

Sorting 2 : Quick Sort & Comparator Problems

AGENDA

- Pivot Partition
- Quick Sort
- Comparator problems

Question :- Partition the array based on the first element

Given an integer array, consider first element as pivot, re-arrange the elements such that for all i :

→ if $A[i] < p$ then it should be present on left side.

→ if $A[i] > p$ then it should be present on right side.

Ex :- $\boxed{54} \ 26 \ 93 \ 17 \ 77 \ 31 \ 44 \ 55 \ 20$

\uparrow
 p

\downarrow
After Partition

$26 \ 17 \ 31 \ 44 \ 20 \ \boxed{54} \ 93 \ 77 \ 55$

Observation

→ Can we say pivot reached at its sorted place?

↳ yes

Brute force

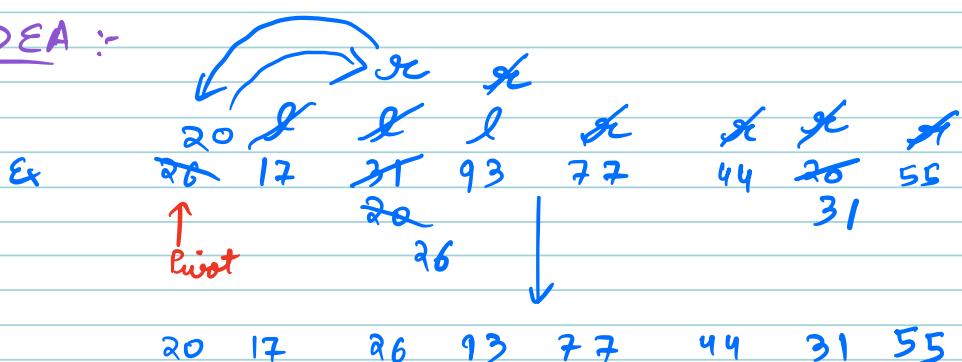
→ sort the array because pivot will reach at its correct place.

$$TC \rightarrow O(N \log N)$$

Optimization

Problem above → we don't want to sort left & right side of pivot.

IDEA :-



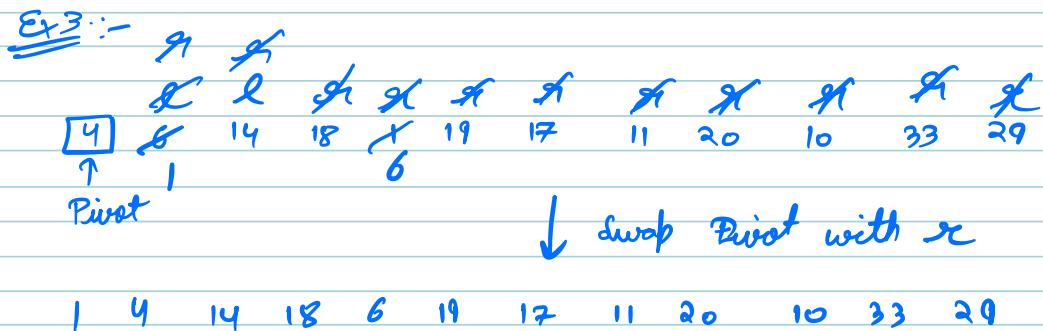
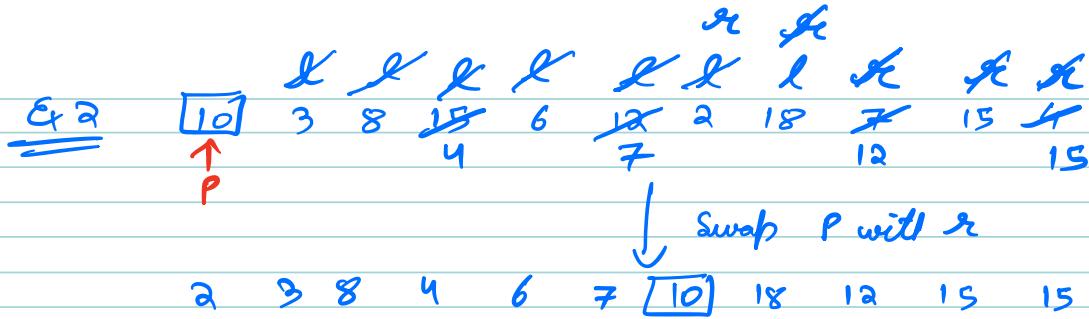
→ Left pointer (l) happy when pointing to something smaller than pivot, then $l++$

→ Right pointer (r) happy when pointing to something greater than pivot, then $r--$

→ when both unhappy we can just swap both of them & do $l++$ & $r--$

→ we will stop when $r < l$

→ At last swap pivot with r



PSEUDO CODE

- Partition (A, first, last) {

 pivot = A[first];

 l = first + 1;
 r = last;

 while (l <= r) {

 if (A[l] <= pivot) {

 l++;

 } else if (A[r] > pivot) {

 r--;

 } else {

 swap (A, l, r);

 l++;

 r--;

 } swap (A, first, r);

$TC \rightarrow OCN$

$SC \rightarrow OCD$

QUICK SORT

x |

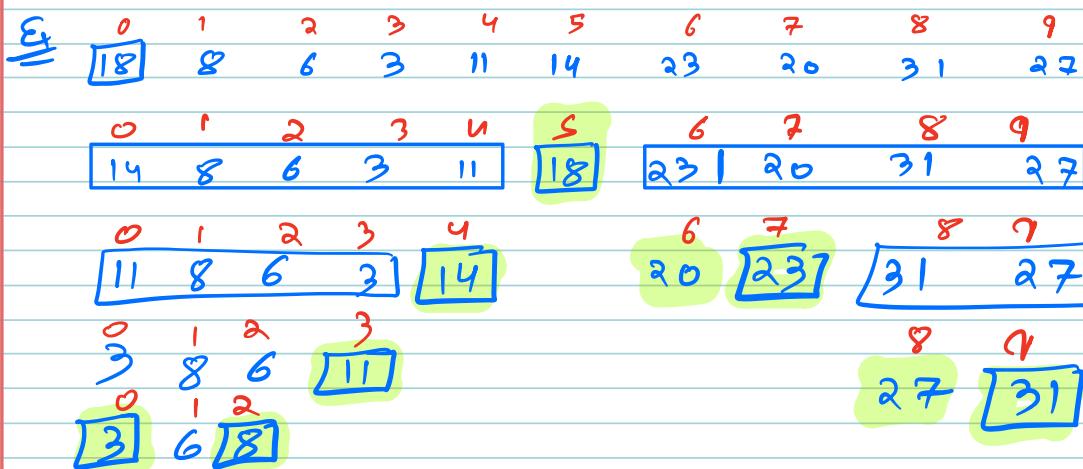
y |

x |

z |



* This is an idea of quick sort.



→ when L & R is same then its base case

→ while writing code we need pivot point for nested calls, so we can rely on for returning Partition point which will be x^{th} index

PSEUDO CODE

```
quick sort ( A , start , end ) {  
    if ( start < end )  
        p = Partition ( A , start , end );  
        quick sort ( A , start , P-1 );  
        quick sort ( A , P+1 , end );  
    }  
}  
It prevents execution when,  
 $s == e$   
 $s > e$   
}
```

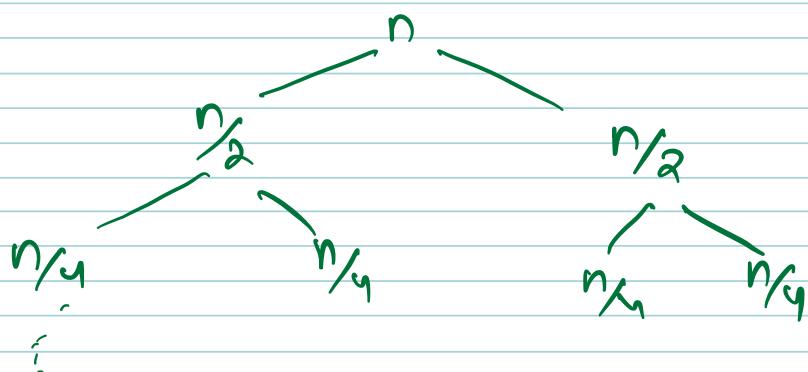
TC

① Best case

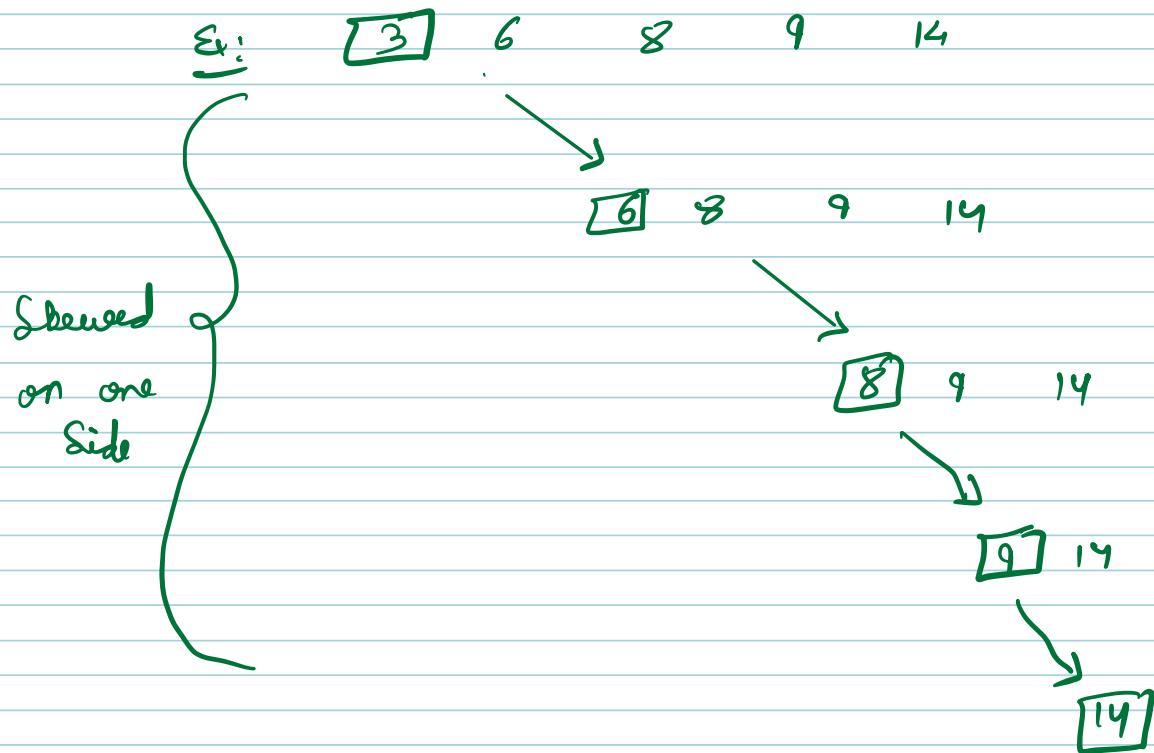
↳ when n divides equally

Recurrence Relation:- $T(n) = 2 T(\frac{n}{2}) + n.$

$$\begin{aligned} TC &\rightarrow N \log N \\ SC &\rightarrow \log N \end{aligned}$$



② worst Case



Recurrence relation $\& T(n) = T(n-1) + n$

$$\begin{aligned}TC &\rightarrow O(N^2) \\SC &\rightarrow O(N)\end{aligned}$$

↓
Recursion stack

⇒ Randomized Quick Sort

→ \boxed{x}

→ $\xrightarrow{< x} \boxed{x} \xleftarrow{> x}$

→ The randomized quicksort is a technique where we randomly pick the pivot element, not necessarily the first & last.

→ There is a random function available in all the languages, to which we can pass array & get random index. Now, we can swap random index element with first element & execute our algorithm as it is.

→ Probability of picking 2 minimums randomly = $\frac{1}{10} \times \frac{1}{9} = \frac{1}{90}$

→ Hence, using randomized quick sort, we can achieve average case of $O(N \log N)$ most of the time.

COMPARATOR

→ In programming, a **comparator** is a function that compares two values & returns a result indicating whether the values are equal, less than or greater than each other.

→ The **comparator** is typically used in sorting algorithm to compare elements in a data structure & arrange them in a specified order.

For languages - Java, Python, JS, C++, Ruby, the following logic is followed.

- ① In sorted form, if first argument should come before second, -ve value is returned.
- ② In sorted form, if second argument should come before first, +ve value is returned.
- ③ If both are same, 0 is returned.

For C++, following logic is followed.

- ① In sorted form, if first argument should come before second, true is returned.
- ② otherwise, false is returned.

question :- Given an array of size n , sort the data in ascending order of count of factors, if count of factors are equal then sort the elements on the bases of their magnitude.

Ex

arr [] = { 9 3 10 6 4 3 }
↓ ↓ ↓ ↓ ↓ ↓
factor → 3 2 4 4 3

Ans = 3 4 4 6 10

PSEUDO CODE

```
collection . sort ( A , new Comparator<Integer>() {
```

 @Override

```
    Public int compare ( Integer v1 , Integer v2 ) {
```

```
        if ( factor ( v1 ) == factor ( v2 ) ) {
```

```
            if ( v1 < v2 ) {
```

```
                return -1;
```

```
            } else if ( v2 < v1 ) {
```

```
                return 1;
```

```
            } else {
```

```
                return 0;
```

```
            } else if ( factors ( v1 ) < factors ( v2 ) ) {
```

```
                return -1;
```

```
            } else {
```

```
                return 1;
```

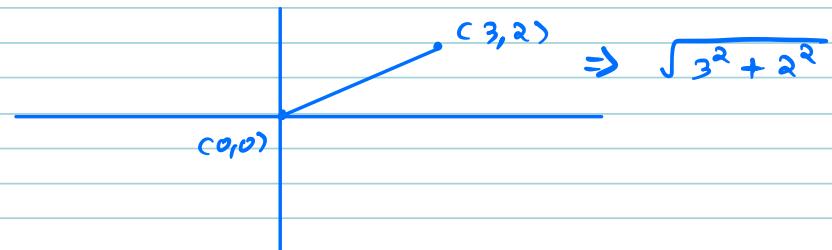
```
    };
```

```
};
```

```
};
```

Question 2 :- Given an array of points where $\text{point}[i] = [x_i, y_i]$ represents a point on the $x-y$ plane & integer k , return k closest point to the origin $(0,0)$.
 The distance b/w two points on the $x-y$ plane is the Euclidean distance

$$\text{C.e. } \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2}$$



Ex1 :- Points = $[[1, 3], [-2, 2]]$ | $B=1$

$$\begin{array}{ccc} \downarrow & \downarrow & \\ \sqrt{1^2 + 3^2} & \sqrt{(-2)^2 + 2^2} & \\ \downarrow & \downarrow & \\ \sqrt{10} & \sqrt{8} & \end{array}$$

Ex2 :- Points = $[[3, 3], [5, -1], [-2, 4]]$ | $B=2$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ \sqrt{3^2 + 3^2} & \sqrt{5^2 + (-1)^2} & \sqrt{(-2)^2 + 4^2} \\ \downarrow & \downarrow & \downarrow \\ \sqrt{18} & \sqrt{26} & \sqrt{20} \end{array}$$

IDEA:- array of points

$$\begin{array}{cc} \downarrow & \\ (x_1, y_1) & (x_2, y_2) \end{array}$$

$$\begin{array}{ll} \text{dist1} = & x_1^2 + y_1^2 \\ \text{dist2} = & x_2^2 + y_2^2 \end{array} \quad \begin{array}{l} \text{can ignore} \\ \text{smaller} \end{array}$$

```
if (dist1 < dist2) {  
    return -1;  
}  
else {  
    return 1;
```

Ques 1 :- Best Case TC of Quick Sort ?

$$\hookrightarrow N \log N$$

Ques 2 :- Worst Case TC of Quick Sort ?

$$\hookrightarrow N^2$$

Ques 3 :- Worst case SC of Quick Sort ?

$$\hookrightarrow N$$

Ques 4 :- Best Case SC of Quick Sort ?

$$\hookrightarrow \log N$$