

Search 1: Binary Search On Array

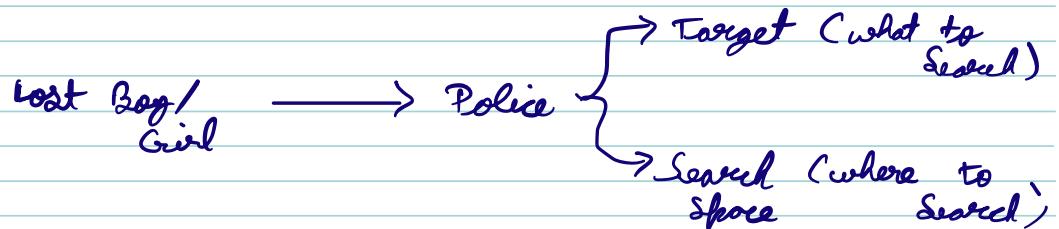
AGENDA

- Introduction
- Search for an element k
- Search first & last occurrence
- Single element in sorted array
- Peak element
- Local minima

* Mock Interview → (30 days)
↳ DSA ends

↳ 5 Mandatory / 7 Additional
↳ SCL → Mock
↳ LLD → Mock

⇒ Search Story



↳ where the search will be faster

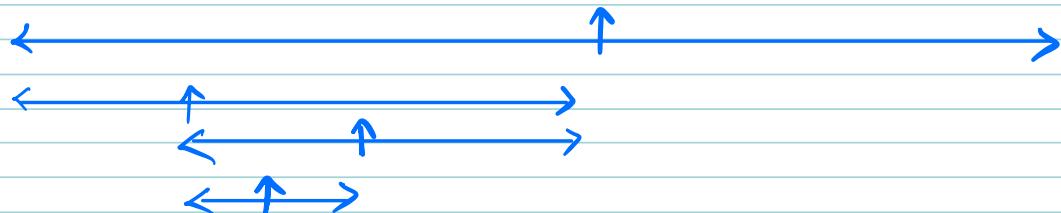
→ word :- In Dictionary, Book, Newspaper {

→ Phone no. :- In Telephone directory, diary }

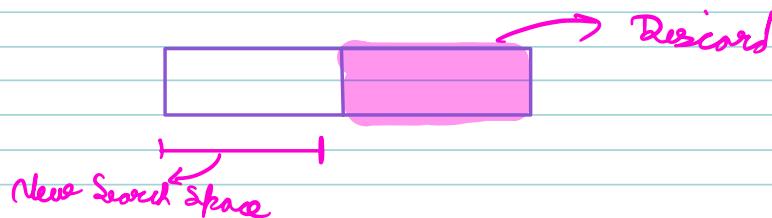
* Learning ⇒ where search space is sorted, then searching becomes easy. But we should not say searching is easy only when search space is sorted. I will learn from examples today

Ex:- Search word (DOG) in Dictionary

A B C D E F M N O X Y Z



⇒ IDEA of Binary search



Quiz :- In binary search, at each step, the search range is typically :-
↳ Halved

BINARY SEARCH

To apply Binary Search :-

- ① Target
- ② Search space
- ③ Some condition to discard 1 half of search space.

Definition :- Divide search space into 2 parts & repeatedly keep on neglecting one half of the search space.

Question :- Given a sorted array with distinct elements, search the index of an element k , if k is not present return -1.

Ex :- $A[] = \begin{matrix} 0 \\ 3 \end{matrix} \begin{matrix} 1 \\ 6 \end{matrix} \begin{matrix} 2 \\ 9 \end{matrix} \begin{matrix} 3 \\ 12 \end{matrix} \begin{matrix} 4 \\ 14 \end{matrix} \begin{matrix} 5 \\ 19 \end{matrix} \begin{matrix} 6 \\ 20 \end{matrix} \begin{matrix} 7 \\ 23 \end{matrix} \begin{matrix} 8 \\ 25 \end{matrix} \begin{matrix} 9 \\ 27 \end{matrix}$

if, $k = 14 \rightarrow 4$ Ans

if, $k = 99 \rightarrow -1$ Ans

Brute force

↳ Linear search

$$TC \rightarrow O(N)$$

Ex :- $A[] = \begin{matrix} 0 \\ 3 \\ 6 \\ 9 \\ 12 \\ 14 \\ 19 \\ 20 \\ 23 \\ 25 \\ 27 \end{matrix}$

⇒ OPTIMIZATION

↳ Binary Search

① Target :- K

② Search space :- Entire array

③ Discard Condition.

↳ i) arr[mid] == K

return mid.

ii) arr[mid] < K

goto right

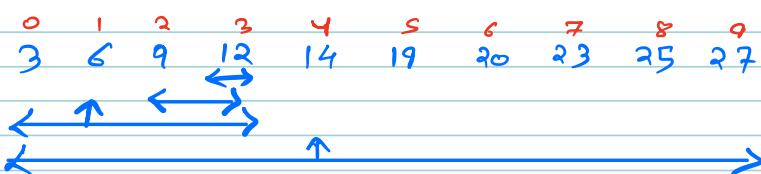
iii) arr[mid] > K

goto left

DRY RUN

$K=12$

Ex :- $A[] = \begin{matrix} 0 \\ 3 \\ 6 \\ 9 \\ 12 \\ 14 \\ 19 \\ 20 \\ 23 \\ 25 \\ 27 \end{matrix}$



Left right mid

0 9 4

goto left
↳ (right = mid + 1)

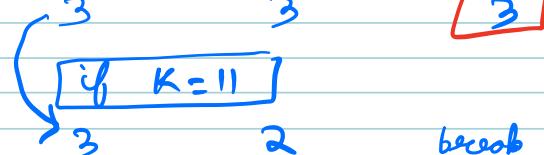
0 3 1

goto right
↳ (left = mid + 1)

2 3 2

goto right
↳ (left = mid + 1)
break;

if $K=11$



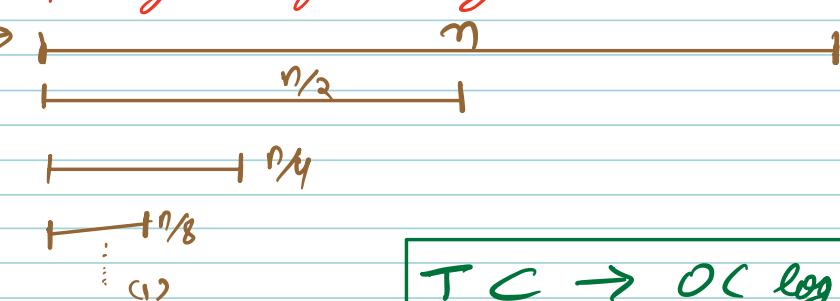
break

goto left
↳ right = mid - 1

PSEUDO CODE

```
int Search ( int []A, int N, int K) {  
    L = 0;  
    R = N-1;  
    while ( L <= R ) {  
        mid =  $\frac{L+R}{2}$   
        if ( A [mid] == K ) {  
            return mid;  
        } else if ( A [mid] < K ) {  
            L = mid + 1;  
            // move right;  
        } else {  
            R = mid - 1;  
            // move left  
        }  
    }  
    return -1;  
}
```

Ques 2 :- If the element 'K' is found in a sorted array, what will be the time complexity of binary search



$$\begin{array}{|c|} \hline T C \rightarrow O(\log n) \\ \hline S C \rightarrow O(1) \\ \hline \end{array}$$

\Rightarrow Best practice

$$\text{mid} = \frac{\text{Left} + \text{Right}}{2}$$

$$\Rightarrow \boxed{\text{mid} = \frac{\text{Left} + (\text{Right} - \text{Left})}{2}}$$

Mathematically

$$\begin{aligned} & \text{Left} + \frac{(\text{Right} - \text{Left})}{2} \\ \Rightarrow & \frac{2\text{Left} + \text{Right} - \text{Left}}{2} \\ \Rightarrow & \frac{\text{Left} + \text{Right}}{2} \end{aligned}$$

\Leftrightarrow why?

Explanation

$$\text{int} \rightarrow 2^{31} - 1$$

Left assume int can store max 100

$$y \quad \text{Left} = 98, \quad \text{Right} = 99$$

① then, $\text{mid} = \frac{\text{Left} + \text{Right}}{2} \Rightarrow \frac{98+99}{2} \rightarrow \text{overflow}$

② while, $\text{mid} = \text{Left} + \frac{(\text{Right} - \text{Left})}{2} \Rightarrow 98 + \frac{99-98}{2}$

$$\Rightarrow 98 + \frac{1}{2} \checkmark$$

Question :- Given a sorted array of n elements.
Find the index of first occurrence of K .

Ex:-

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18
-5 -5 -3 0 0 1 1 5 5 5 5 5 5 8 10 15 17 18
 $\Rightarrow K = 5 \rightarrow 7 \underline{\text{Ans}}$

Brute force

↳ Linear search

TC $\rightarrow O(N)$

↳ OPTIMIZATION

↳ Binary Search

① Target :- K

② Search space :- entire array.

③ Conditions to Discard -

↳ ① $A[\text{mid}] == K$

Ques :- when searching for the first occurrence of an element in a sorted array, what should you do if the current element matches the target K ?

ans = mid;

goto left

right = mid - 1;

② $A[\text{mid}] < K$

goto right;

Left = mid + 1

iii) $A[\text{mid}] > k$

goto left;
 $\text{Right} = \text{mid} - 1;$

DRY RUN

$k = 5$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18
-5 -3 0 0 1 1 5 5 5 5 5 5 8 10 15 17 18

Left	Right	mid	Ans	
0	18	9	9	$\rightarrow \text{Go to Left, Right} = \text{mid} - 1$
0	8	4	4	\downarrow $\rightarrow \text{Go to Right, Left} = \text{mid} + 1$
5	8	6	6	\downarrow $\rightarrow \text{Go to Right, Left} = \text{mid} + 1$
7	8	7	7	$\rightarrow \text{Go to Left, Right} = \text{mid} - 1$
7	6			$\rightarrow \text{break;}$

int search (int [] A, int N, int k) {

L = 0 ;
R = N-1 ;
ans = -1 ;
while (L <= R) {
 mid = $\frac{L+R}{2}$

 if (A [mid] == k) {
 ans = mid ;
 right = mid - 1 ;
 }
 else if (A [mid] < k) {

 L = mid + 1 ;
 // Move right ;
 }
 else {
 R = mid - 1 ;
 // Move left

 return ans ;

Ques :- what is the TC of finding the first occurrence of an element in a sorted array using binary Search?



$$\begin{aligned} \text{TC} &\rightarrow O(C \log N) \\ \text{SC} &\rightarrow O(C1) \end{aligned}$$

Q :- Find the last occurrence?

↳ H.W.

10:29

— 10:39

Question :- Every element occurs twice except for 1, find the unique element.

Note :- Duplicate elements are adjacent to each other but the array is not sorted.

Ex :-

$$A = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 8 & 8 & 5 & 5 & 6 & 2 & 2 \end{bmatrix}$$

$\hookrightarrow \underline{\text{Ans}}$

Brute force

↳ Using XOR over array

TC $\rightarrow O(N)$

SC $\rightarrow O(1)$

↳ OPTIMIZATION

↳ Binary Search.

Ex :-

0	1	2	3	4	5	6	7	8	9	10	11	12
3	3	1	1	8	8	10	10	19	6	6	24	24

↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑

(i) Target \rightarrow unique element

(ii) Search space \rightarrow entire array

(iii) conditions to discard

Observation

(i) For repeated elements what you can observe.

(2) Difference b/w the pair on left & right of unique?

↳ First occurrence of Repeated on left is even indexed while on right its odd indexed.

Ex :-

0 1	2 3	4 5	6 7	8	9 10	11 12
3 3	1 1	8 8	10 10	19	6 6	24 24

mid
↓

(L) (R)
Left Right

(M)
Mid

isUnique

(check Am I first)

occurrence

$A[\text{mid}] == A[\text{mid}-1]$

mid%2

0 12 6 *

*

yes, Go to
Right.
 $\hookrightarrow L = M + 2$

8 12 10 9 *

mid = mid - 1
9

No, Go to
Left
 $\hookrightarrow R = M - 1$

8 8 8 yes

return;

PSEUDO CODE

find Unique C int[] A, int N {

$$\begin{aligned} L &= 0 \\ R &= N - 1 \end{aligned}$$

if ($N == 1$) { return 0; }

if ($A[0] != A[1]$) { return 0; }

if ($A[N-1] != A[N-2]$) { return N-1; }

while ($L <= R$) {

$$M = L + \frac{R-L}{2}$$

if ($A[M] != A[M-1]$ & &
 $A[M] != A[M+1]$) {

return m;

3

$TC \rightarrow O(\log N)$

$SC \rightarrow O(1)$

if $C A[m] == A[m-1]$ {

$m = m-1$;

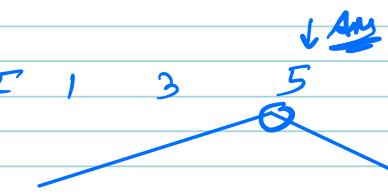
if $C m \% 2 == 0$ {

$i = m+2$;

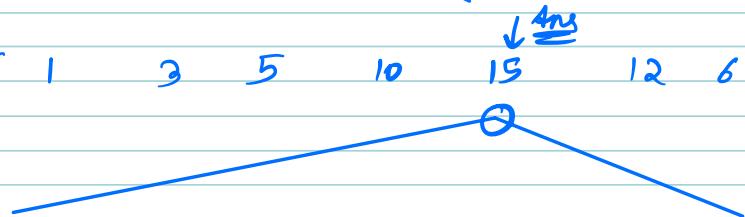
$R = m-1$;

Question :- Given an increasing decreasing array with distinct elements. Find max element.

Ex1:- $A = [1 \quad 3 \quad 5 \quad 2]$



Ex2:- $A = [1 \quad 3 \quad 5 \quad 10 \quad 15 \quad 12 \quad 6]$



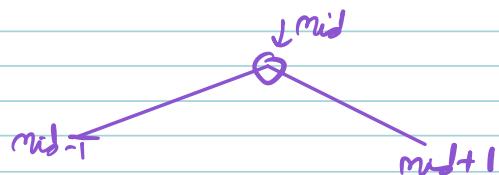
APPROACH

① Target :- peak element

② Search space :- array

③ Condition to discard :

case I

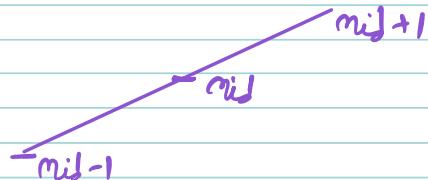


if ($A[mid] > A[m-1]$ &
 $A[mid] > A[mid+1]$) {

 return mid;

}

Case II

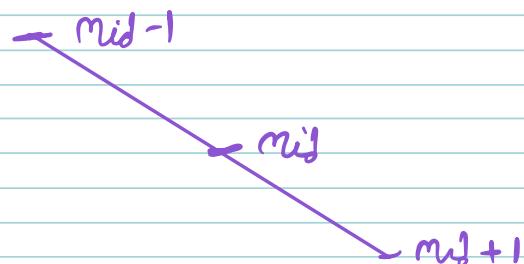


if ($A[mid] > A[m-1]$ &
 $A[mid] < A[mid+1]$) {

 L = mid + 1;
 // Go right

}

Case III



else

{ if ($A[mid] < A[mid-1]$ &
 $A[mid] > A[mid+1]$) {

 R = mid - 1;

 // move left;

}

Edge Case

① 1 3 5 10

② 15 12 10 8

③ single element

Question:- Given an array of N distinct elements, find any local minima in the array.

Local Minima :- a no. which is smaller than its adjacent neighbours.

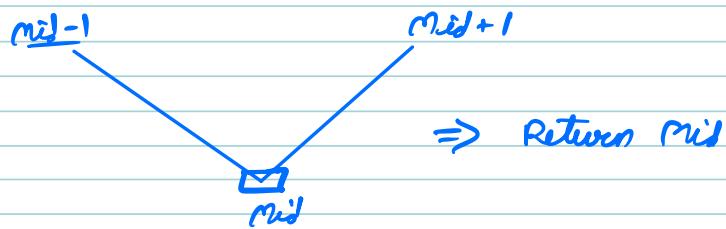
Ex:- $A \Rightarrow 4 \ 3 \ 6 \ 1 \ 0 \ 9 \ 15 \ 8$

$B \Rightarrow 21 \ 20 \ 19 \ 17 \ 15 \ 9 \ 7$

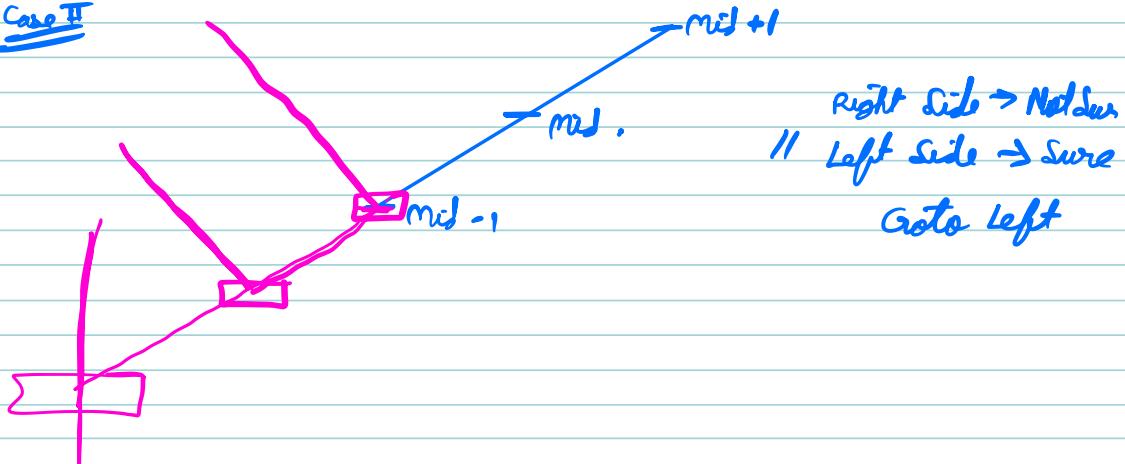
$C \Rightarrow 5 \ 4 \ 15 \ 16 \ 20 \ 21$

$D \Rightarrow 5 \ 8 \ 12 \ 3$

Case I



Case II



Case III

CLUBBING
Together

Case III

mid-1

mid

mid+1

\Rightarrow Left \rightarrow Not Sure

\Rightarrow Right \rightarrow Sure.

mid

mid-1

mid+1

\Rightarrow Left \rightarrow Sure

\Rightarrow Right \rightarrow Sure

[Anywhere]

PSEUDO CODE

$$L = 0, R = N - 1$$

$\text{if } (N == 1) \{ \text{return } A[0] \}$

$\text{if } (A[0] < A[1]) \{ \text{return } A[0] \}$

$\text{if } (A[N-1] < A[N-2]) \{ \text{return } A[N-1] \}$

$\text{while } (L <= R) \{$

$$M = L + \frac{R-L}{2}$$

$\text{if } (A[M] < A[M-1] \& A[M] < A[M+1]) \{$

$\text{return } A[M];$

$\} \text{ else if } (A[M] < A[M-1]) \{$

$L = M+1;$

$} \text{ else } \{$

$R = M-1;$

3

DRY RUN

	x	0	1	2	3	4	5	6	7
	arr	4	8	2	7	6	4	1	5

lo hi mid $A[\text{mid}-1]$ $A[\text{mid}]$ $A[\text{mid}+1]$

0 7 3 2 7 6 → Go to right

4 7 5 6 4 1 → Go to right

6 7 6 4 1 5 return 1

$$\begin{aligned} \text{TC} &\rightarrow O(\log N) \\ \text{SC} &\rightarrow O(1) \end{aligned}$$