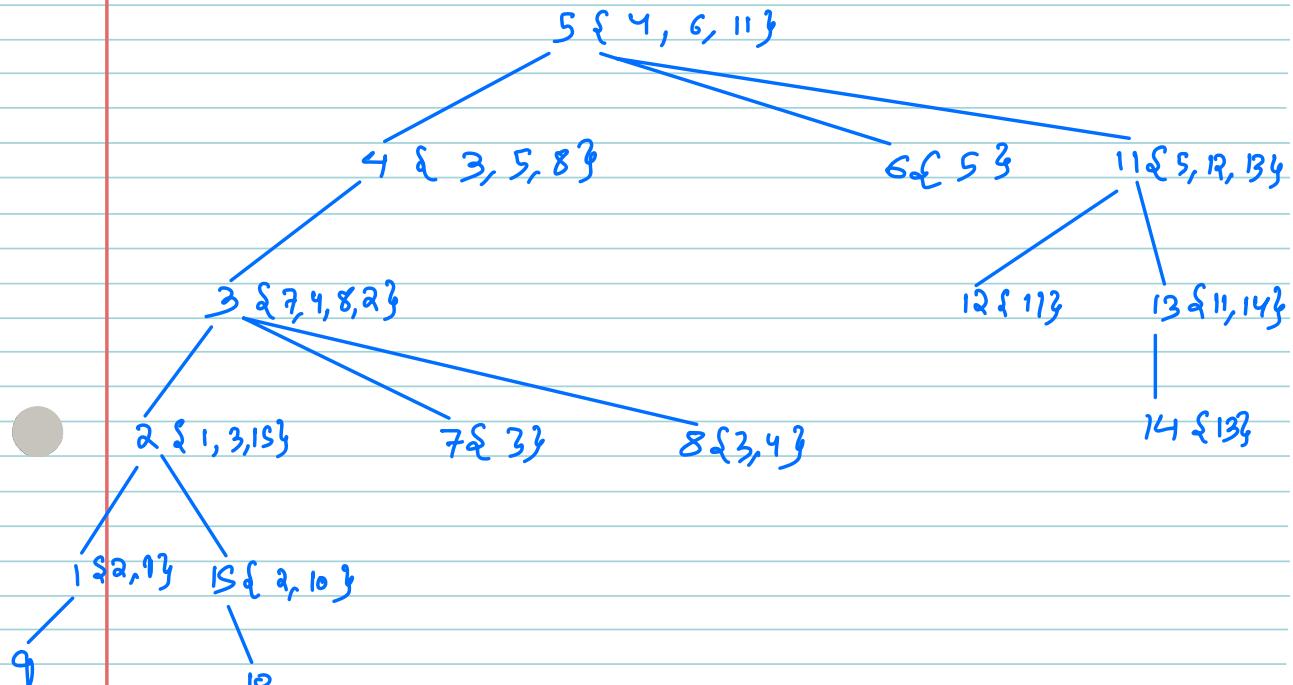


5 { 6, 4, 11 }  
4 { 5, 8, 3 }  
5 ← This a problem.  
So we need to  
maintain Visited Nodes.

## DRY RUN

visited[16] = 

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T



## PSEUDO CODE

```
bool Path C int N, int E, int[u], int[v], int[s],  
int[d]{
```

① Create Adjacency list // TODO

② bool Vis[N+1] = {F};  
DFS C s, g[], vis)

return Vis[d];

```
void DFS C int u , list<int> g , bool[v] vis);
```

Vis[u] = true;

list<int> neigh = g[u];

```

for( int i : neigh ) {
    if( visit[i] == False ) {
        DFS( i, gT, Vis );
    }
}

```

Ques 2 :- Time Complexity for DFS ?

Q How many times you are touching each Node?

↳ 1 time for each node

$TC \rightarrow O(N + E)$  No of times in total for loop will run

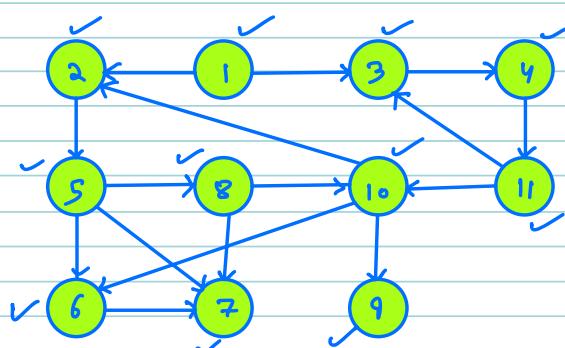
No of times DFS will run

$SC \rightarrow O(N + (N+E) + N)$  Recursion Stack.

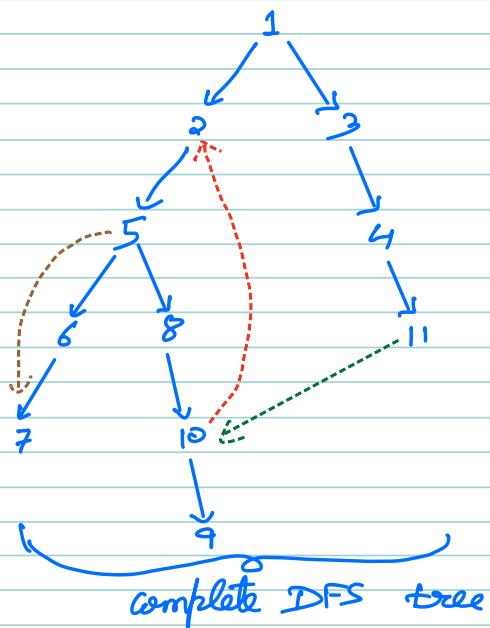
$\approx O(N+E)$  Adjacency List

question :- DFS over Directed graph

Vis[12] =	0	1	2	3	4	5	6	7	8	9	10	11
	T	T	T	T	T	T	T	T	T	T	T	T



DRY RUN



{Tree Edges} → DFS Tree

{Forward Edge} → {Any Node → Descendant}

{backward Edge} → {Any Node → Ancestor}

{Cross Edge} → {Nodes not in a Same path}

**FACT :-** If Backward edge is present, there is cycle

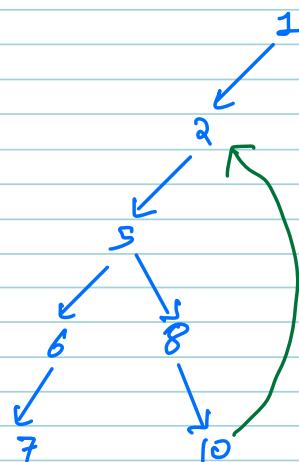
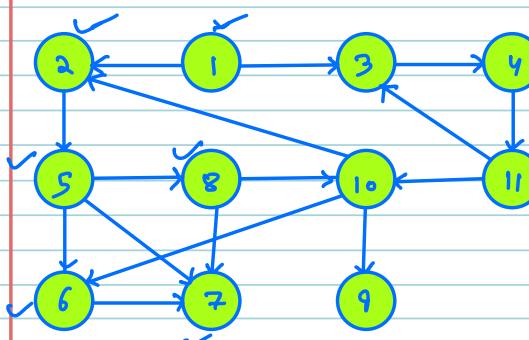
$\Leftrightarrow$  Identify If there is a cycle in Directed Graph.  
 ↳ We need to identify Backward Edge.

### DRY RUN

$v_{is}[12]$ =	0	1	2	3	4	5	6	7	8	9	10	11
	T	T				T	T	TT	T			T

Stack =	T	T	T	T	T	T
---------	---	---	---	---	---	---

stores Element in path



Caller need to insert source in Stack.

### PSEUDO CODE

```
bool isCycle( list<int> g[], int s, bool vis[],  
bool st[] ) {
```

```
    vis[s] = True;
```

```
    for ( i=0; i < g[s].size(); i++ ) {
```

```
        v = g[s].get(i);
```

```
        if ( st[v] == True ) {
```

```
            return True;
```

```
        if ( vis[v] == False ) {
```

```
            st[v] = True;
```

```
            if ( isCycle(g, v, vis, st) ) {
```

```
                return True;
```

```
            st[v] = False;
```

```
        }
```

```
    return False;
```

TC —  $O(N+E)$

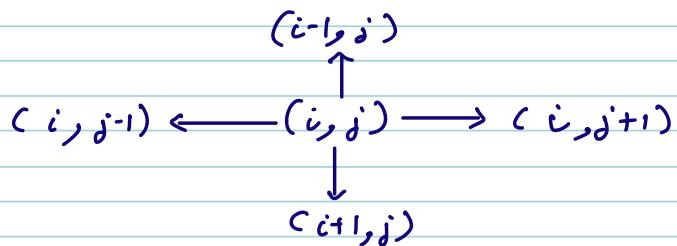
SC —  $O(N+E)$

Question :- No. of Islands?

Given a matrix of 1's & 0's. Find the No. of Islands present?

mat[ ][ ] → 1: land  
0: water

Island :- Water in all directions



Note:- Outside mat[ ][ ] , assume water in all 4 directions.

Ex:-

1	1	0	0	1
0	1	0	1	0
1	0	0	1	1
1	1	0	0	0
1	0	1	1	1

⇒ Total 5 Islands

Q How this question is mapping to Graphs?

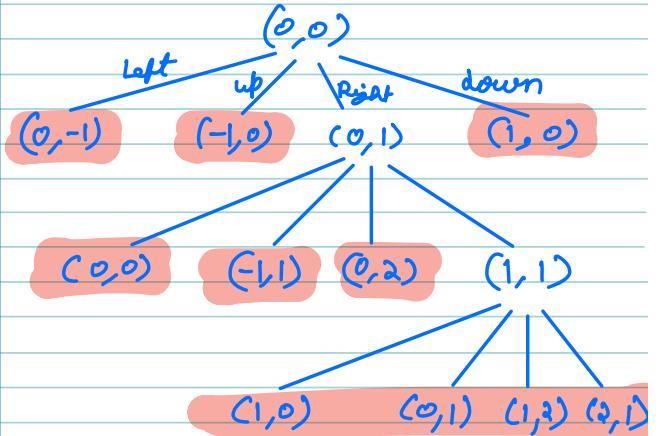
Note :- 1. 1 is indirectly representing nodes.

2. Adjacency List → Any node can atomx 4 neighbour.

Q How to traverse over such Graph?

1	1	0	0	1	
0	1	0	1	0	
1	0	0	1	1	
1	1	0	0	0	
1	0	1	1	1	

will make it 2 islands



→ So we will try to form Island from all the cells & see how many Islands are there

### PSEUDO CODE

```
int Islands( int mat[ ][ ] ) {  
    int c = 0  
    for ( i = 0; i < n; i++ ) {  
        for ( j = 0; j < m; j++ ) {  
            if ( mat[i][j] == 1 ) {  
                c++  
                DFS( mat, i, j );  
            }  
        }  
    }  
}
```

void DFS( int Mat[ ][ ], int i, int j){

if  $C[i < 0] \text{ || } j < 0 \text{ || } i > m \text{ || } j > m \text{ || } Mat[i][j] == 0$   
 $\text{|| } Mat[i][j] == 2 \} \}$

$$\text{mat } [c] [j] = 2;$$

$\text{DFS}(i, j-1);$

$\text{DFS}(i-1, j);$

$\text{DFS}(i, j+1);$

$\text{DFS}(i+1, j);$

$T \leq O(N * n)$

$SC \rightarrow OC$   $N \times m$ )

 DFS calls

## Question

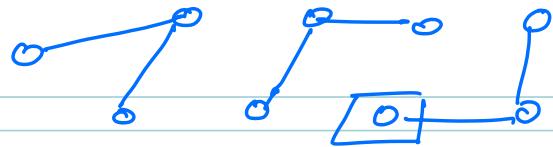
## Problem

In the LinkedIn ecosystem, understanding the structure of professional networks is crucial for both users and the platform itself.

A new feature is being developed to visualize the network of a user in terms of "**clusters**". These clusters represent groups of LinkedIn users who are all connected to each other, either directly or through mutual connections, but do not have connections to users outside their cluster. This visualization aims to help users to increase their **Reach**.

## Problem Statement

Given **A** denoting the total number of people and matrix **B** of size  $M \times 2$ , denoting the bidirectional connections between LinkedIn users, and an Integer **C** denoting the user ID of [REDACTED] the target person, find out the number of connections that this person should make in order to connect with all the networks.



⇒ This is exactly an Island<sup>↑</sup> problem. We just  
Need to subtract 1 from No. of cluster  
present